

Lawrence Livermore National Laboratory

From EFT interactions to a unified *ab initio* description of light nuclei



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INT Program on Effective Field Theory and the Many-body Problem, 4/8/2009

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Outline

- Motivation
- Chiral NNN interactions and NCSM
- Determination of chiral NNN LECs c_D and c_E
 - $A=3$ binding energy
 - $A=4$ binding energy and radius
 - ^{10}B states
 - Triton half life
- p -shell results with chiral NN+NNN
- Scattering and coupling to continuum: Combining NCSM and RGM
 - n - ^4He
 - p - ^{12}C
 - n - ^{16}O
- Outlook

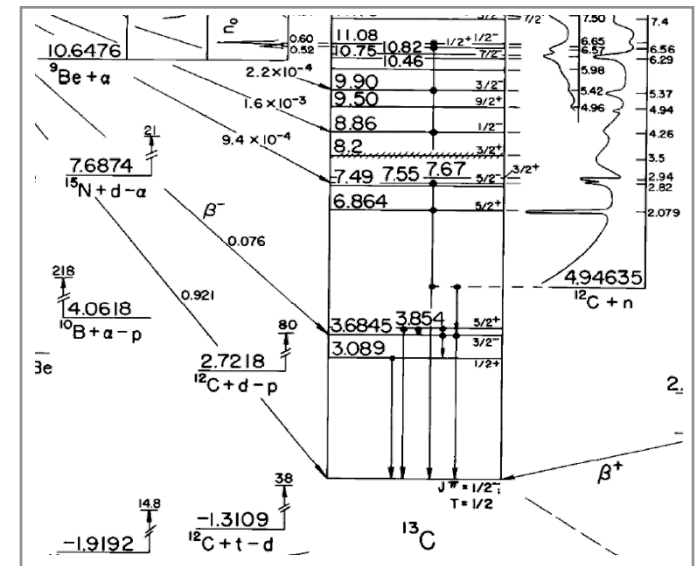


Low-Energy nuclear physics

- Overarching goal:

To arrive at a comprehensive and unified microscopic description of all nuclei and their low-energy reactions from the basic interactions between the constituent protons and neutrons

- This is an ambitious goal
 - Nuclei are self-bound, quantum many-fermion system
 - Complicated interaction with at least two- and three-nucleon components
 - Bound states, resonances, scattering states



Our goal is to arrive at an *ab initio* picture for light nuclei and their reactions



Where do we start?

- Quantum chromodynamics (QCD) is the underlying theory for the strong interaction
 - Lattice QCD calculations are too difficult to do complex nuclei
 - They are not yet capable of providing an accurate nucleon-nucleon or three-nucleon interaction
 - But they can verify that QCD is the correct theory for the strong interaction between hadrons
- We need a theory with point-like nucleons and an interaction based on QCD
 - Effective field theory (EFT) based on the properties of QCD provides an elegant solution with broad predictive power



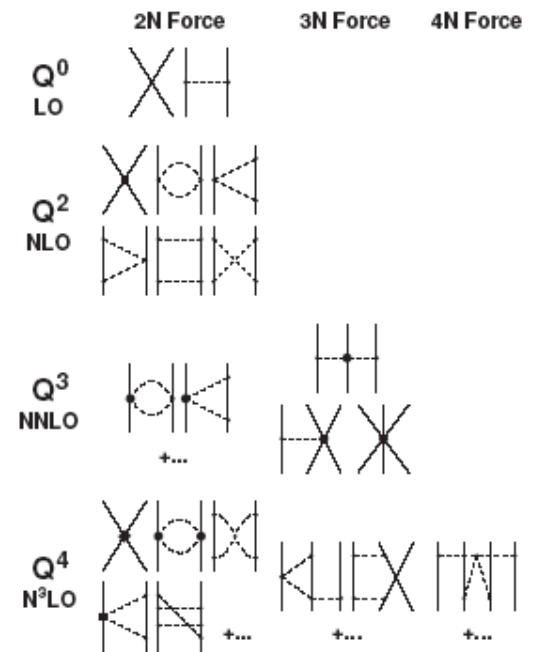
Effective Field Theory

- Based on the symmetries of QCD
 - Degrees of freedom: nucleons + pions
 - Describes pion-pion, pion-nucleon and inter-nucleon interactions at low energies
- Systematic low-momentum expansion to a given order



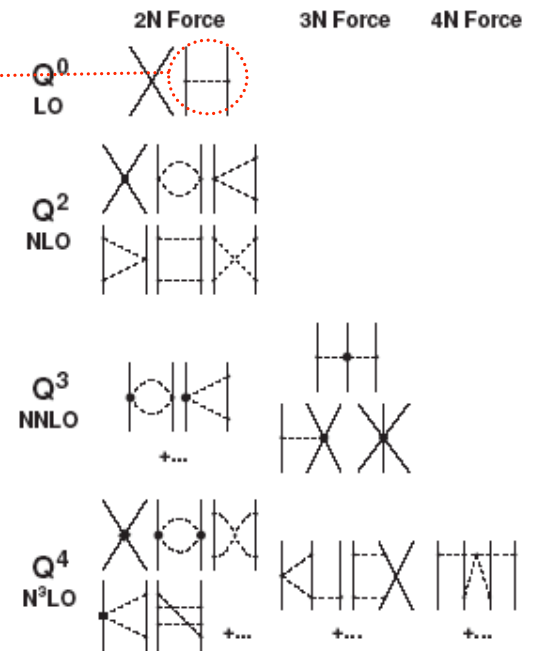
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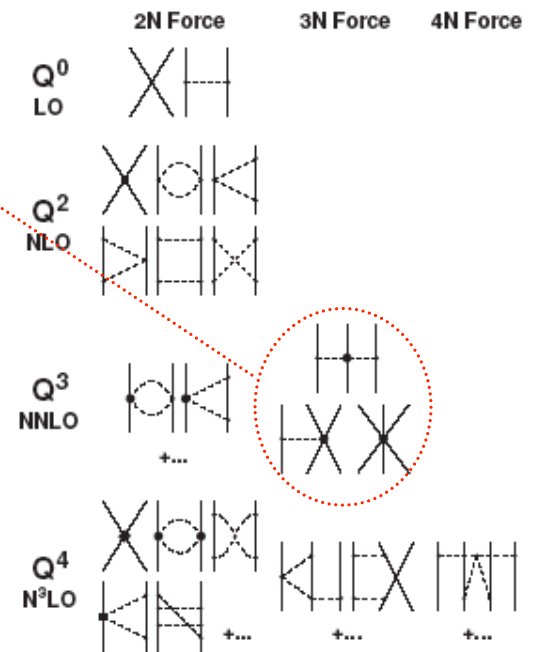
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- Leading order (LO)
 - One-pion exchange



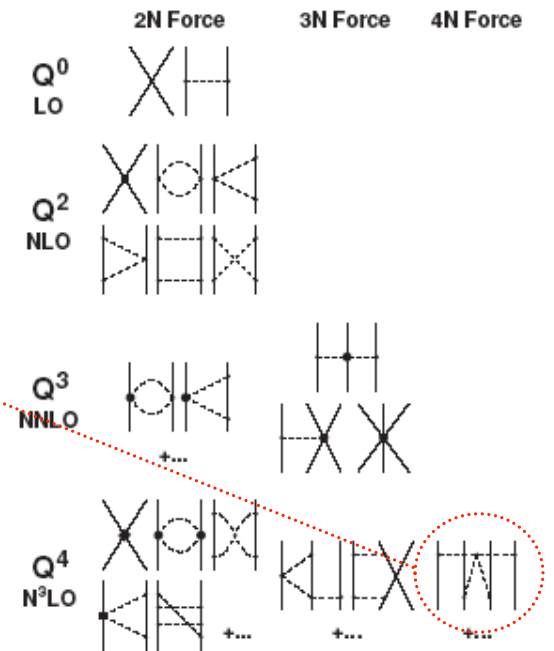
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- NNN interaction appears at next-to-next-to-leading order (N²LO)



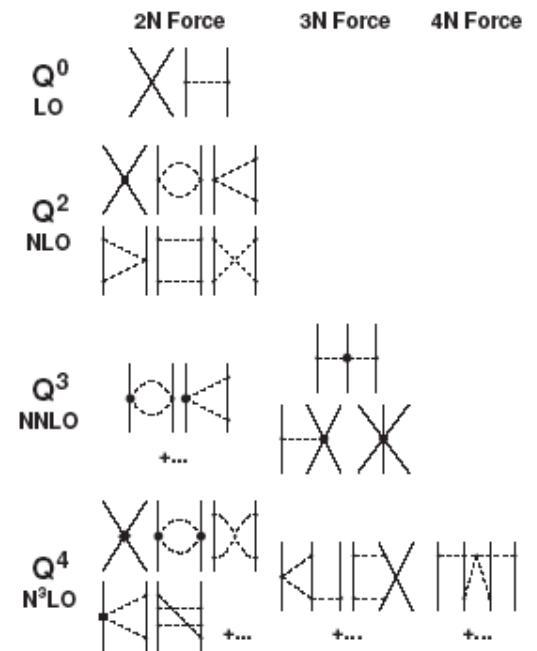
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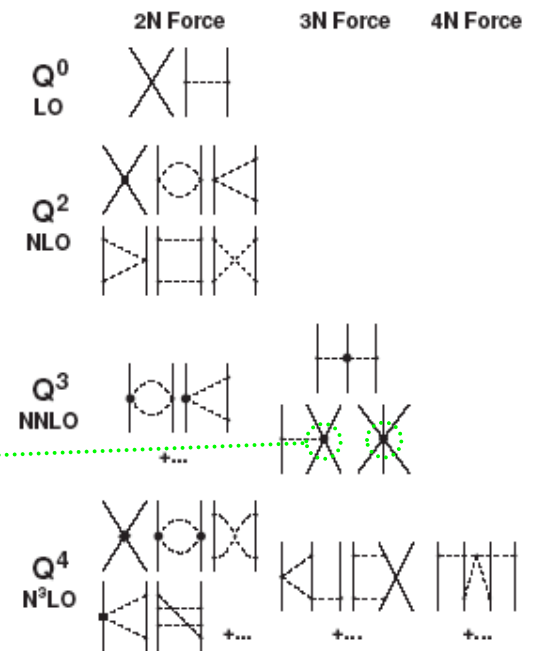
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 - NN parameters enter in the NNN terms etc.



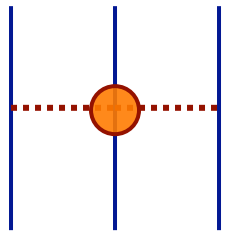
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- Consistency between NN, NNN and NNNN terms
 - NN parameters enter in the NNN terms etc.
- Low-energy constants (LECs)
 - Low-energy theory, integrates out short-range physics
 - Only two NNN and none NNNN low-energy constants up to N³LO



The NNN interaction

N²LO

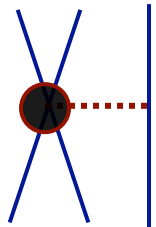


Two-pion exchange

c_1, c_3, c_4 LECs appear in the chiral NN interaction

- Determined in the $A=2$ system

c_1, c_3, c_4

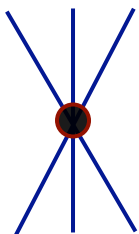


One-pion-exchange-contact

New c_D LEC

New!

c_D



Contact

New c_E LEC

New!

c_E

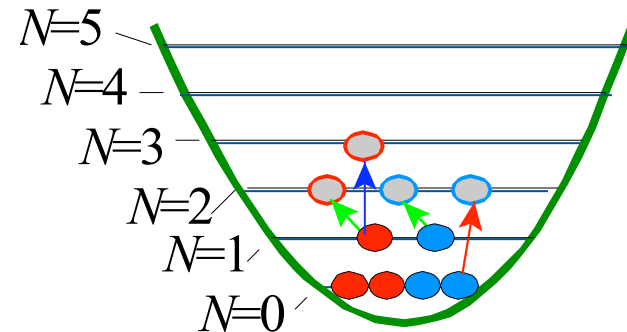
Must be determined
in $A \geq 3$ system

Nontrivial to include in many-body calculations



The *ab initio* NCSM in brief

- The NCSM is a technique for the solution of the A -nucleon bound-state problem
- Hamiltonian (in this talk)
 - Standard high-precision nucleon-nucleon potentials:
 - Idaho chiral $N^3\text{LO}$ with 500 MeV cutoff
 - Soft low-momentum interactions
 - SRG- $N^3\text{LO}$
 - Three-nucleon interactions:
 - Local chiral $N^2\text{LO}$
- Finite harmonic oscillator (HO) basis
 - A -nucleon HO basis states
 - Jacobi relative coordinates
 - Cartesian single-particle coordinates
 - complete $N_{\max} \hbar\Omega$ model space
 - Translational invariance preserved even with single-particle coordinate Slater-determinant (SD) basis
- Effective interaction tailored to model-space truncation for standard potentials
 - Unitary transformation in n -body cluster approximation ($n=2,3$)
- Importance-truncated $N_{\max} \hbar\Omega$ basis
 - Second-order many-body perturbation theory
 - Dimension reduction from billions to tens of millions
 - Access to nuclei beyond p -shell



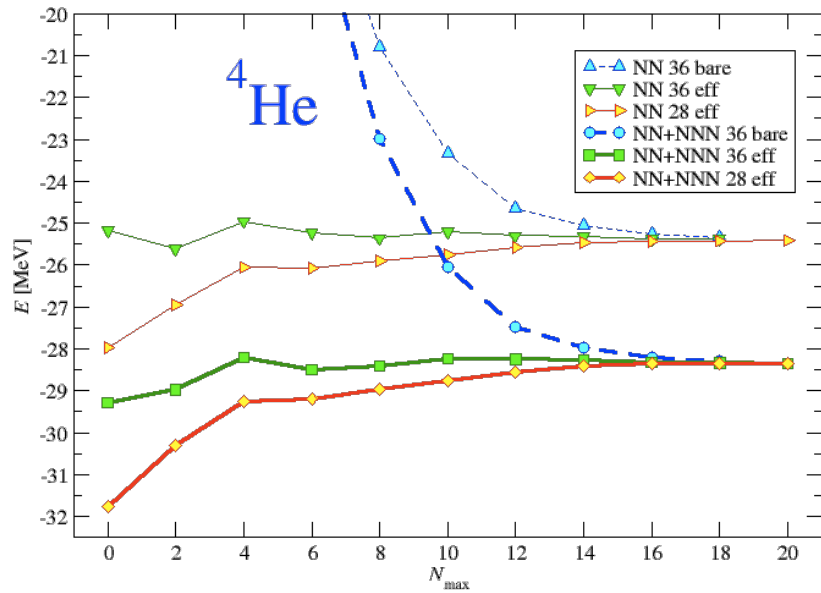
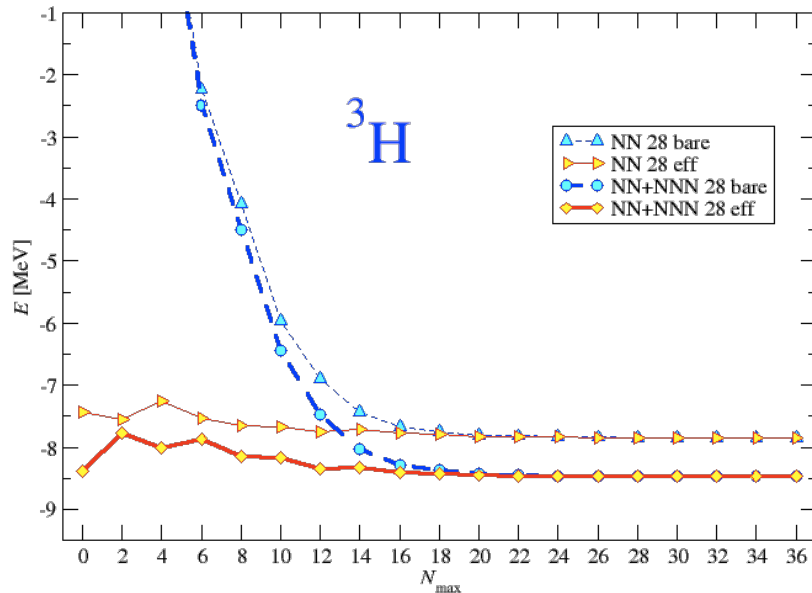
$$\kappa_\nu = - \frac{\langle \Phi_\nu | H' | \Psi_{\text{ref}} \rangle}{E_\nu^{(0)} - E_{\text{ref}}^{(0)}}$$

$$|\kappa_\nu| \geq \kappa_{\text{min}}$$

Convergence to exact solution with increasing N_{\max} for bound states.
No coupling to continuum.



^3H and ^4He with chiral interactions



^3H

^4He

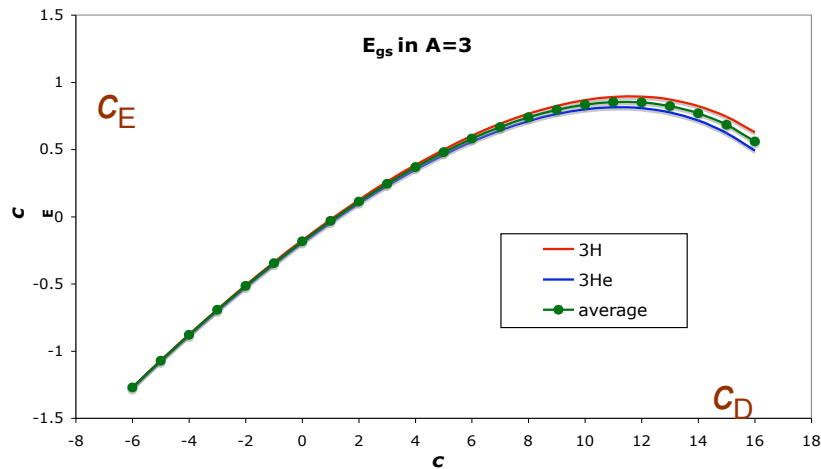
		$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$
NN	NCSM [71]	-7.852(5)	1.650(5)
NN	HH [17]	-7.854	1.655
NN+NNN	NCSM [71]	-8.473(5)	1.608(5)
NN+NNN	HH [17]	-8.474	1.611
Expt.		-8.482	1.60

		$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$
NN	NCSM [71]	-25.39(1)	1.515(2)
NN	HH [17]	-25.38	1.518
NN+NNN	NCSM [71]	-28.34(2)	1.475(2)
NN+NNN	HH [17]	-28.36	1.476
Expt.		-28.296	1.467(13)



Determining the three-nucleon interaction: $A=3$ & ${}^4\text{He}$

- Constrain c_D , c_E to $A=3$ binding energy

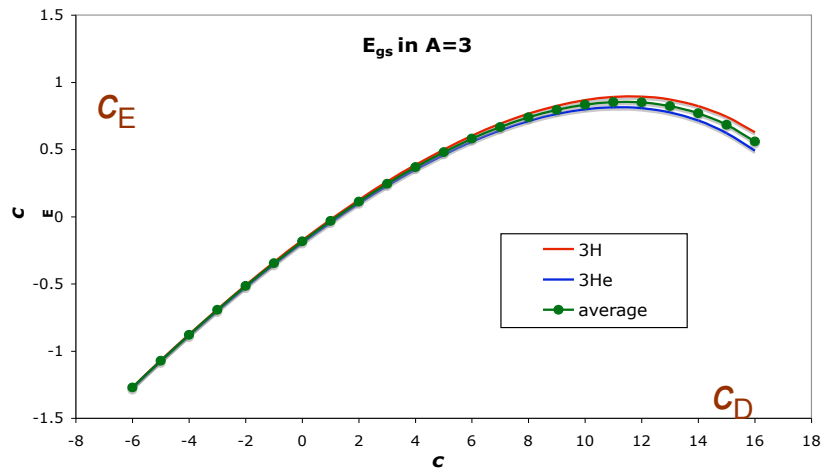


$c_D - c_E$ dependence that fits $A=3$ binding energy



Determining the three-nucleon interaction: $A=3$ & ${}^4\text{He}$

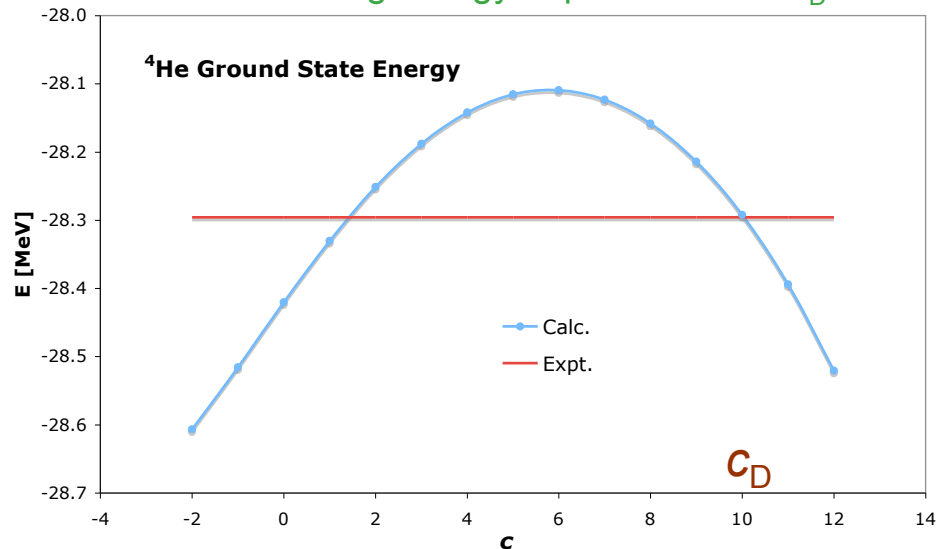
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$c_D - c_E$ dependence that fits $A=3$ binding energy

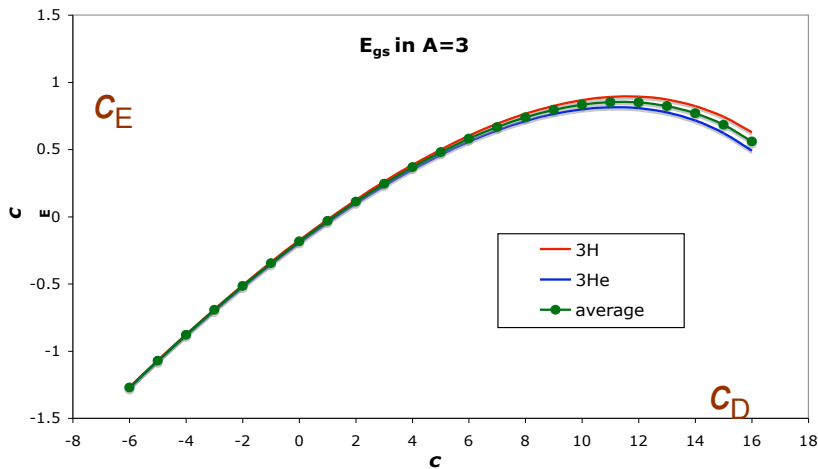
- Other observables are needed:
 - N-d doublet scattering length
 - Correlated with E_{gs}
- ${}^4\text{He}$ binding energy
- Two combinations of $c_D - c_E$ that fit both $A=3$ and ${}^4\text{He}$ binding energies
 - ${}^4\text{He}$ E_{gs} dependence on c_D weak
 - ${}^4\text{He}$ and $A=3$ binding energies correlated

${}^4\text{He}$ binding energy dependence on c_D



Determining the three-nucleon interaction: A=3 & ^4He

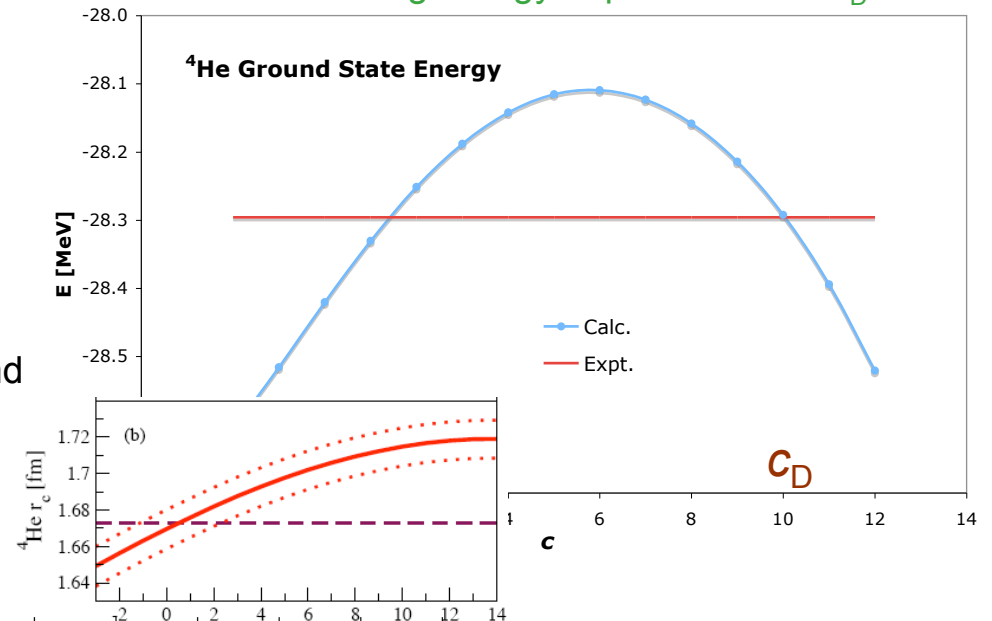
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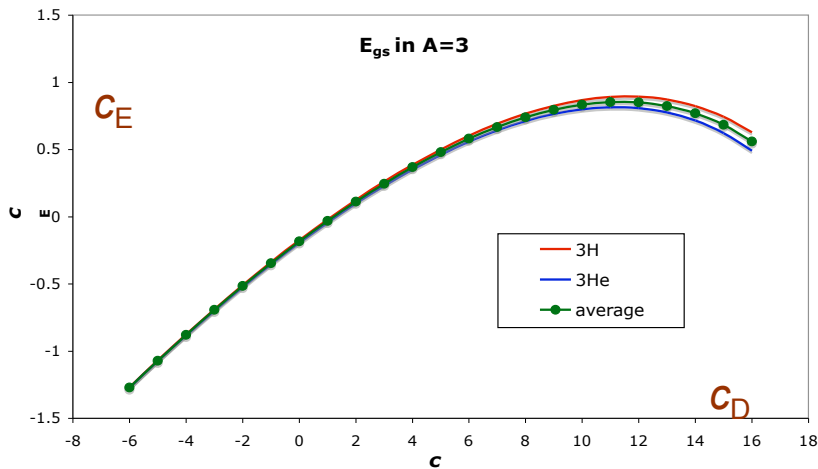
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^4He binding energy dependence on c_D



Determining the three-nucleon interaction: A=3 & ^4He

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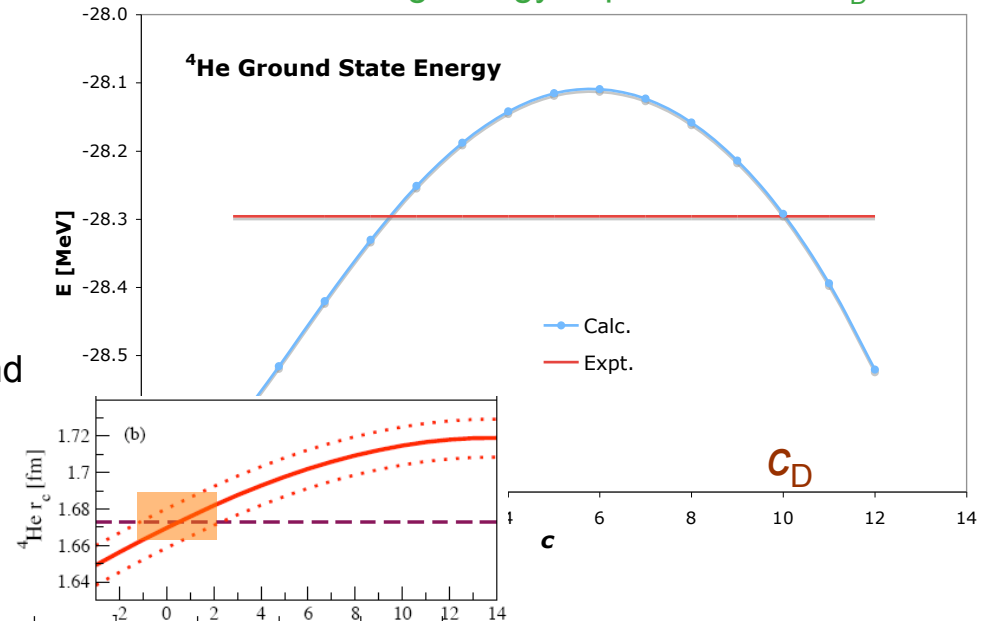


$c_D - c_E$ dependence that fits A=3 binding energy

$-2 < c_D < 2$
preferred

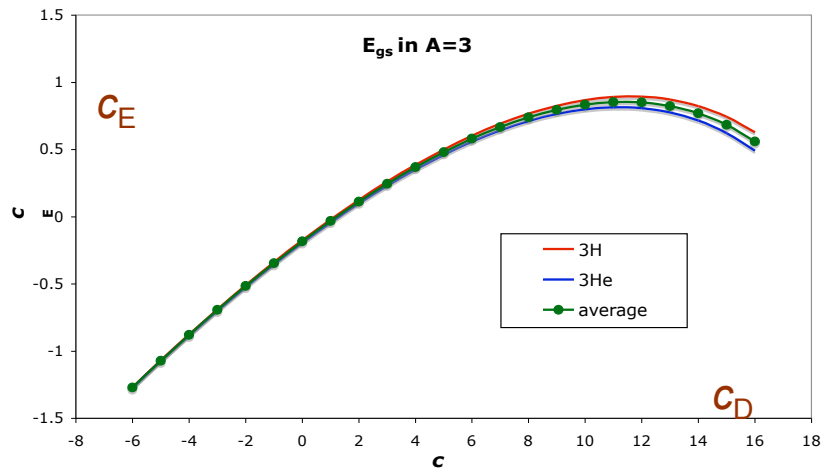
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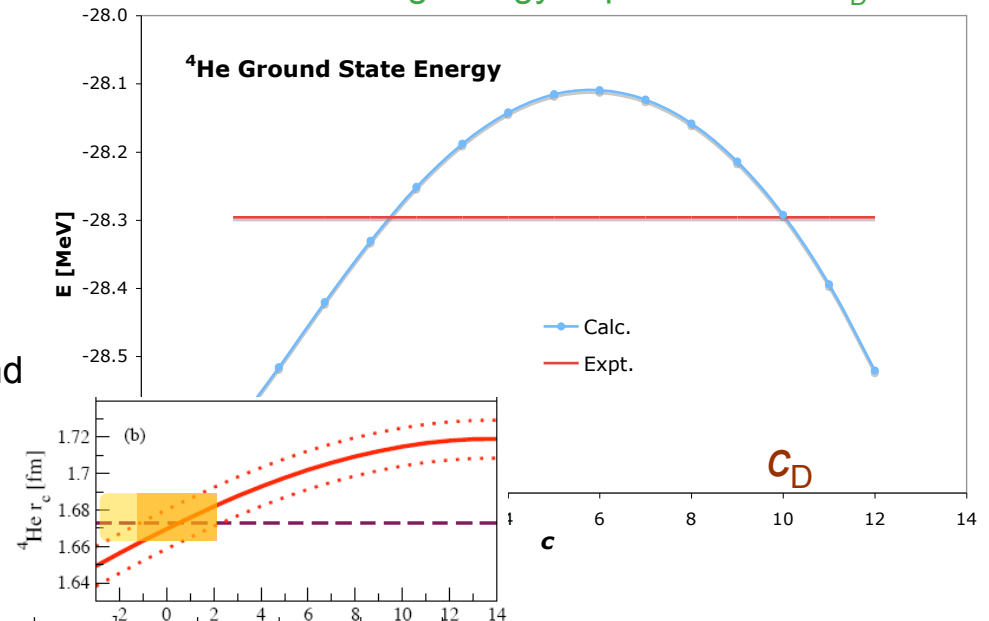


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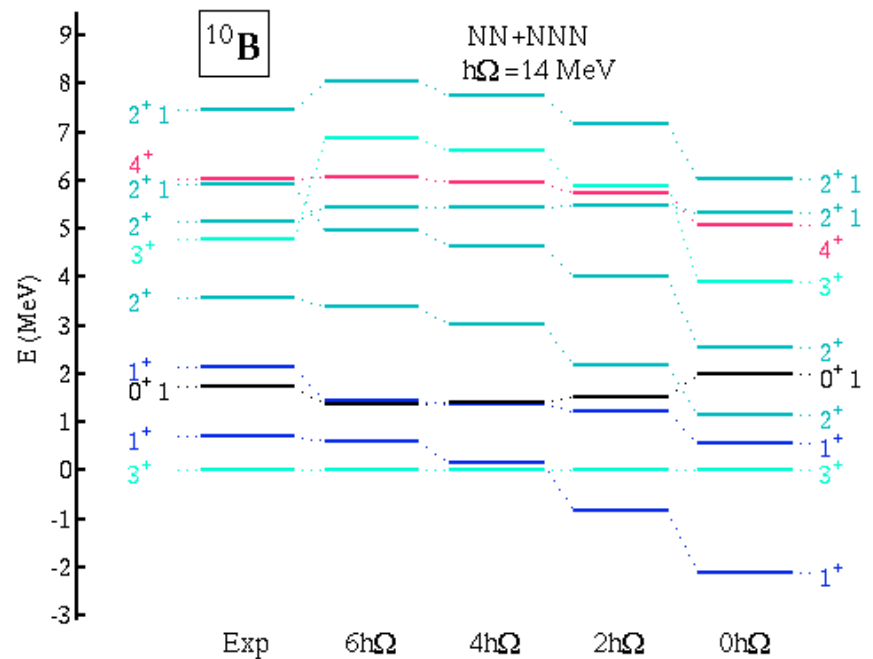
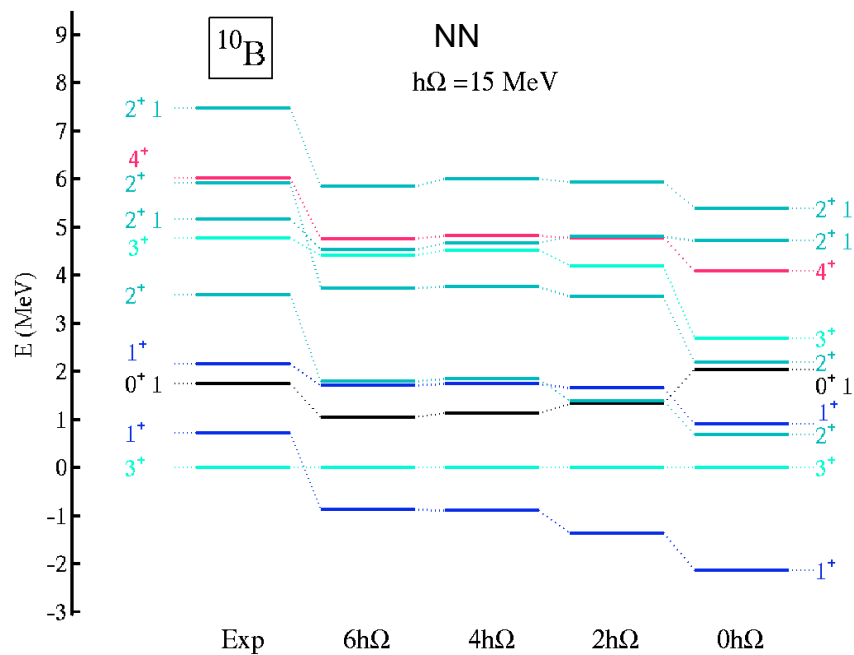
^4He binding energy dependence on c_D



What about the structure of p -shell nuclei?

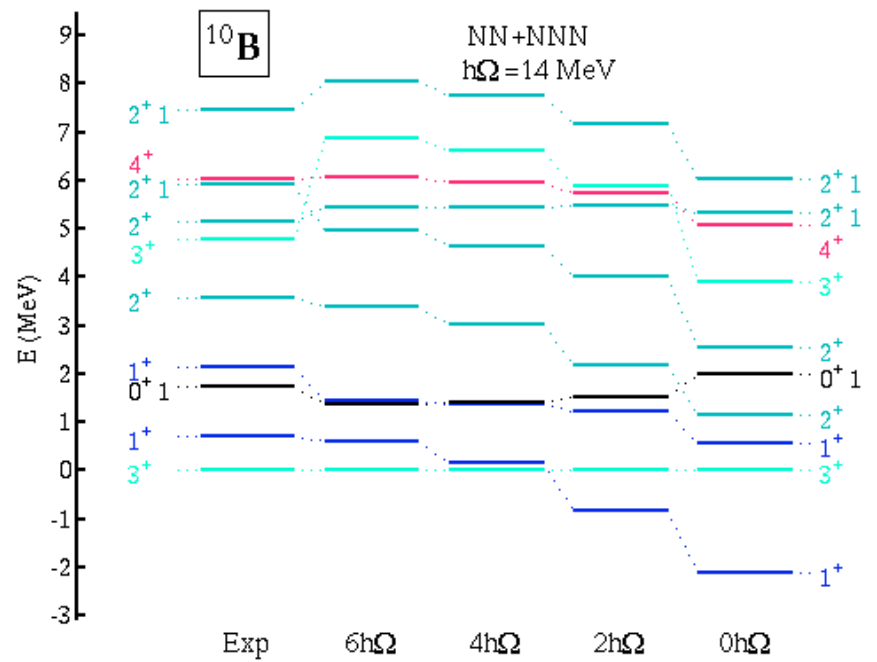
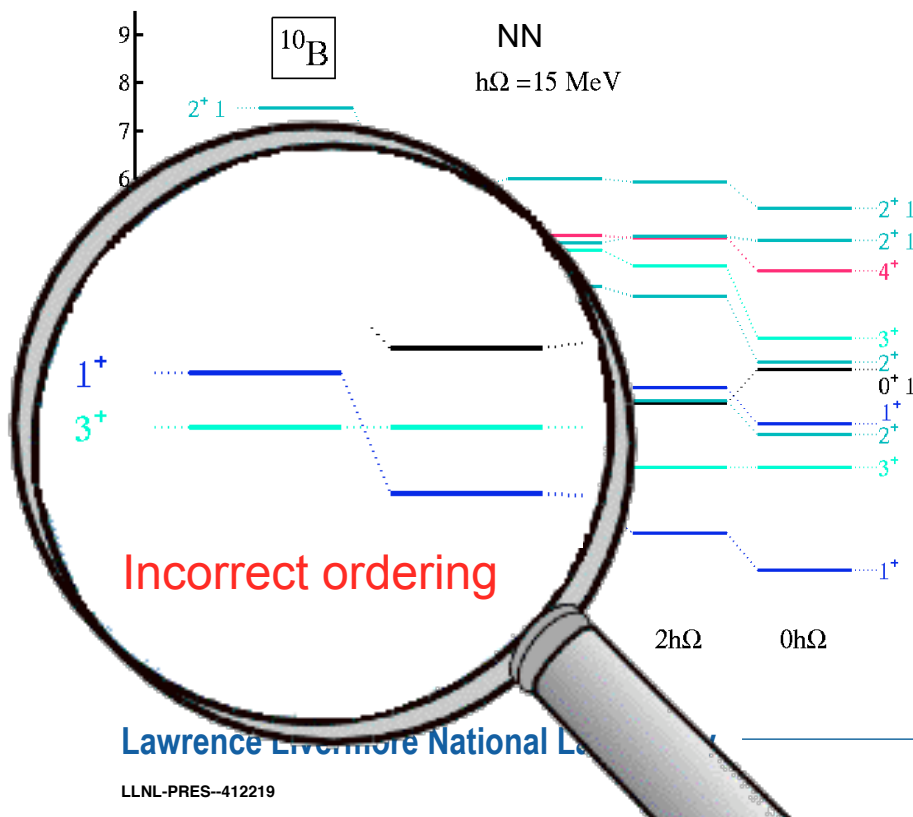
NN is important for heavier *p*-shell nuclei: ^{10}B

- ^{10}B known to be poorly described by standard NN interaction
 - Predicted ground state $1^+ 0$
 - Experiment $3^+ 0$



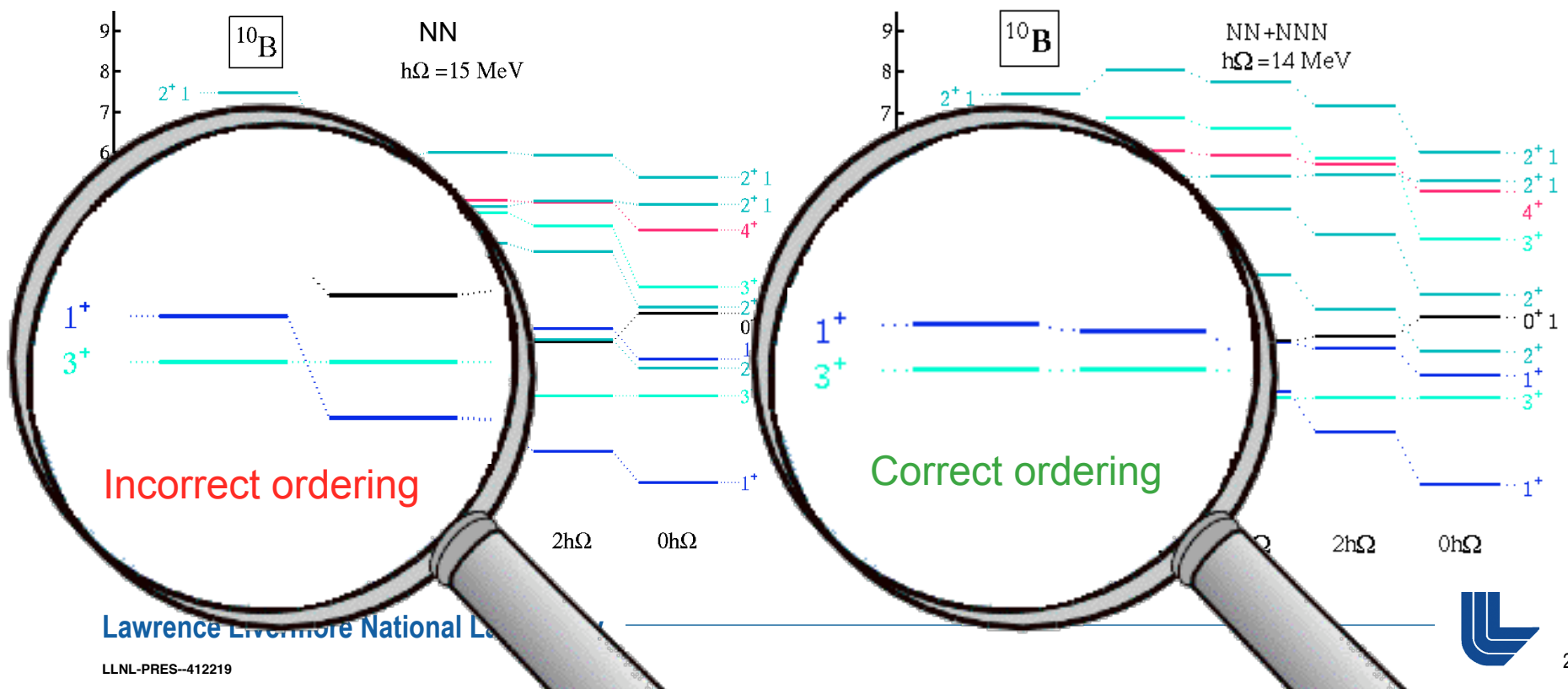
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NN important for heavier *p*-shell nuclei: ^{10}B

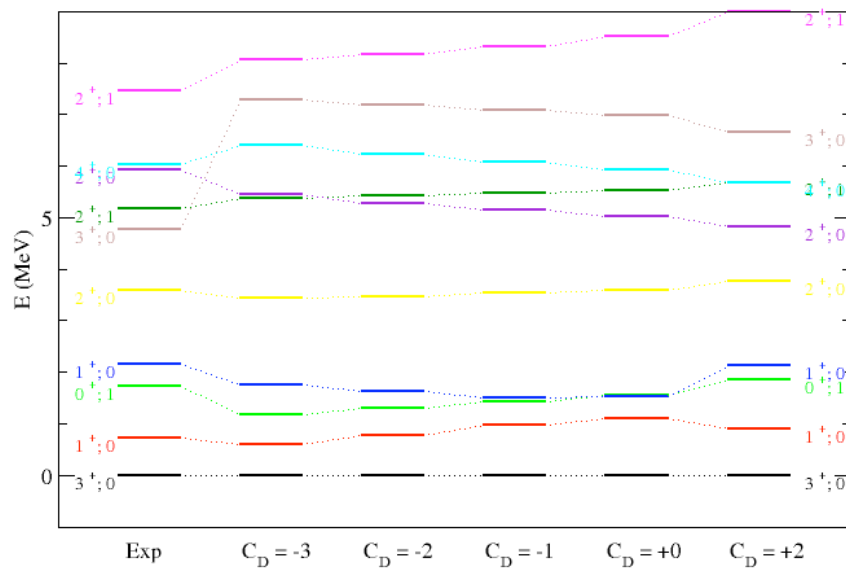
- ^{10}B known to be poorly described by standard NN interaction
 - Predicted ground state $1^+ 0$
 - Experiment $3^+ 0$
- Chiral NNN fixes this problem



Using the NCSM to determine c_D, c_E : ^{10}B

- ^{10}B properties not correlated with $A=3$ binding energy
- Spectrum shows weak dependence on c_D

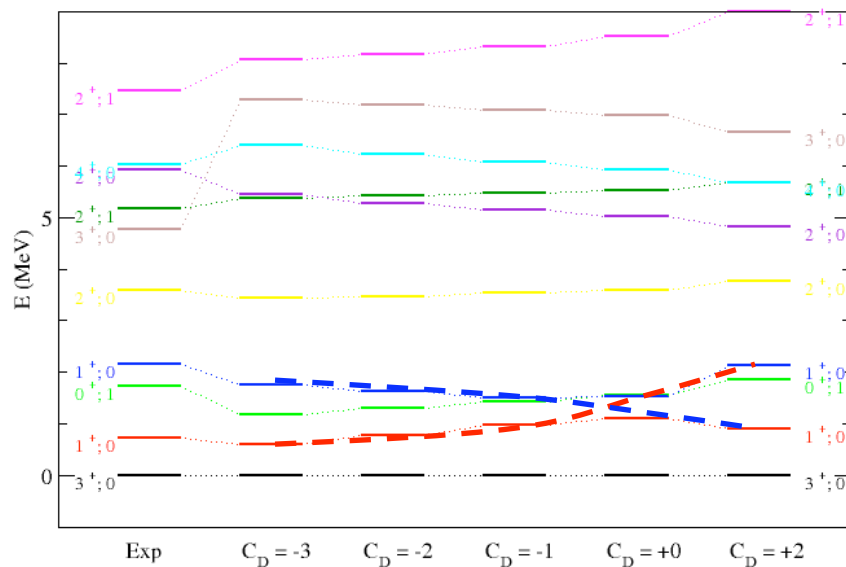
^{10}B NN+NNN c_D dependence for $N_{\text{max}} = 6, h\Omega = 15 \text{ MeV}$



Using the NCSM to determine c_D, c_E : ^{10}B

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- However: Order of 1^+_1 and 1^+_2 changes depending on c_D

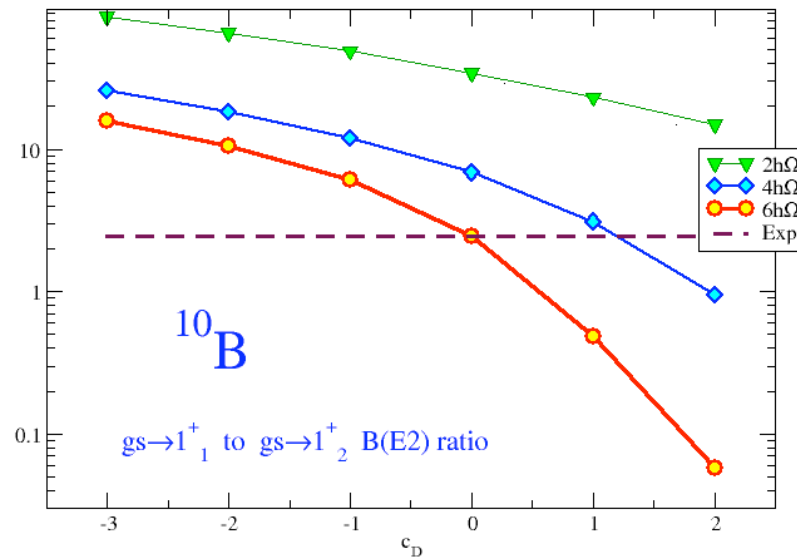
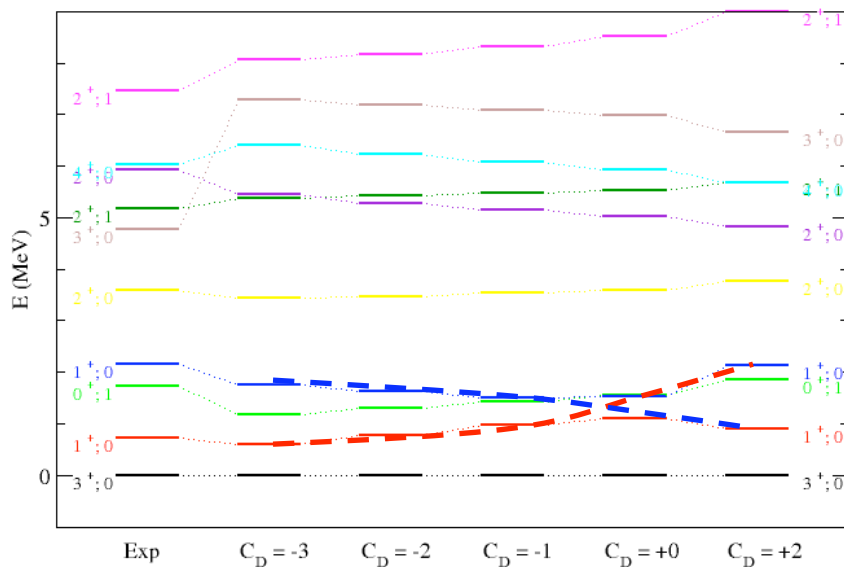
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 - This is seen in ratio of E2 transitions from ground state to 1^+_1 and 1^+_2

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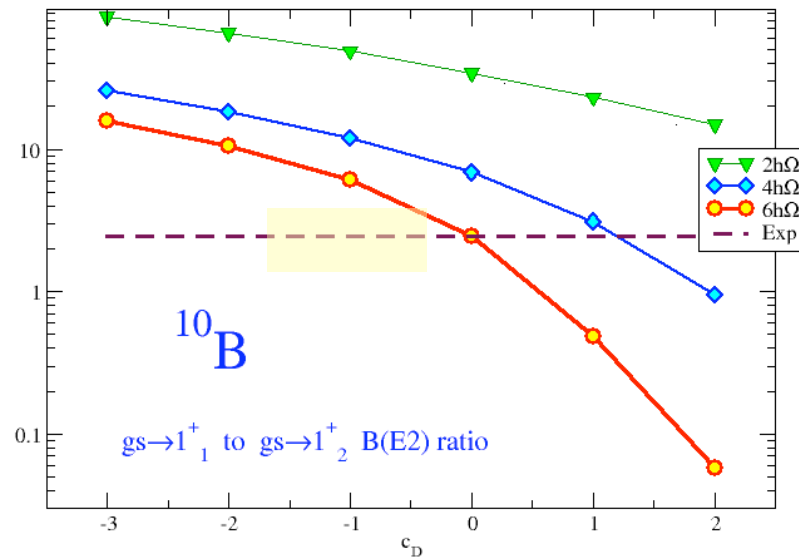
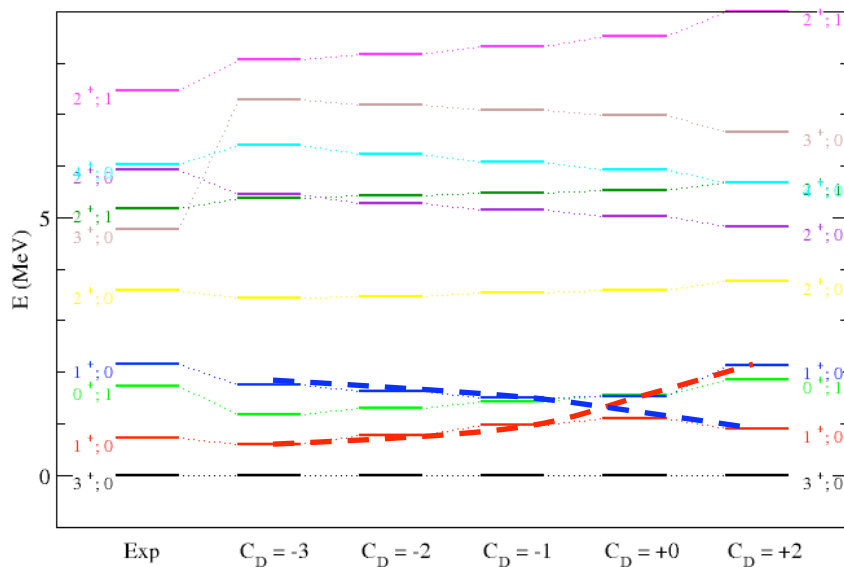


Using the NCSM to determine c_D, c_E : ^{10}B

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- Spectrum shows weak dependence on c_D
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 - This is seen in ratio of E2 transitions from ground state to 1^+_1 and 1^+_2

$-2 < c_D < 0$
preferred

^{10}B NN+NNN c_D dependence for $N_{\text{max}} = 6, \hbar\Omega = 15 \text{ MeV}$



Determination of c_D (and c_E) from the triton half life

- c_D also in the two-nucleon contact vertex with an external probe
- Calculate

$$\langle E_1^A \rangle = |\langle {}^3\text{He} || E_1^A || {}^3\text{H} \rangle|$$

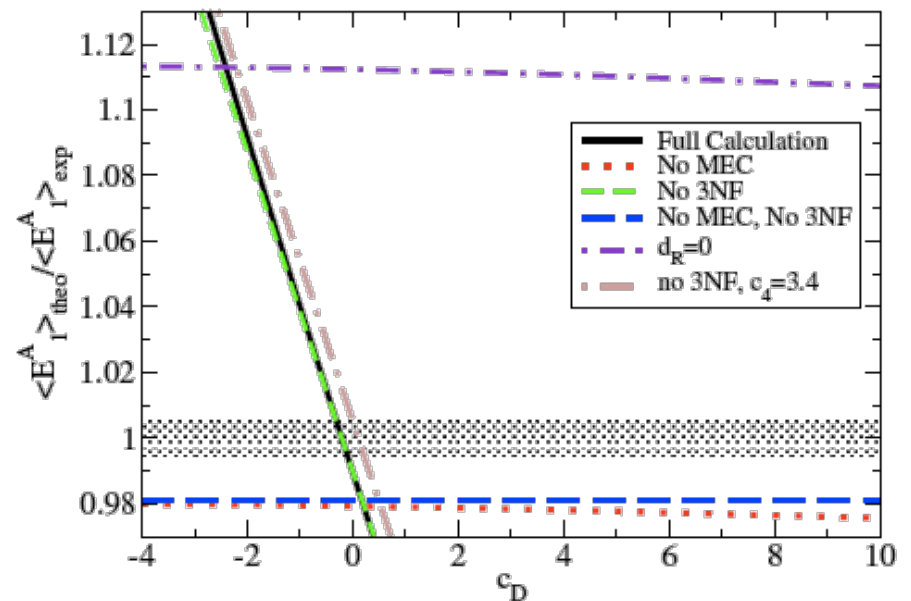
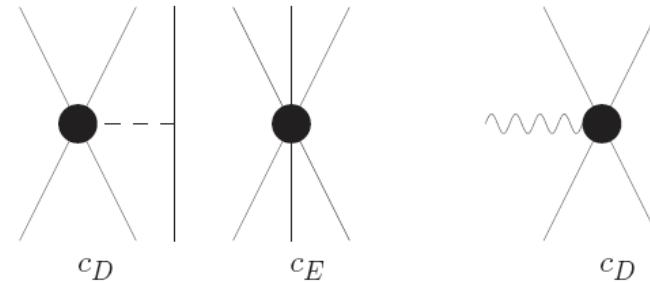
- Leading order GT

$$E_1^A|_{\text{LO}} = i g_A (6\pi)^{-1/2} \sum_{i=1}^A \sigma_i \tau_i^+$$

- N²LO: one-pion exchange plus contact

$$\hat{d}_R \equiv \frac{M_N}{\Lambda_\chi g_A} c_D + \frac{1}{3} M_N (c_3 + 2c_4) + \frac{1}{6}$$

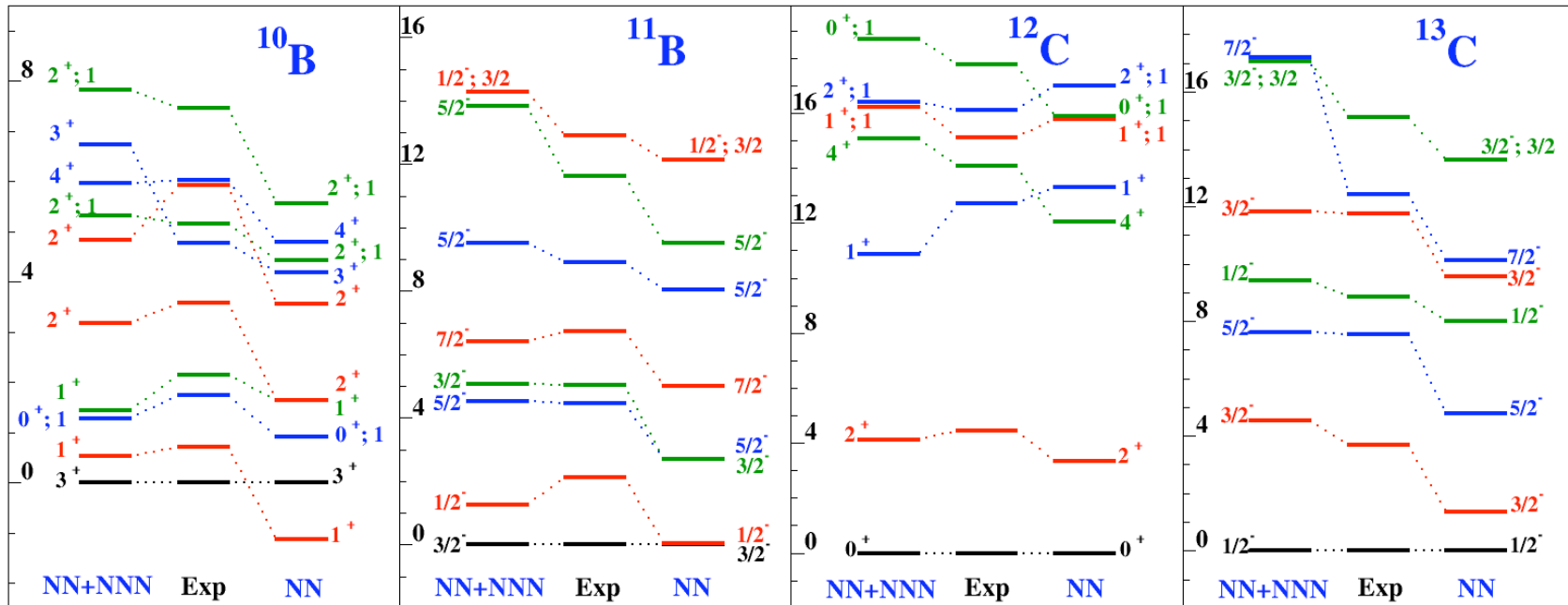
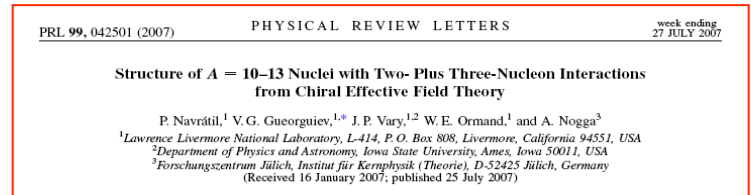
- With the $A=3$ binding energy constraint a robust determination of $c_D = -0.2 \pm 0.1$ and $c_E = -0.205 \pm 0.015$



Not inconsistent with the $A>3$ determination $c_D \approx -1 \pm 1$

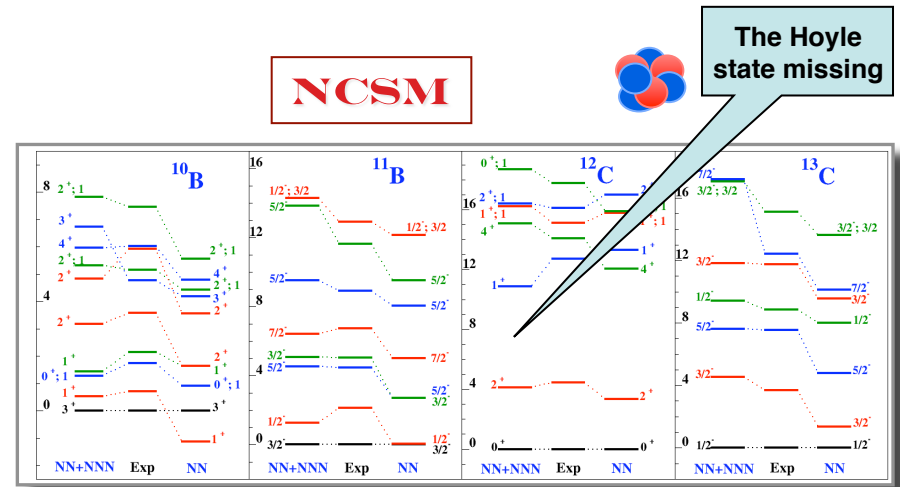
Structure of p-shell nuclei with NN+NNN interactions

- NCSM is only method capable to apply the EFT NN+NNN interactions
 - Technically challenging, large-scale computational problem
 - ~4000 processors used in $^{12,13}\text{C}$ calculations
- Applied to constrain the NNN interaction
 - Investigation of $A=3$, ^4He and p -shell nuclei
 - Globally the best results with $c_D \sim -1$
- NNN interaction essential to describe structure of light nuclei

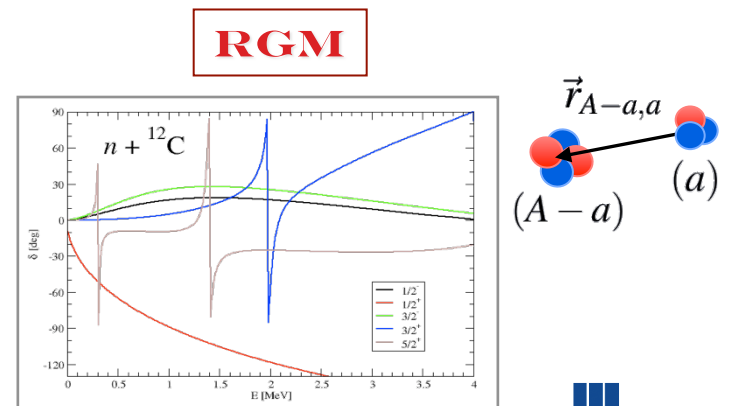


Our goal is to develop an *ab initio* theory to understand nuclear structure....

- ... and **reactions** in light nuclei
- How? - Combining the *ab initio* no-core shell model (NCSM) with the resonating group method (RGM)
 - ⇒ ***ab initio* NCSM/RGM**
 - NCSM - single-particle degrees of freedom
 - RGM - clusters and their relative motion



- Ab initio* theory of nuclear reactions for $A > 4$ is new:**
 - Lisbon:** $p+^3\text{He}$ scattering published in 2007 (PRL **98**, 162502 (2007)); **$A=4$ is their limit**
 - ANL:** $n+^4\text{He}$ scattering published only recently (PRL **99**, 022502 (2007))
- Our approach: readily extendable**
 - $p-^3\text{He}$, $n-^4\text{He}$ & $p-^4\text{He}$ scattering already calculated
 - promising results for p -shell nuclei: $n-^{10}\text{Be}$, $p-^{12}\text{C}$, $n-^{16}\text{O}$
 - Inclusion of d , ^3H , ^3He and α clusters under way

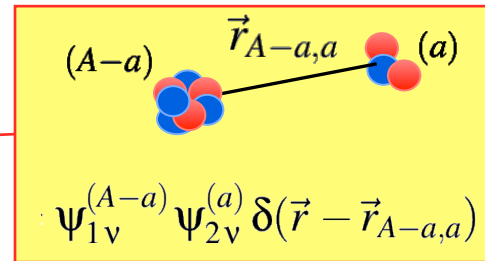


Preserves **Pauli principle** and **translational invariance**
 ↓
 Important as nucleons are **fermions** and nuclei **self-bound**



The *ab initio* NCSM/RGM in a snapshot

- Ansatz: $\Psi^{(A)} = \sum_{\nu} \int d\vec{r} \phi_{\nu}(\vec{r}) \hat{\mathcal{A}} \Phi_{\nu\vec{r}}^{(A-a,a)}$



eigenstates of $H_{(A-a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$$T_{\text{rel}}(r) + \mathcal{V}_{\text{rel}} + \bar{V}_{\text{Coul}}(r) + H_{(A-a)} + H_{(a)}$$

$$\sum_{\nu} \int d\vec{r} \left[\mathcal{H}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) - E \mathcal{N}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) \right] \phi_{\nu}(\vec{r}) = 0$$

either bare interaction or NCSM effective interaction

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi_{\nu\vec{r}}^{(A-a,a)} \rangle$$

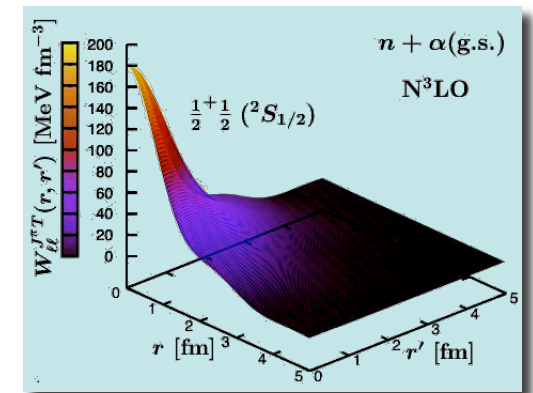
Hamiltonian kernel

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}}^2 | \Phi_{\nu\vec{r}}^{(A-a,a)} \rangle$$

Norm kernel

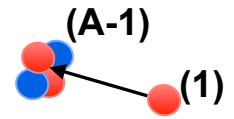
- Non-local integro-differential coupled-channel equations:

$$[\hat{T}_{\text{rel}}(r) + \bar{V}_{\text{C}}(r) - (E - E_{\nu})] u_{\nu}(r) + \sum_{\nu'} \int dr' r' W_{\nu\nu'}(r, r') u_{\nu'}(r') = 0$$



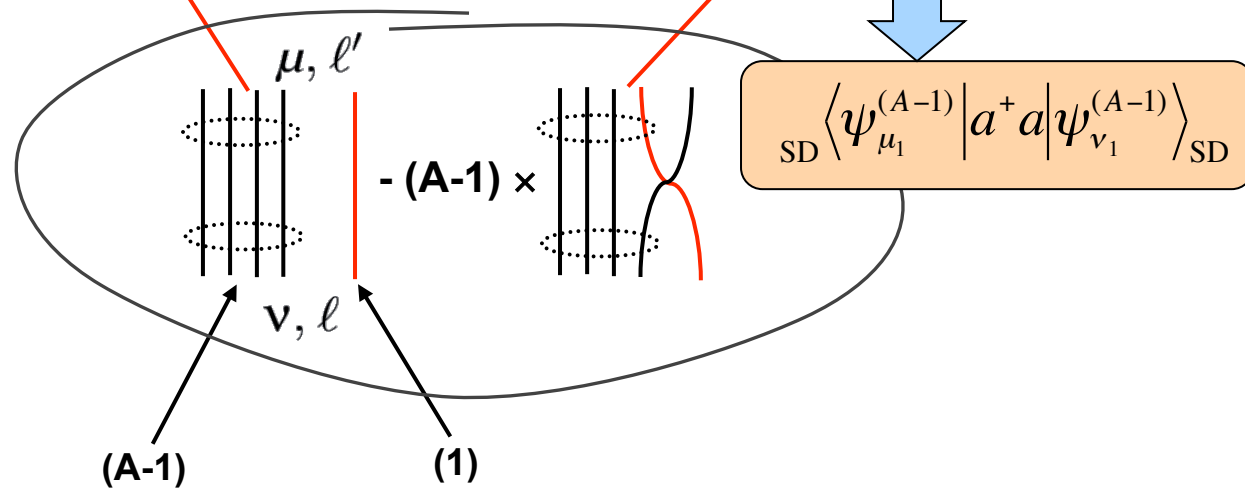
Fully implemented and tested for **single-nucleon projectile** (nucleon-nucleus) basis

Single-nucleon projectile: the norm kernel

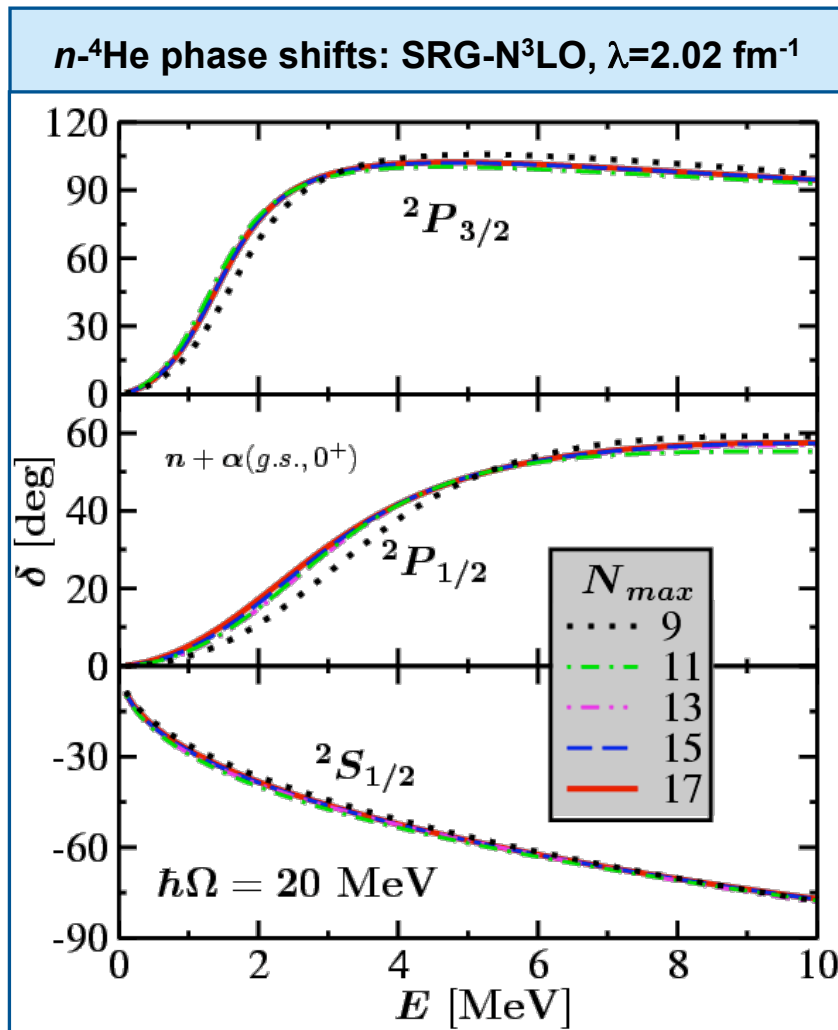
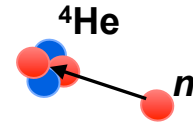


$$\left\langle \begin{array}{c} (1, \dots, A-1) \\ \text{red, blue, red} \\ \swarrow \quad \searrow \\ r' \quad (A) \end{array} \middle| 1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \middle| \begin{array}{c} (1, \dots, A-1) \\ \text{red, blue, red} \\ \swarrow \quad \searrow \\ (A) \quad r \end{array} \right\rangle$$

$$\mathcal{N}_{\mu\ell', \nu\ell}^{(A-1,1)}(r', r) = \delta_{\mu\nu} \delta_{\ell\ell'} \frac{\delta(r' - r)}{r'r} - (A-1) \sum_{n'n} R_{n'\ell'}(r') \langle \Phi_{\mu n'\ell'}^{(A-1,1)JT} | P_{A,A-1} | \Phi_{\nu n\ell}^{(A-1,1)JT} \rangle R_{n\ell}(r)$$



NCSM/RGM *ab initio* calculation of n - ^4He phase shifts



- Similarity-renormalization-group (SRG) evolved chiral N^3LO NN interaction (R. Roth)
- Low-momentum V_{lowk} NN potential
- convergence reached with **bare** interaction

N_{max}	^4He		$\frac{3}{2}^- (^2P_{3/2})$
	$E_{g.s.}$	$\frac{1}{2}^+$	
9	-27.00		81.8
11	-27.41		86.1
13	-27.57		85.7
15	-27.75		84.6
17	-27.77		84.8

Fully *ab initio* calculation. No free parameters.

Good convergence with respect to N_{max}

Is everything else under control? ... need

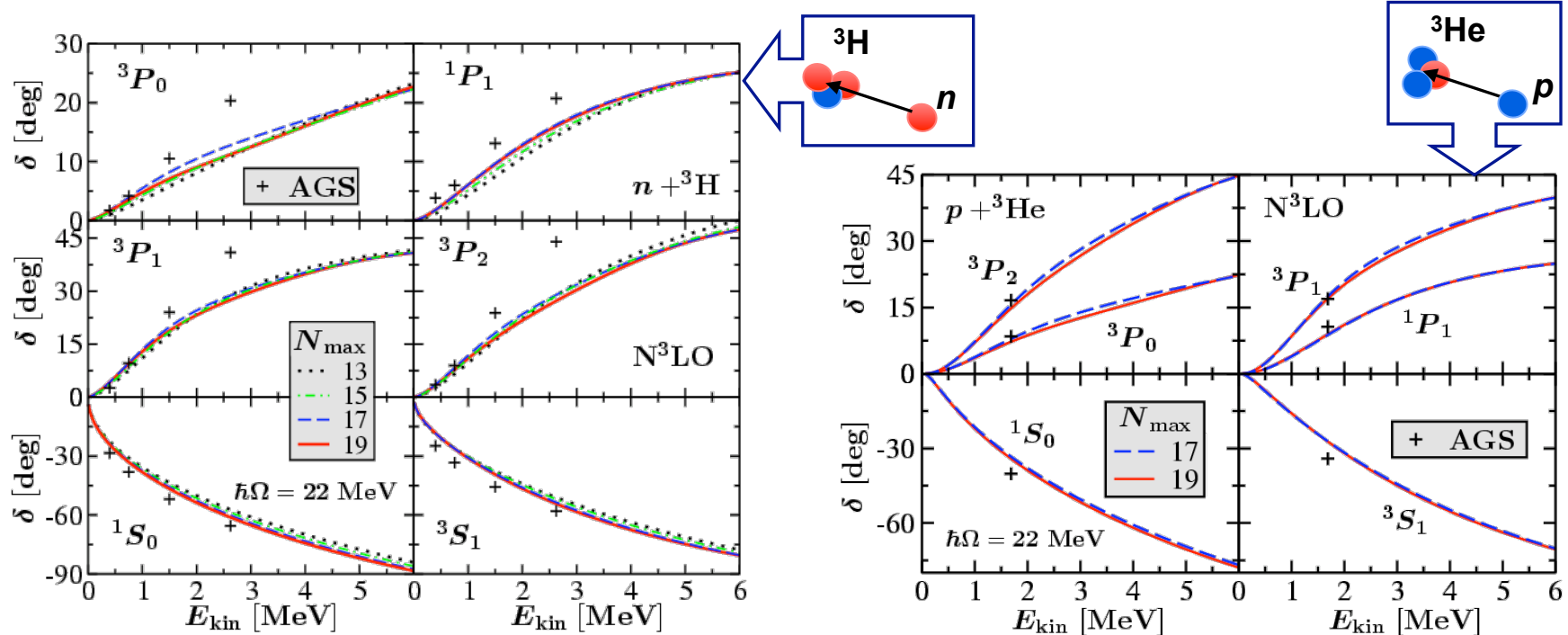
verification against independent *ab initio*

calculations!



NCSM/RGM *ab initio* calculation of n - ^3H and p - ^3He phase shifts

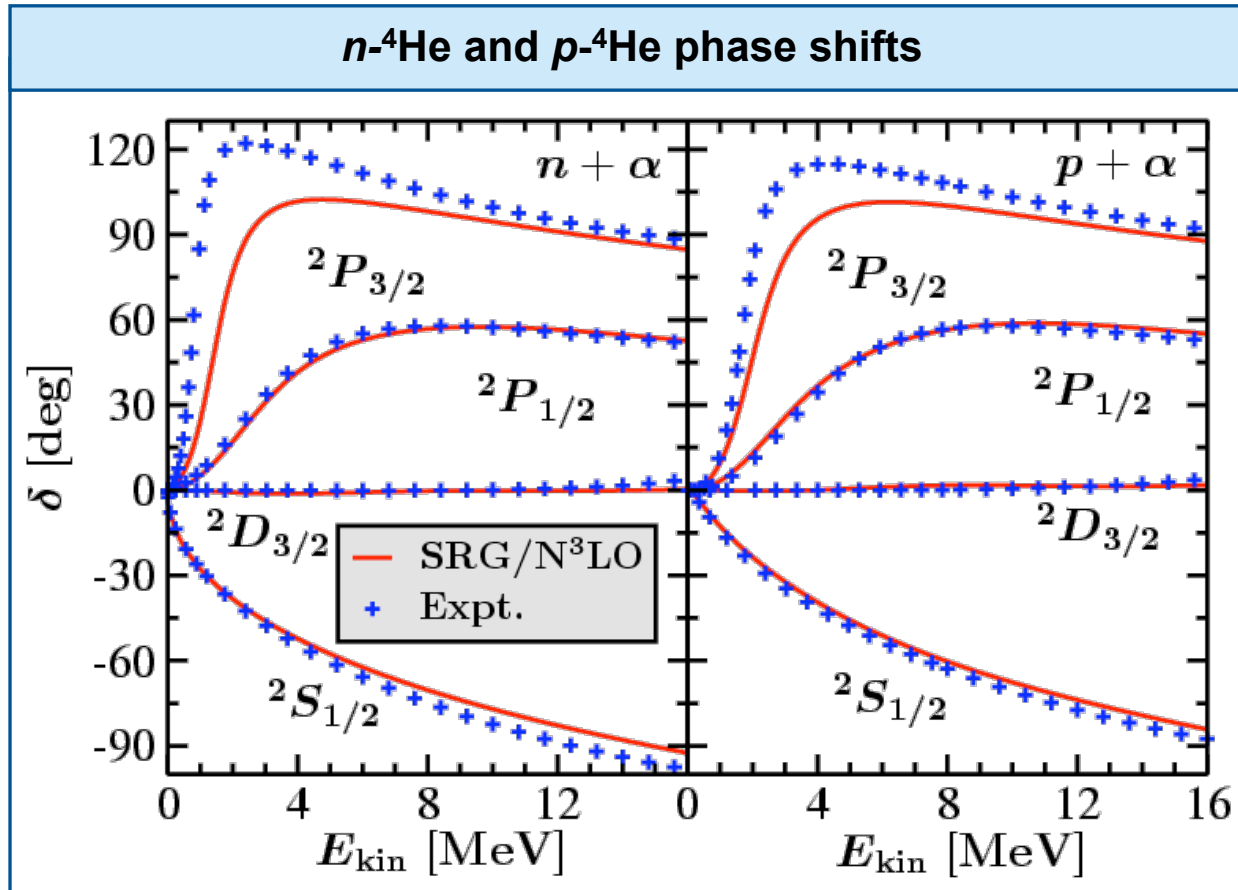
- NCSM/RGM calculations with $n+^3\text{H}(\text{g.s.})$ and $p+^3\text{He}(\text{g.s.})$, respectively.
- $\chi\text{EFT N}^3\text{LO NN}$ potential: convergence reached with **two-body effective** interaction
- Benchmark with Alt, Grassberger and Sandhas (AGS) results [[PRC75, 014005\(2007\)](#)]
 - **What is missing?** - $n+^3\text{H}(\text{ex})$, ^2n+d , $p-^3\text{He}(\text{ex})$, ^2p+d configurations



The omission of **three-nucleon partial waves with $1/2 < J \leq 5/2$** leads to effects of comparable magnitude on the AGS results. **Need to include target excited states!**



n - ^4He phase shifts from SRG-evolved NN interactions

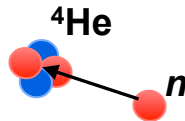


- SRG-evolved interactions (R. Roth)
 - SRG-N³LO
 - SRG-AV18
- convergence reached with **bare** interaction
- ^4He states: g.s., 0^+0
- SRG-AV18 phase shifts present unphysical oscillations

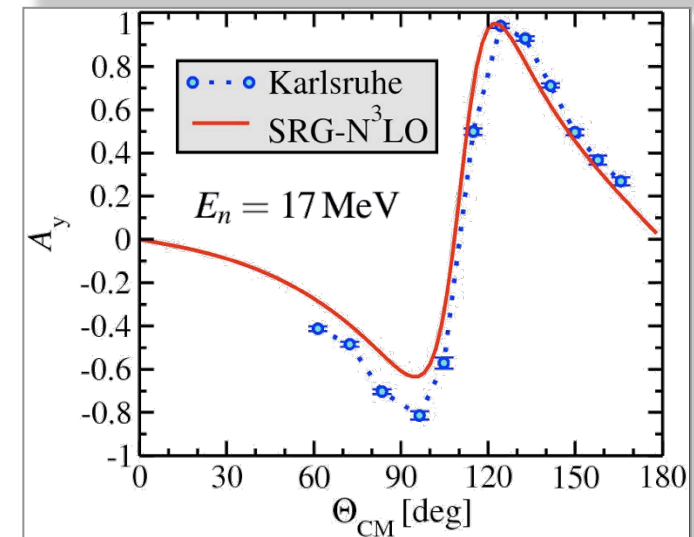
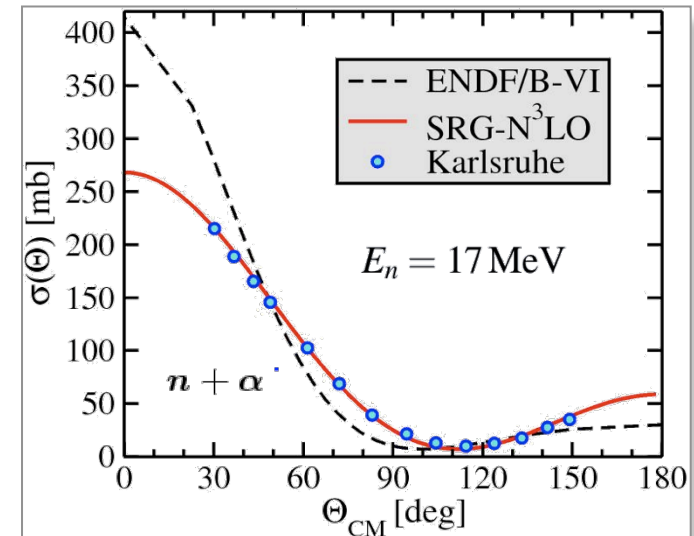
Insufficient spin-orbit strength: $^2P_{3/2}$ underestimated → **NNN needed**

$n+^4\text{He}$ differential cross section and analyzing power

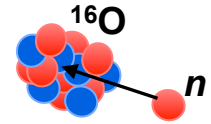
- Neutron energy of 17 MeV
 - beyond low-lying resonances
- Polarized neutron experiment at Karlsruhe
- NCSM/RGM calculations
 - $n+^4\text{He}(g.s.,0+0)$
 - SRG-evolved $N^3\text{LO}$ NN potential
- Good agreement for angular distribution
- Differences for analyzing power
 - A_y puzzle for $A=5$?



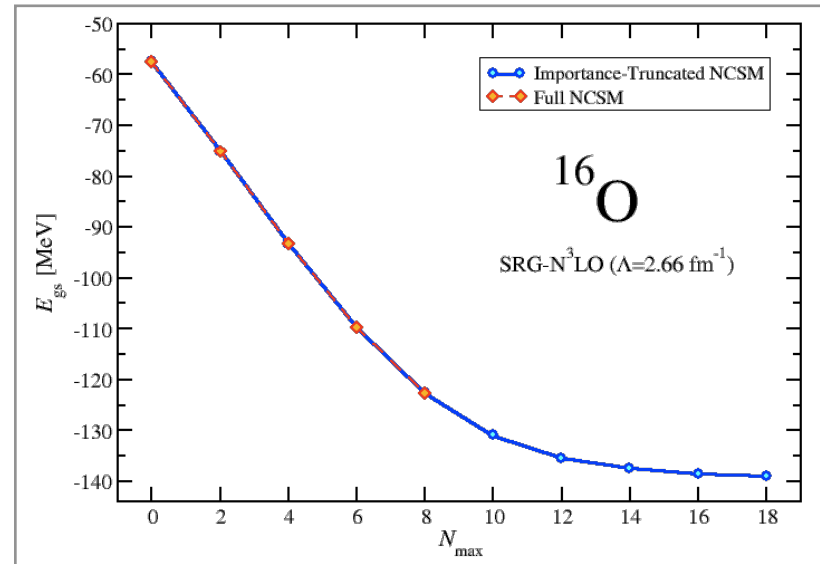
First ever *ab initio* calculation of A_y in for a $A=5$ system. [Strict test of inter-nucleon interactions.](#)



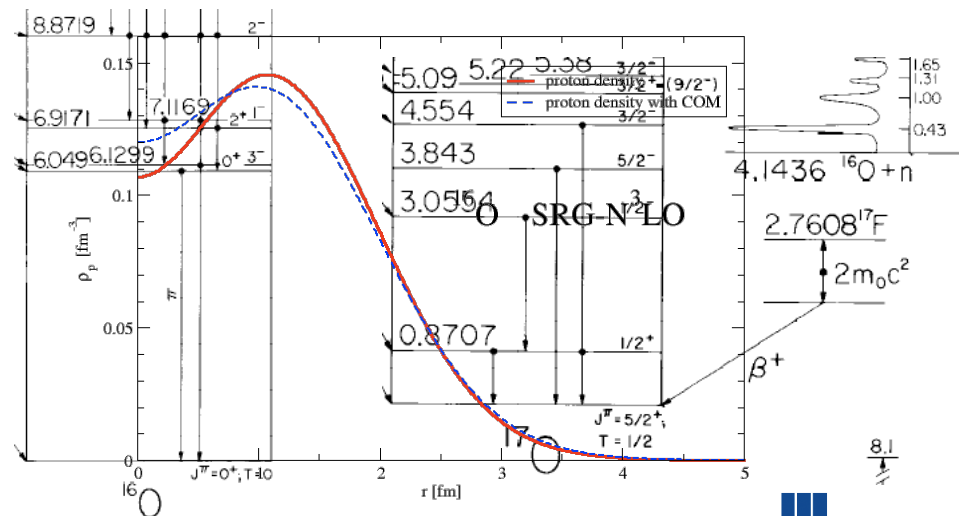
^{16}O ground state, ^{17}O bound states



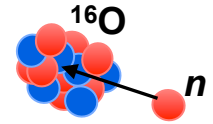
- ^{16}O ground state calculated within importance-truncated NCSM
 - $\geq 4p-4h$ up to $N_{\text{max}}=18$ ($N_{\text{max}}=22$ possible!?), $h\Omega=24$ MeV
 - SRG- N^3LO with $\Lambda=2.66$ fm^{-1}
 - Less overbinding: $E_{\infty} \approx -140$ MeV
 - Benchmarking with full NCSM
 - ^{16}O binding energy up to $N_{\text{max}}=8$
 - **Perfect agreement**



- ^{17}O within *ab initio* NCSM/RGM
 - $1/2^+$ bound: $E_b = -0.88$ MeV wrt ^{16}O
 - $5/2^+$ bound: $E_b = -0.41$ MeV wrt ^{16}O
 - $N_{\text{max}}=19$, $h\Omega=24$ MeV
 - Only ^{16}O ground-state included

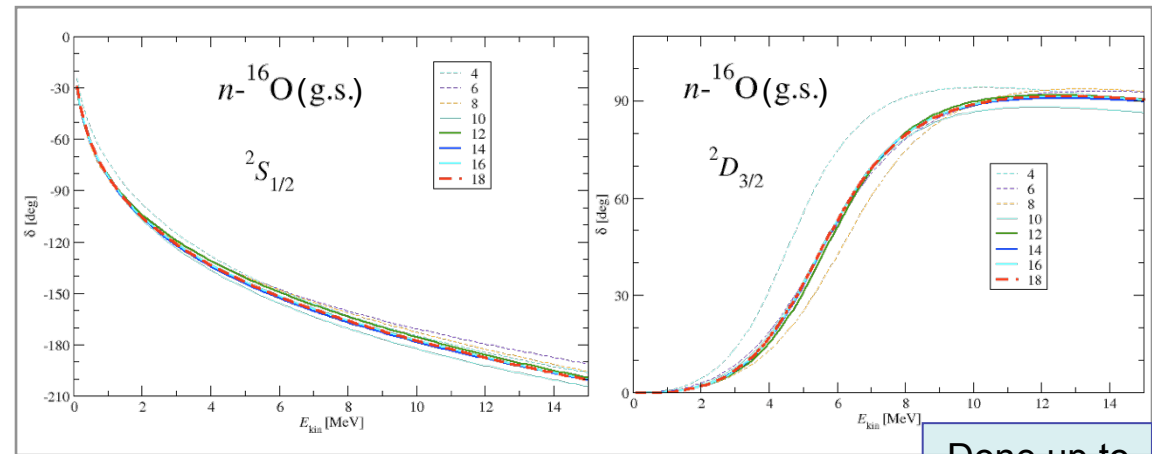


n - ^{16}O scattering with SRG- N^3LO NN potential

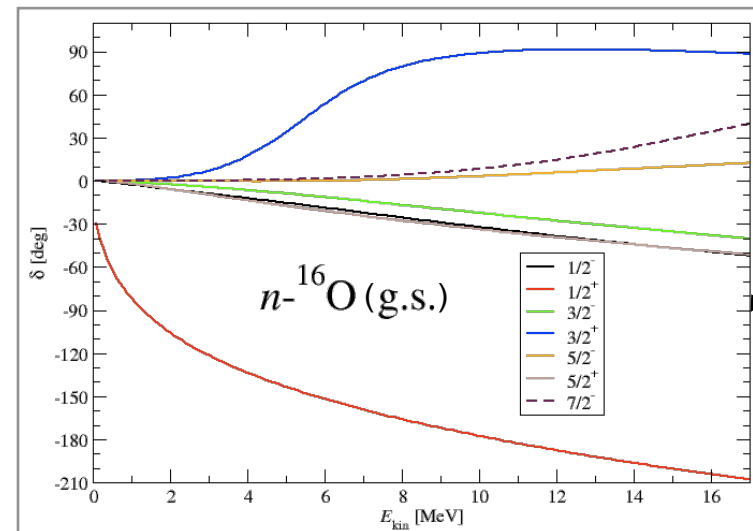


- ^{16}O ground state only
- Phase-shift convergence very good
- Essential to use **large** N_{max}
 - Target wave function
 - Expansion of short-range parts of kernels
 - **IT NCSM** for the target makes it possible

Combining the *ab initio* NCSM/RGM with the **importance-truncated NCSM** highly promising. Access to medium mass nuclei.



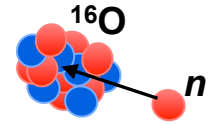
Done up to $N_{\text{max}}=18$ converged



$N_{\text{max}}=18$



n - ^{16}O scattering: Effect of ^{16}O excited states

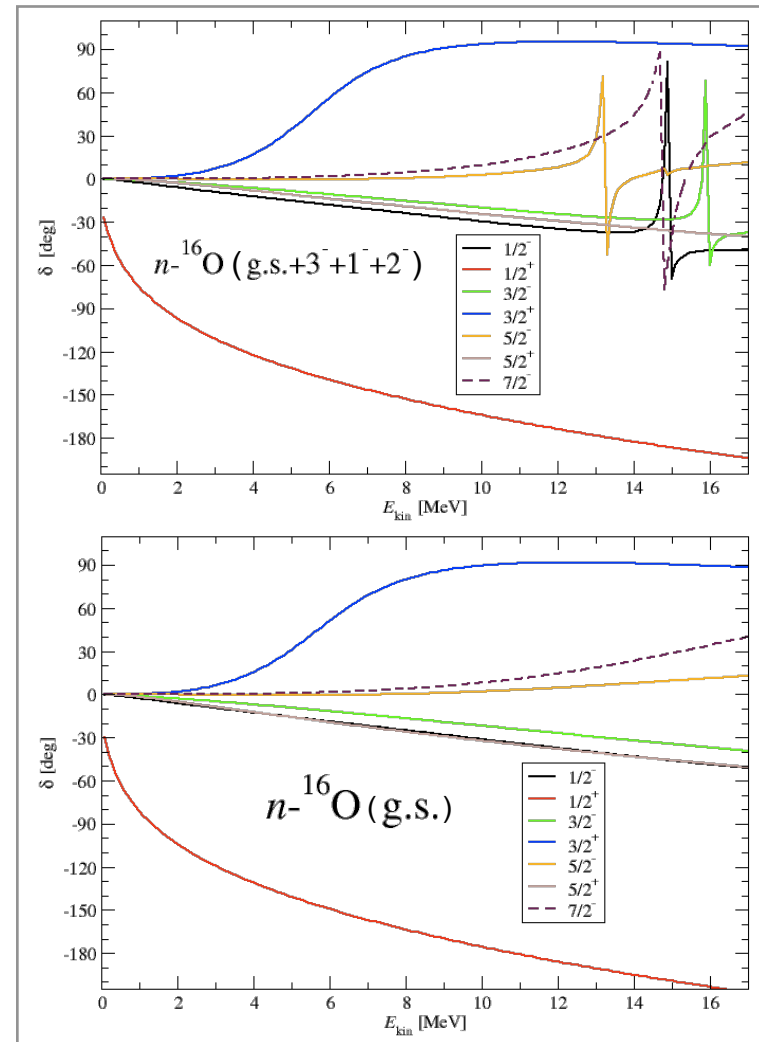
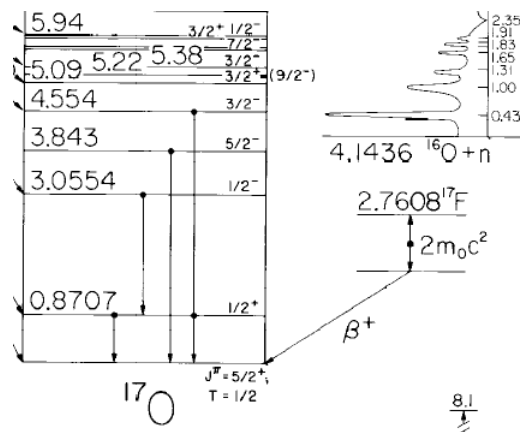


- Need to include ^{16}O excited states (1p-1h...)
- IT NCSM for both the ground state & excited states
- Done up to $N_{\text{max}}=12/13$

Good stability

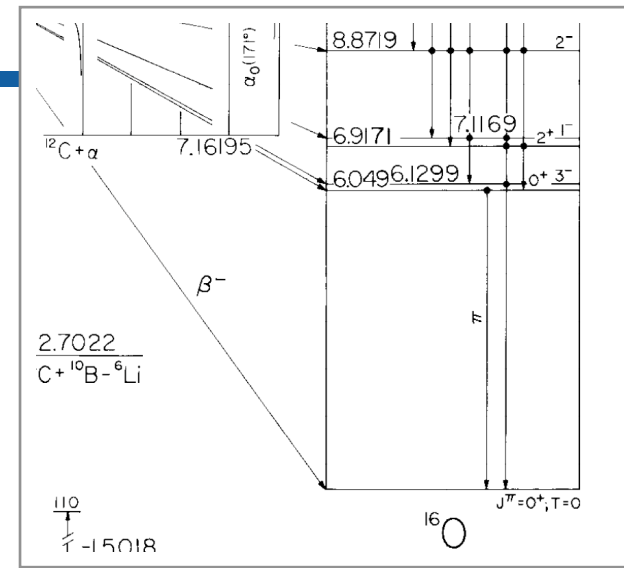
 - g.s. in $N_{\text{max}}=12$
 - $3^-, 1^-, 2^-$ in $N_{\text{max}}=13$
 - Significant increase of binding energies
 - $1/2^+$: $-0.78 \rightarrow -1.03$
 - $5/2^+$: $-0.37 \rightarrow -1.32$ ←

Correct order
 - Appearance of sharp resonances

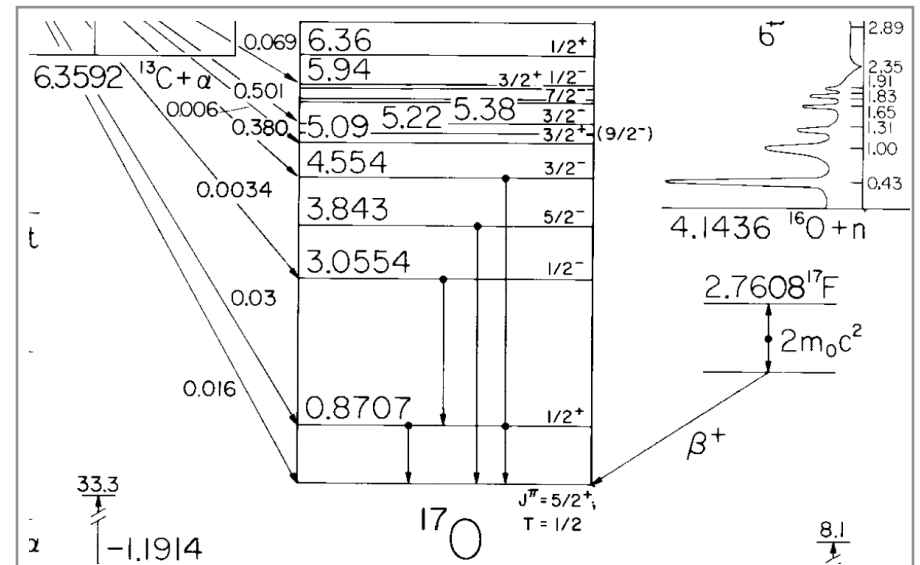


n - ^{16}O scattering: Open issues

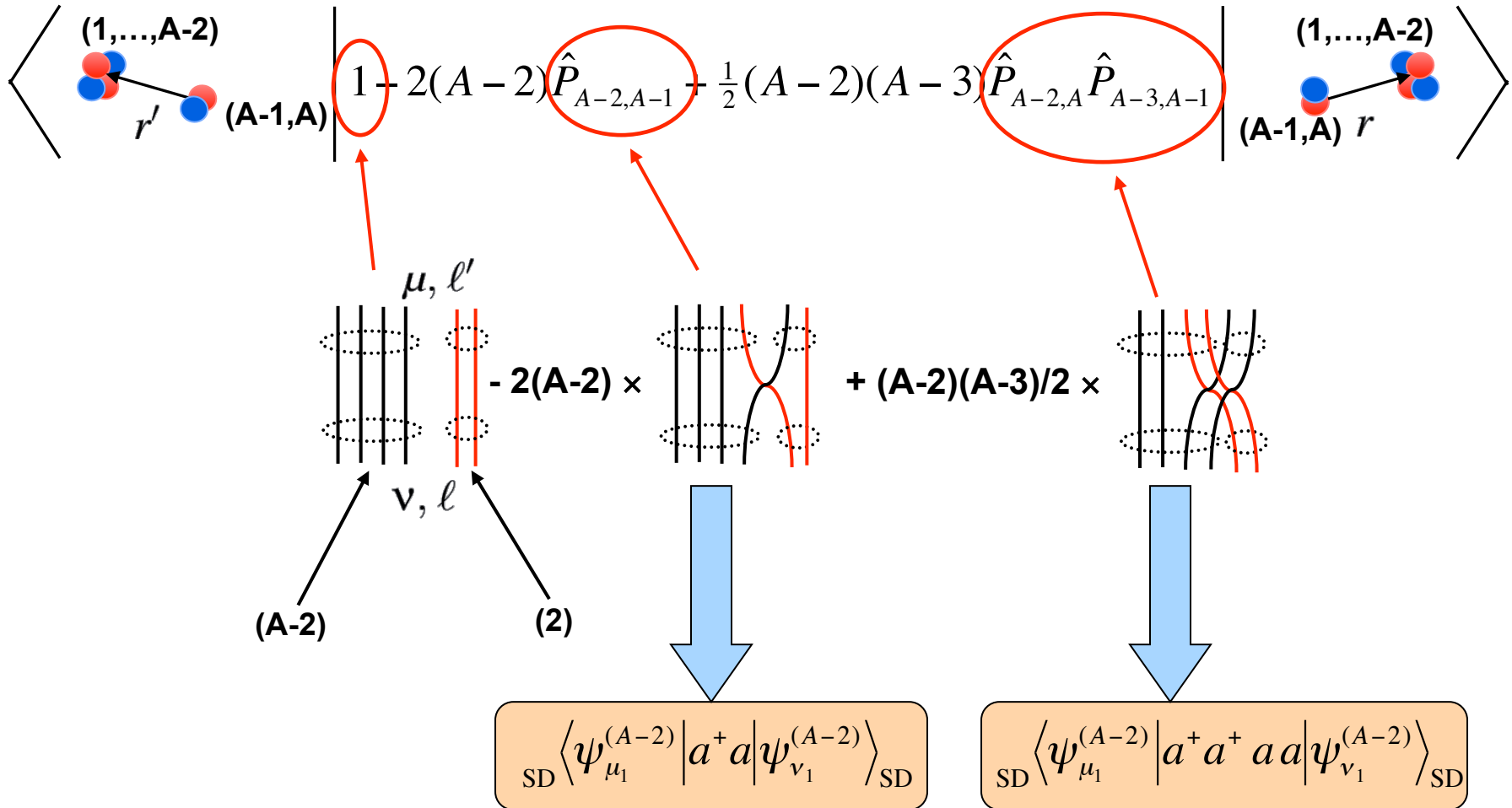
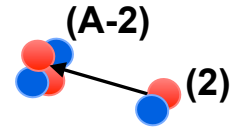
- ^{16}O excited states with the SRG- N^3LO NN potential too high**
 - 3-, 1-, 2- calculated: $\approx 13.3, 15.9, 16.3$ MeV
 - 3-, 1-, 2- experiment: 6.13, 7.12, 8.87 MeV
 - Importance of 3-body force?
 - Density too high?
 - $^{12}\text{C}+\alpha$ not included at present



- $n+^{16}\text{O}$ with the SRG- N^3LO NN potential**
 - 5/2+, 1/2+ underbound
 - 1/2-, 5/2- not bound
 - Resonances too high
 - Impact of incomplete ^{16}O description
 - $^{13}\text{C}+\alpha$ not taken into account

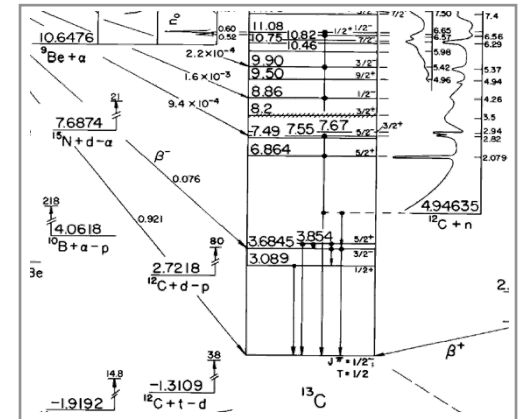


The deuteron projectile: Norm kernel



Conclusions and outlook

- We are extending the *ab initio* NCSM to treat low-energy light-ion reactions
- Our recent achievements:
 - n - ^3H , n - ^4He , n - ^{10}Be and p - $^3,4\text{He}$ scattering phase-shifts with realistic NN potentials (PRL 101, 092501 (2008))
- n - ^{16}O under way:
 - Breakthrough due to the importance-truncated NCSM approach
- Coming next:
 - inclusion of NNN potential terms
 - d , ^3H and ^3He , ^4He projectiles
- Nuclei - complex open many-body systems
 - Bound states, resonances, continuum
- A correct and efficient theoretical description must include all these features
 - Coupling of bound-state theory with cluster theory



(A)

$$|\Psi_A^J\rangle = \sum c_\lambda |A\lambda J\rangle + \sum \int d\vec{r} \varphi_\nu(\vec{r}) \hat{\mathcal{A}} \Phi_{\nu\vec{r}}^{(A-a,a)}$$

$$\begin{pmatrix} H & h \\ h & \mathcal{H} \end{pmatrix} \begin{pmatrix} c \\ \varphi \end{pmatrix} = E \begin{pmatrix} 1 & g \\ g & \mathcal{N} \end{pmatrix} \begin{pmatrix} c \\ \varphi \end{pmatrix}$$

Ab Initio No-Core Shell Model with Continuum

