Neutrino-nucleus cross section in RPA models

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Effective Field Theories and the Many-Body Problem (INT 09-01) – Seattle-WA- 2009

Outline

<u>Motivation</u>

- Neutrino physics
- Detection of supernovae neutrinos.

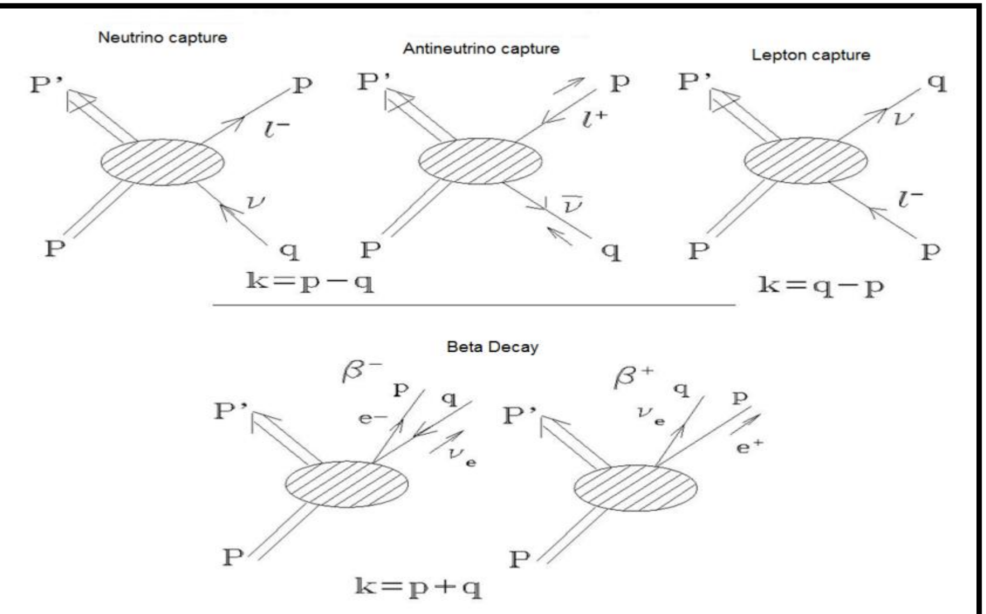
<u>Nuclear Models</u>

- QRPA & PQRPA

<u>Numerical tool</u>

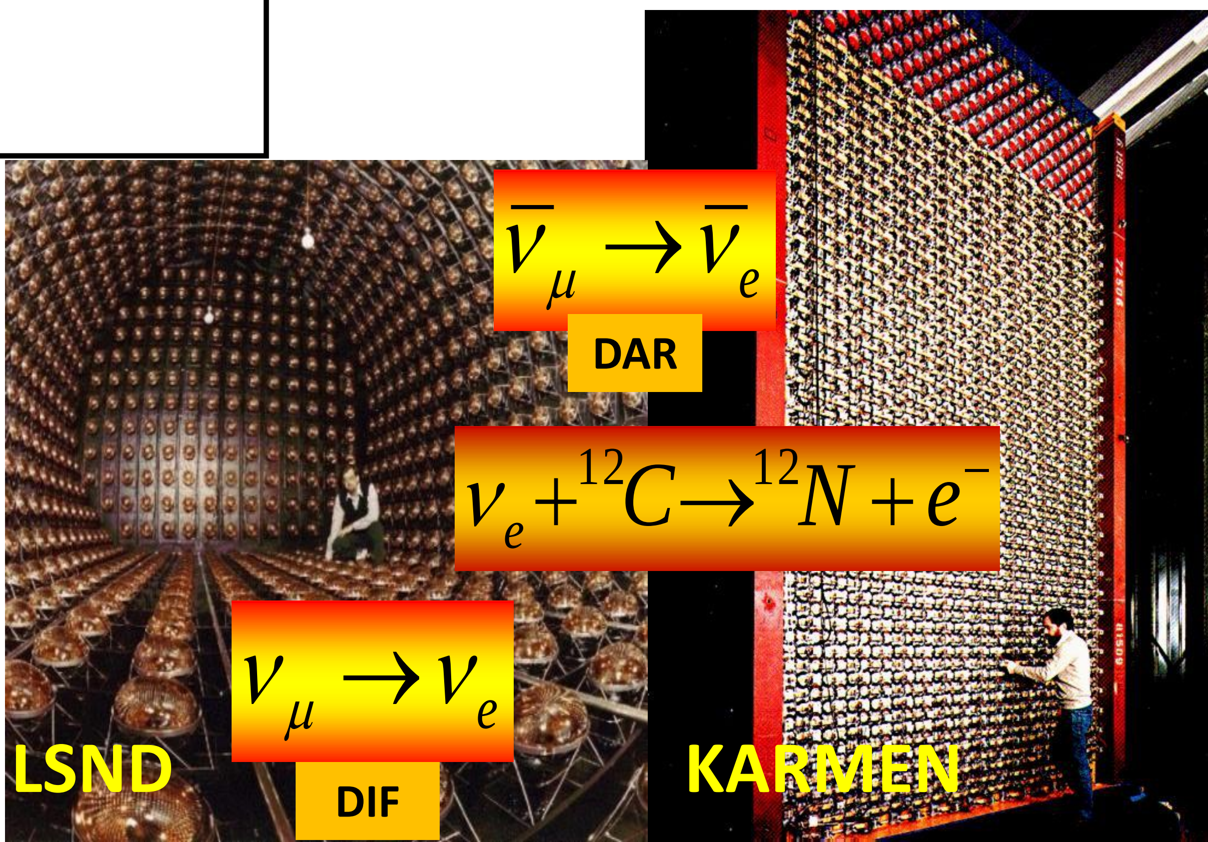
- Numerical code
- <u>Results and Summary</u>

Motivation

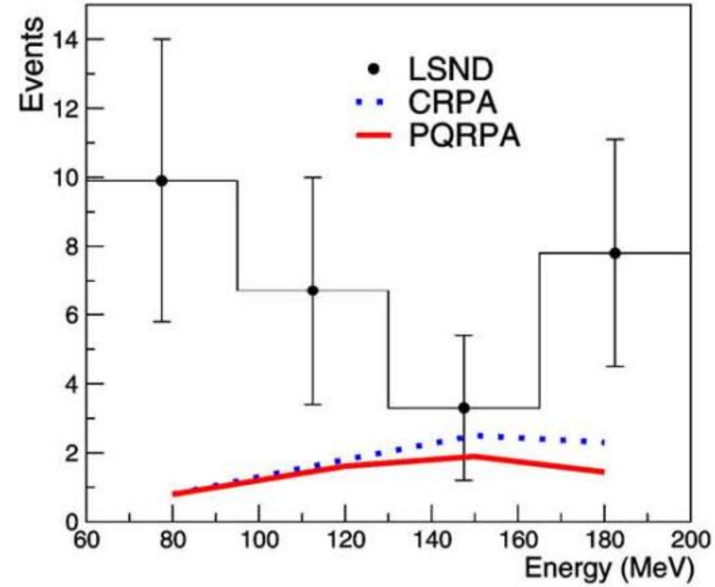


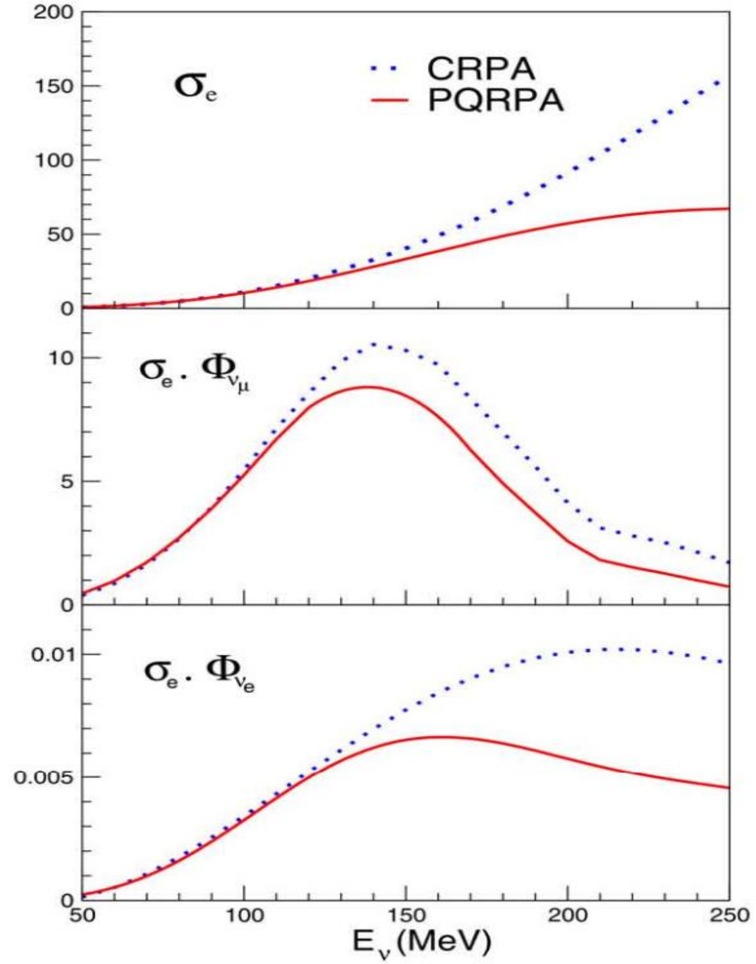
Exotic
Properties
ν-oscillations
& massiveness

Semileptonic
weak
processes



Motivation





$$\tilde{N}_\nu^{osc}(i) = \epsilon f_n \int_{E_\nu(i)} \sigma(E_\nu)\mathcal{R}(E_\nu)\Phi_{\nu_\mu}(E_\nu)P_{\nu_\mu\to\nu_e}\, dE_\nu$$

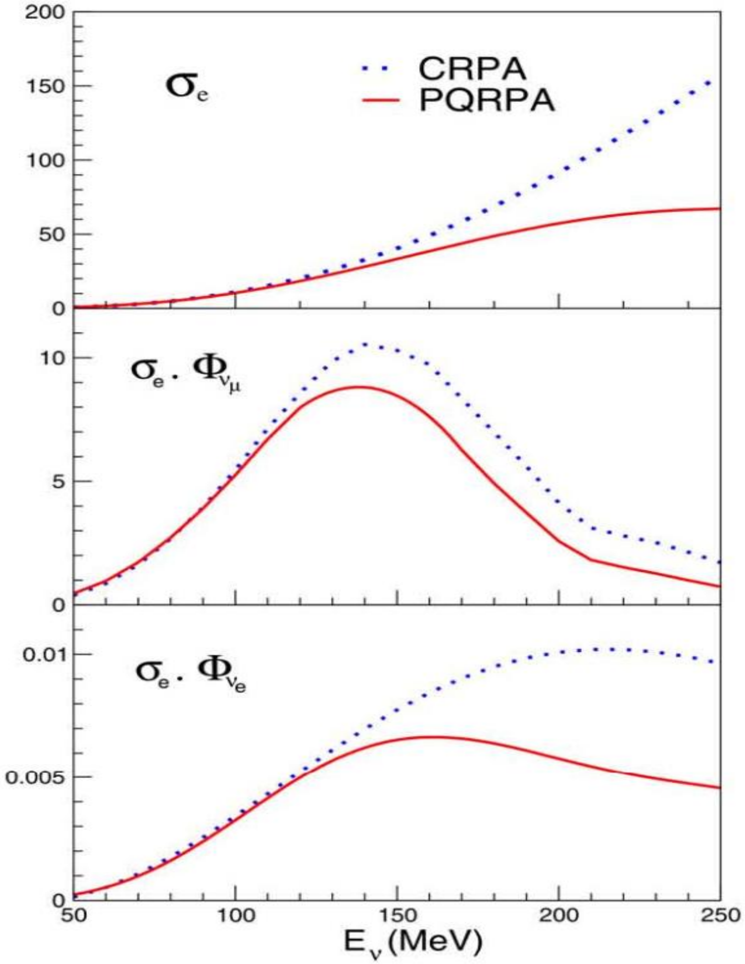
$$\tilde{N}_\nu^{bg}(i) = \epsilon f_n \int_{E_\nu(i)} \sigma(E_\nu)\mathcal{R}(E_\nu)\Phi_{\nu_e}^{bg}(E_\nu)\, dE_\nu,$$

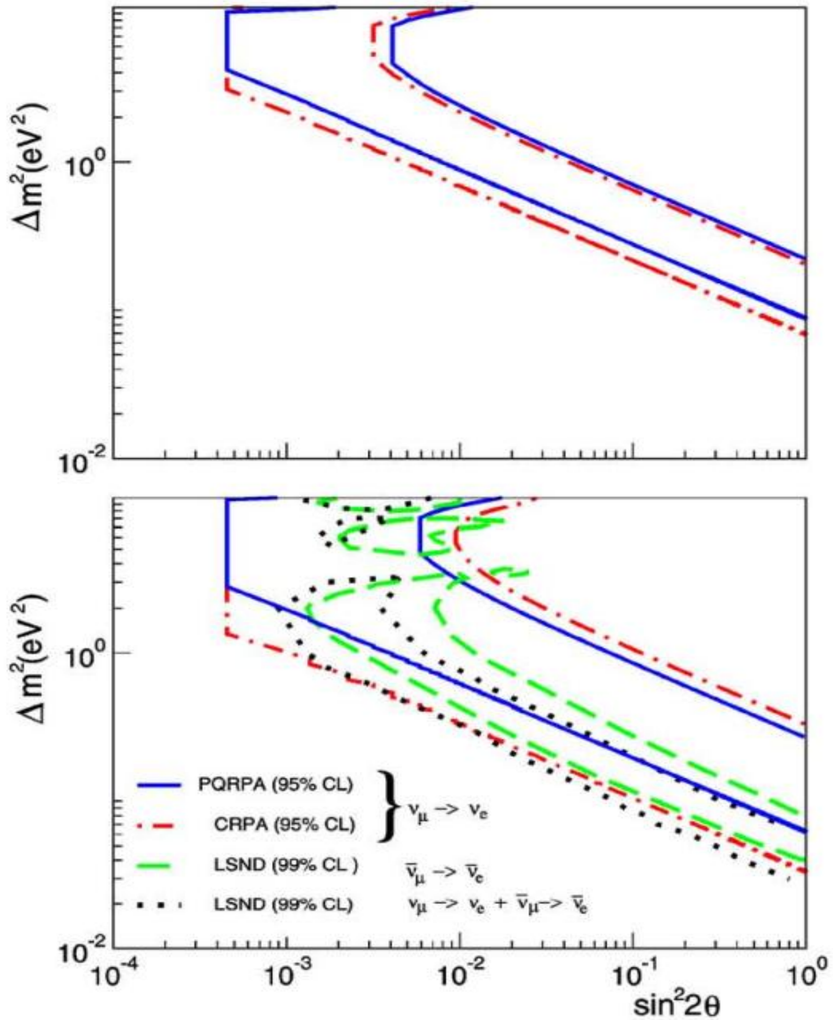
$$\chi^2 = \sum_{i=1}^{4}\left[\frac{N_\nu(i) - \tilde{N}_\nu(i)}{\delta N_\nu(i)}\right]^2$$

Neutrino-nucleus cross section is important to constrain parameters in neutrino oscillations.

$\bar{\nu}_\mu \to \bar{\nu}_e$ $\nu_\mu \to \nu_e$ * PLB642 (2005) 100

Motivation





Neutrino-nucleus cross section is important to constrain parameters in neutrino oscillations.

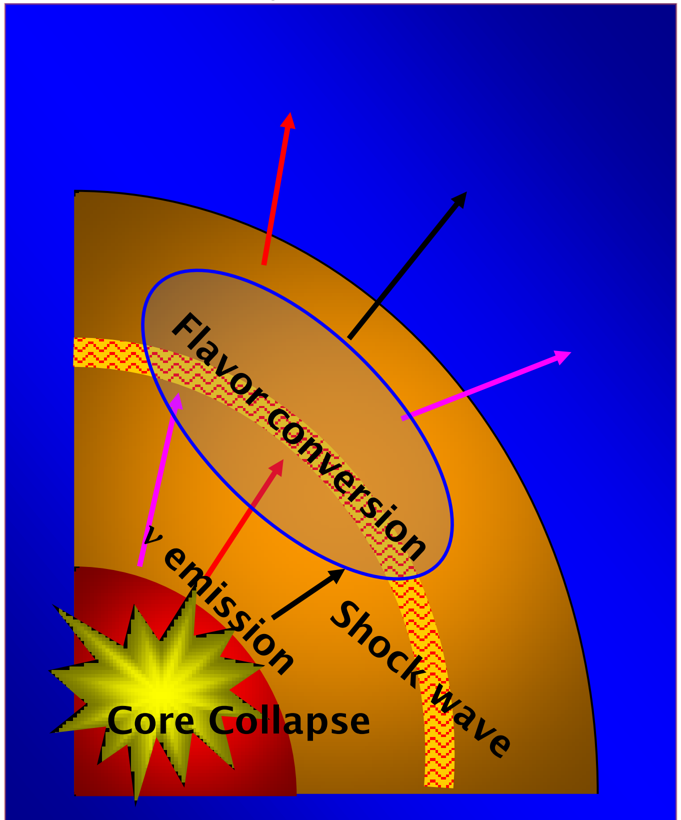
$\bar{\nu}_\mu \to \bar{\nu}_e$ $\qquad \nu_\mu \to \nu_e$

* Increase probability oscillations.
* Confidence level region is diminished by difference in σ_e between PQRPA and CRPA, PLB (2005) 100

Motivation

- **ENERGY SCALES:** 99% of the released energy (~ 10^{53} erg) is emitted by ν and $\underline{\nu}$ of all flavors
- **TIME SCALES:** Neutrino emission lasts **~10 s**
- **EXPECTED: 1-3 SN/century** in galaxy

  (d ≈ O (10) kpc).



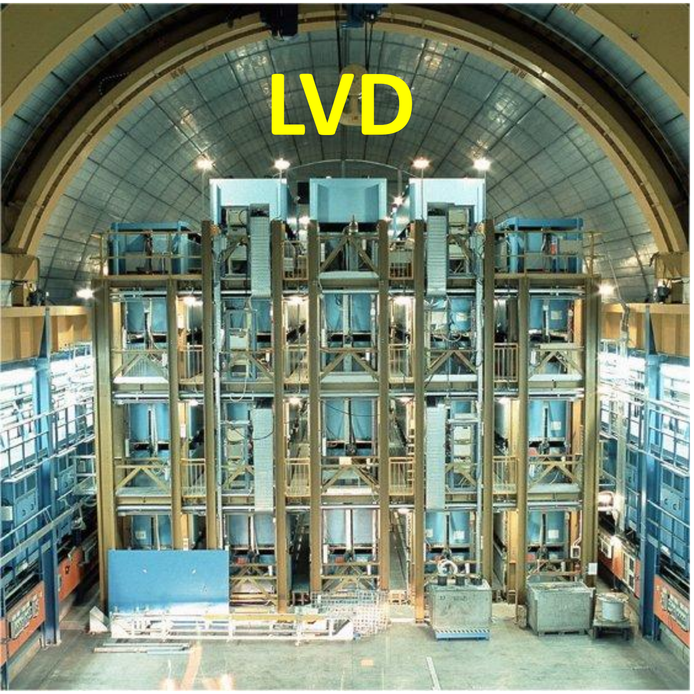
- Y.-Z Qian etal. PRC55, 1532, 1997, Neutrino-induced neutron spallation and supernova r-process nucleosynthesis. ``*r* process would then occur in an intense flux of neutrinos.''
- Y.-Z. Qian etal., Where, oh where has the *r*-process gone? It's a review of the possible sources for *r*-process nuclei (*r*-nuclei).

Supernovae Neutrinos – Detectors

Most of supernova neutrino detectors are designed primarily for other purpose:

- Proton decay search
- Solar and atmospheric neutrino physics
- Accelerator neutrino oscillation studies
- High energy neutrino source search

NFN Gran Sasso National Laboratory (Italy).
Main detection channel:



$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \rightarrow n + p \rightarrow D + \gamma$$

Charged current interaction

$$\nu_e + {}^{12}C \rightarrow {}^{12}N + e^-$$

$$\bar{\nu}_e + {}^{12}C \rightarrow {}^{12}B + e^+$$

$$\nu_e + {}^{56}Fe \rightarrow {}^{56}Co + e^-$$

$$\bar{\nu}_e + {}^{56}Fe \rightarrow {}^{56}Mn + e^+$$

Neutral current interaction

$$\nu_{(e,\mu,\tau)} + e^- \rightarrow \nu_{(e,\mu,\tau)} + e^-$$

$$\bar{\nu}_{(e,\mu,\tau)} + e^- \rightarrow \bar{\nu}_{(e,\mu,\tau)} + e^-$$

$$\nu_{(e,\mu,\tau)} + {}^{12}C \rightarrow {}^{12}C^* + \nu_{(e,\mu,\tau)}$$

$$\bar{\nu}_{(e,\mu,\tau)} + {}^{12}C \rightarrow {}^{12}C^* + \bar{\nu}_{(e,\mu,\tau)}$$

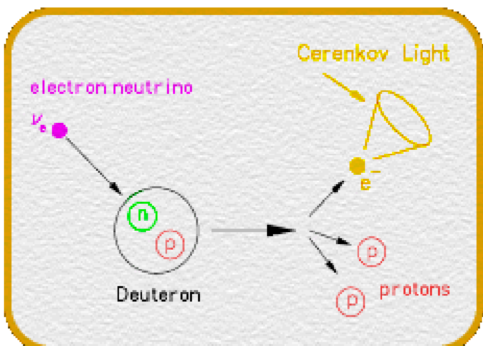
Supernovae Neutrinos – Signal Detection

Number of target nuclei **Neutrino flux** **Interaction cross section** **Efficiency**

$$N_{ev} = N_t \int_0^{\infty} F(E_\nu) \cdot \sigma(E_\nu) \cdot \varepsilon(E_\nu) dE_\nu$$

SNO

$$\nu_e + d \rightarrow p + p + e^-$$



Super-K

$$\nu_e + {}^{16}O \rightarrow {}^{16}F + e^-$$

$$\bar{\nu}_e + {}^{16}O \rightarrow {}^{16}N + e^+$$

$$\bar{\nu}_e + p \rightarrow n + e^+$$

LVD

$$\nu_e + {}^{12}C \rightarrow {}^{12}N + e^-$$

$$\bar{\nu}_e + {}^{12}C \rightarrow {}^{12}B + e^+$$

$$\nu_e + {}^{56}Fe \rightarrow {}^{56}Co + e^-$$

$$\bar{\nu}_e + {}^{56}Fe \rightarrow {}^{56}Mn + e^+$$

$$\sigma(E_\nu)$$

$$F(E_\nu)$$

Neutrino – nucleus interaction

$$\sigma(E_l, J_f) = \frac{p_l E_l}{2\pi} F(Z+1, E_l) \int_{-1}^{1} d(\cos\theta) T_\sigma(|\vec{k}|, J_f) \qquad H_W(\vec{r}) = \frac{G}{\sqrt{2}} J_\alpha l_\alpha e^{-i\vec{k}\cdot\vec{r}}$$

$$\mathrm{T}_\sigma(|\vec{k}|, J_f) \equiv \frac{1}{2J_i+1} \sum_{s_l s_\nu} \sum_{M_f M_i} \left| \left\langle J_f M_f \,|\, H_W \,|\, J_i M_i \right\rangle \right|^2 = \frac{G^2}{2J_i+1} \sum_{M_f M_i} \mathrm{O}_\alpha \mathrm{O}^*_\beta L_{\alpha\beta}$$

$\mathrm{O}_\alpha = \left\langle J_f \,\|\, J_\alpha e^{-i\vec{k}\cdot\vec{r}} \,\|\, J_i \right\rangle,$ Nuclear Matrix Element , Lepton traces $L_{\alpha\beta}$

$k = (\vec{k}, k_\varnothing), \rho = \kappa.r = |\vec{k}|.r$ Transfer momentum, with **k**= |**k|** ž.

$$J_\emptyset = g_V + (\overline{g}_A + \overline{g}_{P1})\boldsymbol{\sigma}\cdot\hat{\mathbf{k}} + i g_A \mathrm{M}^{-1}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla},$$

$$\mathbf{J} = -g_A\boldsymbol{\sigma} - i\overline{g}_W\boldsymbol{\sigma}\times\hat{\mathbf{k}} - \overline{g}_V\hat{\mathbf{k}} + \overline{g}_{P2}(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}} - i g_V \mathrm{M}^{-1}\boldsymbol{\nabla},$$

Hadronic current (non-relativistic)

FNS effect:

$$g \rightarrow g\left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)^2$$

Λ=850 MeV

Elementary Operators :

$$\mathcal{M}^V_J = j_J(\rho) Y_J(\hat{\mathbf{r}}),$$

$$\mathcal{M}^A_J = \kappa^{-1} j_J(\rho) Y_J(\hat{\mathbf{r}})(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}),$$

$$\mathcal{M}^A_{MJ} = \sum_L i^{J-L-1} F_{MLJ} j_L(\rho) \left[Y_L(\hat{\mathbf{r}}) \otimes \boldsymbol{\sigma}\right]_J,$$

$$\mathcal{M}^V_{MJ} = \kappa^{-1} \sum_L i^{J-L-1} F_{MLJ} j_L(\rho) [Y_L(\hat{\mathbf{r}}) \otimes \boldsymbol{\nabla}]_J$$

Neutrino – nucleus interaction

☺ For natural parity states with $\pi=(-)^J$, i.e., 0^+, 1^-, 2^+, 3^-….

$$\mathsf{O}_{\emptyset \mathsf{J}} = g_{\mathsf{V}} \mathcal{M}^{V}_{\mathsf{J}} \qquad \mathcal{M}^{V}_{\mathsf{J}} = j_{\mathsf{J}}(\rho) Y_{\mathsf{J}}(\hat{\mathbf{r}})$$

$$\mathsf{O}^{CVC}_{0\mathsf{J}} = \frac{k_\emptyset}{\kappa} g_{\mathsf{V}} \mathcal{M}^{V}_{\mathsf{J}} \qquad \tilde{\mathcal{M}}^{V}_{1\mathsf{J}} = \kappa^{-1} \frac{1}{\sqrt{2}} \left[\sqrt{\frac{J+1}{2J+1}} j_{\mathsf{J}-1}(\rho) \left[Y_{\mathsf{J}-1}(\hat{\mathbf{r}}) \otimes \boldsymbol{\nabla} \right]_{\mathsf{J}} - \sqrt{\frac{J}{2J+1}} j_{\mathsf{J}+1}(\rho) \left[Y_{\mathsf{J}+1}(\hat{\mathbf{r}}) \otimes \boldsymbol{\nabla} \right]_{\mathsf{J}} \right]$$

$$\mathsf{O}_{0\mathsf{J}} = 2\overline{g}_{\mathsf{V}} \mathcal{M}^{V}_{0\mathsf{J}} - \overline{g}_{\mathsf{V}} \mathcal{M}^{V}_{\mathsf{J}}$$

$$\mathsf{O}_{\mathsf{M}\neq 0\mathsf{J}} = (\mathsf{M} g_{\mathsf{A}} - \overline{g}_{\mathsf{W}}) \hat{\mathcal{M}}^{A}_{1\mathsf{J}} + 2\overline{g}_{\mathsf{V}} \tilde{\mathcal{M}}^{V}_{1\mathsf{J}} \qquad \hat{\mathcal{M}}^{A}_{1\mathsf{J}} = \frac{1}{\sqrt{2}} j_{\mathsf{J}}(\rho) \left[Y_{\mathsf{J}}(\hat{\mathbf{r}}) \otimes \boldsymbol{\sigma} \right]_{\mathsf{J}},$$

☺ For unnatural parity states with $\pi=(-)^{J+1}$, i.e., 0^-, 1^+, 2^-, 3^+….

$$-i\mathsf{O}_{\emptyset \mathsf{J}} = 2\overline{g}_{\mathsf{A}} \mathcal{M}^{A}_{\mathsf{J}} + (\overline{g}_{\mathsf{A}} + \overline{g}_{\mathsf{P1}}) \mathcal{M}^{A}_{0\mathsf{J}} \qquad \mathcal{M}^{A}_{\mathsf{J}} = \kappa^{-1} j_{\mathsf{J}}(\rho) Y_{\mathsf{J}}(\hat{\mathbf{r}}) (\boldsymbol{\sigma} \cdot \boldsymbol{\nabla})$$

$$-i\mathsf{O}_{0\mathsf{J}} = (\overline{g}_{\mathsf{P2}} - g_{\mathsf{A}}) \mathcal{M}^{A}_{0\mathsf{J}} \qquad \tilde{\mathcal{M}}^{A}_{1\mathsf{J}} = \frac{1}{\sqrt{2}} \left[\sqrt{\frac{J+1}{2J+1}} j_{\mathsf{J}-1}(\rho) \left[Y_{\mathsf{J}-1}(\hat{\mathbf{r}}) \otimes \boldsymbol{\sigma} \right]_{\mathsf{J}} - \sqrt{\frac{J}{2J+1}} j_{\mathsf{J}+1}(\rho) \left[Y_{\mathsf{J}+1}(\hat{\mathbf{r}}) \otimes \boldsymbol{\sigma} \right]_{\mathsf{J}} \right]$$

$$-i\mathsf{O}_{\mathsf{M}\neq 0\mathsf{J}} = (-g_{\mathsf{A}} + \mathsf{M}\overline{g}_{\mathsf{W}}) \tilde{\mathcal{M}}^{A}_{1\mathsf{J}} + 2\mathsf{M}\overline{g}_{\mathsf{V}} \hat{\mathcal{M}}^{V}_{1\mathsf{J}}$$

$$\mathcal{T}_{\sigma}(\kappa, J_f) = \frac{4\pi G^2}{2J_i+1} \sum_{\mathsf{J}} \Big[|\langle J_f || \mathsf{O}_{\emptyset \mathsf{J}} || J_i \rangle|^2 \mathcal{L}_{\emptyset} + \sum_{\mathsf{M}=0,\pm 1} |\langle J_f || \mathsf{O}_{\mathsf{MJ}} || J_i \rangle|^2 \mathcal{L}_{\mathsf{M}} - 2\Re\left(|\langle J_f || \mathsf{O}_{\emptyset \mathsf{J}} || J_i \rangle \langle J_f || \mathsf{O}_{0\mathsf{J}} || J_i \rangle \right) \mathcal{L}_{\emptyset 0} \Big].$$

$\mathcal{L}_{\emptyset}$ $\mathcal{L}_{\mathsf{M}}$ $\mathcal{L}_{\emptyset 0}$ Lepton Traces

(i) deForest Jr.& Walecka, Adv.Phys15, 1(1966)
(ii) Kuramoto etal. NPA 512, 711 (1990)
(iii) Luyten etal. NP41,236 (1963)(μ-capture)
(iv) Krmpotic etal. PRC71, 044319(2005).
≈ all are equivalents.

$$\mathsf{O}_{\emptyset \mathsf{J}} = \hat{\mathcal{M}}_{\mathsf{J}},$$

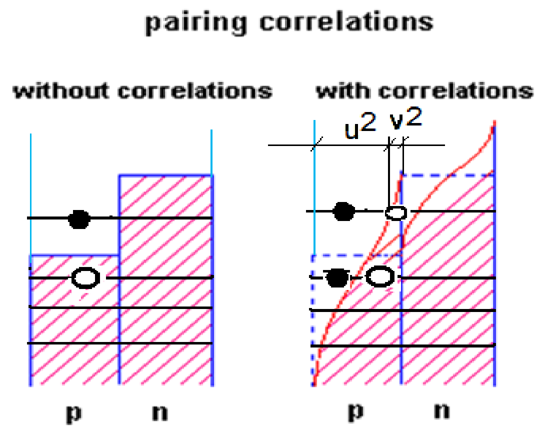
$$\mathsf{O}_{\mathsf{MJ}} = \begin{cases} \hat{\mathcal{L}}_{\mathsf{J}}, & \text{for} \quad \mathsf{M}=0 \\ -\frac{1}{\sqrt{2}} \left[\mathsf{M} \hat{\mathcal{T}}^{MAG}_{\mathsf{J}} + \hat{\mathcal{T}}^{EL}_{\mathsf{J}} \right], & \text{for} \quad \mathsf{M}=\pm 1 \end{cases}$$

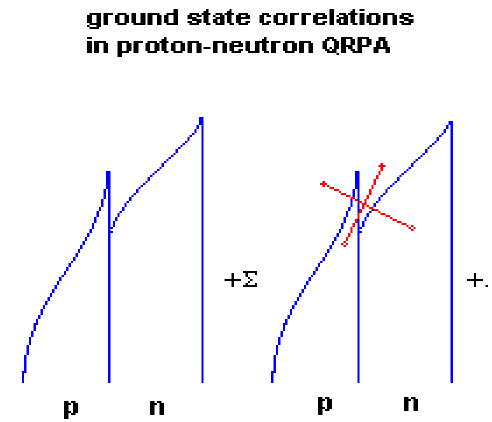
Nuclear Structure Models:

QRPA: Quasiparticle Random Phase Approximation

$$(e_t - \lambda_t)(u_t^2 - v_t^2) + u_t v_t \Delta_t = 0,$$

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ -Y \end{pmatrix},$$





$$\langle BCS|\hat{N}|BCS\rangle \equiv \sum_{t=n(p)} (2j_t+1) v_{j_t}^2 = N(Z),$$

PQRPA: Projected QRPA

$$2\hat{e}_p u_p v_p - \Delta_p (u_p^2 - v_p^2) \ = \ 0,$$

$$\begin{pmatrix} \mathcal{A}_\mu & \mathcal{B} \\ -\mathcal{B}^\dagger & -\mathcal{A}^*_{-\mu} \end{pmatrix} \begin{pmatrix} \mathcal{X}_\mu \\ \mathcal{Y}_\mu \end{pmatrix} = \Omega_\mu \begin{pmatrix} \mathcal{X}_\mu \\ \mathcal{Y}_\mu \end{pmatrix},$$

Particle number is conserved exactly .

Krmpotic etal. PLB319(1993)393.

$$V = -4\pi \left(v_s P_s + v_t P_t \right) \delta(r),$$

Numerical tool: **Quasiparticle Random APproximation**

QRAP code

Single particle States,. 1 to 6 ħϖ H.O.

$$(e_t-\lambda_t)(u_t^2-v_t^2)+u_t v_t \Delta_t = 0,$$

$$\left\langle BCS \,|\, \hat{N} \,|\, BCS \right\rangle = \sum_{t=n(p)} (2j_t+1) v_t^2 = N(Z)$$

QRPA

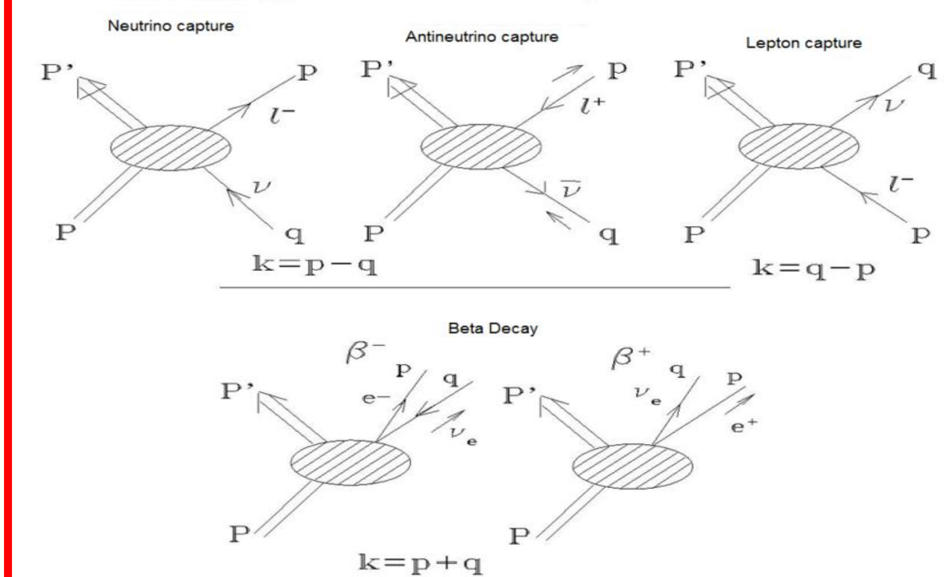
$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ -Y \end{pmatrix},$$

$$V = -4\pi\,(v_s P_s + v_t P_t)\,\delta(r),$$

PQRPA

$$2\hat{e}_p u_p v_p - \Delta_p(u_p^2 - v_p^2) \;=\; 0,$$

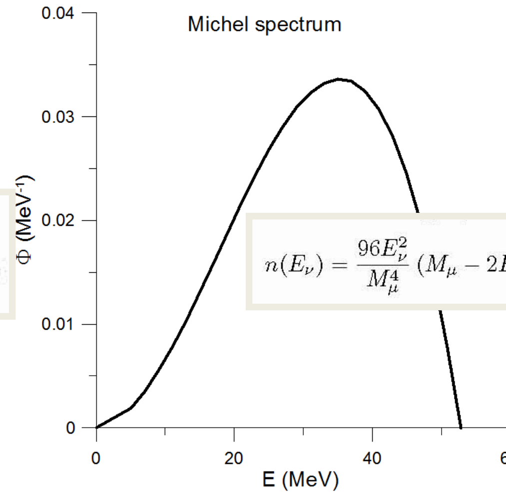
$$\begin{pmatrix} \mathcal{A}_\mu & \mathcal{B} \\ -\mathcal{B}^\dagger & -\mathcal{A}^*_{-\mu} \end{pmatrix} \begin{pmatrix} \mathcal{X}_\mu \\ \mathcal{Y}_\mu \end{pmatrix} = \Omega_\mu \begin{pmatrix} \mathcal{X}_\mu \\ \mathcal{Y}_\mu \end{pmatrix},$$



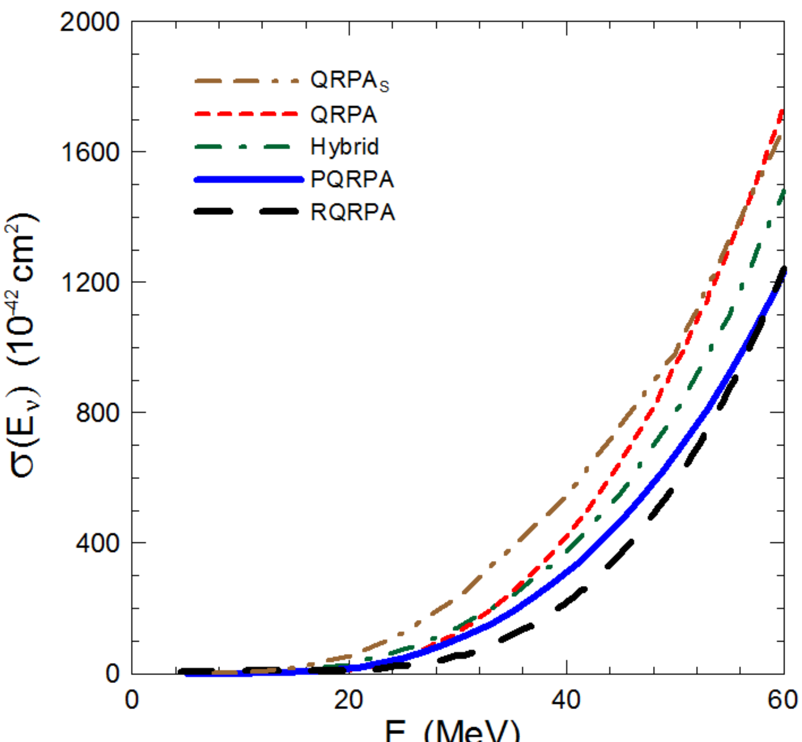
Neutrino-Iron CS

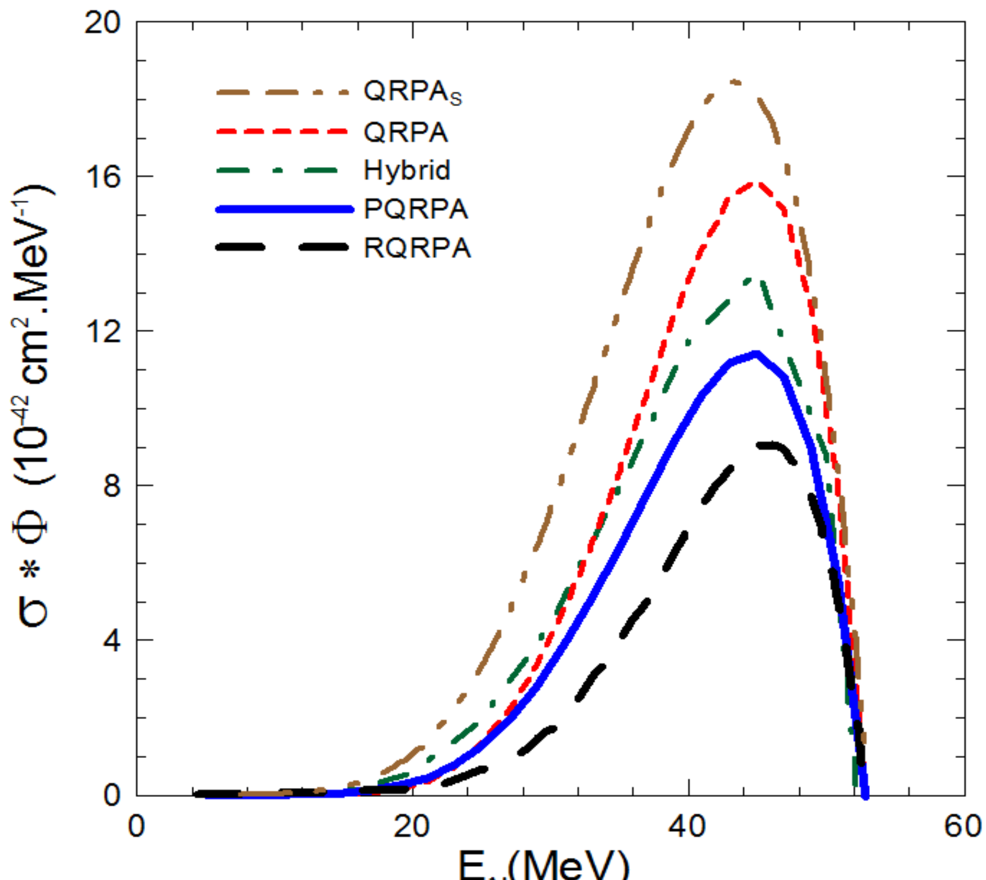
- ♣ 12 s.p. levels: 2, 3 and 4 $\hbar\varpi$,
- ♣ $3\hbar\varpi$, s.p.e of ^{56}Ni, 2&4 s.p.e. H.O.
- ♣ v^{pair}_s (p,n) to Δ(p,n) experimental.
- ♣ v^{ph}_s =24, v^{ph}_t =64,(MeV.fm^3) GT resonance in ^{48}Ca [NPA572,329(1994)].
- ♣ $t = 2\, v^{ph}_t / (v^{pair}_s(p) + v^{pair}_s(n)) = 0$, B(GT-) =17.7 ~ B(GT-)=18.68 Skyrme [NPA716,230(2003)] overestimates exp. 9.9±2.4 [NPA410,371(1983)].

$$\langle \sigma_e \rangle = \int dE_\nu \sigma(E_\nu) n(E_\nu),$$



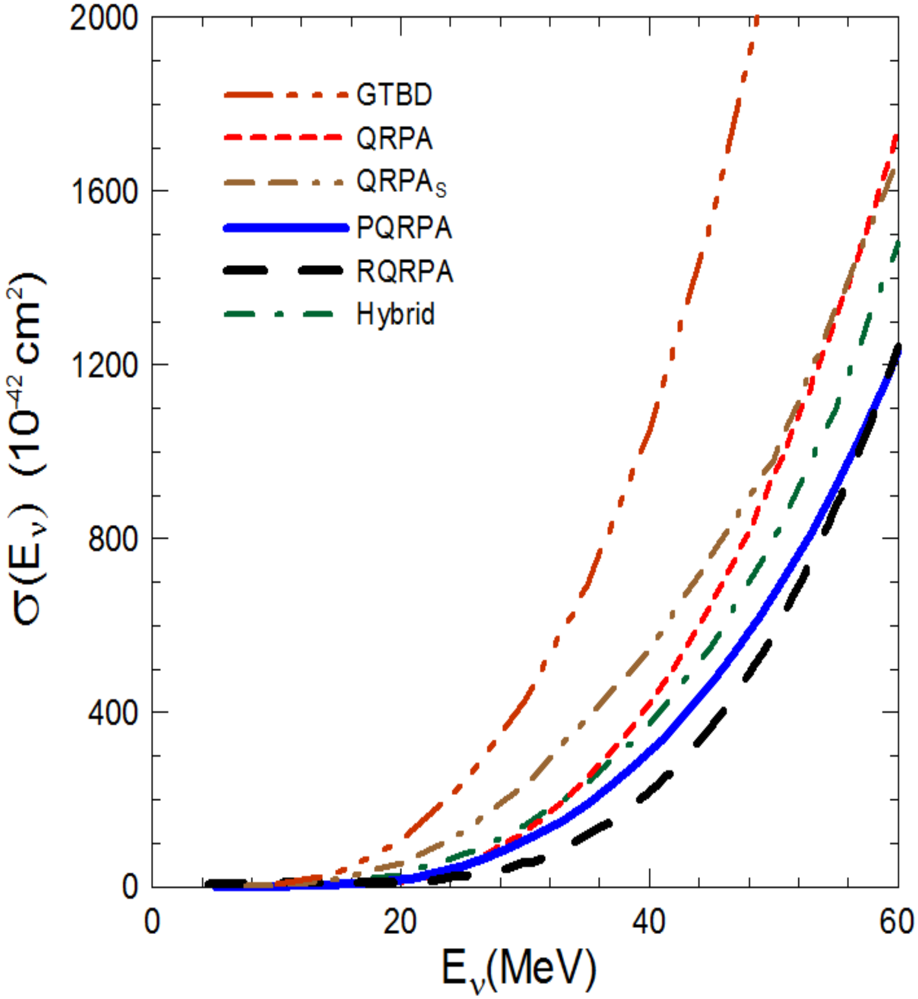
$$n(E_\nu) = \frac{96 E_\nu^2}{M_\mu^4}(M_\mu - 2E_\nu),$$





Neutrino-Iron CS

$$\sigma(E_\nu) \propto E_\nu^2$$



A.R. Samana and C.A. Bertulani, PRC78, 024312 (2008)

Model	$\langle \sigma_e \rangle$
QRPA	264.6
PQRPA	197.3
Hybrid$^{(a)}$ [14]	228.9
Hybrid$^{(b)}$ [14]	238.1
TM [26]	214
RPA [27]	277
QRPA$_S$ [15]	352
RQRPA [16]	140
Exp[5]	$256 \pm 108 \pm 43$

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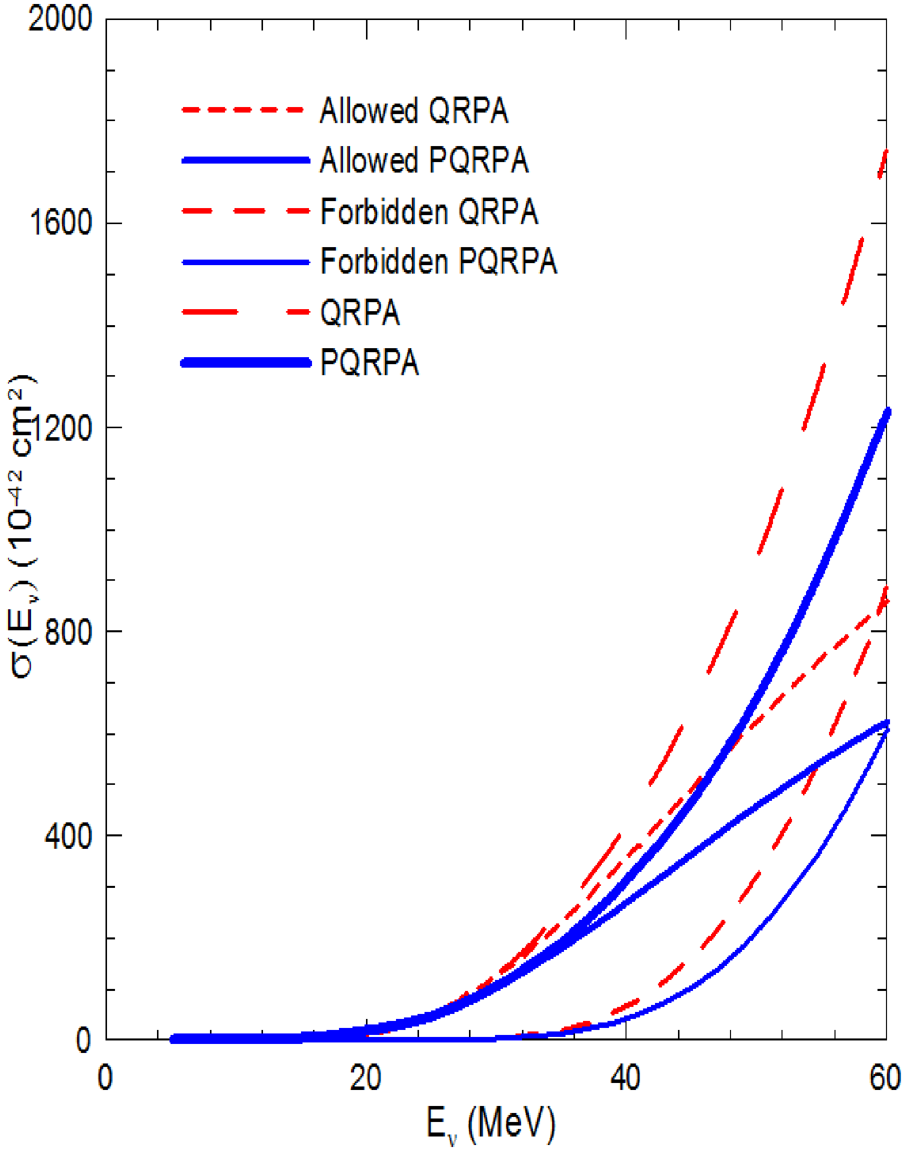
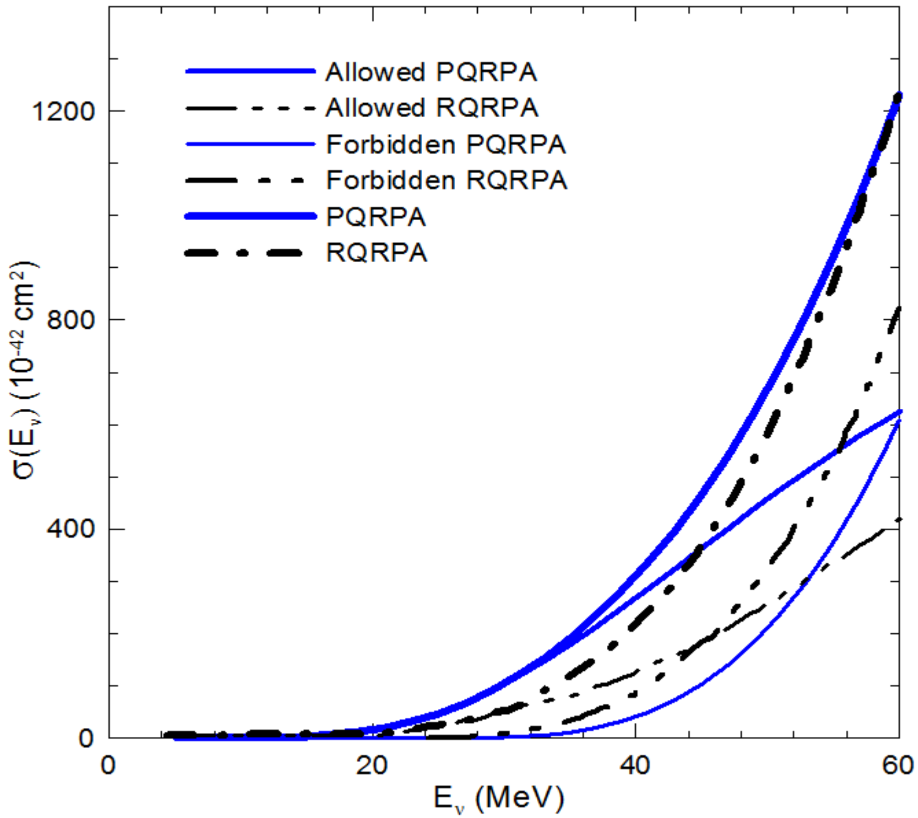
Neutrino-Iron CS (preliminary)



PQRPA & QRPA, PRC 78, 024312 (2008)

RQRPA DD-ME2 , PRC 77,024608 (2008)



Neutrino-Iron CS (preliminary)

RQRPA (N. Paar, private com. 05-29-09)

J^{π}	Hybrid[(a)]	Hybrid[(b)]	QRPA	PQRPA	RQPRA		
0^+	45.7	52.7	41.1	56.6			
1^+	112.9	112.1	172.7	108.1			
allowed	158.6	164.8	213.8	164.7	~ 78.3		
2^+	4.0	4.1	2.9	1.8			
3^+	4.4	4.2	2.5	1.4			
0^-	0.4	0.4	1.1	0.7			
1^-	29.3	29.4	20.7	14.0			
2^-	32.0	35.0	22.4	13.9			
3^-	0.2	0.2	1.2	0.8			
Forbidden	70.3	73.3	50.8	32.6	~ 61.4		
Total	228.9	238.1	264.6	197.3	140.0	360	257
	$g_A \sim 0.74$		$g_A = 1.0$		$g_A \sim 0.9$	g_A=1.262	g_A=1.0

256± 116
KARMEN

Supernovae Neutrinos – Neutrino-Fe cross section to estimate Events in supernova detectors.

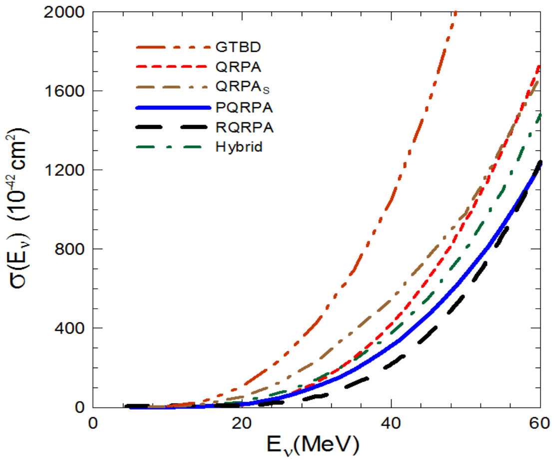
$$N_e \equiv N_e(T_{\nu_e}) = N_t \int_0^\infty F_e^0(E_\nu, T_{\nu_e})\sigma(E_\nu)\varepsilon(E_\nu)dE_\nu,$$

$$\tilde{N}_e \equiv \tilde{N}_e(T_{\nu_x}) = N_t \int_0^\infty F_x^0(E_\nu, T_{\nu_x})\sigma(E_\nu)\varepsilon(E_\nu)dE_\nu.$$

Effective temperature — Time-integrated energy — Neutrino energy

$$F_\alpha^0(E, T_\alpha, \eta_\alpha = 0, L_\alpha, D) = \frac{L_\alpha}{4\pi D^2 T_\alpha^4 F_3(0)} \frac{E^2}{e^{E/T_\alpha}+1},$$

Distance to supernova — Norm.factor — Pinching parameter



D~ 10 kpc,
$L_\alpha = E_b \cdot 1/6$,
$E_b = 3\times10^{53}$ erg,
$\alpha = \nu_\alpha = \{\nu_e, \nu_\mu, \nu_\tau\}$
$T(\nu_x)/T(\underline{\nu}_e) = 1.5$,
$T(\nu_e)/T(\underline{\nu}_e) = 0.8$,
$T(\underline{\nu}_e) = 5$ MeV

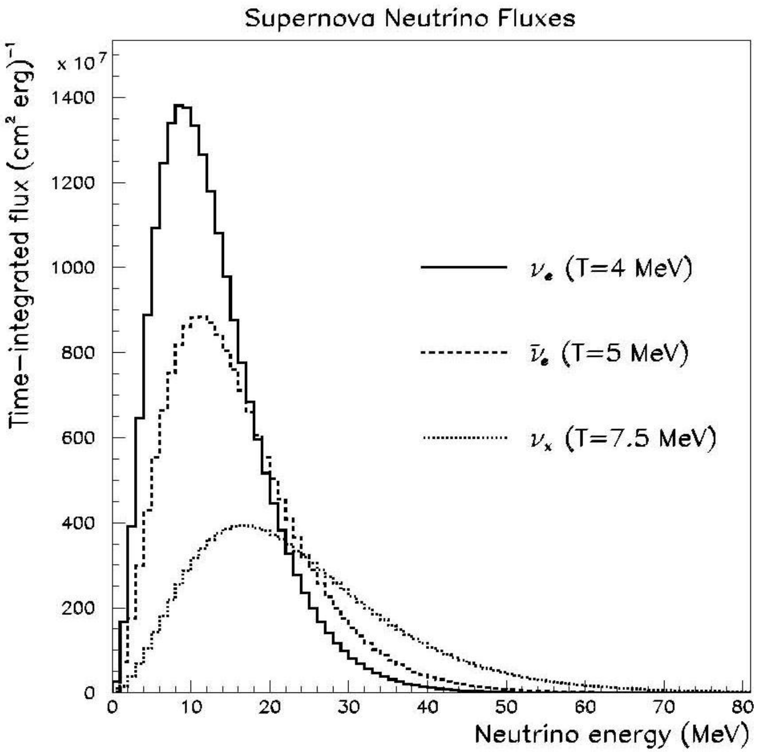
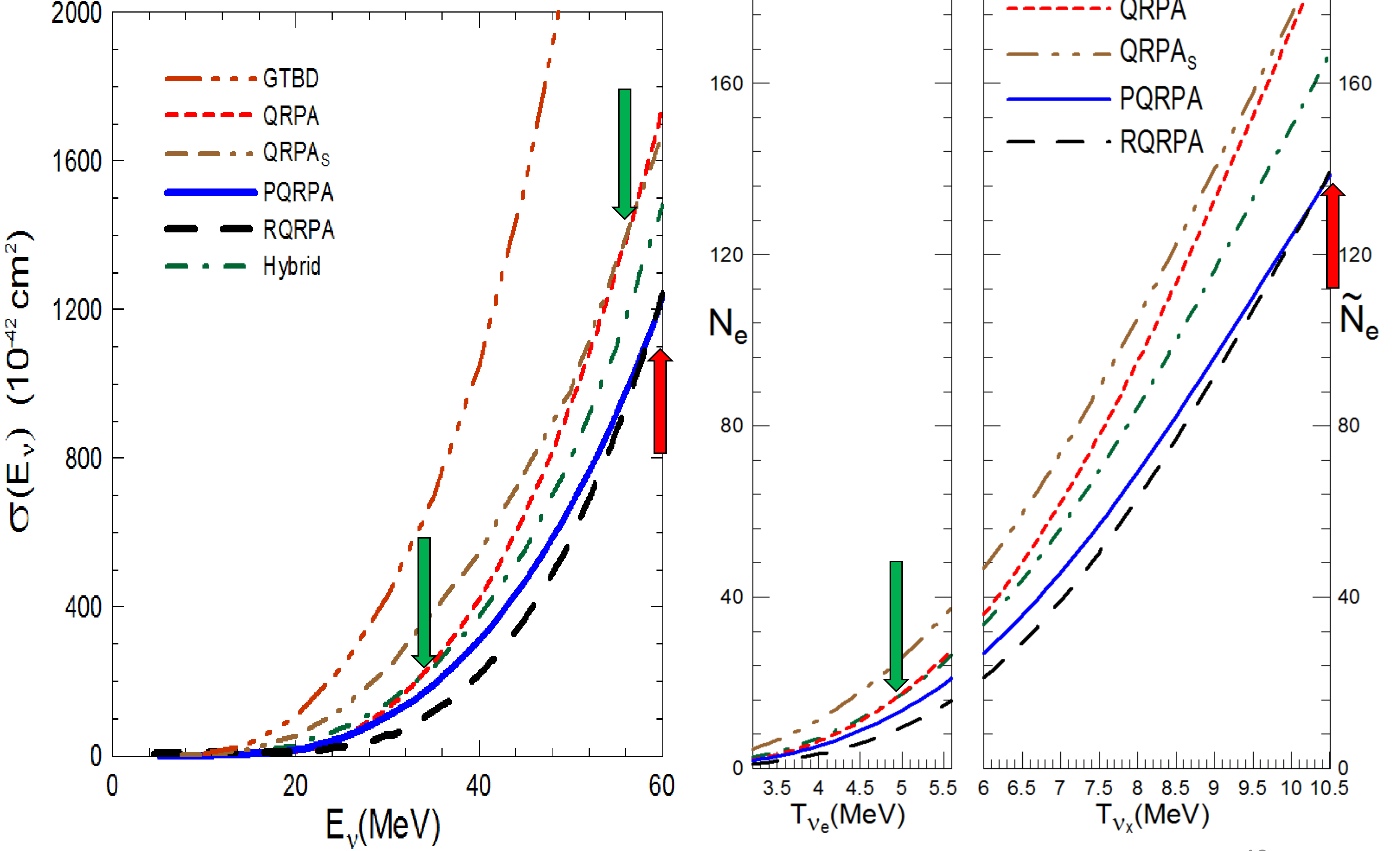


Fig. 1. Neutrino energy spectra at the neutrino-sphere.

Supernovae Neutrinos – Estimate Events

$$\nu_e + {}^{56}Fe \Rightarrow {}^{56}Co^* + e^-$$

Samana & Bertulani, PRC 78, 024312 (2008)



Next studies

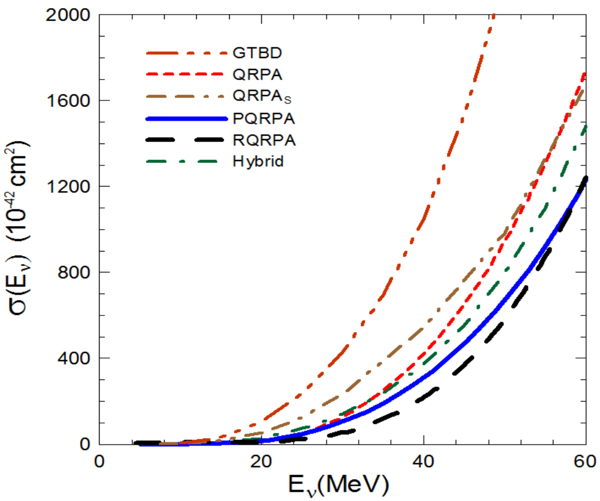
♣ LVD Collaboration, Astrop. Phys. 27 (2007)

Table 5

Fractional variations in the expected results if the $\bar{\nu}_e$ neutrino-sphere temperature is equal to 4 MeV (left value) or 7 MeV (right value), with respect to the chosen value 5 MeV

	No oscillation	Non adiabatic	Adiabatic NH	Adiabatic IH
$\bar{\nu}_e p$	−20%, +35%	−19%, +33%	−19%, +33%	−17%, +29%
$\langle E_{\bar{\nu}_e} \rangle$ in $\bar{\nu}_e p$	−19%, +37%	−19%, +36%	−19%, +36%	−19%, +37%
CC with ^{12}C	−56%, +164%	−44%, +94%	−43%, +85%	−41%, +79%
CC with ^{56}Fe	−55%, +193%	−51%, +164%	−51%, +161%	−49%, +152%
NC with ^{12}C	−40%, +77%	−40%, +77%	−40%, +77%	−40%, +77%

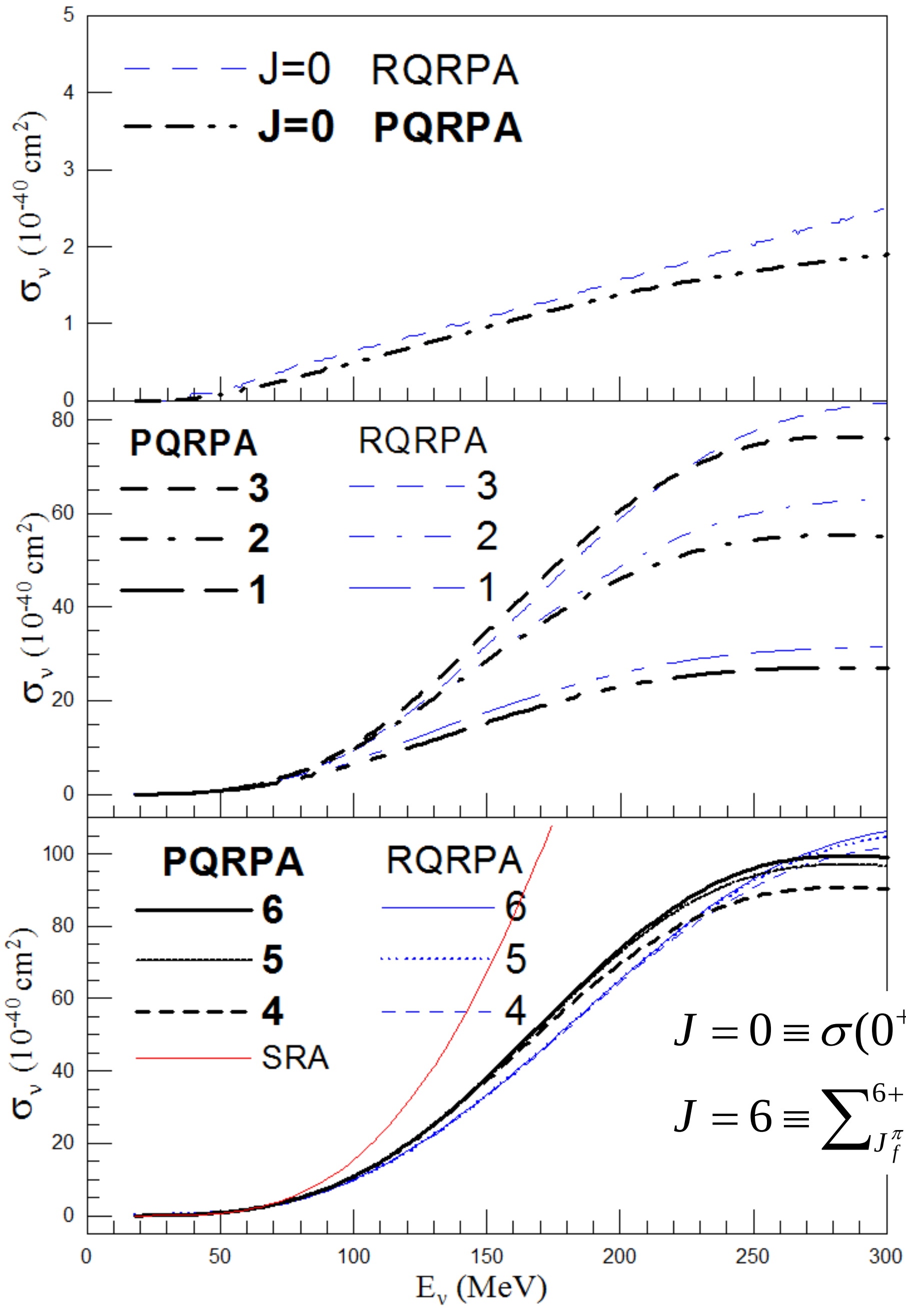
$$\nu_e + {}^{12}C \rightarrow {}^{12}N + e^-$$
$$\bar{\nu}_e + {}^{12}C \rightarrow {}^{12}B + e^+$$
$$\nu_e + {}^{56}Fe \rightarrow {}^{56}Co + e^-$$
$$\bar{\nu}_e + {}^{56}Fe \rightarrow {}^{56}Mn + e^+$$

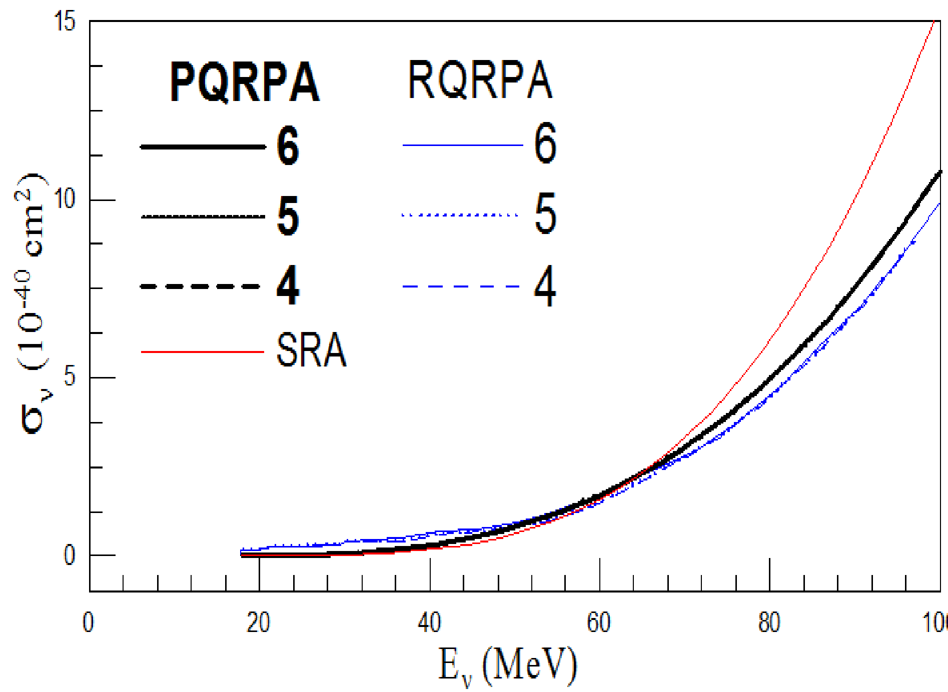


How much change these results with other cross sections?

Next studies

PQRPA (δ-force) and RQRPA (DD-ME2)





$$J = 0 \equiv \sigma(0^+) + \sigma(0^-),\ldots$$

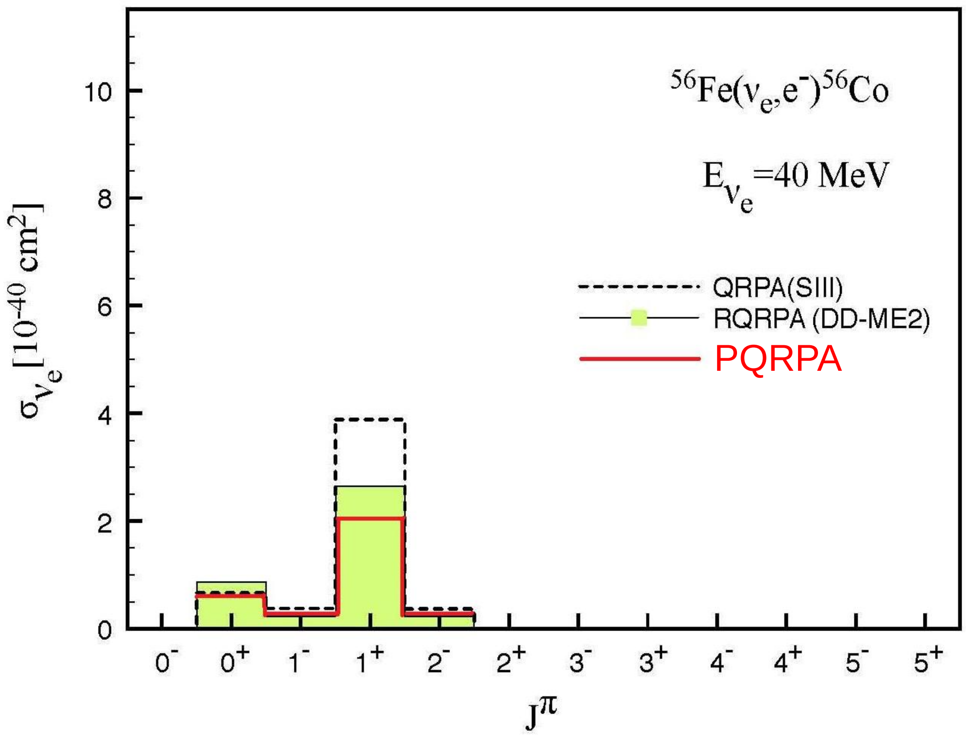
$$J = 6 \equiv \sum_{J_f^\pi = 0+,0-}^{6+,6-} \sigma(J_f^\pi)$$

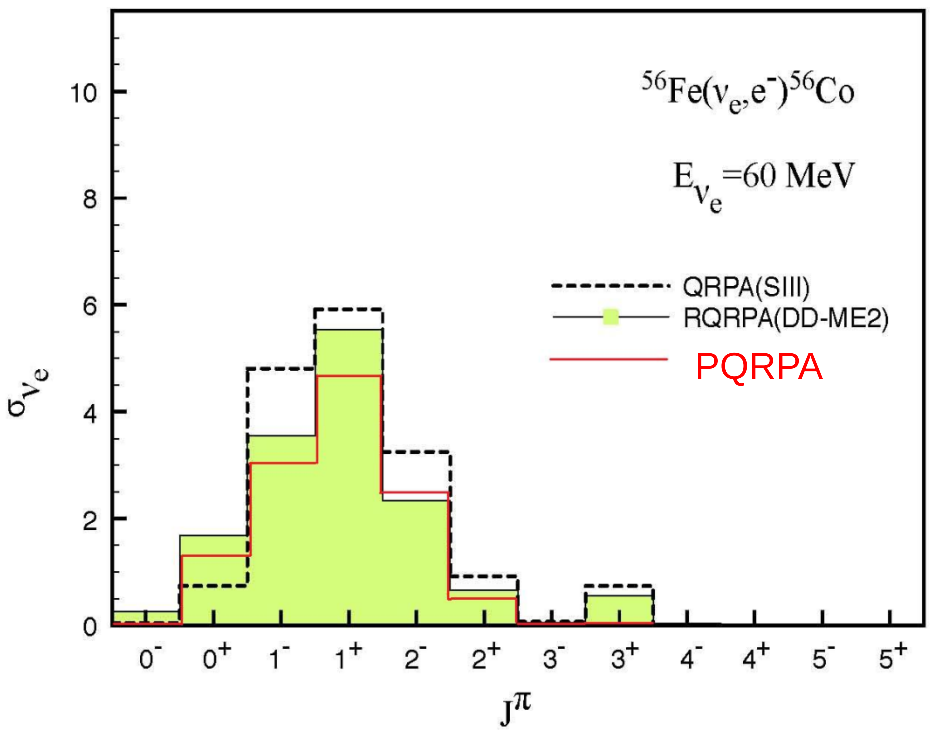
Krmpotic, Samana and Bertulani, arXiv:0808.1317(2008)

Next studies

PQRPA (δ-force) and RQRPA (DD-ME2)

PQRPA [PRC78, 024312(2008)]
RQRPA (N. Paar, private com. 05-29-09)





Summary

We show

- Importance of neutrino-nucleus cross section.
- Expected events are strongly dependent of Neutrino-iron CS & for characteristic supernovae $T\nu$ → interval $E\nu$ < 60 MeV.
- QRPA & PQRPA results show that particle number conservation is important in light and medium mass nuclei 56Fe.

We need

- Extension of PQRPA to charge conserving to evaluate neutral current , important for astrophysical applications.
- Check the energy-weight sum rules for different operators.
- Change the interaction.
- Increase the space in 56Fe.