INT Program

Effective Field Theories and the Many-Body Problem

March 23 – June 5, 2009

The missing three-nucleon forces: Where are they?

R. Machleidt

University of Idaho

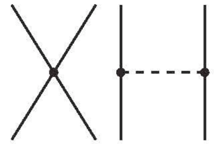
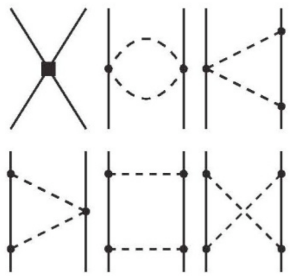
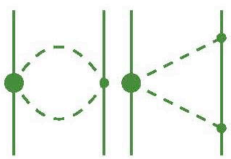
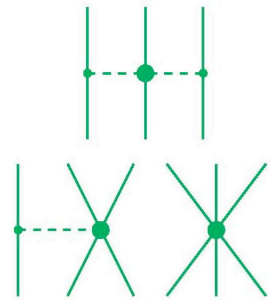
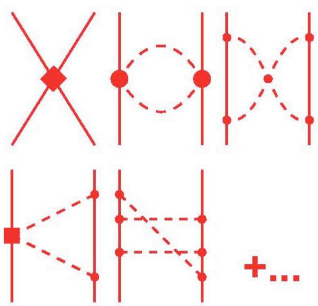
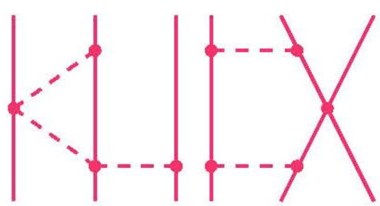
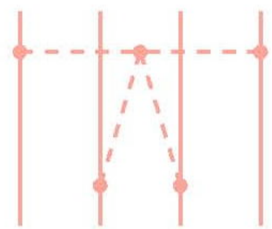
Outline

- Nuclear forces from Chiral Perturbation Theory (ChPT): Current status
- The 3NF at N2LO
- The 3NF at N3LO: Weak expectations
- N4LO or Δ-isobars?
- Summary

The ChPT approach to nuclear forces

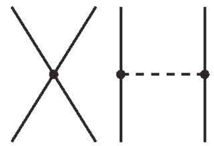
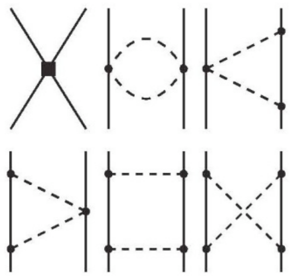
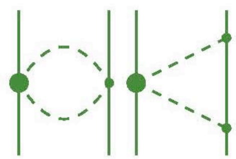
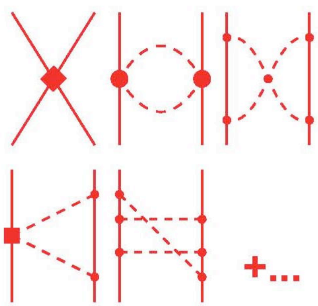
- Clear connection to QCD: via symmetries
- Degrees of freedom relevant to (low-energy) nuclear physics: pions and nucleons
- Systematic expansion: Q/Λ
- Controlled error, predictive power.
- Explains the empirically known hierarchy of nuclear forces

The Hierarchy of Nuclear Forces

		2N forces	3N forces	4N forces
Leading Order	Q^0 LO			
Next-to Leading Order	Q^2 NLO			
Next-to-Next-to Leading Order	Q^3 N^2LO	 +...		
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO	 +...	 +...	 +...

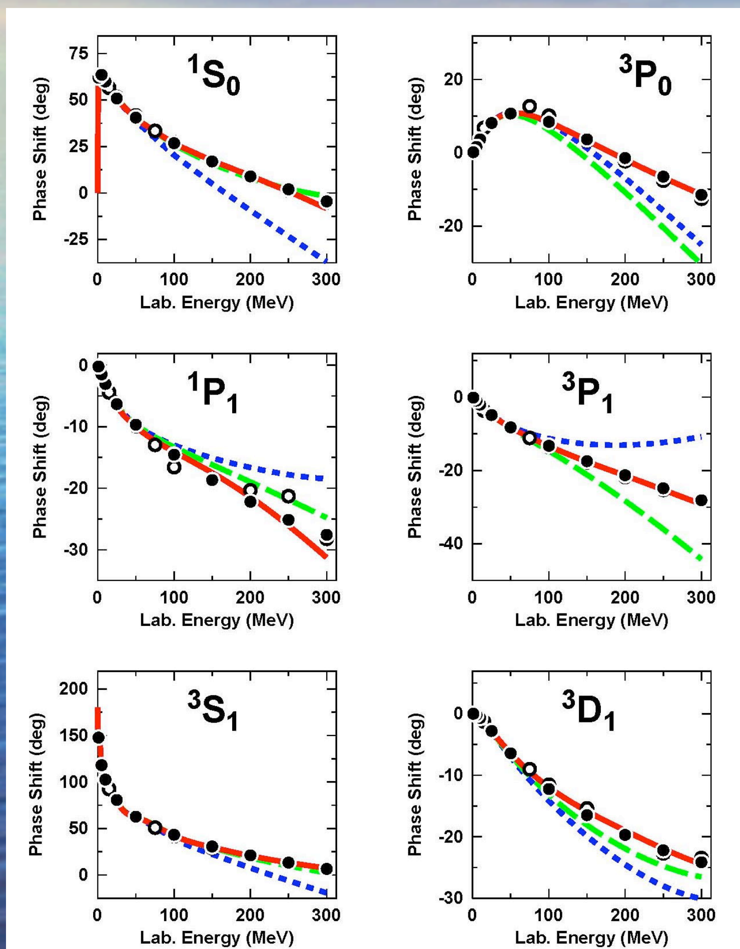
The Hierarchy of Nuclear Forces

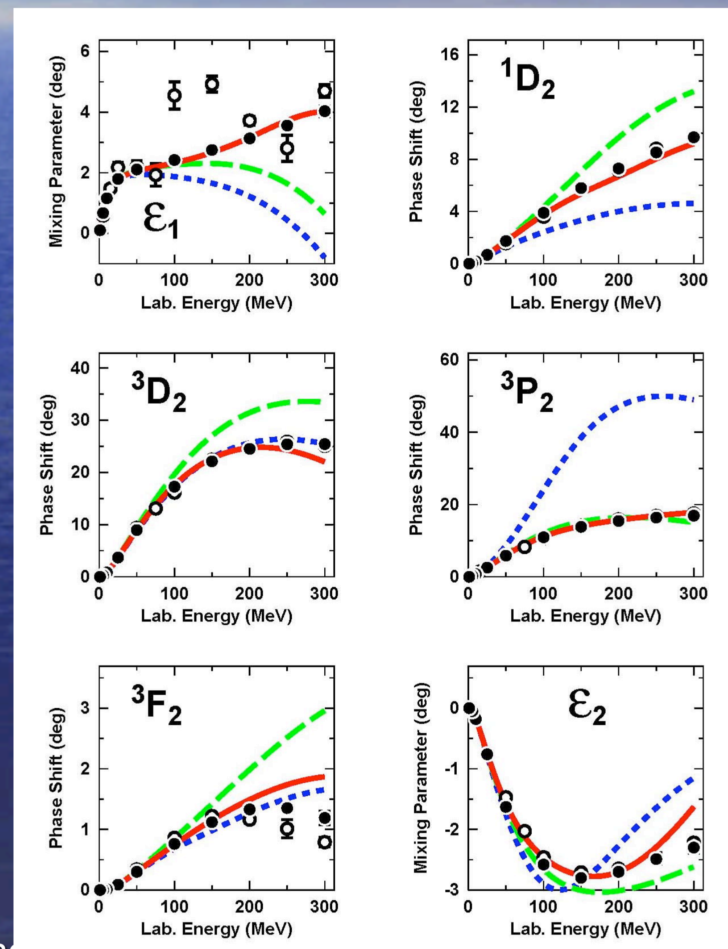
2N forces

Leading Order	Q^0 LO	
Next-to Leading Order	Q^2 NLO	
Next-to-Next-to Leading Order	Q^3 N^2LO	 +...
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO	 +...

Phase shifts up to 300 MeV

Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
Green dashed line: NNLO Potential, and
blue dotted line: NLO Potential
by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).





χ^2/datum for the reproduction of the 1999 np database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.7	4.5	0.95
100–190	501	1.08	22	100	1.10
190–290	843	1.15	47	180	1.11
0–290	2402	1.10	20	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

Deuteron Properties

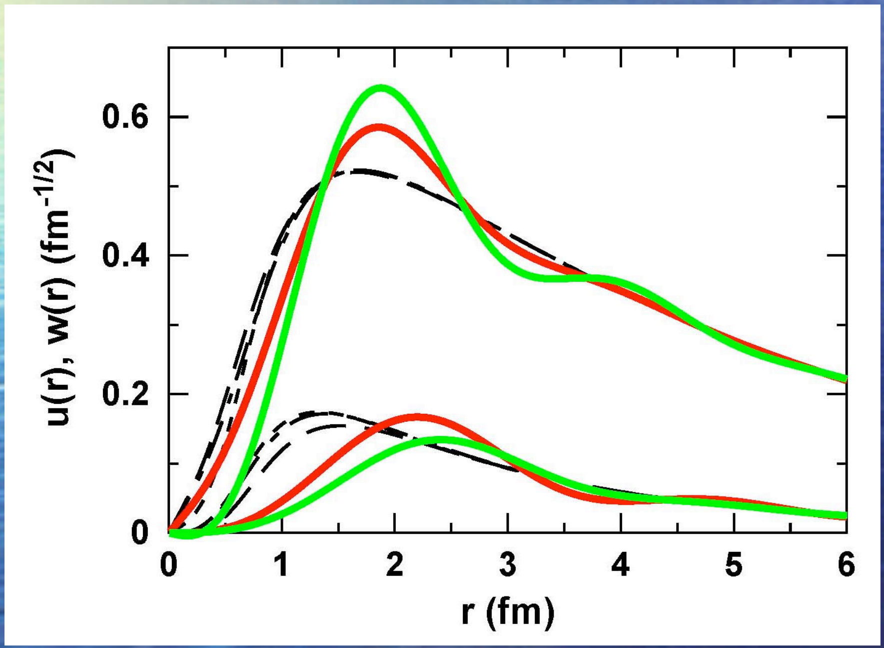
	Idaho N3LO (500)	Bochum/Juelich N3LO (550/600)	CD-Bonn	AV18	Empirical
Binding energy B_d (MeV)	2.224575	2.218279	2.224575	2.224575	2.224575(9)
Asymptotic S state A_S (fm$^{-1/2}$)	0.8843	0.8820	0.8846	0.8850	0.8846(9)
Asymptotic D/S state η	0.0256	0.0254	0.0256	0.0250	0.0256(4)
Matter radius r_d (fm)	1.978	1.977	1.966	1.967	1.9754(9)
Quadrupole moment Q_d (fm^2)	0.275	0.266	0.270	0.270	0.286(1)
D-state probability P_D (%)	4.51	3.28	4.85	5.76	

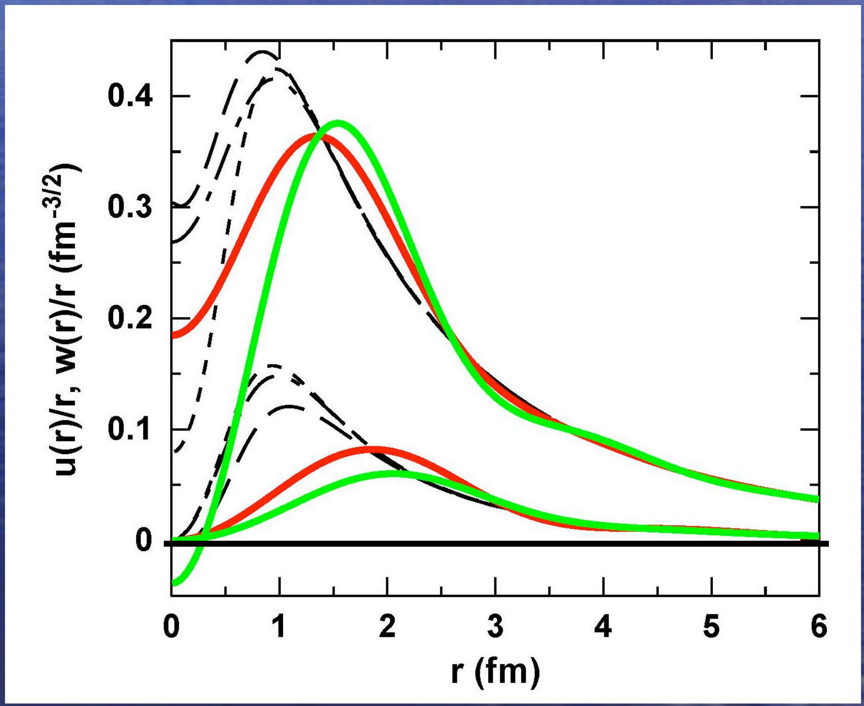
Deuteron wave functions

of N3LO potentials

red = Idaho, green = Bochum/Juelich

Black lines = high-precision pots.





Results from applications of chiral two-nucleon forces

- Latest Coupled Cluster results for A≤48 using N3LO

PRL 101, 092502 (2008) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2008

Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions

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[2]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[3]*Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316 Oslo, Norway*

(Received 20 June 2008; published 29 August 2008)

We compute the binding energies, radii, and densities for selected medium-mass nuclei within coupled-cluster theory and employ a bare chiral nucleon-nucleon interaction at next-to-next-to-next-to-leading order. We find rather well-converged results in model spaces consisting of 15 oscillator shells, and the doubly magic nuclei ^{40}Ca, ^{48}Ca, and the exotic ^{48}Ni are underbound by about 1 MeV per nucleon within the coupled-cluster singles-doubles approximation. The binding-energy difference between the mirror nuclei ^{48}Ca and ^{48}Ni is close to theoretical mass table evaluations. Our computation of the one-body density matrices and the corresponding natural orbitals and occupation numbers provides a first step to a microscopic foundation of the nuclear shell model.

Chiral NN potential at N^{3}LO underbinds by ~1MeV/nucleon. (Size extensivity at its best.)

Nucleus	DE / A [MeV]
^{4}He	1.08 (0.73^{FY})
^{16}O	1.25
^{40}Ca	0.84
^{48}Ca	1.27
^{48}Ni	1.21

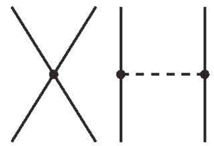
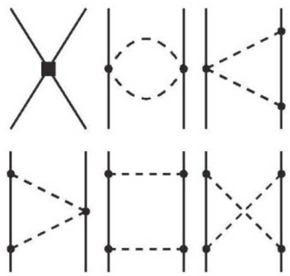
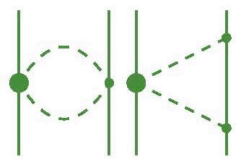
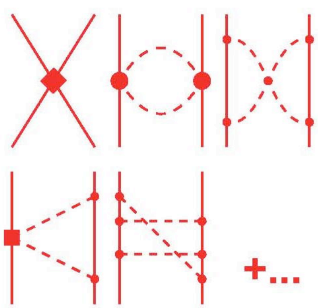
V_low-k

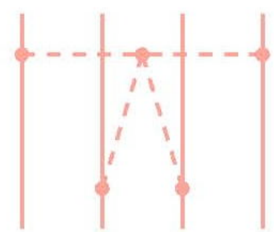
?

λ	E/A	[illegible]	Q	$\Delta E/A$
1.9	-15.37	-47.59	[illegible]5	-6.82
2.2	-13.67	-44[illegible]	1.23	-4.82
2.5	-12.23	-42.39	1.21	-3.68
$\Lambda \approx 3.5$	-7.68	-30.[illegible]	1.14	0.87

2N forces **4N forces**

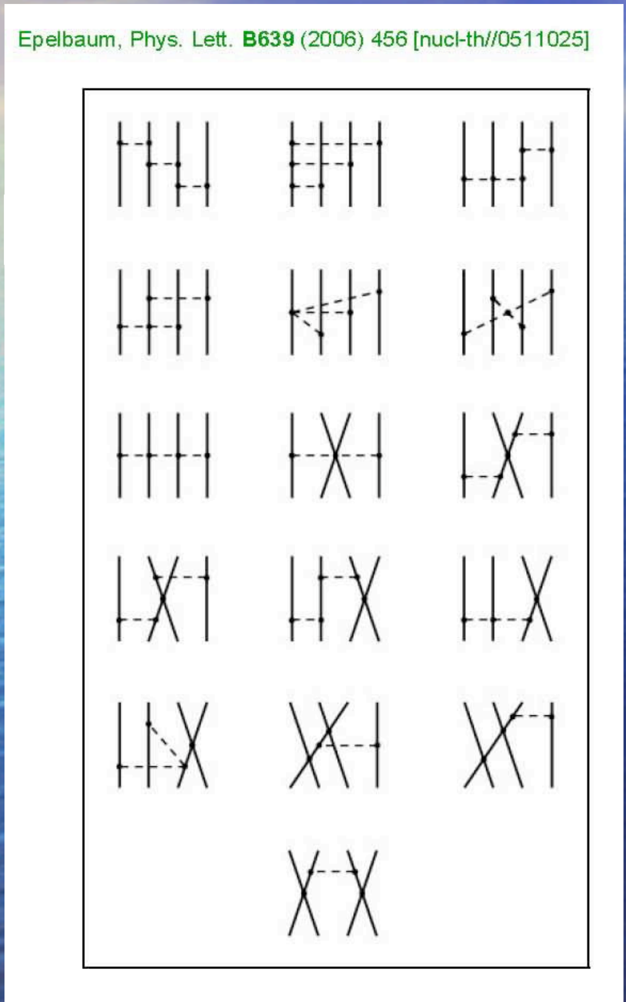
The Hierarchy of Nuclear Forces

Leading Order	Q^0 LO	
Next-to Leading Order	Q^2 NLO	
Next-to-Next-to Leading Order	Q^3 N^2LO	 +...
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO	 +...



+...

4NF at N3LO (leading order)



Note that only vertices from

$\mathcal{L}^{(2)}_{\pi\pi}$, $\mathcal{L}^{(1)}_{\pi N}$ and $\mathcal{L}^{(0)}_{NN}$ are involved,

- no new parameters,
- weak.

First rough estimate:

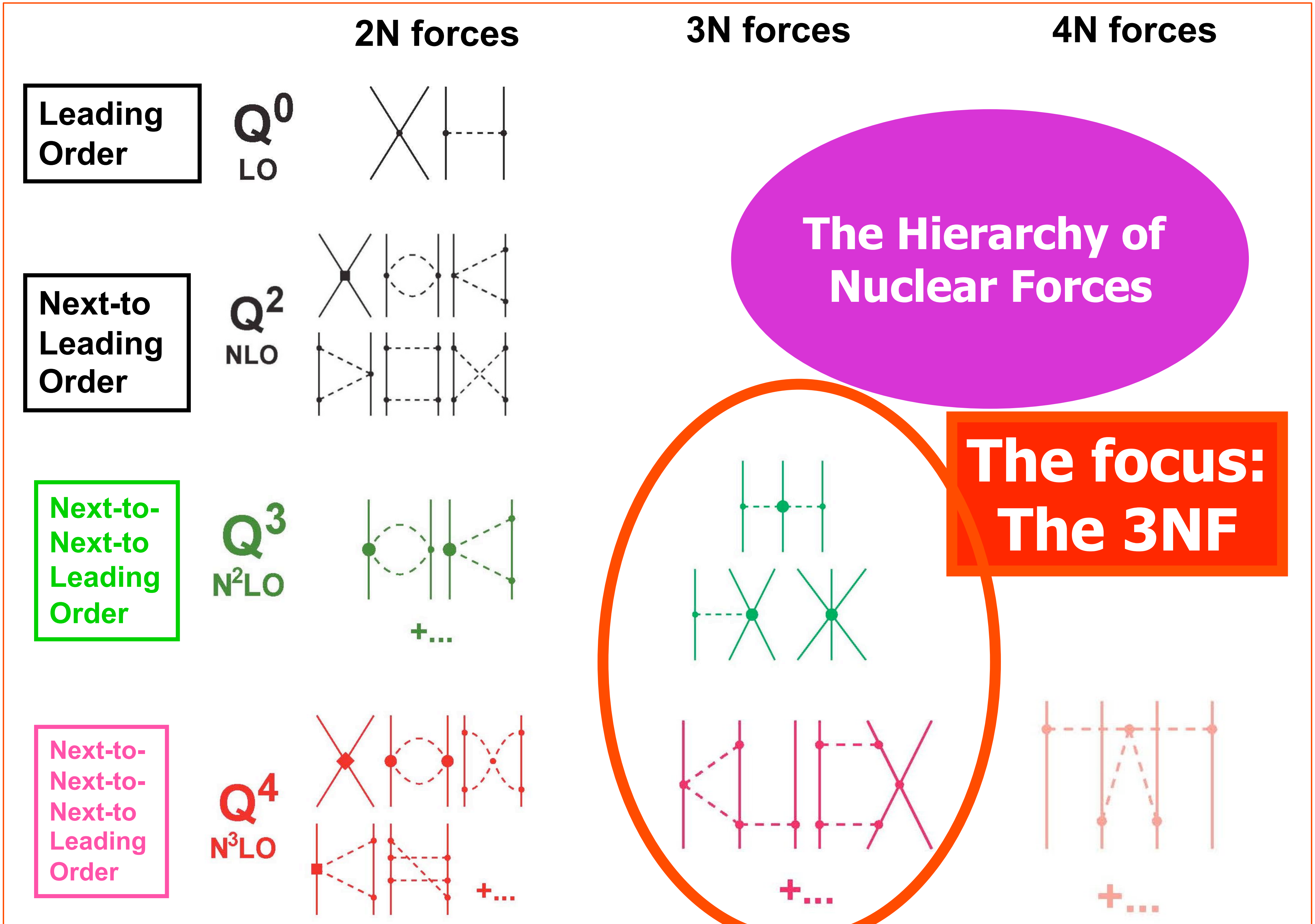
$\approx$ 0.1 MeV to α binding.

Essentially negligible, as to be expected.

Interim summary

- 2NF o.k.

  (except for the issue of non-perturbative renormalization)
- 4NF (negligibly) small …


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...
The Hierarchy of Nuclear Forces
The focus: The 3NF

The chiral 3NF

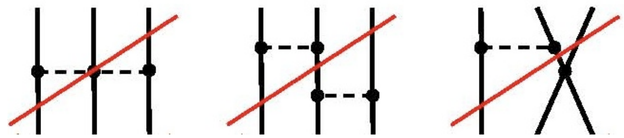
Basic Diagrams

$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

Leading Order (Q^0)

Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)

Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)

The chiral 3NF

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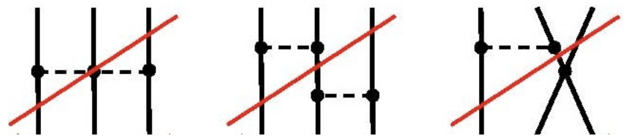
$(1/M)^n$ corrections

Leading Order (Q^0)

Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)

Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)

Coon & Friar (1986)

The chiral 3NF

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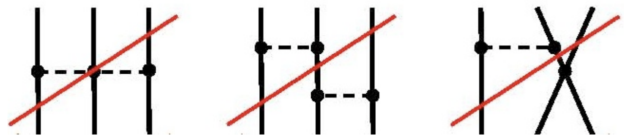
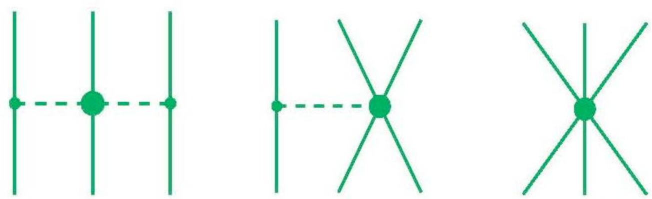
$(1/M)^n$ corrections

Leading Order (Q^0)

Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)

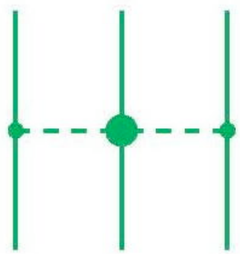


Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)

Three-nucleon forces at N2LO

TPE-3NF



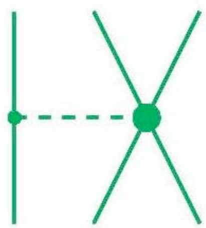
$$V^{\rm 3NF}_{\rm TPE} = \left(\frac{g_A}{2f_\pi}\right)^2 \frac{1}{2} \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i \cdot \vec{q}_i)(\vec{\sigma}_j \cdot \vec{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)} F^{\alpha\beta}_{ijk}\, \tau_i^\alpha \tau_j^\beta$$

with $\vec{q}_i \equiv \vec{p_i}' - \vec{p}_i$, where $\vec{p}_i$ and $\vec{p_i}'$ are the initial and final momenta of nucleon i, respectively, and

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\vec{q}_i \cdot \vec{q}_j\right] + \frac{c_4}{f_\pi^2}\sum_\gamma \epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$

No new parameters!

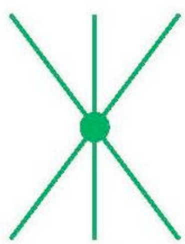
OPE-3NF



$$V^{\rm 3NF}_{\rm OPE} = D\,\frac{g_A}{8f_\pi^2} \sum_{i\neq j\neq k} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{q_j^2+m_\pi^2} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)(\vec{\sigma}_i \cdot \vec{q}_j)$$

One new parameter, D.

Contact-3NF



$$V^{\rm 3NF}_{\rm ct} = E\,\frac{1}{2}\sum_{j\neq k} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k \,.$$

One new parameter, E.

Strategy: Adjust D and E to two few-nucleon observables, e.g., the triton and alpha-particle binding energies. Then predict properties of other light nuclei.

The chiral 3NF

Basic Diagrams

$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

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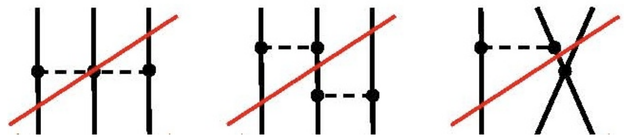
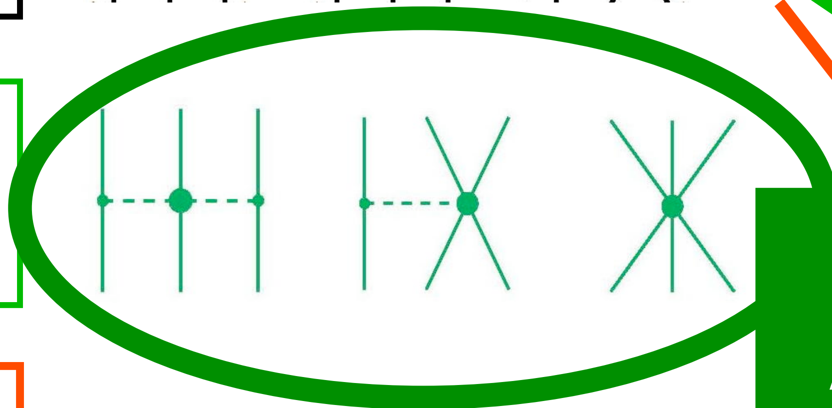
Leading Order (Q^0)

Next-to Leading Order (Q^2)



$(1/M)^n$ corrections

Next-to-Next-to Leading Order (Q^3)



The 3NF At N2LO :

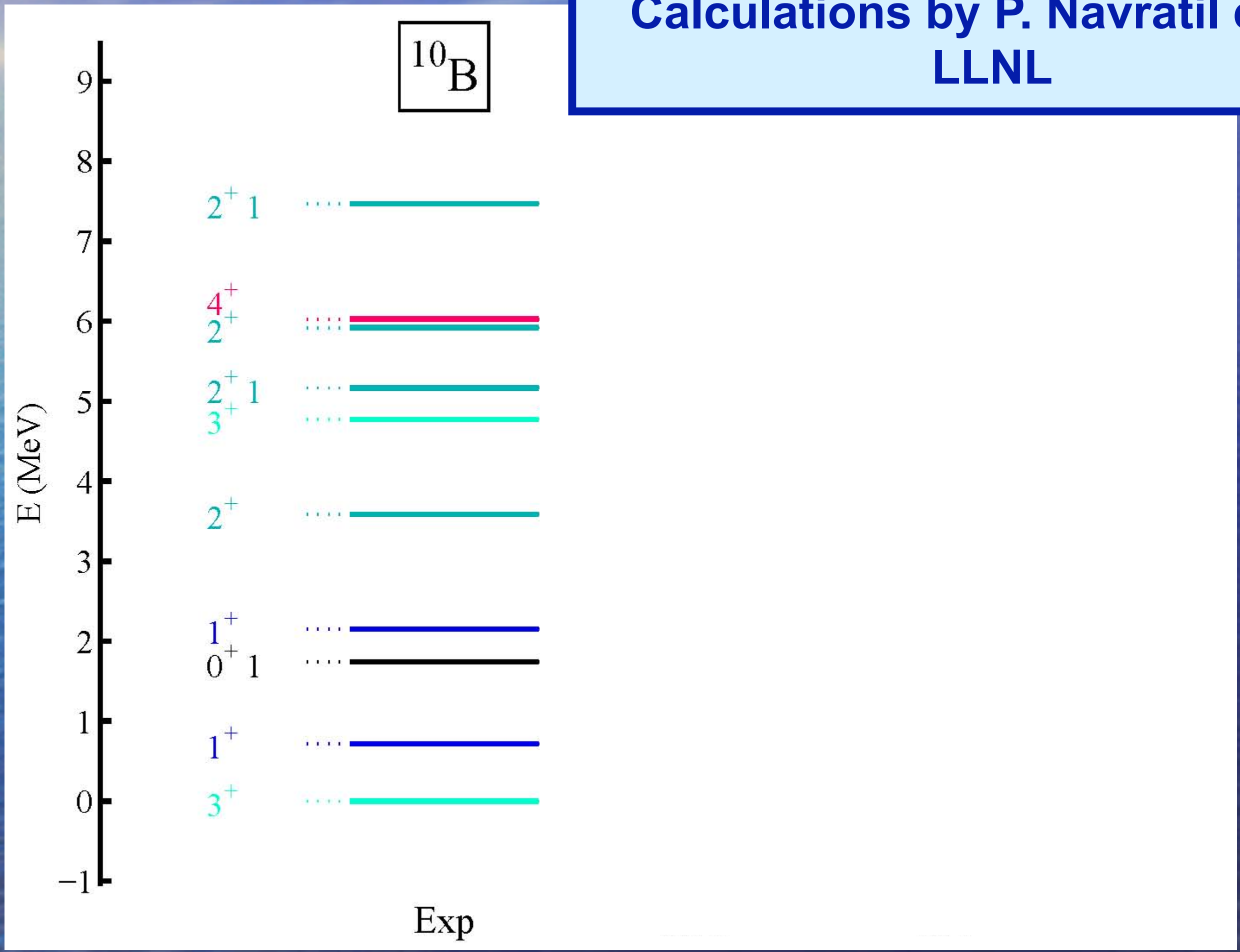
How well does it do?

Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)

Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



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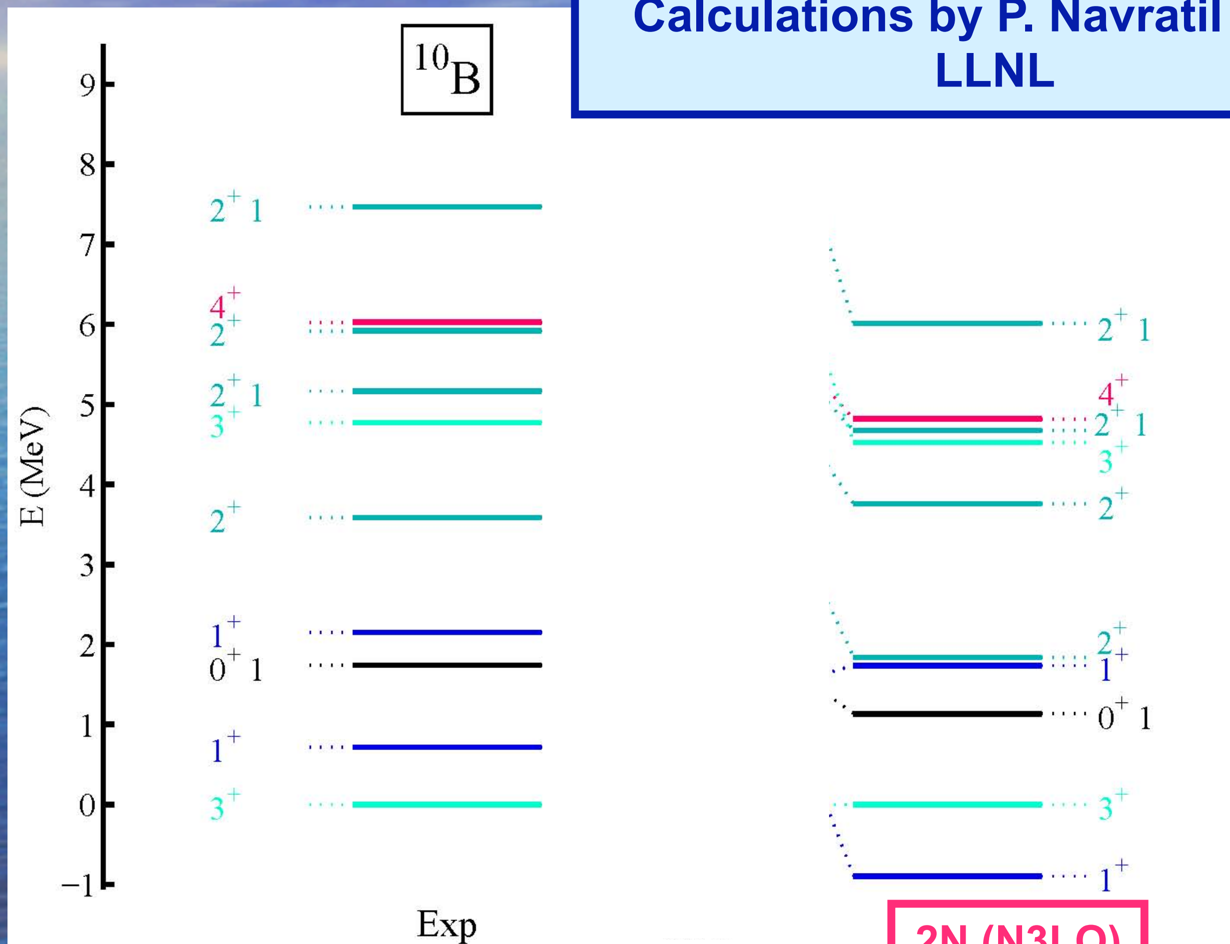
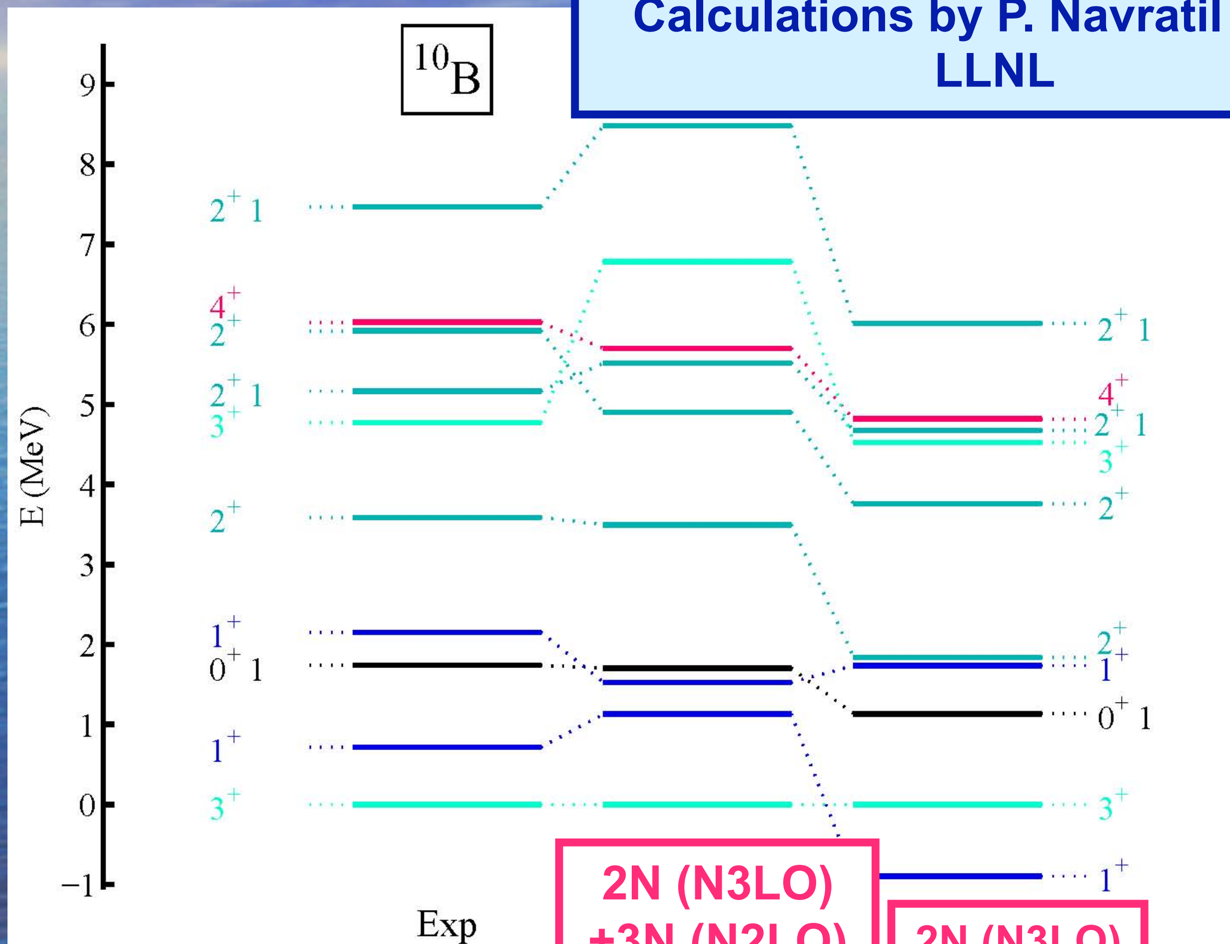


2N (N3LO) force only

Calculating the properties of light nuclei using chiral 2N and 3N forces

“No-Core Shell Model “
Calculations by P. Navratil et al., LLNL

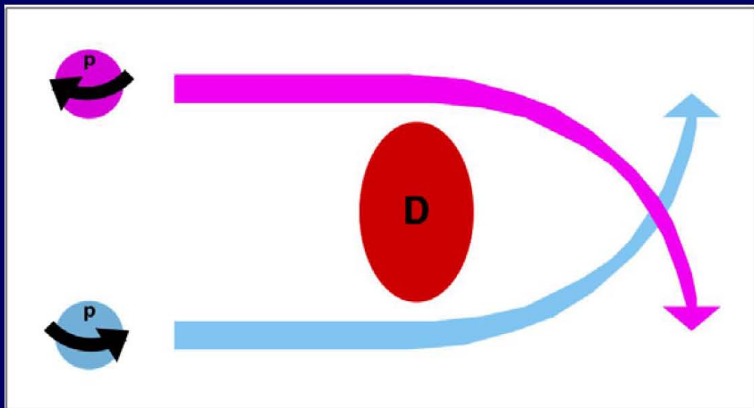


2N (N3LO) +3N (N2LO) forces

2N (N3LO) force only

The A_y puzzle at low energies

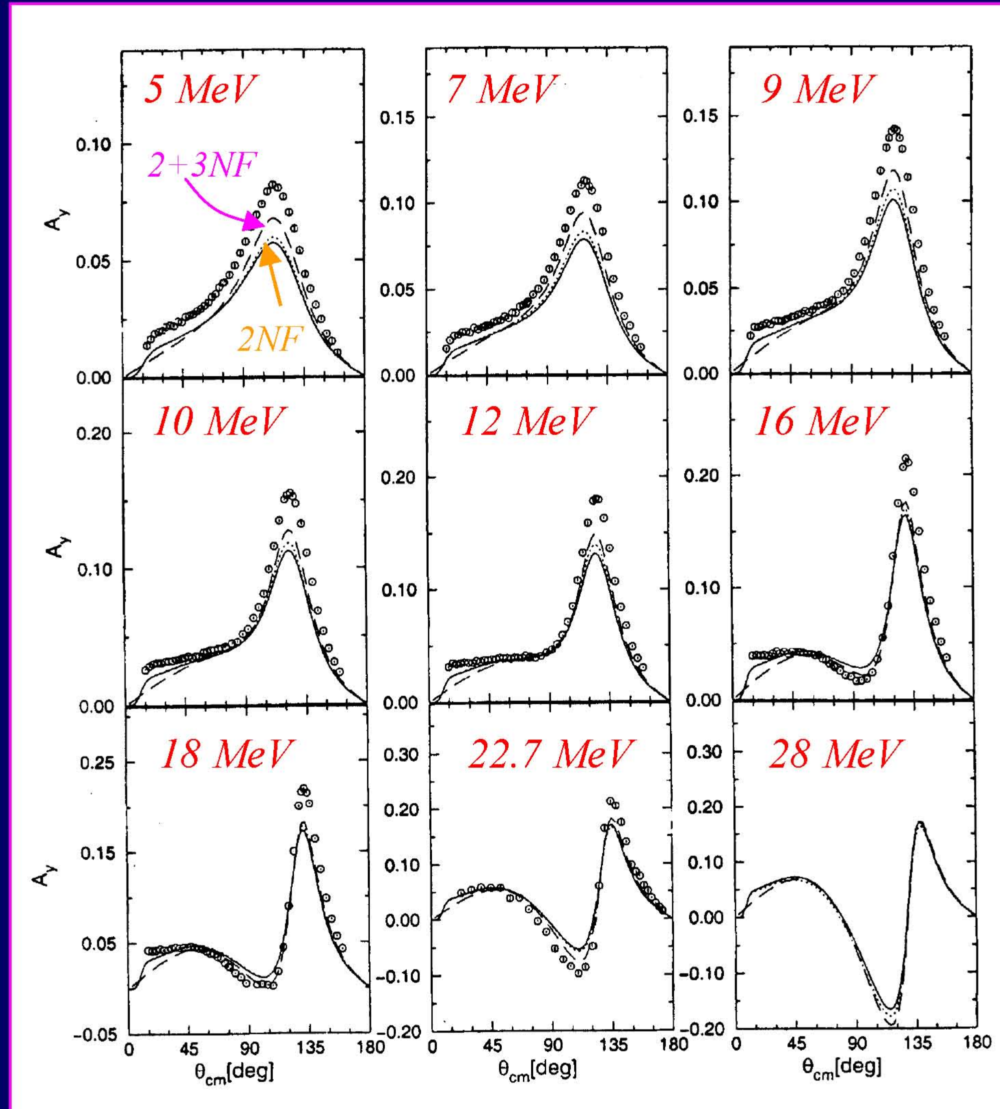
$$\vec{p} + d \rightarrow p + d$$



$$\sigma = \sigma_0(1 + pA_y\cos\phi)$$

$$\Rightarrow A_y = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Calculations by Pisa group

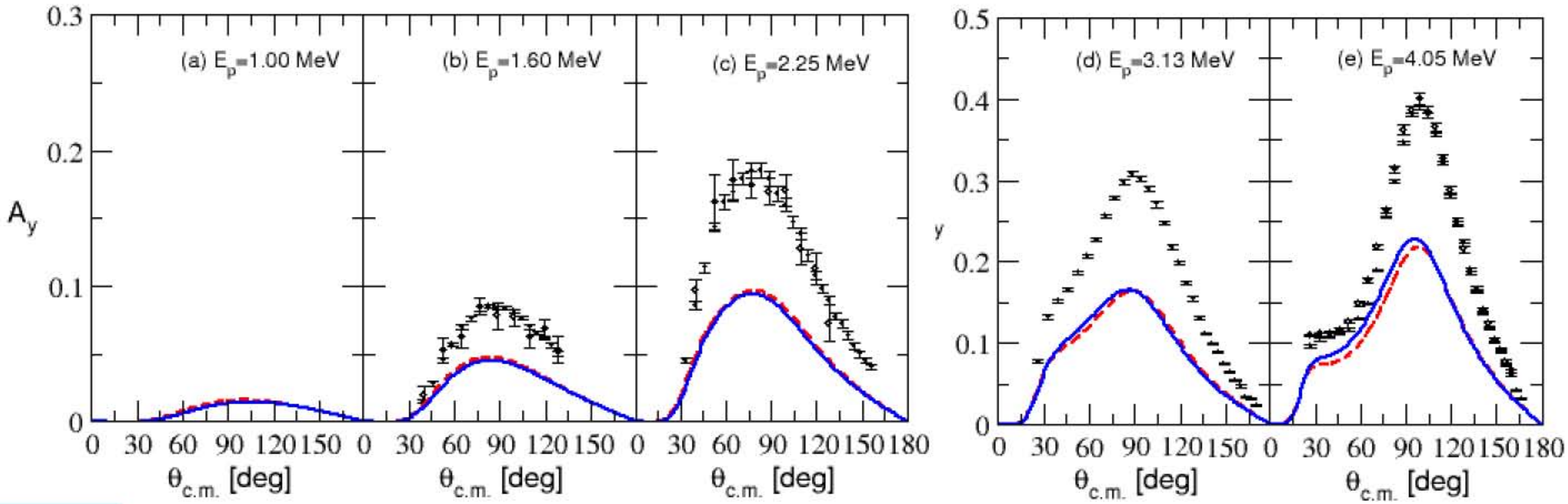


Very small effects from the (standard) 3N forces

p-^{3}He elastic scattering

$$\vec{p}+{}^3He \rightarrow p+{}^3He$$

Analyzing power



----- AV18

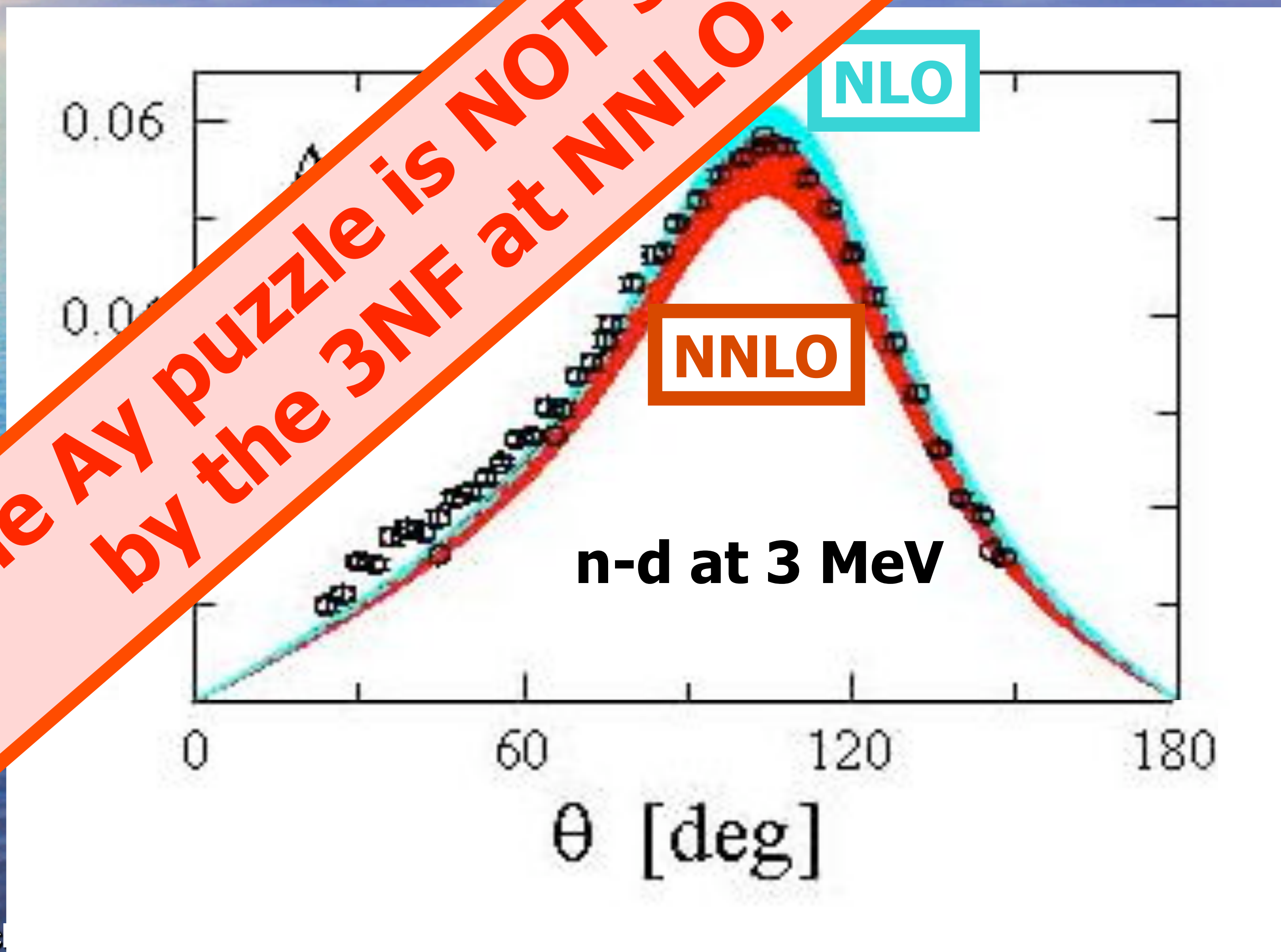
—— AV18+UIX

Courtesy of M. Viviani; latest data from TUNL.

MV, Marcucci, Kievsky & Rosati, PRC in press

Ay puzzle at N and NNLO (2N

The Ay puzzle is NOT solved by the 3NF at NNLO.



Why do we need 3NFs beyond NNLO?

- The 2NF is N3LO;
  consistency requires that all contributions are at the same order.

- There are unresolved problems in 3N, 4N scattering and nuclear structure.

What 3NFs are generated by ChPT beyond NNLO?

The chiral 3NF

Basic Diagrams

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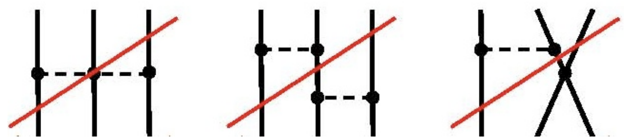
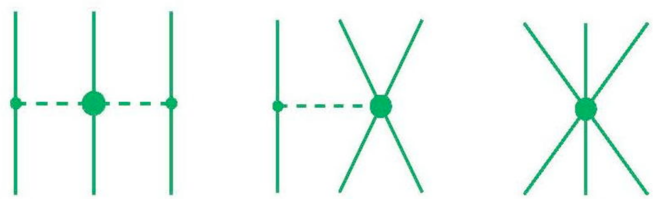
$(1/M)^n$ corrections

Leading Order (Q^0)

Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)



Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)

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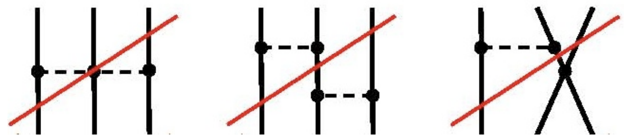

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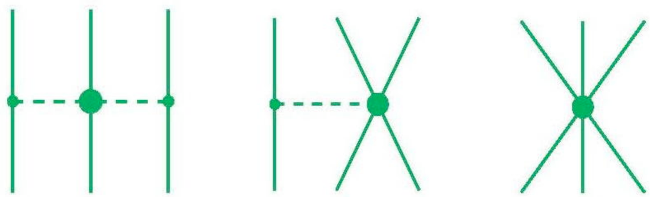

Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)

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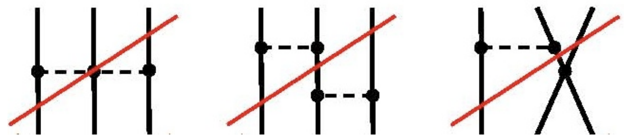
$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

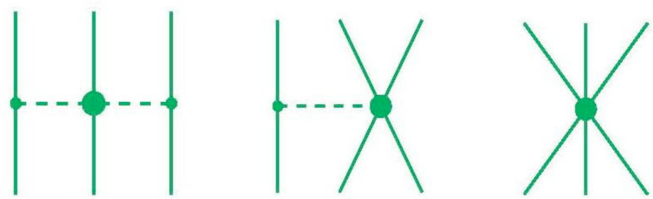
$(1/M)^n$ corrections

Leading Order (Q^0)

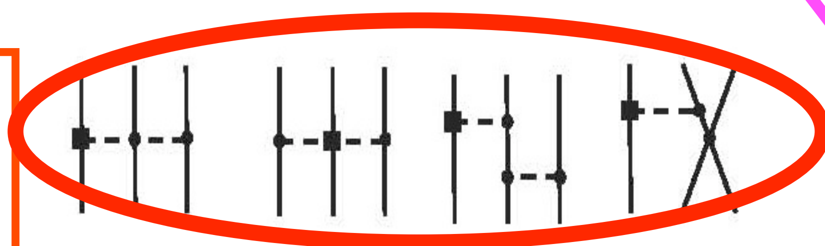
Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)



Next-to-Next-to-Next-to Leading Order (Q^4)



N4LO (Q^5)

The chiral 3NF

Basic Diagrams

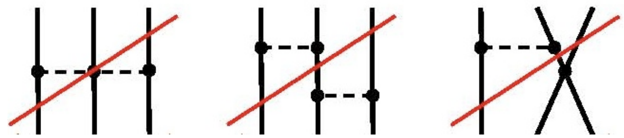
$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

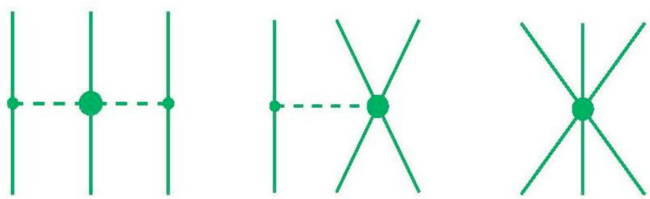
$(1/M)^n$ corrections

Leading Order (Q^0)

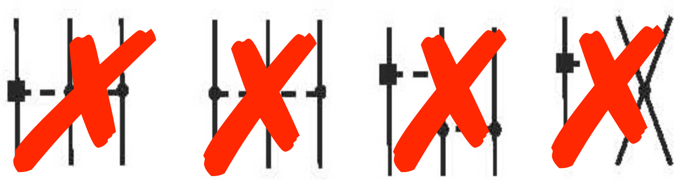
Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)



Next-to-Next-to-Next-to Leading Order (Q^4)



Time derivative

N4LO (Q^5)

The chiral 3NF

Basic Diagrams

$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

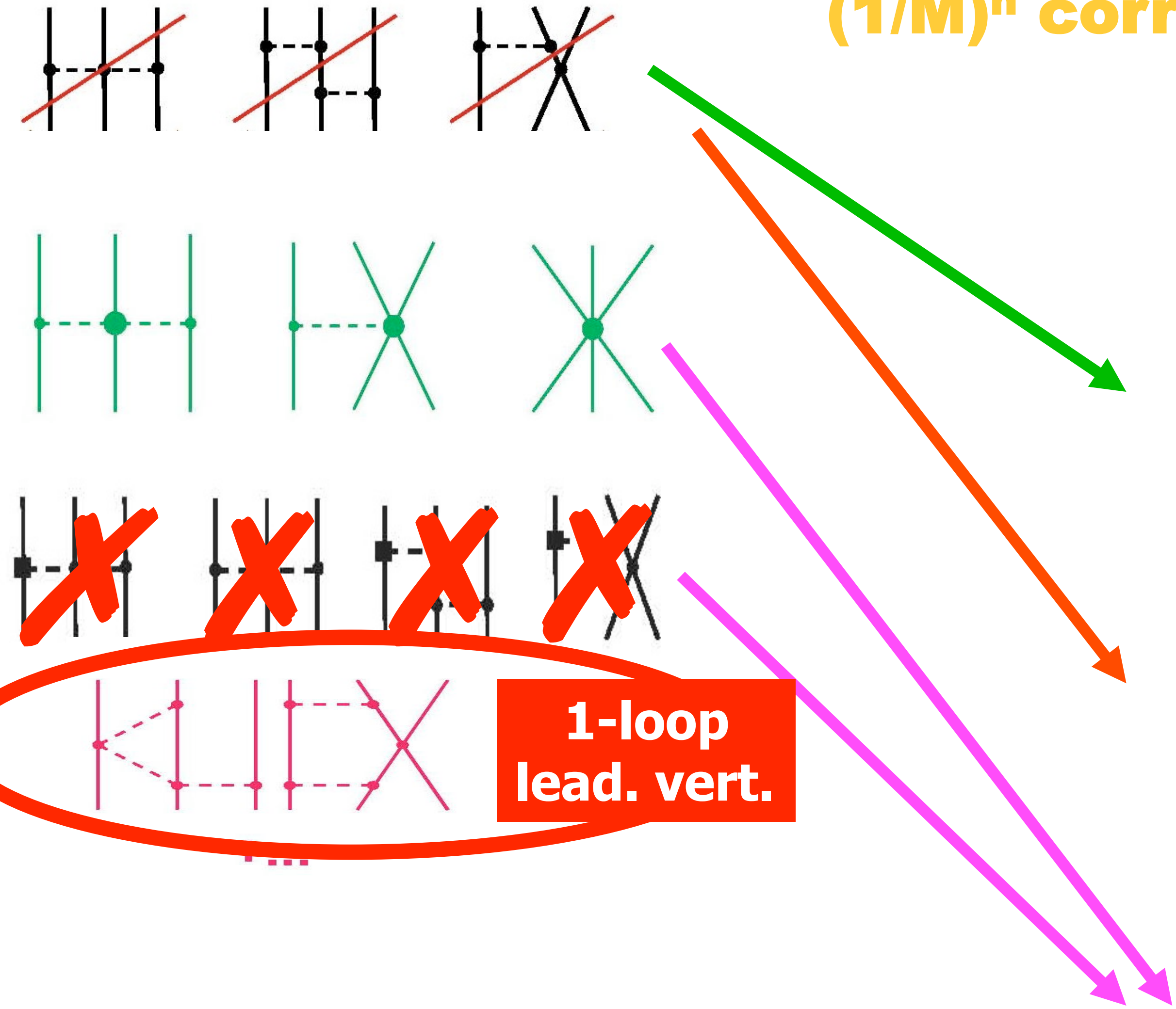
$(1/M)^n$ corrections

Leading Order (Q^0)

Next-to Leading Order (Q^2)

Next-to-Next-to Leading Order (Q^3)

Next-to-Next-to-Next-to Leading Order (Q^4)

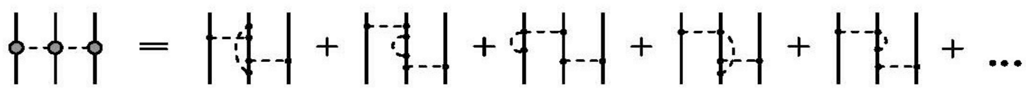


N4LO (Q^5)

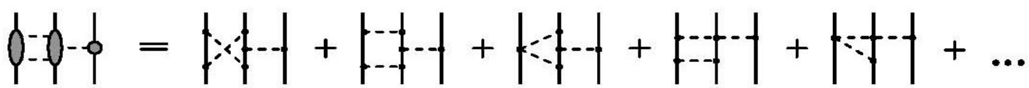
The 3NF at N3LO explicitly

One-loop, leading vertices

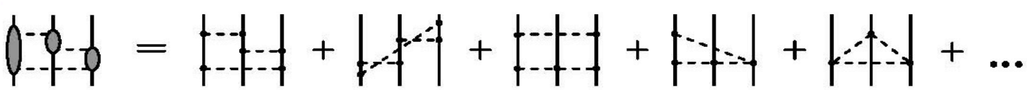
2π–exchange



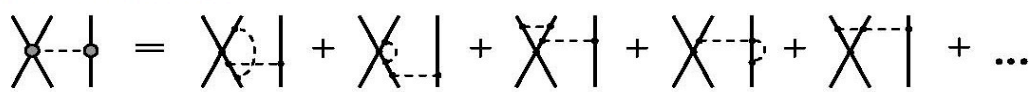
2π-1π–exchange



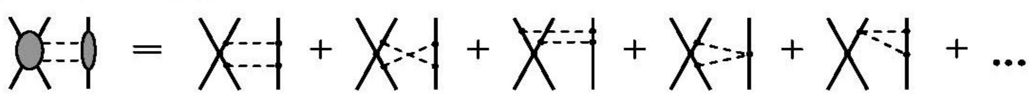
ring diagrams



contact-1π–exchange



contact-2π–exchange



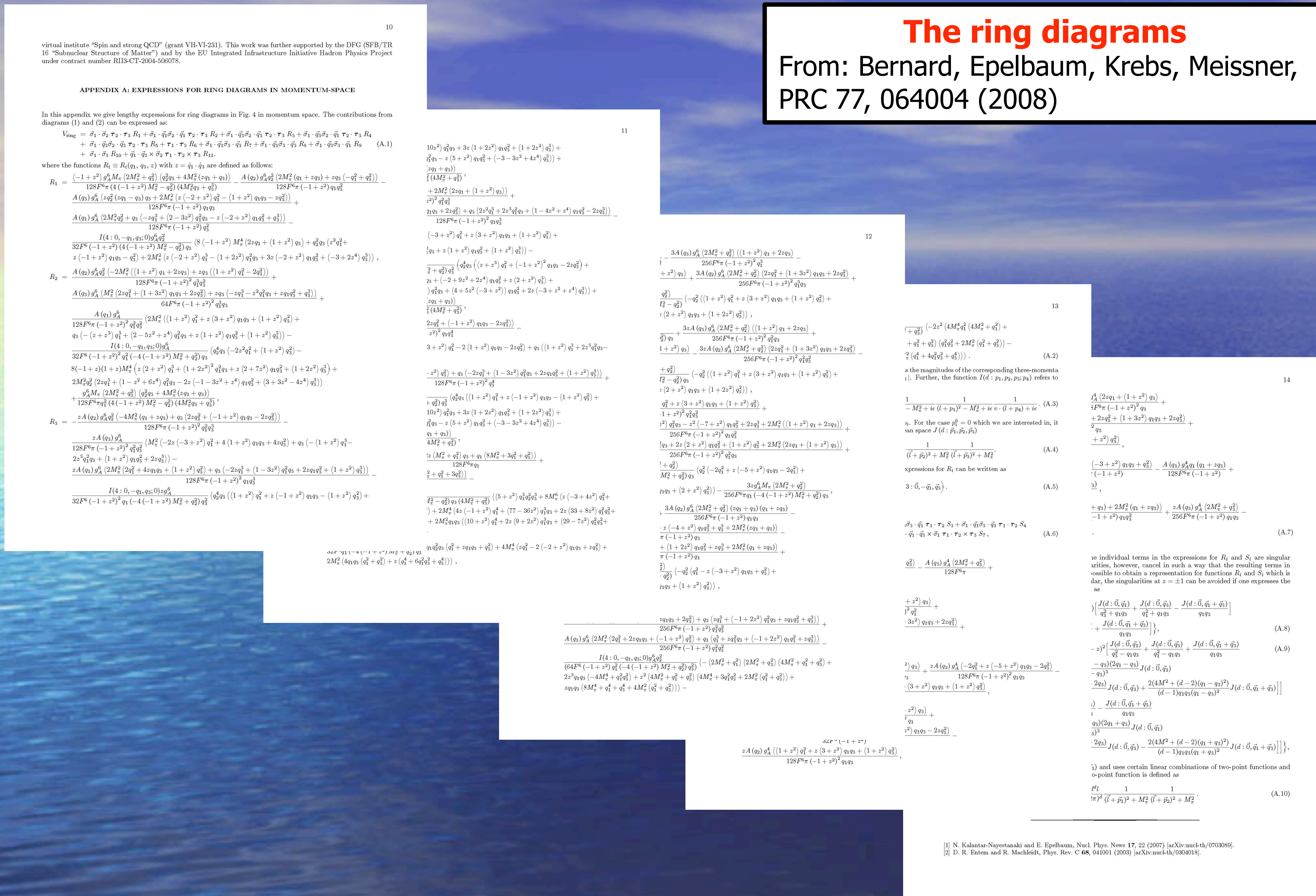
Ishikawa & Robilotta,
PRC 76, 014006 (2007)

Bernard,
Epelbaum,
Krebs,
Meissner,
PRC 77, 064004 (2008)

In progress

The ring diagrams

From: Bernard, Epelbaum, Krebs, Meissner, PRC 77, 064004 (2008)



virtual institute "Spin and strong QCD" (grant VH-VI-231). This work was further supported by the DFG (SFB/TR 16 "Subnuclear Structure of Matter") and by the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RII3-CT-2004-506078.

APPENDIX A: EXPRESSIONS FOR RING DIAGRAMS IN MOMENTUM-SPACE

In this appendix we give lengthy expressions for ring diagrams in Fig. 4 in momentum space. The contributions from diagrams (1) and (2) can be expressed as:

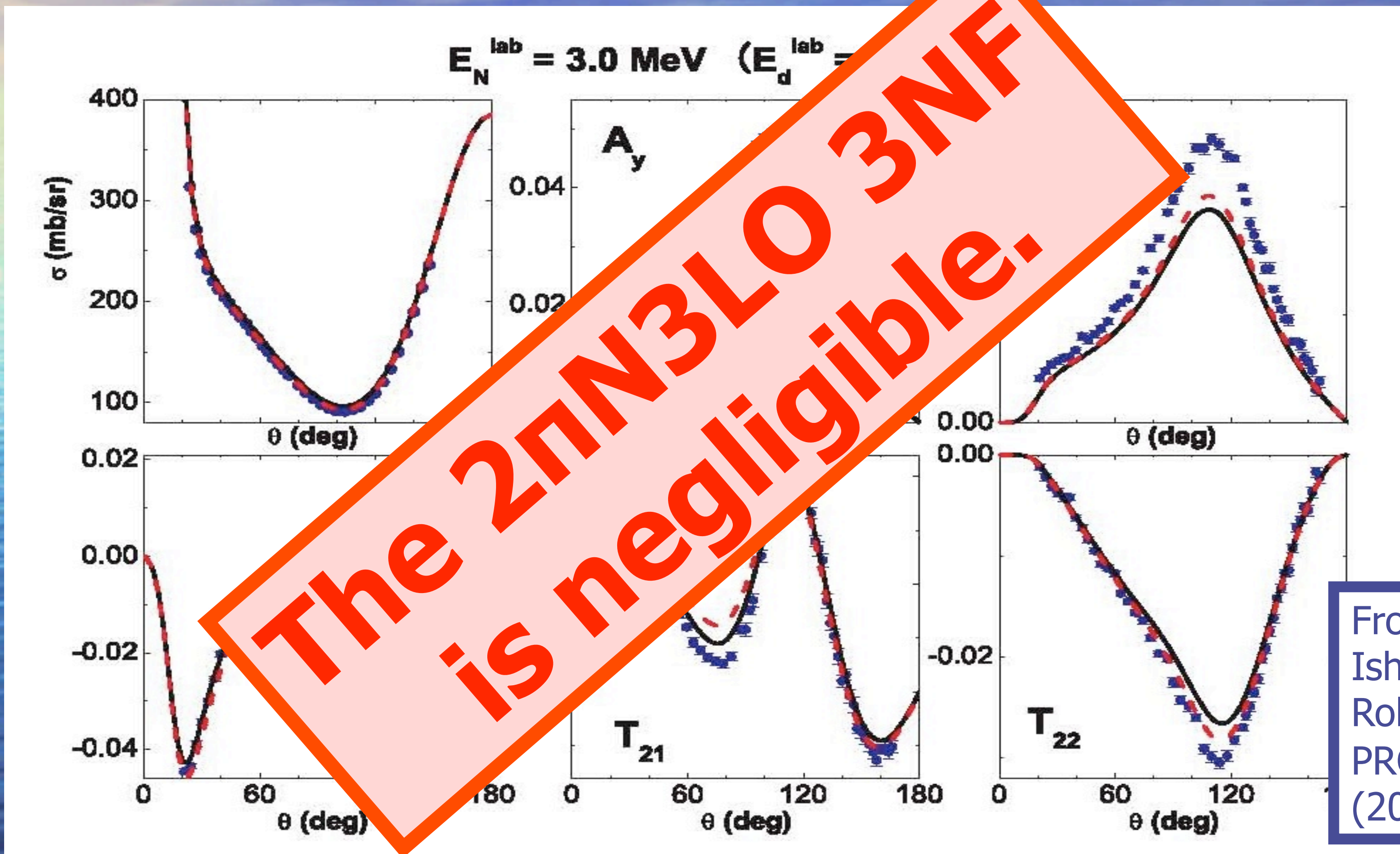
where the functions $R_i \equiv R_i(q_1, q_3, z)$ with $z = \hat{q}_1 \cdot \hat{q}_3$ are defined as follows:

[1] N. Kalantar-Nayestanaki and E. Epelbaum, Nucl. Phys. News **17**, 22 (2007) [arXiv:nucl-th/0703089].
[2] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003) [arXiv:nucl-th/0304018].

Proton-deuteron elastic scattering

Black line: 2NF only

Red dashed line: 2NF + 3NF(N2LO+2πN3LO) ≈ 2NF + 3NF(N2LO)



The 2πN3LO 3NF is negligible.

From:
Ishikawa & Robilotta,
PRC 76, 014006 (2007)

Interim balance

- The 3NF at N3LO may be generally weak.
- Reason: only “leading” vertices are involved, which are known to be weak (e.g., from NN).
- Thus, the 3NF at N3LO may not solve any of the outstanding problems.

What now?

Go to the next order!

The chiral 3NF

Basic Diagrams

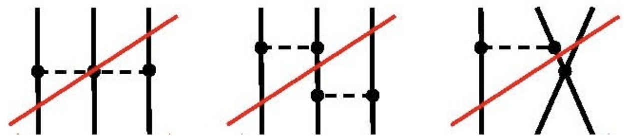
$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

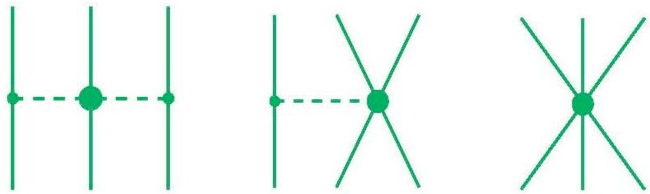
$(1/M)^n$ corrections

Leading Order (Q^0)

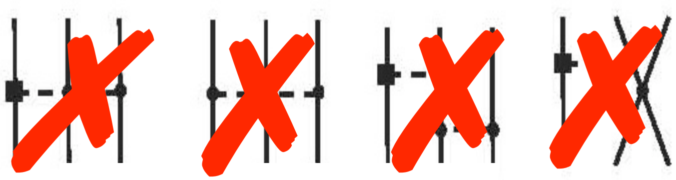
Next-to Leading Order (Q^2)

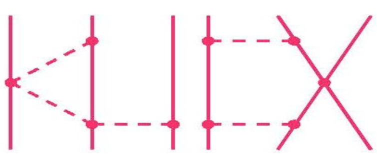


Next-to-Next-to Leading Order (Q^3)



Next-to-Next-to-Next-to Leading Order (Q^4)

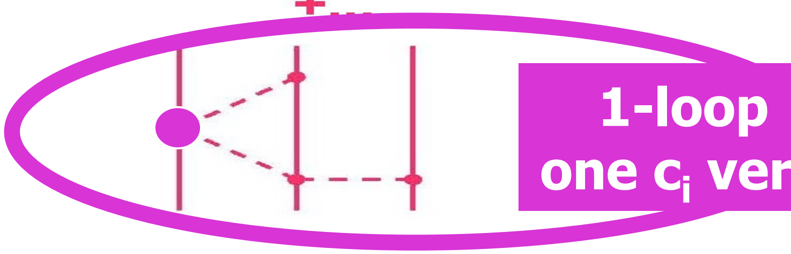




1-loop lead. vert.

+ ...

N4LO (Q^5)

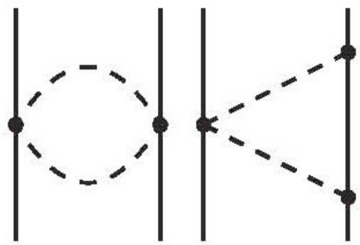
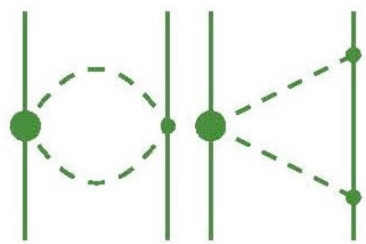


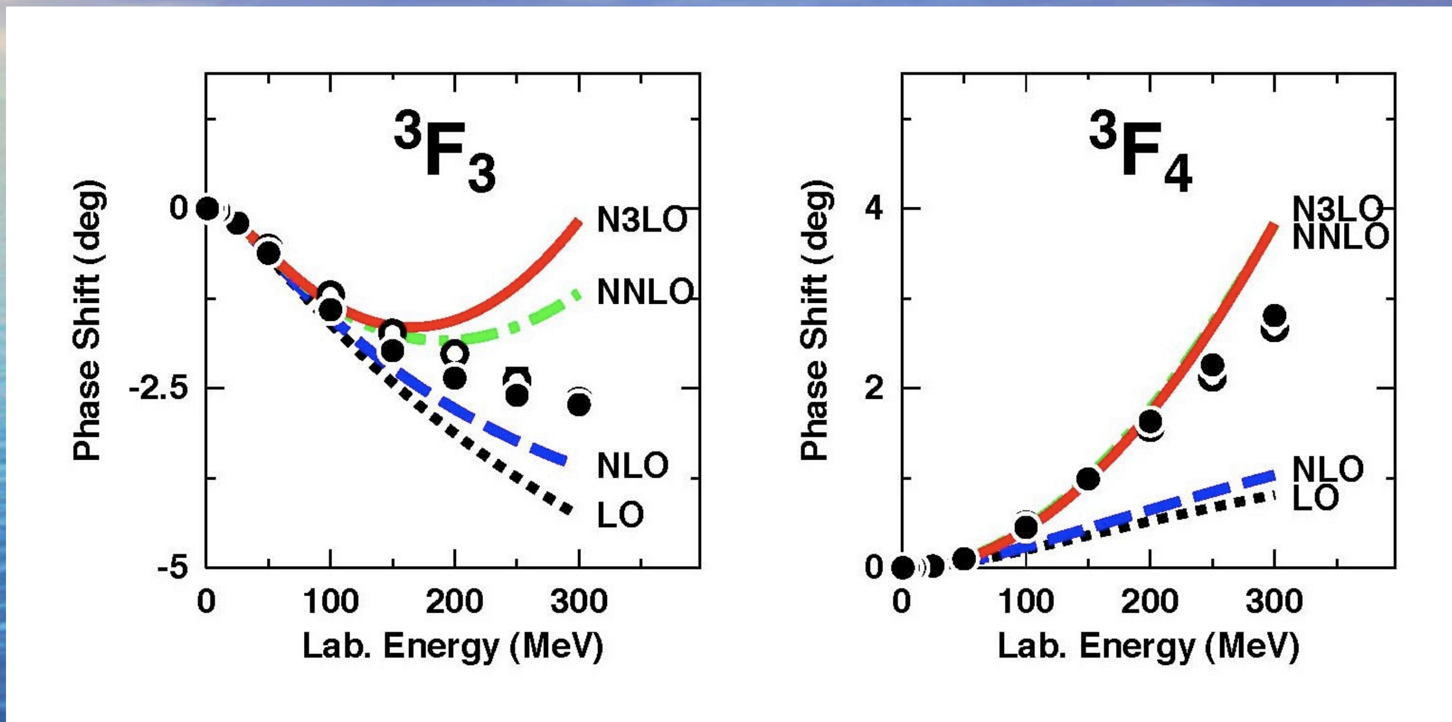
1-loop one c_i vert.

What to expect from those N4LO 1-loop diagrams?

Compare to “similar” 2NF diagrams.

Corresponding 2NF contributions

	2π
1-loop lead. vert.	
1-loop One c_i vert.	



$^3F_3$
Phase Shift (deg)
0
-2.5
-5
N3LO
NNLO
NLO
LO
0
100
200
300
Lab. Energy (MeV)
$^3F_4$
Phase Shift (deg)
4
2
0
N3LO
NNLO
NLO
LO
0
100
200
300
Lab. Energy (MeV)

Corresponding 2NF contributions

2π | **3π**

1-loop
lead. vert.

Kaiser (2000)

1-loop
One c_i vert.

Kaiser (2001)

The chiral 3NF

Basic Diagrams

$$\text{Power} = 2 + 2L + \sum_{\text{all vertices}} \Delta_i$$

$$\text{with } \Delta_i = d_i + \frac{n_i}{2} - 2$$

$(1/M)^n$ corrections

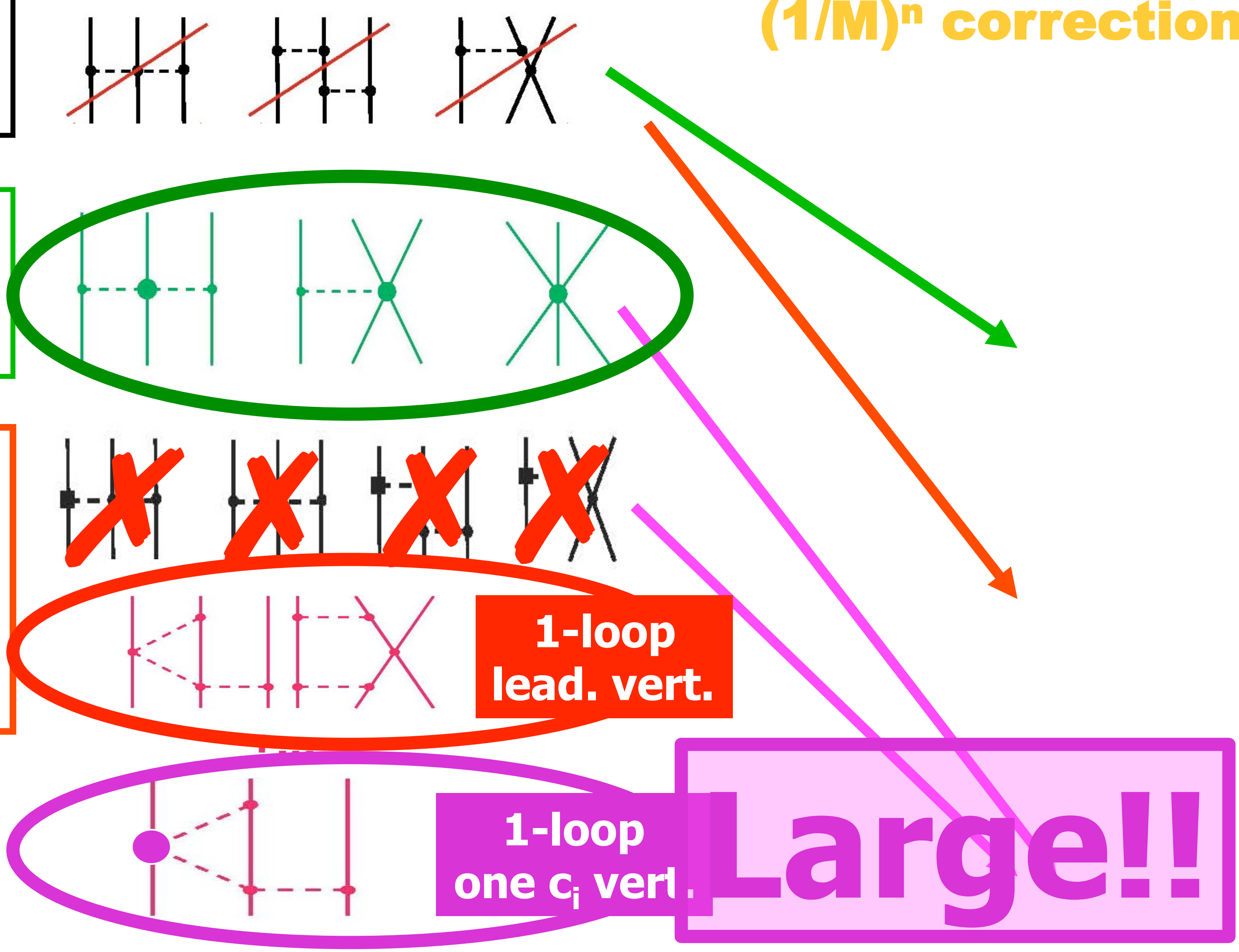
Leading Order (Q^0)

Next-to Leading Order (Q^2)

Next-to-Next-to Leading Order (Q^3)

Next-to-Next-to-Next-to Leading Order (Q^4)

N4LO (Q^5)



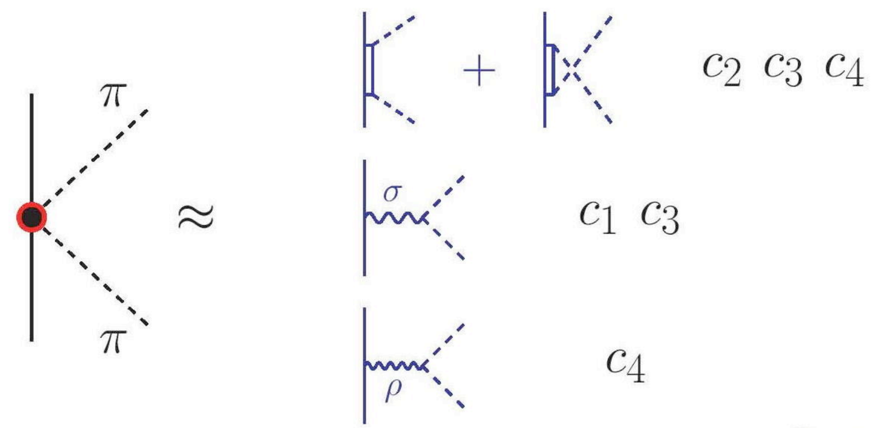
So, we will have to go to N4LO!?

- No, not necessarily.
- There is an alternative:

  go to N3LO, but include $\Delta(1232)$ isobars explicitly.

pi-N Lagrangian with two derivatives ("next-to-leading" order)

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\pi N, c_i} = \; & \bar{N} \Big[2\, c_1\, m_\pi^2 \,(U + U^\dagger) \\
& + \left(c_2 - \frac{g_A^2}{8 M_N} \right) u_0^2 \\
& + c_3 \, u_\mu \, u^\mu \\
& + \frac{i}{2} \left(c_4 + \frac{1}{4 M_N} \right) \vec{\sigma} \cdot (\vec{u} \times \vec{u}) \Big] N
\end{aligned}
$$



Bernard et al. '97

Chiral 2NF

Δ less

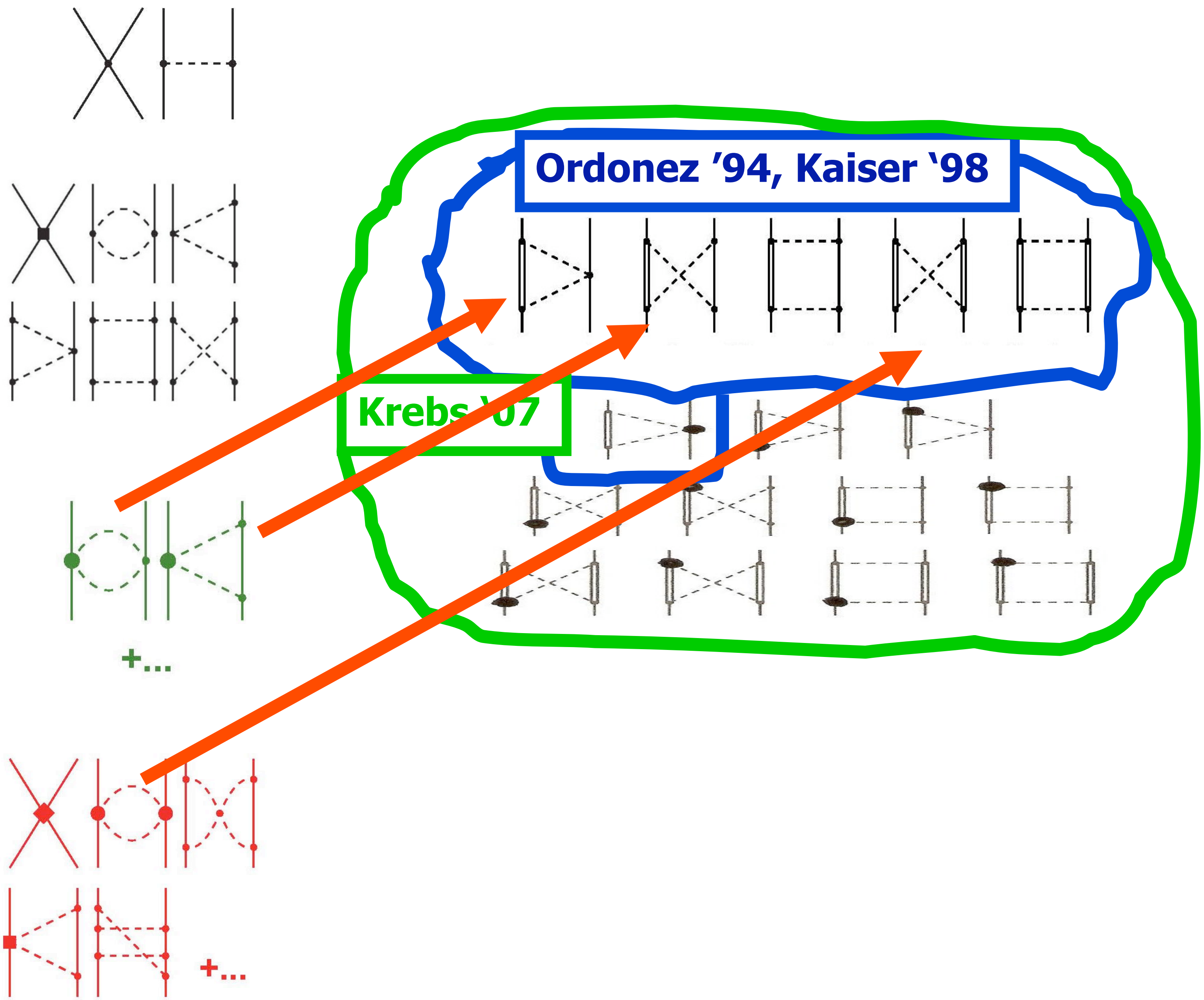
Additional in Δ full

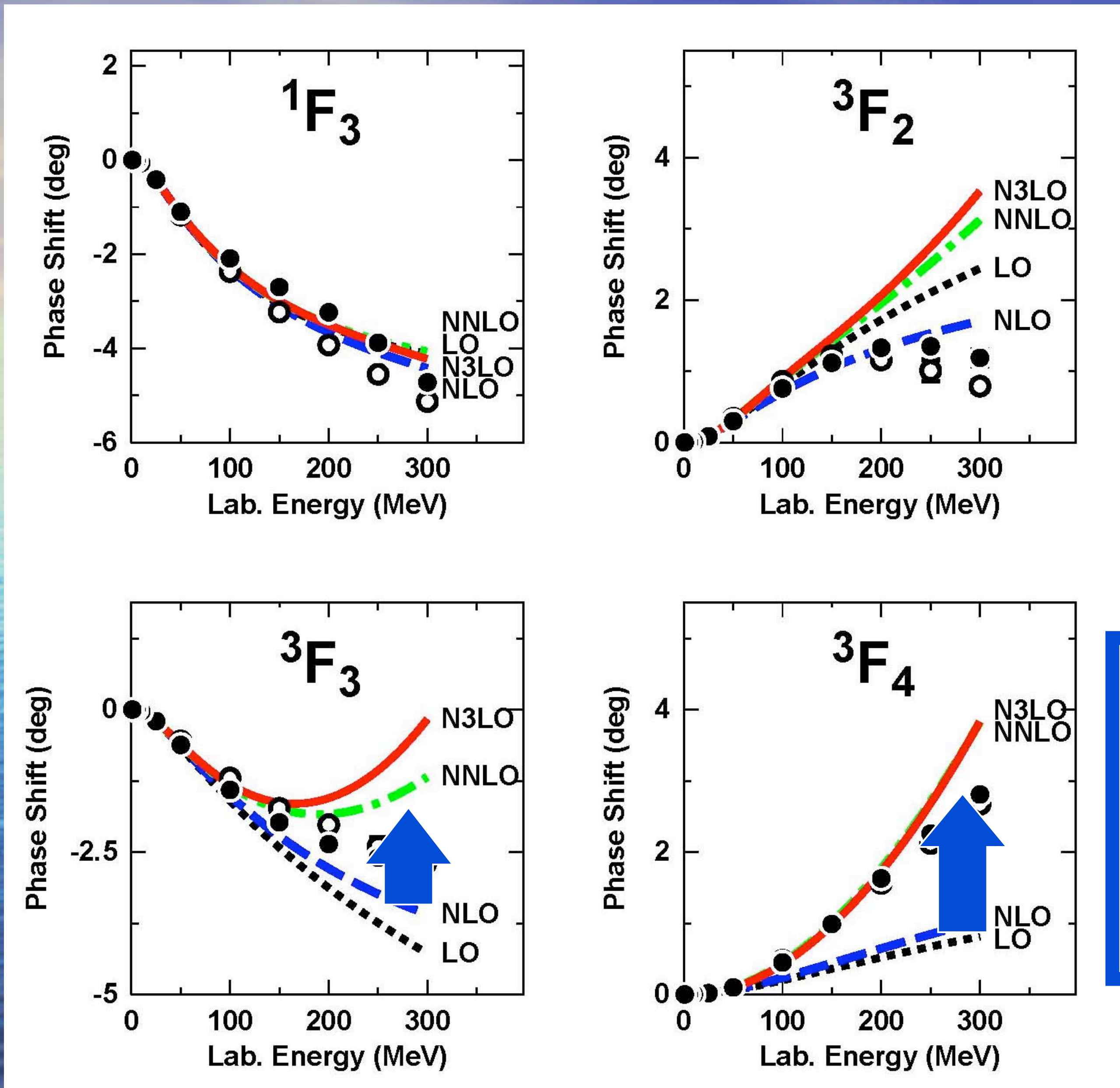
Leading Order — Q^0 LO

Next-to Leading Order — Q^2 NLO

Next-to-Next-to Leading Order — Q^3 N^{2}LO

Next-to-Next-to-Next-to Leading Order — Q^4 N^{3}LO





Δ-full theory Raises NLO, Better convergence

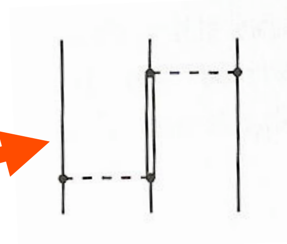
The 3NF in the Δ-full theory

The chiral 3NF: Δ-less vs. Δ-full

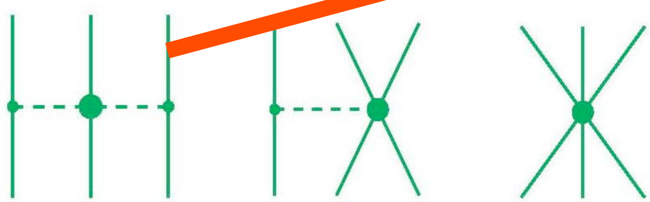
Δ less | **Additional in Δ full**

Leading Order (Q^0)

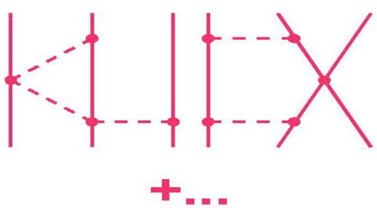
Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)

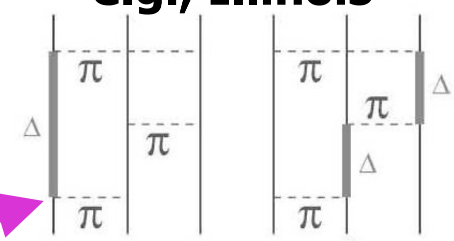


Next-to-Next-to-Next-to Leading Order (Q^4)

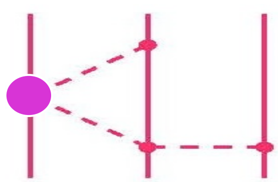


1-loops with Δ

e.g., Illinois



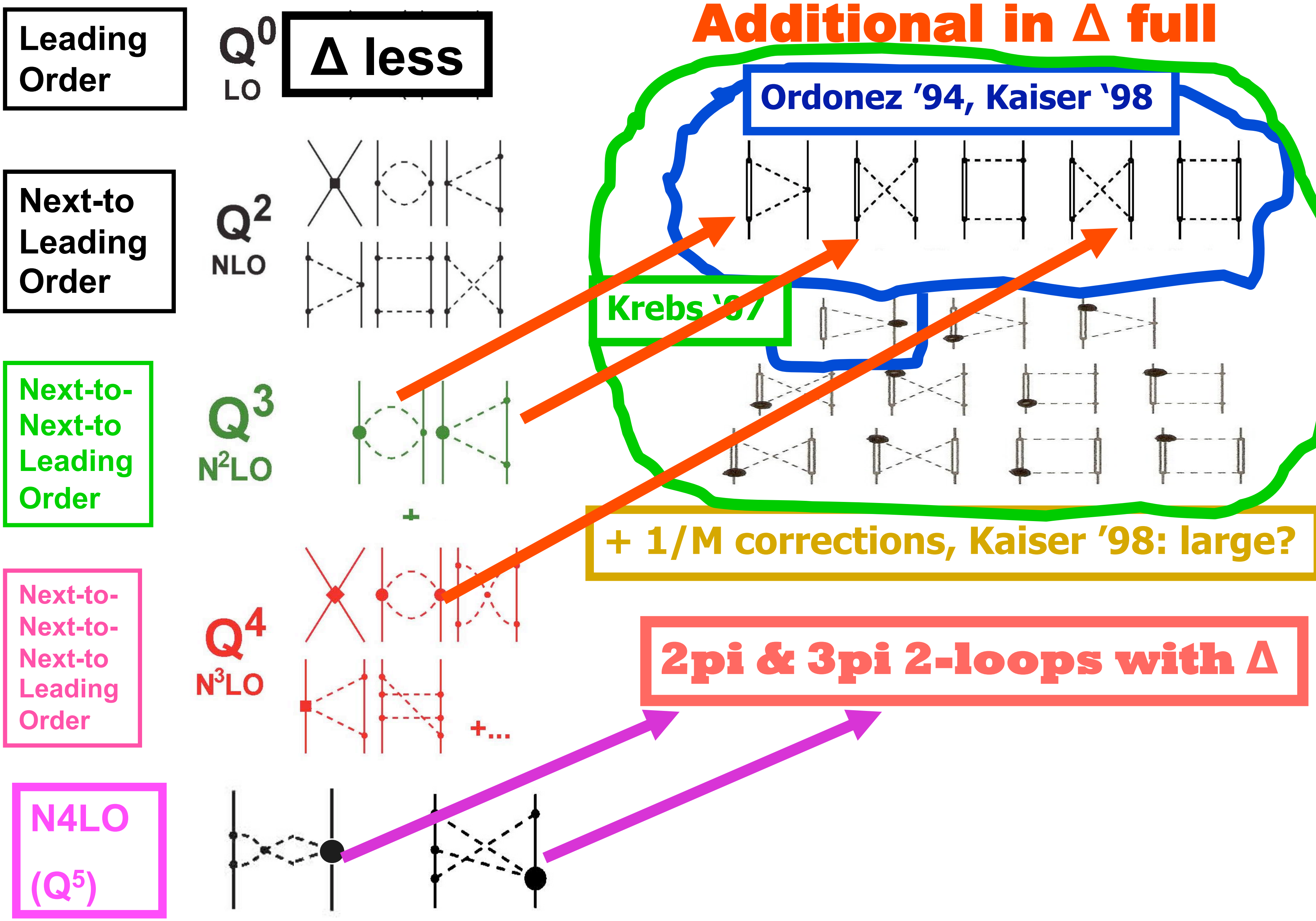
N4LO (Q^5)



However, there is a however …

- For reasons of consistency, the 2NF must then also include Δ's---up to N3LO!
- So, what does that look like?

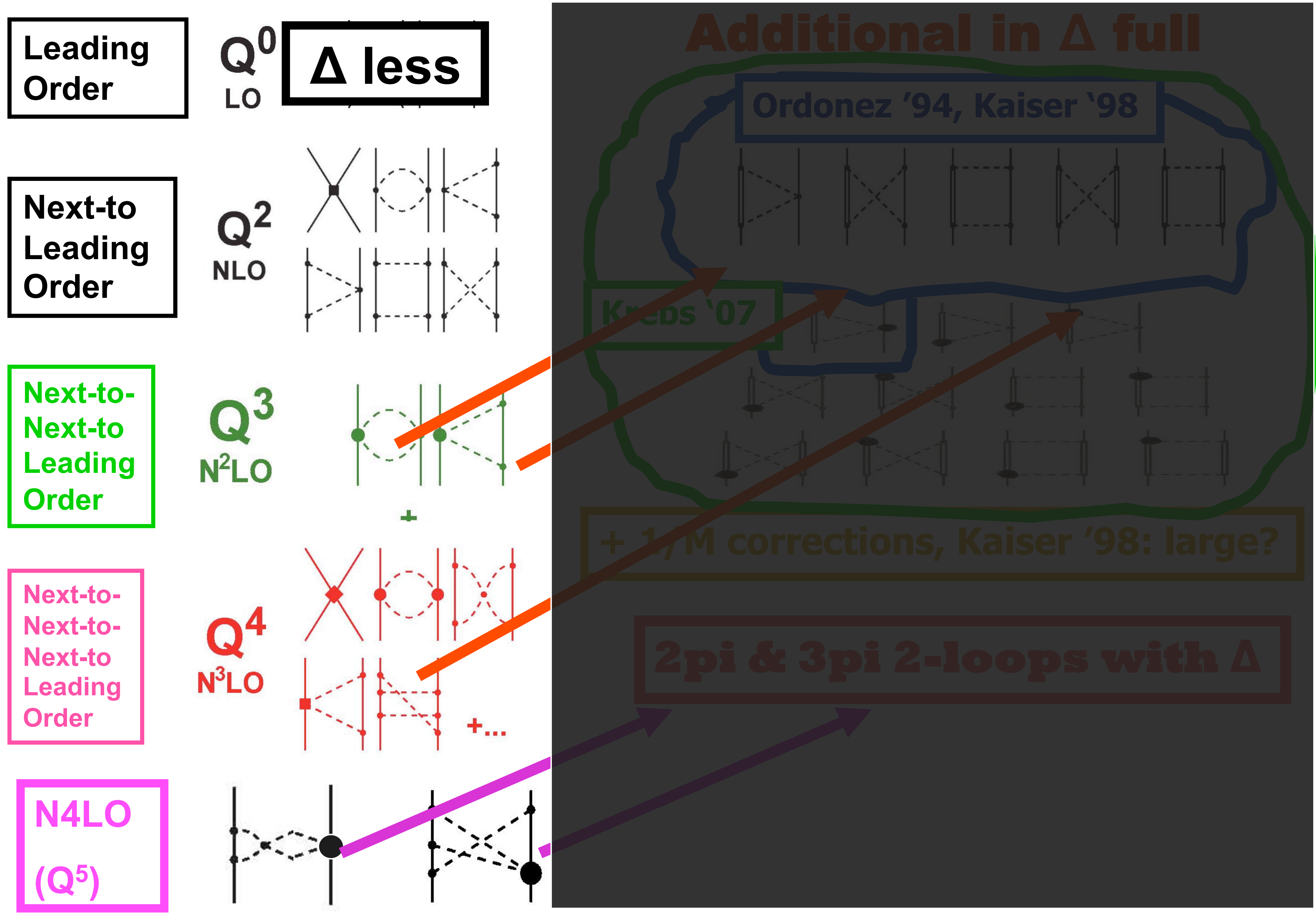
The chiral 2NF: Δ-less vs. Δ-full



In summary:

Either

The chiral 2NF: Δ-less vs. Δ-full



The chiral 3NF: Δ-less vs. Δ-full

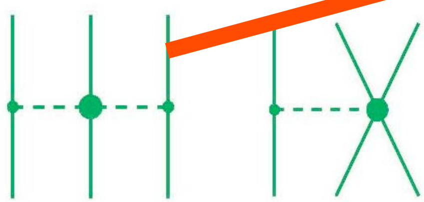
Leading Order (Q^0)

Δ less

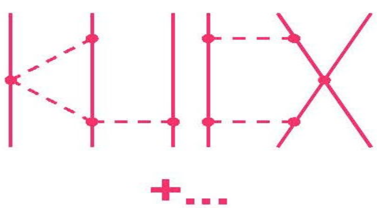
[redacted]

Next-to Leading Order (Q^2)

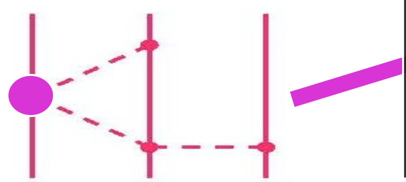
Next-to-Next-to Leading Order (Q^3)



Next-to-Next-to-Next-to Leading Order (Q^4)

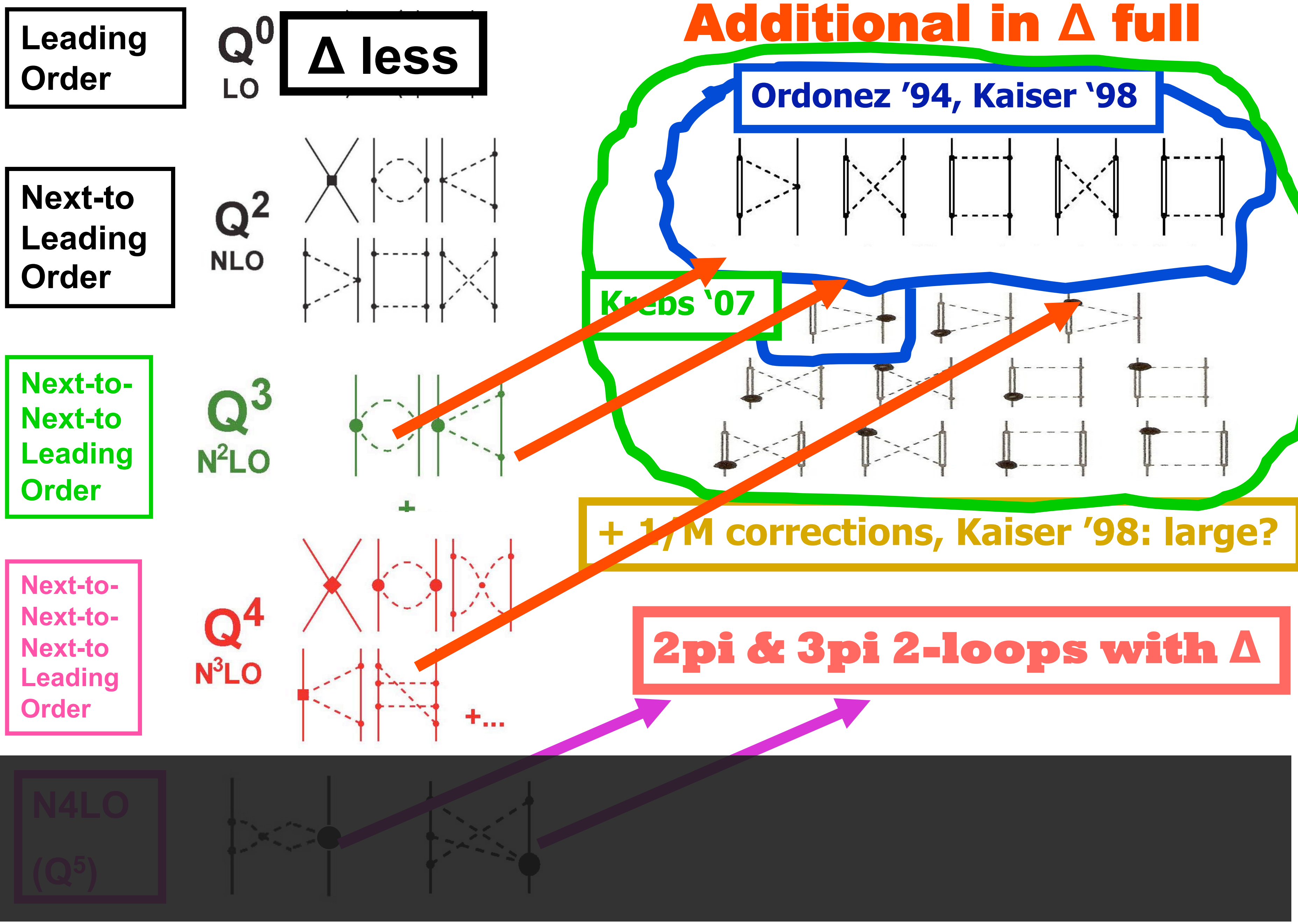


N4LO (Q^5)



Or

The chiral 2NF: Δ-less vs. Δ-full

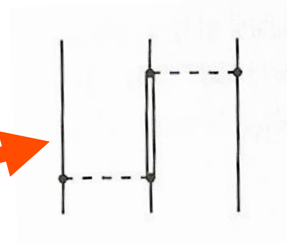


The chiral 3NF: Δ-less vs. Δ-full

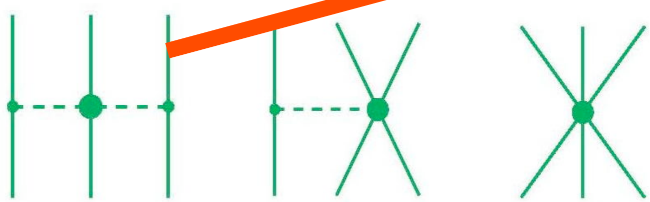
Δ less | **Additional in Δ full**

Leading Order (Q^0)

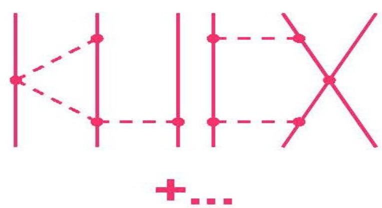
Next-to Leading Order (Q^2)



Next-to-Next-to Leading Order (Q^3)

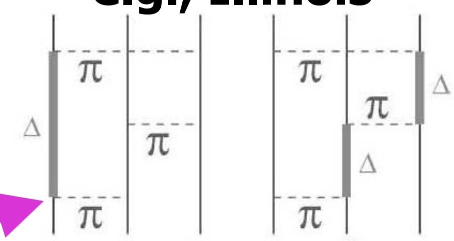


Next-to-Next-to-Next-to Leading Order (Q^4)



1-loops with Δ

e.g., Illinois



[redacted]

Whatever we decide (Δ-less or Δ-full), it will be impossible to do a complete calculation anytime soon; so what's a good strategy?

- Assume "resonance saturation" for the 2NF, i.e., stay with the present Δ-less N3LO 2NF (this has flaws!)
- Add the Δ-full 3NF at N3LO.

Conclusions

- **3NFs beyond N2LO are needed!**
- **The 3NF at N3LO of the Δ-less theory may be weak and useless.**
- **This calls for further 3NF contributions:**

  - Δ-less N4LO or

  - Δ-full N3LO,

  they will be large, but none will be easy.
- **ChPT of nuclear forces has been fairly easy and straightforward, so far; but what's left won't be easy, no matter how you do it.**

The missing three-nucleon forces: Where are they?

They are

- at N4LO in Δ-less or
- at N3LO in Δ-full.