INT Program *Effective Field Theories and the Many-Body Problem* March 23 – June 5, 2009

The missing three-nucleon forces: Where are they?

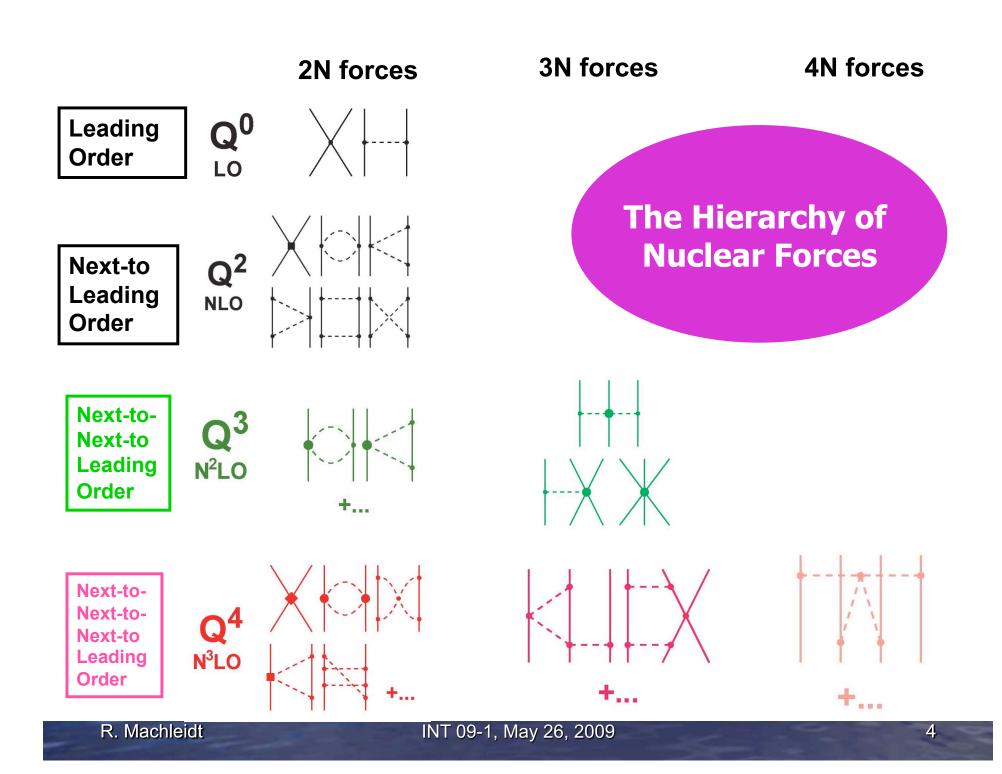
R. Machleidt University of Idaho

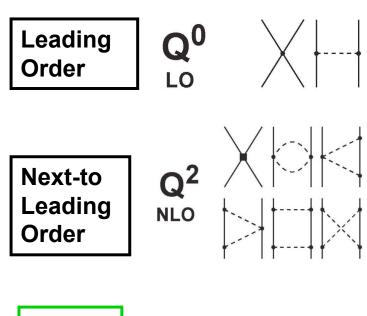
Outline

Nuclear forces from Chiral Perturbation Theory (ChPT): Current status
The 3NF at N2LO
The 3NF at N3LO: Weak expectations
N4LO or Δ-isobars?
Summary

The ChPT approach to nuclear forces

Clear connection to QCD: via symmetries
Degrees of freedom relevant to (lowenergy) nuclear physics: pions and nucleons
Systematic expansion: Q/Λ
Controlled error, predictive power.
Explains the empirically known hierarchy of nuclear forces



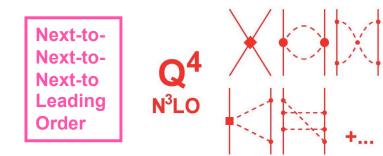


The Hierarchy of Nuclear Forces

Next-to-Next-to Leading Order



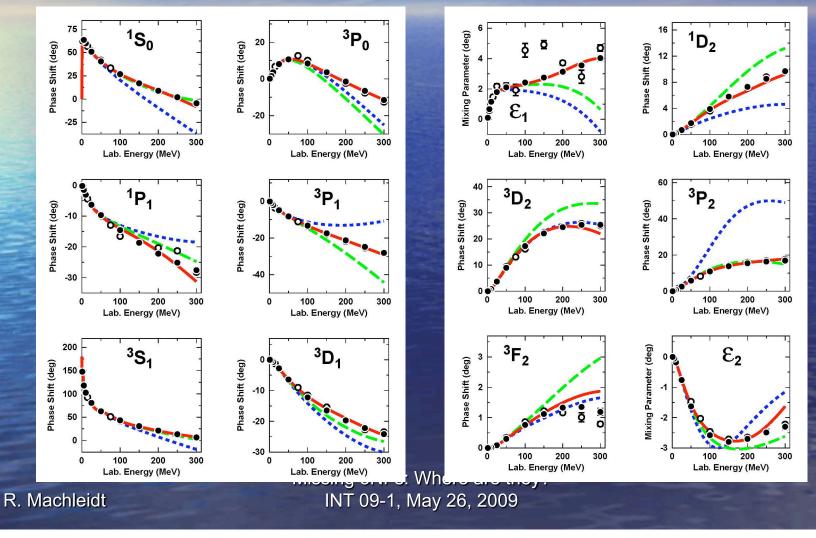
2N forces



R. Machleidt

Phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003). Green dashed line: NNLO Potential, and blue dotted line: NLO Potential by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).



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$\chi^2/{ m datum}$ for the reproduction of the

1999np database

Bin (MeV)	# of data	N ³ LO	NNLO	NLO	AV18
0-100	1058	1.05	1.7	4.5	0.95
100 - 190	501	1.08	22	100	1.10
190 - 290	843	1.15	47	180	1.11
0-290	2402	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003). NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

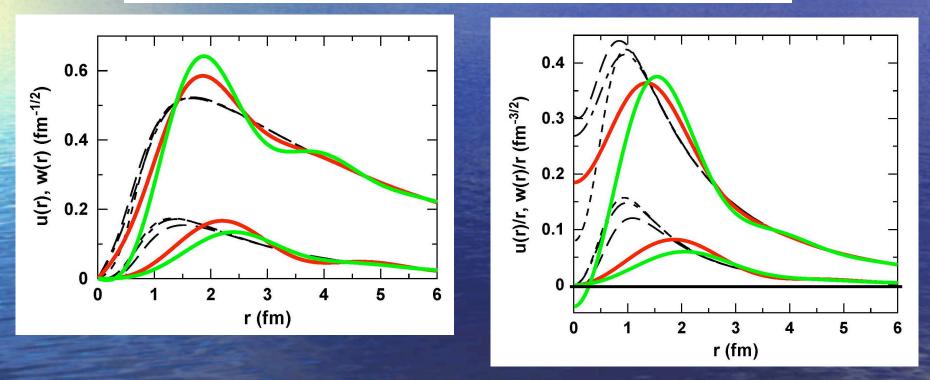
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Deuteron Properties

	Idaho N3LO (500)	Bochum/Juelich N3LO (550/600)	CD-Bonn	AV18	Empirical
Binding energy B_d (MeV)	2.224575	2.218279	2.224575	2.224575	2.224575(9)
Asymptotic S state A_S (fm $^{-1/2}$)	0.8843	0.8820	0.8846	0.8850	0.8846(9)
Asymptotic D/S state η	0.0256	0.0254	0.0256	0.0250	0.0256(4)
Matter radius r_d (fm)	1.978	1.977	1.966	1.967	1.9754(9)
Quadrupole moment Q_d (fm ²)	0.275	0.266	0.270	0.270	0.286(1)
D -state probability P_D (%)	4.51	3.28	4.85	5.76	

Deuteron wave functions of N3LO potentials

red = Idaho, green = Bochum/Juelich Black lines = high-precision pots.



Results from applications of chiral twonucleon forces

Latest Coupled Cluster results for A ≤ 48 using N3LO

Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions

G. Hagen,¹ T. Papenbrock,^{2,1} D. J. Dean,¹ and M. Hjorth-Jensen³

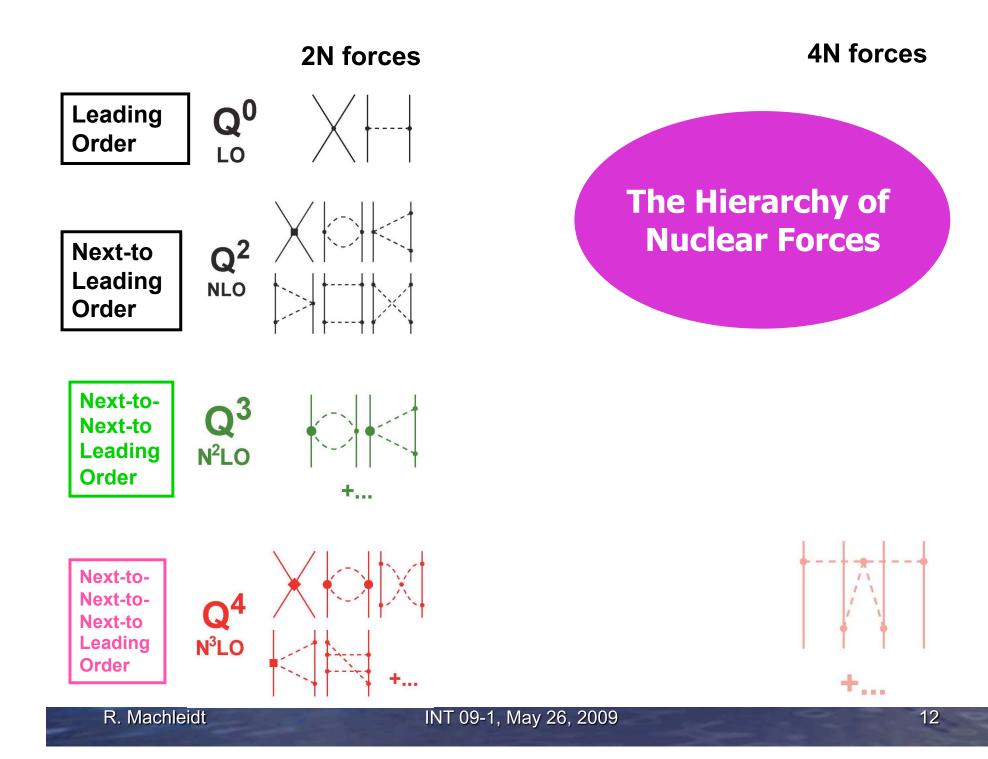
¹Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA ²Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA ³Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway (Received 20 June 2008; published 29 August 2008)

We compute the binding energies, radii, and densities for selected medium-mass nuclei within coupledcluster theory and employ a bare chiral nucleon-nucleon interaction at next-to-next-to-next-to-leading order. We find rather well-converged results in model spaces consisting of 15 oscillator shells, and the doubly magic nuclei ⁴⁰Ca, ⁴⁸Ca, and the exotic ⁴⁸Ni are underbound by about 1 MeV per nucleon within the coupled-cluster singles-doubles approximation. The binding-energy difference between the mirror nuclei ⁴⁸Ca and ⁴⁸Ni is close to theoretical mass table evaluations. Our computation of the one-body density matrices and the corresponding natural orbitals and occupation numbers provides a first step to a microscopic foundation of the nuclear shell model.

Chiral NN potential at N³LO underbinds by ~1MeV/nucleon. (Size extensivity at its best.)

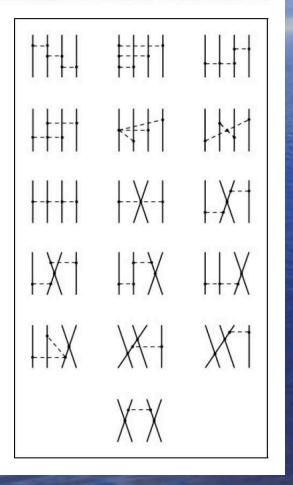
Nucleus	DE / A [MeV]
⁴ He	1.08 (0.73 ^{FY})
¹⁶ O	1.25
⁴⁰ Ca	0.84
⁴⁸ Ca	1.27
⁴⁸ Ni	1.21

	V.	low	/-k			
	λ	E/A		Q	$\Delta E/A$	
	1.9	-15.37	-47.59	.5	-6.82	
	2.2	-13.67	-44	1.23	-4.82	
	2.5	-12.23	-42.39	1.21	-3.68	
3NF	$1 \approx 3.5$	-7.68	-3 6	1.14	0.87	
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4NF at N3LO (leading order)

Epelbaum, Phys. Lett. B639 (2006) 456 [nucl-th//0511025]

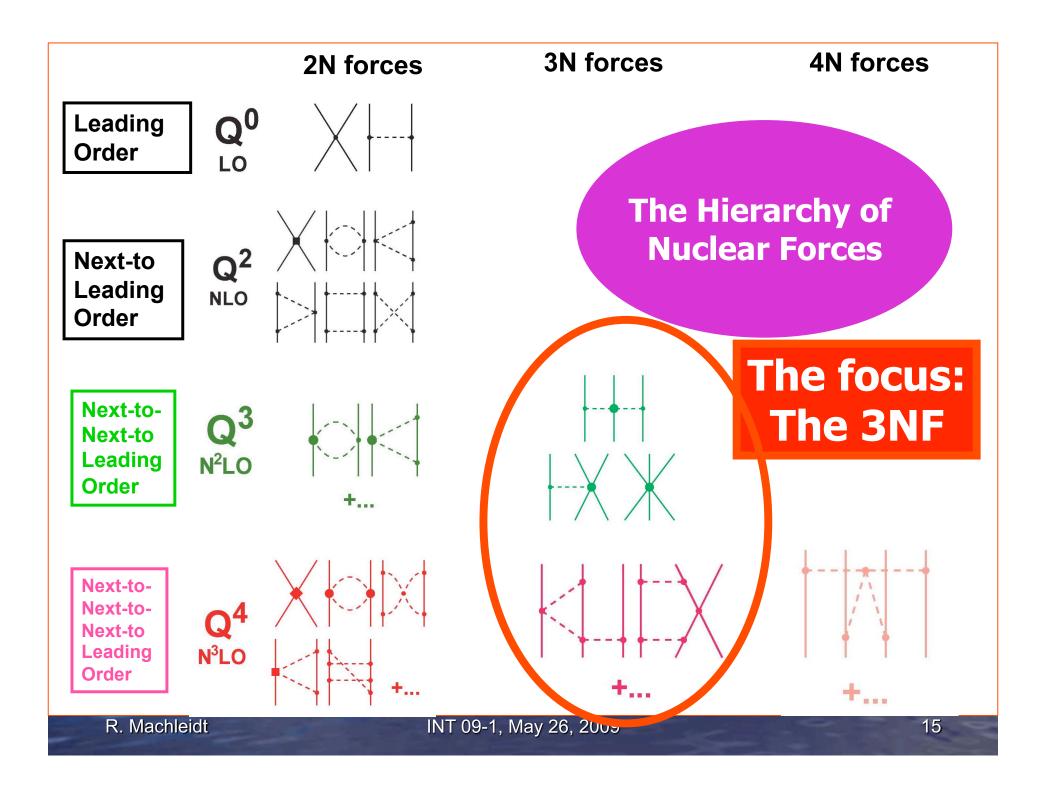


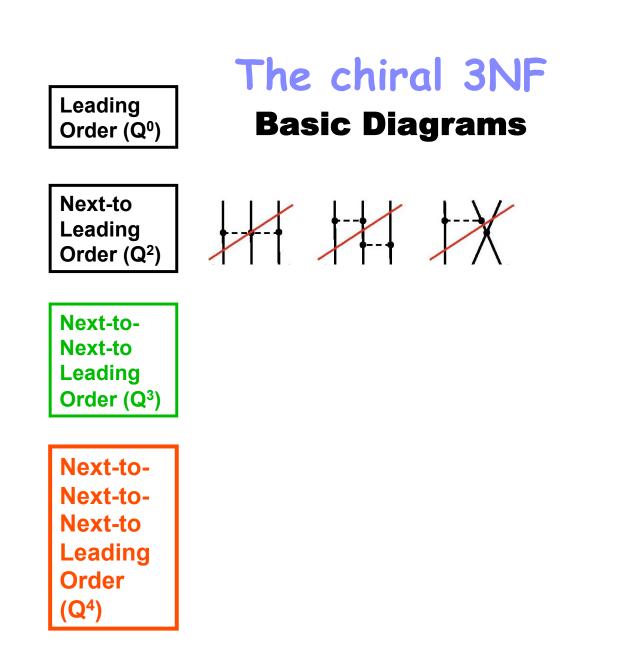
Note that only vertices from $\mathcal{L}_{\pi\pi}^{(2)}, \mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{NN}^{(0)}$ are involved, • no new parameters, • weak. First rough estimate: ≈ 0.1 MeV to α binding.

Essentially negligible, as to be expected.

Interim summary

 2NF o.k. (except for the issue of non-perturbative renormalization)
 4NF (negligibly) small ...





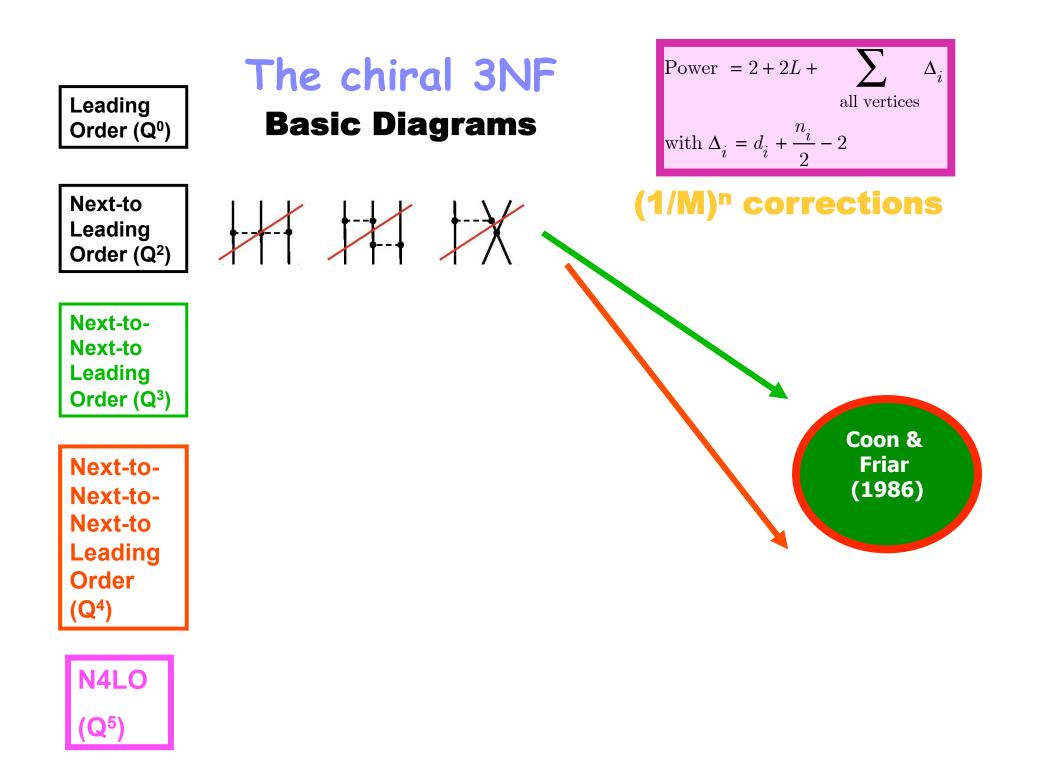
Power = 2 + 2L +

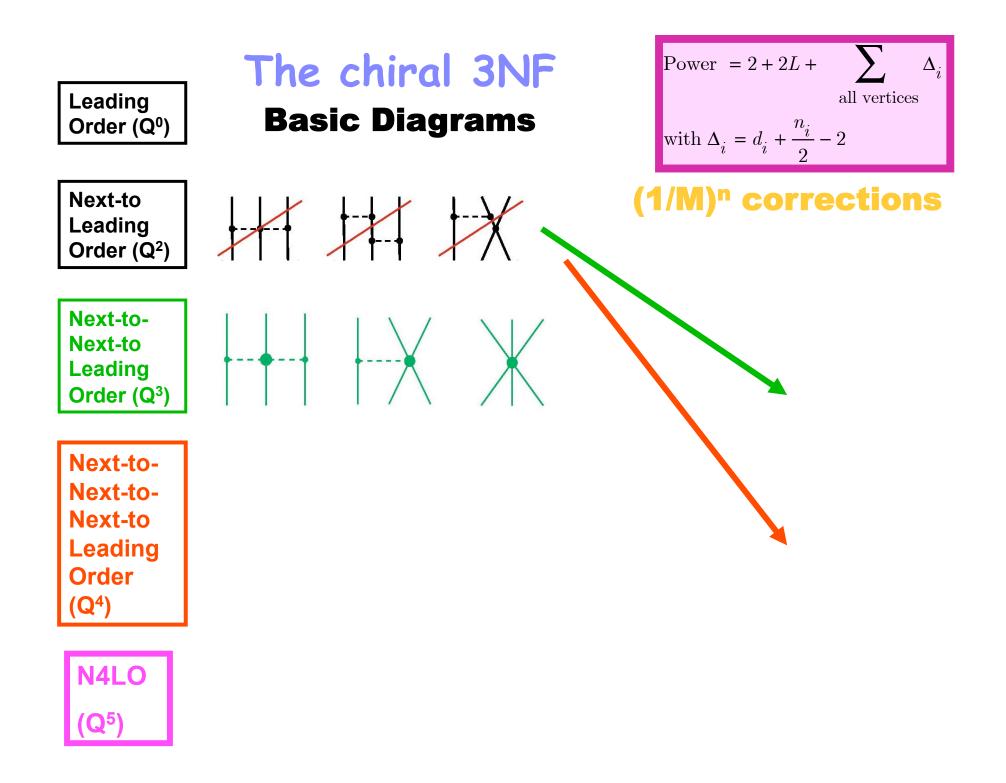
with $\Delta_i = d_i + \frac{n_i}{2} - 2$

 Δ_i

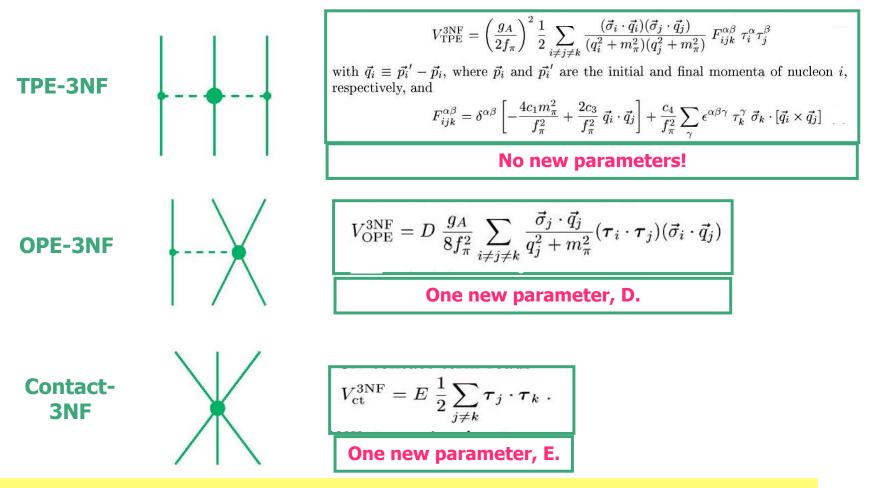
all vertices

N4LO (Q⁵)

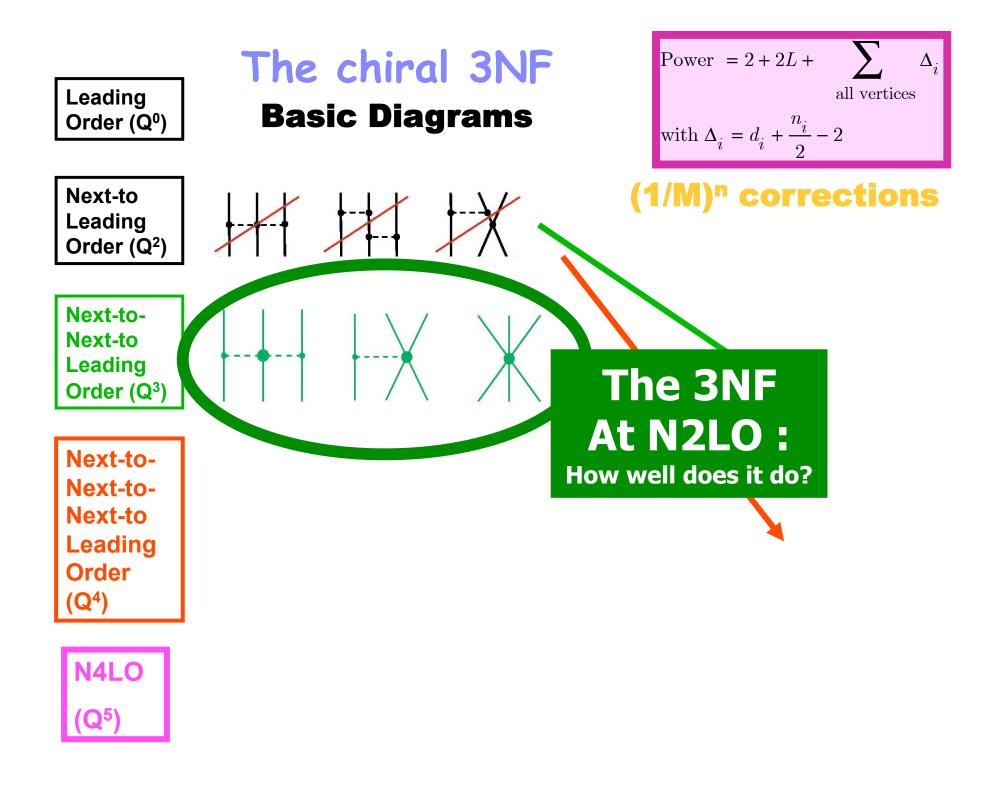




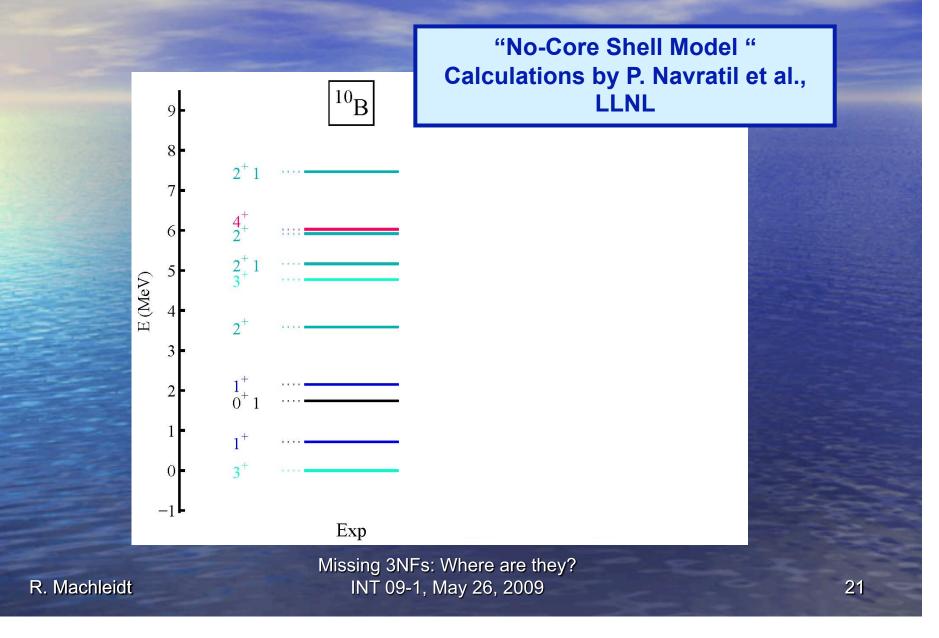
Three-nucleon forces at N2LO



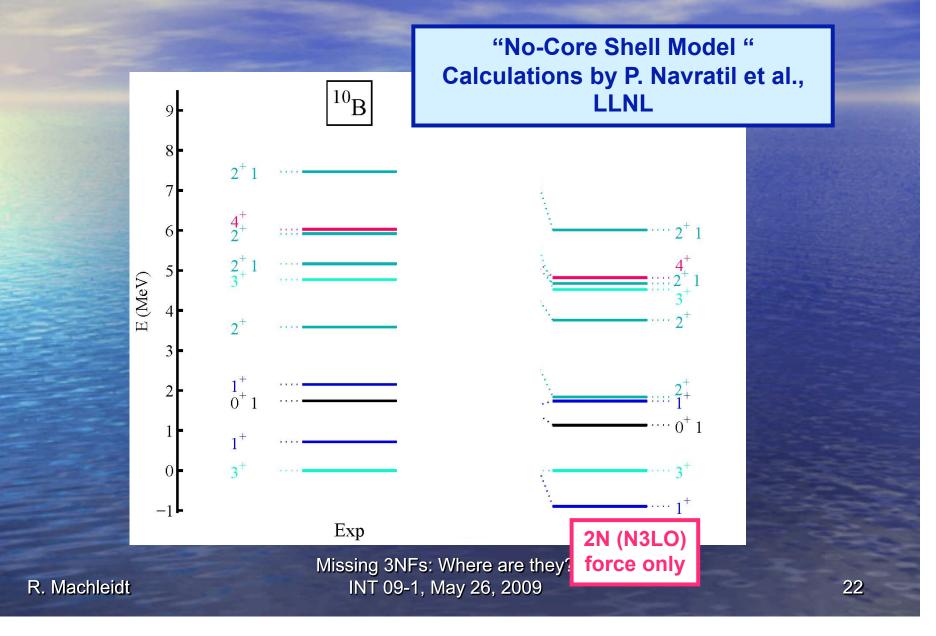
Strategy: Adjust D and E to two few-nucleon observables, e.g., the triton and alpha-particle binding energies. Then predict properties of other light nuclei.



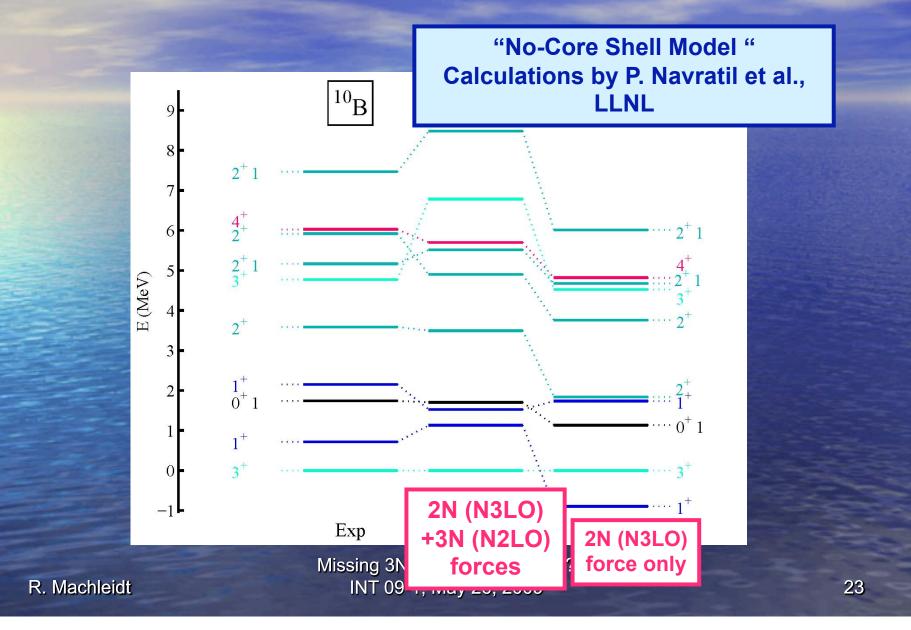
Calculating the properties of light nuclei using chiral 2N and 3N forces



Calculating the properties of light nuclei using chiral 2N and 3N forces

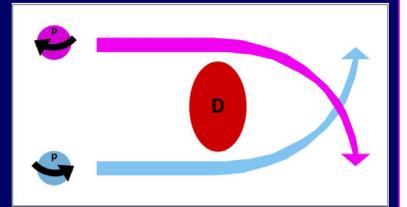


Calculating the properties of light nuclei using chiral 2N and 3N forces



The A_y puzzle at low energies

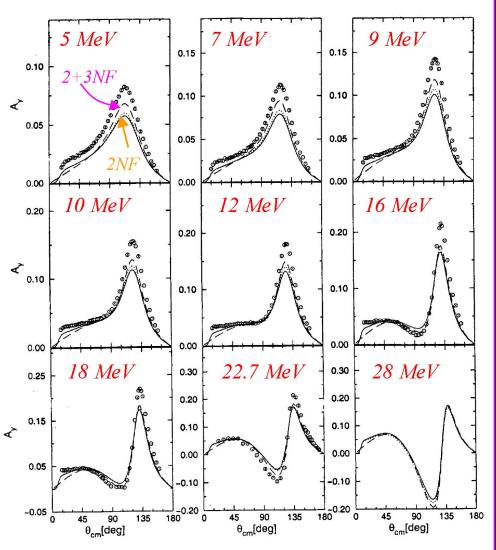




$$\sigma = \sigma_0(1 + pA_y \cos\phi)$$

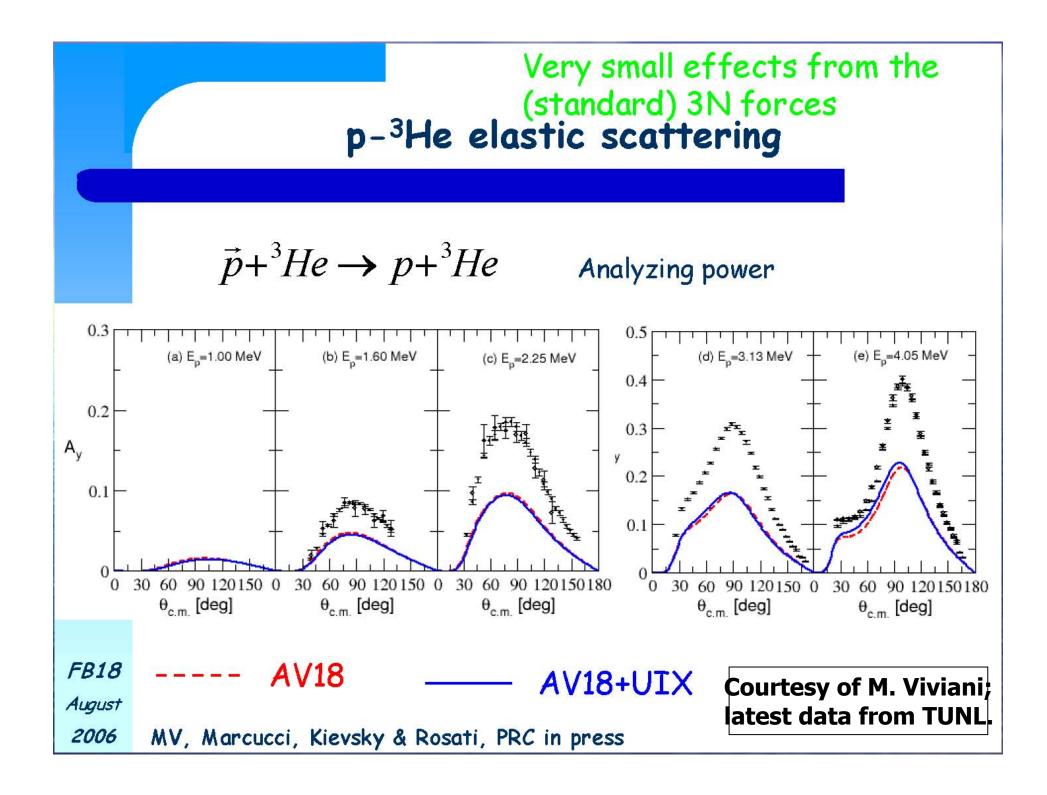
$$\Rightarrow A_{y} = \frac{\sigma_{L} - \sigma_{R}}{\sigma_{L} + \sigma_{R}}$$

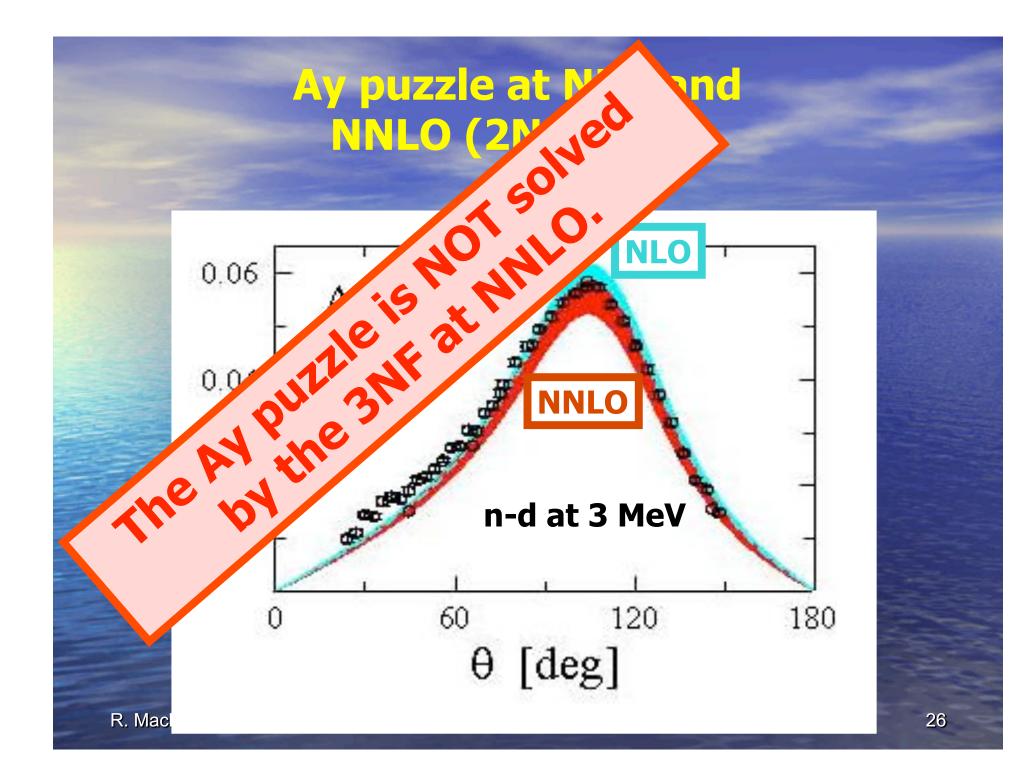
Calculations by Pisa group





Study of three-body systems at KVI





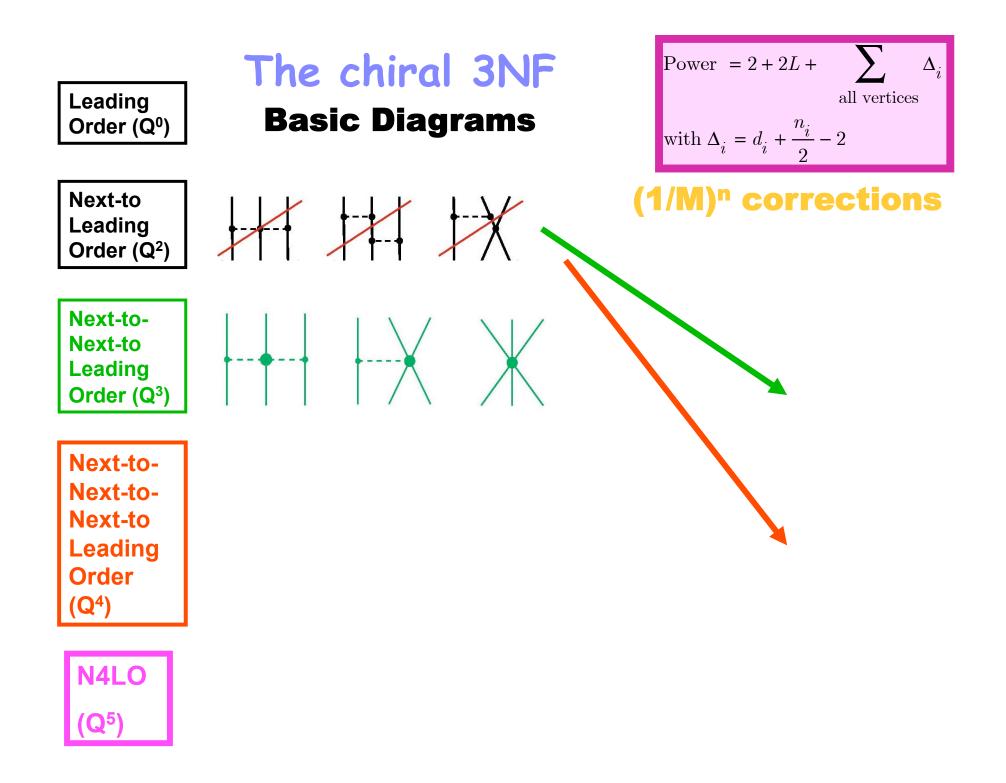
Why do we need 3NFs beyond NNLO?

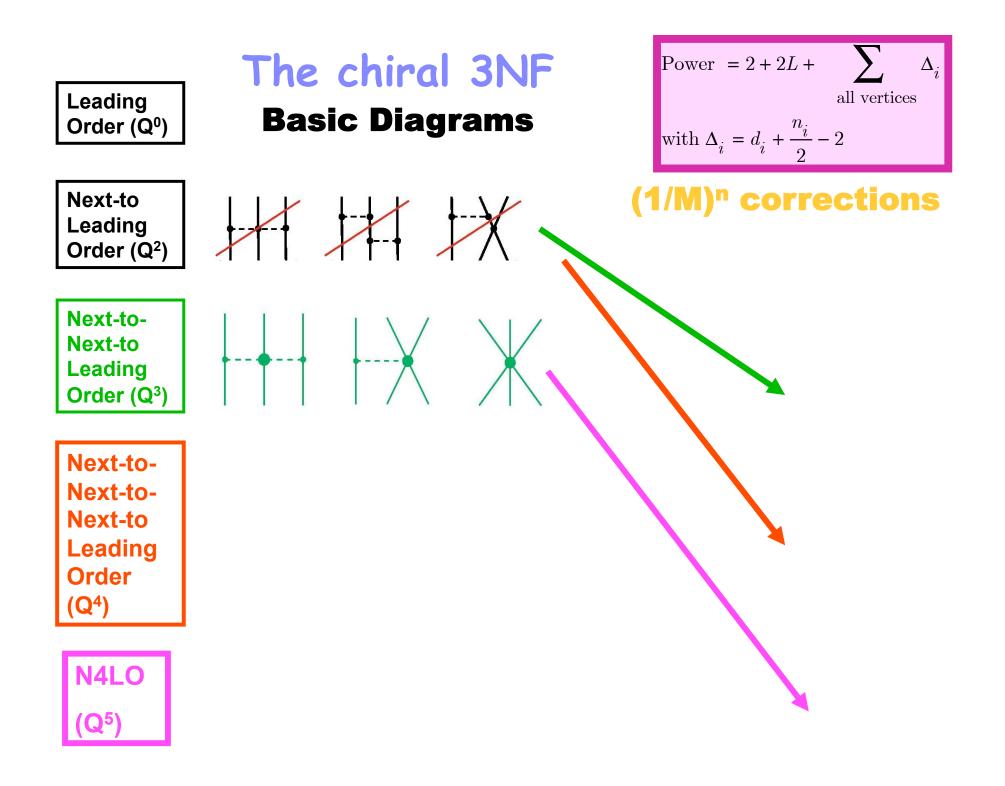
 The 2NF is N3LO; consistency requires that all contributions are at the same order.

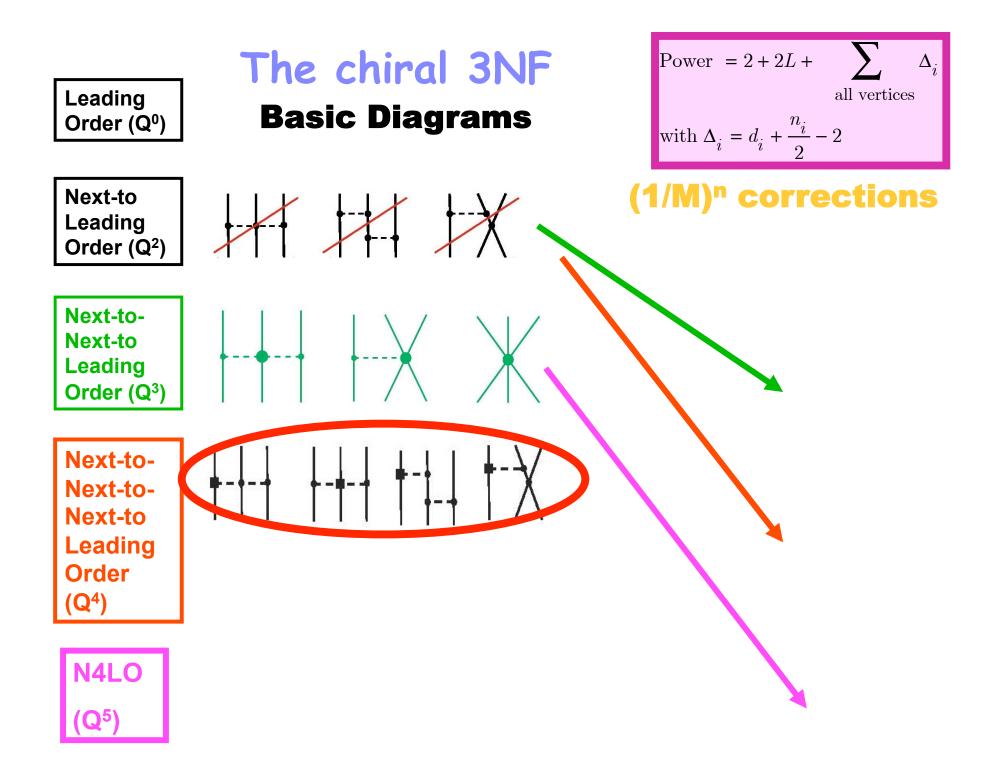
There are unresolved problems in 3N, 4N scattering and nuclear structure.

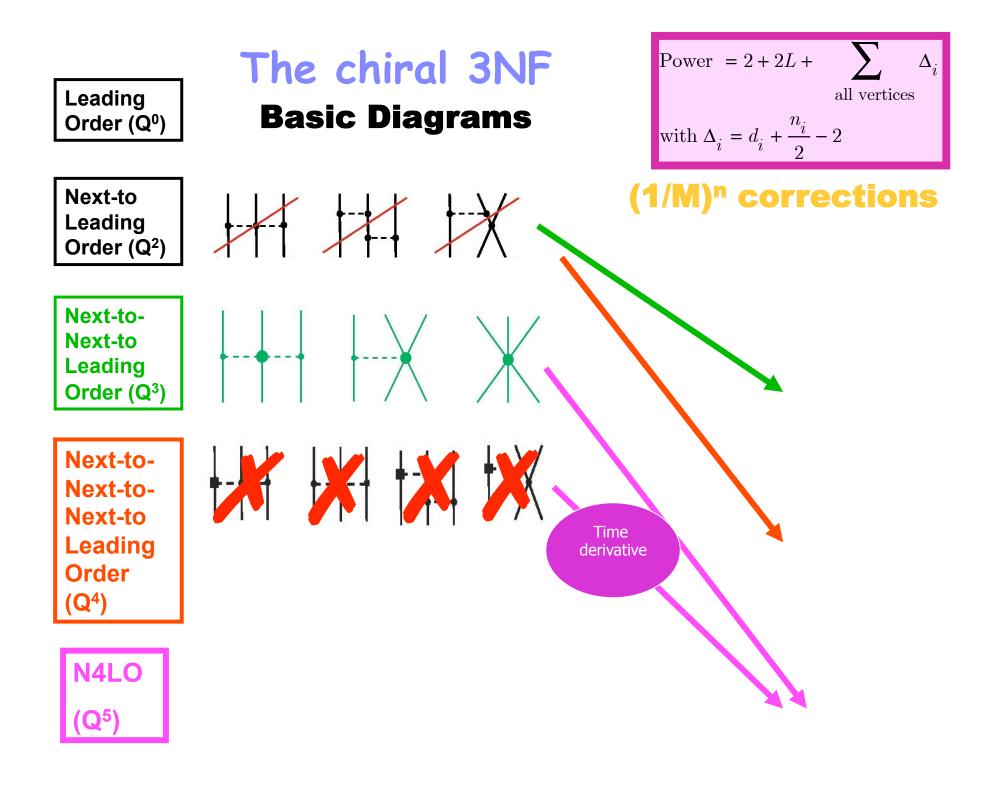
What 3NFs are generated by ChPT beyond NNLO?

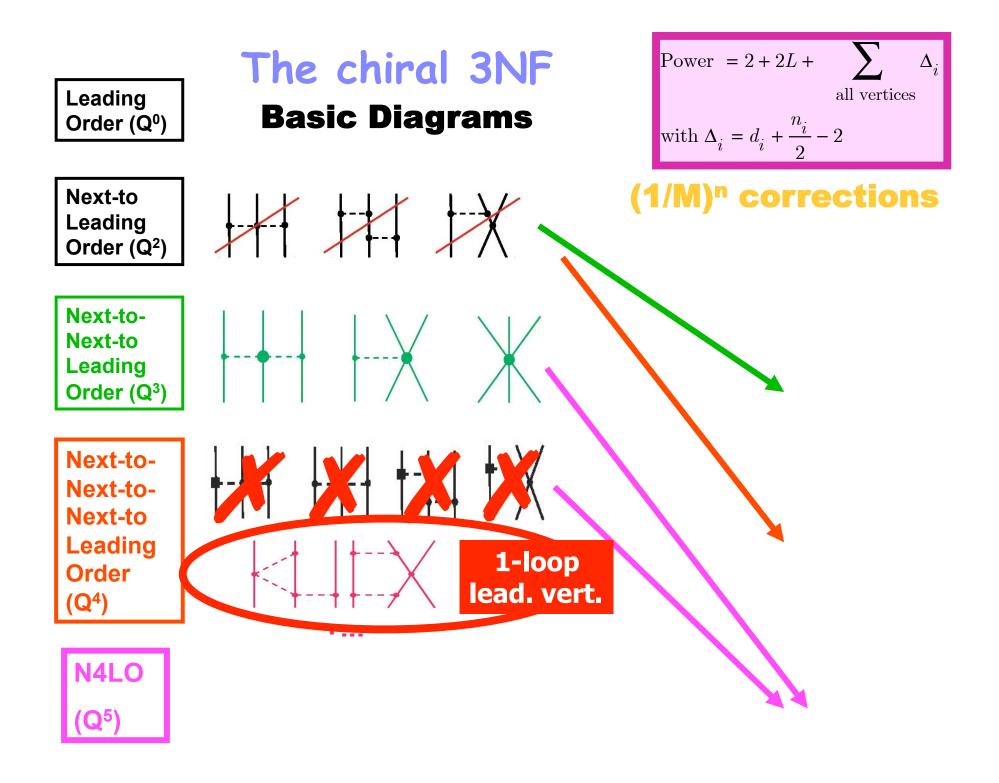
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The 3NF at N3LO explicitly

One-loop, leading vertices

 2π -exchange

$$\phi \cdot \phi \cdot \phi = \left\{ -\frac{1}{4} \right\} + \left[-\frac{1}{4} \right] + \cdots$$

 2π - 1π -exchange

ring diagrams

$$\left| \frac{1}{2} - \frac{1}{2} - \frac{1}{2} \right| = \left| \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}$$

contact- 1π -exchange

contact- 2π -exchange

$$\left| \left\langle \frac{1}{2} \right\rangle = \left| \left\langle \frac{1}{2} \right\rangle + \left| \left\langle \frac{1}{$$

Missing 3NFs: Where are they? INT 09-1, May 26, 2009

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Ishikawa & Robilotta, PRC 76, 014006 (2007)

> Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008)

> > In progress

virtual institute "Spin and strong QCD" (grant VH-VI-231). This work was further supported by the DFG (SFB/TR 16 "Subnuclear Structure of Matter") and by the EU Integrated Infrastructure Initiative Hadron Physics Project under context number RII3-CT-2004-500078.

APPENDIX A: EXPRESSIONS FOR RING DIAGRAMS IN MOMENTUM-SPACE

In this appendix we give lengthy expressions for ring diagrams in Fig. 4 in momentum space. The contributions from diagrams (1) and (2) can be expressed as:

$$\begin{split} V_{\rm freg} &= \vec{a}_1 \cdot \vec{a}_2 \, \tau_3 \cdot \tau_3 \, R_1 + \vec{a}_1 \cdot \vec{q}_1 \cdot \vec{a}_2 \cdot \vec{q}_1 \, \tau_3 \, \tau_3 \, R_2 + \vec{a}_1 \cdot \vec{q}_1 \vec{a}_2 \cdot \vec{q}_1 \, \tau_2 \cdot \tau_3 \, R_3 + \vec{a}_1 \cdot \vec{q}_1 \vec{a}_2 \cdot \vec{q}_1 \, \tau_2 \cdot \tau_3 \, R_4 \\ &+ \vec{a}_1 \cdot \vec{q}_1 \vec{a}_2 \cdot \vec{q}_2 \, \tau_3 \cdot \tau_3 \, R_3 + \tau_1 \cdot \tau_3 \, R_3 + \vec{a}_1 \cdot \vec{q}_1 \vec{a}_2 \cdot \vec{q}_1 \, R_1 + \vec{a}_1 \cdot \vec{q}_1 \vec{a}_2 \cdot \vec{q}_1 \, R_4 + \vec{a}_1 \cdot \vec{q}_2 \vec{a}_2 \cdot \vec{q}_1 \, R_6 \\ &+ \vec{a}_1 \cdot \vec{a}_2 \, \vec{a}_2 \cdot \vec{q}_2 \, \tau_2 \, \tau_3 \, \vec{n}_3 \, \tau_1 \cdot \tau_3 \, \tau_3 \, \vec{n}_1 \, R_1 + \vec{a}_1 \cdot \vec{q}_1 \cdot \vec{q}_1 \cdot \vec{q}_2 \cdot \vec{q}_1 \, \vec{q}_4 + \vec{a}_1 \cdot \vec{q}_2 \vec{a}_2 \cdot \vec{q}_1 \, R_6 \end{split} \tag{A.1}$$

where the functions $R_i = R_i(q_1, q_3, z)$ with $z = \hat{q}_1 \cdot \hat{q}_5$ are defined as follows:

- $$\begin{split} R_1 &= \frac{(-1+z^2)g_A^2M_\pi\left(2M_\pi^2+q_B^2\right)(q_{2}^2q_3+4M_\pi^2\left(zq_1+q_3\right))}{128F^{q_\pi}\left(4(-1+z^2)M_\pi^2-q_B^2\right)(M_\pi^2q_1+q_3^2)} \frac{A(q_2)g_A^2q_2^2\left(2M_\pi^2\left(q_1+zq_3\right)+zq_1\left(-q_1^2+q_3^2\right)\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} \frac{A(q_3)g_A^2q_2^2\left(2M_\pi^2\left(q_1+zq_3\right)+zq_3\left(-q_1^2+q_3^2\right)\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} \frac{A(q_3)g_A^2q_2^2\left(2M_\pi^2\left(q_1+zq_3\right)+zq_3\left(-q_1^2+q_3^2\right)\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2\left(2M_\pi^2\left(q_1+zq_3\right)+zq_3\left(-q_1^2+q_3^2\right)\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2\left(2M_\pi^2\left(q_1+zq_3\right)+zq_3\left(-q_1^2+q_3^2\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2\left(2M_\pi^2\left(q_1+zq_3\right)+zq_3\left(-q_1^2+q_3^2\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2\left(2M_\pi^2\left(q_1+q_3^2\right)+zq_3\left(-q_1^2+q_3^2\right)}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2\left(2M_\pi^2\left(q_1+q_3^2\right)+zq_3^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2q_3^2\left(2M_\pi^2\left(q_1+q_3^2\right)+zq_3^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2} + \frac{A(q_3)g_A^2}{128F^{q_\pi}\left(-1+z^2\right)q_1q_3^2}$$
 - $\frac{A\left(q_{1}\right)g_{A}^{5}\left(2M_{\pi}^{2}q_{2}^{2}+q_{5}\left(-zq_{1}^{2}+\left(2-3z^{2}\right)q_{1}^{2}q_{5}-z\left(-2+z^{2}\right)q_{1}q_{5}^{2}+q_{5}^{3}\right)\right)}{128F^{6}\pi\left(-1+z^{2}\right)q_{5}^{2}}-$

 $\begin{array}{c} I(4:0,-q_1,q_2,0)g_1^2q_2^2 \\ \hline 32P^6(-1+z^2)\left(4(-1+z^2)M_2^2-q_2^2\right)q_3 \left(8\left(-1+z^2\right)M_3^4\left(2zq_1+\left(1+z^2\right)q_3\right)+q_2^2q_3\left(z^2q_1^2+\left(-1+z^2\right)q_1q_3-q_2^2\right)+2M_3^2\left(z\left(-2+z^2\right)q_1^2q_1+\left(1+2z^2\right)q_1^2q_3+3z\left(-2+z^2\right)q_1q_3^2+\left(-3+2z^4\right)q_3^2\right)\right), \end{array} \right)$

$$\begin{split} R_2 &= \frac{A\left(q_2\right)g_A^4q_2^2\left(-2M_{+}^2\left(\left(1+z^2\right)q_1+2zq_3\right)+zq_5\left(\left(1+z^2\right)q_1^2-2q_3^2\right)\right)}{12SF^4\left(-1+z^2\right)^2q_1^2q_2} + \\ & - \frac{A\left(q_5\right)g_A^4\left(M_{\pi}^2\left(2zq_1^2+\left(1+3z^2\right)q_1q_2+2zq_3^2\right)+zq_5\left(-zq_1^2-z^2q_1^2q_3+zq_1q_2^2+q_3^2\right)\right)}{64F^4\left(-1+z^2\right)^2q_1^2q_3} \end{split}$$

 $\begin{array}{c} \frac{4 G(\eta) \delta_{n}^{2}}{12 \delta^{2} r^{2}_{0}(1+2^{2})} \frac{1}{q^{2} q^{2}_{0}} \left[2 M_{n}^{2} \left((1+z^{2}) q^{2}_{0}+z \left((1+z^{2}) q^{2}_{0}+z \left((1+z^{2}) q^{2}_{0}+z \left((1+z^{2}) q^{2}_{0}\right)+\right. \\ \left. q_{1} \left(-(z+z^{2}) q^{2}_{0}+(2-zz^{2}+z^{4}) q^{2}_{0}q_{1}+z \left((1+z^{2}) q^{2}_{0}\right)\right) - \\ \frac{1}{32 \delta^{2} \left(-(1+z^{2}) q^{2}_{0}-(1+z^{2}) q^{2}_{0}+q^{2}_{0}) q^{2}_{0}} \left(q^{2}_{0} \left(-2z^{2} q^{2}_{0}+1 \left((1+z^{2}) q^{2}_{0}\right)-\right. \\ \left. 8 \left(-(1+z) \left((1+z^{2}) q^{2}_{0}+(1+z^{2}) q^{2}_{0}+z \left((1+z^{2}) q^{2}_{$

 $\begin{array}{l} & +\frac{1}{128F^{6}\pi q_{1}^{2}\left(4\left(-1+z^{2}\right)M_{\pi}^{2}-q_{2}^{2}\right)\left(4M_{\pi}^{2}q_{3}+q_{3}^{2}\right)},\\ R_{5} & = & -\frac{zA\left(q_{2}\right)g_{A}^{2}q_{2}^{2}\left(-4M_{\pi}^{2}\left(q_{1}+zq_{3}\right)+q_{3}\left(2zq_{1}^{2}+\left(-1+z^{2}\right)q_{1}q_{3}-2zq_{3}^{2}\right)\right)}{128F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}^{2}} & - \end{array}$

 $\frac{zz(q_{1})d_{1}^{2}}{12gW^{2}(1+z^{2})^{2}q_{1}^{2}q_{1}^{2}}\left(M_{z}^{2}\left(-2z\left(-3+z^{2}\right)q_{1}^{2}+4\left(1+z^{2}\right)q_{1}q_{2}+4zq_{1}^{2}\right)+q_{1}\left(-\left(1+z^{2}\right)q_{1}^{2}-2zq_{1}^{2}q_{2}^{2}+1\left(1+z^{2}\right)q_{2}^{2}+2q_{1}^{2}\left(1+z^{2}\right)q_{1}^{2}+2zq_{1}^{2}q_{1}^{2}+2zq_{1}^{2}\left(1+z^{2}\right)q_{1}^{2}+q_{1}\left(-2zq_{1}^{2}+\left(1-3z^{2}\right)q_{1}^{2}q_{1}^{2}+2zq_{1}q_{1}^{2}+\left(1+z^{2}\right)q_{2}^{2}\right)}{2W^{2}(1-q_{1}^{2})d_{1}^{2}+2W^{2}(1+z^{2})q_{2}^{2}+q_{1}^{2}\left(1+z^{2}\right)q_{1}^{2}+q_{1}^{2}\left(1+z^{2}\right)q_{1}^{2}+2zq_{1}^{2}q_{1}^{2}+2zq_{1}^{2}q_{1}^{2}+2zq_{1}^{2}q_{1}^{2}+2zq_{1}^{2}q_{1}^{2}+2zq_{1}^{$

 $\frac{I(4:0,-q_1,q_5;0)zg_A^6}{32P^6\left(-1+z^2\right)^2q_1\left(-4\left(-1+z^2\right)M_x^2+q_5^2\right)q_4^2}\left(q_2^4q_3\left(\left(1+z^2\right)q_1^2+z\left(-1+z^2\right)q_1q_3-\left(1+z^2\right)q_3^2\right)+2q_4^2+q_5^2\right)q_4^2+q_5^2+qq_5^2+q_5^2+q_5^2+q_5^2+q_5^2+q_5^2+q_5^2+q_5^2+q_5^2+q_5^2+q_5$

$$\begin{split} & 10t^3 \, q_{11}^2 n + 3s \left(1 + 2s^2\right) q_{12}^2 + \left(1 + 2s^2\right) q_{11}^2 + \\ & \tilde{q}_{13} - z \left(5 + s^2\right) q_{11}^2 + \left(-3 - 3s^2 + 4s^4\right) q_{11}^2\right) \right) + \\ & \frac{z_{12} - q_{12}}{(z_{11} - q_{11})} \\ & \frac{z_{12} - q_{12}}{(z_{11} - q_{11})} \\ & \frac{z_{12} + 2sq_{12}^2}{(z_{12} - q_{11} + (1 + 2s^2) q_{12})} \\ & - \frac{z_{12} + 2sq_{12}^2}{(z_{12} - q_{11})} \\ & \frac{z_{12} + sq_{12} + q_{12} + q_{12} q_{12} q_{12} \\ & \frac{z_{12} + sq_{12} + q_{12} q_{12} q_{12} \\ & \frac{z_{12} + sq_{12} + q_{12} q_{12} q_{12} \\ & \frac{z_{12} + sq_{12} + q_{12} q_{12} q_{12} \\ & \frac{z_{12} + sq_{12} + q_{12} q_{12} q_{12} \\ & \frac{z_{12} + sq_{12} + q_{12} q_{12} q_{12} \\ & \frac{z_{12} + sq_{12} + sq_{12} q_{12} \\ & \frac{z_{12} + (z_{12} + s^2 + 2s^2 + q_{12} + (z_{12} + s^2 - q_{12} + z_{12} + z_{12} + q_{12} + z_{12} \\ & \frac{z_{12} + (z_{12} + z_{12} + 2s^2 + 2s^2 + q_{12} + z_{12} + z_{12} + z_{12} + q_{12} + z_{12} \\ & \frac{z_{12} + (z_{12} + z_{12} + 2s^2 + 2s^2 + q_{12} + z_{12} + z_{12} + z_{12} + z_{12} + z_{12} + z_{12} \\ & \frac{z_{12} + (z_{12} + z_{12} + 2s^2 + 2s^2 + 2s^2 + q_{12} + z_{12} \\ & \frac{z_{12} + (z_{12} + z_{12} - z_{12} + z$$

$$\begin{split} & -\frac{s^2}{9}(\frac{g}{g} + \frac{g}{2g}(-2sg^2 + (1-3s^2)\frac{g}{4g}n + 2sg_4g^2 + (1+s^2)\frac{g}{2})) \\ & +\frac{1}{128P^{5q}(-1+s^2)\frac{g}{4q}} + s(-1+s^2)g_4g_9 - (1+s^2)\frac{g}{4g}) + \\ & +\frac{g}{4g^2_{1q}}(\frac{g}{4g}n_{1}(1+s^2)g_4^2 + s(-1+s^2)g_4g_9) + \\ & +\frac{1}{10s^2}\frac{g}{4g}n + s(1+2s^2)g_4g^2 + (2-3s^2 + s^4)g_4^2)) - \\ & -\frac{g}{4g}n - s(-1+s^2)g_4g^2 + (-3-3s^2 + s^4)g_4^2)) - \\ & -\frac{g}{4M^2_{1}+q^2_{1}}(g_1 + g_1(SM^2_{1} + Sg^2_{1} + g^2_{1})) + \\ & +\frac{1}{128P^{5q}q_{1}} + \\ & \frac{1}{2s}(M^2_{1} + g^2_{1})g_1 - n \\ & \frac{1}{s}(M^2_{1} + g^2_{1}) - \\ \end{split}$$

 $\begin{array}{l} \overline{d_s^2-q_s^2} g_3(4M_s^2+q_0^2) & (\left[5+z^2\right) q_1^2 q_2^2 q_3^2 +8M_s^6 \left(z\left(-3+4z^2\right) q_1^2+t\right) \\ + \left[2+2M_s^4 \left(4z\left(-1+z^2\right) q_1^4+(77-36z^2) q_1^2 q_3+2z\left(33+8z^2\right) q_1^2 q_2^2+2M_s^2 q_1 q_3+2z\left(10+z^2\right) q_1^4+2z\left(9+2z^2\right) q_1^2 q_3+2y\left(-7z^2\right) q_1^2 q_2^2+ \end{array}\right) \end{array}$

 $\begin{array}{l} \sup_{q_{1} < q_{1} < q_{$

The ring diagrams

From: Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008)

$+ \frac{3A(q_0)g_h^x\left[2M_t^2 + q_0^2\right]\left((1 + z^2)q_1 + 2zq_3\right)}{256F^n\pi(-1 + z^2)^2q_1^2} - \frac{z^2}{25}g_1^x+\frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + 3z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + 3z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + 3z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + 3z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + 3z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + 3z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + (1 + z^2)q_1q_3 + 2zq_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]\left(2zq_1^2 + q_2^2\right)}{226F^n\pi(-1 + z^2)^2q_1q_3} + \frac{3A(q_0)g_h^x\left[2M_t^2 + q_2^2\right]}{226F^n\pi(-1 + z^2)^2} + \frac{3A(q_0)g_h$

 $\begin{array}{l} \frac{q_2^2}{(2-q_2^2)} \left(-q_2^2 \left((1+z^2) q_1^2+z \left(3+z^2\right) q_1 q_1 + \left(1+z^2\right) q_1^2\right) + \right. \\ \left. \left. \left(2+z^2\right) q_1 q_3 + \left(1+2z^2\right) q_5^2\right)\right), \\ \left. \left(2+z^2\right) q_1 q_3 + \left(1+2z^2\right) q_5^2\right)\right), \end{array}$

$$\begin{split} \frac{1}{2} & \int_{1}^{1} \frac{3 z A \left(q_{2}\right) \beta_{1}^{A} \left(2M_{\pi}^{2} + q_{2}^{2}\right) \left(\left(1 + z^{2}\right) q_{1} + 2zq_{3}\right)}{286F^{2} \left(-1 + 1 + 2^{2}\right)^{2} q_{1}^{2} q_{3}} + \\ \frac{1 + z^{2} \left(q_{1}\right)}{286F^{2} \left(-1 + 1 + 2^{2}\right)^{2} q_{1}^{2} q_{3}} - \\ \frac{1 + z^{2} \left(q_{1}\right)}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2} q_{3}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{3}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2} q_{1}^{2} q_{1}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2} q_{1}^{2} q_{1}^{2}}{286F^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2} \left(-1 + z^{2}\right)^{2} q_{1}^{2}}{28} - \\ \frac{1 + z^{2}$$

$$\begin{split} & \frac{+ q \tilde{q}}{(2 - q)_{2R}} - (-q \tilde{q} \left(\left(1 + z^2 \right) q \tilde{q} + z \left(3 + z^2 \right) q q q + \left(1 + z^2 \right) q \tilde{q} \right) + \\ & \left(2 + z^2 \right) q q q + \left(1 + 2z^2 \right) q \tilde{q} \right) \right) , \\ & \left(2 + z^2 \right) q q q + \left(1 + 2z^2 \right) q \tilde{q} \right) + \\ & \left(1 + z^2 \right) q q \tilde{q} + 2z q q + 2Z q + 2Z q q +$$

 $\begin{array}{c} -256F^{6}\pi\left(-1+z^{2}\right)^{2}q_{1}^{2}q_{3}\\ +q_{1}^{2}\\ +q_{2}^{2}\right)\\ M_{\pi}^{2}+q_{2}^{2})q_{3}\left(q_{2}^{2}\left(-2q_{1}^{2}+z\left(-5+z^{2}\right)q_{1}q_{3}-2q_{3}^{2}\right)+\right.\\ \end{array}$

 $\frac{3 z g_A^2 M_\pi \left(2 M_\pi^2 + q_2^2\right)}{2 56 F^6 \pi q_1 \left(-4 \left(-1 + z^2\right) M_\pi^2 + q_2^2\right)} + \frac{3 z g_A^2 M_\pi \left(2 M_\pi^2 + q_2^2\right)}{2 56 F^6 \pi q_1 \left(-4 \left(-1 + z^2\right) M_\pi^2 + q_2^2\right) q_3},$

 $-\frac{3A\left(q_{2}\right)g_{A}^{6}\left(2M_{x}^{2}+q_{2}^{2}\right)\left(zq_{1}+q_{3}\right)\left(q_{1}+zq_{5}\right)}{286F^{6}\pi\left(-1+z^{2}\right)q_{1}q_{3}}-\\ \frac{z\left(-4+z^{2}\right)q_{1}q_{3}^{2}+q_{3}^{2}+2M_{x}^{2}\left(zq_{1}+q_{3}\right)\right)}{\pi\left(-1+z^{2}\right)q_{3}}-$

$$\begin{split} & \frac{1}{\tau(1+z^2)} \frac{1}{qq_2^2 + zq_3^2 + 2M_{\pi}^2 (q_1 + zq_3)]}{\pi (-(1+z^2) q_1} + \\ & \frac{1}{\tau(2)} \frac{1}{q_2^2} \left(-\frac{q_2^2 (q_1^2 - z (-3 + z^2) q_1 q_3 + q_3^2) +}{(q_3 + (1+z^2) q_3^2)} \right) \\ & (q_3 + (1+z^2) q_3^2) \right) , \end{split}$$

 $\frac{i\eta_1q_3 + 2q_5^2) + q_1\left(zq_1^2 + (-1 + 2z^2)q_1^2q_1 + zq_1q_2^2 + q_3^2\right)}{266F^{\pi}(-1 + z^2)q_1^2q_2^2} + \frac{1}{2}\left(\frac{1}{2}\left(2q_1^2 + 2q_1q_2 + (-1 + z^2)q_1^2+ q_1(q_1^2 + zq_1q_2 + (-1 + 2z^2)q_1q_2^2 + zq_3^2\right)\right)}{266F^{\pi}(-1 + z^2)q_1^2 + 2q_1^2q_2 + (-1 + 2z^2)q_1q_2^2 + zq_3^2\right)}$

 $\begin{array}{c} \frac{I(4:0,-q_1,q_2;0)g_{12}^{*}g_{12}}{(64F^{*}(-1+\tau^2))g_{12}^{*}(-4(-1+\tau^2))g_{12}^{*}+g_{12}^{*})(g_{12})(-(2M_{\pi}^{*}+q_{1}^{2})(2M_{\pi}^{*}+q_{1}^{2}+q_{1}^{2}+q_{1}^{2})+}\\ 2g_{12}g_{13}g_{13}(-(-M_{\pi}^{4}+q_{1}^{2})g_{12}^{*})+2(M_{\pi}^{4}+q_{1}^{2}+q_{1}^{2})(M_{\pi}^{4}+q_{1}^{2}+q_{1}^{2})+\\ 2g_{13}(8M_{\pi}^{4}+q_{1}^{4}+q_{1}^{4}+M_{\pi}^{4}(q_{\pi}^{4}+q_{1}^{2})))- \end{array}$

 $\frac{3ZP^{\sim}(-1+z^{\circ})}{22R(q_2) g_A^4 \left(\left(1+z^2\right) q_1^2+z \left(3+z^2\right) q_1 q_3+\left(1+z^2\right) q_3^2\right)}{128F^6 \pi \left(-1+z^2\right)^2 q_1 q_3}$

$$\begin{split} & \frac{1}{(+q_1^2)} \left(-2z^2 \left(4M_1^4 q_1^2 + q_1^2 \right) + \right. \\ & + q_1^2 + q_1^2 \right) \left(q_1^2 q_2^2 + 2M_1^2 \left(q_1^2 + q_2^2 \right) \right) - \\ & \left. q_1^2 \left(q_1^4 + 4q_1^2 q_2^2 + q_2^2 \right) \right) \right) \right) . \end{split}$$
(A.2) s the magnitudes of the corresponding three-momenta i. Further, the function $I(d: p_1, p_2, p_3; p_4)$ refers to

$$\begin{split} \frac{1}{1} & -\frac{1}{M_{\pi}^2} + i\epsilon \left(1 + p_i\right)^3 - M_{\pi}^2 + i\epsilon \left(\cdot \cdot (1 + p_i) + i\epsilon \cdot (\Lambda, 3)\right) \\ p_i & \text{ For the case } p_i^2 = 0 \text{ which we are interested in, it san space } d\left(i + p_i p_i^2, p_i\right) \\ \text{ an space } d\left(i + p_i p_i^2, p_i\right) \\ \text{ supposing in For } R_i \text{ can be written as} \\ \text{ 3} : 6, -\tilde{q}_i, \tilde{q}_i\right). \qquad (A.3) \end{split}$$

 $_{i}\vec{\sigma}_{3} \cdot \vec{q}_{1} \tau_{1} \cdot \tau_{2} S_{3} + \vec{\sigma}_{1} \cdot \vec{q}_{1}\vec{\sigma}_{3} \cdot \vec{q}_{3} \tau_{1} \cdot \tau_{2} S_{4}$ $- \vec{q}_{1} \cdot \vec{q}_{3} \times \vec{\sigma}_{1} \tau_{1} \cdot \tau_{2} \times \tau_{3} S_{7},$ (A.6)

 $\frac{q_3^2)}{128F^6\pi} - \frac{A(q_3) g_A^4(2M_\pi^2 + q_3^2)}{128F^6\pi} + -$

 $+\frac{z^2(q_3)}{(r_1^2 q_1^2)} + -\frac{-3z^2(q_1q_3+2zq_3^2)}{(r_1q_3+2zq_3^2)} +$

 $(z^2) q_3) +$

 $(r^2) q_1 q_3 - 2z q_5^2)$

 $\frac{\frac{2}{9}g_3}{g_1} + \frac{zA(q_2)g_4^A\left(-2q_1^2 + z\left(-5 + z^2\right)g_1g_3 - 2q_3^2\right)}{128F^6\pi\left(-1 + z^2\right)^2q_1q_3} - \frac{1}{2}$

 $\frac{12k^{2}\pi(-1+x^{2})\cdot q_{1}q_{2}}{(-1+x^{2})\cdot q_{1}q_{2}},$ $\frac{2q_{1}}{(-1)}J(d;\vec{0},\vec{q}_{1}) + \frac{2(4M^{2}+(d-2)(q_{1}-q_{3})^{2})}{(d-1)q_{1}q_{1}q_{1}(q_{1}-q_{3})^{2}}J(d;\vec{0},\vec{q}_{1}+\vec{q}_{3})]]$ $\frac{1-x^{2}(q_{1}-q_{1})}{(q_{1}-q_{1})},$ $\frac{1-x^{2}(d+1)q_{1}q_{1}q_{1}(q_{1}-q_{3})^{2}}{(d-1)q_{1}q_{1}q_{1}(q_{1}-q_{3})^{2}}J(d;\vec{0},\vec{q}_{1}+\vec{q}_{3})]$

$\begin{array}{l} \frac{q_3)(2q_1+q_3)}{q_3^{3}}J(d:\vec{0},\vec{q}_1) \\ \frac{(-2q_3)}{q_3}J(d:\vec{0},\vec{q}_3) - \frac{2(4M^2+(d-2)(q_1+q_3)^2)}{(d-1)q_1q_3(q_1+q_3)^2}J(d:\vec{0},\vec{q}_1+\vec{q}_3)\Big]\Big\}, \end{array}$

 $\eta_A^4 (2zq_1 + (1 + z^2) q_3) +$

 $+2zq_1^2+(1+3z^2)q_1q_3+2zq_3^2)$

 $\frac{(-3 + z^2) q_1 q_3 + q_3^2}{(-1 + z^2)} - \frac{A (q_1) g_A^4 q_1 (q_1 + zq_3)}{128 F^6 \pi (-1 + z^2)} +$

 $\frac{+q_3) + 2M_{\pi}^2(q_1 + zq_3)}{-1 + z^2)q_1q_3^2} + \frac{zA(q_3)g_A^4(2M_{\pi}^2 + q_3^2)}{256F^6\pi(-1 + z^2)q_1q_3}$

$$\begin{split} & \big[\frac{J(d:\vec{0},\vec{q}_{1})}{q_{5}^{2}+q_{1}q_{3}} + \frac{J(d:\vec{0},\vec{q}_{5})}{q_{1}^{2}+q_{1}q_{3}} - \frac{J(d:\vec{0},\vec{q}_{1}+\vec{q}_{3})}{q_{1}q_{3}} \big] \\ & + \frac{J(d:\vec{0},\vec{q}_{1}+\vec{q}_{3})}{q_{1}q_{3}} \Big] \Big\}, \end{split}$$

 $\frac{-q_3)(2q_1 - q_3)}{-q_5)^3}J(d:\vec{0},\vec{q_1})$

 $(-z)^2 \left[\frac{J(d:\vec{0},\vec{q_1})}{q_3^2 - q_1q_3} + \frac{J(d:\vec{0},\vec{q_3})}{q_1^2 - q_1q_3} + \frac{J(d:\vec{0},\vec{q_1} + \vec{q_8})}{q_1q_3} \right]$

re individual terms in the expressions for R_i and S_i are singular virities, however, cancel in such a way that the resulting terms in usofille to obtain a representation for functions R_i and S_i which is lar, the singularities at $z=\pm 1$ can be avoided if one expresses the

 $8F^6\pi (-1+z^2)^2 q_3$

 $+ z^{2}) q_{3}^{2})$

<u>s)</u>,

⁵) and uses certain linear combinations of two-point functions and o-point function is defined as $\frac{P_1^{\ell}}{(\vec{k} - \vec{p}_1)^2 + M_2^2} \frac{1}{(\vec{k} - \vec{p}_2)^2 + M_2^2}$. (A.10)

N. Kalantar-Nayestanaki and E. Epelbaum, Nucl. Phys. News 17, 22 (2007) [arXiv:nucl-th/0703089]
 D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001 (2003) [arXiv:nucl-th/0304018].

Missing 3NFs: Where are they? INT 09-1, May 26, 2009

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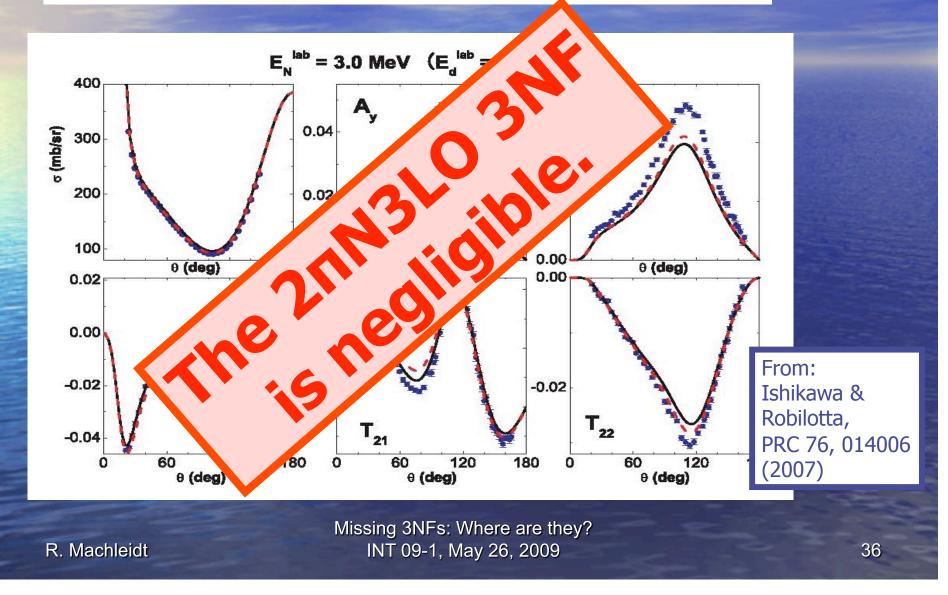
(A.7)

(A.8)

(A.9)

Proton-deuteron elastic scattering

Black line: 2NF only Red dashed line: $2NF + 3NF(N2LO+2\pi N3LO) \approx 2NF + 3NF(N2LO)$



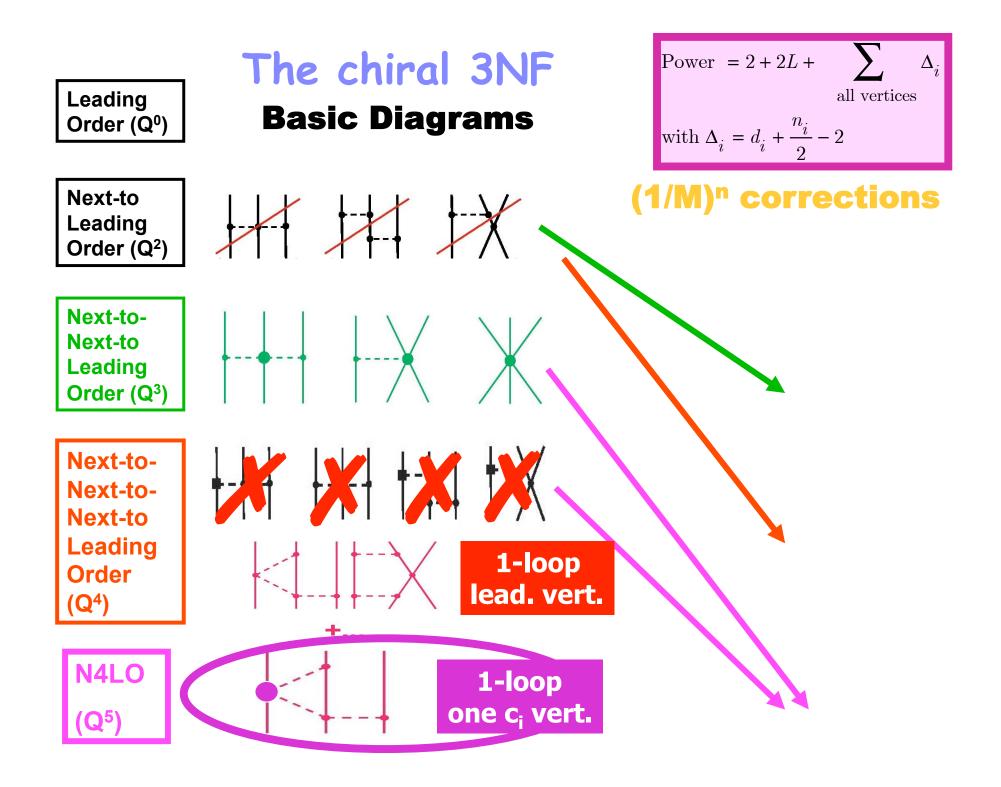
Interim balance

The 3NF at N3LO may be generally weak.
Reason: only "leading" vertices are involved, which are known to be weak (e.g., from NN).
Thus, the 3NF at N3LO may not solve any of the outstanding problems.

What now?

Go to the next order!

R. Machleidt



What to expect from those N4LO 1-loop diagrams?

Compare to "similar" 2NF diagrams.



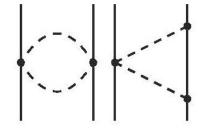
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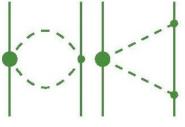
Corresponding 2NF contributions

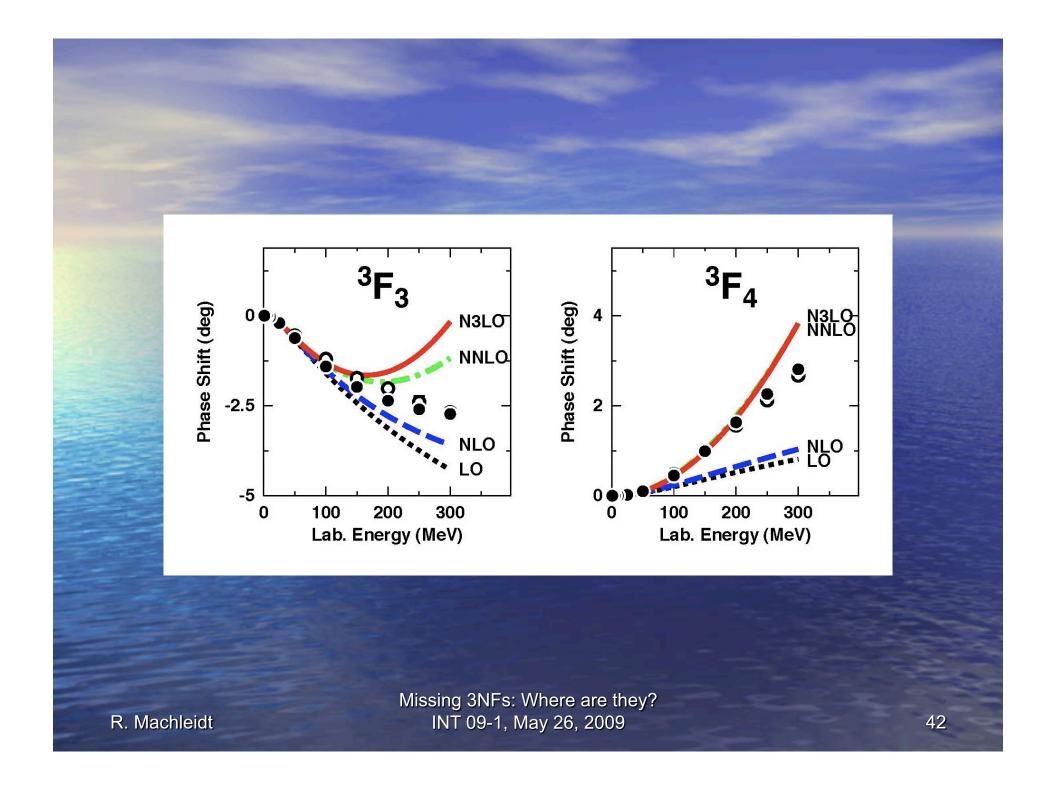




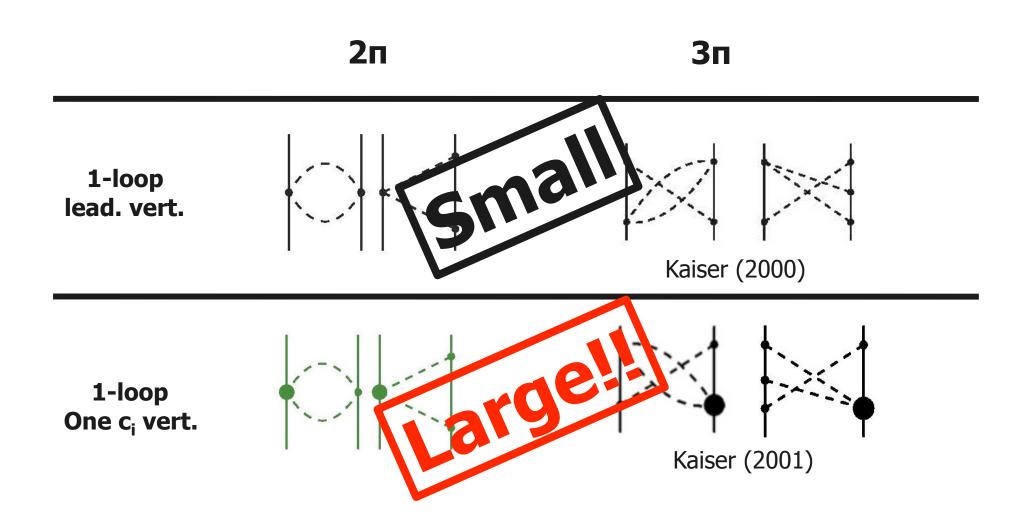


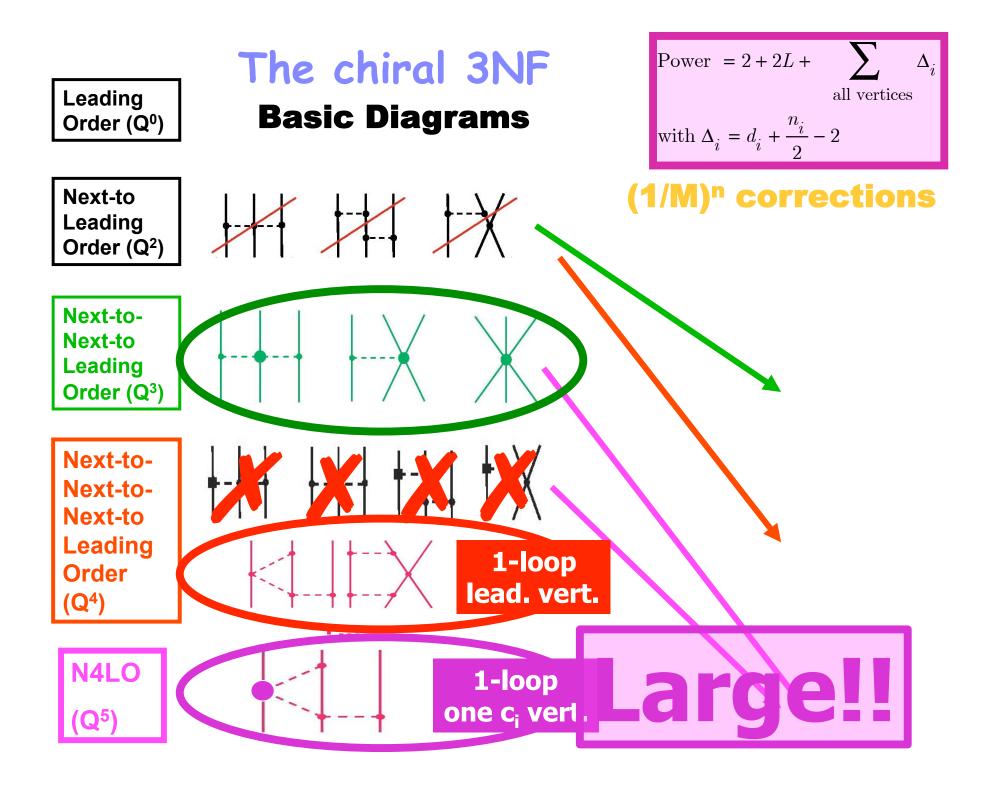






Corresponding 2NF contributions



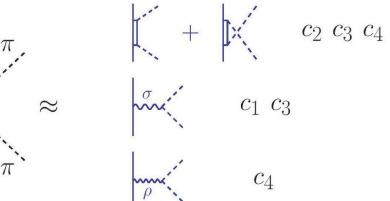


So, we will have to go to N4LO!?

 No, not necessarily.
 There is an alternative: go to N3LO, but include Δ(1232) isobars explicitly.

pi-N Lagrangian with two derivatives ("next-to-leading" order)

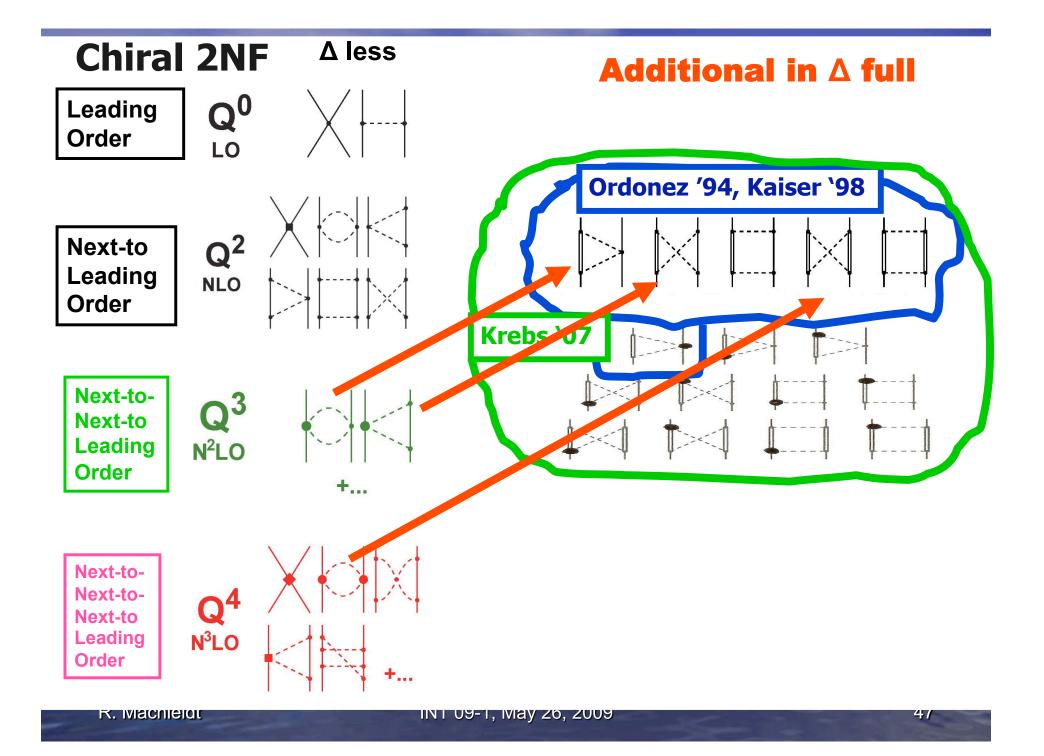
$$\mathcal{L}_{\pi N,c_{i}}^{(2)} = \bar{N} \left[2 c_{1} m_{\pi}^{2} (U + U^{\dagger}) + \left(c_{2} - \frac{g_{A}^{2}}{8M_{N}} \right) u_{0}^{2} + c_{3} u_{\mu} u^{\mu} + \frac{i}{2} \left(c_{4} + \frac{1}{4M_{N}} \right) \vec{\sigma} \cdot (\vec{u} \times \vec{u}) \right] N$$

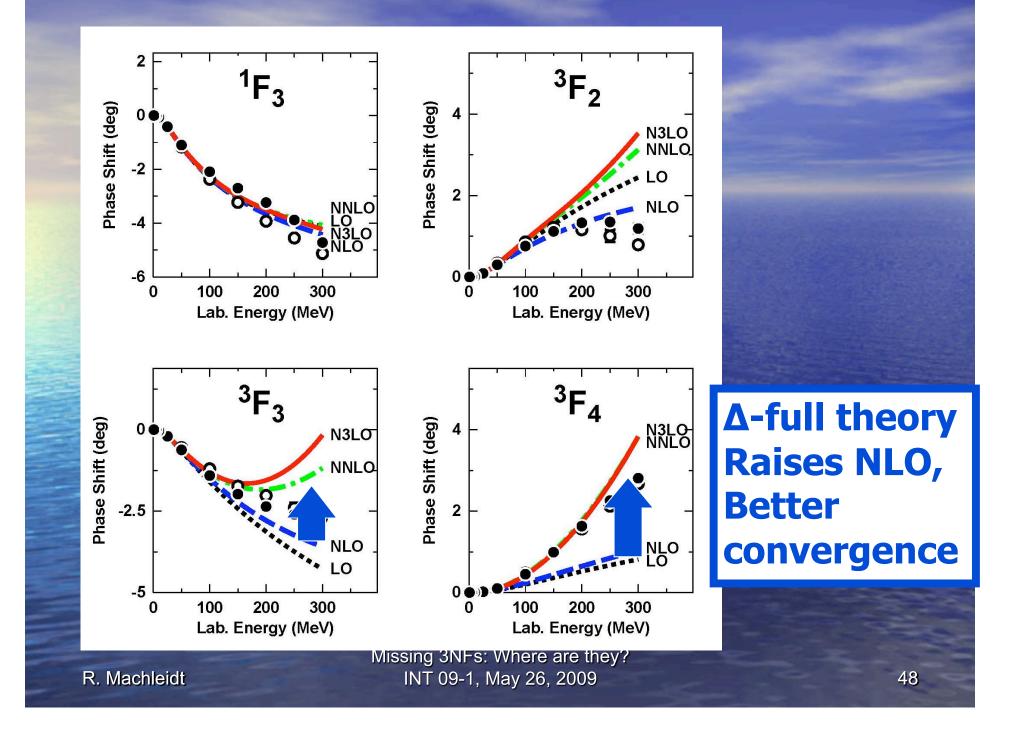


Bernard et al. '97

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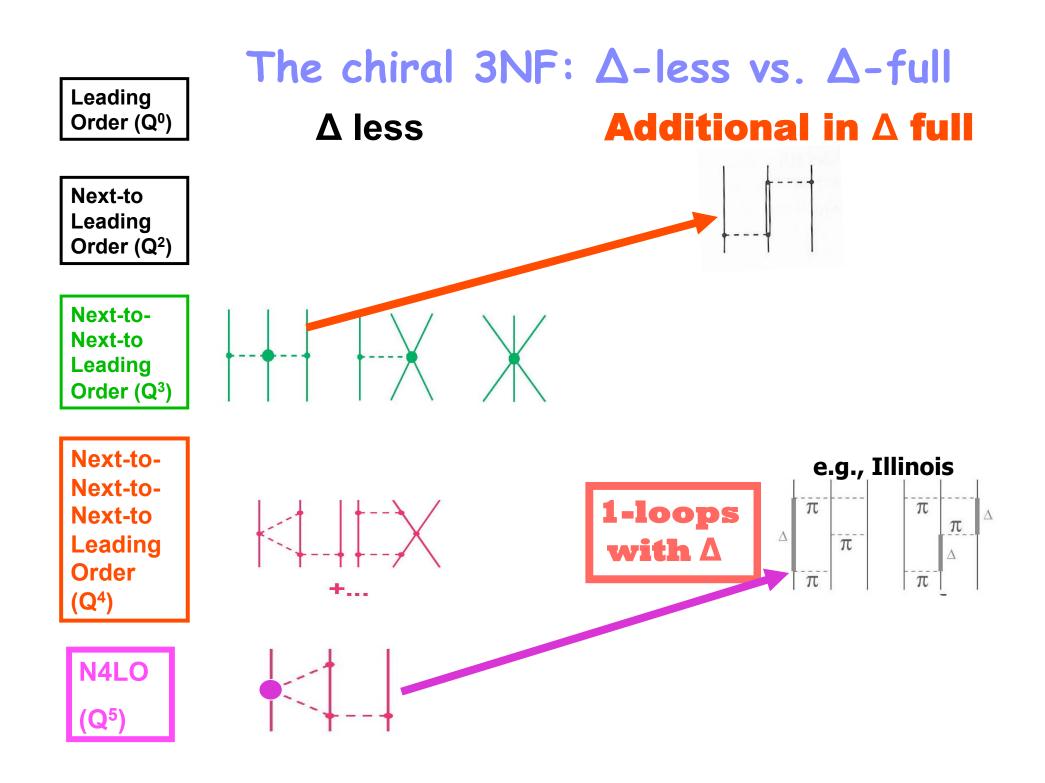
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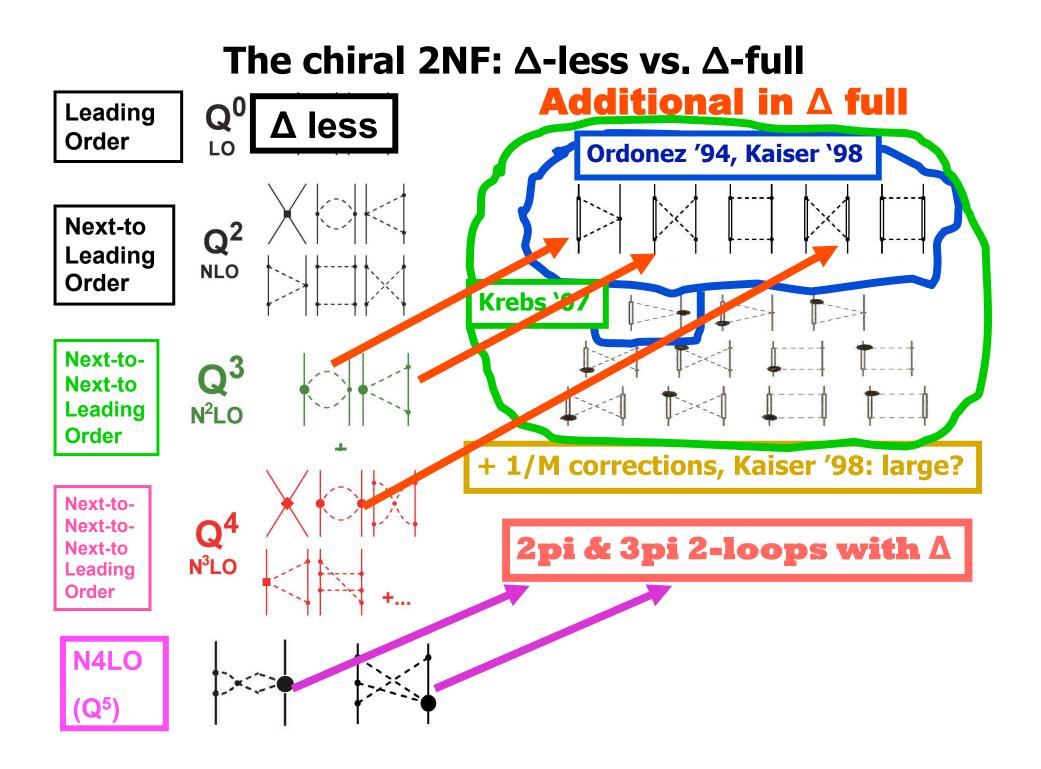
The 3NF in the Δ -full theory

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However, there is a however ...

For reasons of consistency, the 2NF must then also include ∆'s---up to N3LO!
 So, what does that look like?

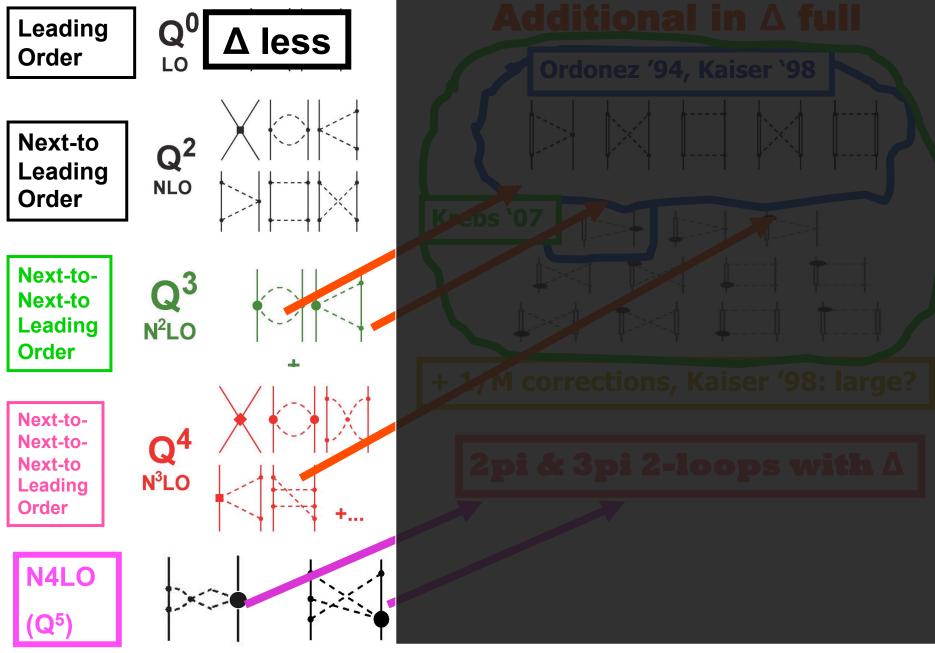


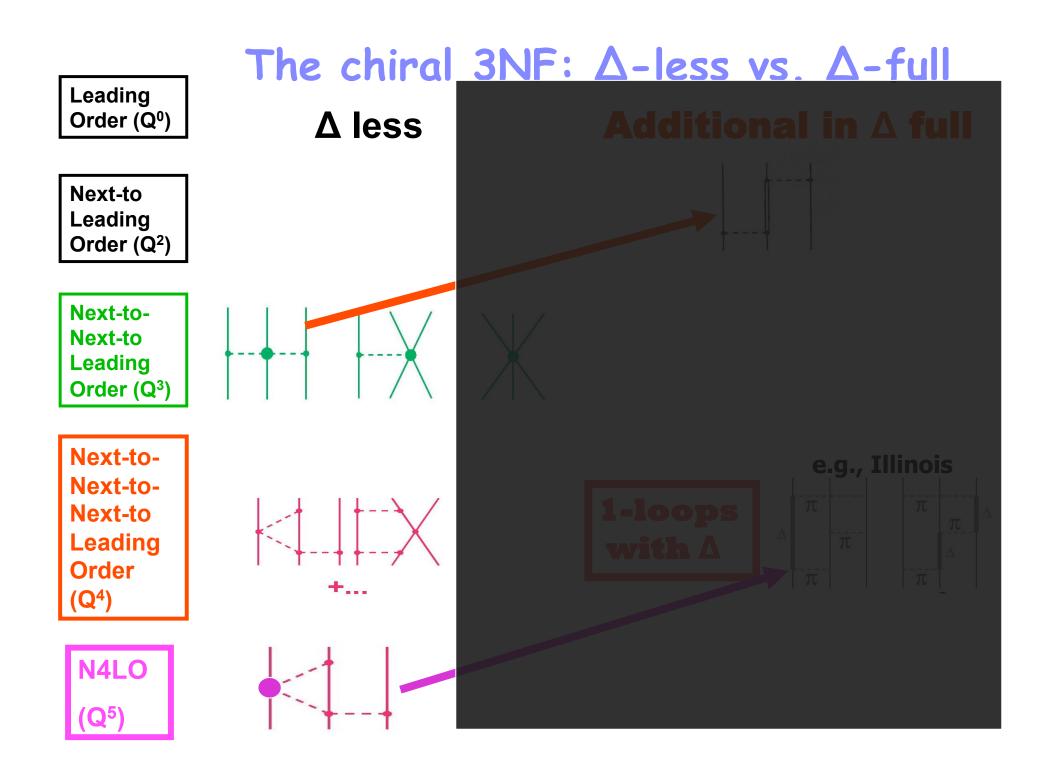
In summary:



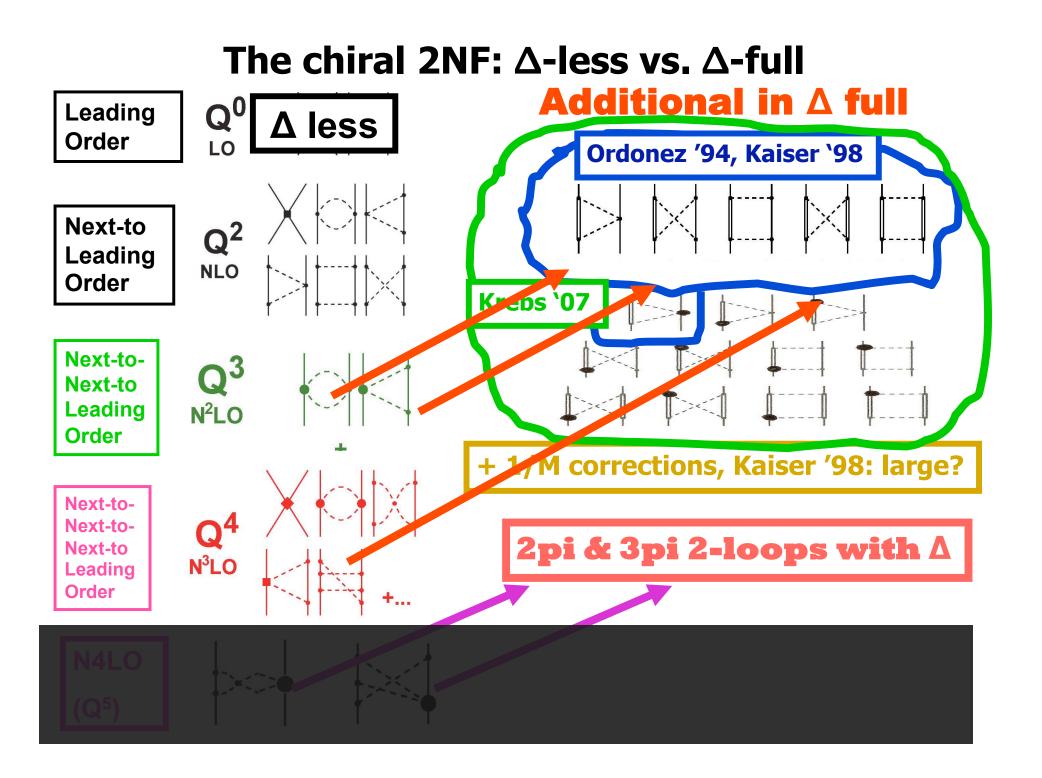
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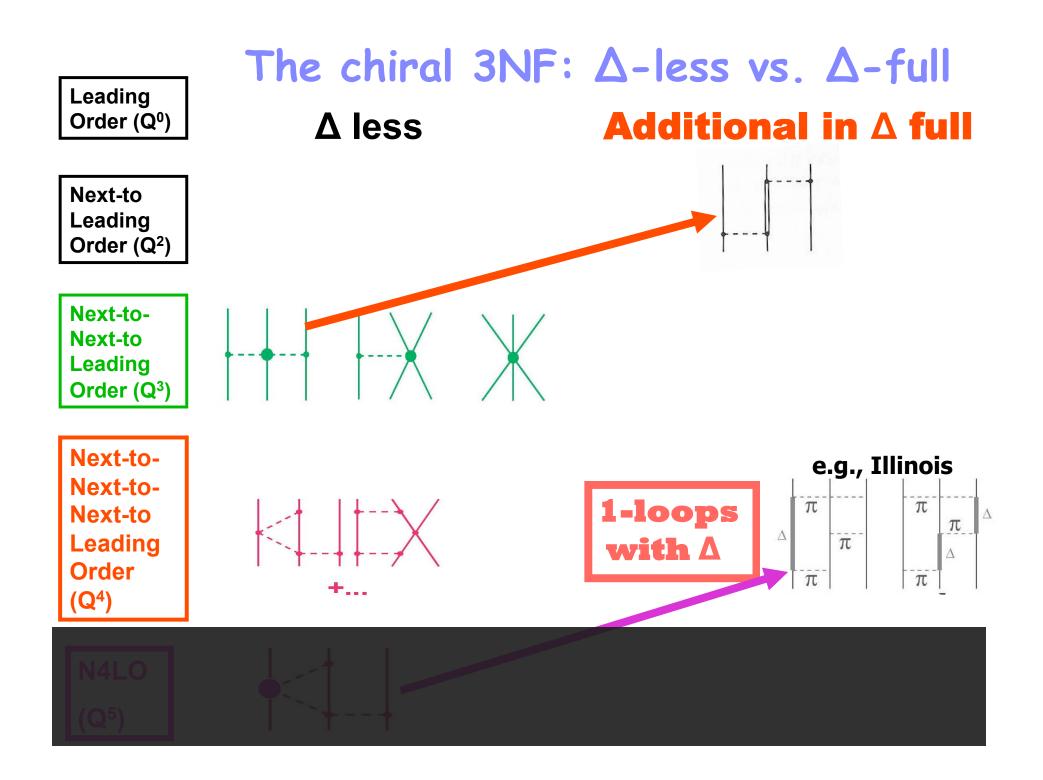
The chiral 2NF: Δ -less vs. Δ -full











Whatever we decide (Δ -less or Δ -full), it will be impossible to do a complete calculation anytime soon; so what's a good strategy?

Assume "resonance saturation" for the 2NF, i.e., stay with the present Δ-less N3LO 2NF (this has flaws!)
Add the Δ-full 3NF at N3LO.

Conclusions

• 3NFs beyond N2LO are needed! The 3NF at N3LO of the Δ-less theory may be weak and useless. This calls for further 3NF contributions: - A-less N4LO or - A-full N3LO, they will be large, but none will be easy. ChPT of nuclear forces has been fairly easy and straightforward, so far; but what's left won't be easy, no matter how you do it.

The missing three-nucleon forces: Where are they?

They are

at N4LO in Δ-less or
at N3LO in Δ-full.

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Missing 3NFs: Where are they? INT 09-1, May 26, 2009

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