Extended EDF with Density-Dependent Coupling Constants

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Outline

- Skyrme EDF
- Extensions of Skyrme EDF
- Density dependent coupling constants
- Single particle energies
- Unstable region
- Conclusions

Skyrme EDF

$$E_t^{even} = C_t^{\rho}\rho_t^2 + C_t^{\tau}\rho_t\tau_t + C_t^{\Delta\rho}\rho_t\Delta\rho_t^2 + C_t^{\nabla J}\rho_t\nabla J_t + C_t^{J}J_t^2 \text{ , } t=0,1$$

$$C_t^{\rho} = C_{t0}^{\rho} + C_{tD}^{\rho}\rho_0^{\sigma}$$

- Uses local density approximation
- Density dependence included only in ρ_t^2 term
- Derivatives of densities up to second order

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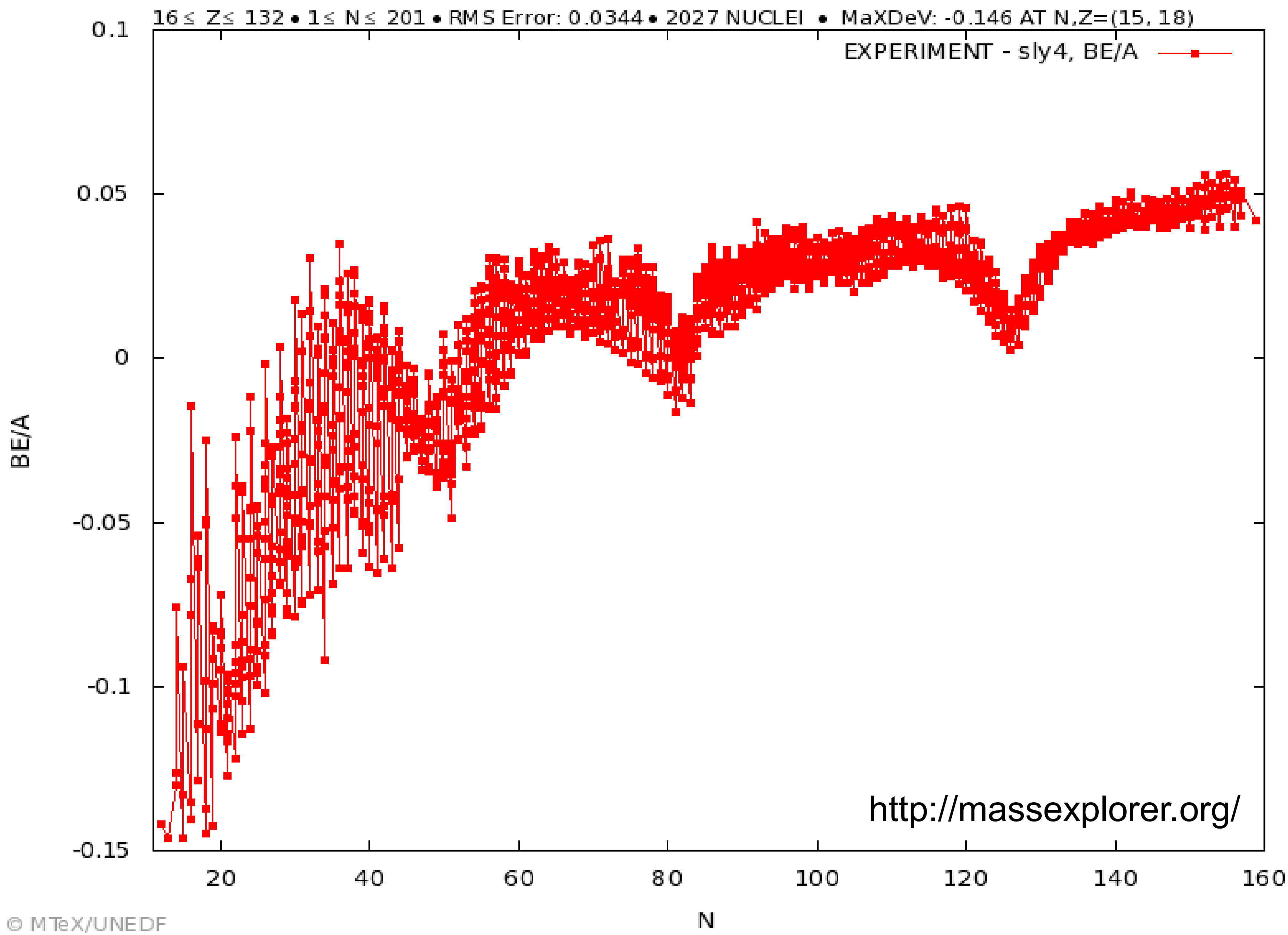
Skyrme EDF



16 ≤ Z ≤ 132 • 1 ≤ N ≤ 201 • RMS Error: 0.0344 • 2027 NUCLEI • MaXDeV: -0.146 AT N,Z=(15, 18)
EXPERIMENT - sly4, BE/A
BE/A
N
0.1
0.05
0
-0.05
-0.1
-0.15
20
40
60
80
100
120
140
160
http://massexplorer.org/
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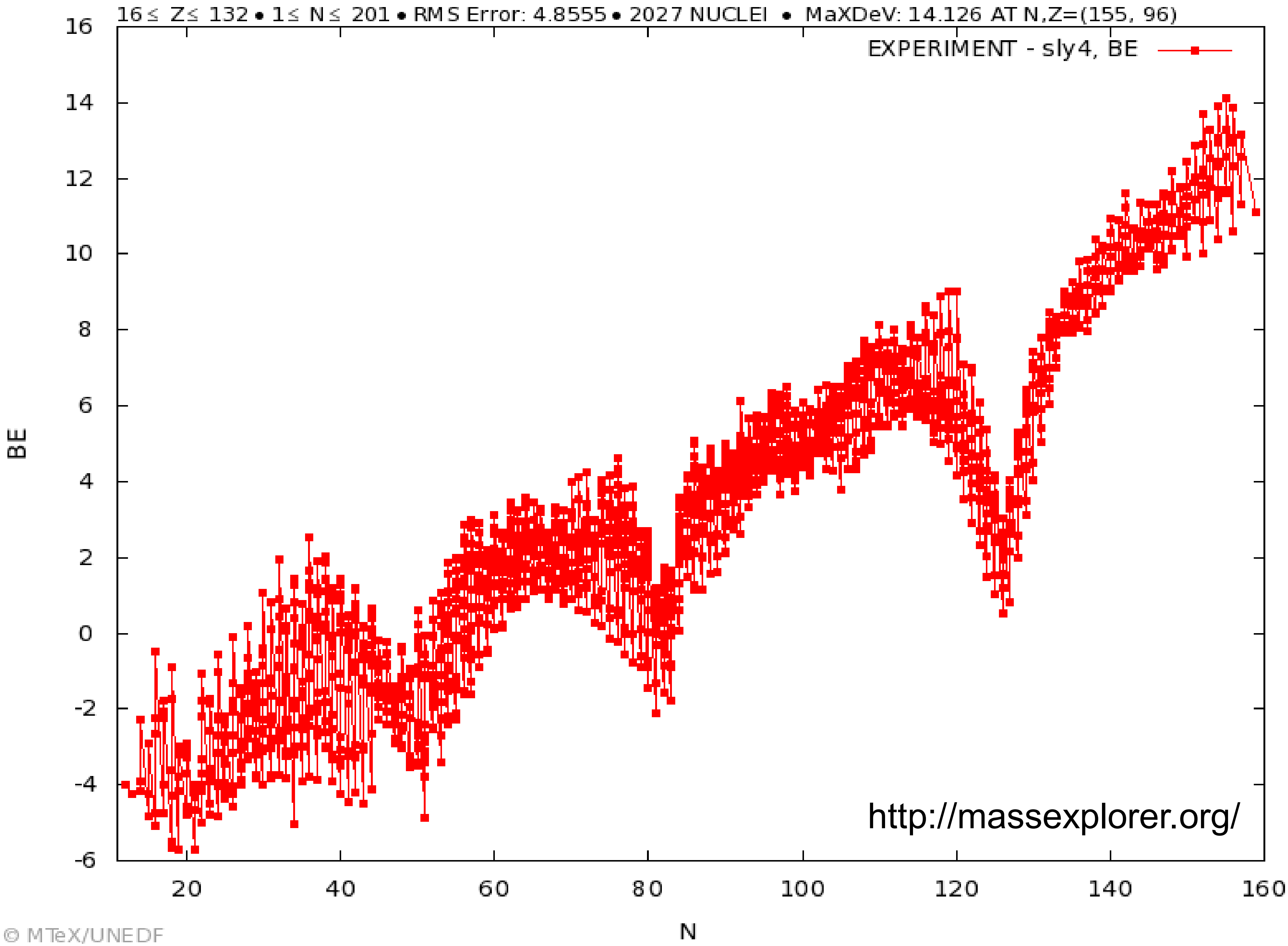
Skyrme EDF



16 ≤ Z ≤ 132 • 1 ≤ N ≤ 201 • RMS Error: 4.8555 • 2027 NUCLEI • MaXDeV: 14.126 AT N,Z=(155, 96)
EXPERIMENT - sly4, BE
BE
N
16
14
12
10
8
6
4
2
0
-2
-4
-6
20
40
60
80
100
120
140
160
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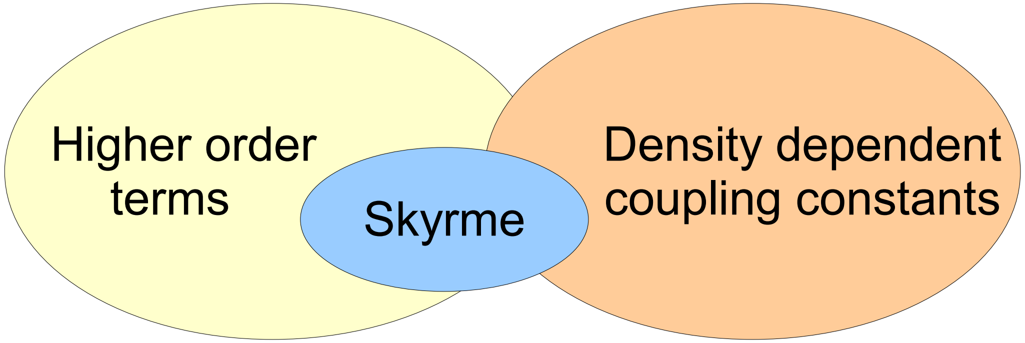
Possible extensions of EDF

- Higher order terms
- Density dependent coupling constants
- A mixture of these two
- All extended functionals should include also Skyrme

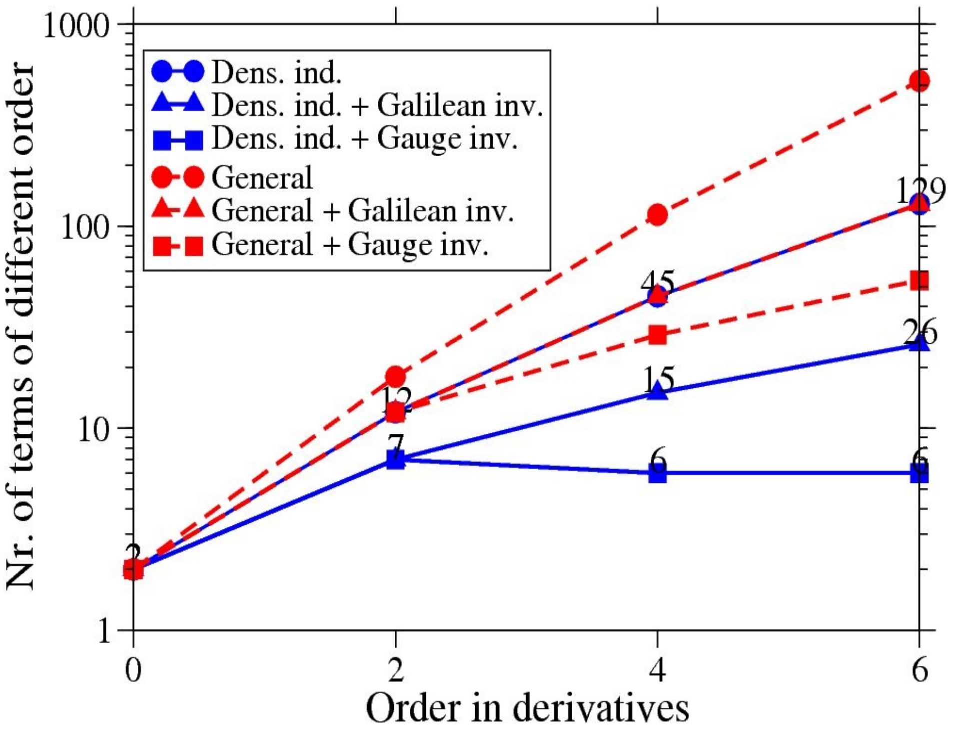


Higher order terms

- Expand EDF to higher order derivatives

$$T^{n'L'v'J'}_{mI,nLvJ}(\boldsymbol{r}) = [\rho_{n'L'v'J'}(\boldsymbol{r})[D_{mI}\rho_{nLvJ}(\boldsymbol{r})]_{J'}]_0$$

$$\boldsymbol{\rho_{mInLvJQ}} = \left((\vec{\nabla}^m)_I \left((\vec{k}^n)_L \rho_v\right)_J\right)_Q \text{ , m+n} \le 6$$



- Some of the 4th order terms:

No.		$C^{n'L'v'J'}_{mI,nLvJ}$	$\rho_{n'L'v'J'}$ Time-even	D_{mI}	ρ_{nLvJ} Time-even
1	•	$C^{0000}_{40,0000}$	$[\rho]_0$	$[\nabla\nabla]^2_0$	$[\rho]_0$
2	•	$C^{0000}_{20,2000}$	$[\rho]_0$	$[\nabla\nabla]_0$	$[[kk]_0\rho]_0$
3	•	$C^{0000}_{22,2202}$	$[\rho]_0$	$[\nabla\nabla]_2$	$[[kk]_2\rho]_2$
4	•	$C^{0000}_{00,4000}$	$[\rho]_0$	1	$[[kk]^2_0\rho]_0$
5	•	$C^{2000}_{00,2000}$	$[[kk]_0\rho]_0$	1	$[[kk]_0\rho]_0$
6	•	$C^{2202}_{00,2202}$	$[[kk]_2\rho]_2$	1	$[[kk]_2\rho]_2$

B. G. Carlsson et. al. Phys. Rev. C78, 044326 (2008)

Density-dependent coupling constants

I. Density dependence of all the coupling constants

For the time-reversal and spherical symmetries imposed, the extended EDF reads

$$\mathcal{H}_t(r) = C_t^{\rho}\rho_t^2 + C_t^{\tau}\rho_t\tau_t + C_t^{\Delta\rho}\rho_t\Delta\rho_t + \tfrac{1}{2}C_t^{J}J_t^2 + C_t^{\nabla J}\rho_t\nabla\cdot J_t$$
$$+ C_t^{\nabla\rho}(\nabla\rho_t)^2 + C_t^{\nabla\rho'}(\nabla\rho_t)\cdot J_t$$

and depends linearly on 38 coupling constants,

$$C_t^{\rho},\quad C_t^{\tau},\quad C_t^{\Delta\rho},\quad C_t^{\mathrm{J}},\quad \text{and}\quad C_t^{\nabla\mathrm{J}},$$

$$\alpha_t^{\rho},\quad \alpha_t^{\tau},\quad \alpha_t^{\Delta\rho},\quad \alpha_t^{\mathrm{J}},\quad \alpha_t^{\nabla\mathrm{J}},\quad \alpha_t^{\nabla\rho},\quad \text{and}\quad \alpha_t^{\nabla\rho'},$$

$$\beta_t^{\rho},\quad \beta_t^{\tau},\quad \beta_t^{\Delta\rho},\quad \beta_t^{\mathrm{J}},\quad \beta_t^{\nabla\mathrm{J}},\quad \beta_t^{\nabla\rho},\quad \text{and}\quad \beta_t^{\nabla\rho'},$$

for $t = 0$ and 1, i.e.,

$$C_t^m(\rho_0,\rho_1) = C_t^m\left[1 + \alpha_t^m\left(1 - \left(\frac{\rho_0}{\rho_{\mathrm{sat}}}\right)^{\gamma_t^m}\right) + \beta_t^m\left(\left(\frac{\rho_1}{\rho_{\mathrm{sat}}}\right)^2\right)^{\eta_t^m}\right]$$

and on 28 powers γ_t^m and η_t^m.

M. Kortelainen *et al.*, to be published

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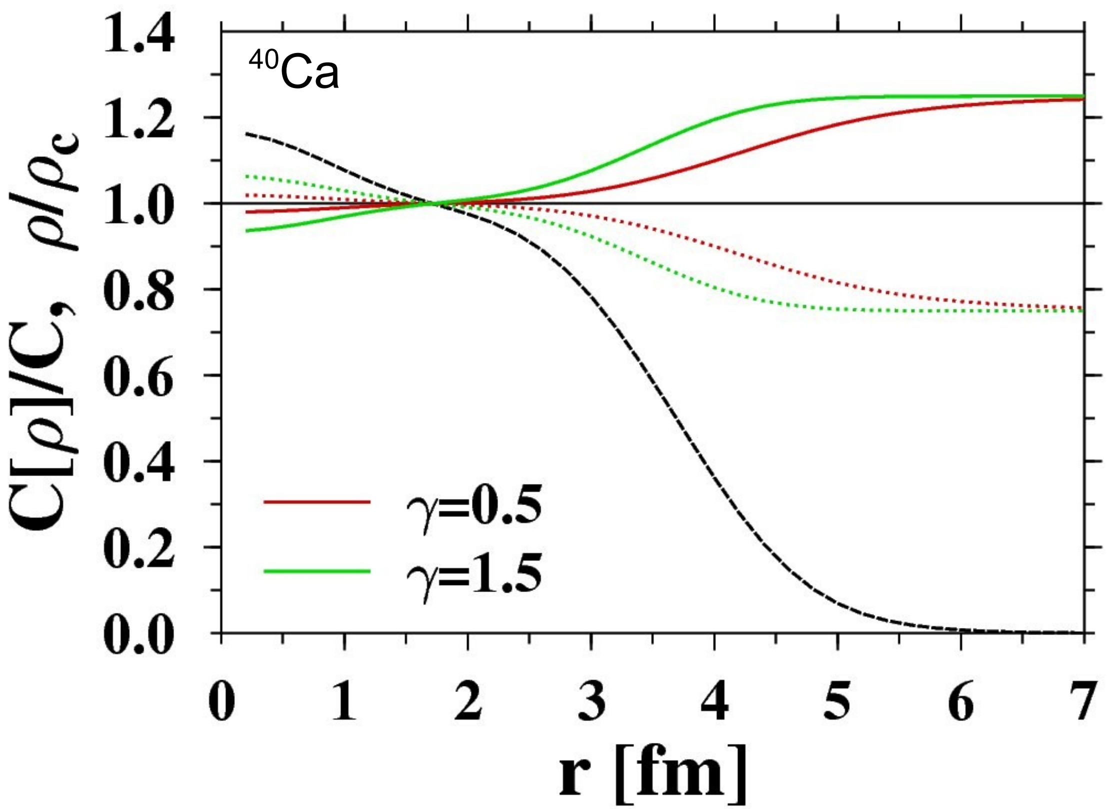
Density-dependent coupling constants

- Inside the nucleus coupling constant remains close to its original value
- On surface the value of the coupling constant changes



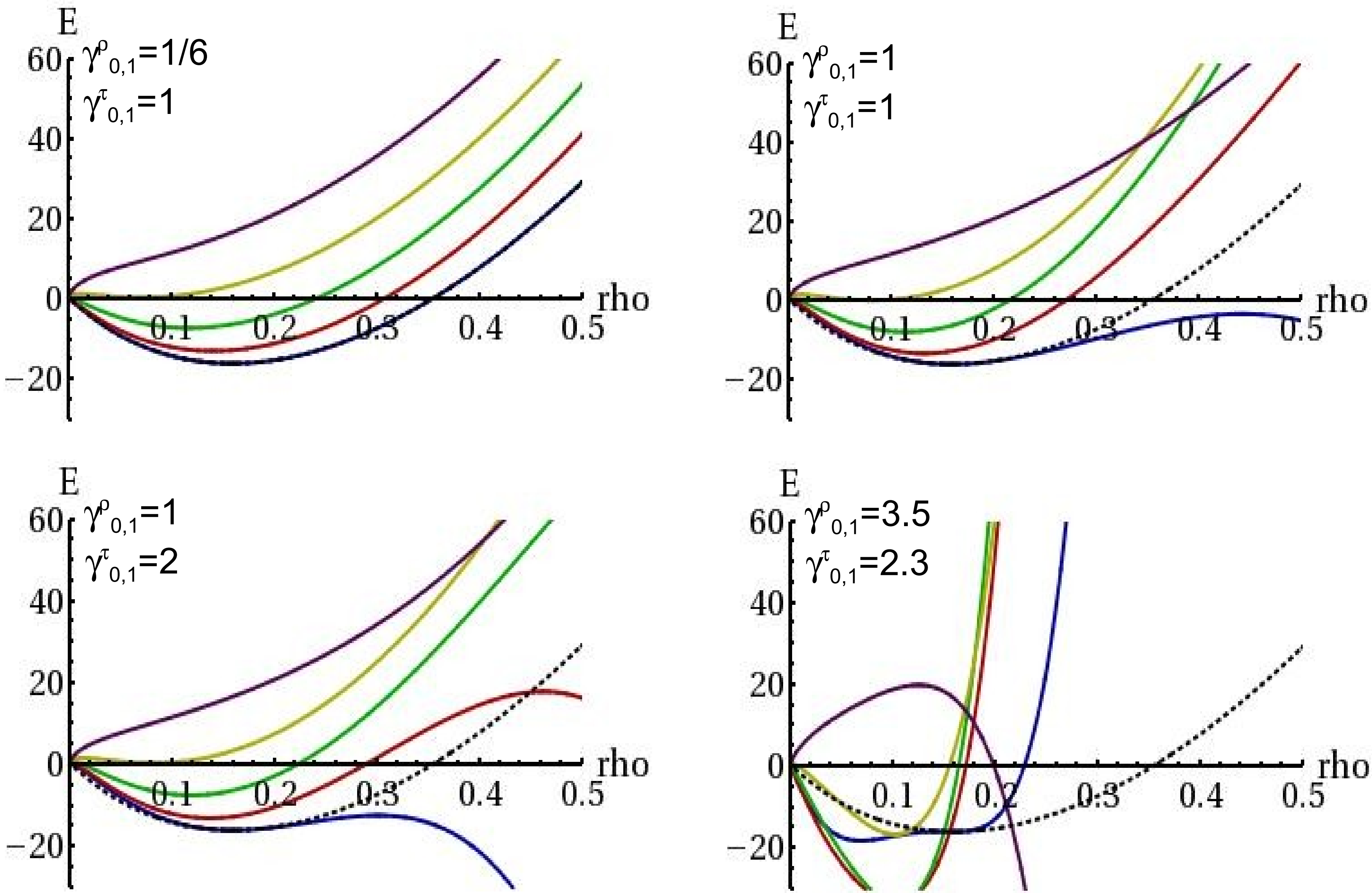
Nuclear matter properties

- 8 nuclear matter parameters
- Symmetric nuclear matter:
  E/A, ρ_{sat}, K_∞, m^*_s
- Asymmetric nuclear matter:
  a_{sym}, L_{NM}, ΔK_{NM}, m^*_v

→

- 8 EDF parameters fixed:
  C^ρ_0, C^ρ_1, C^τ_0, C^τ_1,
  $\alpha^\rho_{0,}$ α^ρ_1, α^τ_0, α^τ_1
- Powers $\gamma^\rho_{0,}$ γ^ρ_1, γ^τ_0, γ^τ_1 free parameters
- β^ρ_0,β^ρ_1,β^τ_0,β^τ_1, and η^ρ_0,η^ρ_1,η^τ_0,η^τ_1 free parameters in volume part

Saturation curves



Powers γ^ρ and γ^τ in finite nuclei

- Nuclear matter properties fix the saturation curve at saturation density
- Below saturation density the effect of γ^ρ and γ^τ on the shape of the saturation curve is small
- Effect of γ^ρ and γ^τ on the finite nuclei is also small on masses

RMS of the masses [MeV] with c.m. Included. SLy4: 1.99MeV

PRELIMINARY RESULTS

$\gamma^\rho_{0,1}$ \ $\gamma^\tau_{0,1}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	1.56	1.57	1.55	1.57	1.54	1.56	1.55	1.53	1.50	1.54
0.2	1.62	1.61	1.61	1.59	1.63	1.61	1.64	1.62	1.61	1.63
0.3	1.62	1.63	1.66	1.67	1.69	1.68	1.71	1.70	1.72	1.73
0.4	1.62	1.66	1.68	1.71	1.72	1.75	1.77	1.80	1.82	1.83
0.5	1.64	1.67	1.69	1.73	1.76	1.78	1.82	1.86	1.89	1.92
0.6	1.56	1.67	1.69	1.74	1.78	1.81	1.84	1.91	1.95	1.99

- Removing the c.m. correction makes dependency on powers even smaller in masses

Density dependence on $C^{\nabla\rho}_0$

- Density dependent $C^{\nabla\rho}_0$ seems to improve fitted masses

$\gamma^{\tau}_{0,1}, \gamma^{\nabla\rho}_0$

	0.25	0.50	0.75	1.00	1.25	1.50
RMS:	1.21	1.14	1.11	1.10	1.09	1.08

SLy4: 1.99 [MeV]

PRELIMINARY RESULTS

- On charge radii improvement seems to be even better

$\gamma^{\tau}_{0,1}, \gamma^{\nabla\rho}_0$

	0.25	0.50	0.75	1.00	1.25	1.50
RMS:	0.019	0.014	0.013	0.012	0.012	0.011

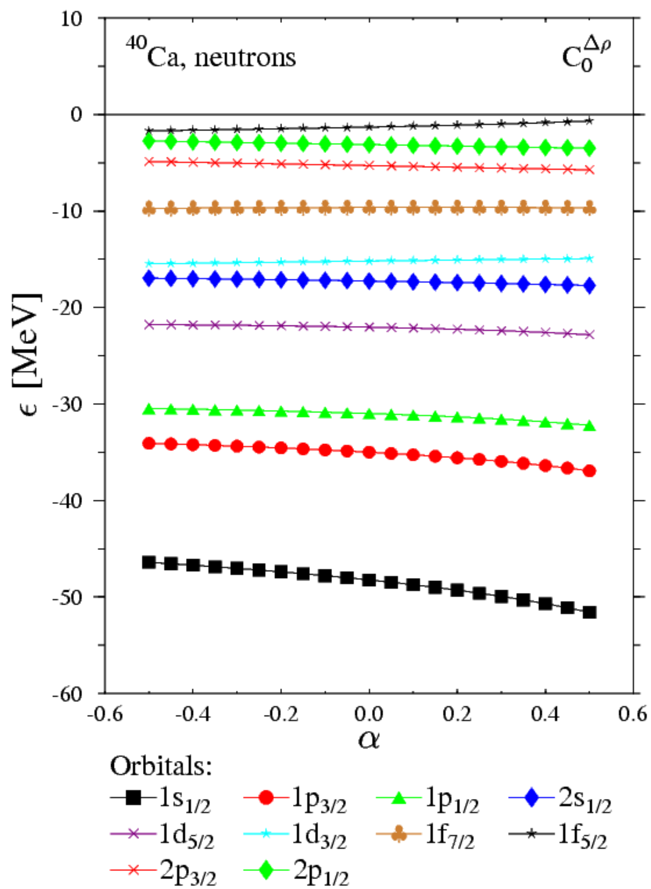
SLy4: 0.030 [fm]

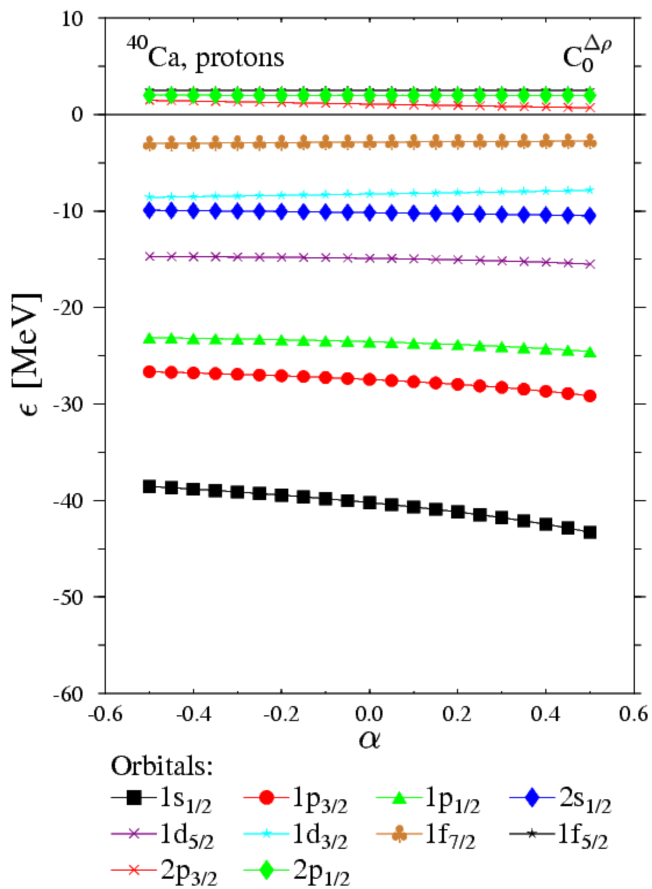
PRELIMINARY RESULTS

$\gamma^{\rho}_{0,1}=1/6$

Single particle energies

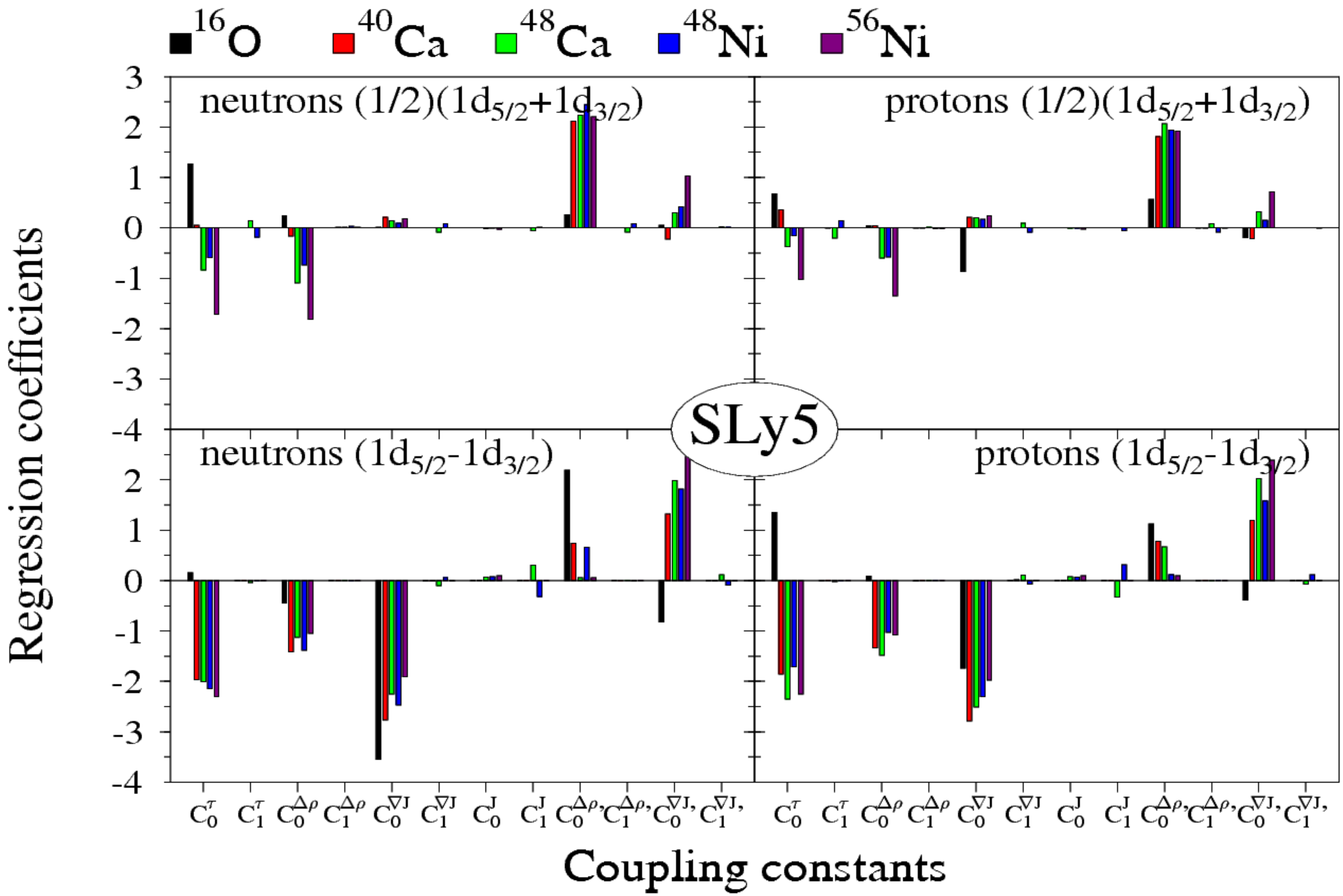
- Instead of considering nuclear bulk properties one could also fit only to the single-particle energies
- s.p. energies in ^{16}O, $^{40,48}Ca$, ^{56}Ni, ^{132}Sn, and ^{208}Pb studied



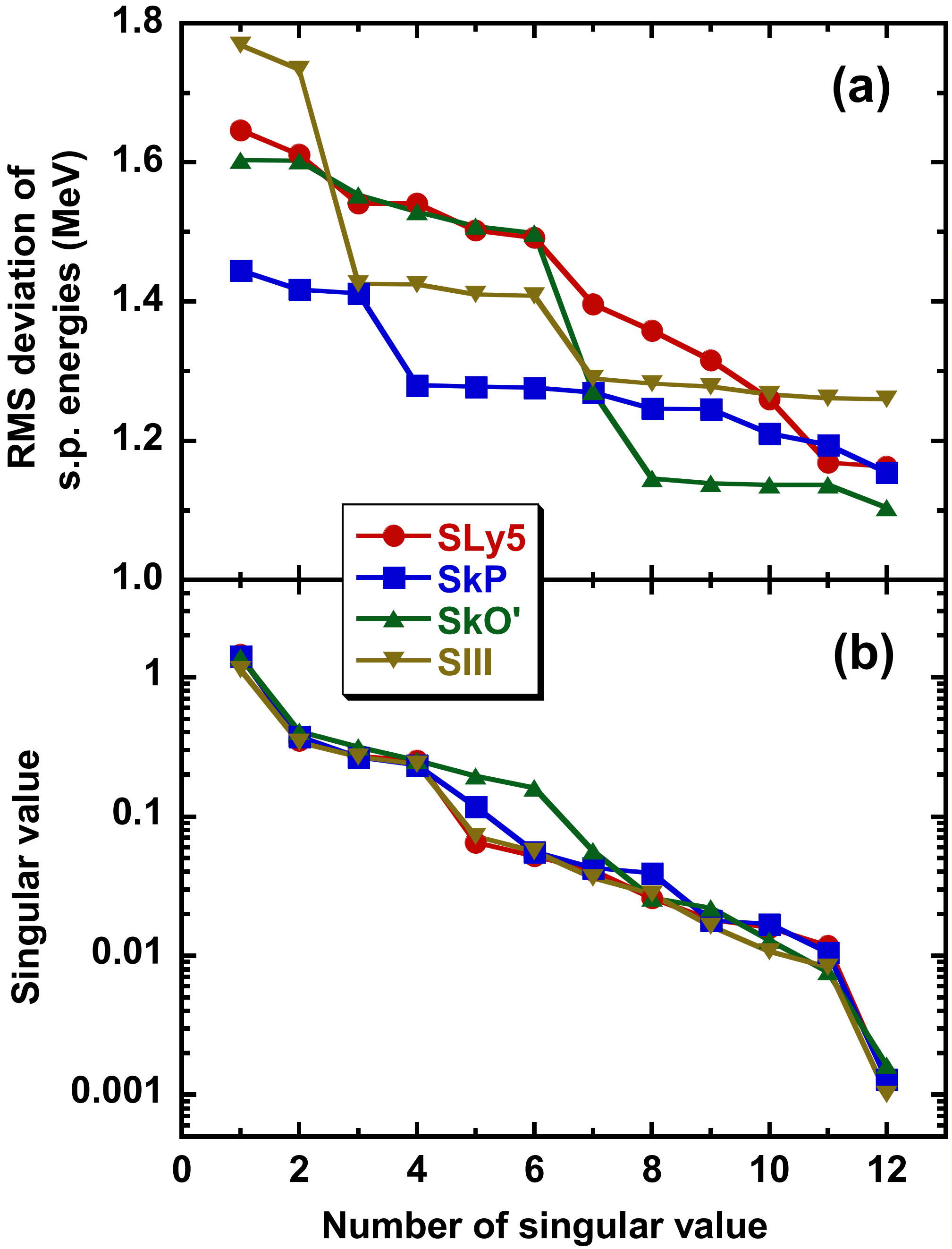


Single particle energies

- For linear one-step fit one needs to know the linear regression coefficients



Fitting coupling-constants to s.p. energies



M. Kortelainen *et al.*, Phys. Rev. C77, 064307 (2008)

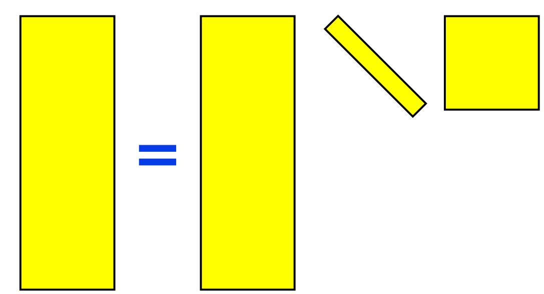
Fits of s.p. energies

$$\epsilon_i - \epsilon_i^{\text{EXP}} = -\sum_m \beta_{im} \Delta C_m,$$

EXP: M.N. Schwierz, I. Wiedenhover, and A. Volya, arXiv:0709.3525

Singular value decomposition

$$\beta_{im} = \sum_\mu V_{i\mu} d_\mu U^T_{\mu m},$$

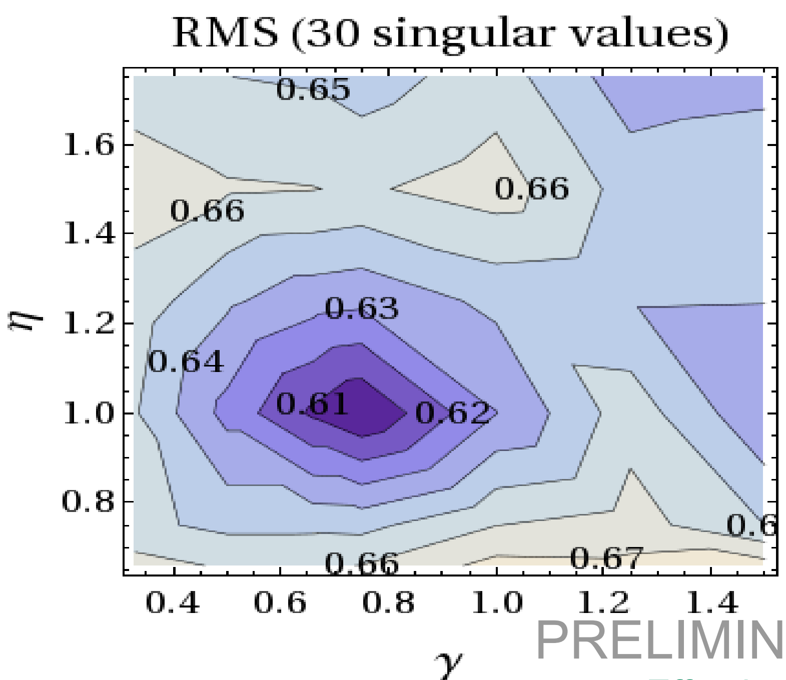


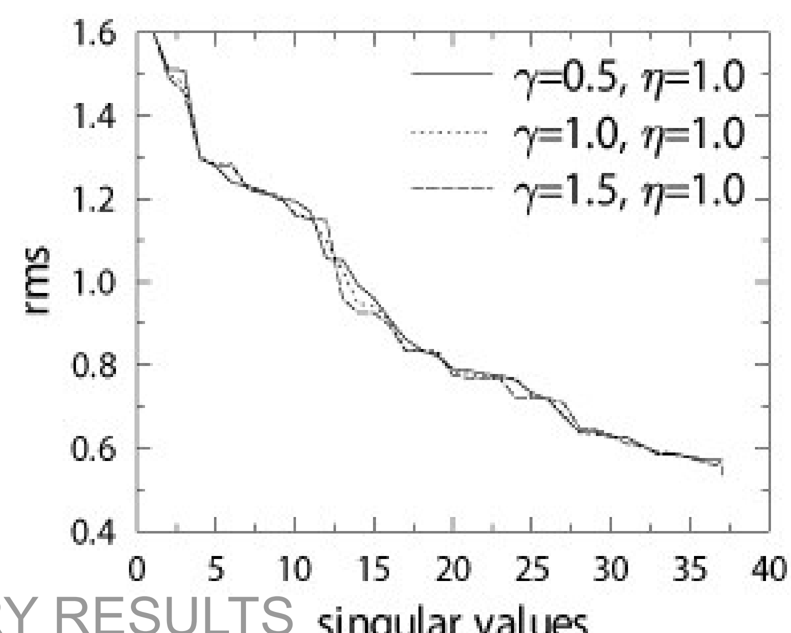
$$\sum_i V_{i\mu} V_{i\nu} = \delta_{\mu\nu},$$

$$\sum_m U_{m\mu} U_{m\nu} = \delta_{\mu\nu},$$

Single particle energies

- Fit only to the single-particle energies, all other properties disregarded
- Linear one-step fit predicts rms for s.p. energies ≈ 0.6 - 0.7MeV, when all C's, α's, and β's adjusted (Sly5: rms ≈ 1.6 Mev)
- Very small dependency on the powers γ and η

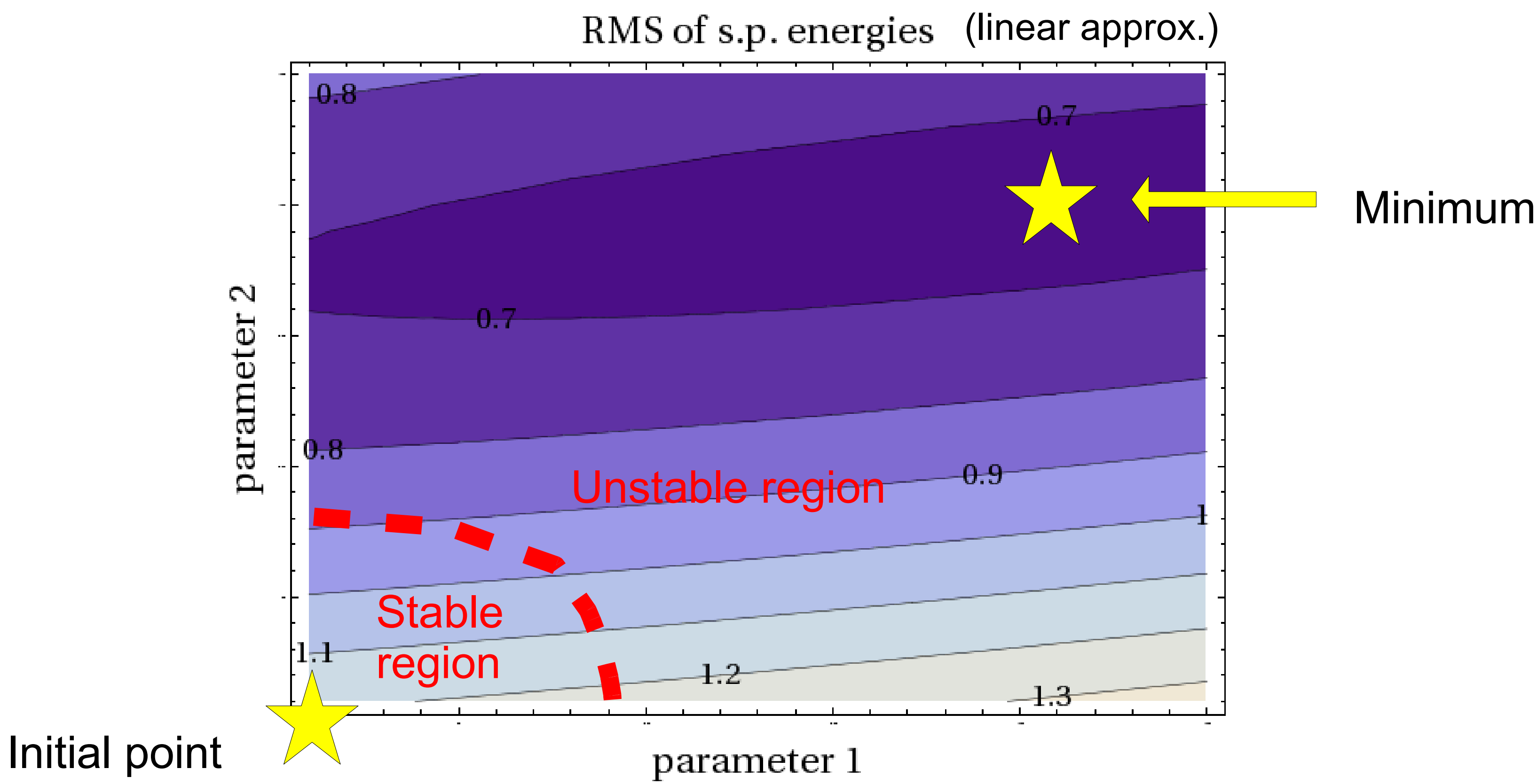




PRELIMINARY RESULTS

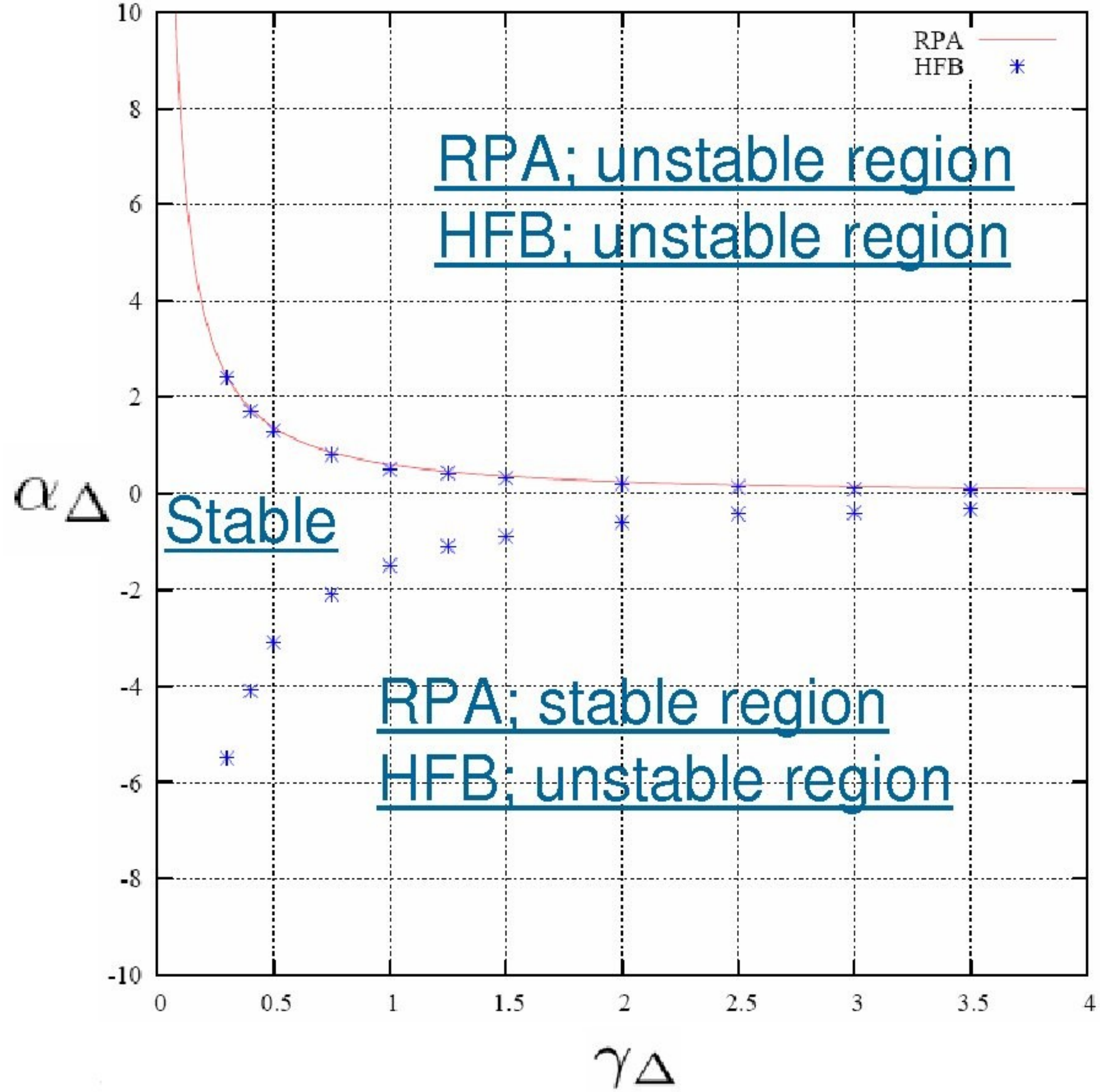
Single particle energies and unstable region

- Parameters coming from the one-step linear fit outside the stable region of the HFB solution
- Real rms probably around 0.9 MeV. By adjusting only Cc's one can get rms around 1.1 MeV



Unstable region of the HFB solution

- Some areas in the parameter space are unstable
- Nuclear matter RPA can give some of the stability limits, but not all of them
- Limits coming from the RPA are in good agreement with HFB calculations
- The lower limit of $\gamma^{\Delta\rho}$ is probably related to surface instabilities



K. Mizuyama, to be published

Conclusions

- Possible expansions of Skyrme EDF include higher order terms and density dependent coupling constants
- For infinite nuclear matter only certain powers γ^{ρ} and γ^{τ} give acceptable saturation curves
- γ^{ρ} and γ^{τ} (when reasonable) can not affect much to the saturation curve below saturation density. Also, effect on finite nuclei is small
- Density dependence on $C^{\nabla\rho}_{0}$ improves masses and charge radii
- Density dependency can not improve much single particle energies
- Parameter space is limited by unstable region

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Open questions and future plans

- Effect of other density-dependent volume terms in functional
- Which of the 38 coupling constant parameters are important
- Values of the powers γ and η
- Mapping of the stable region of the HFB solution
- Density dependence on deformed nuclei
- Other density-dependent functionals (DME, Fayans, ...)

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