Electromagnetic two-body currents of one- and two-pion range $*$

S. Pastore - Seminar @ INT

May 2009

$*$ with R. Schiavilla and J.L. Goity - PRC**78**, 064002 (2008)

Outline

- Status of the calculation
  - Currents from nuclear interactions
- Currents in χEFT: preliminaries
  - Time-ordered perturbation theory (TOPT)
  - Interaction Hamiltonians H_1 from χEFT $\mathscr{L}$
  - Power counting
- Calculation up to one loop (N^{3}LO)
  - Currents up to N^{3}LO
  - Technical issues: recoil corrections at N^{2}LO and N^{3}LO
  - Current conservation
- Electromagnetic observables at N^{2}LO (no loop): results
  - $A = 2, 3$ magnetic moments
  - *n*-*p*, *n*-*d* radiative capture cross sections at thermal neutron energies
- N^{3}LO calculation: preliminaries
  - EM currents up to N^{3}LO (without Δ's)
  - LEC's from NN data fitting

Section I

- Status of the calculation
  - Currents from nuclear interactions

Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

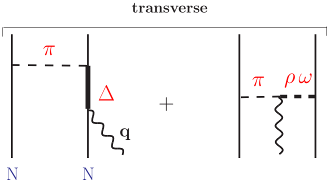
- Current operator **j** constructed so as to satisfy the continuity equation with a realistic Hamiltonian

$$\mathbf{q}\cdot\mathbf{j} = [H, \rho]_-$$

$$H = \sum_i K_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk}$$

υ and V realistic two- and three-body interactions

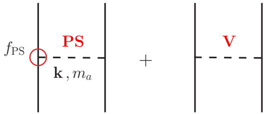
$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(\upsilon) + \ldots + \mathbf{j}^{(3)}(V)$$



- $\mathbf{j}^{(2)}$ and $\mathbf{j}^{(3)}$ derived from υ and V

Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

$v^{\mathrm{ME}} \quad = $

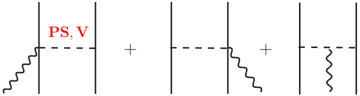


- Exploiting the meson exchange (ME) mechanism, one assumes that the static part υ_0 of υ is due to pseudoscalar (PS) and vector (V) exchanges
- υ^{ME} is expressed in terms of 'effective propagators' υ_{PS}, υ_V, υ_{VS}, fixed such to reproduce υ_0, for example

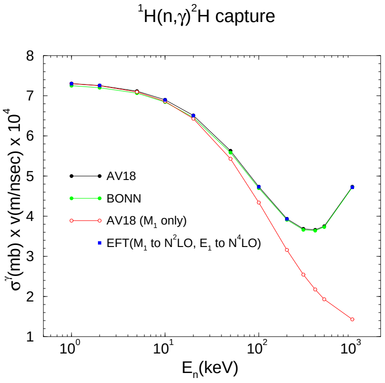
$$\upsilon_{PS} = [\upsilon^{\sigma\tau}(k) - 2\upsilon^{t\tau}(k)]/3$$

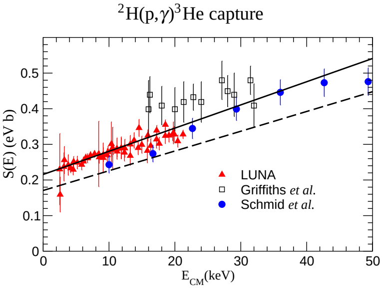
  with $\upsilon^{\sigma\tau}$ and $\upsilon^{t\tau}$ components of υ_0
- The current operator is obtained by taking the non relativistic reduction of the ME Feynman amplitudes and replacing the bare propagators with the 'effective' ones

$\mathbf{j}^{(2)}(v_0) \quad =$



Satisfactory description of a variety of nuclear EM properties [see Marcucci *et al.* (2005) and (2008)]





■ π̸ EFT - Rupak NPA**678**, 405 (2000)

... but ...

- Thermal neutron capture cross sections on ^{2}H and ^{3}He overpredicted by theory

σ	EXP (mb)	THEORY (mb)	
^{2}H(n,γ) ^{3}H	0.508 ± 0.015	0.556	Marcucci *et al.* (2005)
^{3}He(n,γ) ^{4}He	0.055 ± 0.003	0.086	Schiavilla *et al.* (1992)

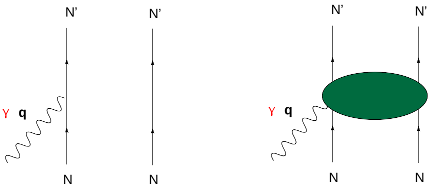
  - ▷ *n-d*, *p-d*, *n*-^{3}He, and *p*-^{3}H radiative captures are very sensitive to many-body terms in the electromagnetic current operators

$$\mu_z^{1-\text{body}} \mid \Psi(^3\text{H})\rangle \simeq \mu_p \mid \Psi(^3\text{H})\rangle \quad \longrightarrow \quad \langle\Psi(^3\text{H}) \mid \mu_z^{1-\text{body}} \mid \Psi(n,d)\rangle \simeq \mu_p \langle\Psi(^3\text{H}) \mid \Psi(n,d)\rangle = 0$$

$$\mu_z^{1-\text{body}} \mid \Psi(^4\text{He})\rangle \simeq 0 \quad \longrightarrow \quad \langle\Psi(^4\text{He}) \mid \mu_z^{1-\text{body}} \mid \Psi(n,{}^3\text{He})\rangle \simeq 0$$

- Isoscalar magnetic moments are a few % off (10% in A=7 nuclei)

- Currents in χEFT: preliminaries
  - Time-ordered perturbation theory (TOPT)
  - Interaction Hamiltonians H_1 from χEFT $\mathscr{L}$
  - Power counting



- 1-body: describes the current of a free nucleon
- 2-body: includes the effect of the NN interaction on the currents of a nucleon pair

EM current operator related to the transition amplitude via

$$T_{fi} = \langle N'N' \mid T \mid NN;\gamma\rangle|_{'\text{irreducible}'} = -\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\,\omega_q}} \cdot \mathbf{j}$$

Transition amplitude in TOPT

Relevant degrees of freedom:

- non relativistic nucleons (N)
- pions (π); mediators of the NN interaction at large interparticle distances
- non relativistic Delta-isobars (Δ)
  $m_\Delta \sim m_N + 2m_\pi$

Transition amplitude in time-ordered perturbation theory

$$T_{fi} = \langle N'N' \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid NN; \gamma \rangle$$

$H_0 =$ free π, N, Δ Hamiltonians
$H_1 =$ interacting π, N, Δ, γ Hamiltonians

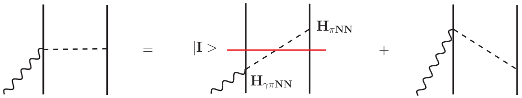
Transition amplitude in TOPT

In practice, insert complete sets of eigenstates of H_0 between successive terms of H_1

$$T_{fi} = \langle N'N' \mid H_1 \mid NN;\gamma\rangle + \sum_{|I\rangle} \langle N'N' \mid H_1 \mid I\rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid NN;\gamma\rangle + \ldots$$

The contributions to the T_{fi} are represented by time ordered diagrams

Example: seagull pion exchange current



N number of H_1's (vertices)
$\rightarrow$ N! time-ordered diagrams

H_1's are derived from the Chiral Effective Field Theory Lagrangians ($\mathscr{L}_{eff}$)

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- QCD is the underlying theory of strong interaction; on this basis π, N, and Δ interactions are completely determined by the underlying quark-gluon dynamics
- At low energies perturbative techniques (expansion in α_S) cannot be applied to solve QCD and we are far from a quantitative understanding of the low-energy physics by ab initio calculations from QCD
- χEFT exploits the χ symmetry exhibited by QCD at low energy to restrict the form of the interactions of pions among themselves and with other particles

χEFT Interaction Hamiltonians

- The pion couples by powers of its momentum $Q \rightarrow \mathscr{L}_{eff}$ can by systematically expanded in powers of Q/M
$$\mathscr{L}_{eff} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots$$
$M \sim 1$ GeV is the hard scale where χEFT will break down and characterizes the convergence of the expansion $\rightarrow$ we are limited to kinematic regions with $Q \ll M$
- χEFT allows for a perturbative treatment in terms of Q - as opposed to a coupling constant - expansion
- The coefficients of the expansion, Low Energy Constants (LEC's) are unknown and need to be fixed by comparison with exp data

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- χEFT allows for a perturbative treatment in terms of Q - as opposed to a coupling constant - expansion
- The coefficients of the expansion, Low Energy Constants (LEC's) are unknown and need to be fixed by comparison with exp data
- Due to the chiral expansion, T_{fi} can be expanded as

$$T_{fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N^2LO} + \ldots$$

$$T^{\rm NLO} \sim \frac{Q}{M} T^{\rm LO}$$

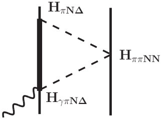
$$T^{\rm N^2LO} \sim \left(\frac{Q}{M}\right)^2 T^{\rm LO}$$

- The power counting scheme allows us to arrange the contributions of T_{fi} in powers of a small momentum Q

Each contribution to the T_{fi} scales as

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$

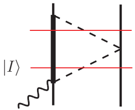
α_i = number of derivatives (momenta) in H_1
β_i = number of π's at each vertex
($Q^{-\beta_i/2}$ takes in account $\frac{1}{\sqrt{2\,\omega_\pi}}$ energy factor in the π field)



$$H_1\ \text{scaling} \sim \underbrace{Q^1 \times Q^{-1/2}}_{H_{\pi N\Delta}} \times \underbrace{Q^1 \times Q^{-1}}_{H_{\pi\pi NN}} \times \underbrace{e\,Q^0 \times Q^{-1/2}}_{H_{\pi\gamma N\Delta}} \sim e\,Q^0$$

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{e\,Q^0} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$

$N =$ number of vertices
$N-1 =$ number of intermediate states
$L =$ number of loops
(Q^{3L} takes in account $\int d^3Q$)



- Two energy denominators, each scaling as Q^{-1} in the static limit

$$E_N = m_N + \frac{p^2}{2m_N} \sim m_N \; ; \qquad E_\Delta = m_\Delta + \frac{p^2}{2m_\Delta} \sim m_\Delta$$
$$m_\Delta - m_N \sim Q$$
$$\omega_\pi \sim Q$$

$$\frac{1}{E_i - H_0}|I\rangle \sim \frac{1}{2m_N - (m_\Delta + m_N + \omega_\pi)}|I\rangle = -\frac{1}{m_\Delta - m_N + \omega_\pi}|I\rangle \sim \frac{1}{Q}|I\rangle$$

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{e\,Q^0} \times \underbrace{Q^{-(N-1)}}_{Q^{-2}} \times \underbrace{Q^{3L}}_{Q^3} = e\,Q^1$$

$N =$ number of vertices
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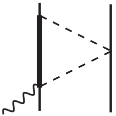

$\sim$ **e Q**

- Two energy denominators, each scaling as Q^{-1} in the static limit

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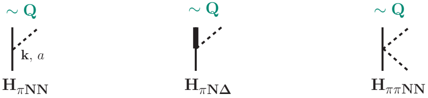
- This power counting also follows from considering Feynman diagrams, where loop integrations are in four dimensions



$$H_{\pi NN} = \frac{g_A}{F_\pi} \int d\mathbf{x} N^\dagger(\mathbf{x}) \left[\boldsymbol{\sigma} \cdot \nabla \pi_a(\mathbf{x})\right] \tau_a N(\mathbf{x}) \quad \longrightarrow \quad V_{\pi NN} = -i \frac{g_A}{F_\pi} \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\sqrt{2\,\omega_k}} \tau_a \sim Q^1 \times Q^{-1/2}$$

$$H_{\pi N\Delta} = \frac{h_A}{F_\pi} \int d\mathbf{x} \Delta^\dagger(\mathbf{x}) \left[\mathbf{S} \cdot \nabla \pi_a(\mathbf{x})\right] T_a N(\mathbf{x}) \quad \longrightarrow \quad V_{\pi N\Delta} = -i \frac{h_A}{F_\pi} \frac{\mathbf{S} \cdot \mathbf{k}}{\sqrt{2\,\omega_k}} T_a \sim Q^1 \times Q^{-1/2}$$

- $H_{\pi NN}$: $\left(\frac{m_\pi g_A}{F_\pi}\right)^2 \frac{1}{4\pi} = 0.075$ from Nijmegen analysis on NN scattering data
- $H_{\pi N\Delta}$: $h_A \sim 2.77$ fixed by reproducing the width of the Δ resonance

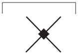
$\sim \mathbf{Q^0}$

$C_S,\ C_T$ — $H_{CT,1}$ | D_T — $H_{CT,2}$ | $C'_S,\ C'_T$ — $H_{CT,3}$ | D'_T — $H_{CT,4}$ | D''_T — $H_{CT,5}$

$$H_{\rm CT,5} = D''_T \int \mathrm{d}\mathbf{x}\, \left[\Delta^\dagger(\mathbf{x})\mathbf{S}\,T_a N(\mathbf{x})\right] \cdot \left[N^\dagger(\mathbf{x})\mathbf{S}^\dagger\, T_a^\dagger \Delta(\mathbf{x})\right]$$

- $H_{\rm CT,1}$: 4-nucleon contact terms, 2 LEC's
- $H_{\rm CT,2-5}$: contact terms involving one or two Δ's, 5 LEC's

$\sim \mathbf{Q^2}$

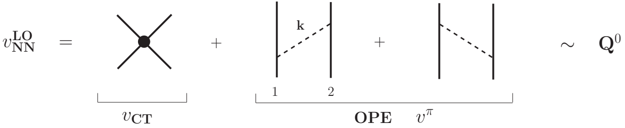


$C_i,\ i = 1, 14$

$H_{\rm CT2D}$

$$H_{\rm CT2D,2} = C'_2 \int \mathrm{d}\mathbf{x}\, \left[N^\dagger(\mathbf{x})\nabla N(\mathbf{x})\right] \cdot \left[\left[\nabla N(\mathbf{x})\right]^\dagger N(\mathbf{x})\right]$$

- $H_{\rm CT2D}$: 4-nucleon contact terms with two derivatives acting on N, 14 LEC's C_i

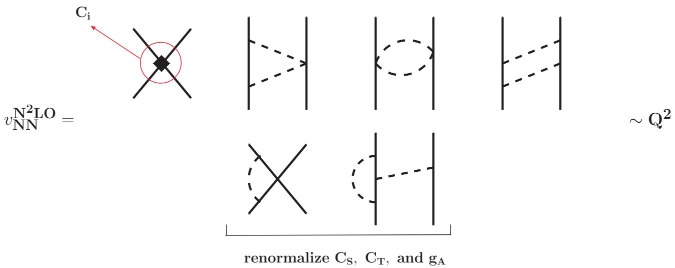


$$T_{fi}^{\mathrm{LO}} = \langle N'N' \mid H_{\mathrm{CT},1} \mid NN\rangle + \sum_{|I\rangle} \langle N'N' \mid H_{\pi NN} \mid I\rangle \frac{1}{E_i - E_I} \langle I \mid H_{\pi NN} \mid NN\rangle$$

Leading order NN potential in χEFT

$$\upsilon_{\mathrm{NN}}^{\mathrm{LO}} = C_S + C_T\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{F_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

χEFT NN potential at N^{2}LO (without Δ's)

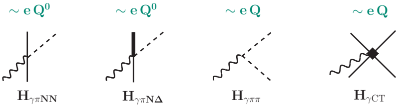


- At N^{2}LO there are 7 LEC's, C$_i$, fixed so as to reproduce NN scattering data
- Loop-integrals contain ultraviolet divergences reabsorbed into g_A, C$_S$, C$_T$, and C$_i$'s

EM Vertices

- EM H_1 obtained by minimal substitution in the π- and N-derivative couplings

$$\begin{aligned} \nabla \pi_{\mp}(\mathbf{x}) &\rightarrow [\nabla \mp ie\mathbf{A}(\mathbf{x})]\,\pi_{\mp}(\mathbf{x}) \\ \nabla N(\mathbf{x}) &\rightarrow [\nabla - iee_N\mathbf{A}(\mathbf{x})]\,N(\mathbf{x})\ , \qquad e_N = (1+\tau_z)/2 \end{aligned}$$



- EM H_1 of individual N's and Δ's obtained by non-relativistic reduction of the effective Hamiltonians (non-minimal couplings)

  - $H_{\gamma NN}$: $\mu_p = 2.793$ n.m. and $\mu_n = -1.913$ n.m. magnetic moments
  - $H_{\gamma N\Delta}$: $\mu^* \simeq 3$ n.m. from $\gamma N\Delta$ data

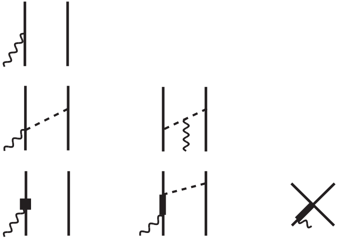
- Calculation up to one loop (N^3LO)
  - Currents up to N^3LO
  - Technical issues: recoil corrections at N^2LO and N^3LO
  - Current conservation

- Up to N^{2}LO

LO : eQ^{-2}

NLO : eQ^{-1}

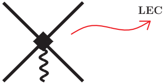
N^{2}LO : eQ^{0}



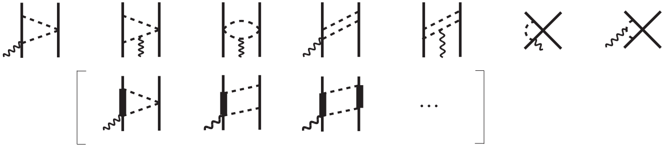
N^{2}LO – RC   N^{2}LO–Δ N^{2}LO–Δ_c

- One-loop corrections to the one-body current (absorbed into μ_N and $\langle r_N^2 \rangle$)

- Currents from $(NN)(NN)$ contact interactions with two gradients involving a number of LEC's (+ non-minimal terms)



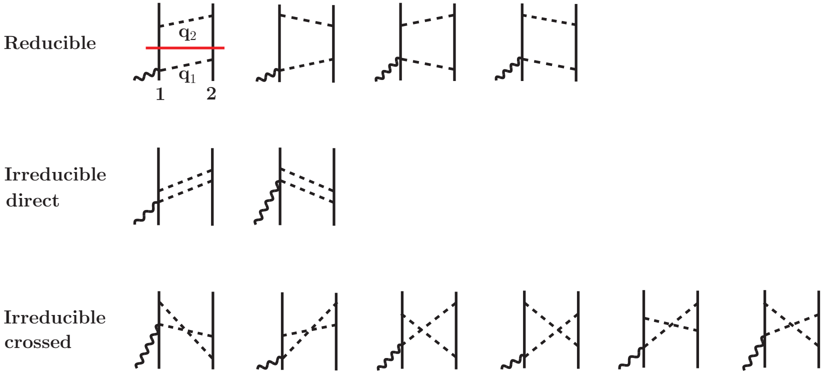
- One-loop corrections at N^{3}LO (eQ)



- One-loop renormalization of to the tree-level currents

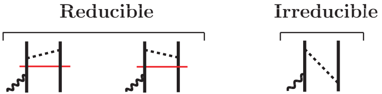
The box diagram: an example at N^{3}LO

4 interaction Hamiltonians $\longrightarrow$ 4! time ordered diagrams

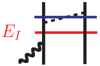


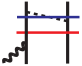
12 diagrams + (same with γ hooked up π with momentum $\mathbf{q}_2$) = 24

- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$ 

- Recoil corrections to the reducible contribution obtained by expanding in powers of E_N/ω_π the propagators



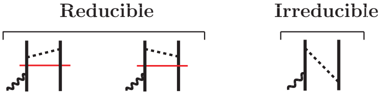

$$\simeq \upsilon^\pi \frac{1}{E_i - E_I} \mathbf{j}^{\mathrm{LO}} + \frac{\upsilon^\pi}{2\,\omega_\pi} \mathbf{j}^{\mathrm{LO}}$$


$$= -\frac{\upsilon^\pi}{2\,\omega_\pi} \mathbf{j}^{\mathrm{LO}}$$

$$|\Psi\rangle \simeq |\phi\rangle + \frac{1}{E_i - H_0} \upsilon^\pi |\phi\rangle + \dots$$

$$\langle \Psi_f | \mathbf{j}^{\mathrm{LO}} | \Psi_i \rangle \simeq \langle \phi_f | \mathbf{j}^{\mathrm{LO}} | \phi_i \rangle + \langle \phi_f | \upsilon^\pi \frac{1}{E_i - H_0} \mathbf{j}^{\mathrm{LO}} + \mathrm{h.c.} | \phi_i \rangle + \dots$$

- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$ 

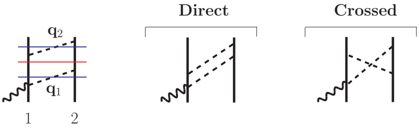
- Recoil corrections to the reducible contribution obtained by expanding in powers of E_N/ω_π the propagators

$$\simeq \; v^\pi \frac{1}{E_i - E_I}\,\mathbf{j}^{\rm LO} + \frac{v^\pi}{2\,\omega_\pi}\,\mathbf{j}^{\rm LO}$$

$$= \; -\frac{v^\pi}{2\,\omega_\pi}\,\mathbf{j}^{\rm LO}$$

- Recoil corrections to the reducible diagrams cancel irreducible contribution

$$\mathbf{j}^{\mathrm{N^3LO}} \quad = $$



- Reducible contributions

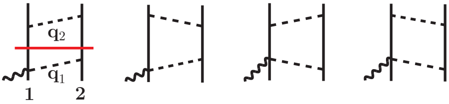
$$\begin{aligned}\mathbf{j}_{\mathrm{red}} \quad &\sim \quad \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1) \\ &- \quad \int 2\,\frac{\omega_1 + \omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)\end{aligned}$$

- Irreducible contributions

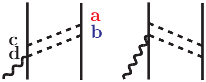
$$\begin{aligned}\mathbf{j}_{\mathrm{irr}} \quad &= \quad \int 2\,\frac{\omega_1 + \omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1) \\ &- \quad \int 2\,\frac{\omega_1^2 + \omega_2^2 + \omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1 + \omega_2)}[V_{\pi NN}(2,\mathbf{q}_2), V_{\pi NN}(2,\mathbf{q}_1)]_-\,V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)\end{aligned}$$

- Observed partial cancellations at N^{3}LO between recoil corrections to reducible diagrams and irreducible contributions (valid at all orders?)

Reducible



Irreducible direct

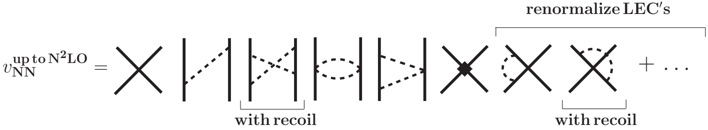


Irreducible crossed

$$\begin{aligned} \text{direct} &= f_d(\omega_1,\omega_2)\, V_a\, V_b\, V_c\, V_d \\ \text{crossed} &= f_c(\omega_1,\omega_2)\, V_b\, V_a\, V_c\, V_d \end{aligned} \qquad V_b V_a = V_a V_b - [V_a, V_b]_-$$

$$\begin{aligned} \text{irreducible} &= [f_d(\omega_1,\omega_2)+f_c(\omega_1,\omega_2)]V_a\, V_b\, V_c\, V_d \\ &- f_c(\omega_1,\omega_2)[V_a, V_b]_-\, V_c\, V_d \end{aligned}$$

- Potential at N^{2}LO inχEFT (without Δ's)



- It is in agreement with that obtained by the method of unitary transformations [Epelbaum *et al.*, NPA**637**, 107 (1998)]
- Retaining recoil corrections in both $\upsilon_{\rm NN}^{\chi\rm EFT}$ and $\mathbf{j}$ ensures current conservation up to N^{3}LO included

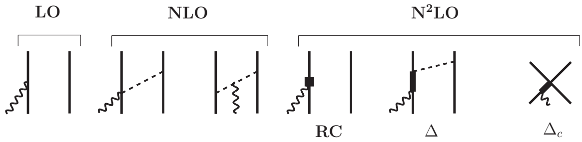
$$\mathbf{q}\cdot\mathbf{j} = \left[\frac{p_1^2}{2\,m_{\rm N}} + \frac{p_2^2}{2\,m_{\rm N}} + \upsilon_{\rm NN}^{\chi\rm EFT}\,,\,\rho\right]_-$$

- In hybrid calculations, the continuity equation is not strictly satisfied . . .

$$\upsilon_{\rm NN}^{\rm realistic} = \upsilon_{\rm NN}^{\chi\rm EFT} + \underbrace{\upsilon_{\rm NN}^{\rm realistic} - \upsilon_{\rm NN}^{\chi\rm EFT}}_{\text{higher order than N}^2\text{LO}}$$

- Electromagnetic observables at N^{2}LO (no loop): results
  - $A = 2, 3$ magnetic moments
  - *n-p*, *n-d* radiative capture cross sections at thermal neutron energies

$$\mathbf{j}^{\text{up to N}^2\text{LO}} =$$



- The calculation of EM observables is carried out in r-space
  $\longrightarrow$ we need configuration-space representation of the current operators
- At NLO and N^{2}LO the operator present $1/r^2$ and $1/r^3$ singularities
  $\longrightarrow$ regularize them by introducing a momentum cutoff

$$C_\Lambda(p) = \mathrm{e}^{-(p/\Lambda)^2}, \qquad \Lambda \leq M$$

- Λ=(500–800) MeV

- Hybrid approach

The current operator is used in transition matrix elements between w.f.'s obtained from realistic Hamiltonians with two- and three-body potentials

▷ $A = 2$ w.f.'s from AV18 or CDB potentials[1]

  - long-range NN interaction via OPE
  - fitted to reproduce NN scattering data
  - reproduce *d* properties

▷ $A = 3$ w.f.'s (HH) from AV18-UIX or CDB-UIX* potentials[2]

  - reproduce ^{3}H binging energy and a variety of N-*d* scattering data

[1] R.B. Wiringa *et al.* PRC**51**, 38 (1995); R.Machleidt PRC**63**, 024001 (2001)

[2] A. Kievsky *et al.* JPG**35**, 063101 (2008); B.S. Pudliner *et al.* PRC**56**, 1720 (1997)

- Isoscalar observables
  - μ_d deuteron magnetic moment
  - μ_S isoscalar combination of the trinucleon magnetic moments

$$\mu_S = \frac{1}{2}\left[\mu(^3\text{He}) + \mu(^3\text{H})\right]$$

Isoscalar currents :

- d magnetic moment (μ_d) and isoscalar combination (μ_S) of ^{3}H/^{3}He magnetic moments

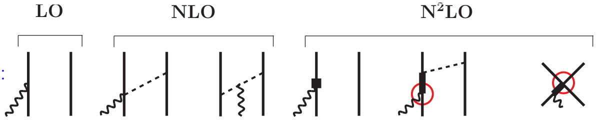
	μ_d (n.m.)		μ_S (n.m.)	
	AV18	CDB	AV18/UIX	CDB/UIX*
LO	+ 0.8469	+ 0.8521	+ 0.4104	+ 0.4183
N^{2}LO-RC	– 0.0082	– 0.0080	– 0.0045	– 0.0052
EXP	+0.8574		+0.426	

- N^{2}LO contribution (cutoff Λ independent) is 1% of LO, but of opposite sign

- Isovector observables
  - $n + p \rightarrow d + \gamma$ cross-section at thermal neutron energies, $\upsilon_n \sim 2200$ m/s
  - μ_V isovector combination of the trinucleon magnetic moments

$$\mu_V = \frac{1}{2}\left[\mu(^3\text{He}) - \mu(^3\text{H})\right]$$

Isovector currents :



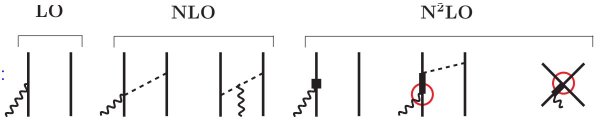
$C_\Delta = f\mu^* \frac{h_A g_A}{F_\pi^2}$ $C_{\Delta_C} = f'\mu^* D_T$

- ^{1}H$(n,\gamma)^2$H x-section
  - N^{2}LO-Δ_c gives no contribution to the x-section, ($\mu_{\Delta_c}^{N^2LO}|\Psi_{NN}; L = \text{even}\rangle = 0$)
  - C_Δ : h_A from Δ width, μ^* from $N\gamma$ data

	m.e. (mb$^{1/2}$)
	AV18
Λ (MeV)	600
LO	17.45
NLO	+ 0.42
N^{2}LO-RC	– 0.05
N^{2}LO-Δ	+ 0.16
Sum	17.99
EXP	18.24

- x-section is underpredicted by ~ 2.5 %: fix $C_\Delta(\Lambda)$ by reproducing the exp values of ^{1}H$(n,\gamma)^2$H x-section

Isovector currents :



$C_{\Delta} = f \mu^* \frac{h_A g_A}{F_\pi^2}$ $C_{\Delta_C} = f' \mu^* D_T$

- μ_V in ^{3}H/^{3}He
  - with C_Δ (Λ) fixed to reproduce $\sigma(np \rightarrow d\gamma)$

	μ_V(n.m.)
	AV18
Λ (MeV)	600
LO	– 2.159
NLO	– 0.197
N^{2}LO-RC	+ 0.029
N^{2}LO-Δ	– 0.253
Sum	– 2.580
EXP	– 2.533

$f = 1$ → – 0.102

- N^{2}LO correction larger than NLO
- μ_V underpredicted by ~ 2 %
- N^{2}LO current completely determined with C_{Δ_c} (Λ) fixed by reproducing μ_V

- $n + d \rightarrow {}^3\mathrm{H} + \gamma$ cross-section (σ_T) at thermal neutron energies
- $\vec{n} + d \rightarrow {}^3\mathrm{H} + \gamma$ photon circular polarization factor R_c

$\mathbf{P}_N$ = neutron polarization
P_γ = photon circular polarization

$$P_\Gamma = \frac{\sigma(\mathbf{P}_N, P_\gamma = 1) - \sigma(\mathbf{P}_N, P_\gamma = -1)}{2\,\sigma_T} = R_c\,\mathbf{P}_N \cdot \hat{\mathbf{q}}$$

^{2}H(n,γ)^{3}H and ^{2}H($\vec{n}$,γ)^{3}H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

- N^{2}LO theory underpredicts exp
- Strong Λ-dependence
- Expected chiral convergence not observed

^{2}H$(n,\gamma)^3$H and ^{2}H$(\vec{n},\gamma)^3$H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

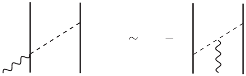
- LO $<$ 50% exp: 1-body currents suppressed due to pseudo-orthogonality between initial final and states (well known)

$$\mu_z^{\rm LO} \mid \Psi(^3{\rm H})\rangle \simeq \mu_p \mid \Psi(^3{\rm H})\rangle \rightarrow \langle\Psi(^3{\rm H}) \mid \mu_z^{\rm LO} \mid \Psi(n,d)\rangle \simeq \mu_p \langle\Psi(^3{\rm H}) \mid \Psi(n,d)\rangle$$

^{2}H$(n,\gamma)^3$H and ^{2}H$(\vec{n},\gamma)^3$H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO (seagull only)	0.425			– 0.425		
LO+NLO (full)	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

- NLO seagull- and in-flight-contributions nearly cancel out



Predictions for *n*-*d* capture at thermal energy

^{2}H$(n,\gamma)^3$H and ^{2}H$(\vec{n},\gamma)^3$H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

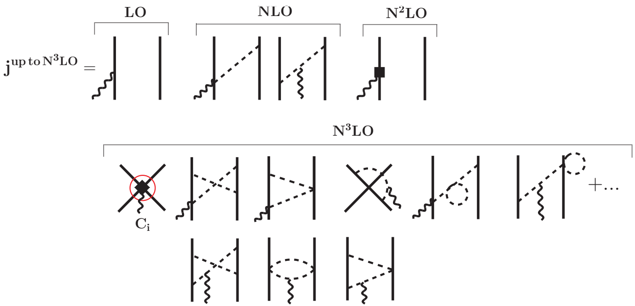
- N^{2}LO theory $\sim$ 25% smaller than exp: strong Λ-dependence
  - mainly due to short-range behavior of N^{2}LO-Δ_c contact current governed by a Gaussian of half-width $2/\Lambda$
  - Stable, cut-off independent results, when contact terms are included [Song *et al.* arXiv:0812.3834]

^{2}H$(n,\gamma)^3$H and ^{2}H$(\vec{n},\gamma)^3$H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

- N^{2}LO theory $\sim$ 25% smaller than exp: strong Λ-dependence
  - mainly due to short-range behavior of N^{2}LO-Δ_c contact current governed by a Gaussian of half-width $2/\Lambda$
  - Stable, cut-off independent results, when contact terms are included [Song *et al.* arXiv:0812.3834]
- N^{2}LO contribution much larger than NLO
  - NLO cancellations
  - N^{2}LO makes up for missing loop corrections at N^{3}LO

- N^{3}LO calculation: preliminaries
  - EM currents up to N^{3}LO (without Δ's)
  - LEC's from NN data fitting



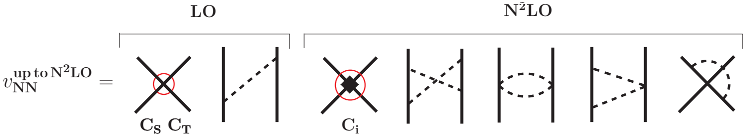
- Derive magnetic moment operators generated by two-pion-exchange currents via

$$\boldsymbol{\mu}(\mathbf{R},\mathbf{k}) = -\frac{i}{2}\left[e\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\,\mathbf{R}\times\nabla_k\,\upsilon^{2\pi}(\mathbf{k}) + \nabla_q\times\mathbf{j}^{2\pi}(\mathbf{q},\mathbf{k})\,|_{\mathbf{q}=0}\right]$$

where $\upsilon^{2\pi}(\mathbf{k})$ is the Fourier transform of $V^{2\pi}(\mathbf{r})$

$$V^{TPE} = \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,V^{2\pi}(\mathbf{r})$$

- Potential at N^{2}LO in χEFT (without Δ's) $*$

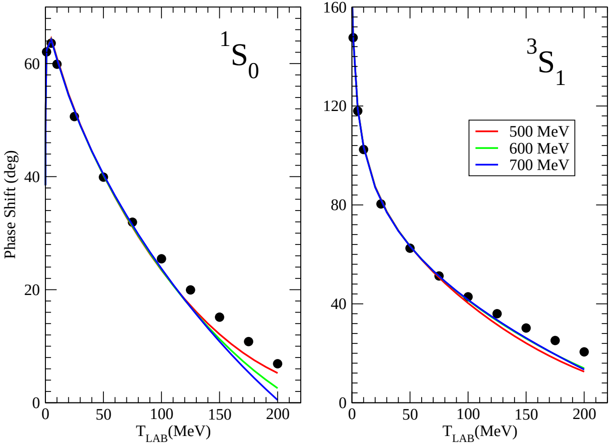


- Fix LEC's
- Obtain w.f.'s for a self-consistent calculation

$*$ with R.B. Wiringa

N^3LO calculation (without Δ's): preliminaries

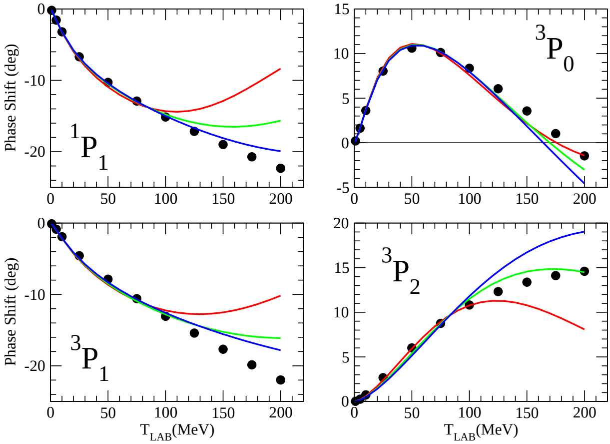
Phase shifts in S-wave *



* F.Gross and A.Stadler PRC **78**, 104405 (2008)

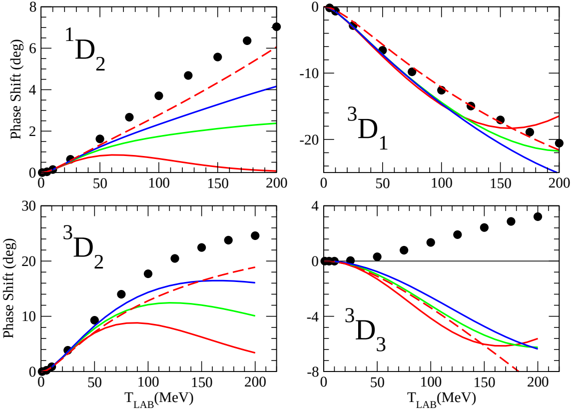
N^3LO calculation (without Δ's): preliminaries

Phase shifts in P-wave *



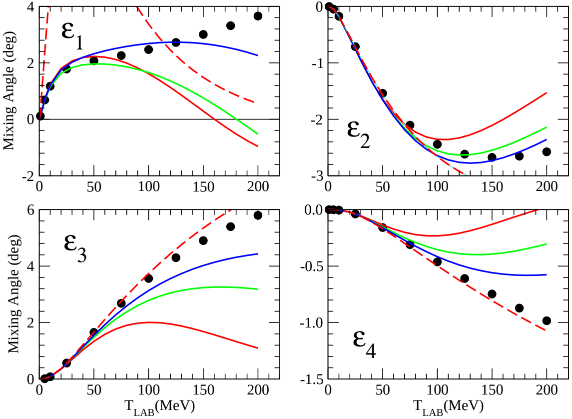
* F.Gross and A.Stadler PRC **78**, 104405 (2008)

Phase shifts in D-wave ∗



∗ F.Gross and A.Stadler PRC **78**, 104405 (2008)

Mixing angle *



* F.Gross and A.Stadler PRC **78**, 104405 (2008)

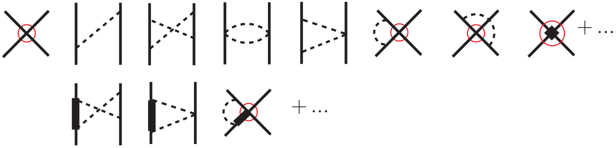
Summary

- Currents up to N^{3}LO have been derived in χEFT
- Currents up to N^{2}LO have been determined by reproducing exp values of the ^{1}H$(n,\gamma)^2$H x-section and μ_V
- At N^{2}LO the ^{2}H$(n,\gamma)^3$H x-section and R_c are unpredicted by theory
- A strong cutoff dependence has been observed

Outlook

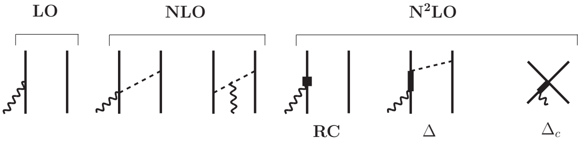
- Incorporate the N^3LO operators into the calculations of the captures and the magnetic moments of light nuclei ($A < 8$)
- Include contributions involving Δ's up to N^3LO (fix LEC's from the corresponding NN potential)

$v_{NN}^{\chi EFT} =$



- N^3LO 3-body currents also need to be derived

$\mathbf{j}^{\text{up to N}^2\text{LO}} =$



$$\mathbf{j}^{\rm LO} = \frac{e}{2\,m_N}\left[2\,e_{N,1}\,\mathbf{K}_1 + i\,\mu_{N,1}\,\boldsymbol{\sigma}_1\times\mathbf{q}\right]$$

$$\mathbf{j}^{\rm NLO} = -i\,e\,\frac{g_A^2}{F_\pi^2}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\,\frac{1}{k_2^2+m_\pi^2}\,\boldsymbol{\sigma}_1\,(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2) + i\,e\,\frac{g_A^2}{F_\pi^2}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\,\frac{\mathbf{k}_1-\mathbf{k}_2}{(k_1^2+m_\pi^2)(k_2^2+m_\pi^2)}\,(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\,(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)$$

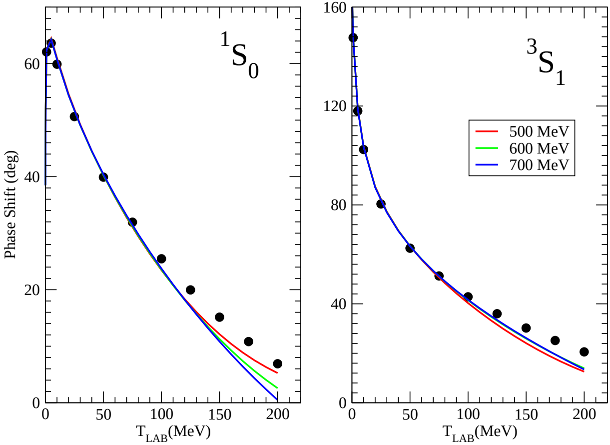
$$\mathbf{j}^{\rm N^2LO}_{\rm RC} = -\frac{e}{8\,m_N^3}\,e_{N,1}\left[2\,\left(K_1^2+q^2/4\right)(2\,\mathbf{K}_1+i\,\boldsymbol{\sigma}_1\times\mathbf{q}) + \mathbf{K}_1\cdot\mathbf{q}\,(\mathbf{q}+2i\,\boldsymbol{\sigma}_1\times\mathbf{K}_1)\right]$$

$$- \frac{i\,e}{8\,m_N^3}\,\kappa_{N,1}\left[\mathbf{K}_1\cdot\mathbf{q}\,(4\,\boldsymbol{\sigma}_1\times\mathbf{K}_1 - i\,\mathbf{q}) - (2\,i\,\mathbf{K}_1 - \boldsymbol{\sigma}_1\times\mathbf{q})\,q^2/2 + 2\,(\mathbf{K}_1\times\mathbf{q})\,\boldsymbol{\sigma}_1\cdot\mathbf{K}_1\right]$$

$$\mathbf{j}^{\rm N^2LO}_{\Delta} = i\,\frac{e\,\mu^*}{9\,m_N}\,\frac{g_A\,h_A}{\Delta F_\pi^2}\,\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{k_2^2+m_\pi^2}\left[4\,\tau_{2,z}\,\mathbf{k}_2 - (\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\,\boldsymbol{\sigma}_1\times\mathbf{k}_2\right]\times\mathbf{q}$$

$$\mathbf{j}^{\rm N^2LO}_{\Delta_c} = -i\,\frac{e\,\mu^*}{9\,m_N}\,\frac{D_T}{\Delta}\left[4\,\tau_{2,z}\,\boldsymbol{\sigma}_2 - (\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z\,\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\right]\times\mathbf{q}$$

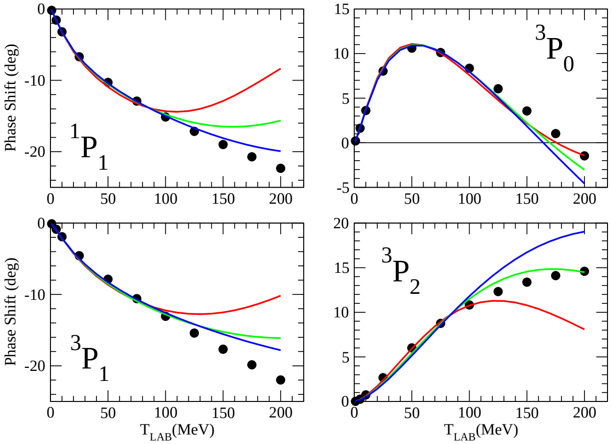
Phase shifts in S-wave *



* F.Gross and A.Stadler PRC **78**, 104405 (2008)

N^3LO calculation (without Δ's): preliminaries

Phase shifts in P-wave *



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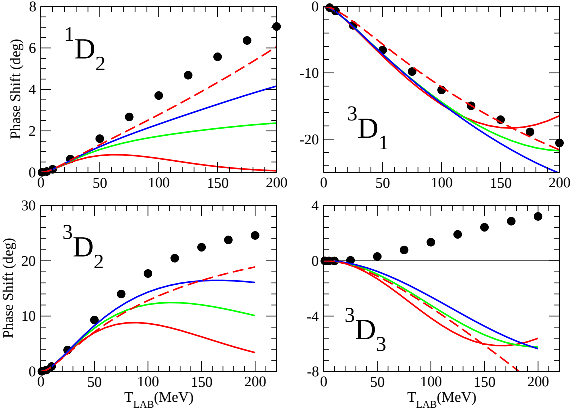
N^3LO calculation (without Δ's): preliminaries

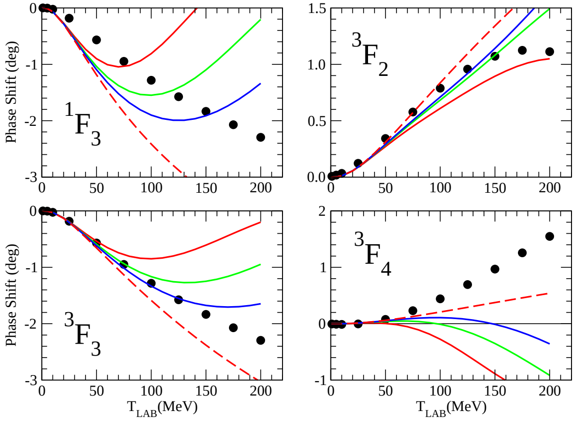
Phase shifts in D-wave *



* F.Gross and A.Stadler PRC **78**, 104405 (2008)

N^3LO calculation (without Δ's): preliminaries

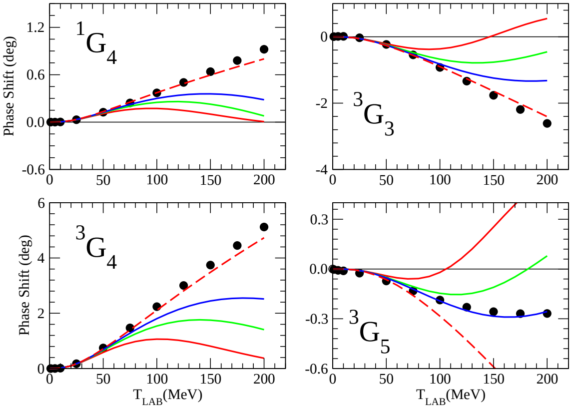
Phase shifts in F-wave ∗



∗ F.Gross and A.Stadler PRC **78**, 104405 (2008)

N^3LO calculation (without Δ's): preliminaries

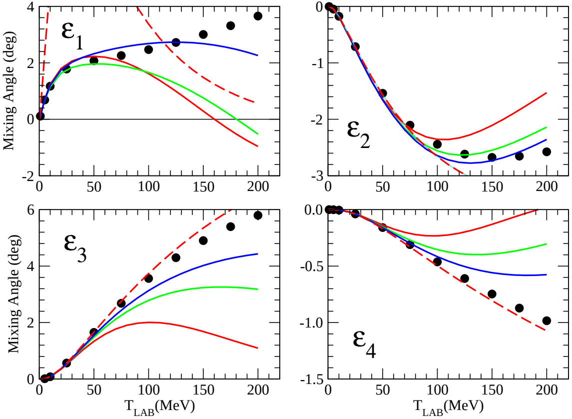
Phase shifts in G-wave *



* F.Gross and A.Stadler PRC **78**, 104405 (2008)

N^3LO calculation (without Δ's): preliminaries

Mixing angle *



* F.Gross and A.Stadler PRC **78**, 104405 (2008)

Deuteron w.f.'s