π N scattering in the delta-isobar region

--- Towards delta-ful nuclear forces

Bingwei Long (ECT*, U. Arizona)

U. van Kolck (U. Arizona)

Outline

Why delta?

Power counting

Fitting phase shifts (preliminary)

Conclusion

Delta in nuclear forces

$$\delta \equiv m_\Delta - m_N \approx 300\,\text{MeV} \approx 2\,m_\pi << M_{QCD}(\sim 1\,\text{GeV})$$

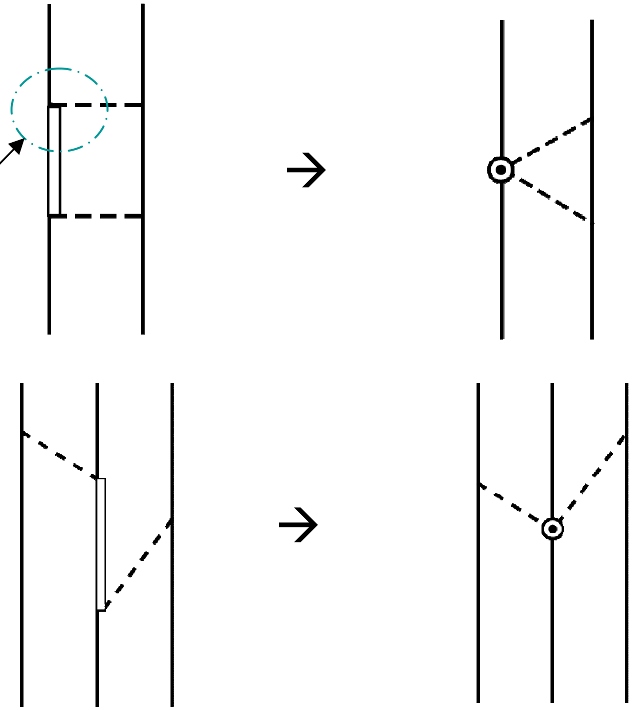
Delta-ful Delta-less

Delta as an explicit DOF

→better convergence of EFT expansions

WANTED!

via π N scattering around the delta peak



Delta becomes short-range physics

EFT expansion in powers of $m_\pi / \delta \sim 0.5$

Delta-less 3NF makes error of ~25%

Pandharipande, Phillips & van Kolck (2005)

An incomplete "recipe" for EFT

- Relevant ***degrees of freedom*** at low energies
- ***Symmetries***
- ***Power counting***: a scheme to weigh numerous contributions
- Renormalization

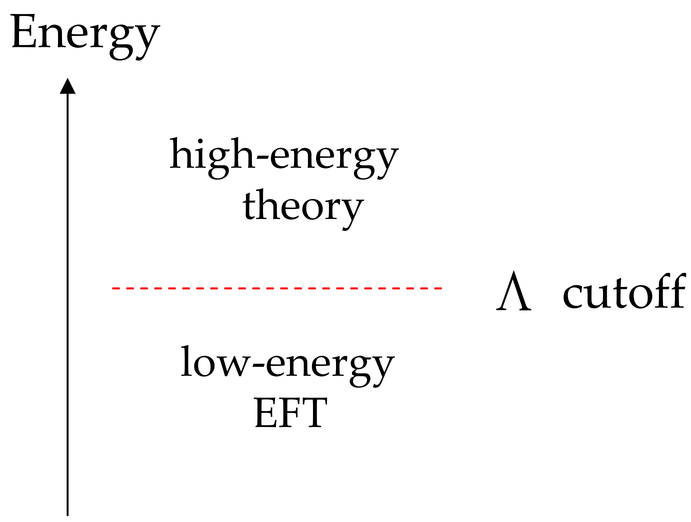
⇓

observables independent of Λ

renormalization group (RG) invariance

⇓

Model independence



EFT of nuclear physics

Few GeV — Perturbative QCD

~ 1GeV — Lattice QCD

M_{QCD} {
m_N , m_Δ ... Nucleons and excitations
m_ρ , m_ϖ ... Non-Goldstone mesons
...

~ 100MeV — Nuclear EFT

M_{Nuc} ~ ?

Systematic Expansion in Q/M_{QCD}

Dofs

- Nucleons
- Delta isobars
- Pions
- ……

Symmetries of QCD

Lorentz invariance, P, T, Δ B=0…

$SU(3)_c$ gauge : trivially preserved

all hadrons are color singlets

chiral symmetry : $SU(2)_L \times SU(2)_R$

1. spontaneously broken
2. approximate ($m_{u,\,d} \neq 0$)

Pseudo-Goldstone bosons (→ Pions)

non-linear realization of chiral symmtry

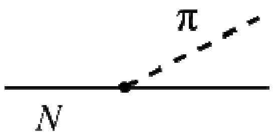
Callen, Coleman, Wess & Zumino (1969)

Standard ChPT counting

- **one-nucleon** (or purely mesonic) processes

N- π couplings : infinitely many but well organized

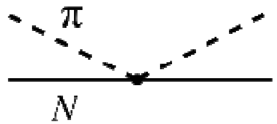
low-energy constants (LECs)
assume naturally sized



$O(1)\frac{Q}{f_\pi}$

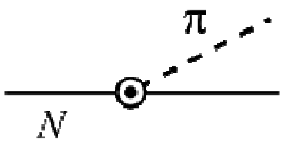
Q: k_π , m_π ...

f_π : pion decay constant ~ 92MeV

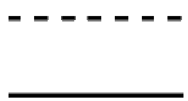


. . .

once-circled = 1-more derivative



$O(1)\frac{Q}{f_\pi}\frac{Q}{M_{QCD}}$



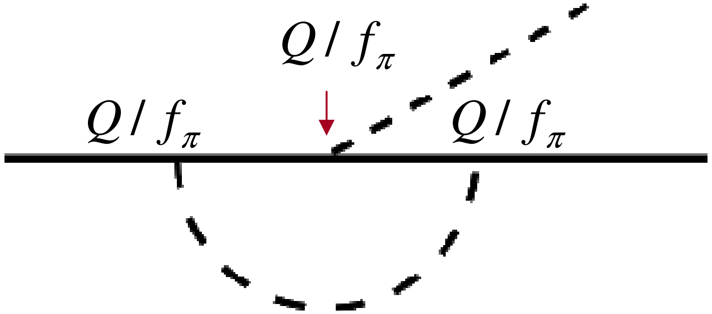
pion propagator $\sim 1/Q^2$

nucleon propagator $\sim 1/Q$

Truncation → approximation of N- π interactions

Counting pion loops

Assume: renormalization has been properly done.



$$\sim O(1)\,\frac{Q}{f_\pi}\left(\frac{Q}{4\pi f_\pi}\right)^2$$

pion propagator	$\sim 1/Q^2$
nucleon propagtor	$\sim 1/Q$
loop integral	$\sim \int \frac{d^4 l}{(2\pi)^4} \sim \frac{Q^4}{(4\pi)^2}$

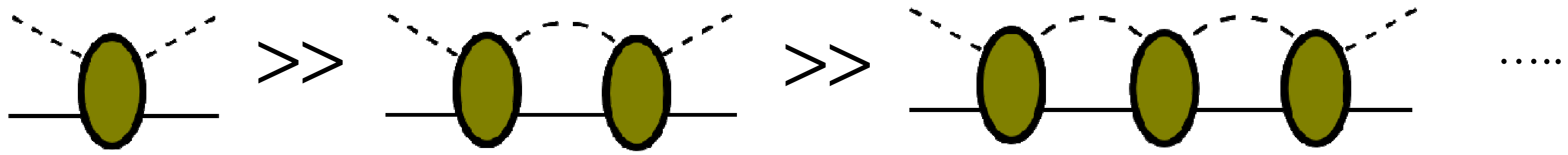
suppressed by $\left(\frac{Q}{M_{\text{QCD}}}\right)^2$

$4\pi f_\pi \sim 1\text{GeV} \sim M_{\text{QCD}}$

Check: the established counting provides enough counterterms for renormalization.

Where comes the delta resonance?

πN "potential" once-iterated twice-iterated



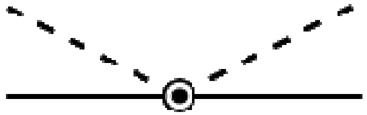
can't be broken by cutting a pion *and* nucleon line

suppressed due to the pion loop

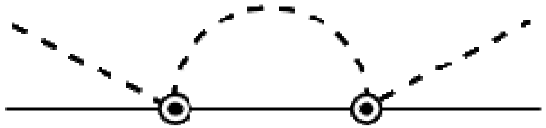
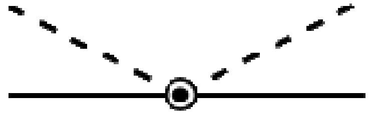
suppressed due to the pion loops

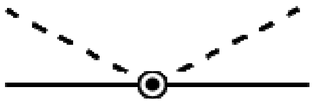
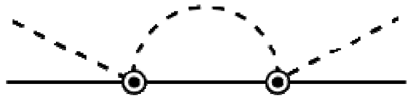
A resonance *cannot* be generated by a perturbative series → change standard ChPT power counting to generate the delta resonance

In P_{33} channel, the LECs are unnatural

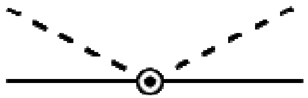


$$\gg \frac{O(1)}{M_{QCD}} \frac{Q^2}{f_\pi^2}$$

so that: when $Q \sim \delta$  $\sim$ 

therefore  +  + … in order to generate the delta resonance

But…

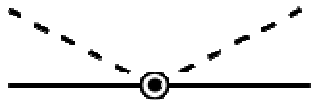
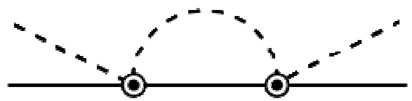


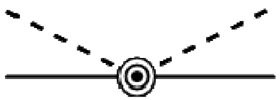
$N^\dagger N \nabla\pi\nabla\pi$

non-renormalizable interactions (NR)

- Isn't EFT supposed to make use of NR interactions?
- Not necessarily so when NR interactions are non-perturbative.

What would happen is…

To remove cutoff dependence in  +  + …

One needs to promote  → new cutoff dependence

→ promote yet another counterterm → ……

An incomplete "recipe" for EFT

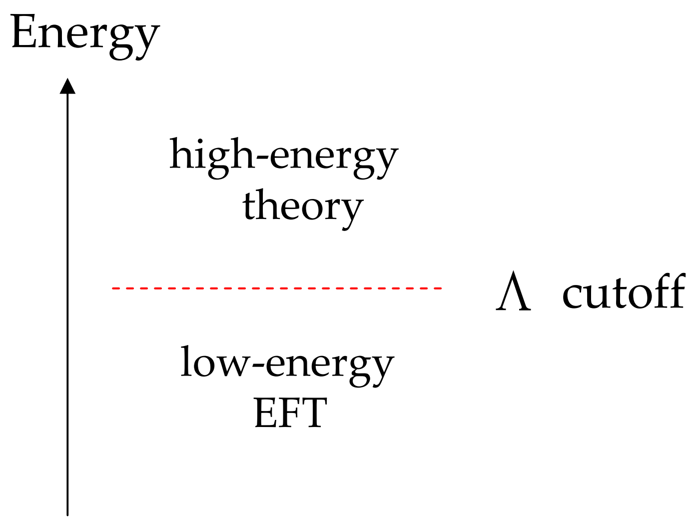
- Relevant ***degrees of freedom*** at low energies
- Symmetries
- Power counting: a scheme to weigh numerous contributions
- Renormalization

⇩

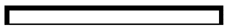
observables
independent of Λ
renormalization group
(RG) invariance

⇩

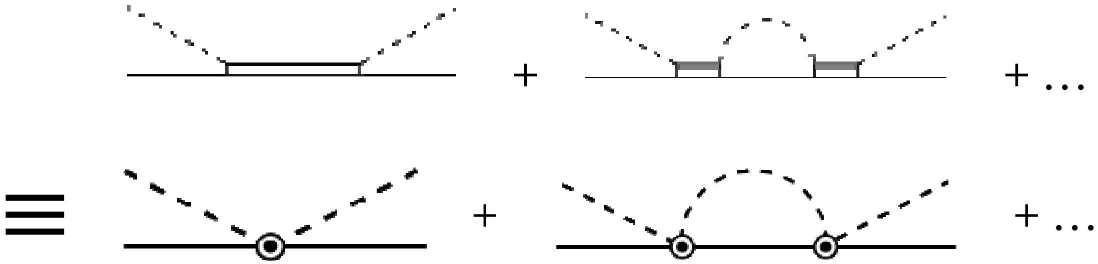
Model independence



Explicit Delta DOF

Delta propagator  $\sim \dfrac{1}{E-\delta}$ E : center-of-mass energy

To produce the delta resonance,



in the deltaless theory

With explicit delta: - Natural LECs

- Resonance arises due to $E-\delta$ diverging

What have been done

Ellis & Tang (1998)

pion-nucleon scattering & explicit delta but very different power counting

Fettes & Meissner (2001)

pion-nucleon scattering & explicit delta but below delta

Pascalutsa & Phillips (2003)

photon-nucleon scattering & explicit delta, similar power counting but $m_\pi << \delta << M_{QCD}$

$\vdots$

Delta as a nonrelativistic baryon

- $Q \ll M_N$, M_Δ → nucleon and delta are nonrelativistic (heavy baryons)

  If $Q \sim M_N$, ChPT already breaks down

- Perturbative Lorentz invariance in powers of Q/M

- A bottom-up approach (the one we use)

  - Rotational invariant (RI) operators

    Delta field: 4-component spinor (spin 3/2)

    e.g. $N^\dagger \vec{S}\Delta \cdot \vec{\nabla}\pi$ (isospin suppressed)

  - Order-by-order boost transformation rules constrains coefficients of RI operators

Foldy-Wouthuysen rep.

$\chi(t,\vec{x})$ (2s+1)-component spinor

An infinitesimal boost $\chi'(t,\vec{x}) \equiv (1 - i\vec{\xi}\cdot\vec{K})\chi(t,\vec{x})$ rapidity $\xi \sim Q/m$

Boost generators

$$\vec{K} = \frac{1}{2}(\vec{x}\omega + \omega\vec{x}) - \frac{\vec{s}\times\vec{p}}{m+\omega} - t\vec{p}$$

$$\vec{p} \equiv -i\vec{\nabla} \quad \omega \equiv (m^2 + p^2)^{\frac{1}{2}} \quad \vec{s}: \text{spin operators}$$

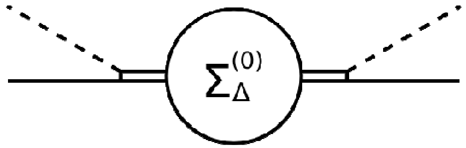
- ω is nonlocal → not so easy to construct a fully relativistic theory
- A formal expansion in p/m → order-by-order trans. rules for boosts

$$\vec{K}^{(0)} = m\vec{x} - t\vec{p} \qquad \text{(Galilean transformation)}$$

$$\vec{K}^{(1)} = \frac{1}{4m}\left(p^2\vec{x} + \vec{x}p^2\right) - \frac{1}{2m}\vec{s}\times\vec{p}$$

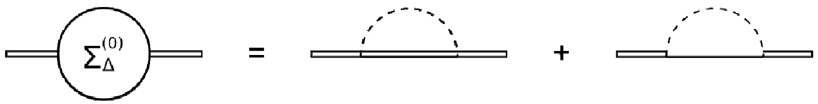
Power counting

one- Δ -reducible : diagrams with a pure delta intermediate state



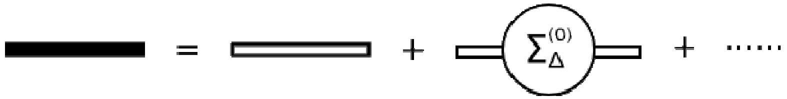
$\cdots\cdots$

$\sim \dfrac{1}{E-\delta}$ $\qquad$ $\sim \dfrac{1}{E-\delta}\dfrac{\Sigma_\Delta^{(0)}}{E-\delta}$



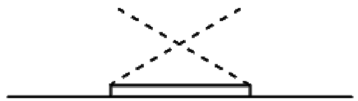
$\sim \dfrac{Q^3}{M_{QCD}^2}$

Dressing needed in one- Δ -reducible diagrams when $|E-\delta| \sim \Sigma_\Delta^{(0)} \sim \dfrac{Q^3}{M_{QCD}^2}$

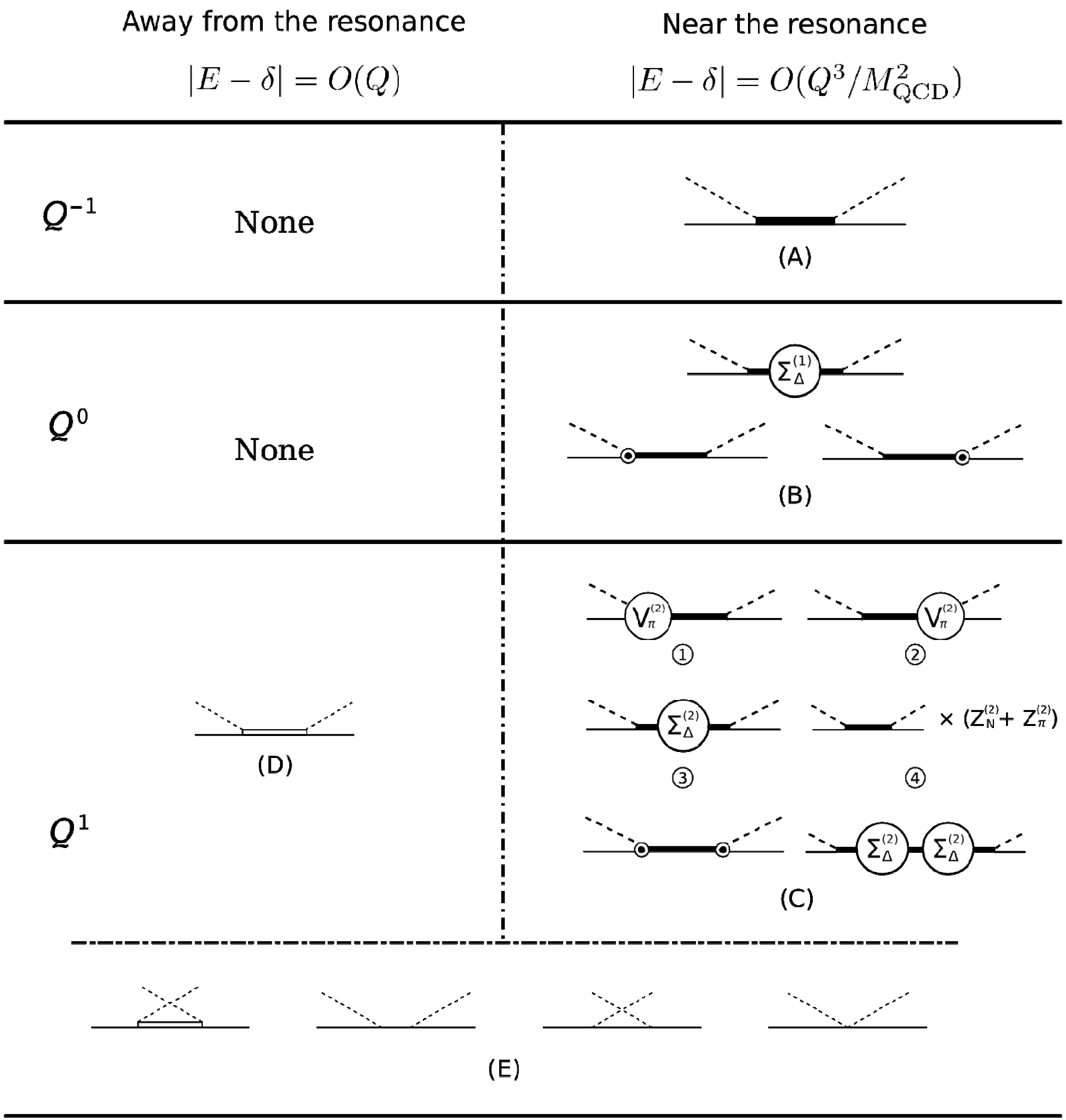


$\sim \dfrac{M_{QCD}^2}{Q^3}$ $\quad$ enhanced by M_{QCD}^2/Q^2 relative to bare propagator

one- Δ -**ir**reducible: no need to dress regardless of E (standard counting applies)



$\sim \dfrac{1}{E+\delta}$ $\quad$ no cancellation between E & δ



Away from the resonance
$|E-\delta| = O(Q)$
Near the resonance
$|E-\delta| = O(Q^3/M_{\rm QCD}^2)$
Q^{-1}
None
(A)
Q^0
None
$\Sigma_\Delta^{(1)}$
(B)
Q^1
(D)
$V_\pi^{(2)}$
①
②
$\Sigma_\Delta^{(2)}$
③
$\times\ (Z_N^{(2)} + Z_\pi^{(2)})$
④
(C)
(E)

Sewing two regions

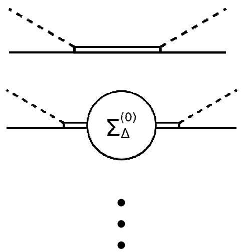
Two countings for two different regions → a piece-wise description ?

	Away from the resonance $|E-\delta| = O(Q)$	Near the resonance $|E-\delta| = O(Q^3/M_{\rm QCD}^2)$
Q^{-1}	0	

What if enforcing  in both regions ?

Q^{-1} 

equivalent to



a ***subset*** of higher orders → leading order

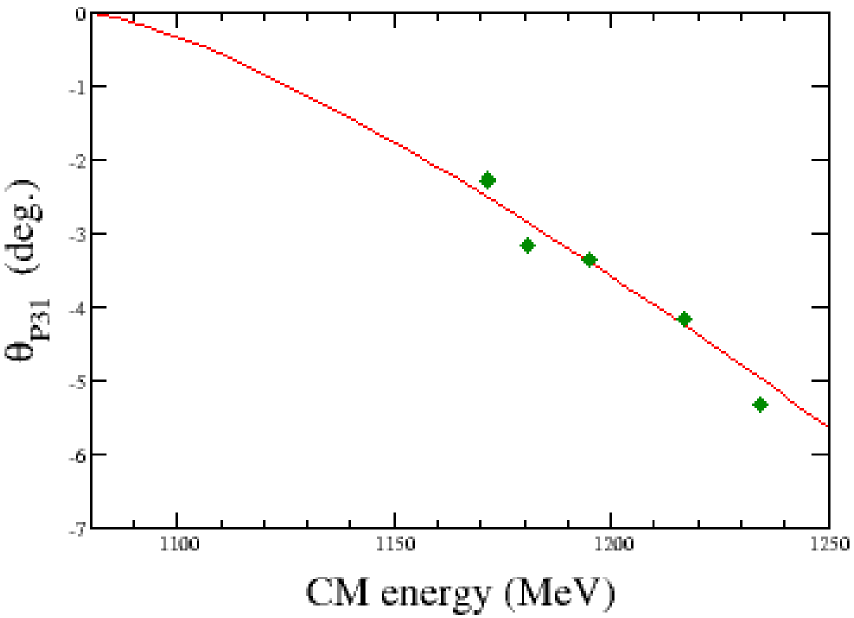
Still the same counting as long as
- not claiming better accuracy
- added terms are renormalized

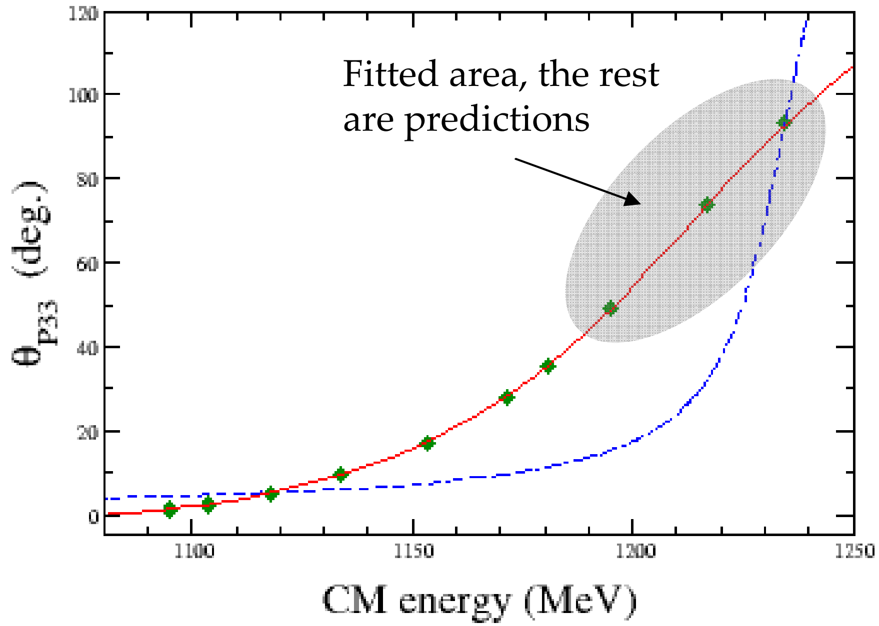
P-wave phase shifts (Preliminary)

Dots - PSA inputs (SAID program, George Washington group)

Blue dashed - LO

Red solid - NNLO





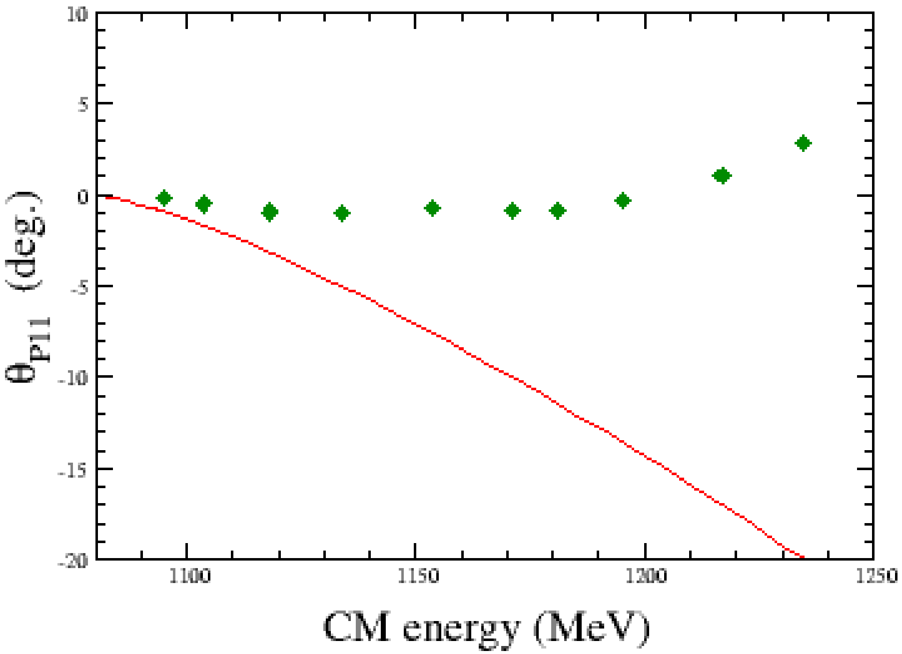
Fitted parameters

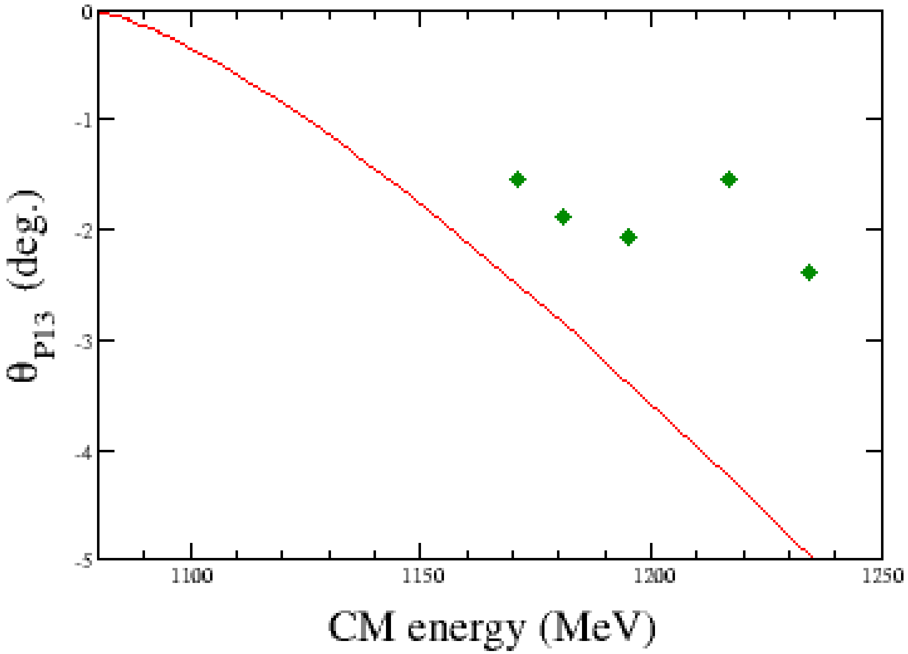
$\delta = 318\,\mathrm{MeV}$ ***not*** Breit-Wigner mass

$h_A = 2.48$ leading $\pi N\Delta$ coupling

$\kappa = 0.131$ related to subleading $\pi N\Delta$ coupling

P-wave phase shifts (Preliminary)





Conclusion

- Delta is important for nuclear forces

  Pion-nucleon scattering around the delta peak → $\pi N\Delta$ couplings

- Explicit delta DOF

- A non-standard ChPT power counting

- A good description of π N scattering with 3 parameters