Field Theoretical Approach to Density Functional Theory and Relativistic Fermi-Liquid Theory

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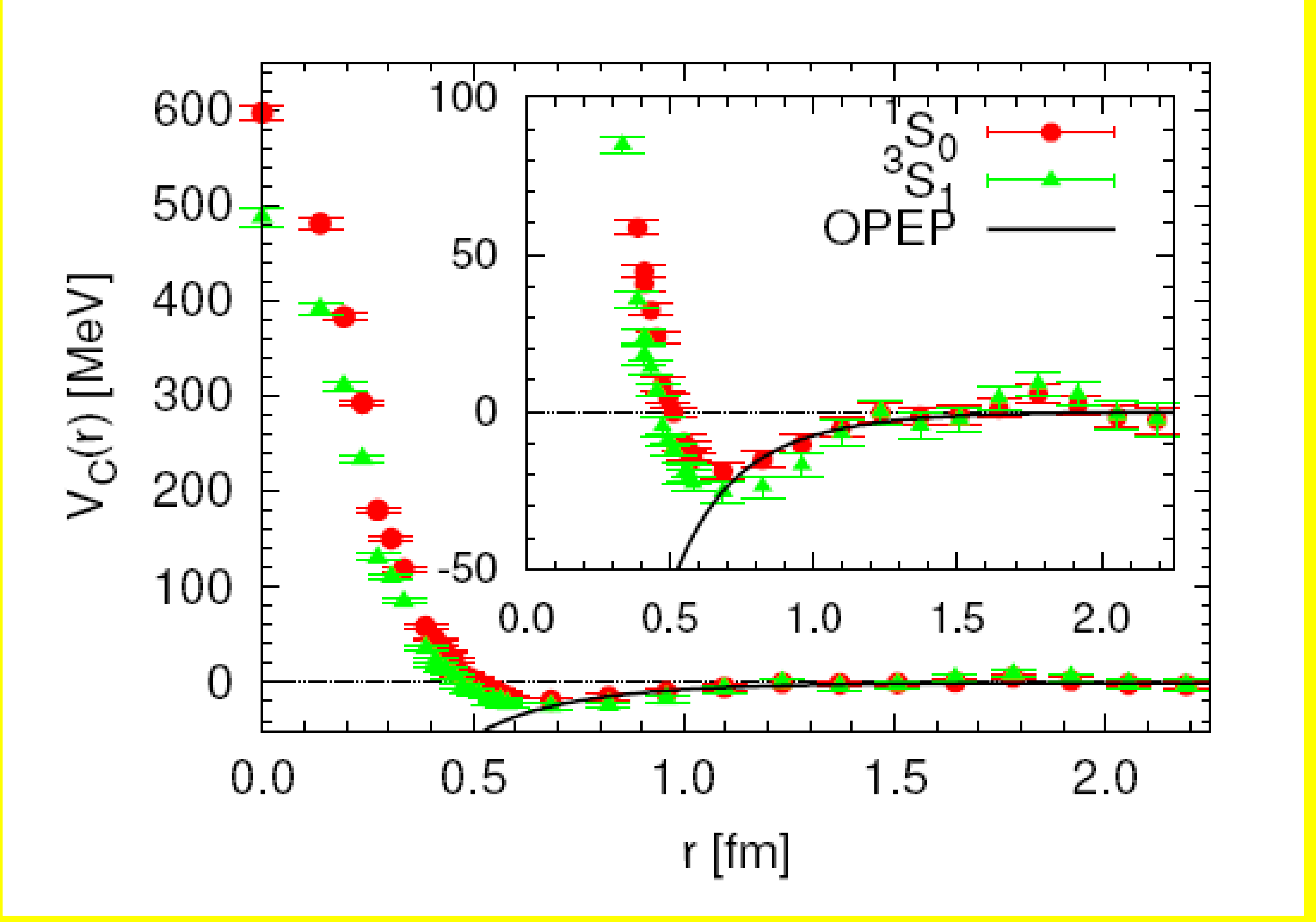
The Topics:

- Hadrons, their Interactions, and Nuclei
- Nuclear Density Functional Theory
- Nuclear Matter and Nuclei
- Dynamics: Relativistic Fermi Liquid Theory
- Quasielastic Response
- Summary and Outlook

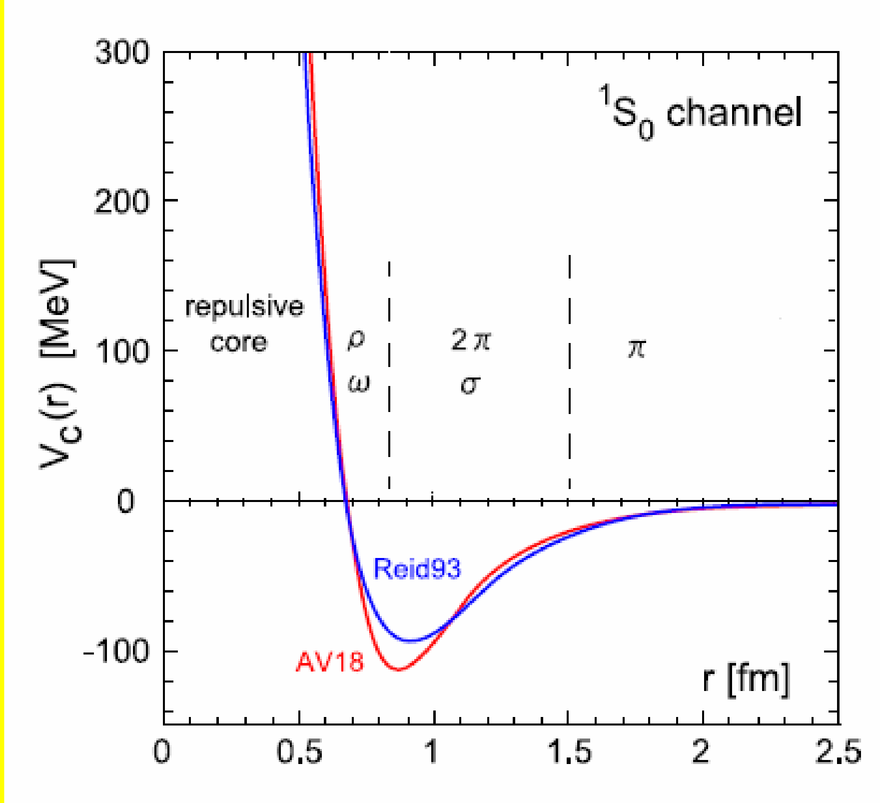
NN-Interaction from the Lattice

N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007).



$m_\pi/m_\rho = 0.595$

Low-Energy QCD: The NN Interaction at Tree-Level



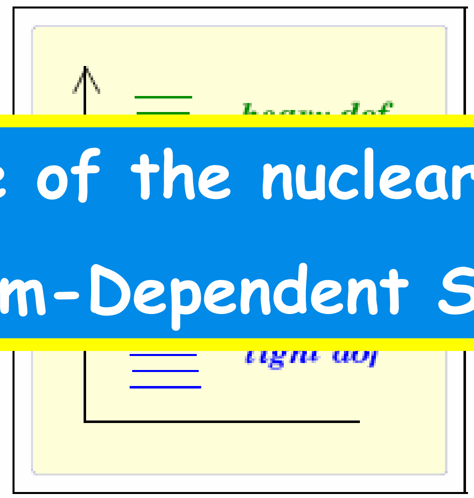
$$K_{SE} \sim \frac{1}{q}\tan\delta_{SE}(q) \sim \frac{1}{-\frac{1}{a_{SE}}+\frac{1}{2}q^2 r_{SE}}$$

At r=0: a_{SE}(S=0,I=1)= -23.8fm; a_{TE}(S=1,I=0)= +5.42fm

r_{SE} = 2.71fm ; r_{TE} = 1.71fm

Weinberg Hypothesis (~1979):

- Nuclear Physics ↔ EFT of Pions and Nucleons
- Symmetries of the underlying fundamental theory of QCD
- Spontaneously broken chiral symmetry
- Low energy theorems
- Order-by-Order expansion in Q/Λ with LEC



Weinberg 1979

Influence of the nuclear medium?

Medium-Dependent Scales?

$$\mathcal{L}_{\mathrm{QCD}} \rightarrow \mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi \mathrm{N}} + \mathcal{L}_{\mathrm{NN}} + \ldots$$

Theorems on the Dynamics of Interacting Quantum Many-Body Systems:

Kohn-Sham (~1960) : QM many-body systems DFT of E[ρ]

Kohn-Hohenberg (~1963) : DFT → E[ρ,τ]

Nuclei : E[$\rho_p,\rho_n,\tau_p,\tau_n,\kappa_p,\kappa_n \ldots$]

$$E[\rho,\tau]/\rho = \tau + \tfrac{1}{2}E_{int}/\rho \sim \tau + \tfrac{1}{2}\rho(3a_{SE}+a_{TE})/4 + \ldots$$

$$\rho = \langle \Psi^{+}\Psi \rangle \sim \frac{1}{3}k_F^3 \; ; \; \tau = \left\langle \frac{\hbar^2}{2m}|\nabla\Psi|^2 \right\rangle \frac{1}{\rho} \sim \frac{3}{5}k_F^2$$

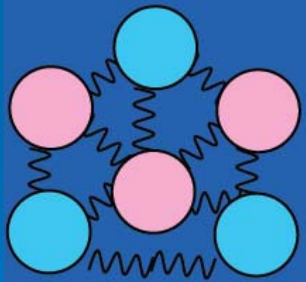
Interactions and Effective Range Expansion

$$K(k,k'|q_s k_F) = V(k,k') + \int \frac{d^3q}{(2\pi)^3} V(k,q)\tilde{g}_{NN}(q,q_s|k_F)Q_F(q,k_F)K(q,k'|q_s k_F)$$

$$\Rightarrow \quad \frac{1}{q_s}\tan(\tilde{\delta}_0) = -\frac{m}{\hbar^2 4\pi}K_0(q_s,q_s|q_s k_F) \rightarrow -\frac{1}{\tilde{a}_0(k_F)} + \frac{1}{2}q_s^2\tilde{r}_0(k_F)$$

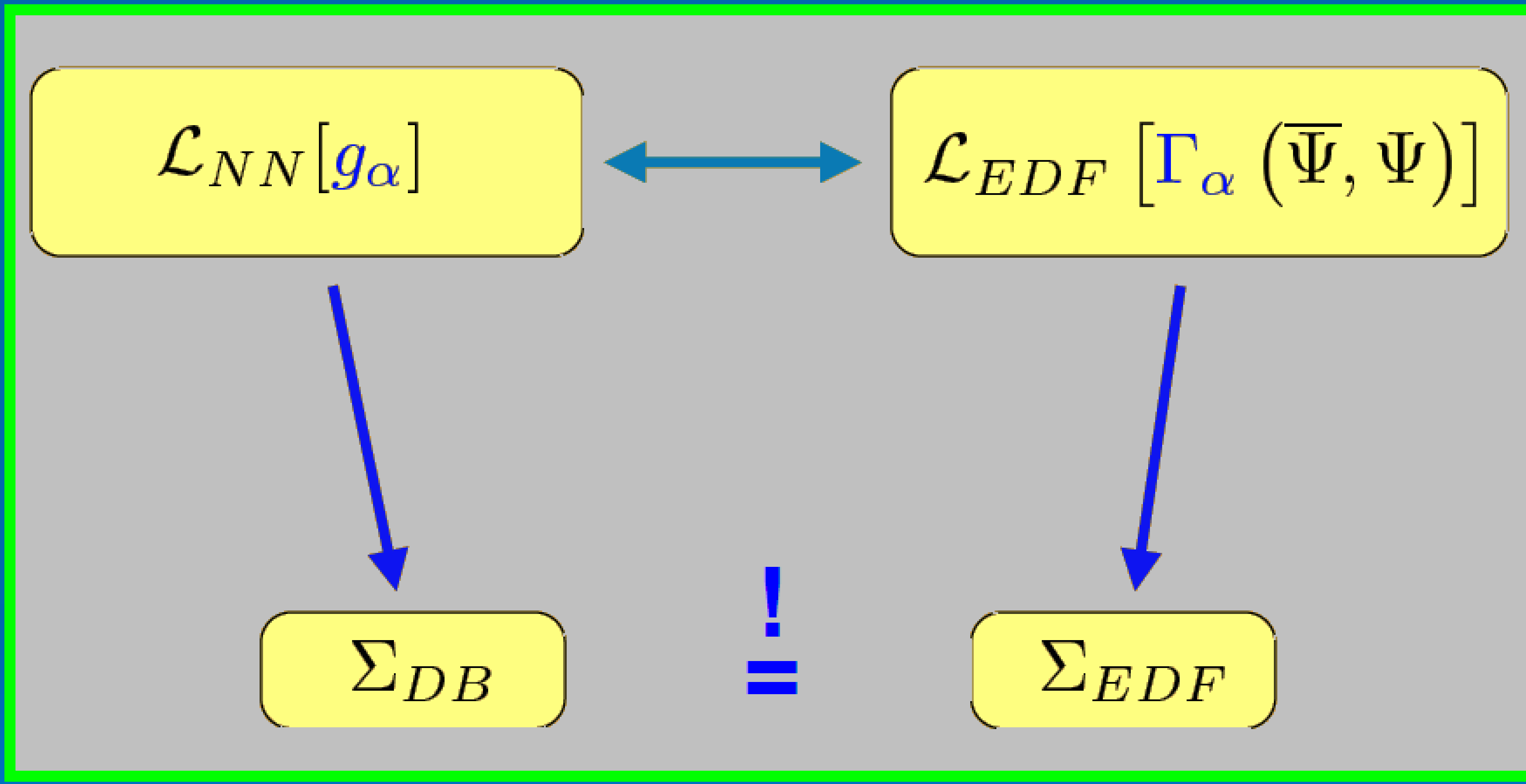
Fieldtheoretical Approach to a Hadronic Density Functional Theory

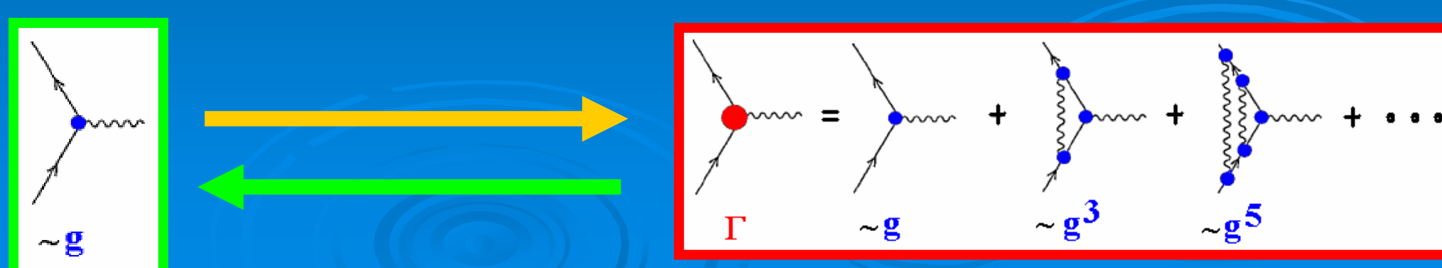
$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$



$$\begin{aligned}\mathcal{L}_{int} = \; & \bar{\psi} g_\sigma \Phi_\sigma \psi + \bar{\psi} g_\delta \boldsymbol{\tau} \boldsymbol{\Phi}_\delta \psi \\ & + \bar{\psi} g_\pi \gamma^5 \boldsymbol{\tau} \boldsymbol{\Phi}_\pi \psi + \bar{\psi} g_\eta \gamma^5 \Phi_\eta \psi \\ & - \bar{\psi} g_\omega \gamma_\mu A^\mu_\omega \psi - \bar{\psi} g_\rho \gamma_\mu \boldsymbol{\tau} \mathbf{A}^\mu_\rho \psi - e \bar{\psi} \hat{Q} \gamma_\mu A^\mu_\gamma \psi\end{aligned}$$

Building blocks for a covariant nuclear DFT…





ab initio Density Dependent Hadron Field Theory: The DDRH Lagrangian

- Covariance of field equations
- Thermodynamical consistency
- Systematic xpansion

- Density Dependent Vertices
- Static Polarization Self-Energies
- Nuclei and Hypernuclei

The Quantum-Field Equations:

$$\left(\partial_\mu \partial^\mu + m^2\right)\Phi_m = -g_m z_m(\hat{\rho})\bar{\Psi}\gamma_m \Psi$$

Full Quantal Structure, Symmetries…:

$$\Psi(x) = \sum_{ks}\left(a^+_{ks} u_{ks}(x) + b^+_{ks} v_{ks}(x)\right)$$

$$\Phi_m \sim D_m \left(\sum_{ks,k's'} \left(a^+_{ks} a_{k's'} \bar{u}_{ks} \gamma_m u_{k's'} + b^+_{ks} b_{k's'} \bar{v}_{ks} \gamma_m v_{k's'} + \ldots\right) + :\bar{\Psi}\gamma_m \Psi: \right)$$

$$D_m(p) \sim \frac{1}{p^2 - m^2} F(p^2, \Lambda^2)$$

General QFT → Nuclear DFT:

Choice of Vacuum/Reference State ↔ g.s. |0>

$$\Phi_m = \phi_m + \chi_m$$

$$\phi_m = \langle 0 | \Phi_m | 0 \rangle \sim \; :\bar{\Psi}\gamma_m \Psi:$$

$$\chi_m = \Phi_m - \phi_m \sim \left[a^+ a\right]_m \rho_m + \left[b^+ b\right]_m \tilde{\rho}_m \ldots$$

Nuclear Configuration ↔ Spontaneous Symmetry Breaking:

- Fixing N≠Z → Isospin Symmetry
- Mean-Field → Translational Symmetry
- Deformation → Rotational Symmetry

The DFT Classical Field Equations

QFT Realization for a Specific Nuclear Case:

$$\left(\partial_\mu \partial^\mu + m^2\right)\phi_m = -G_m(\rho)\rho_m$$

$$\rho_m = \left\langle 0 \right| \bar{\Psi}\gamma_m \Psi \left| 0 \right\rangle$$

$$G_m(\rho) = g_m z_m(\rho)$$

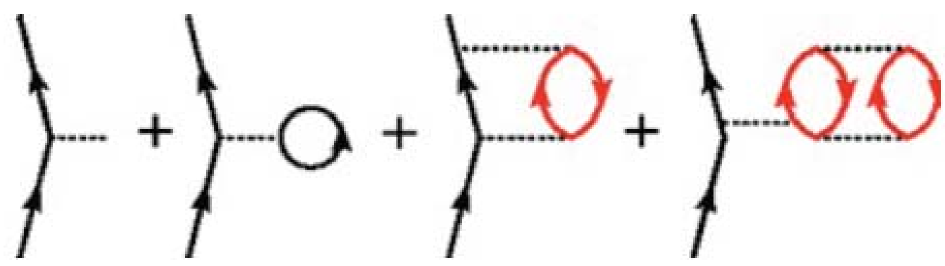
$$\Sigma_m = G_m(\rho)\phi_m(\rho)$$

$$\left(\not{p} - M + \Sigma_s(\rho) - \gamma_\nu \Sigma^\nu(\rho) + \Sigma^{(r)}(\rho)\right)\psi(x) = 0$$

Rearrangement Contributions

$$\frac{\delta \mathcal{L}_{int}}{\delta \bar{\psi}} = \frac{\partial \mathcal{L}_{int}}{\partial \bar{\psi}} + \frac{\partial \mathcal{L}_{int}}{\partial \hat{\rho}} \frac{\delta \hat{\rho}}{\delta \bar{\psi}}$$

$$\hat{\Sigma}^{\mu} = \hat{\Sigma}^{\mu(0)} + \hat{\Sigma}^{\mu(r)}$$

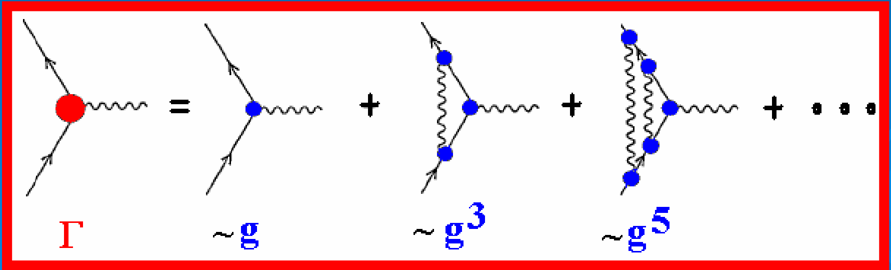


QFT Vertices → DD Vertices:

$$\Gamma_m(\hat{\rho}) = \Gamma_m(\rho) + \delta\Gamma_m$$

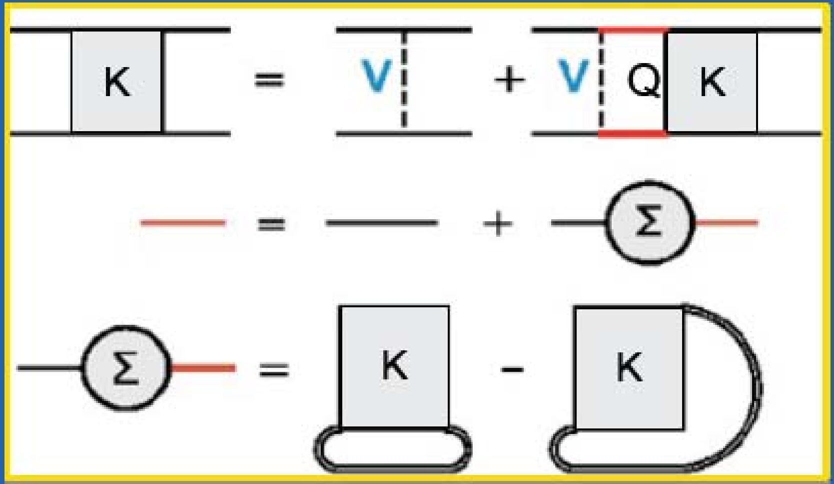
$$\bar{\Psi}\Gamma_m(\hat{\rho})\Psi = \left\langle \bar{\Psi}\gamma_m z_m(\rho)\Psi \right\rangle + \ldots$$

$$\left\langle \bar{\Psi}\gamma_m z_m(\hat{\rho})\Psi \right\rangle = z_m(\rho)\left\langle \bar{\Psi}\gamma_m \Psi \right\rangle$$



Giessen DDRH-Theory: *ab initio* Approach to BB Interactions and covariant DFT…

$$K = V + \int V g_{NN} Q_F K$$



- Ladder Kernel
- Map the ab-initio calculations on an effective Lagrangian
- Medium dependent renormalization

$$V_{OBE} = \sum_\alpha g_\alpha^2 D_\alpha(t) \langle \overline{u}_1 \hat{O}_\alpha u_3 \rangle \langle \overline{u}_2 \hat{O}_\alpha u_4 \rangle$$

Field theoretical concept

- ground state solutions are characterized by
  - meson and nucleon fields
  - vertex functionals
- Density dependence:
  - Effective Masses (dynamic rearrangement)
  - Additional contribution due to intrinsic density dependence

$$T_{\mu\nu} = (\partial_\nu \varphi_i) \frac{\partial \mathcal{L}}{\partial(\partial^\mu \varphi_i)} - g_{\mu\nu} \mathcal{L} \quad \rightarrow E = T_{00}[\varphi_i]$$

Choice of Density Dependence – DBHF Vertices

- Vertices: Lorentz scalars retaining internal symmetries
- cancel certain classes of diagrams
- DDRH choice: Vertices to cancel DBHF Self-Energies

$$\Sigma_m^{DB}(k \mid k_F) \doteq G_m^2(k_F)\rho_m + \sum_{m'} f_{mm'} G_{m'}^2(k_F)\, tr_s tr_q \int_{K_F} d^3k'\, D_{m'}(k,k')\, n_{sq}(k' \mid k_F)\, \bar{u}_{sq}\gamma_m \vec{\tau}_m u_{sq}$$

$$K_{12\to 34}(k,k') = \sum_m G_m^2(k_F) D_m(k,k')\rho_m(1,3)\rho_m(2,4)$$

$$\hat{\rho} \mapsto j_\mu j^\mu$$

Density Dependence of the Vertices (Averaged over the Fermi Sphere)

$G^2_m(k_F) / g^2 = z(k_F/m)$:

$\gamma=(2/\pi)g^2(M/m)^2$, $x=k_F/m \to 0$:

$z(x)\sim$

$$1+\left(-\frac{15}{2}\frac{\gamma}{1+\gamma\pi/4}x+\frac{3}{80}\frac{\gamma\,(64\,\pi+16\,\gamma\,\pi^2+375\,\gamma)}{(1+\gamma\,\pi/4)^2}x^2\right)\frac{1}{1+\gamma\,\pi/4}+\mathcal{O}(x^3)$$

Scales for In-Medium Vertices:

- Fermi-Momentum vs. Meson Mass: $x=k_F/m$
- Nucleon vs. Meson Mass: $\gamma\sim(M/m)^2$

Choice of Density Dependence „Hartree" Vertices

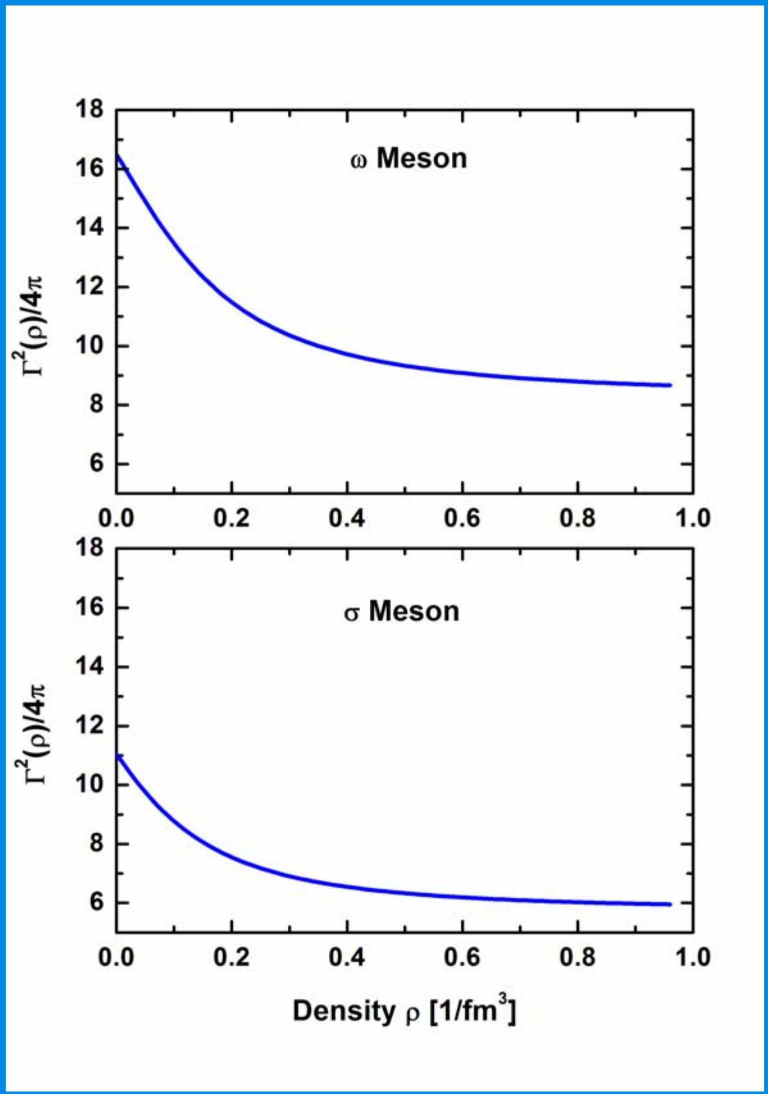
- include Exchange Effects into the Vertices
- Vertices to cancel DBHF Self-Energies averaged over K_F

$$\left\langle \Sigma_m^{DB} \right\rangle_{|K_F} = \overline{\Sigma}_m^{DB}(k_F) \doteq \Sigma_m^{DFT} = \Gamma_m^2(k_F)\rho_m$$

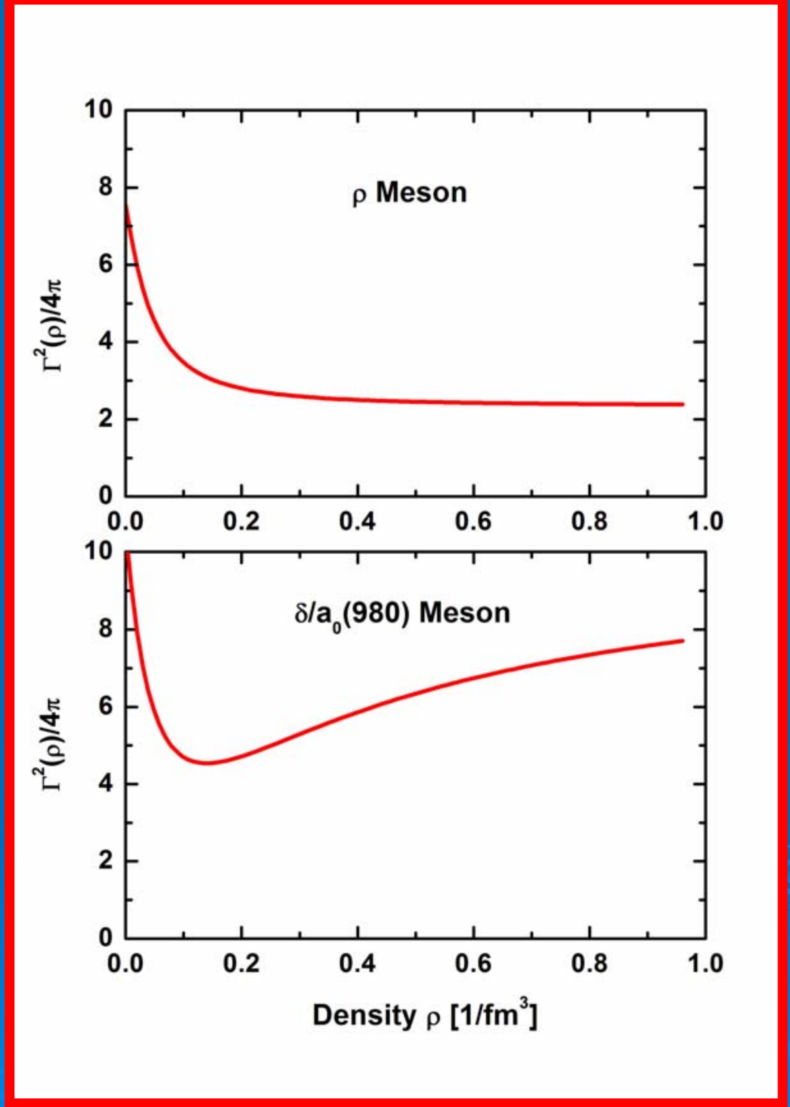
$$\Gamma_m^2(k_F) = \frac{1}{\rho_m}\overline{\Sigma}_m^{DB}(k_F)$$

$$\Gamma_m^2(k_F) = G_m^2(k_F)\left(1 + \sum_{m'} f_{mm'} \frac{g_{m'}^2}{g_m^2}\left\langle D_{m'} \right\rangle_{|K_F}\right)$$

Nuclear Matter DBHF Vertices



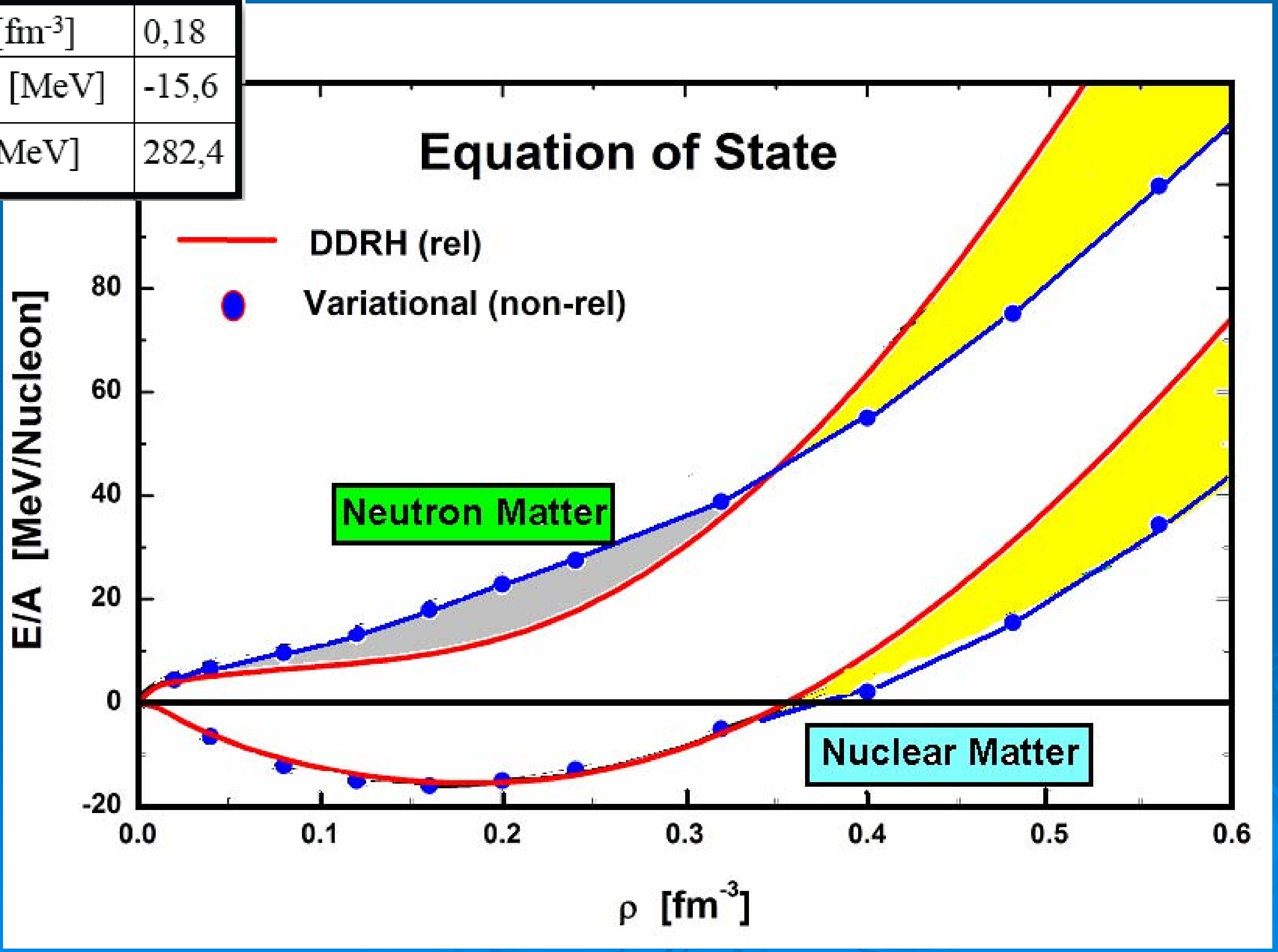
Isoscalar Vertices



Isovector Vertices

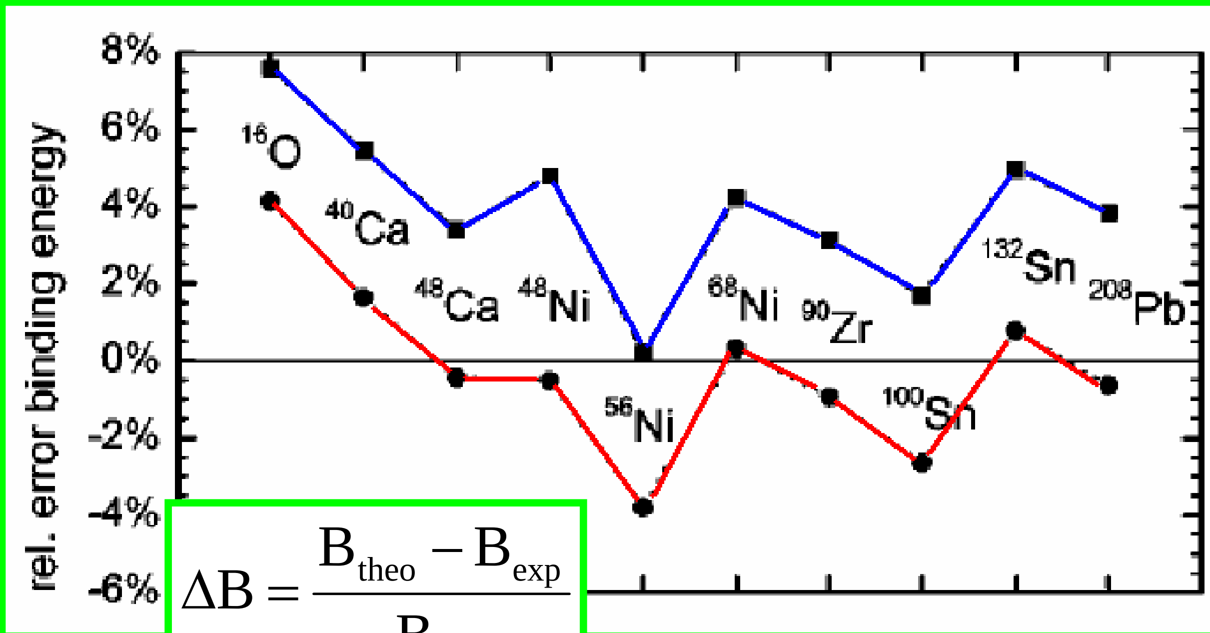
The EoS: DDRH Dirac-Brueckner vs. Urbana V18+UIX

ρ_0 [fm^{-3}]	0,18
ε/ρ [MeV]	-15,6
K[MeV]	282,4



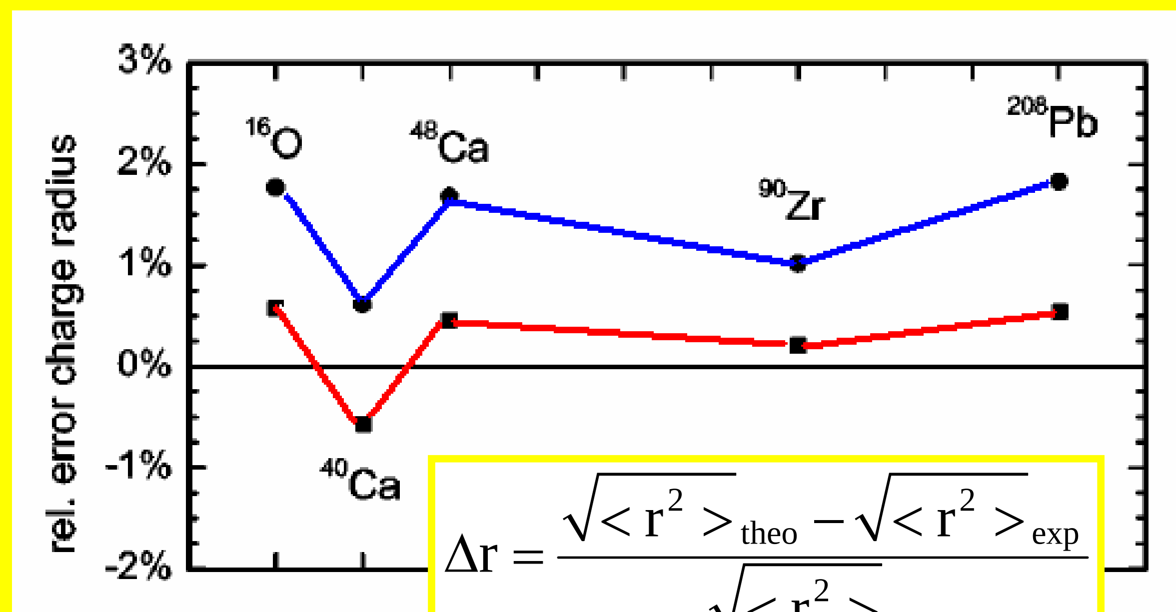
DDRH Results:

B(A) and Charge Radii



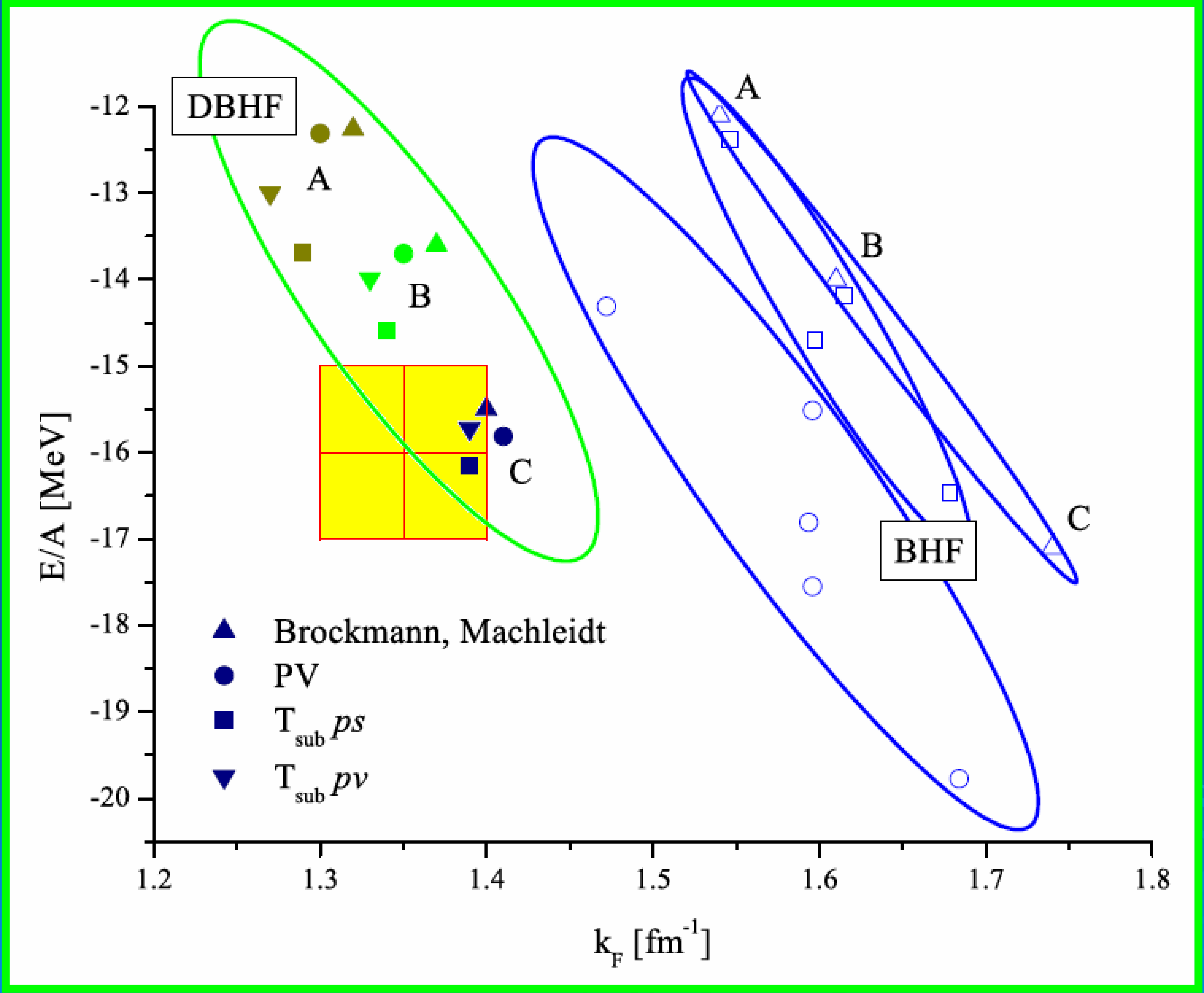
$$\Delta B = \frac{B_{theo} - B_{exp}}{B_{exp}}$$

Pseudo-"Hartree" Vertices

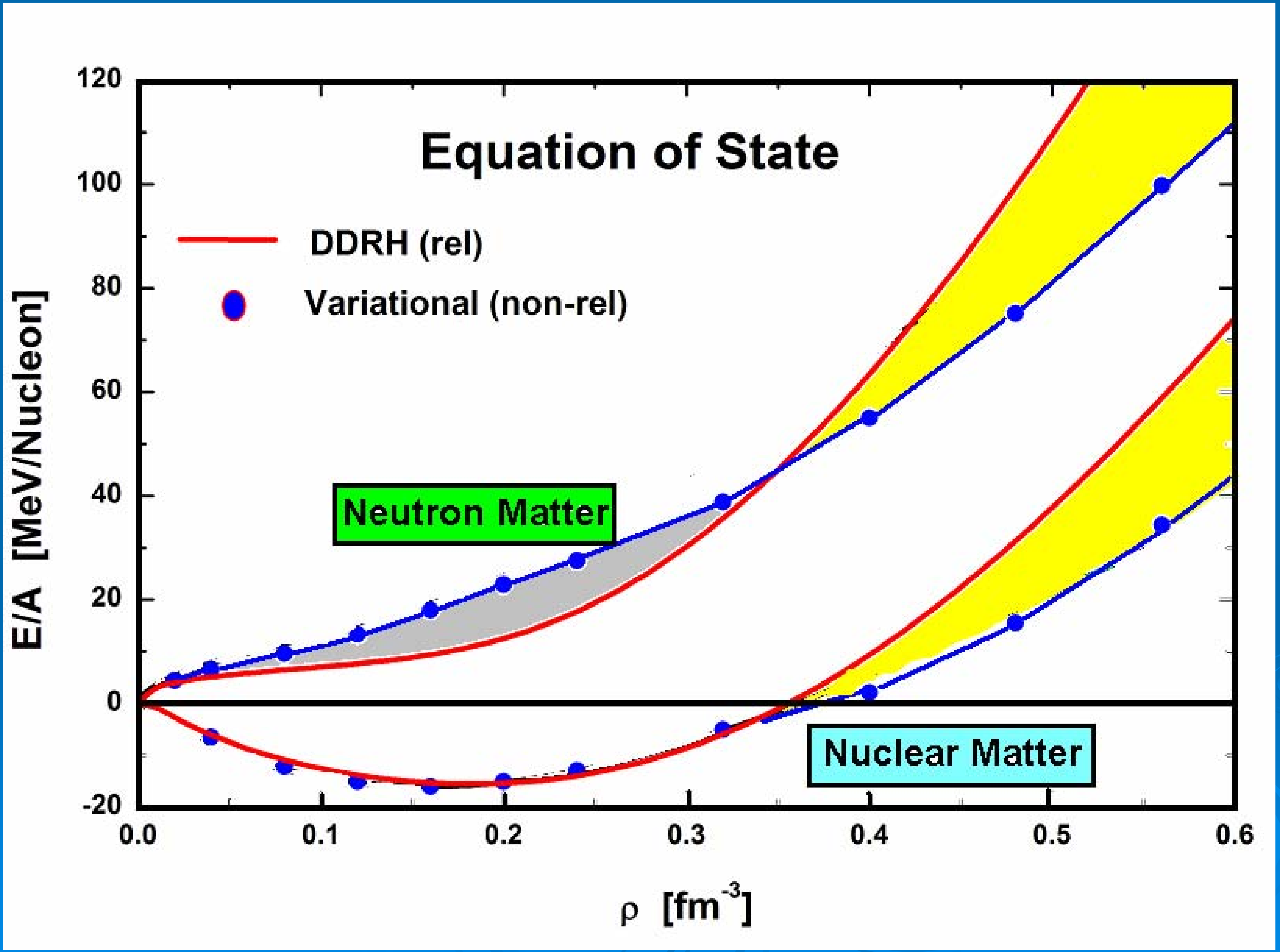


$$\Delta r = \frac{\sqrt{< r^2 >}_{theo} - \sqrt{< r^2 >}_{exp}}{\sqrt{< r^2 >}_{exp}}$$

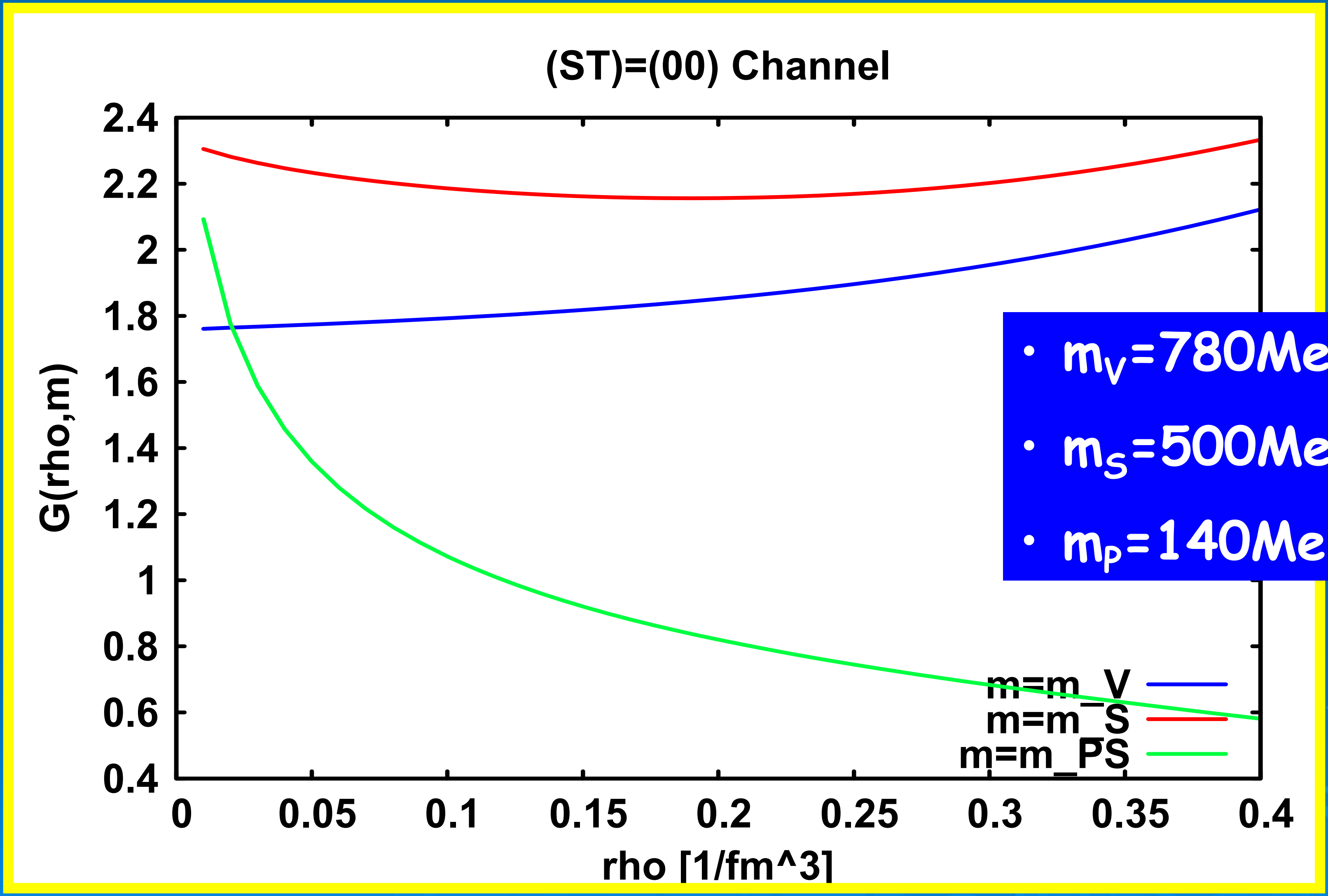
..why using a relativistic Approach?



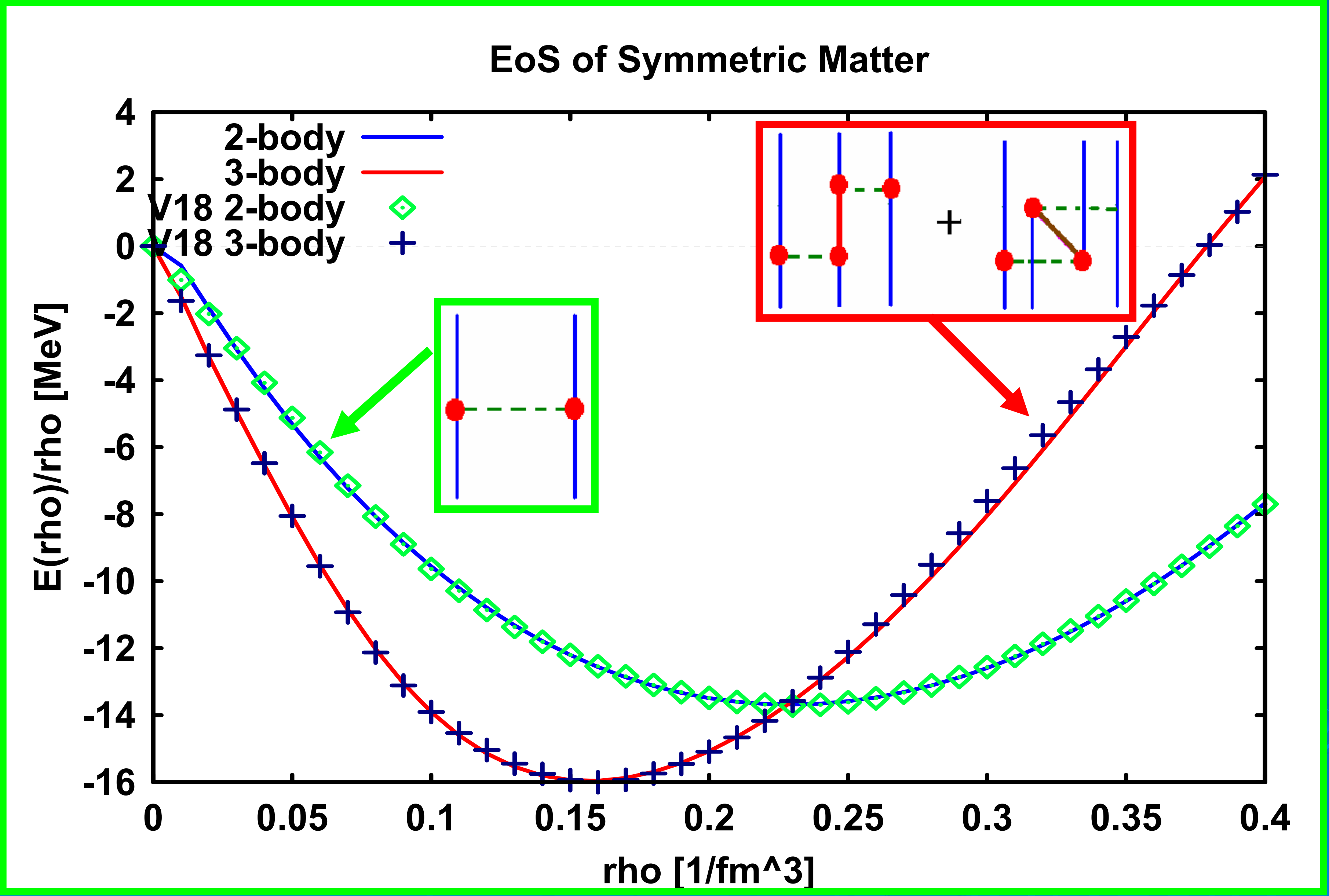
The EoS: DDRH Dirac-Brueckner vs. Urbana V18+UIX



Effective Hartree Vertices from V18+UIX (non-rel.)

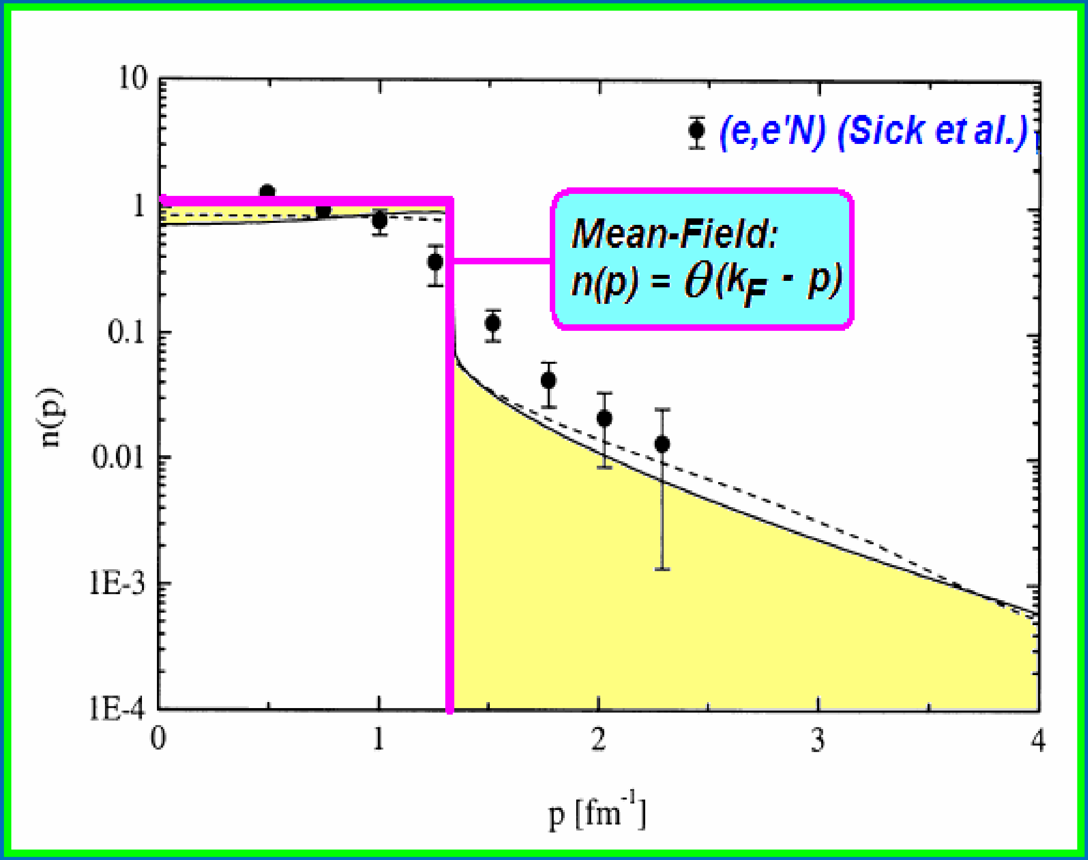


- m_V=780MeV
- m_S=500MeV
- m_P=140MeV


EoS of Symmetric Matter
2-body
3-body
V18 2-body
V18 3-body
E(rho)/rho [MeV]
rho [1/fm^3]
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
+

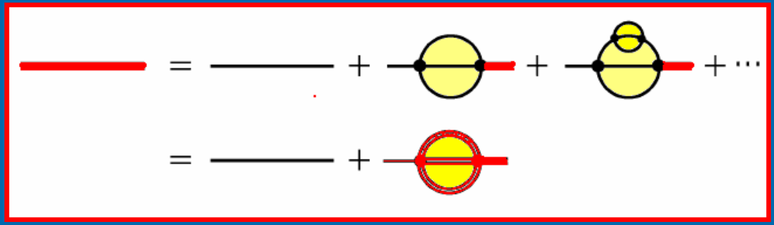
A. Ataie, H.L.

Beyond the Mean-Field:
Short-range Correlations in Nuclear Matter



Momentum Distribution

$$n(p) = N(k_F) \int a(p,\omega)\, d\omega$$



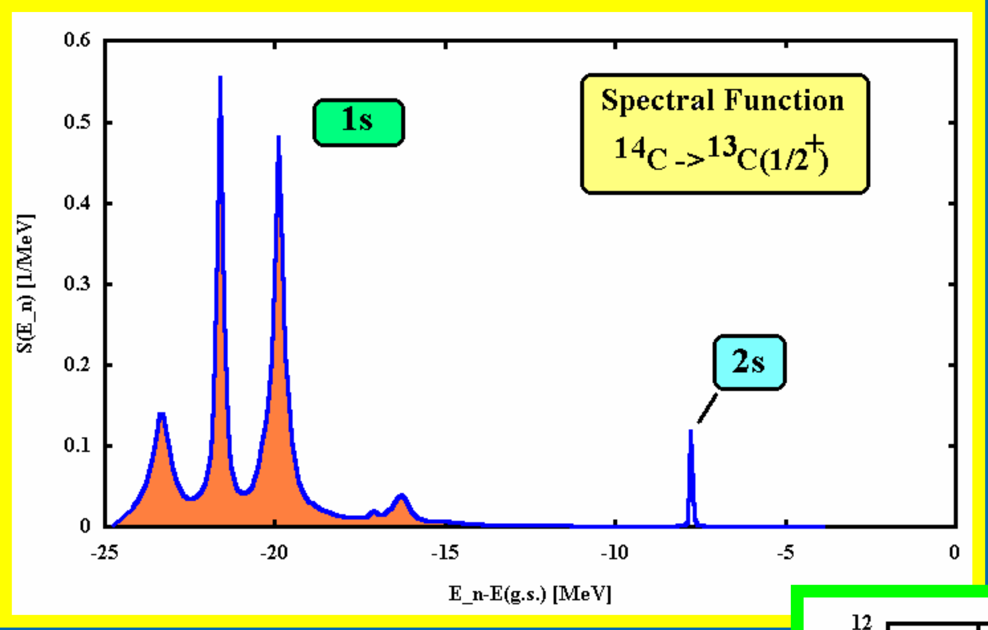
Dynamical Self-Energies:

$$g(\omega,q) = \frac{1}{\varepsilon_{MF}(q) + \Sigma^{(pol)}(\omega,q) - \omega \pm i\eta}$$

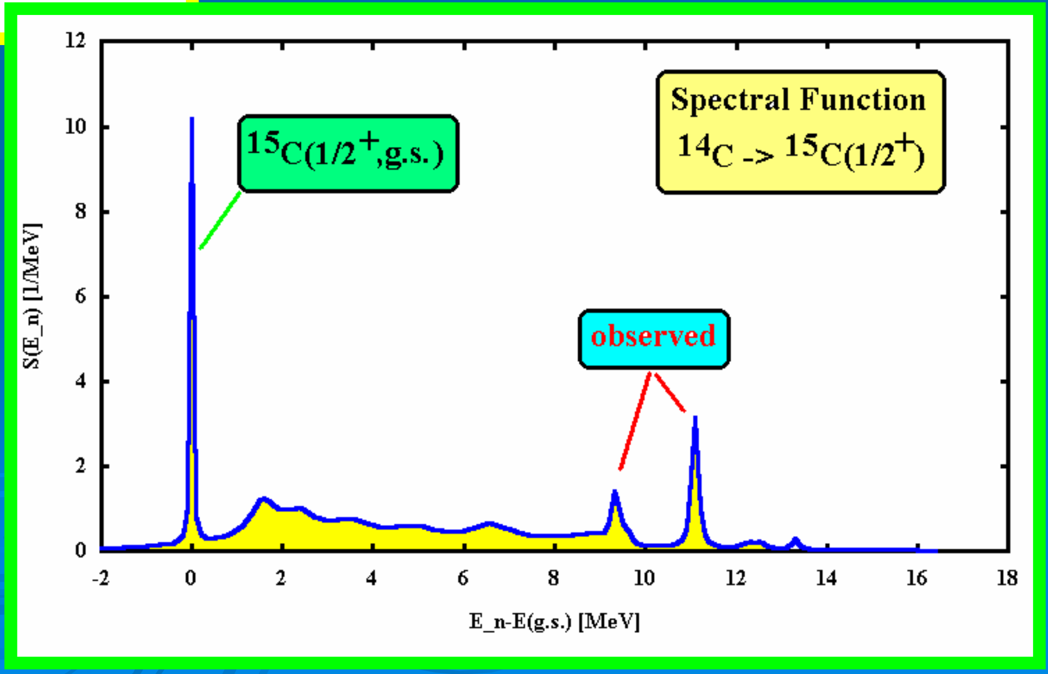
$$\Sigma^{(pol)}(\omega, q \,|\, k_F) \sim V G_{HF} \Pi_{RPA} V^{+}$$

$$\Sigma^{(pol)}(\omega, q \,|\, k_F) = U^{(pol)}(\omega, q \,|\, k_F) - \frac{i}{2}\Gamma^{(pol)}(\omega, q \,|\, k_F)$$

1/2⁺ Particle and Hole Strength Functions in ^{14}C



Hole strength function



Particle strength function

Beyond the Ground State: Adding Dynamics

$$E(\rho) \approx E(\rho_0) + \sum_{q=p,n} \frac{\partial E(\rho)}{\partial \rho_q}\Big|_{\rho_0} \delta\rho_q + \sum_{q,q'=p,n} \frac{\partial^2 E(\rho)}{\partial \rho_q \partial \rho_{q'}}\Big|_{\rho_0} \delta\rho_q \delta\rho_{q'} + \ldots$$

$$E(\rho) \approx E(\rho_0) + \sum_{q=p,n} \left(T_q + U_q(\rho_0)\right)\delta\rho_q + \sum_{q,q'=p,n} f_{qq'}(\rho_0)\delta\rho_q \delta\rho_{q'} + \ldots$$

Fermi-Liquid Theory

Variational Interactions

$$T_{\mu\nu} = (\partial_\nu \varphi_i)\frac{\partial \mathcal{L}}{\partial(\partial^\mu \varphi_i)} - g_{\mu\nu}\mathcal{L} \qquad \rightarrow E = T_{00}[\varphi_i]$$

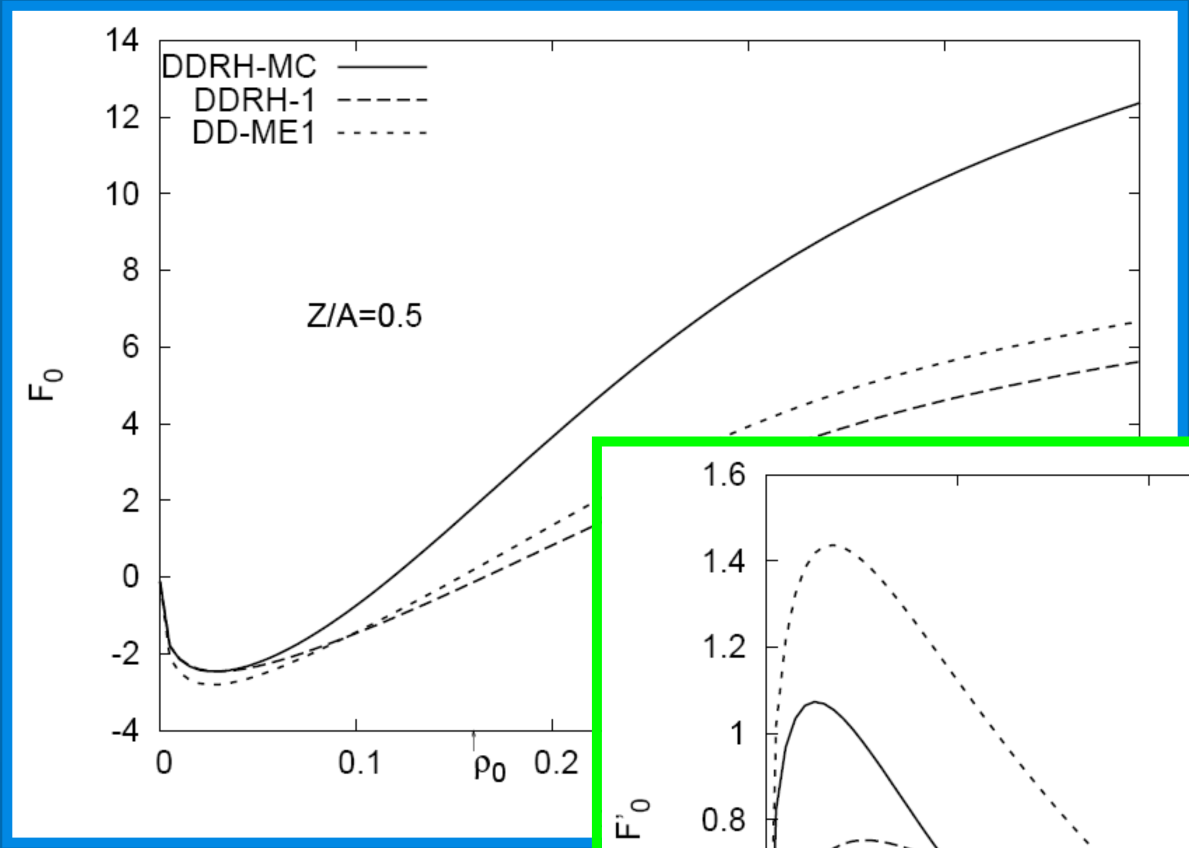
$$\delta\Phi_m = D_m \hat{\Gamma}_m \delta\hat{\rho} + \hat{\rho} D_m \delta\hat{\Gamma}_m$$

$$\delta E = \sum_m \frac{\delta E}{\delta \hat{\rho}_\mu(m)} \delta\hat{\rho}_\mu(m) \equiv \sum_m \hat{\varepsilon}^\mu(m)\delta\hat{\rho}_\mu(m)$$

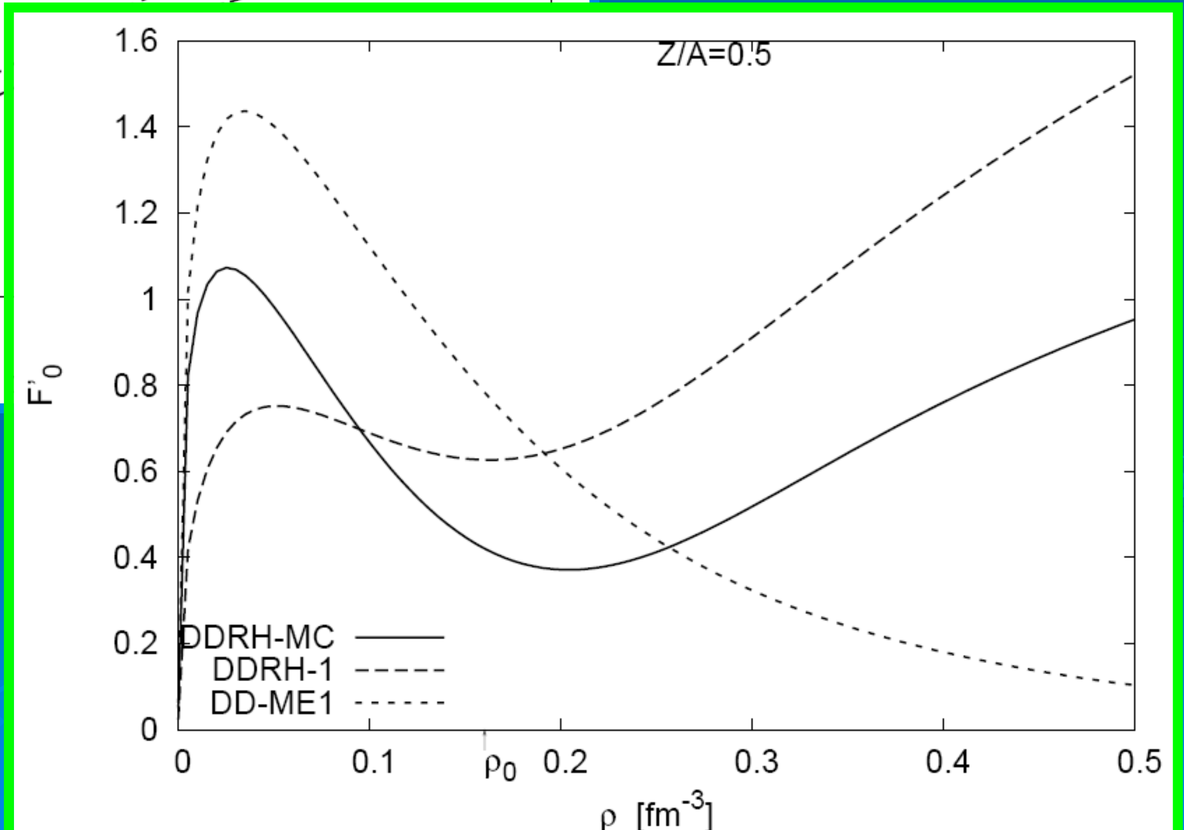
$$\Rightarrow \hat{\varepsilon}^\mu = \hat{\varepsilon}^0_\mu + \hat{\varepsilon}^r_\mu$$

$$\hat{\mathcal{V}}^{\alpha\beta}(m,m') = \frac{\delta^2 E}{\delta\hat{\rho}_\alpha(m)\delta\hat{\rho}_\beta(m')} = \hat{\mathcal{V}}^{\alpha\beta}_0(m,m') + \hat{\mathcal{V}}^{(r)\alpha\beta}(m,m')$$

DDRH Landau-Migdal Parameter in Infinite Nuclear Matter



F_0 in Symmetric Nuclear Matter



F'_0 in Symmetric Nuclear Matter

Landau-Midgal Parameters and Observables

Relations to observables

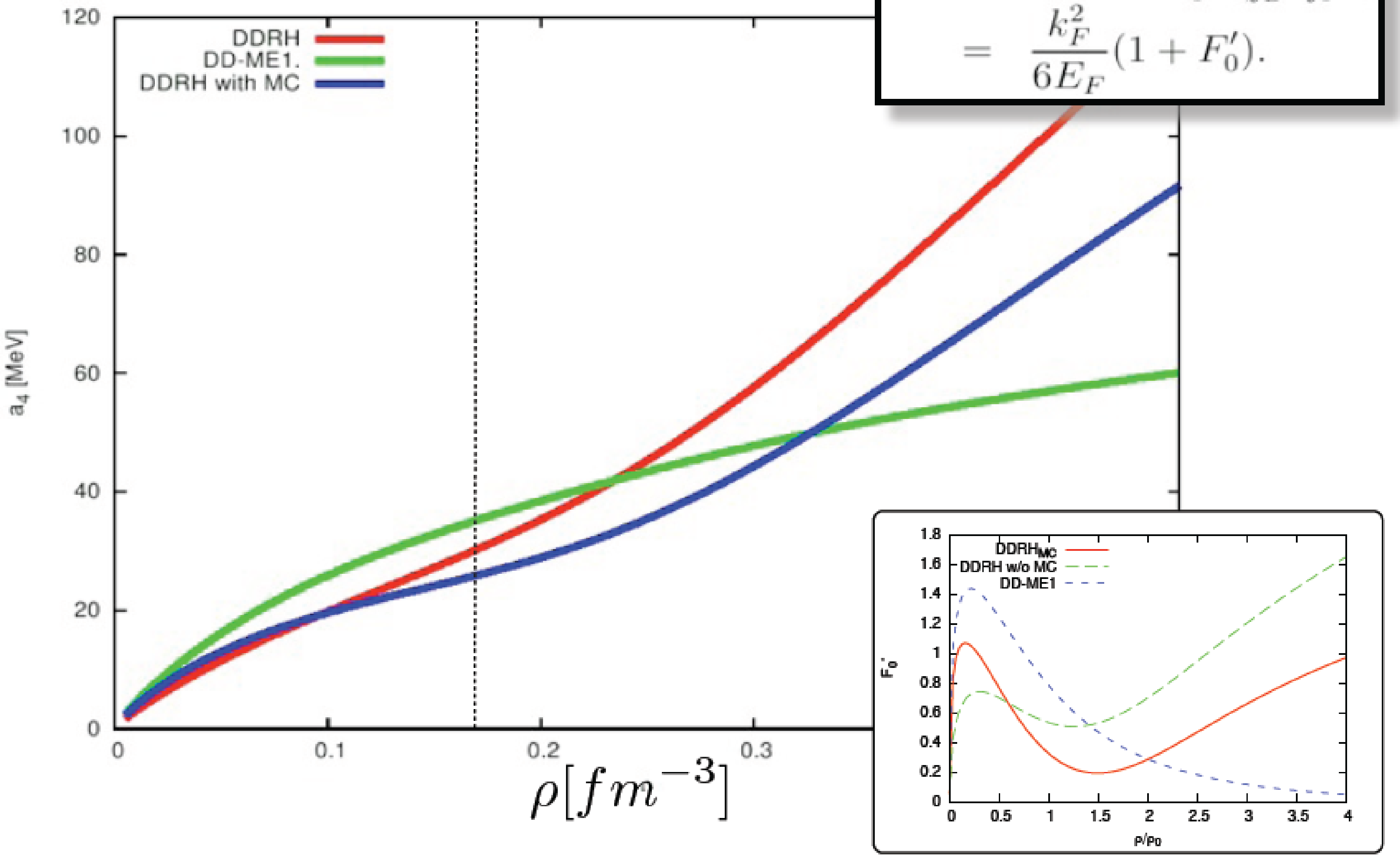
$$E_s(\rho) = \rho \left. \frac{\partial^2 E(\rho,\rho_3)}{\partial \rho_3^2} \right|_{\rho_3=0} = \frac{\rho}{8}\left(\frac{1}{N_p}+\frac{1}{N_n}\right)(1+F_0')_{\xi=0.5}$$

$$K = 9\rho\left(\frac{\partial^2 E}{\partial \rho^2}\right) = 9\rho\left(\frac{\xi^2}{N_p}+\frac{(1-\xi)^2}{N_n}\right)(1+F_0)$$

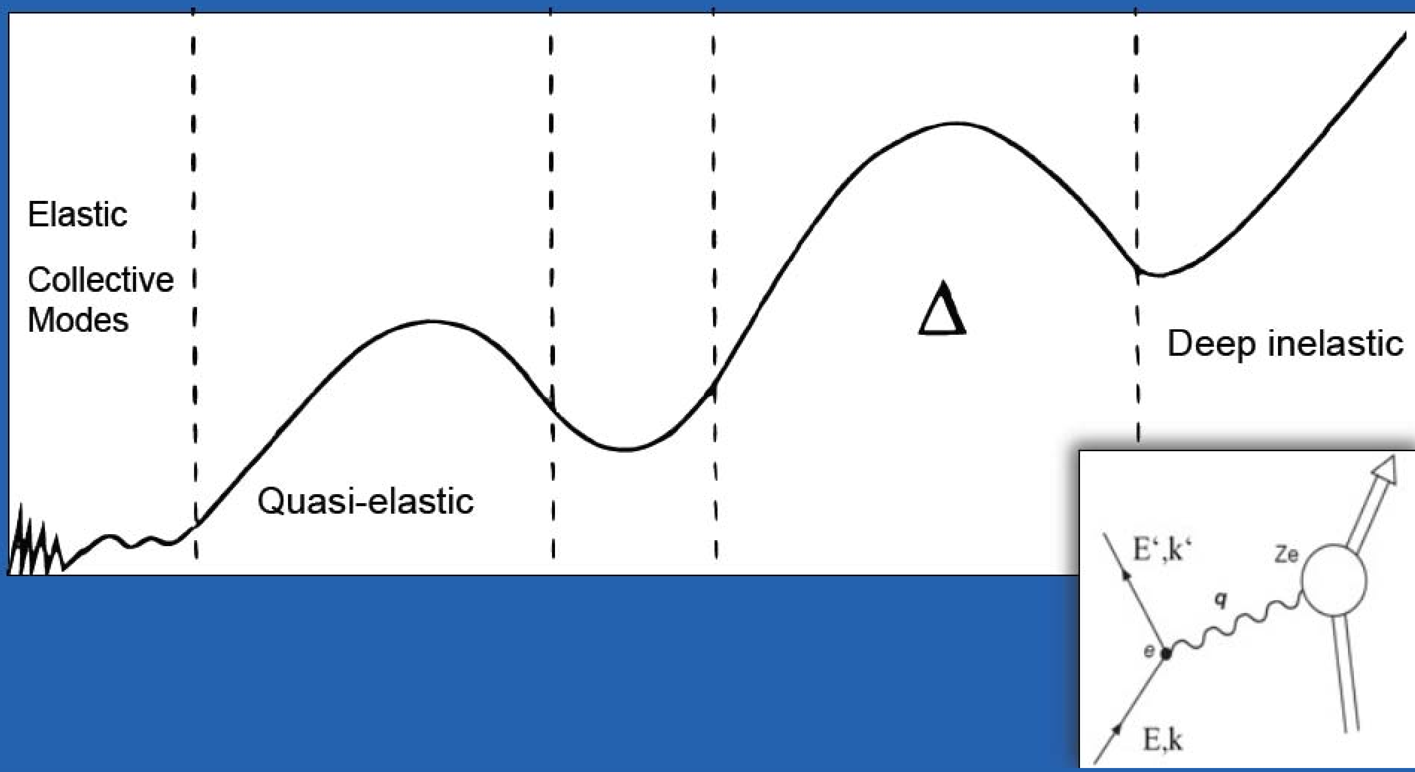
$$\frac{m^*}{m} = 1+\frac{1}{3}F_1$$

Symmetry energy

$$a_4 \equiv \frac{\rho}{2} \frac{\partial^2 E'}{\partial \rho_3^2}\bigg|_{n_3=0, j_B=j_3=0} = \frac{k_F^2}{6E_F}(1+F_0').$$



Quasi-elastic (e,e'p) Scattering on Nuclei



$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_{\mathrm{M}} \left[\left(\frac{Q^2}{q^2} \right) R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2} \right) R_T(q,\omega) \right]$$

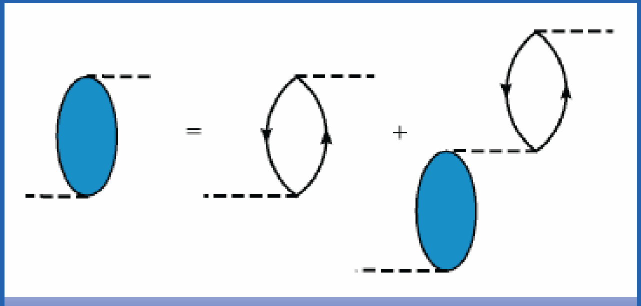
$$R_{L/T} \sim \Im \Pi^{\mathrm{RPA}}_{L/T}$$

Relativistic RPA

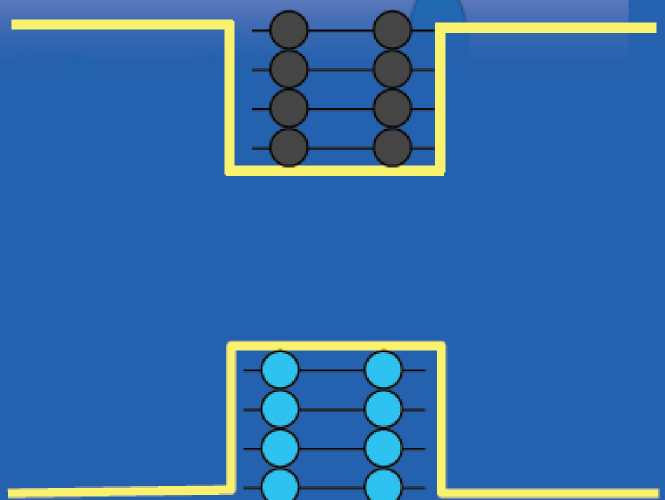
$$\Pi^{RPA} = \Pi^0 + V\Pi^0\Pi^{RPA}$$

$$\Pi_0(\Gamma_A, \Gamma_B) = \frac{1}{i}\int \frac{d^4p}{(2\pi)^4} Tr[\Gamma_A G_0(p+q)\Gamma_B G_0(p)]$$

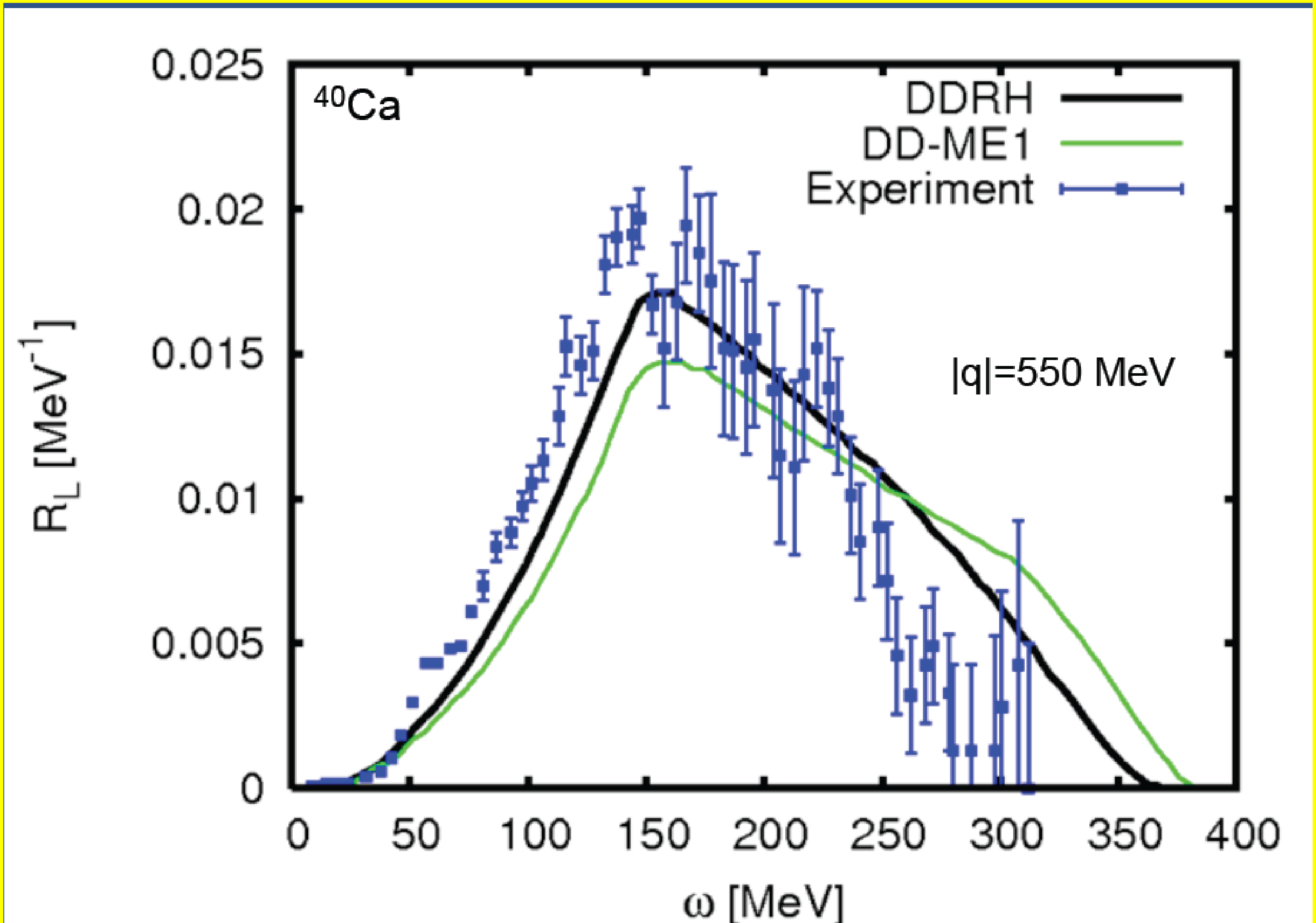
$$\begin{aligned} G_0(p) &= (\not{p} + M)\left[\frac{1}{p^2 - M^2 + i\epsilon} + \frac{i\pi}{E_p}\delta(p_0 - E_p)\Theta(k_F - p)\right] \\ &= G_F + G_D \end{aligned}$$



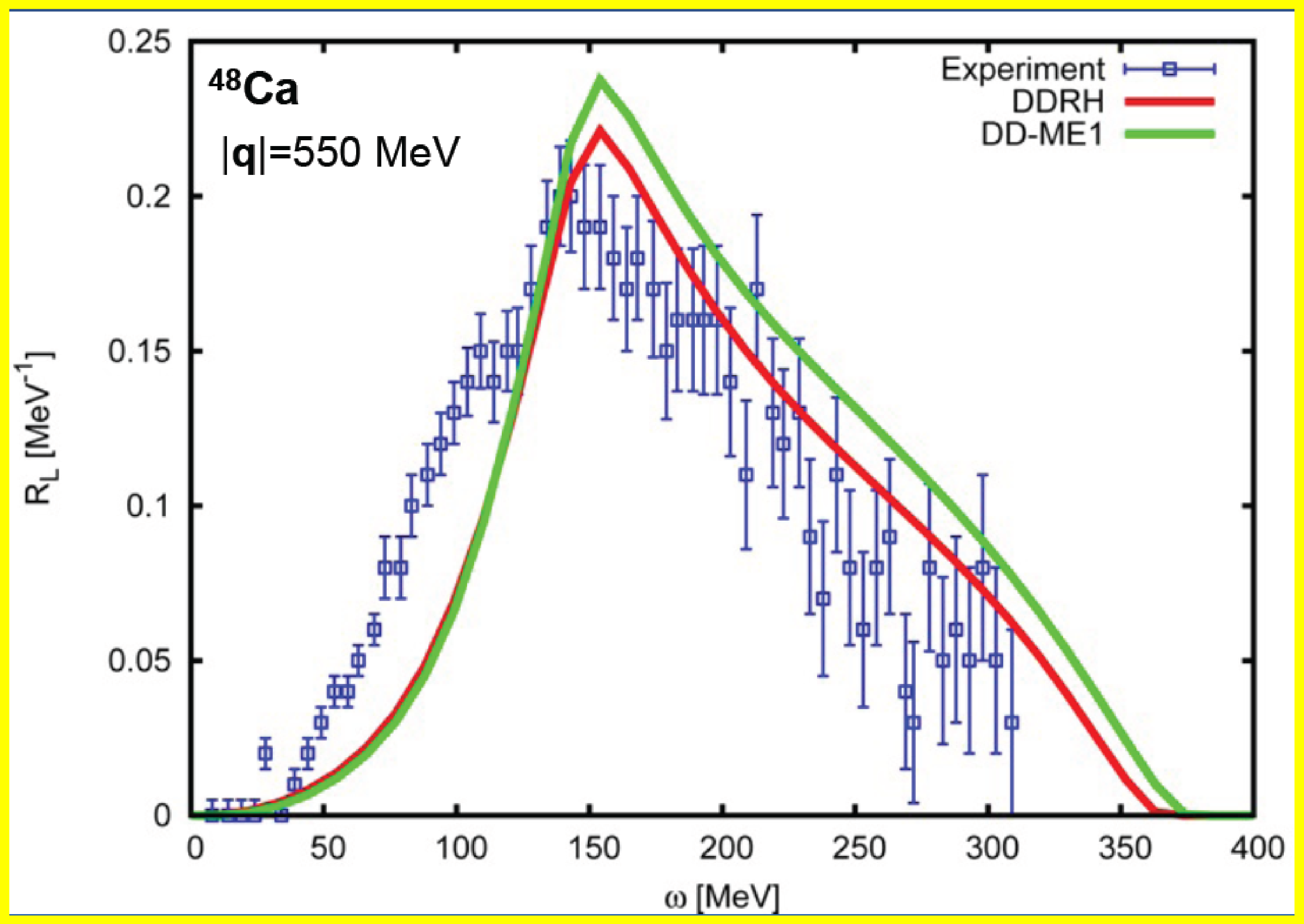
- $\Pi = \Pi_D + \Pi_F$
- Π_D : ph + part of NN excitations
- no-sea approximation
- Calculation of Π with dressed NN-meson vertices



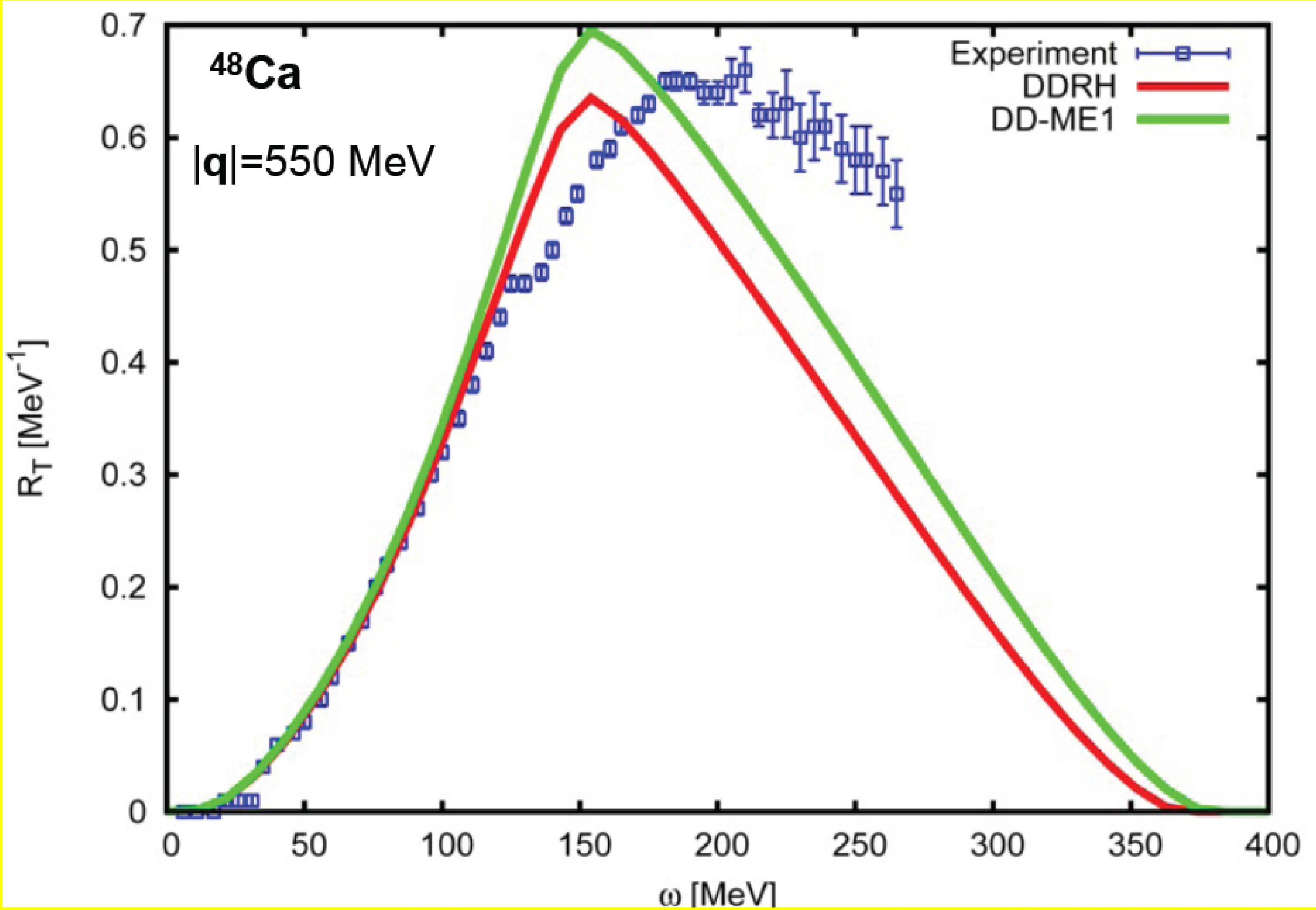
Longitudinal Response Function: ^{40}Ca



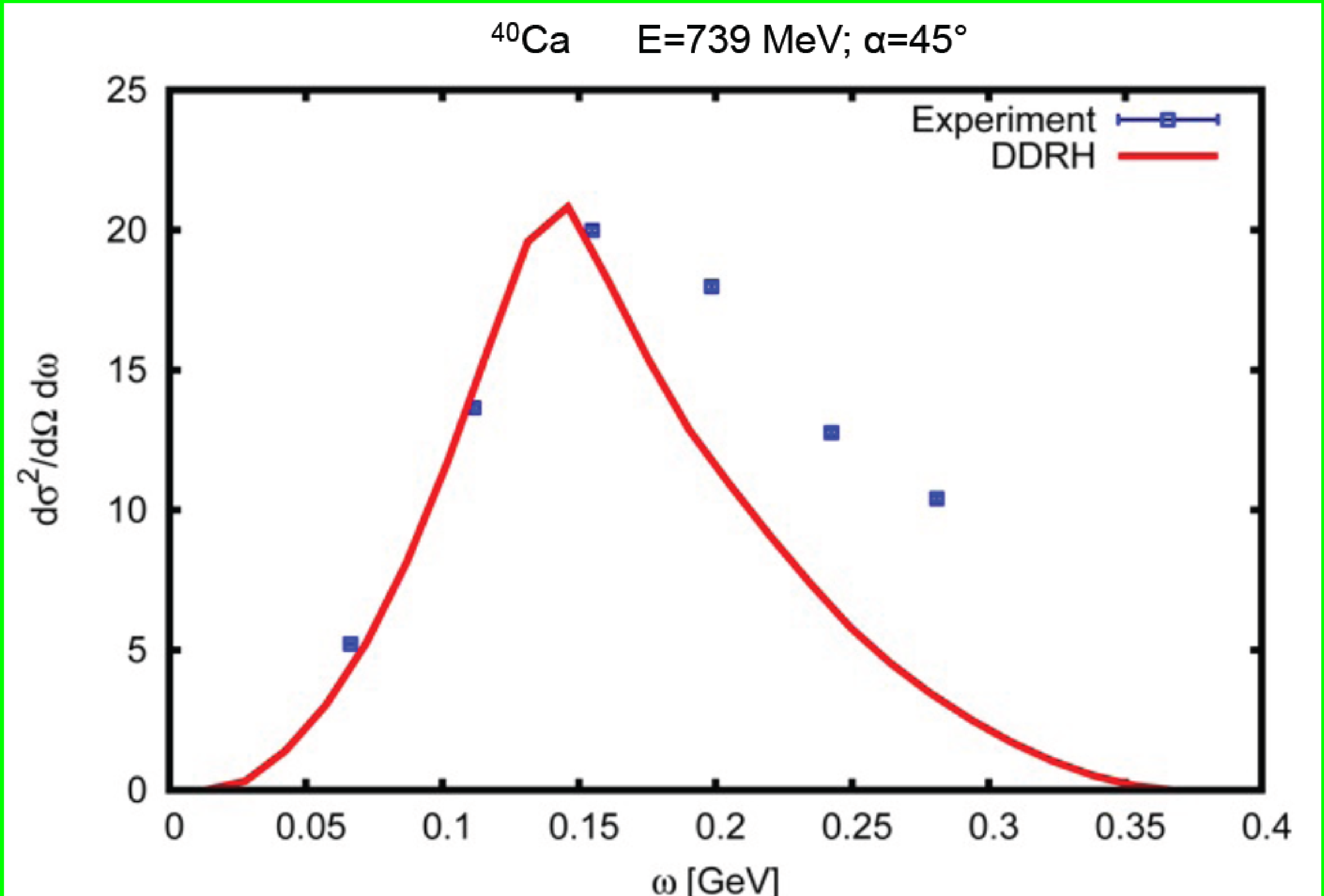
Longitudinal Response Function: ^{48}Ca



Transversal Response Function: ^{48}Ca



The ee'p Cross Section: ^{40}Ca



Summary, Conclusions and Outlook

- Low-energy QCD and NN-interactions
- EFT in free Space and at the Fermi-Momentum Scale
- Relativistic DFT
- Applications: Nuclear Matter, Nuclei, and Neutron Stars
- Dynamics: Fermi-Liquid Theory

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