Spatial Correlations Induced by Pairing in Open Shell Nuclei

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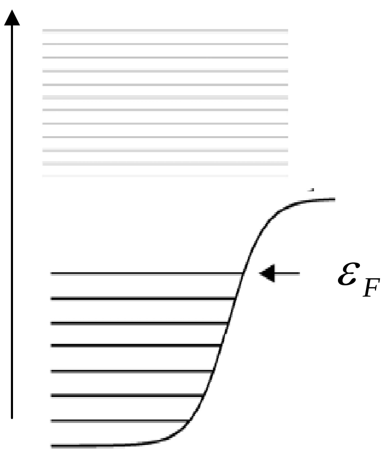
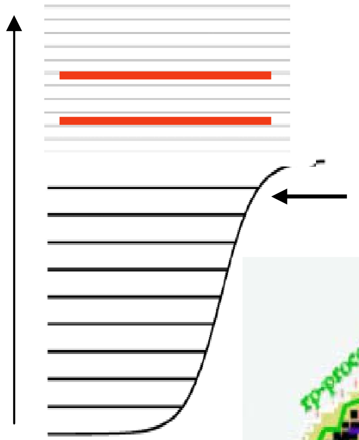
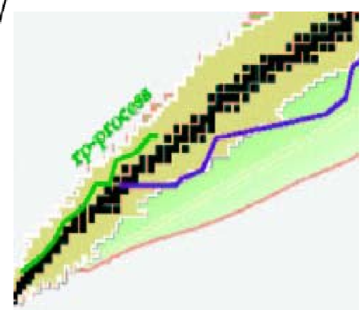
Unified (DFT ?) Description of Pairing in Nuclear Systems

stable nuclei

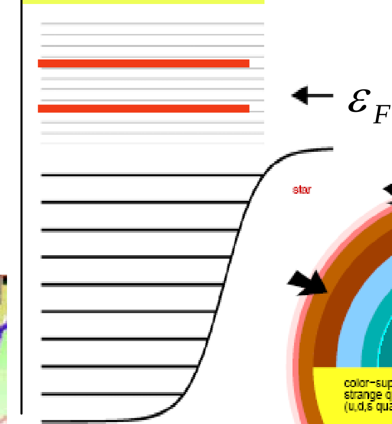


neutron-rich nuclei

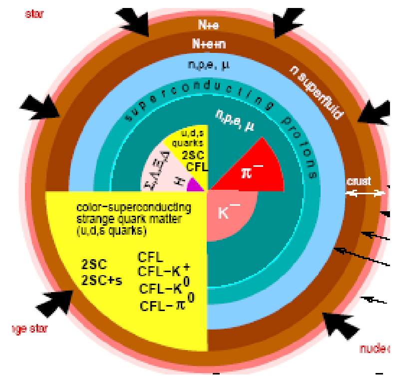




nuclei+ neutrons



nuclear matter



neutrons: superfluidity of $^1S_0$ type

• ***Consequences***

- excitations (energy gap)

- reduction of moment of inertia

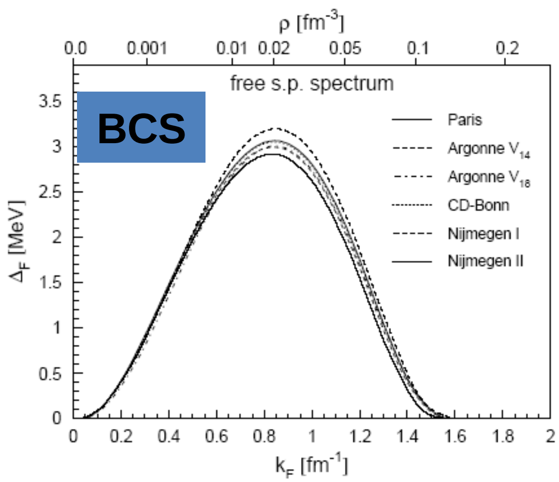
- enhanced pair transfer

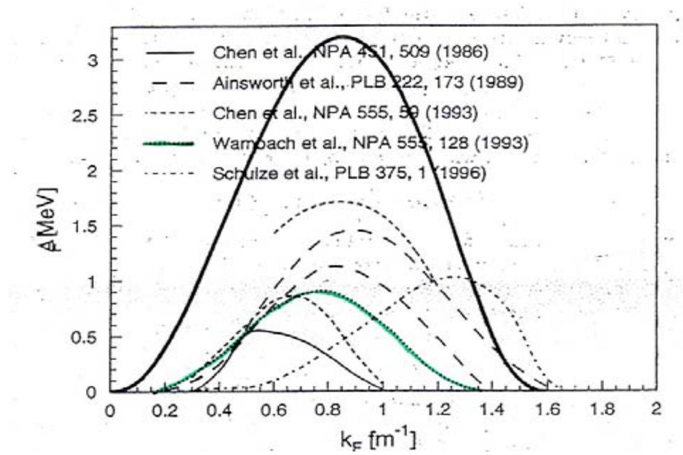
• **Crust**: - neutrons: superfluidity of $^1S_0$ type

• ***Core*** : - neutrons: superfluidity of ^3PF type
- protons: superfluidity of $^1S_0$ type

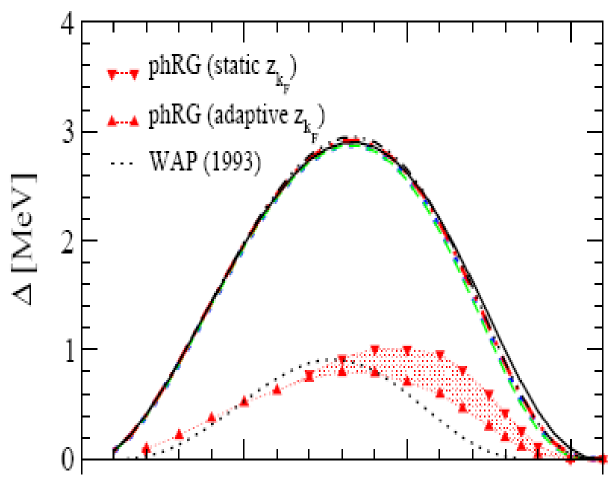
• ***Conequences*** : - geant glitches
- cooling

$^1S_0$ Pairing Gap in Neutron Matter

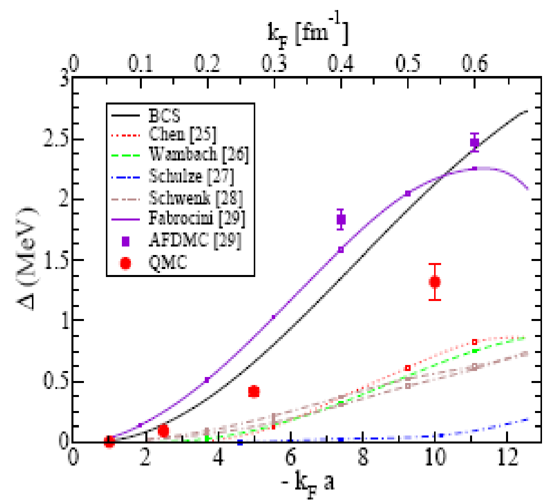




U.Lombardo, H.-J. Schulze (2001)



A. Schwenk et al, NPA713 (2003) 191



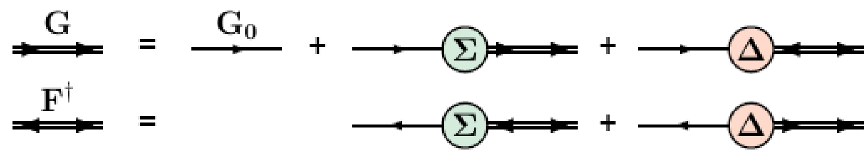
A. Gezerlis, J. Carlson, PRC77(2008)

S. Gandolfi et al, PRL101(2008)

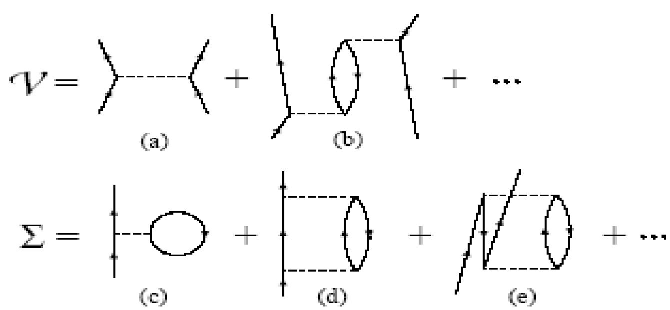
Pairing in neutron matter: Gorkov type calculations

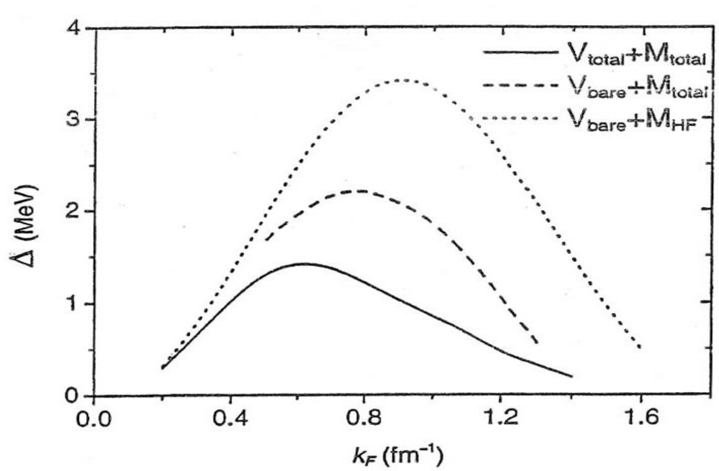
Gorkov equations

$$G = -i < 0 | T(aa^{+}) | 0 >; \qquad F = -i < 0 | T(a^{+}a^{+}) | 0 >$$



$$\Delta_k(\omega) = \sum_{k'} \int d\omega V_{kk'}(\omega, \omega') \frac{\Delta_{k'}(\omega')}{[\omega' - \varepsilon_{k'}(\omega')][\omega' + \varepsilon_{k'}(-\omega')]}$$

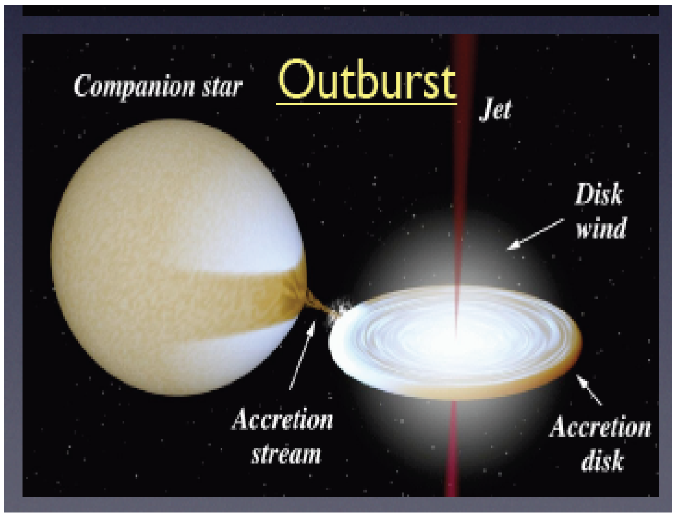


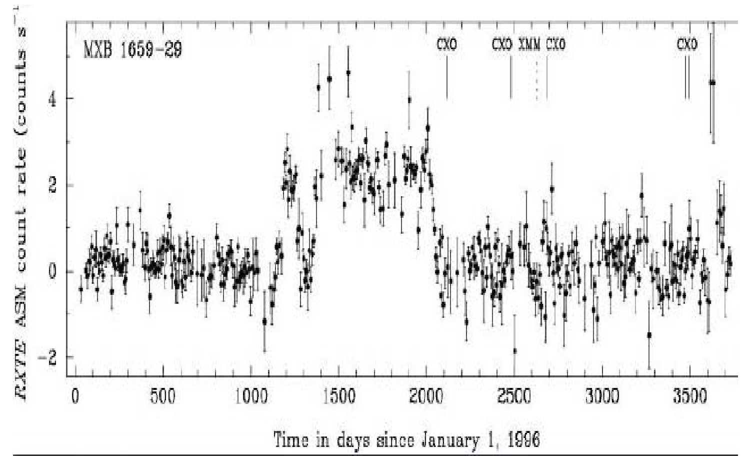


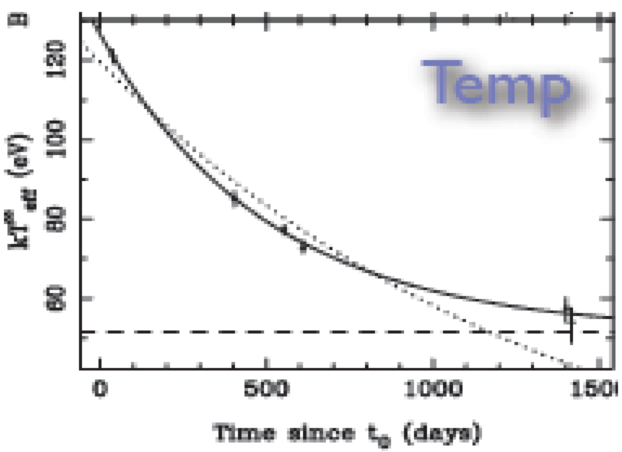
C.Shen, U.Lombardo, P.Schuck, W.Zuo, N.S, PRC.C(2003)
L. Cao, U. Lombardo, P. Schuck, PRC(2006)

constraints from neutron stars properties ?

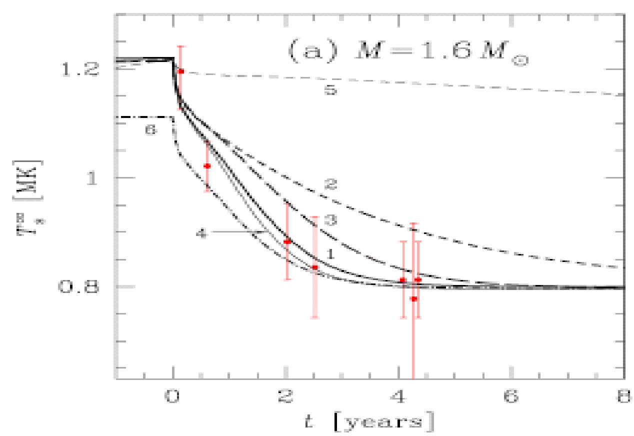
Cooling of Neutron Stars in X- Ray Binaries





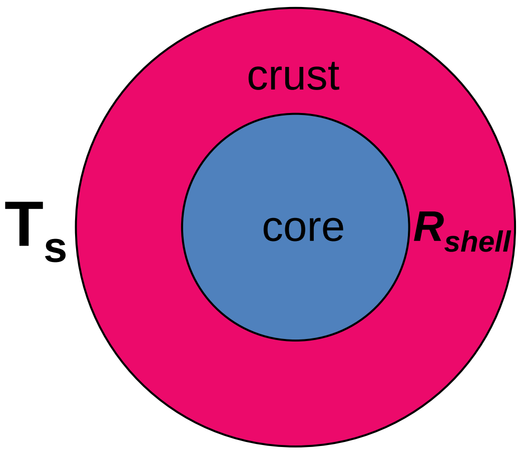


E. Cackett et al, 2006



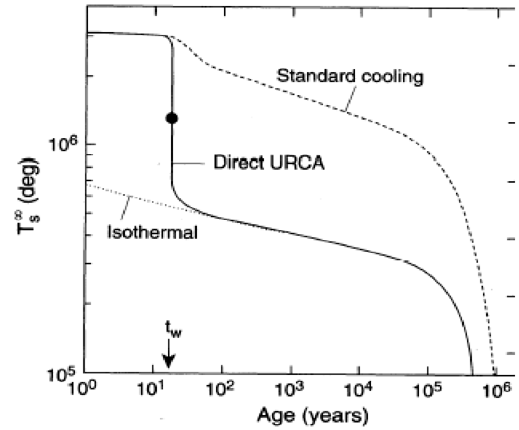
PS Shternin et al, (2007)

Cooling Time and Crust Superfluidity

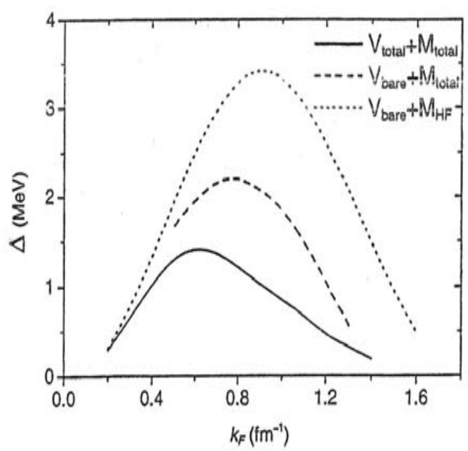


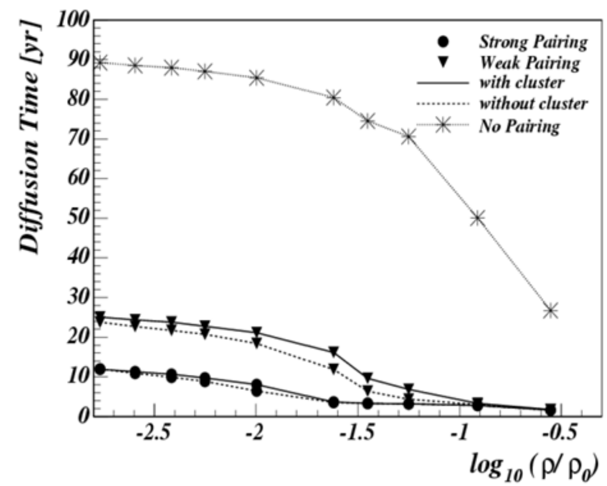
$$t_w \approx R_{shell}^2 \frac{C_V}{k}$$

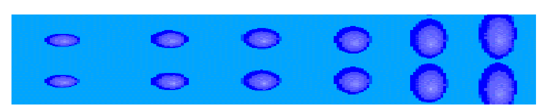
$$C_V \propto C_V^0 \, e^{-\Delta/kT}$$



J.M. Lattimer et al, ApJ 1994



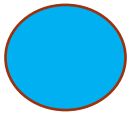


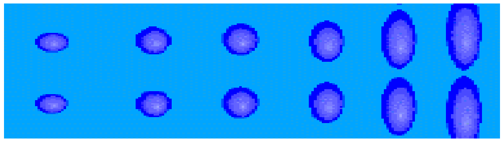


C. Monrozeau, J. Margueron, N. S , PRC67(2007)

Dificulties of an universal DFT with pairing

significant finite size effects





coherence length

ξ

R_N

d

$$\frac{\xi}{R_N}$$

$$\frac{\xi}{d}$$

accuracy of BCS for nuclei: - particle number conservation

Coherence Length in Nuclei

<u>*A. Bohr and B. R. Mottelson :*</u>

*"In nuclei, the pairs **cannot** be localized within dimensions smaller than the nuclear radius R ".*

(Nuclear Structure, vol II,1963)

$$\xi \sim \frac{\hbar^2 k_F}{m\Delta}$$

$$\Delta << \hbar\omega_0 \sim 49/\mathrm{R}_\mathrm{N}$$

$$\xi >> \frac{\hbar^2 k_F}{m}\frac{1}{49}R_N$$

$$\Delta \sim 1\,\mathrm{MeV}$$

$$\xi \sim 40\ \mathrm{fm}$$

no spatial correlations !

Two-body correlations induced by pairing: definitions

- **two-body density** $\rho_2(r_1\sigma_1, r_2\sigma_2) = \sum_{\sigma_2 \ldots \sigma_N} |\Psi(r_1\sigma_1, r_2\sigma_2, \ldots\ldots r_N\sigma_N)|^2 \; dr_3 \ldots dr_N$

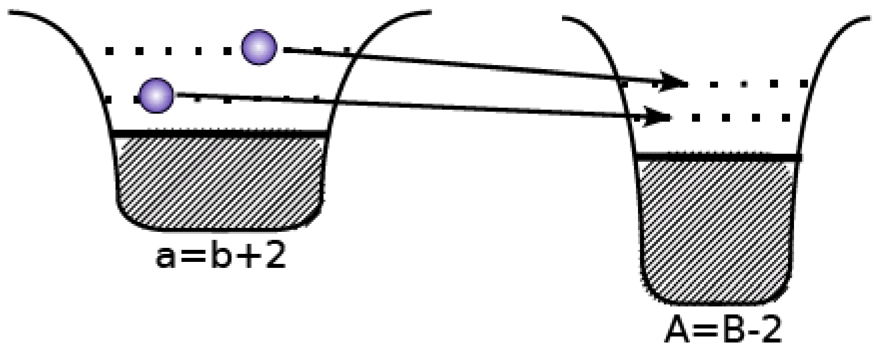
- **two-body correlations** $|k(r_1\sigma, r_2 - \sigma)|^2 = \rho_2(r_1\sigma, r_2 - \sigma) - \rho(r_1\sigma)\rho(r_2 - \sigma)[1 - P_{12}]$

- **two-body correlations in BCS** $k(r_1\sigma, r_2 - \sigma) = <0|\, a(r_2 - \sigma) a(r_1\sigma)\,|0> = \sum_i u_i \mathrm{v}_i \varphi_i(\mathrm{r}_1\sigma)\, \varphi_i(\mathrm{r}_2 - \sigma)$

describes correlations between two generic nucleons

$$\underline{\psi}_0 = A[\chi_{\uparrow\downarrow}(r_1 - r_2)\chi_{\uparrow\downarrow}(r_3 - r_4)\ldots\chi_{\uparrow\downarrow}(r_{N-1} - r_N)]$$

Pairing tensor and two-particle transfer



$$\frac{d\sigma_0}{d\Omega} \propto \; .....|\int \mathrm{k(r)}\ \mathrm{j}_0(qr)r^2dr\,|^2$$

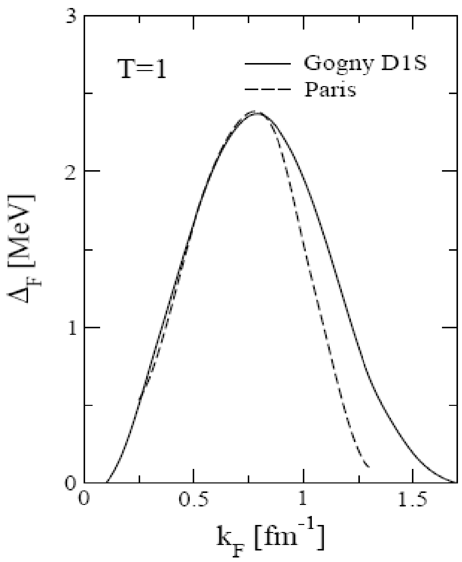
Main Issues

- *Localisation properties of* $k(r_1\sigma, r_2 - \sigma)$

  surface./bulk localisation ?

- *Coherence length in nuclei*

- *Structure of Cooper pairs : BCS vs exact model*



Pairing correlations in HFB approach

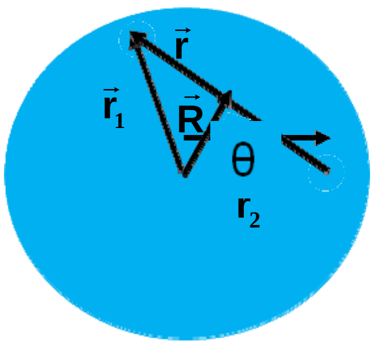
- ***pairing tensor in coordinate representation :***

$$\kappa(r_1\sigma_1, r_2 - \sigma_2) \propto \langle 0 | a(r_2 - \sigma_2) a(r_1, \sigma_1) | 0 \rangle$$

- ***HFB equations***

$$\begin{pmatrix} h(\rho) - \lambda & \Delta(\kappa) \\ \Delta(\kappa) & -h(\rho) + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

calculations with Gogny force D1S

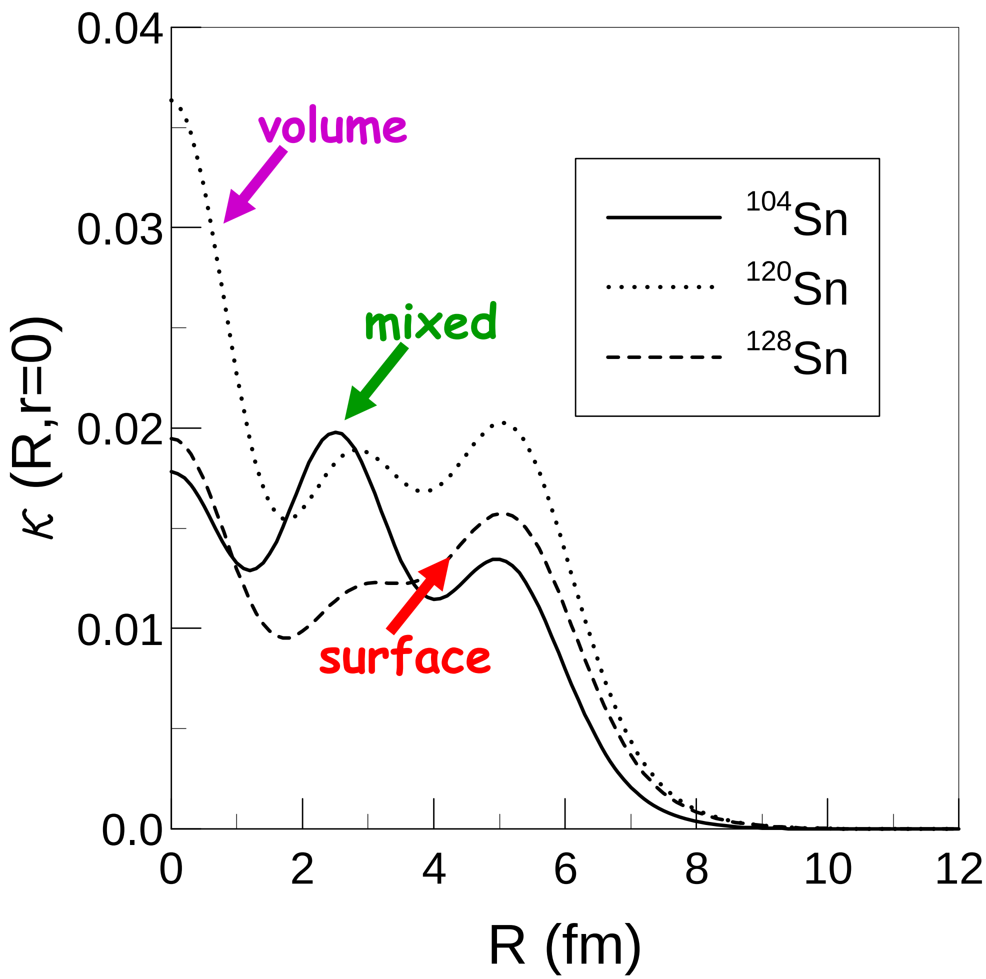


$$\kappa(\vec{r}_1, \vec{r}_2)_{S=0} \Rightarrow k(\vec{r}, \vec{R})_{S=0}$$

- ***coherence length***

$$\xi(R) = \frac{\left(\iint r^4 \kappa(R, r, \theta)^2 \, dr \, \sin\theta d\theta \right)^{1/2}}{\left(\iint r^2 \kappa(R, r, \theta)^2 \, dr \, \sin\theta d\theta \right)^{1/2}}$$

Spatial correlations: surface or bulk ?



N. Pillet, N. S, P. Schuck, PRC76(2007)

Localization of pairing correlations: Skyrme-HFB

$$V_{pair} = V_0[1-\eta(\rho/\rho_0)^{\alpha}]\delta(r-r')$$

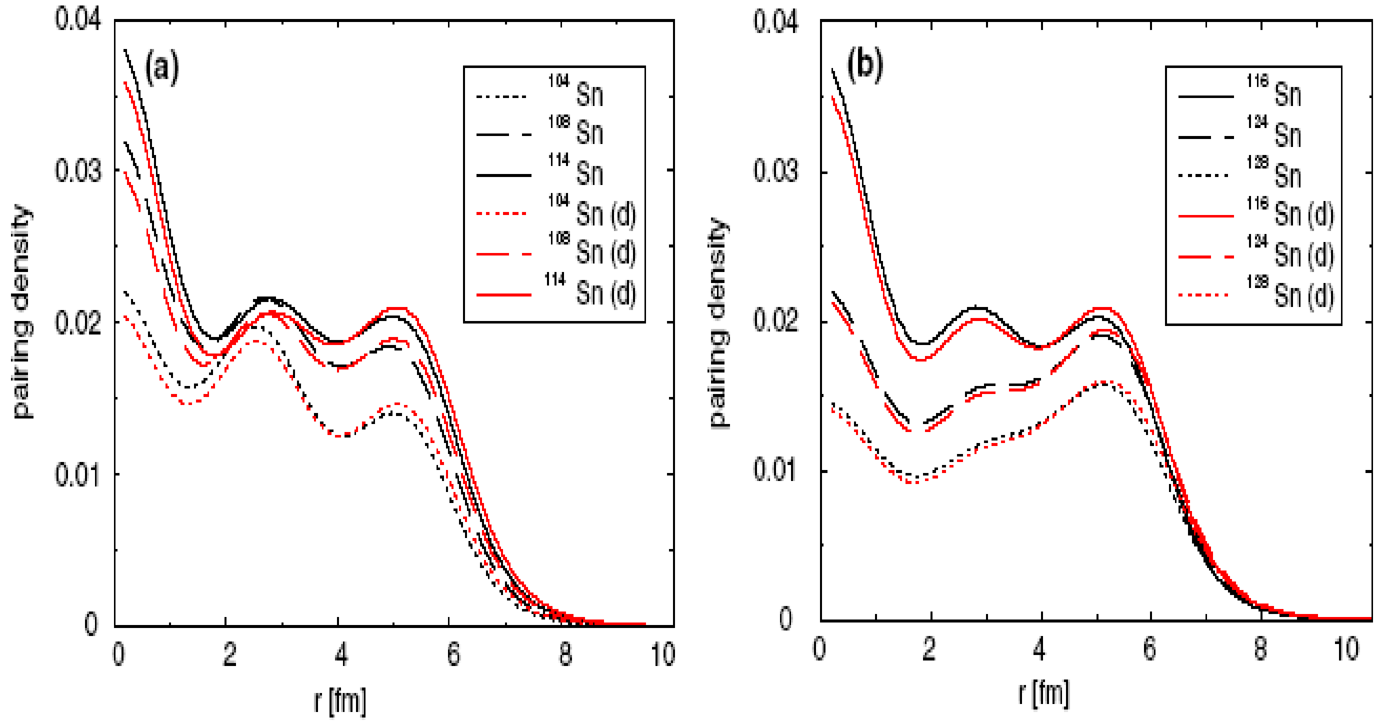
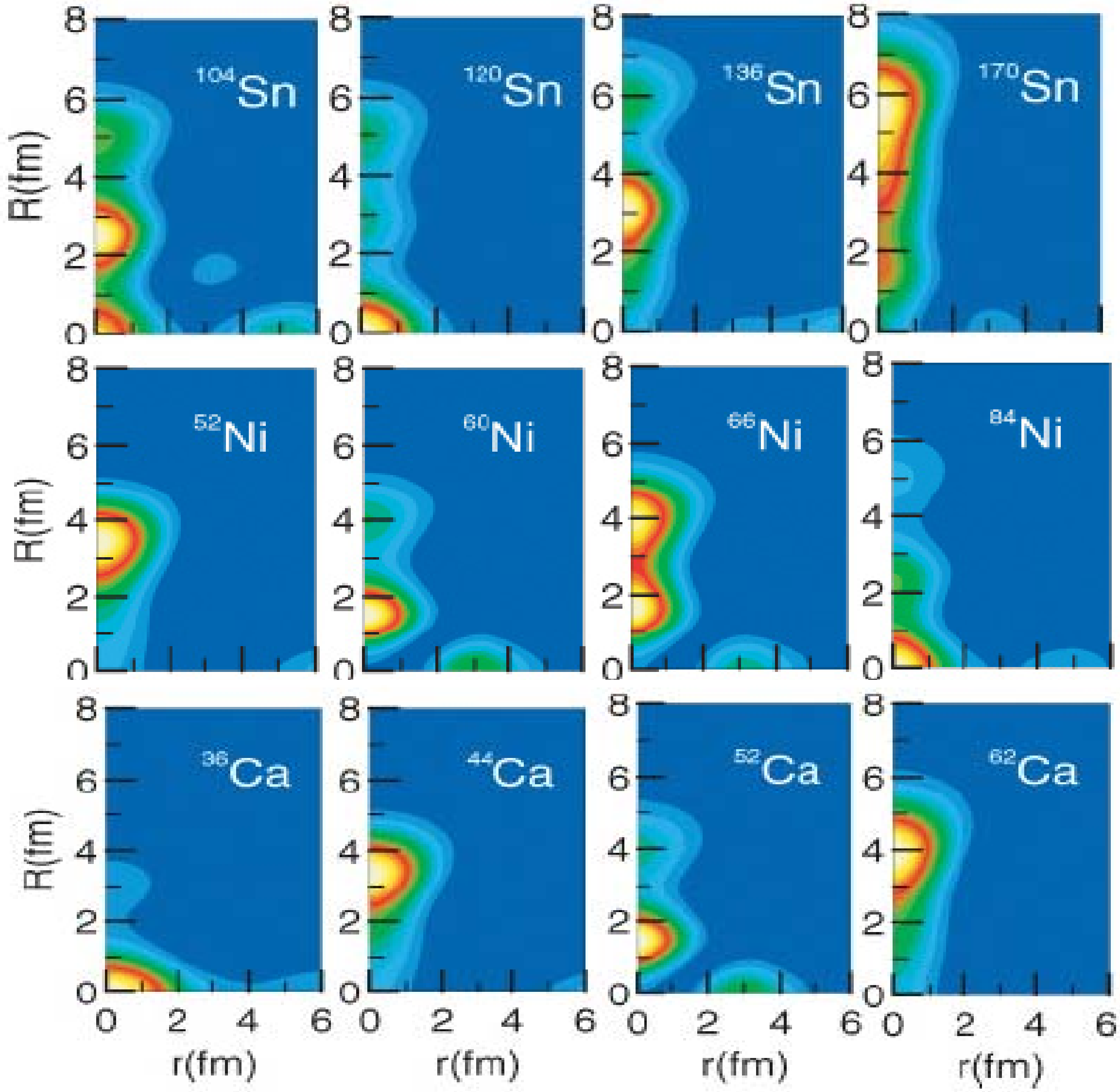


FIG. 2. (Color online) Neutron pairing densities $\kappa(r)$ (in fm^{-3}) for Sn isotopes calculated in the HFB approach. The black (red) curves correspond to the δ force (density-dependent δ force).

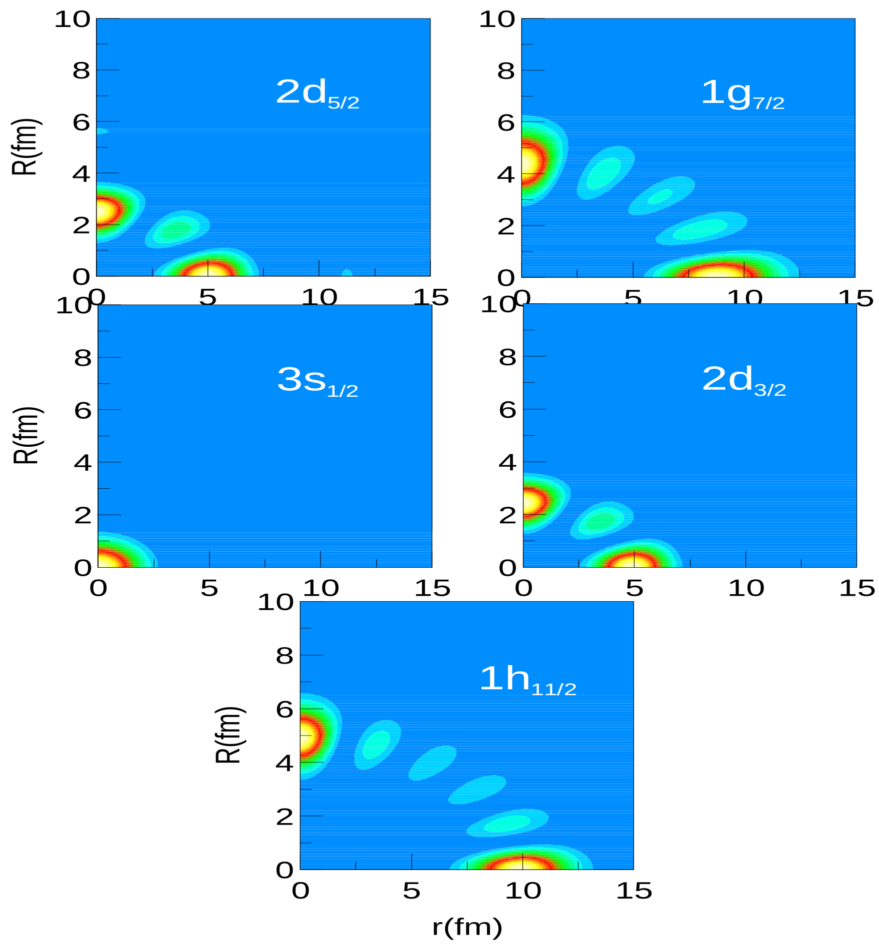
N. S, P. Schuck, X. Vinas, PRC 71 (2005) 054303

$$|\kappa(r,R)|^2$$



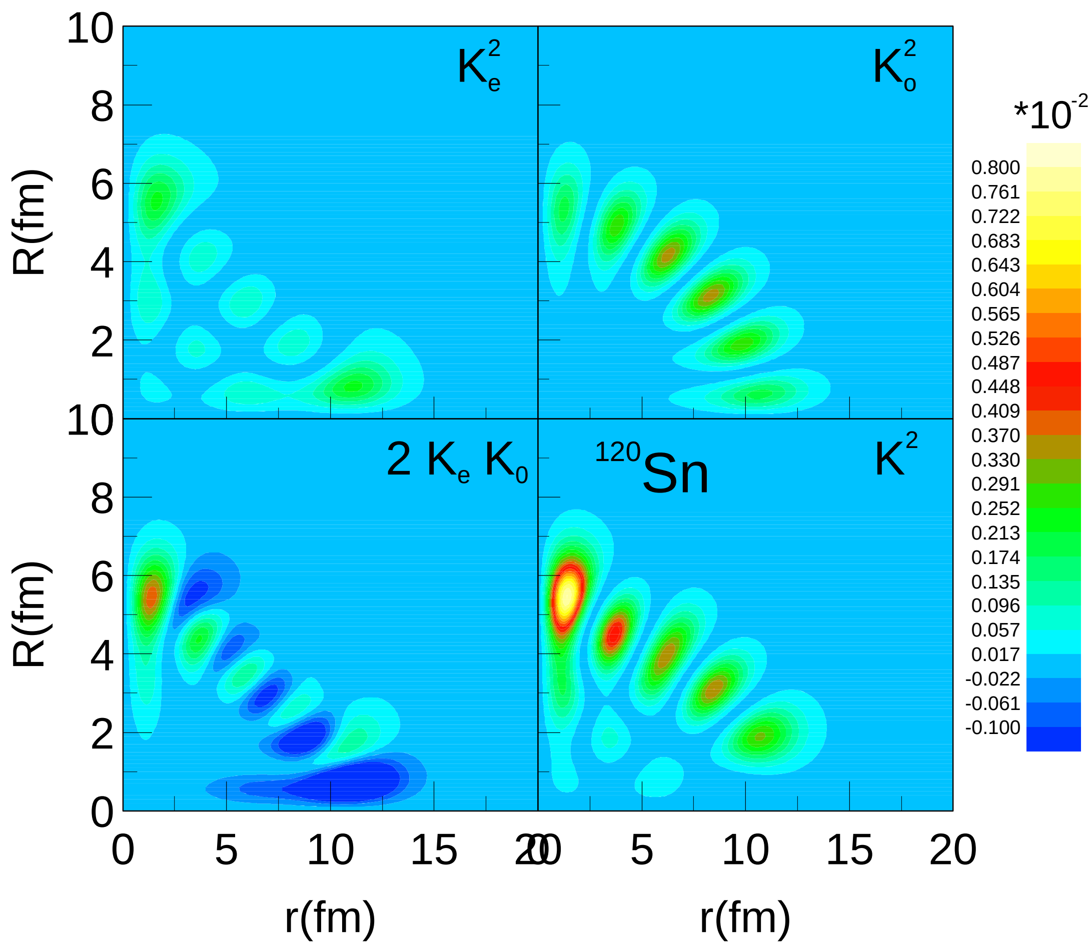
N. Pillet, N.S., P. Schuck, PRC76, 2007

Two-body uncorrelated wave function



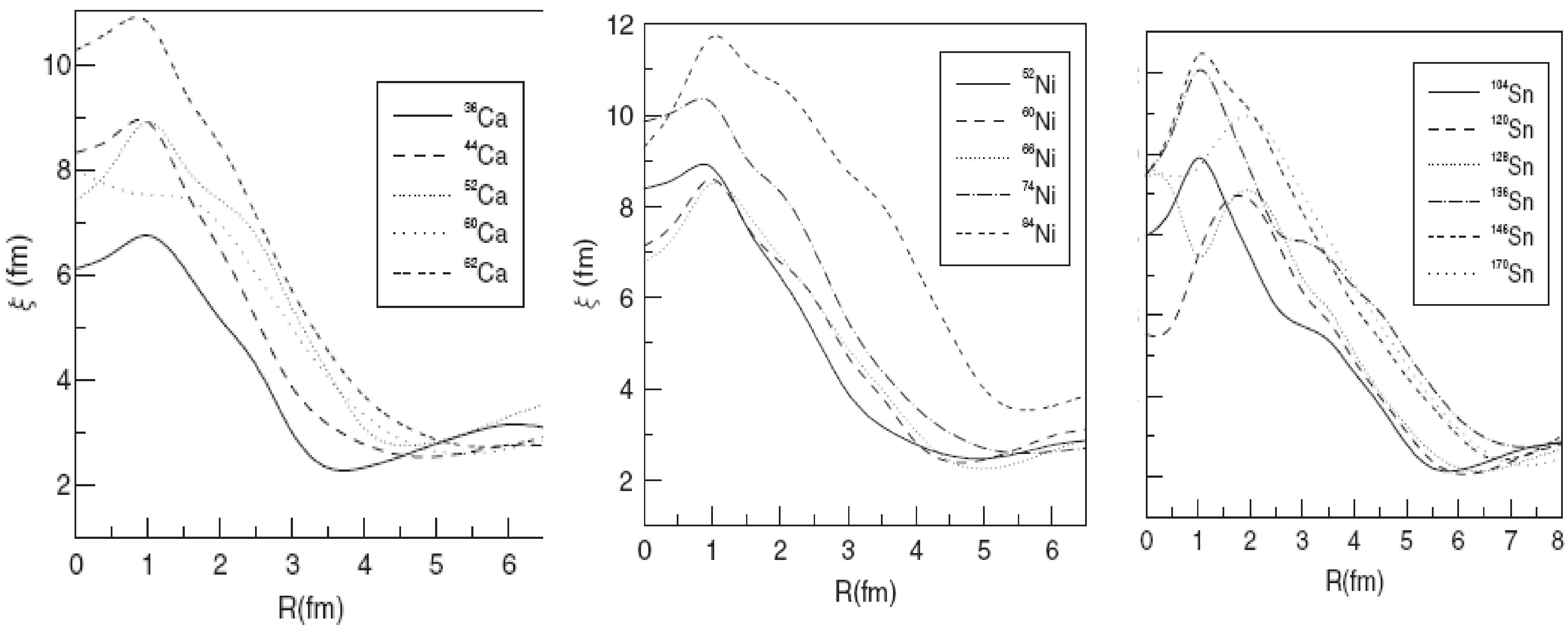
$$\kappa^{BCS}(\vec{r}_1,\vec{r}_2) = \frac{1}{4\pi}\sum_{n_1 j}(2j+1)u_{nlj}\,\mathrm{v}_{\mathrm{nlj}}R_{nl}(r_1)R_{nl}(r_2)\,P_l(\cos\theta)$$

The Effect of Parity Mixing



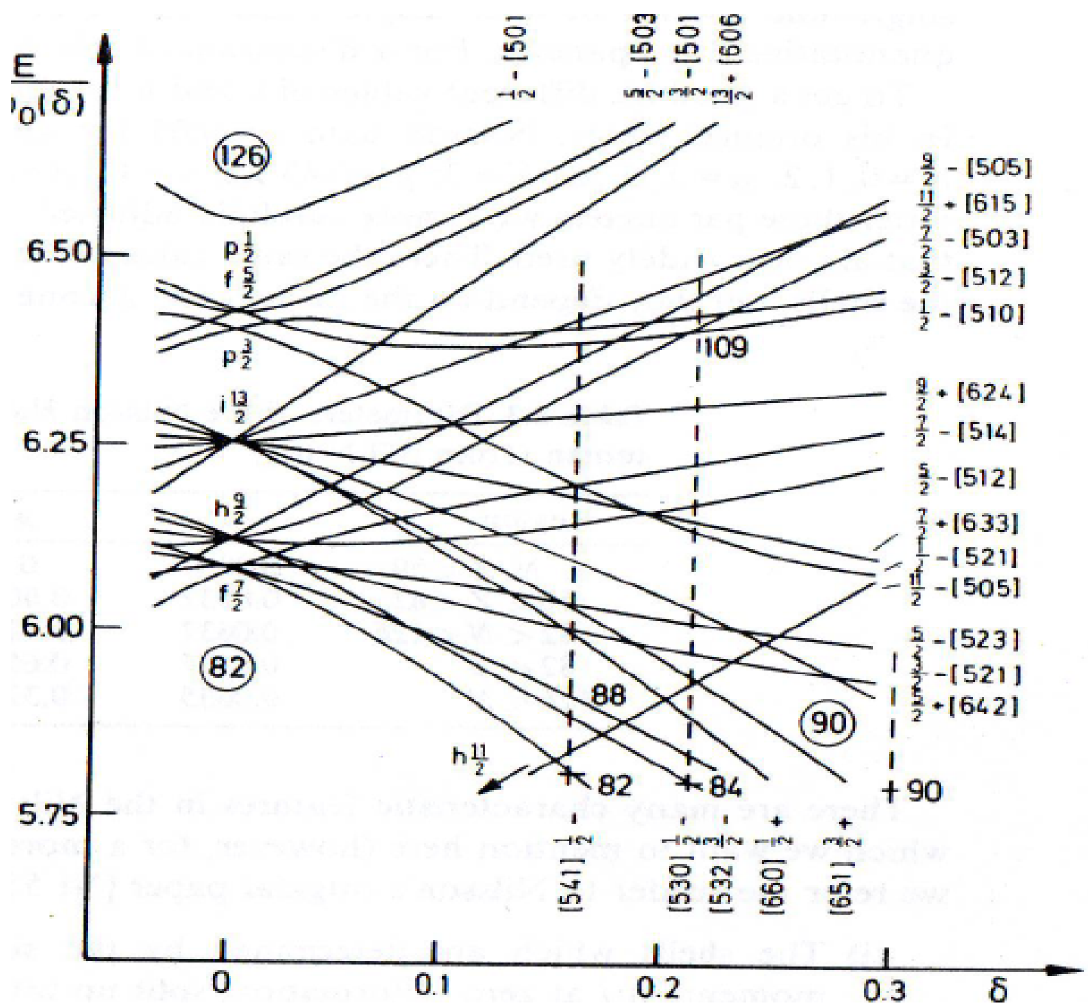
$$\kappa^{BCS}(\vec{r}_1, \vec{r}_2) = \frac{1}{4\pi} \sum_{n_1 j} (2j+1) u_{nlj} \mathrm{v}_{\mathrm{nlj}} R_{nl}(r_1) R_{nl}(r_2)\, P_l(\cos\theta)$$

Coherence length in Ca, Ni and Sn isotopes



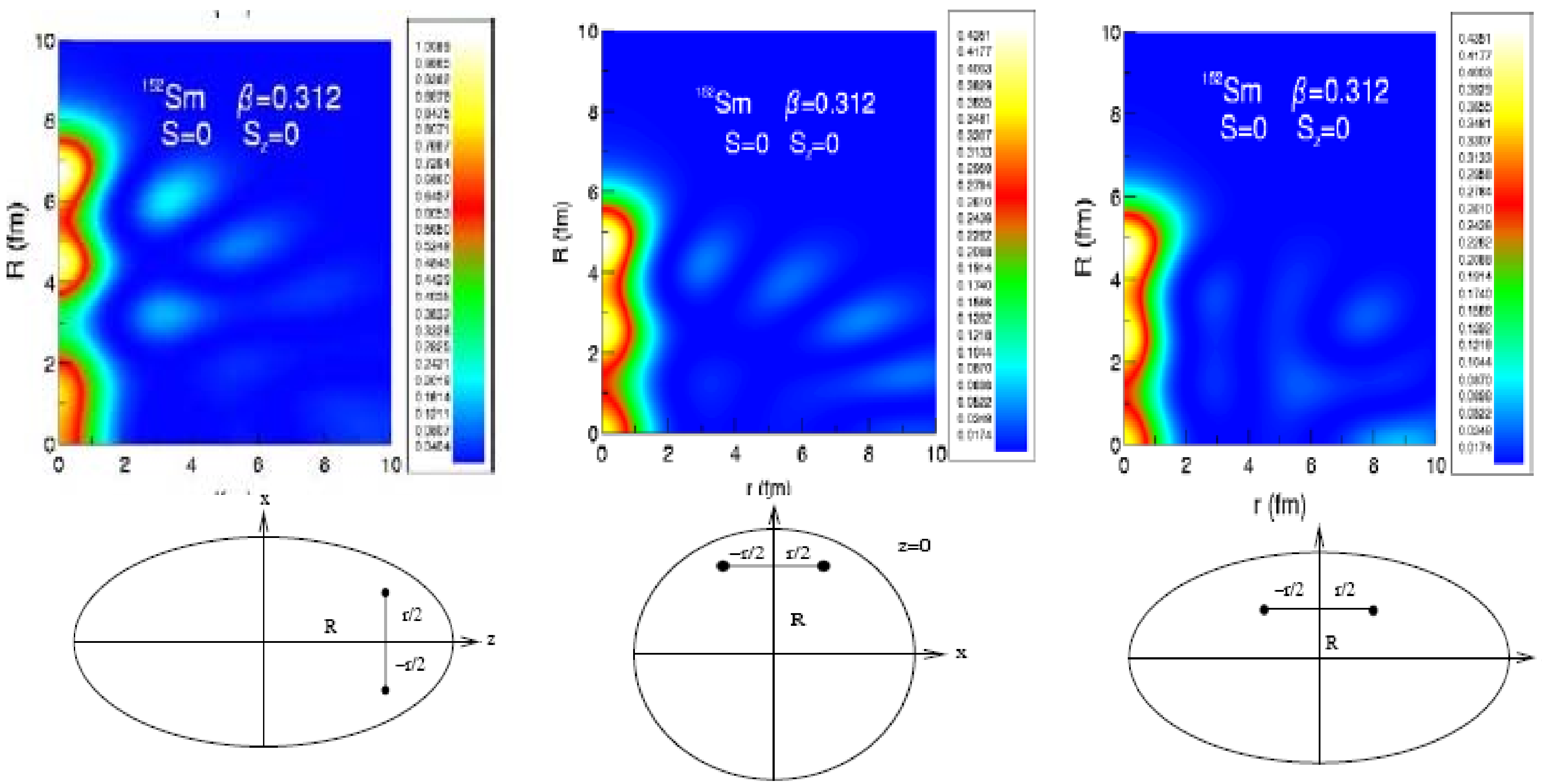
$$\xi(R) = \frac{\left(\iint r^4 \kappa(R, r, \theta)^2 \, dr \, \sin\theta d\theta\right)^{1/2}}{\left(\iint r^2 \kappa(R, r, \theta)^2 \, dr \, \sin\theta d\theta\right)^{1/2}}$$

What happens in deformed nuclei ?



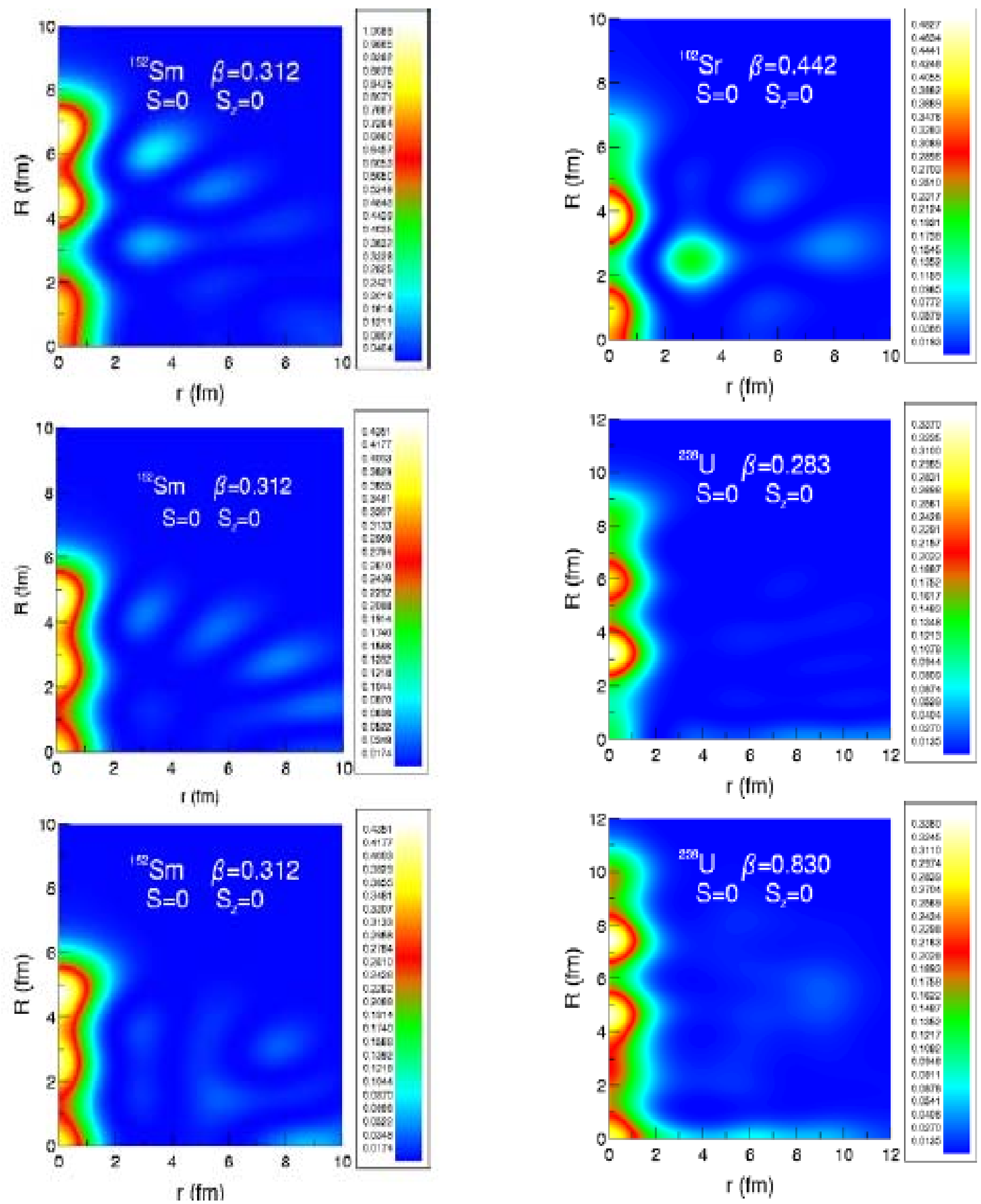
Pairing localization in ^{152}Sm

$$|\kappa(r,R)|^2$$



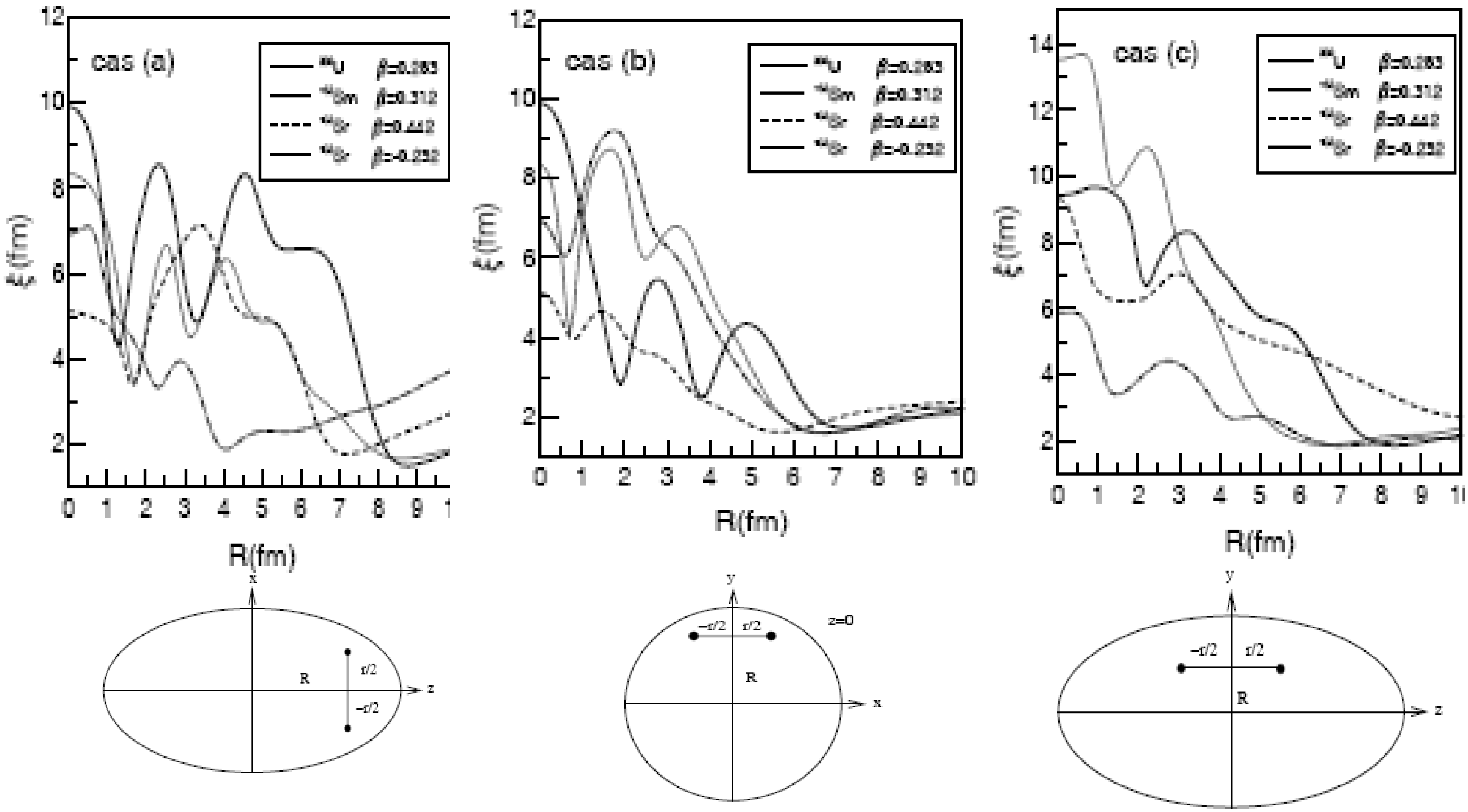
N. Pillet, N.S, P. Schuck, J.F. Berger, in preparation

$$|\kappa(r,R)|^2$$



152Sm β=0.312
S=0 Sz=0
152Sr β=0.442
S=0 Sz=0
228U β=0.283
S=0 Sz=0
228U β=0.830
S=0 Sz=0
R (fm)
r (fm)

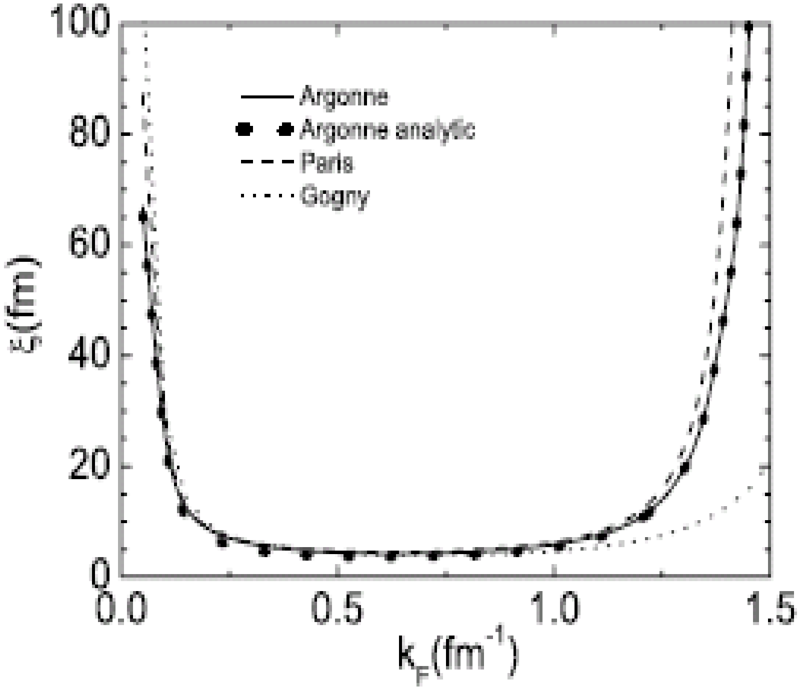
Coherence length in deformed nuclei



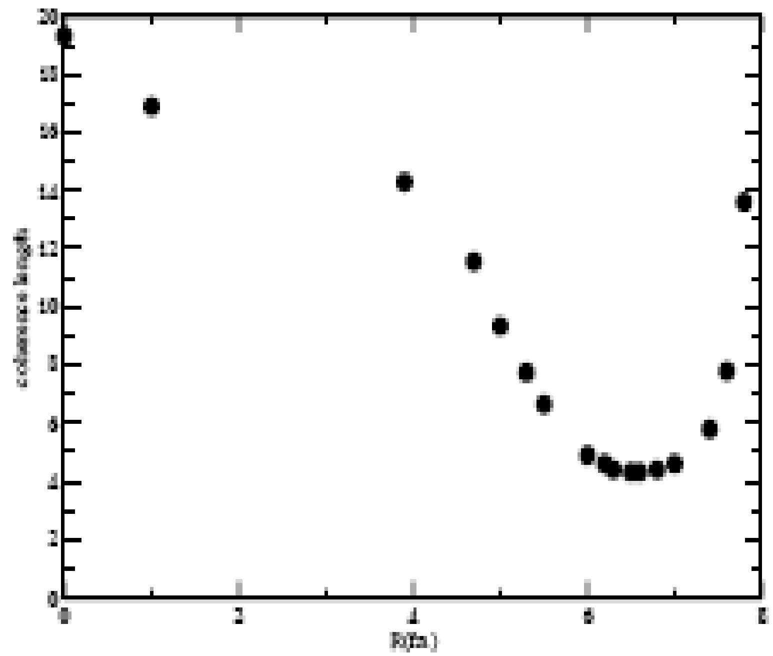
Why the coherence length is so small in the surface ?

Coherence length in neutron matter and nuclei

Infinite neutron matter



^{120}Sn



$$\xi_{nm}(k_F) = \frac{\int \left|\frac{d\kappa(k_F,k)}{dk}\right|^2 k^2 dk}{\int |\kappa(k_F,k)|^2 k^2 dk}$$

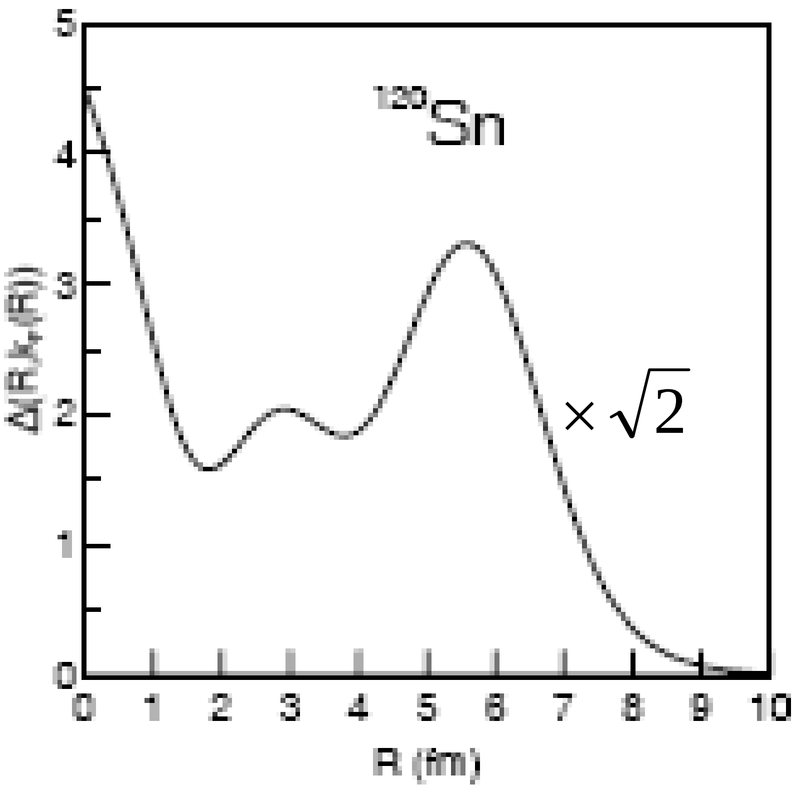
$$\xi(R) = \xi_{nm}(k_F(R))$$

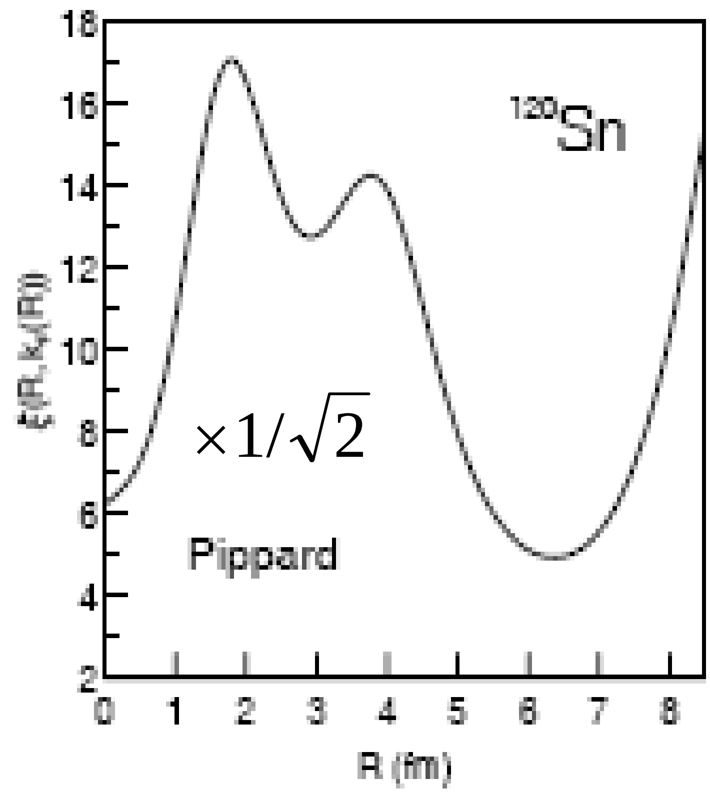
$$\xi = \alpha \frac{1}{\pi} \frac{\hbar^2}{m^*} \frac{k_F}{\Delta_F}$$

Coherence length and Pippard approximation

$$\Delta_F(R) = 4\pi \int \Delta(R,r)\frac{sin(k_F(R))}{k_F(R)r}r^2dr$$

$$\xi = \alpha\frac{1}{\pi}\frac{\hbar^2}{m^*}\frac{k_F}{\Delta_F}$$

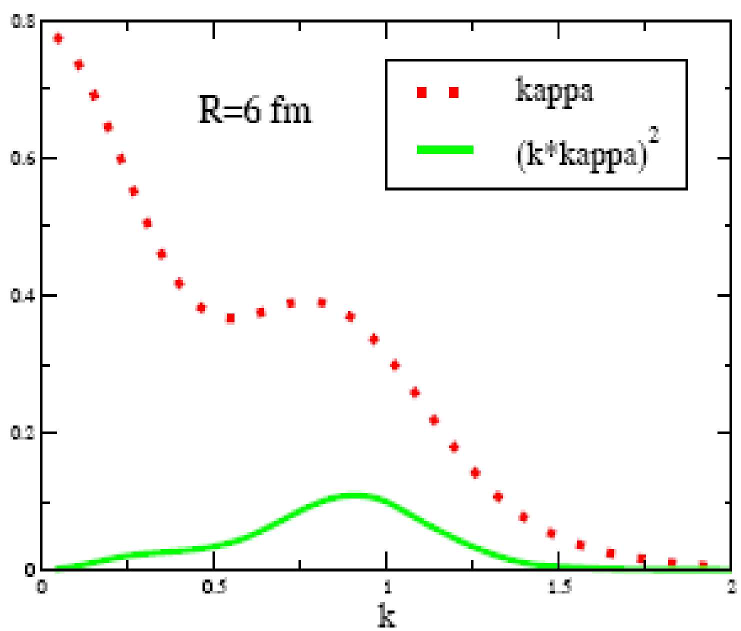


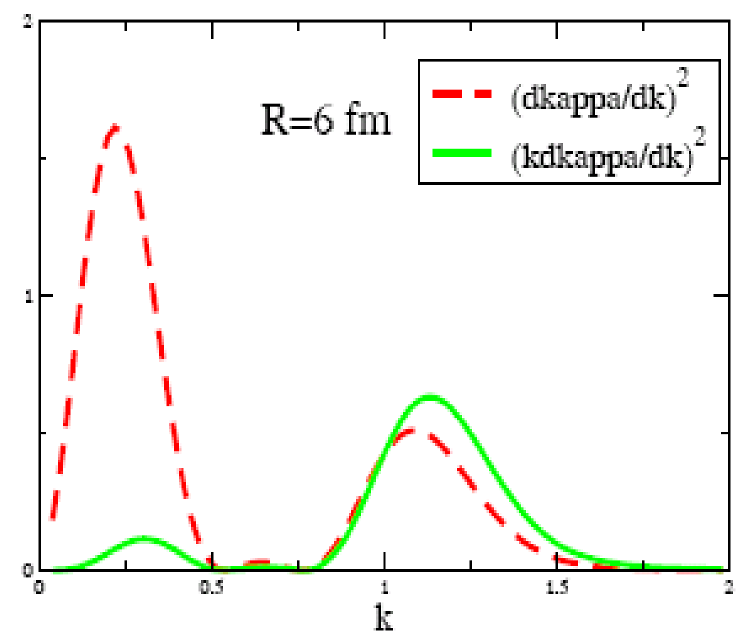


Pairing tensor in momentum representation

$$\xi(R) = \frac{\int \left| \frac{d\kappa(R,k)}{dk} \right|^2 k^2 dk}{\int |\kappa(R,k)|^2 \; k^2 dk}$$

$$\kappa(R,k) = 4\pi \int \kappa(R,r) \frac{\sin(kr)}{kr} r^2 dr$$





Pairing in local density approximation

$$\Delta(\boldsymbol{R},\boldsymbol{p}) = \frac{1}{2}\int \frac{d^3k}{(2\pi\hbar)^3}\, v(\boldsymbol{p}-\boldsymbol{k})$$

$$\times \frac{\Delta(\boldsymbol{R},\boldsymbol{k})}{\{[k^2/2m - \lambda(\boldsymbol{R})]^2 + \Delta^2(\boldsymbol{R},\boldsymbol{k})\}^{1/2}}$$

$$\lambda(\boldsymbol{R}) = \lambda - V(\boldsymbol{R})$$

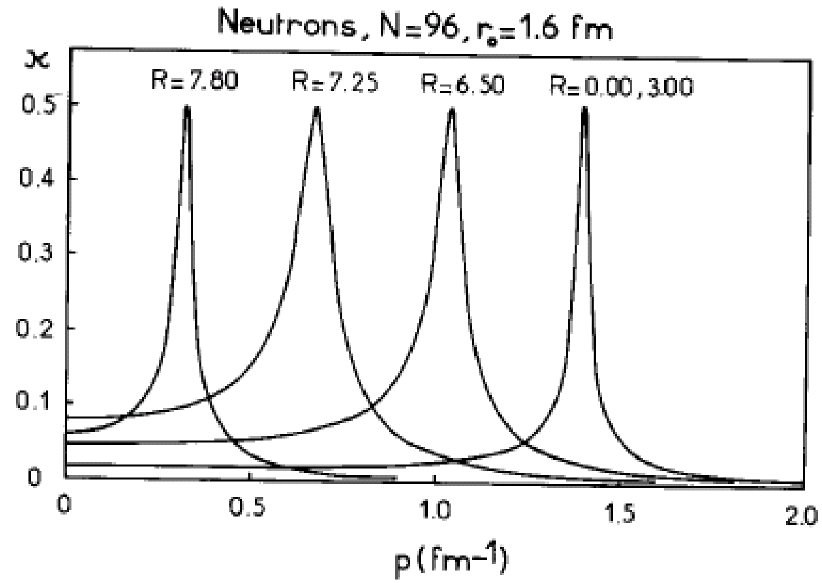


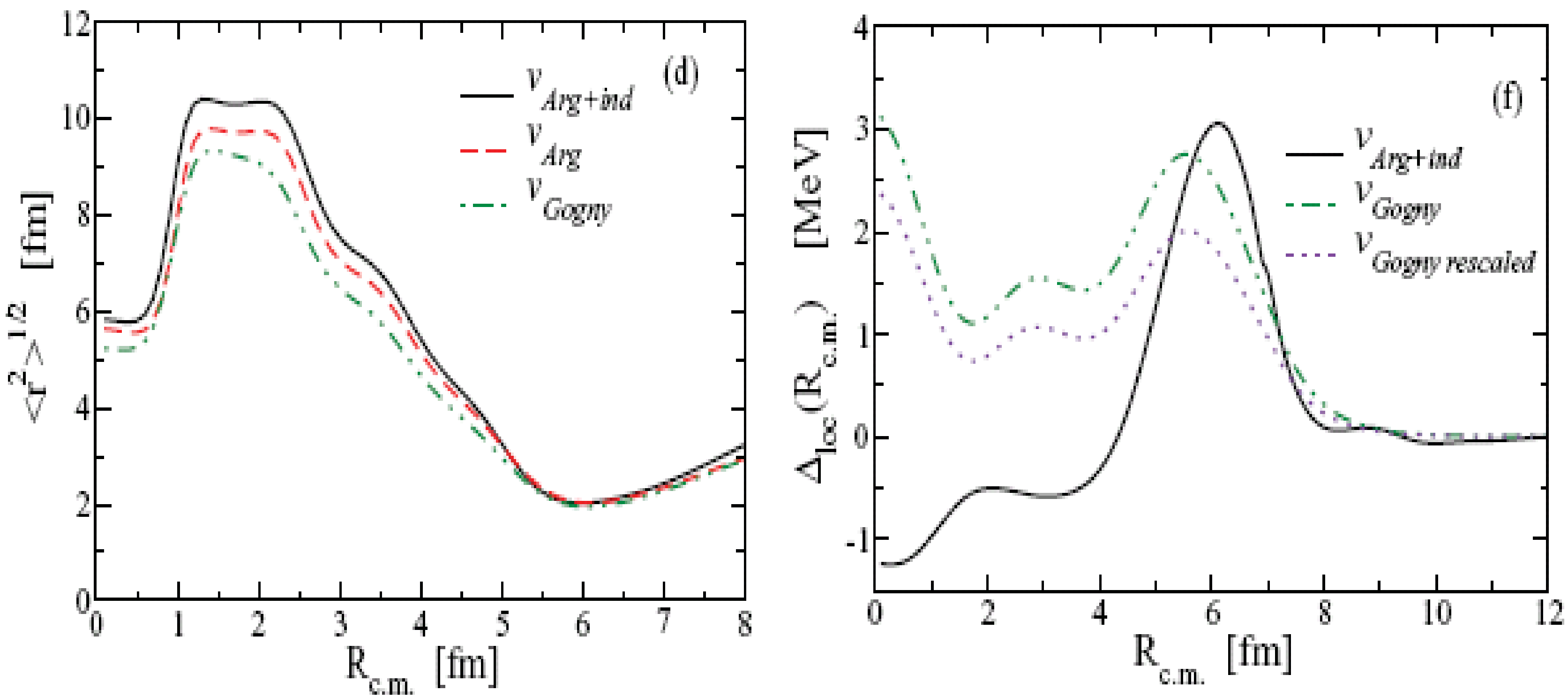
Fig. 2. The abnormal density $\kappa(R, p)$ as a function of p for different R values. Note that κ does not change between 0.0 and ~ 3 fm.

R. Bengtsson, P. Schuck, PLB89(1980)

How much depends the coherence length on pairing force ?

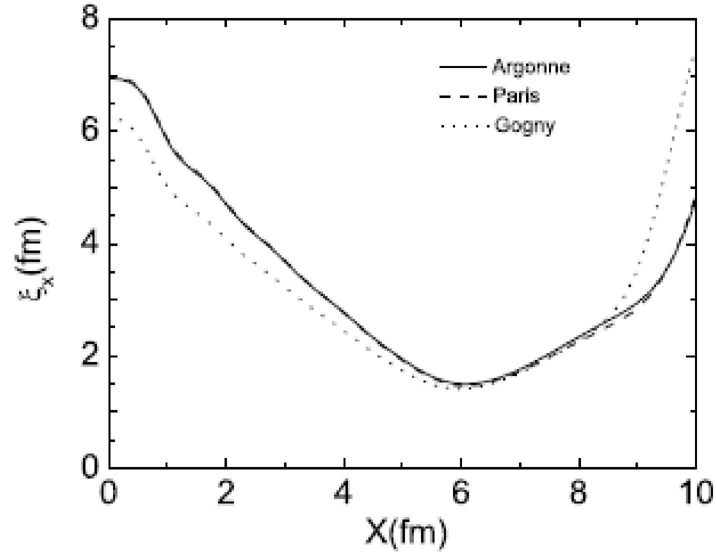
Coherence length versus pairing force

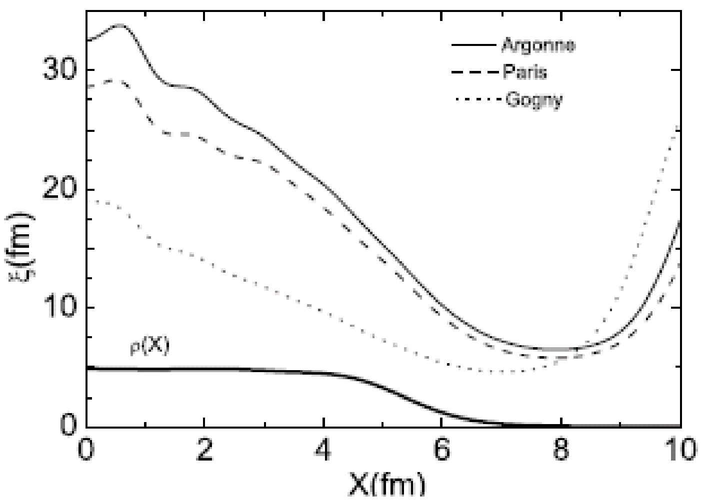
^{120}Sn

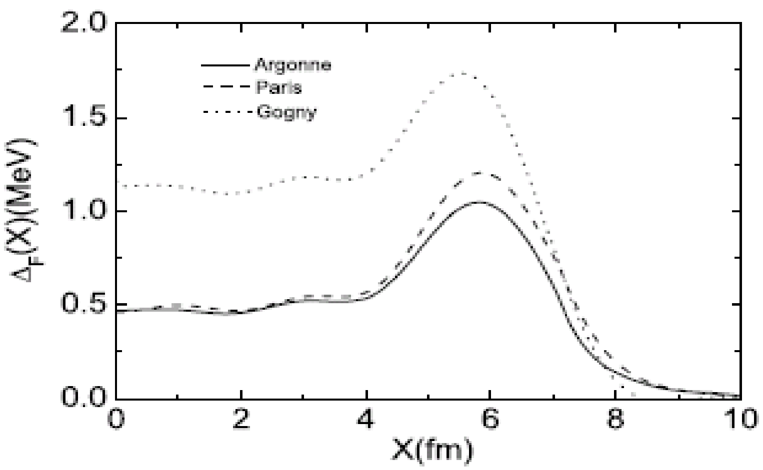


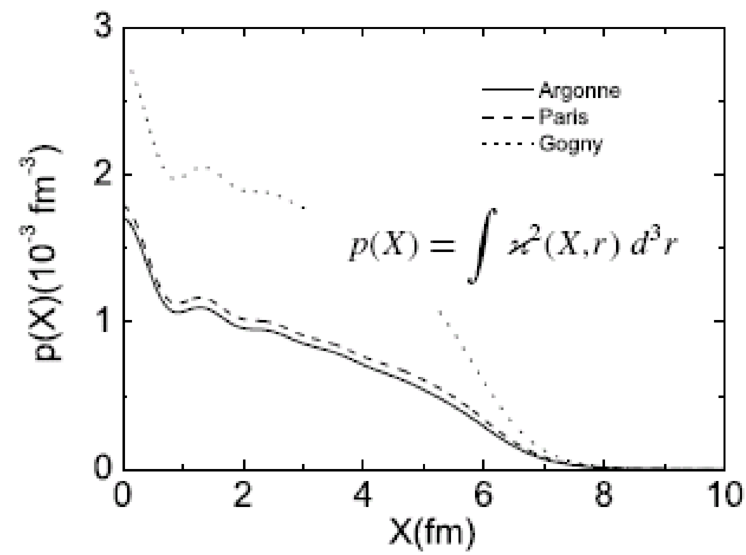
A. Pastore, F. Barranco, R.A.Broglia, E. Vigezzi, PRC(2008)

Coherence length in a slab of neutron matter







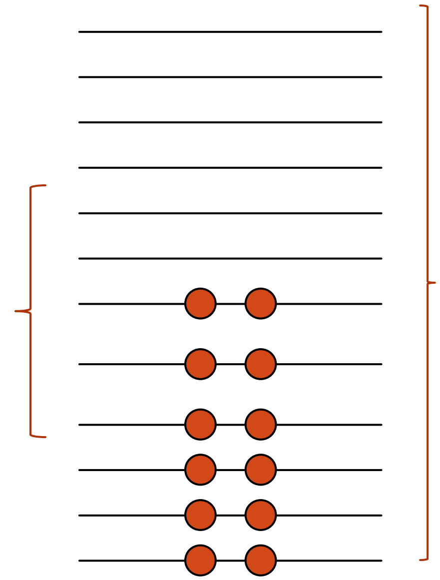


S.Pankratov, E. Saperstein,M.Zerev, M. Baldo,U.Lombardo,PRC79(2009)

How much depend the two-body correlations on pairing treatment ?

Solutions of pairing Hamiltonian

$$H = \sum_{i}^{\Omega} \varepsilon_i (a_i^\dagger a_i + a_{\bar{i}}^\dagger a_{\bar{i}}) - g \sum_{i \geq j}^{\Omega} a_i^\dagger a_{\bar{i}}^\dagger a_{\bar{j}} a_j$$



$$|BCS> \propto \sum_{n} \frac{(\Gamma^+)^n}{n!} |->$$

$$|PBCS> \propto (\Gamma^\dagger)^{N_{pair}} |0> \qquad \Gamma^+ = \sum_i x_i a_i^+ a_{\bar{i}}^+$$

Exact

$$|\Psi> = \prod_{\nu}^{N} B_\nu^\dagger |0> \qquad B_\nu^\dagger = \sum_i \frac{1}{2\varepsilon_i - E_\nu} a_i^\dagger a_{\bar{i}}^\dagger$$

$$\frac{1}{g} - \sum_j \frac{1}{2\varepsilon_j - E_\nu} + \sum_{\mu \neq \nu} \frac{2}{E_\mu - E_\nu} = 0$$

all pairs are different !

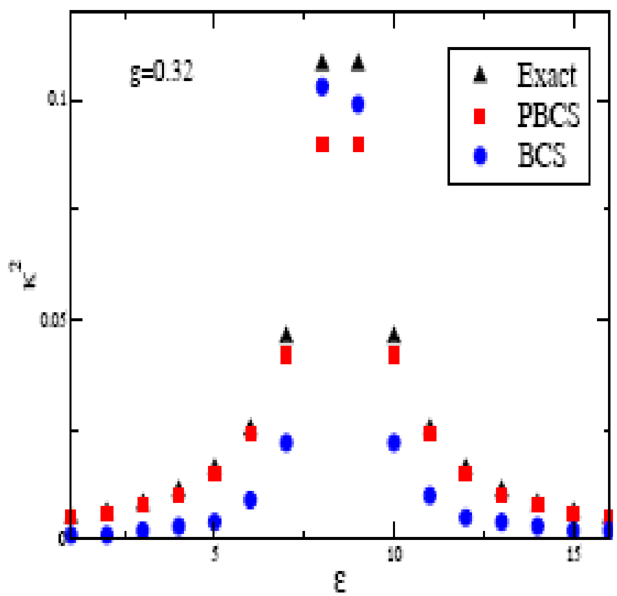
R. W. Richardson and N. Sherman, Nucl. Phys. 52 (1964)221

Two-body correlations in configuration space

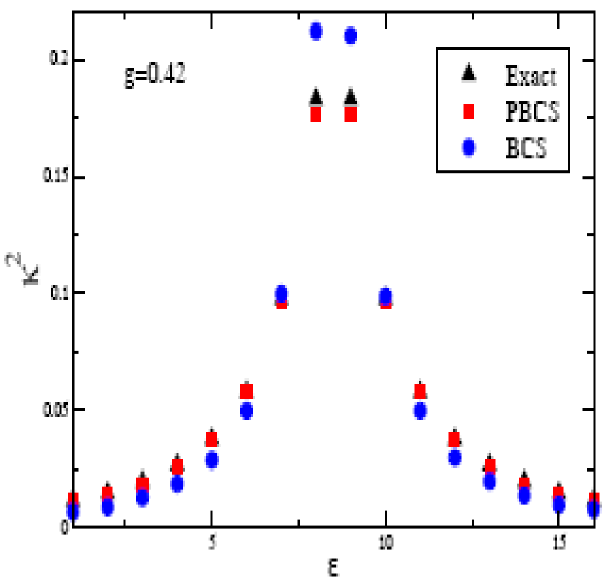
$$| k_{ii} |^2 = |< BCS(N) | c_i c_{\bar{i}} | BCS(N) >|^2 = u_i^2 \mathrm{v}_i^2$$

N_{pair} =8

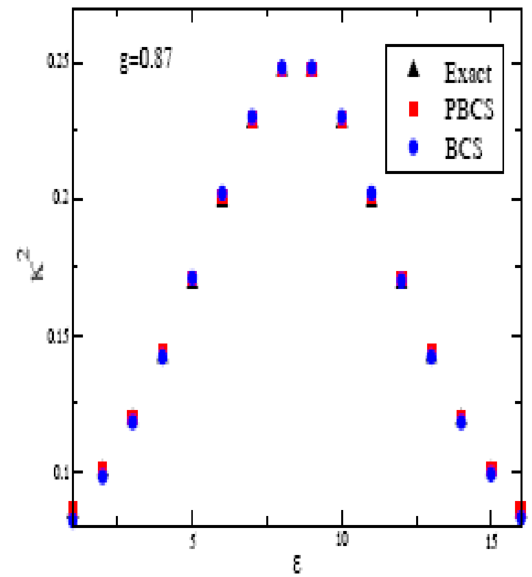
weak coupling



intermediate coupling



strong coupling



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Pair transfer amplitudes

$$|k_{ii}|^2 = |< BCS(N) | c_i c_{\bar{i}} | BCS(N) >|^2 = u_i^2 \mathrm{v}_i^2$$

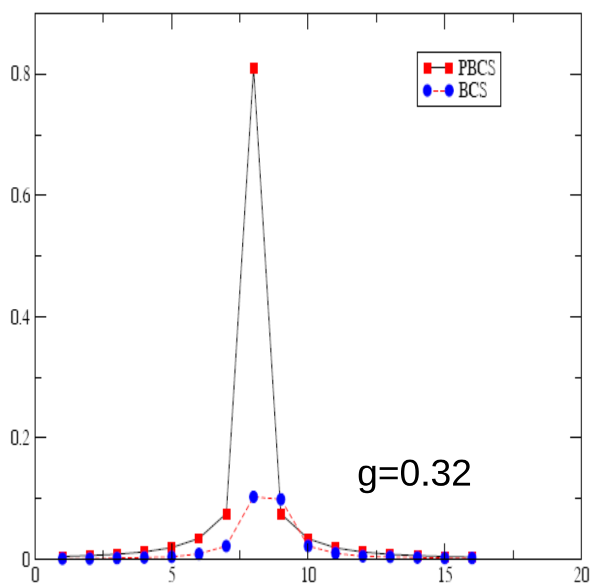
$$|k_{ii}^{PBCS}|^2 = |< PBCS(N-2) | c_i c_{\bar{i}} | PBCS(N) >|^2$$

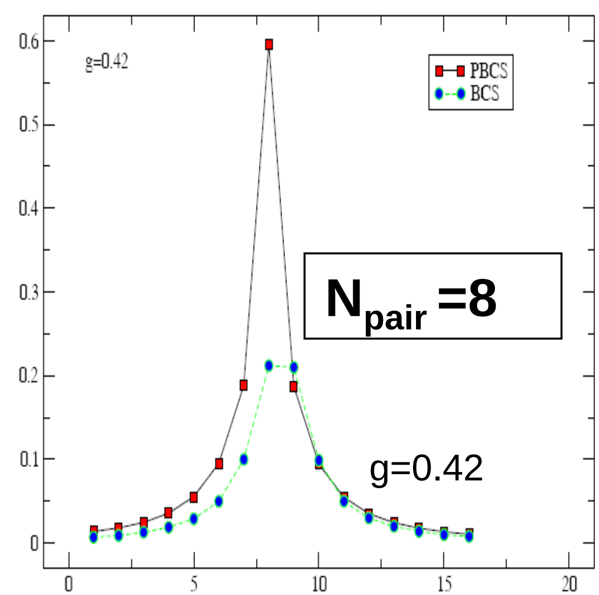
$$T = \left(\frac{\Delta}{2g}\right)^2 = \left(\sum_i k_{ii}\right)^2$$

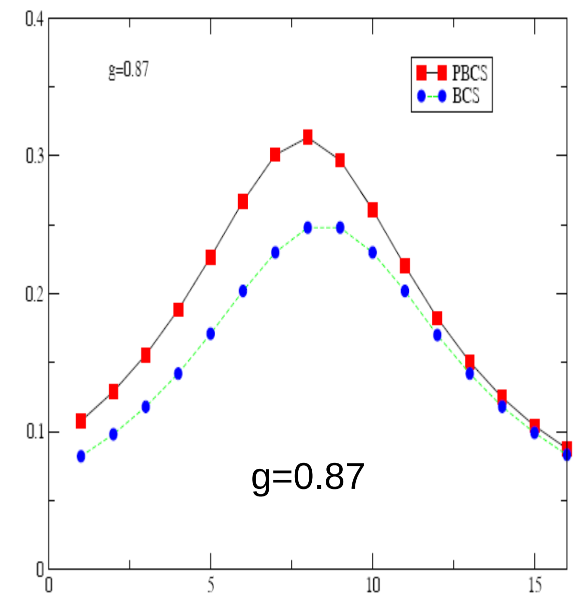
weak coupling

intermediate coupling

strong coupling







$$\left(\frac{T_{PBCS}}{T_{BCS}}\right)^2 = 1.7$$

$$\left(\frac{T_{PBCS}}{T_{BCS}}\right)^2 = 1.2$$

Pair wave functions

exact model

$$\Phi_{E_\nu}(r\sigma, r'-\sigma) \propto \sum \frac{1}{2\varepsilon_i - E_\nu} \varphi_i(r\sigma)\varphi_{\bar{i}}(r'-\sigma)$$

pair wave function

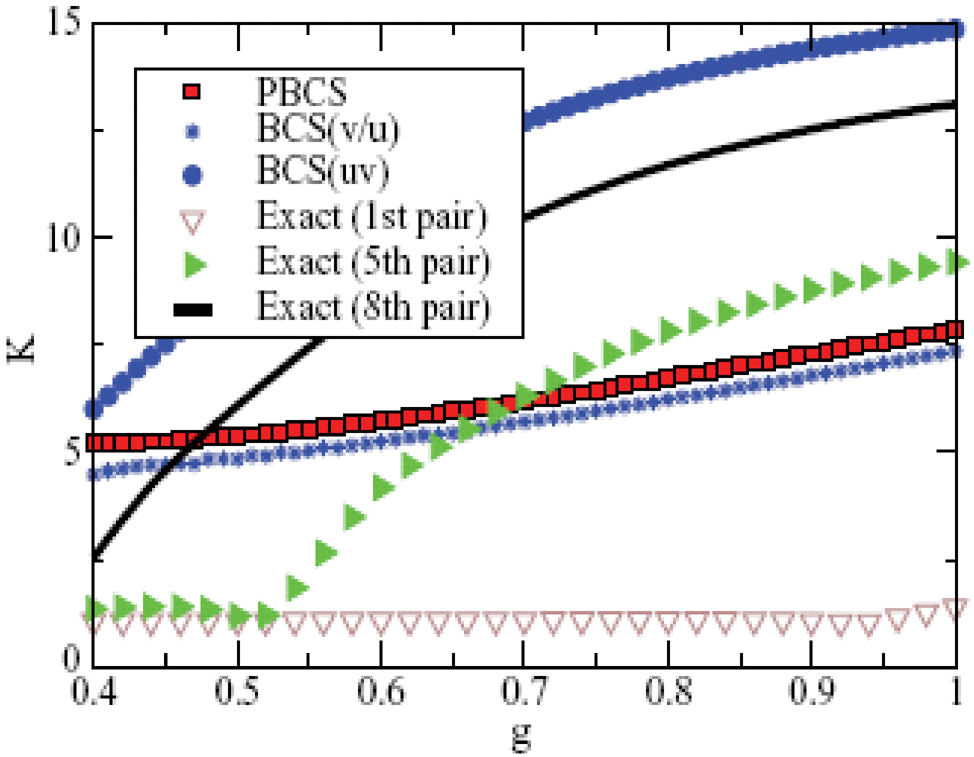
BCS, PBCS

$$\Phi(r\sigma, r'-\sigma) \propto \sum \frac{\mathrm{v_i}}{\mathrm{u_i}} \varphi_i(r\sigma)\varphi_{\bar{i}}(r'-\sigma)$$

pairing tensor

$$k(r\sigma, r'-\sigma) = \sum_i u_i \mathrm{v_i} \varphi_i(r\sigma)\varphi_{\bar{i}}(r'-\sigma)$$

$N_{pair} = 8$

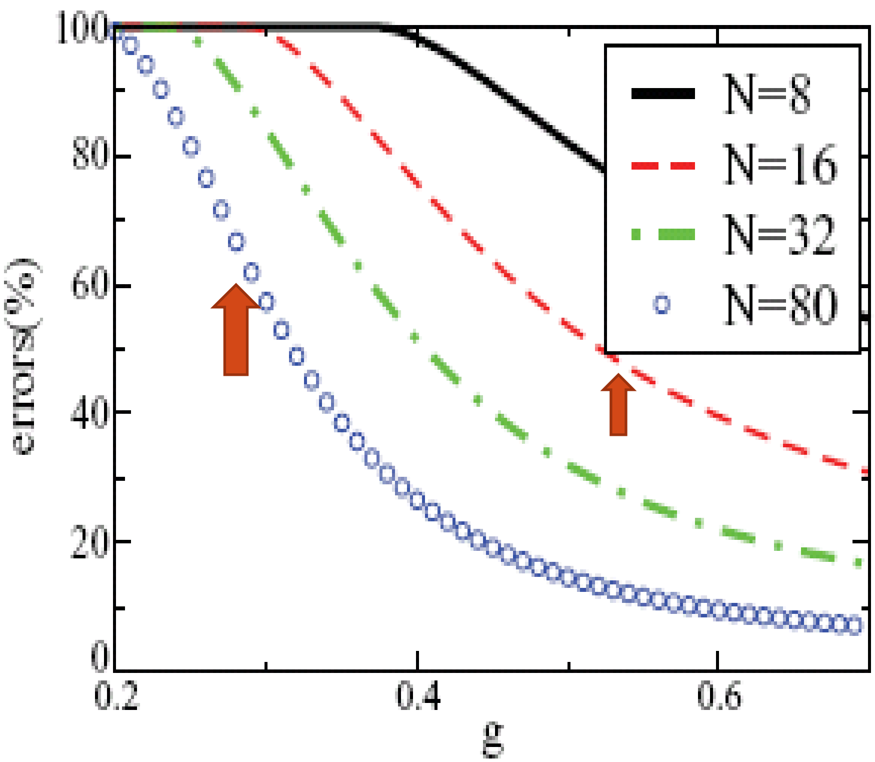


measure of correlations

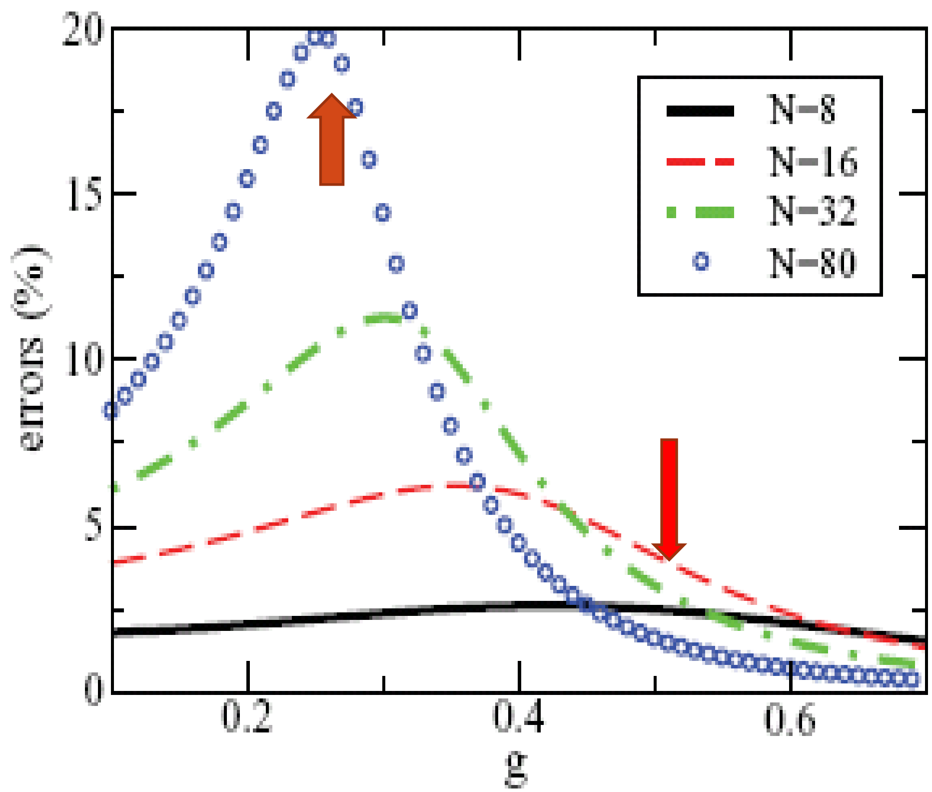
$$K = 1 / \sum_i x_i^4$$

Accuracy and energy cut-off

BCS



PBCS



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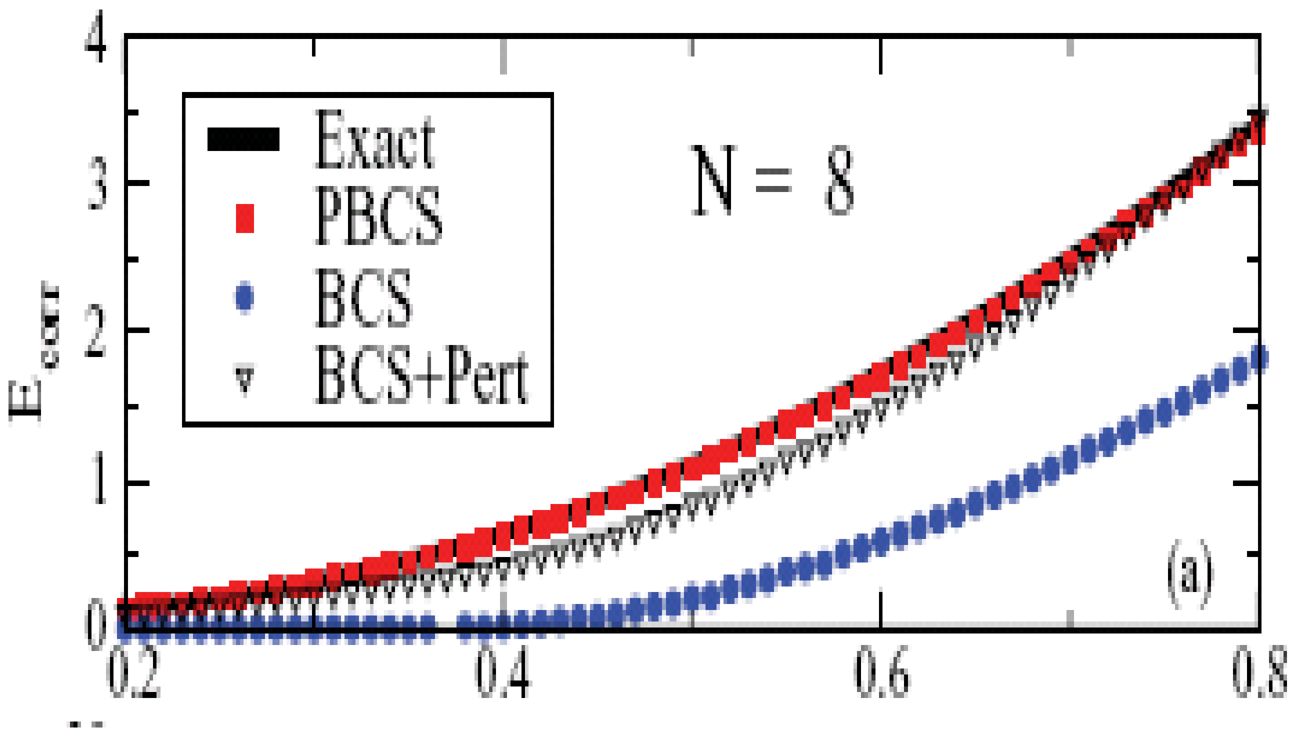
BCS and perturbative corrections

$$E^P_{\rm corr} = \frac{g^2}{2}\sum_{i=1}^{N_{\rm pair}} \sum_{j=N_{\rm pair}+1}^{\Omega} \frac{1}{\varepsilon_j - \varepsilon_i}$$



N. S , G. Bertsch, PRC78(2008)

Summary and Conclusions

- *small coherence length (2-3 fm) in the nuclear surface*

- *localization of pairing correlations: dominated by the shell structure*

- *exact solution : all pairs are different !*

implication: energy cutoff in BCS should be
comparable with the pairing gap