Three-Body Problem; Low Momentum Interactions and Off-shell Effects

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Seattle april 29 2009

Motivation

EFT has brought us into a new era for nuclear many-body theory.

Combine this with what we learnt for the past 50 years, what do we get.

I try to answer some questions, but may generate more questions.

Numerical exercises for nuclear matter and the Triton making things as simple as possible, not looking for a quantitative solution but only extracting what I think is important.

Relevance for shell model calculations?

Topics of concern related to EFT are:

- In-medium ‘effective’ Interactions.
- Momentum cut-offs and off-shell effects.
- Three-body, Many-body…..
- Observables and UN-observables

- Main Topic:
- Triton energy.
- n-Deuteron scattering length.
- Momentum cut-off.
- Effect of short-ranged (high momentum) repulsion on low momentum effective interactions.
- ==
- On-shell NN-potential by Inverse Scattering, exact fits.
- Nuclear Matter, what can we learn?
- S-state potentials are separable.
- Off-shell is unobservable but if separable??!!.
- Off-shell------Three-body connection.
- Effect of high energy off-shell on low momentum Triton and n-D.

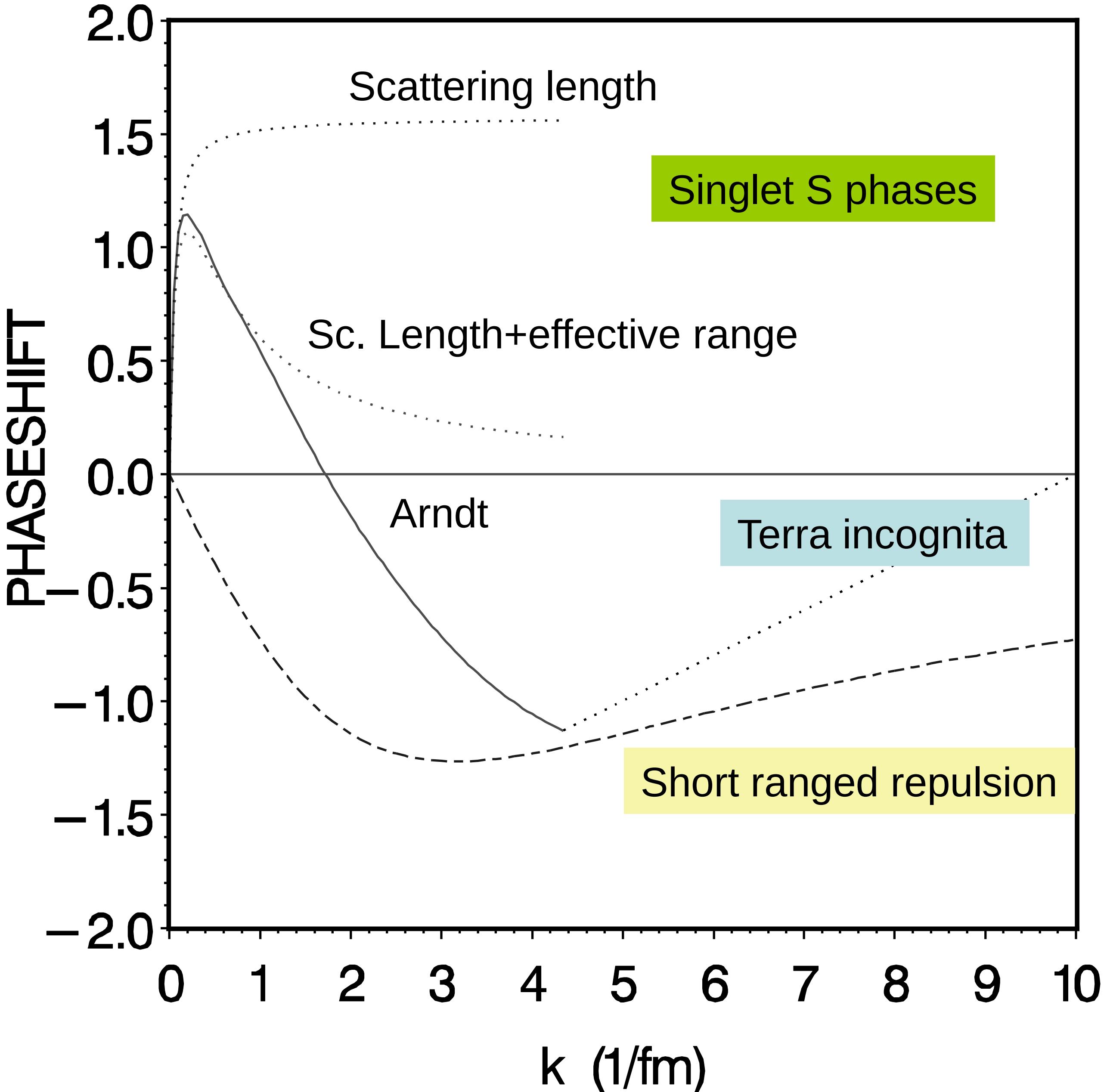
$\Rightarrow$

- I shall not get into the business of constructing QCD (meson) theoretical potentials.
- Instead I take a short-cut by taking the information experiments can give.

- Inputs in my calculations are:
- 2-body scattering phase-shifts
- Deuteron data matched to Bonn-A,B,C

My experimental Aircraft

- Of course this is not enough to uniquely define a potential or T-matrix or …
- Off-shell important in many-body problem
- Two potentials may be ‘phase-shift’ equivalent but off-shell difference gives different many-body results.
- I’ll get back to that.



2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
PHASESHIFT
Scattering length
Singlet S phases
Sc. Length+effective range
Arndt
Terra incognita
Short ranged repulsion
0
1
2
3
4
5
6
7
8
9
10
k (1/fm)

2-body scattering

$$Free space:$$

$$T = V + V \frac{1}{e_0 + i\eta} T \quad \text{scattering } \mathrm{e}^{\mathrm{i}\delta} \sin\delta$$

$$R = V + PV \frac{1}{e_0} R \quad \text{Reactance matrix } \tan\delta / k$$

$$In - medium:$$

$$K = V + V(Q/e)K \quad \text{Brueckner Reaction matrix}$$

$$\text{"Effective Interaction"}$$

Q is Pauli operator

$$\mathrm{e}_0 = k^2 - k'^2$$

$$e = e_0 + mean\ \ field$$

NOTE:

$$\text{In the limit } Q \to 1, e \to e_0$$

$$K \neq R \quad \text{Instead } < \mathrm{k} \,|\, K \,|\, k >= \delta(k)$$

$R-matrix\ \ generalised$ (e.g. for Triton)

$$t(k,k',\omega)=V(k,k')+V(k,k'')\frac{1}{\omega-k''^2}t(k'',k',\omega)$$

Separable potential Inverse Scattering

Assume a rank-1 attractive potential:

V(k,k')=-v(k)v(k')

phaseshifts (input)

$$\mathrm{v}^2(k) = -\frac{(4\pi)^2}{k}\sin\delta(k)\,|D(k^2)|$$

Then by 'standard' inverse scattering techniques:

Principal value

$$D(k^2) = \frac{k^2+E_B}{k^2}\exp\left[\frac{2}{\pi}P\int_0^\infty \frac{k'\,\delta(k')}{k^2-k'^2}dk'\right]. \quad E_B \text{ is binding energy}.$$

D

- Why use separable potentials?

1. Inverse Scattering with exact fit to On-shell data and the Deuteron.
2. ‘Realistic’ for S-states, pole at zero energy
3. Higher rank potentials to adjust Off-shell (Ranks varying from1 to 4 are used here)

Example, Unitary limit:

Infinite scattering length, effective range $= 0$:

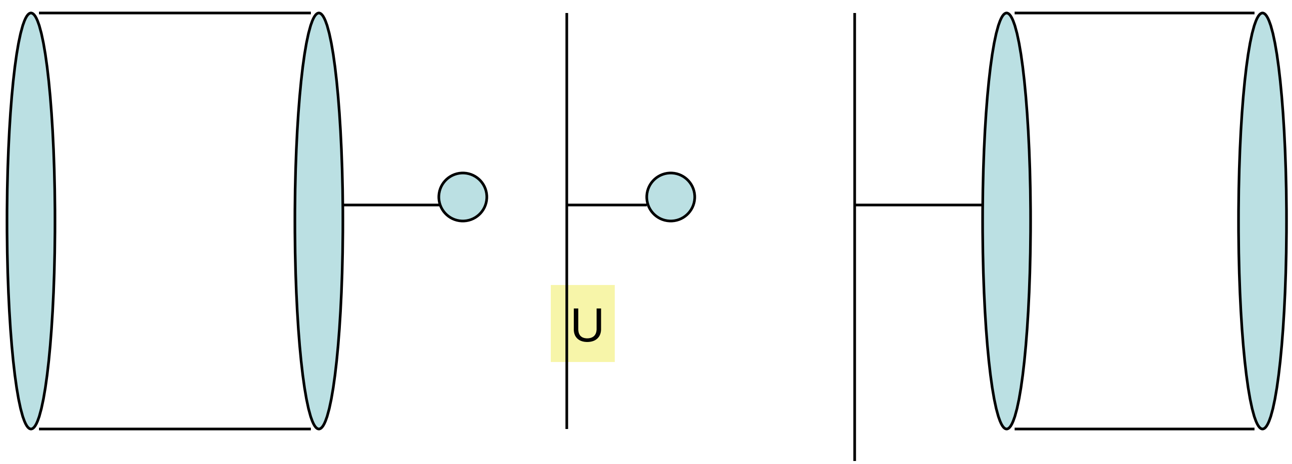
$$\delta(k) = \frac{\pi}{2}$$

$$v^2(k) = -\frac{4\pi}{\sqrt{\Lambda^2 - k^2}}$$

Brueckner Theory

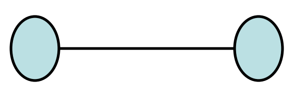
- Brueckner theory in its ‘standard’ form contains the minimal physics needed around saturation density. Less is doomed to failure. More is better.

Two- and three-body diagrams in Brueckner theory



Main 3-body saturation

'rearrangement'
Saturation of finite nuclei



Two-body ladders

For separable potential

In-medium effective Interaction

$$K = V + VGK$$

$$< k | V | k' >= v(k) v(k')$$

$$< k | K | k' >= \frac{v(k) v(k')}{1 + (2\pi)^{-3} \int_0^{\Lambda} v^2(k) G k^2 dk}$$

$$G = \frac{Q}{\omega - k'^2}$$

Compare Bonn-B and Separable Separable fits Arndt phases and Bonn-B Deuteron

	$^1S_0$		$^3S_1$	
k_f	Bonn	Sep	Bonn	Sep
1.35	-16.66	-16.57	-21.34	-21.33
1.60	-22.62	-22.76	-26.59	-26.27
1.90	-28.72	-29.84	-31.36	-31.45

Brueckner Nuclear Matter energies/particle contributions

Brueckner with Bonn-B and separable.

k_F	1.35 Bonn-B	1.35 ↓	1.60 Bonn-B	1.60 ↓	1.90 Bonn-B	1.90 ↓
$^1S_0$	-16.66	-16.57	-22.62	-22.76	-28.72	-29.84
$^3S_1$	-21.34	-21.33	-26.59	-26.27	-31.36	-31.45
$^3P_0$	-3.55	-3.28	-5.24	-5.02	-6.67	-7.00
$^3D_1$	1.48	2.13	2.95	4.35	5.58	8.58
$^1P_1$	4.43	3.59	8.01	6.93	14.47	13.34
$^3P_1$	9.99	11.47	18.47	21.46	34.13	41.00
$^3P_2$	-7.45	-7.80	-14.27	-14.73	-26.68	-27.01
$^3F_2$	-0.55	-0.18	-1.22	-0.39	-2.50	-0.84
$^1D_2$	-2.37	-2.36	-4.82	-4.97	-9.62	-10.22
$^3D_2$	-3.97	-3.50	-7.68	-6.62	-14.45	-12.13
J > 2	1.69	1.10	3.35	2.10	6.23	3.74
Total	-38.30	-36.75	-49.67	-45.91	-59.58	-51.84

- Bonn and separable both fit same on-shell and Deuteron (n-p bound state)
- In-medium off-shell not fitted so maybe different.
- What about it???
- Next slide.

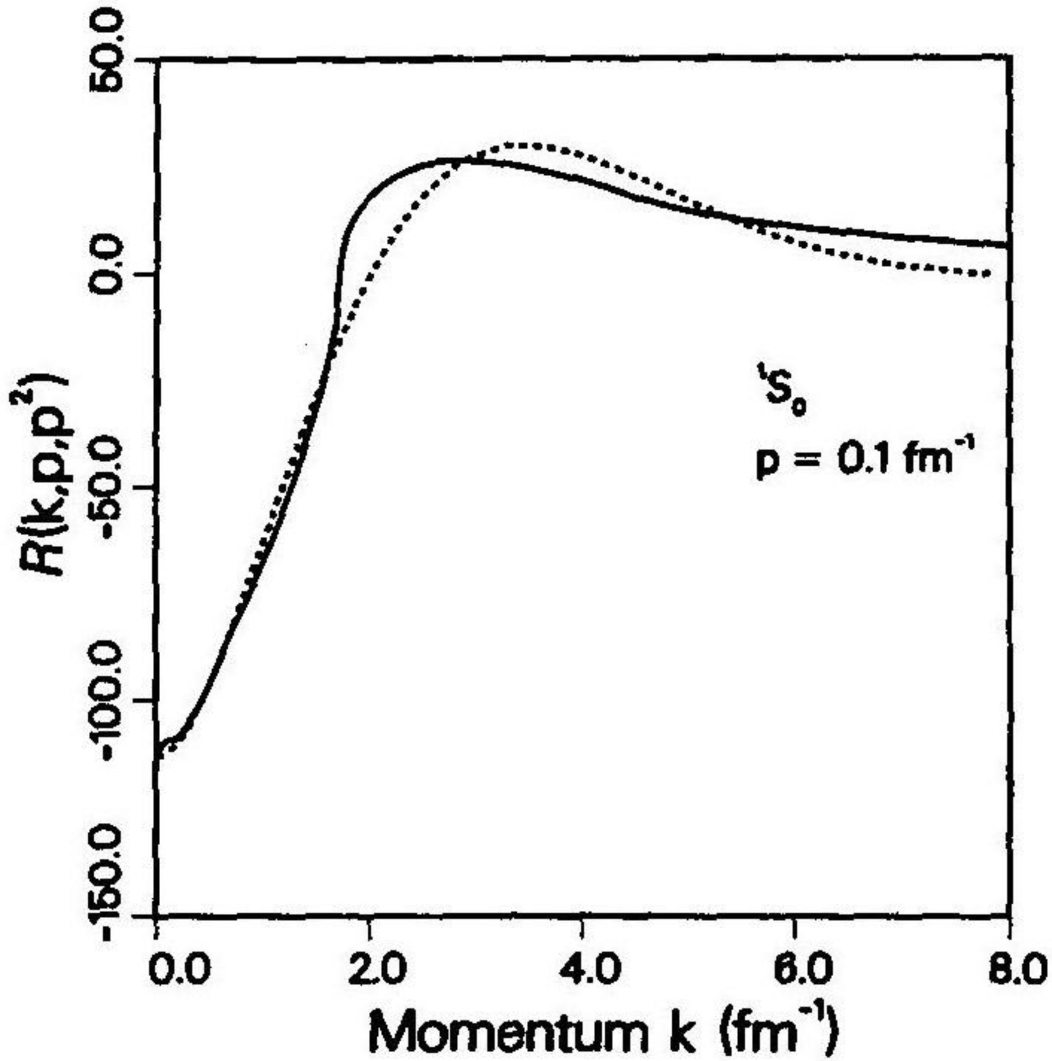


FIG. 4. Half-shell reactance matrix elements in the $^1S_0$ channel calculated with the Bolsterli-MacKenzie rank 2 potential in this work (solid) and the Bonn-B potential (dashed).

- Why the off-shell agreement?
- Answer: S-state potentials ARE separable!!
- Why? Really?
- Answer: Separable for low momenta around the low energy pole.
- OPEP a local potential!!
- Yes, but shown separable approx. good for low momenta.

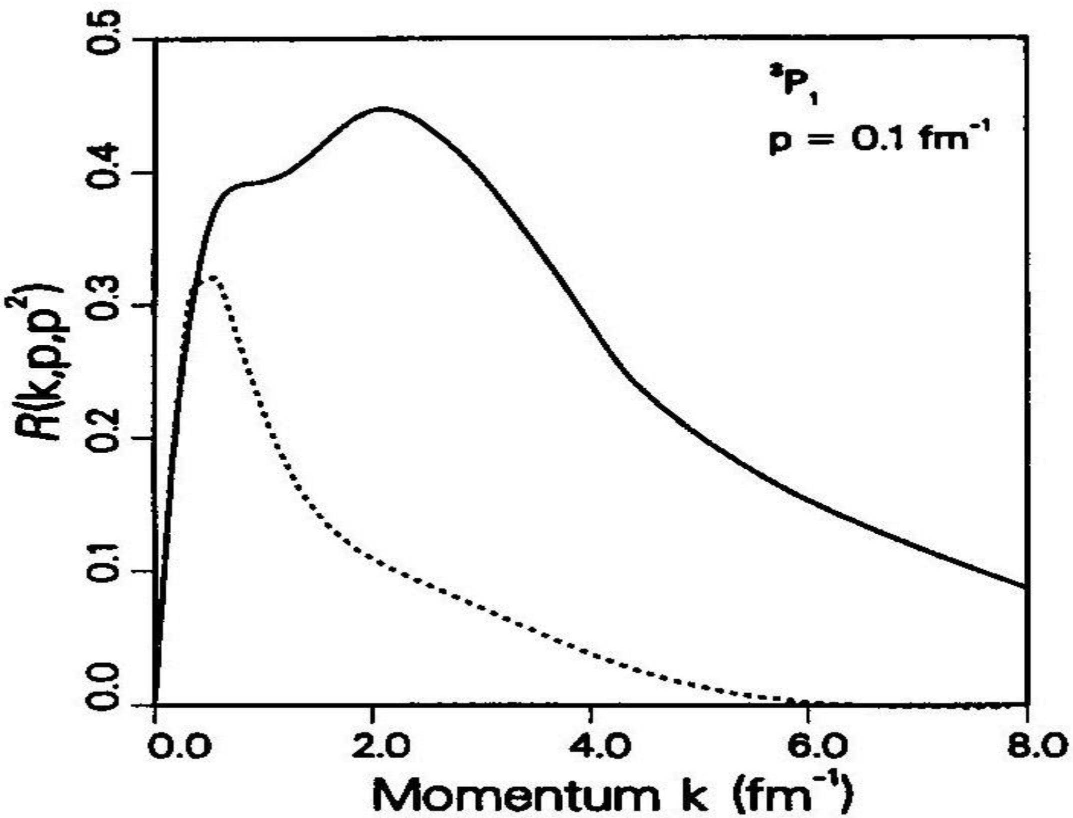
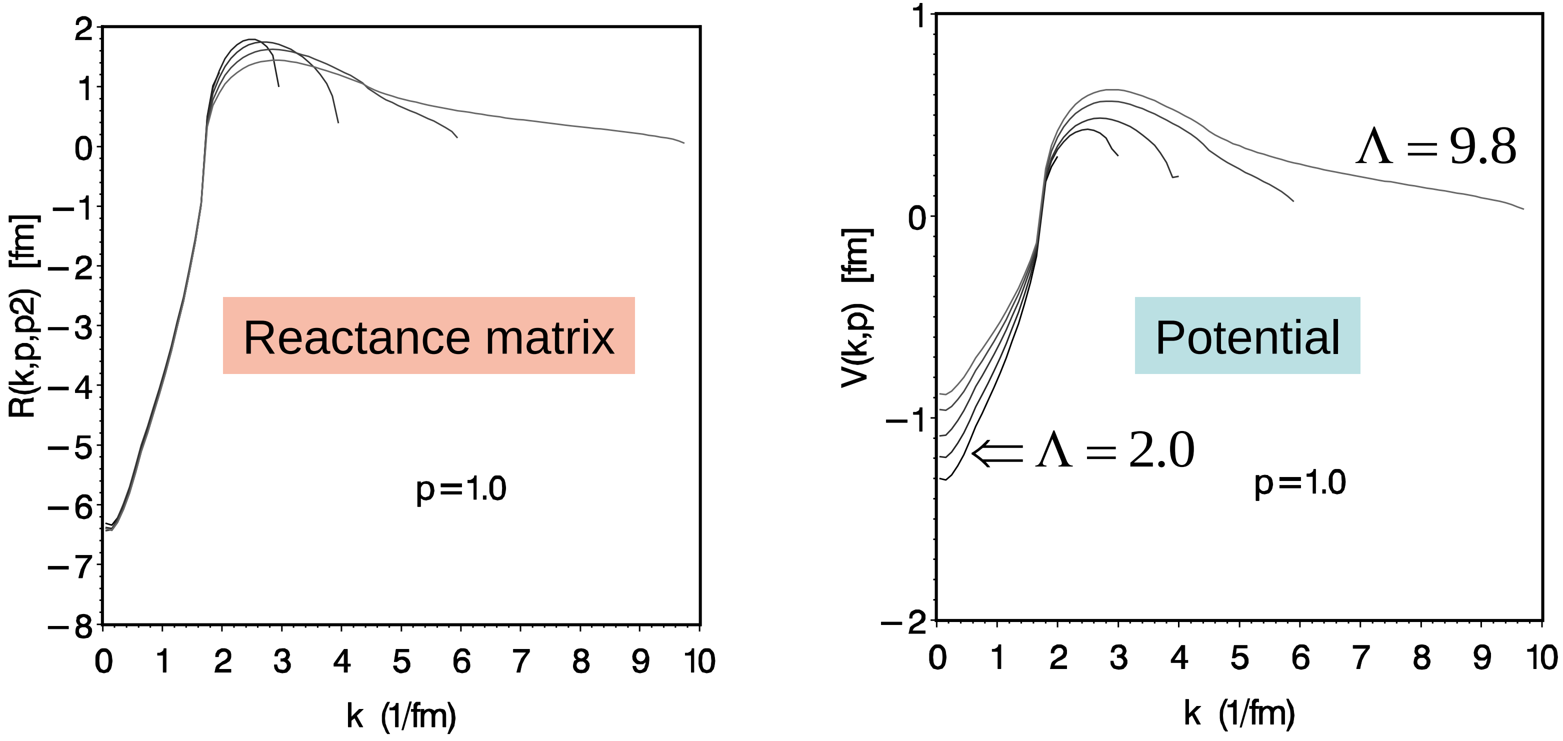


FIG. 5. Half-shell reactance matrix elements in the $^3P_1$ channel calculated with the rank 1 separable potential in this work (solid) and the Bonn-B potential (dashed).

- What about momentum cut-offs Λ

1S0; R and V for different cut-off Lambda

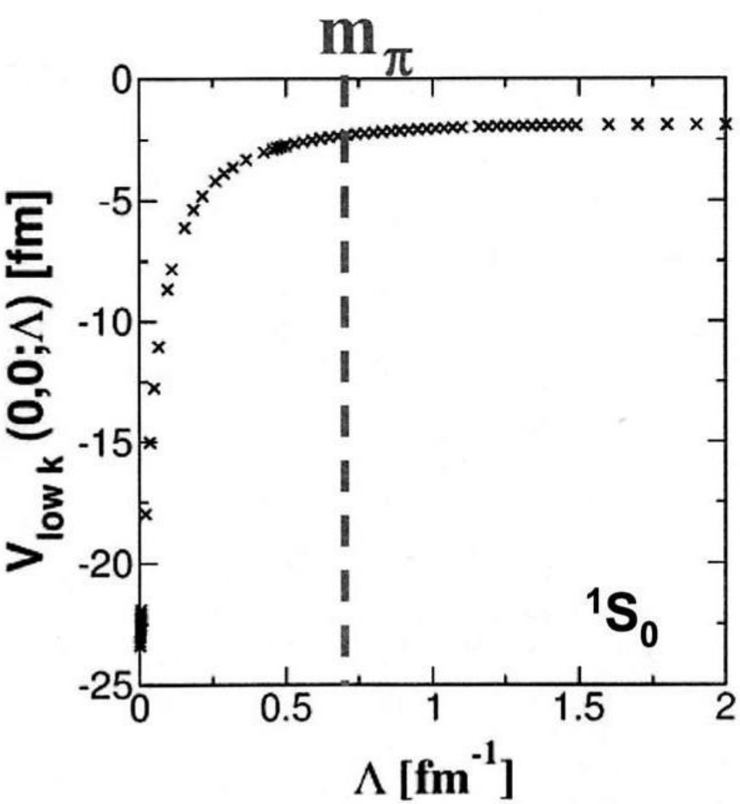
Note that the on-shell R is independent of cut-off. It is fixed by the phase-shifts.

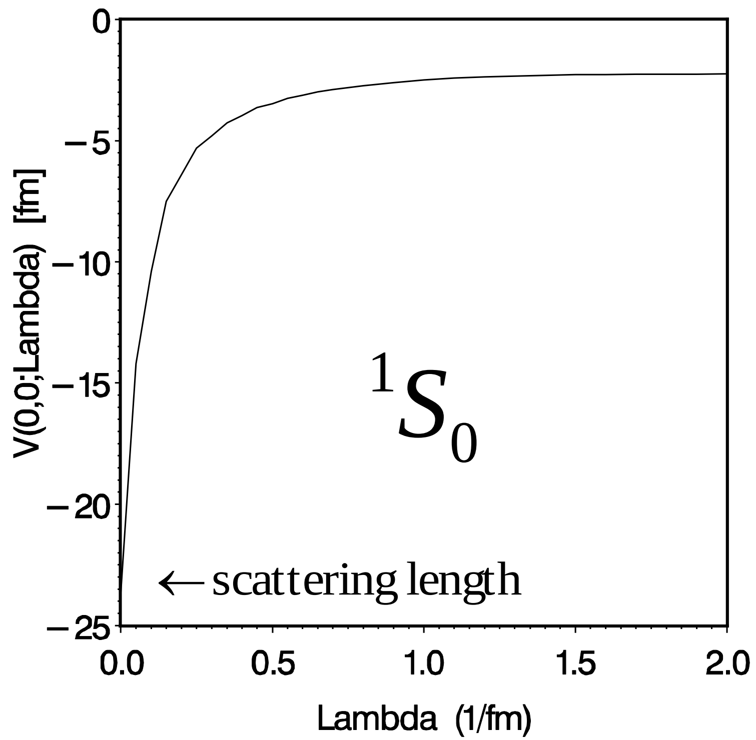


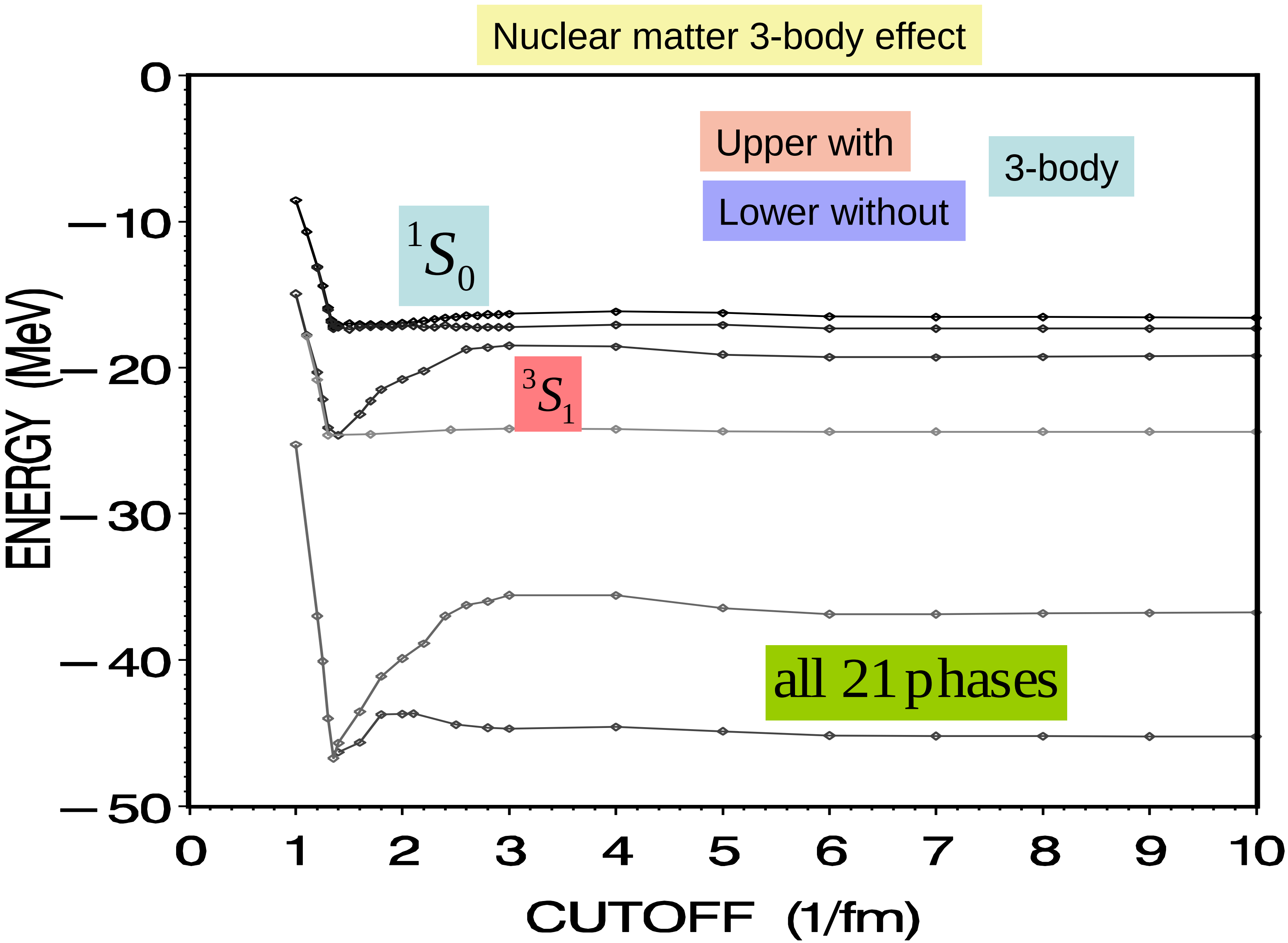
Note that the low momentum part of R is (nearly) independent of cut-off
While the potential is not.

- Last slide relates to a well-known fact.
- R and/or T that are directly related to scattering are much better approximations to in-medium interactions than the potential.
- Many early works used that relationship with some success.

Comparison with V-low-k
Left figure

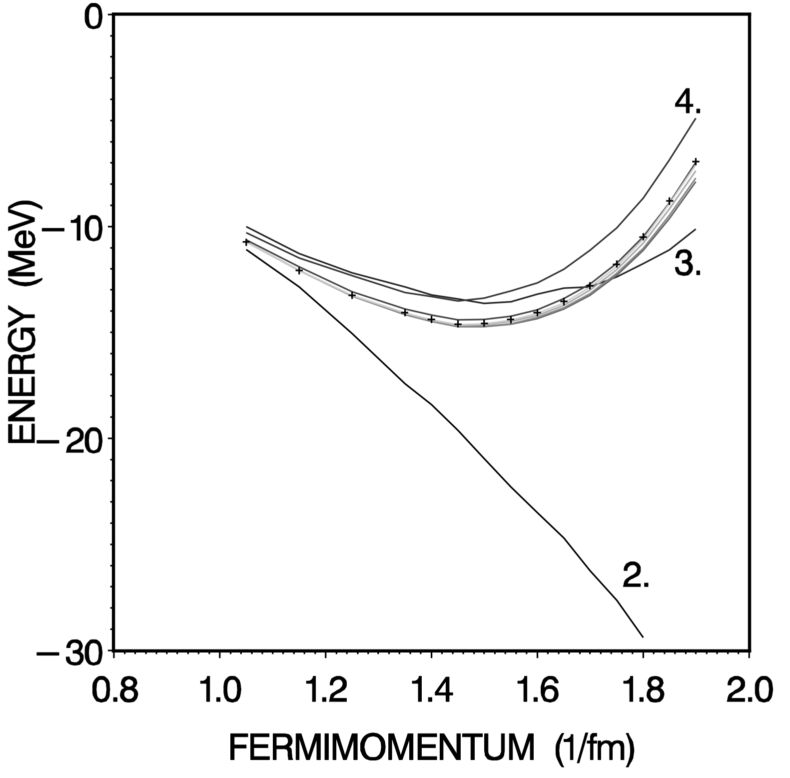






Nuclear matter 3-body effect
Upper with
Lower without
3-body
$^1S_0$
$^3S_1$
all 21 phases
ENERGY (MeV)
CUTOFF (1/fm)
0
−10
−20
−30
−40
−50
0
1
2
3
4
5
6
7
8
9
10

Effect of cut-off (three-body) on saturation



Off-shell (three-body) effect

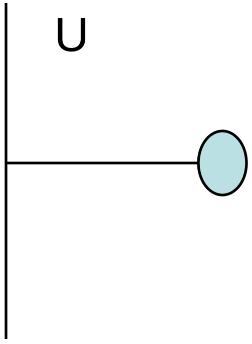
$$K = V + \frac{Q}{\omega - e} K$$

$$K' = V + \frac{Q}{\omega' - e} K$$

$$K' = K + K\left(\frac{Q}{\omega' - e} - \frac{Q}{\omega - e}\right)K'$$

$$K' - K = I_w(\omega' - \omega) \rightarrow I_w * U \text{ (U is mean field)}$$

$$I_w = \int (\Psi - \Phi)^2 dr$$



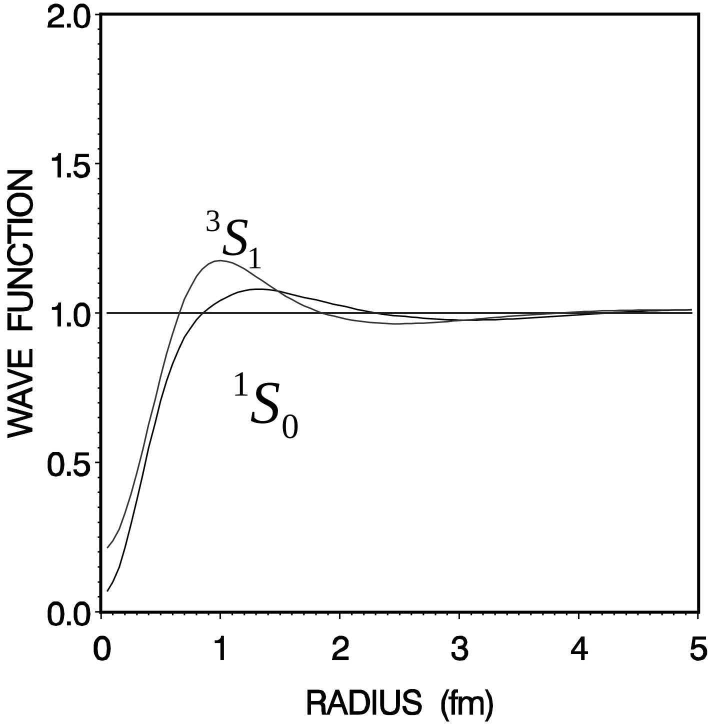
$$\Psi = \Phi + \frac{Q}{\omega - e} K$$

'wound'-integral

- Next slides will show that the decrease at small Lambda is associated with a decrease in 2-body correlations.
  Off-shell---3-body term is lost.

In-medium Wave-function

$$\Lambda = 9.8\, fm^{-1}$$

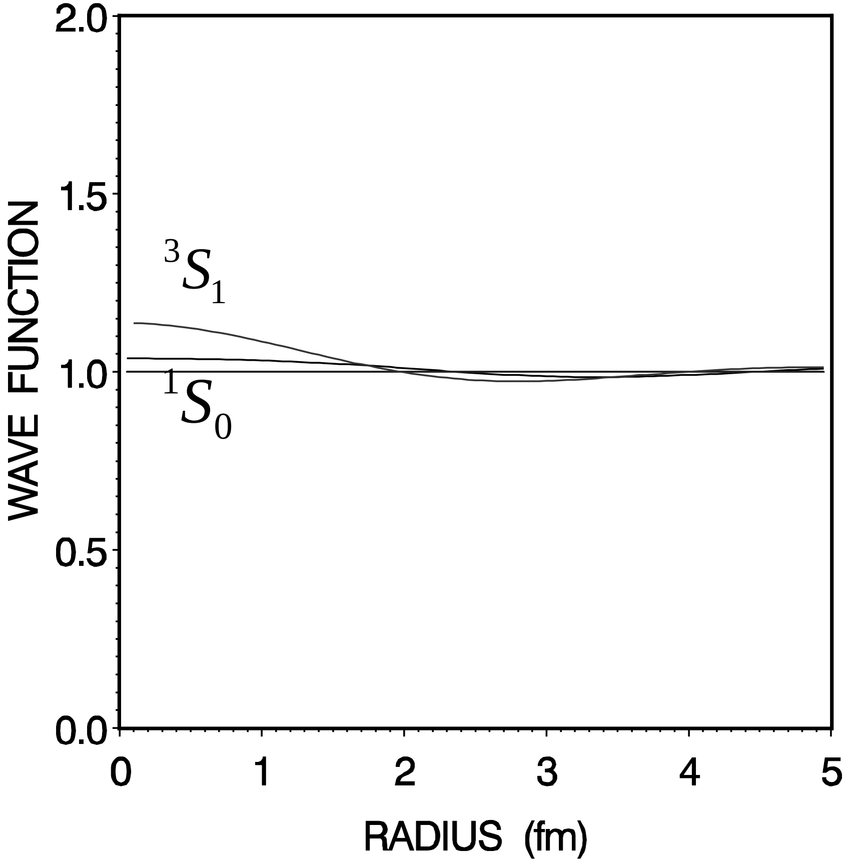


K=0

Kohler-Moszkowski (ARXIV)

In-medium Wave-function

$$\Lambda = 2.0\ fm^{-1}$$



K=0

Kohler-Moszkowski (ARXIV)

Summary of last slides

Independence of cut-off if it is larger than

2-3 1/fm

Mean field (three-body) has a repulsive effect that decreases for small cut-offs.

- Consistent with:

Repulsion ~ wound-integral*mean field because we saw correlations and therefore wound-integral small for cut-off=2 1/fm

Repulsion is due to a 3-body term (not intrinsic 3-body force)

- Triton problem a low momentum problem
- Only S-states (almost).
- So separable approximation should be good here.

Faddeev equation

$$\chi(q) = \frac{2}{D(E_T - \frac{3}{4}q^2)} \int \frac{v(|k + \frac{q}{2}|)v(|q + \frac{k}{2}|)}{q^2 + q.k + k^2 - E_T} \chi(q) dk$$

$$\chi(q) = \frac{2}{D(E_T - \frac{3}{4}q^2)}$$

For separable potential

In-medium effective Interaction

$$K = V + VGK$$

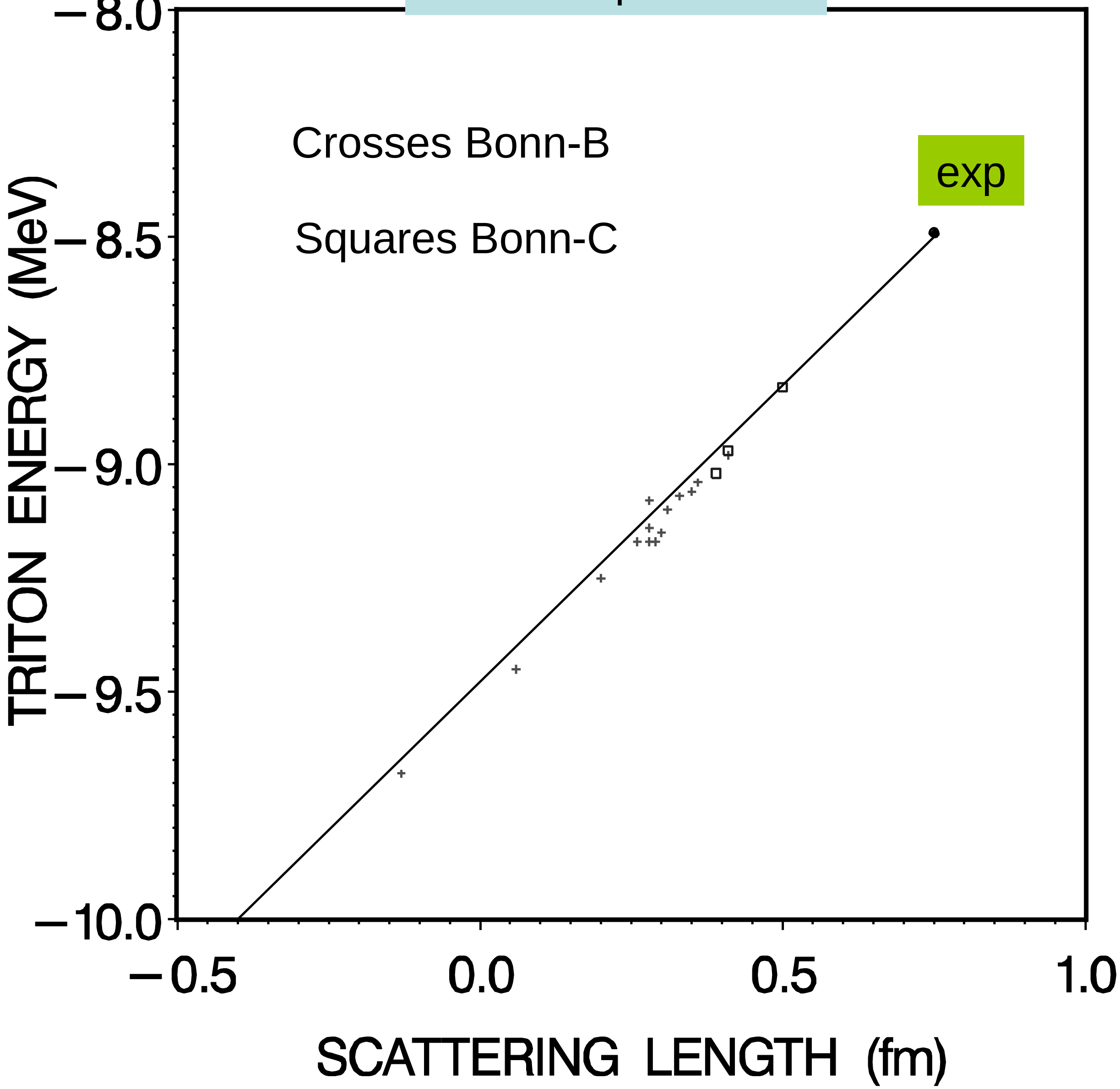
$$< k\,|V\,|\,k' >= v(k)v(k')$$

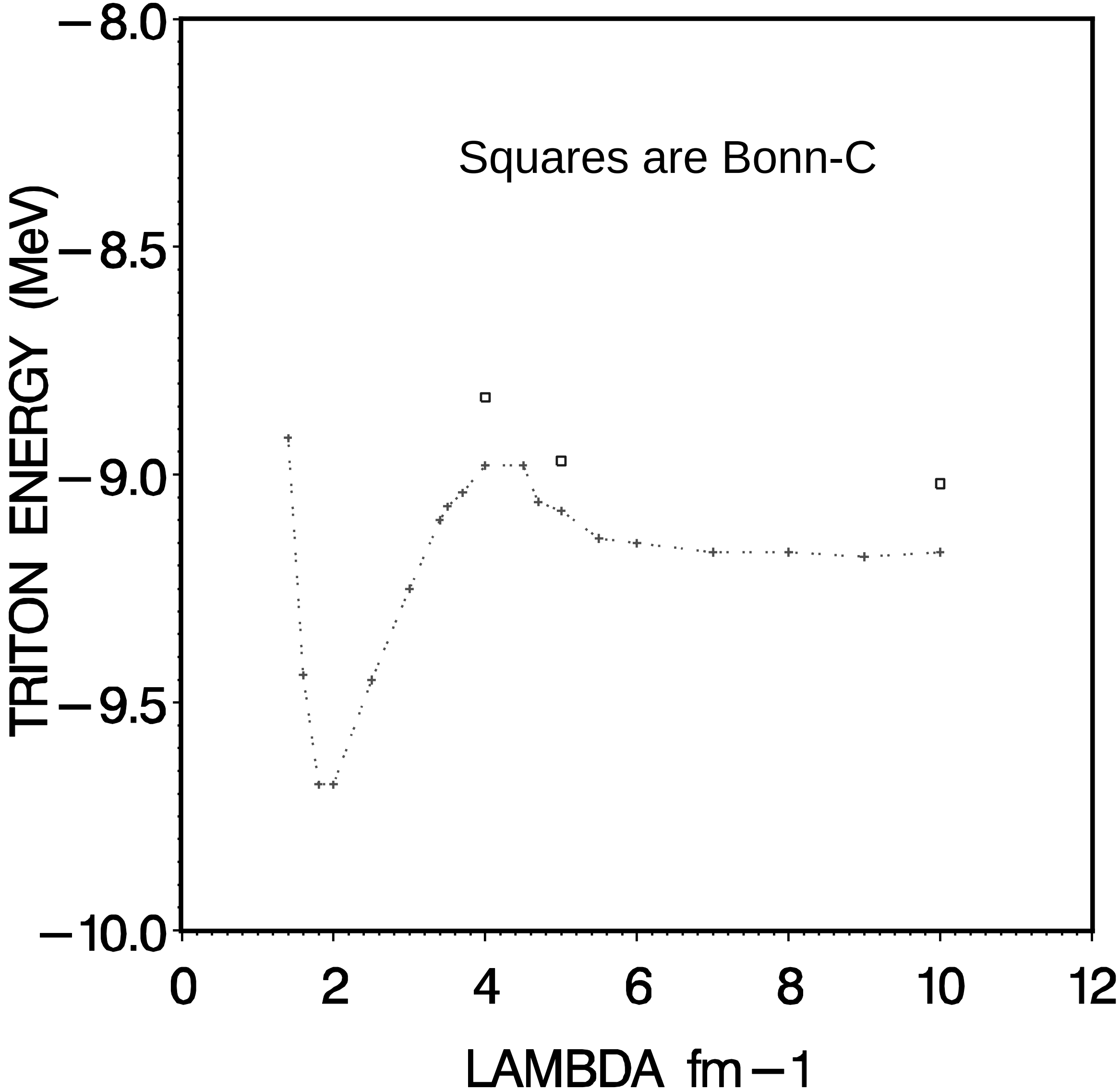
$$< k\,|\,K\,|\,k' >= \frac{v(k)v(k')}{1+(2\pi)^{-3}\int_0^{\Lambda} v^2(k)Gk^2dk}$$

$$G = \frac{Q}{\omega - k'^2}$$

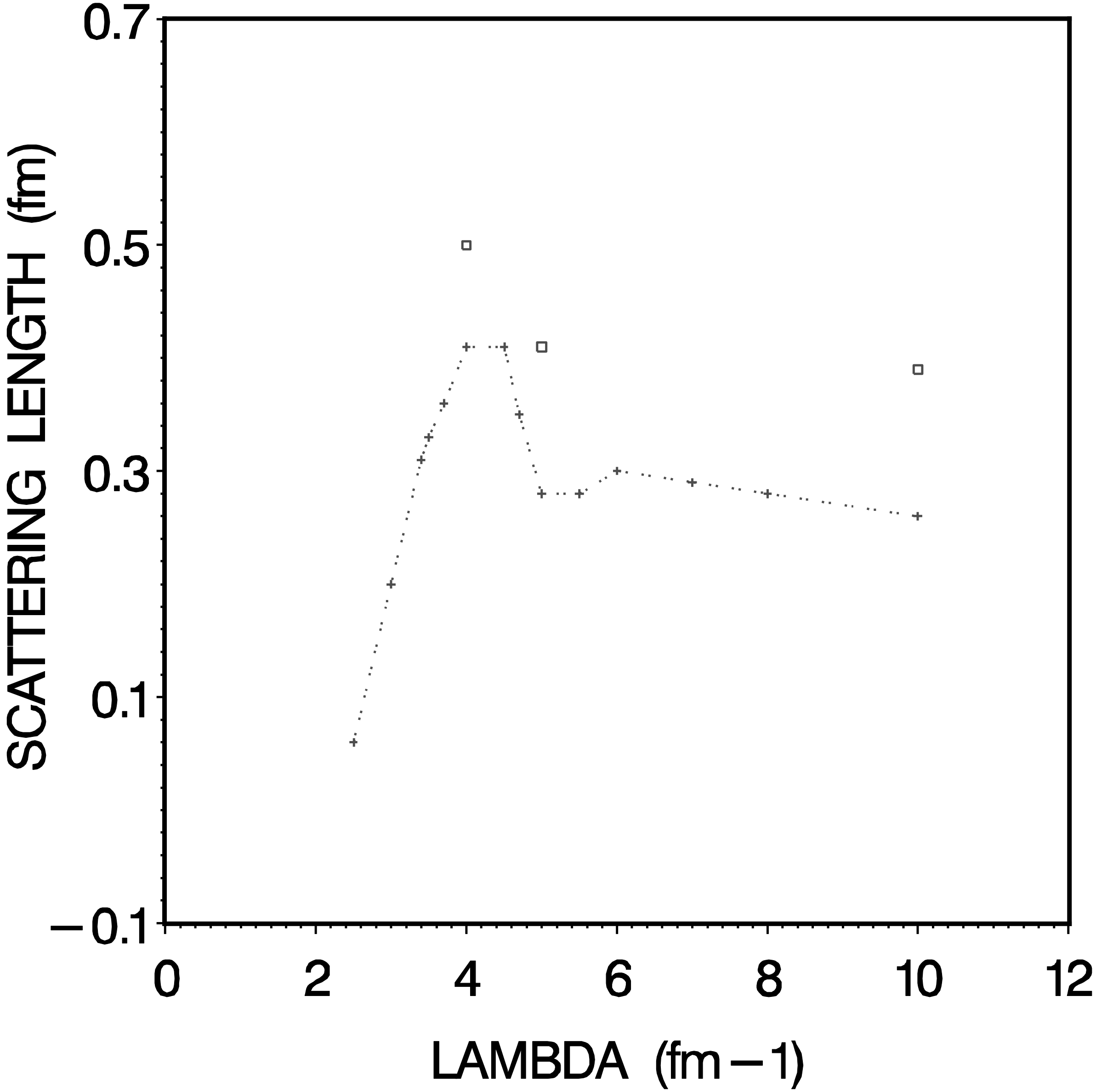
PHILLIPS line

Cut-off dependence





Squares are Bonn-C
TRITON ENERGY (MeV)
−8.0
−8.5
−9.0
−9.5
−10.0
0
2
4
6
8
10
12
LAMBDA fm−1



0.7
0.5
0.3
0.1
−0.1
SCATTERING LENGTH (fm)
0
2
4
6
8
10
12
LAMBDA (fm−1)

THREE-BODY SUMMARY

- Triton energy too low
- n-D scattering length too small

Too low on the Phillips line

Solution:

Three-body force (term?)

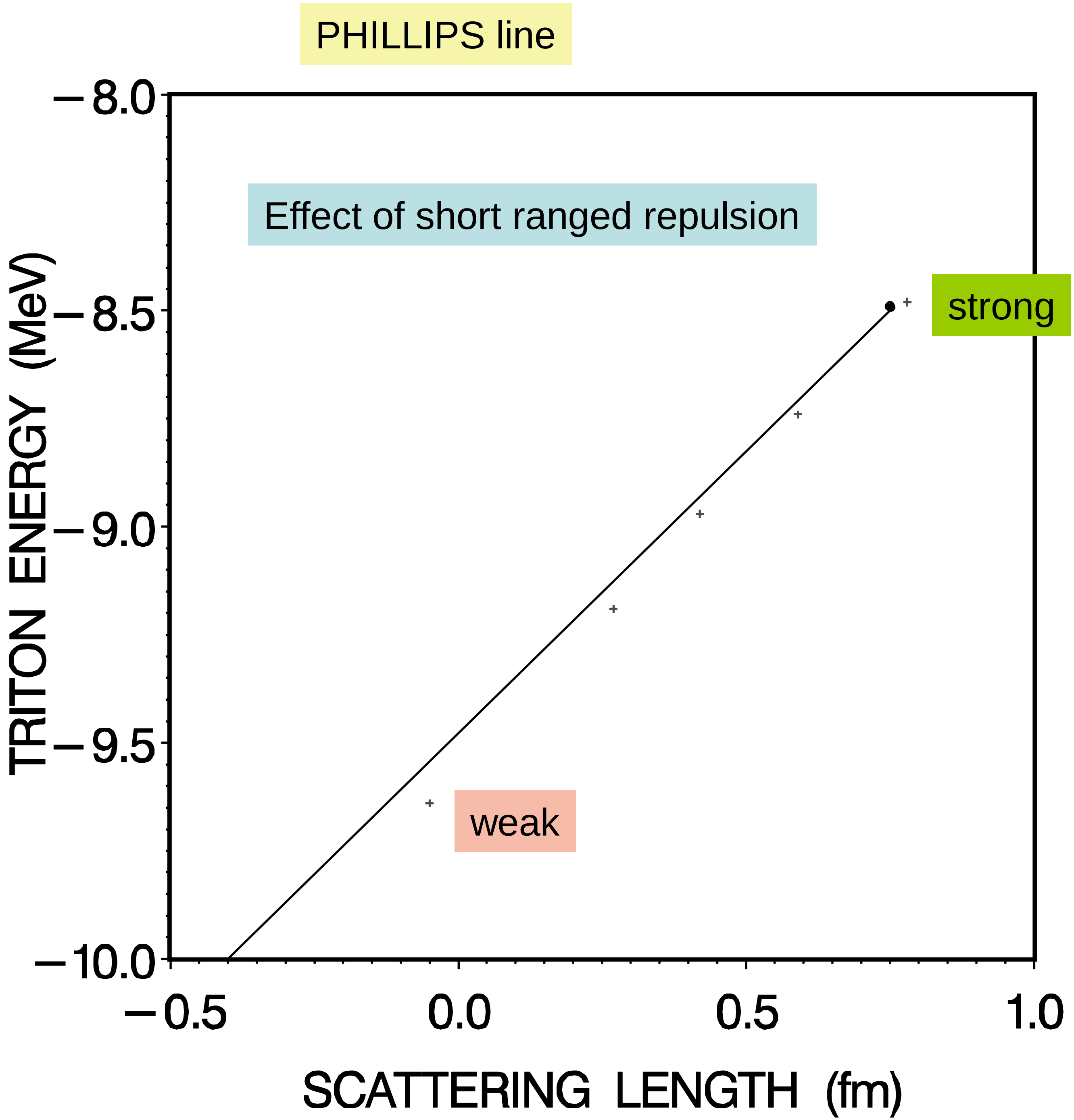
—Off-shell correction

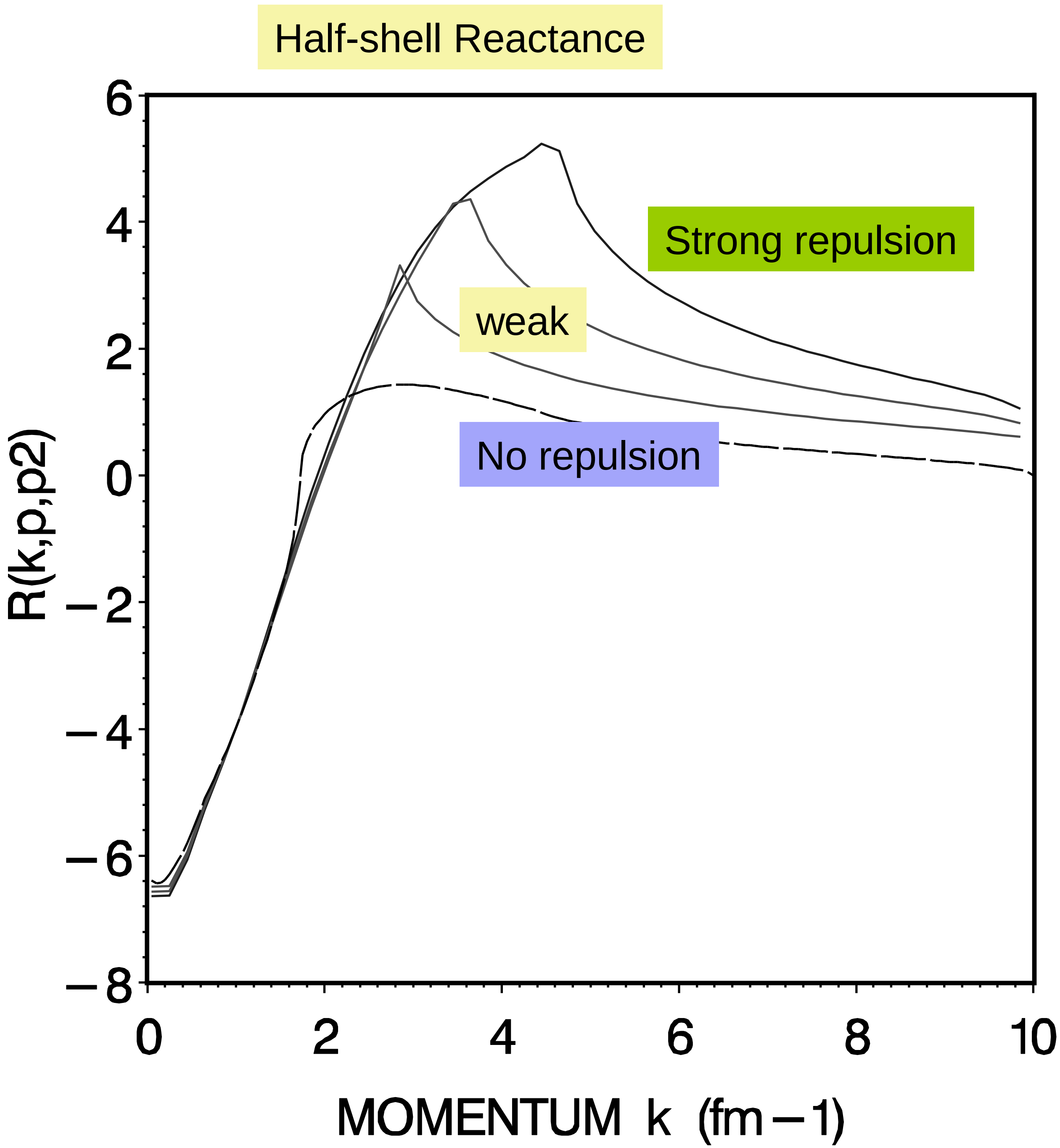
Off-shell correction

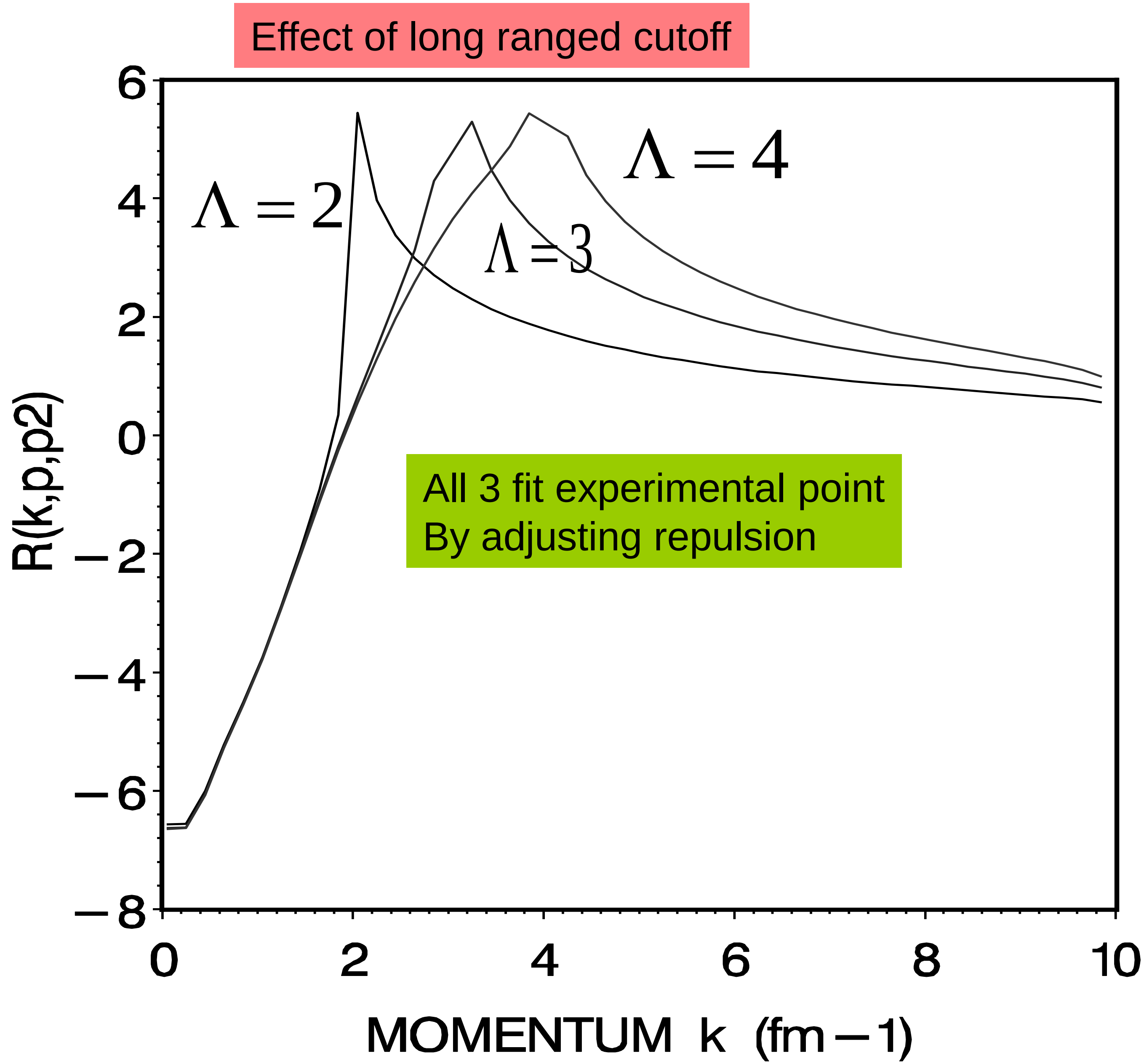
- Increase the rank of the separable potential.
- V=-g(k)g(k')+h(k)h(k')
- g is long-ranged in coordinate space
- h is short ranged

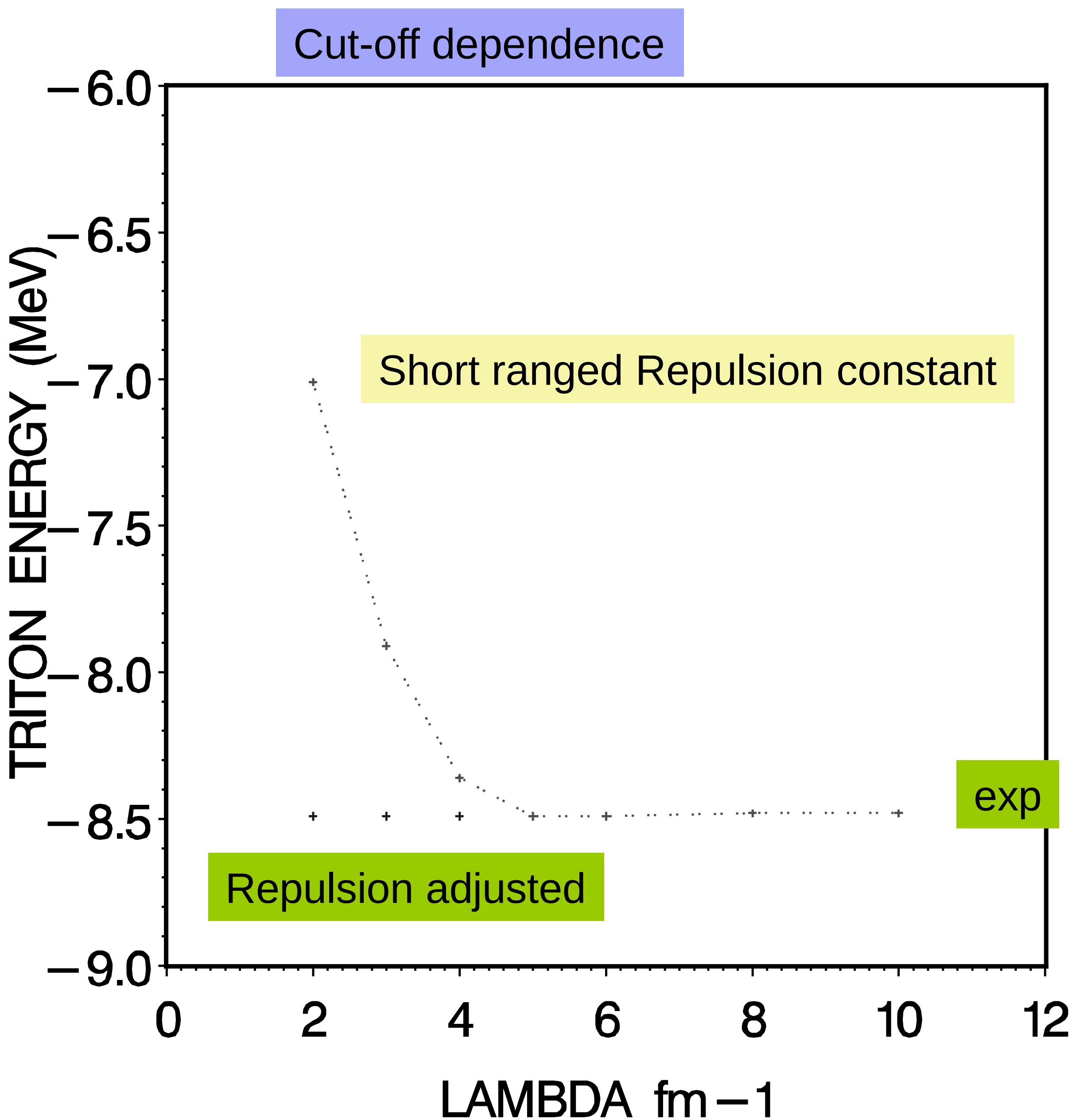
$$\text{define h by phase-shifts}\, \delta_s(k) = kr_c/(1+ck^2)$$

g and h are then obtained by inverse scattering to fit the singlet S phases


PHILLIPS line
Effect of short ranged repulsion
strong
weak
TRITON ENERGY (MeV)
−8.0
−8.5
−9.0
−9.5
−10.0
−0.5
0.0
0.5
1.0
SCATTERING LENGTH (fm)


Half-shell Reactance
Strong repulsion
weak
No repulsion
R(k,p,p2)
MOMENTUM k (fm−1)
6
4
2
0
−2
−4
−6
−8
0
2
4
6
8
10


Effect of long ranged cutoff
Λ = 2
Λ = 3
Λ = 4
All 3 fit experimental point
By adjusting repulsion
R(k,p,p2)
MOMENTUM k (fm−1)
6
4
2
0
−2
−4
−6
−8
0
2
4
6
8
10


Cut-off dependence
Short ranged Repulsion constant
exp
Repulsion adjusted
TRITON ENERGY (MeV)
−6.0
−6.5
−7.0
−7.5
−8.0
−8.5
−9.0
0
2
4
6
8
10
12
LAMBDA fm−1

SUMMARY

On-shell $\delta(k) < \Lambda$

In Many Body System also

Off-shell $\delta(k) > \Lambda$

Use:

Attractive force $\delta(k) < \Lambda$

Repulsive force for off-shell

Contact force Λ – dependent

2-body Correlations generate a 3-body term

Conclusion

- Three-body term important for Nuclear Matter and Finite Nuclei.
- Three-body term generated by 2-body correlations.