Universal Correlations in Pion-less EFT with the Resonating Group Model

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Ultimate Goals

- analysis of **universal correlations** between bound & scattering observables in $A > 4$ systems

  $\Rightarrow$ objects which **differ on a microscopic scale** exhibit the **same low-energy behavior**

- How far in A (and density) can one **push the pionless EFT**?

  $\Rightarrow$ development of the most **simple** theory, **rooted in QCD**, appropriate for the description of low-energy nuclear properties

The Effective Field Theory "without pions"

see e.g. P.F. Bedaque, U. van Kolck,
Ann. Rev. Nucl. Part. Sci. **52**, 2002

- **appropriate** theory for nuclear systems with $p_{\text{typ}} \ll m_\pi$
- **systematic** expansion of observables in $\frac{p_{\text{typ}}}{\Lambda_{\not\pi}}$
- provides theoretical **uncertainties**
- systematic analysis of **universal** aspects of few-nucleon systems feasible
- **model independent** statements about the consistency between theory & experiment

(Refined) Resonating Group Model

see e.g. H. M. Hofmann, proceedings of Models and Methods
in Few-Body Physics, 1986.

- **versatile** method:
  - suited for calculations of **bound** & **scattering** observables
  - applicable to other **degrees of freedom**, *e.g.*, (α, n) & $\left({}^9\text{Li}, t, n\right)$
  - Coulomb interaction included
- with $\text{EFT}_{\not\pi}^{\text{NLO}}$ relatively modest **CPU time** requirements

The RRGM model space

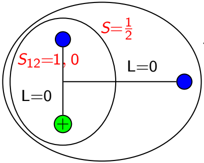
Ritz variation: $\delta\left(\langle\psi|\hat{\mathrm{H}} - E|\psi\rangle\right) = 0$

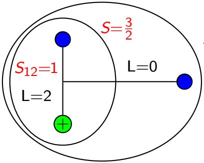
width parameters

$$\psi^{J^\pi}_{\mathrm{BS}}(\vec{\rho}_m, \vec{s}_m) = \mathcal{A}\left\{\sum_{d,i,j} c_{dij}\left[\left[\prod_{k=1}^{N-1} e^{-\gamma_{dk}\vec{\rho}_k^2}\mathcal{Y}_{l_{ki}}(\vec{\rho}_k)\right]^{L_i} \otimes \Xi^{S_j}\right]^J \cdot \Upsilon\right\}$$

antisymmetrizer

Jacobi coordinates

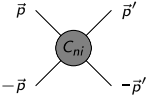
$\left|{}^3\mathrm{H}\right\rangle^{J^\pi=\frac{1}{2}^+} = \Bigg|$ 

$\Bigg\rangle^{J^\pi=\frac{1}{2}^+} + \Bigg|$ 

$\Bigg\rangle^{J^\pi=\frac{1}{2}^+} + \quad \cdots$

NN potential & LEC determination

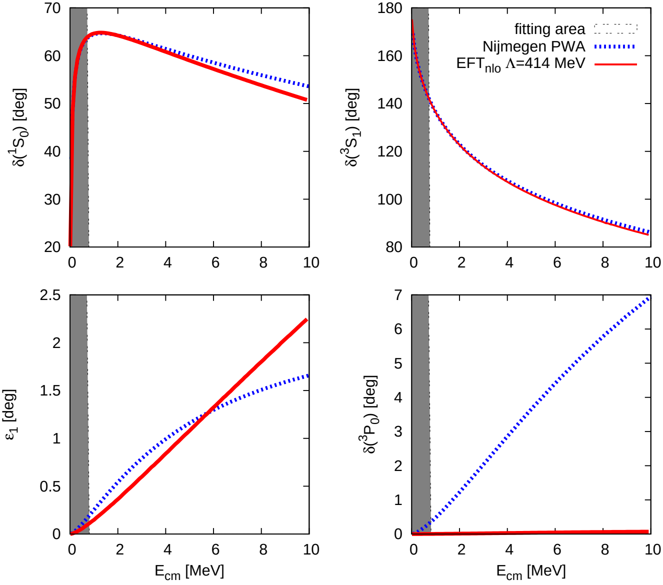
$$
\begin{aligned}
V_{\not\pi}^{\mathrm{NLO}}(\vec{r}) \quad = \quad & l_0(r)\left(A_1 + A_2\vec{\sigma}_1\cdot\vec{\sigma}_2\right) + \left(A_3 + A_4\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\left\{e^{-\frac{\Lambda^2}{4}\vec{r}^2}, \vec{\nabla}^2\right\} + \\
& l_0(r)\left(A_5 + A_6\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\vec{r}^2 + l_0(r)A_7\vec{L}\cdot\vec{S} + l_0(r)A_8\left[\vec{\sigma}_1\cdot\vec{r}\,\vec{\sigma}_2\cdot\vec{r} - \frac{1}{3}\vec{r}^2\vec{\sigma}_1\cdot\vec{\sigma}_2\right] \\
& -A_9\left\{e^{-\frac{\Lambda^2}{4}\vec{r}^2}, \left[\left[\partial^r\otimes\partial^s\right]^2 \otimes \left[\sigma_1^p\otimes\sigma_2^q\right]^2\right]^{00}\right\}
\end{aligned}
$$



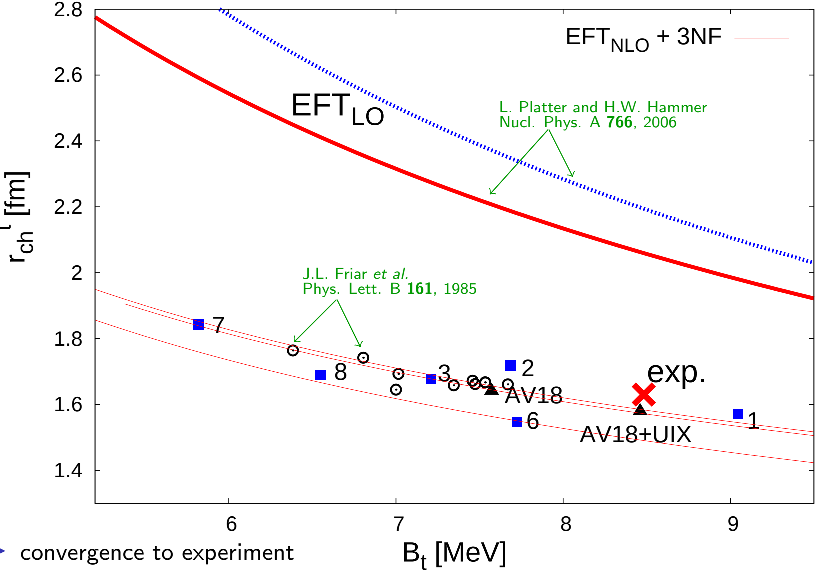
- Gaussian regulator functions $l_0(r,\Lambda) \propto e^{-\frac{\Lambda^2}{4}\vec{r}^2}$
- low-energy-constants depend on cutoff, $A_i = A_i(\Lambda)$
- LEC of leading order 3NF **not** fitted

- low-energy input data: $B_d,\ \delta\left({}^1S_0, {}^3S_1, {}^{1,3}P_{0,1,2}\right), \epsilon_1$ for $E_{\mathrm{cm}} < 1$ MeV
- two methods to obtain different LEC sets $\leftrightarrow$ different short-range physics:
  - variation of **cutoff** parameter Λ
  - variation of **low-energy input**

NN phase shifts at NLO

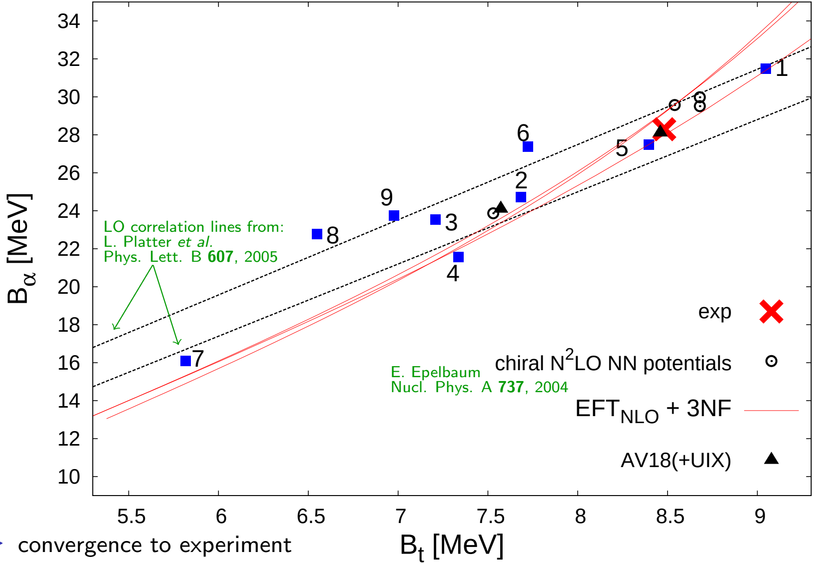


- deviation from $\delta^{\text{Nij}}(^{1,3}S_{0,1})$ is $\lesssim 10\%$
- deviation from ϵ_1^{Nij} is $\lesssim 30\%$
- P-wave phase shifts fitted to various values $< \delta^{\text{Nij}}(P)$

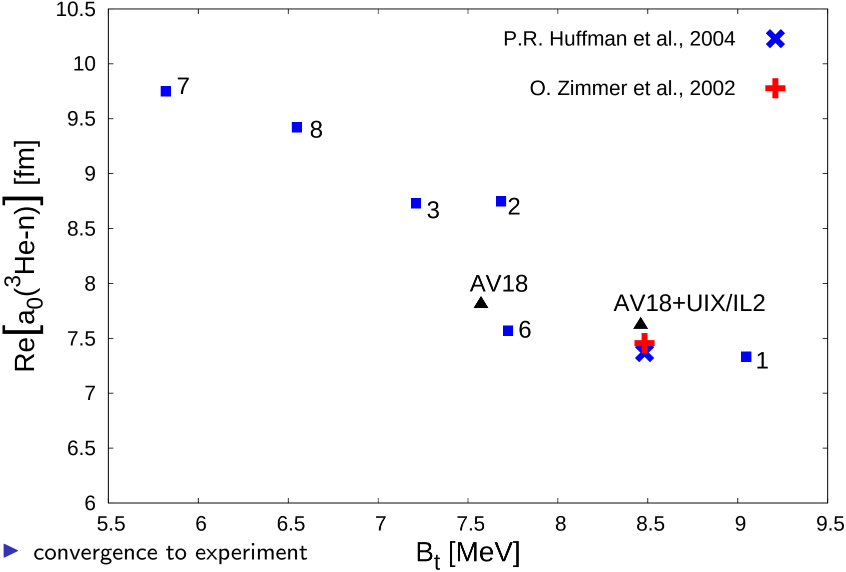


- convergence to experiment
- LO → NLO ⇒ $Q \approx \frac{1}{3}$
- NLO band width ⇒ $Q \approx \frac{1}{3}$

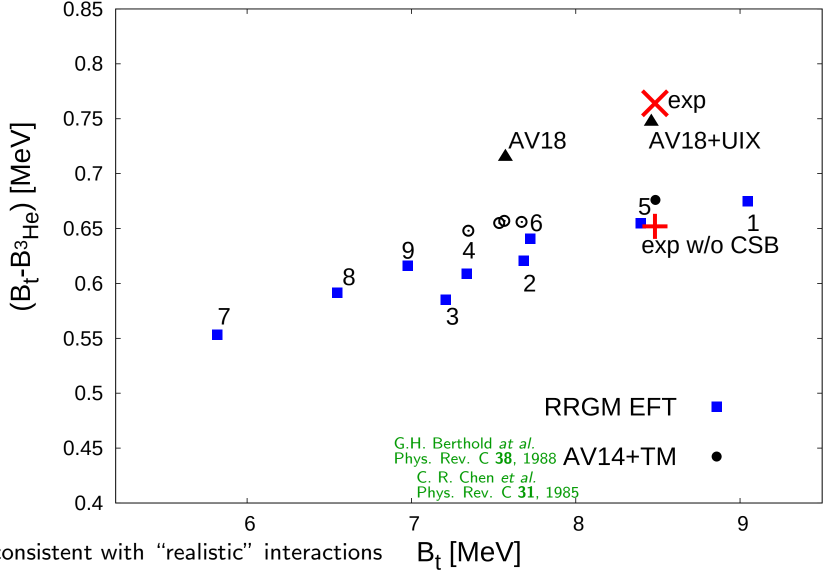
- one three-body parameter needed for a result independent of short distance physics
- EFT$_{\not\pi}$ works up to NLO



- convergence to experiment
- NLO band width ⇒ $Q \approx \frac{1}{3}$
- **no** four-body force needed at NLO
- one three-body parameter needed for a result independent of short distance physics
- $EFT_{\not\pi}$ still works up to NLO



- convergence to experiment
- NLO band width ⇒ $Q \approx \frac{1}{3}$
- **no** four-body force needed at NLO
- calculation **not accurate enough** to discriminate between conflicting data
- EFT$_{\not\pi}$ still^2 works up to NLO



- consistent with “realistic” interactions
- discrepancy to experiment due to higher order CSB interactions
- missing contribution consistent with χPT calculations — Friar, Payne, van Kolck, Phys. Rev. C **71**, 2005

Conclusions

- evidence for the **applicability** of EFT$_{(\not\pi)}$ at NLO in the ^{4}He-channel
  - **convergence** from LO to NLO with $Q \approx \frac{1}{3}$
  - new correlation for $a_0\left({}^3\text{He} - n\right)$
  - consistency with high precision models & experiment
  - **four-nucleon** contact interaction **not** necessary at NLO
- model independent calculation of CIB/CSB effect of the Coulomb interaction in ^{3}H and ^{3}He
- RRGM and EFT$_{(\not\pi)}$ *resonate* well

The "next order"

- universality in **halo nuclei**
- **electro-weak** processes in the four-nucleon system

RRGM for scattering states: $\delta\left(\langle\psi_\lambda|\hat{\mathrm{H}} - E|\psi_\lambda\rangle - \frac{1}{2}a_{\lambda\lambda}\right) = 0$

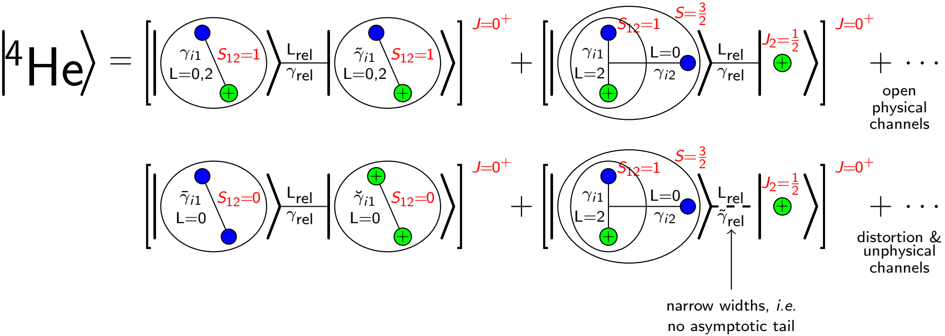
fragment wave functions

$$\psi^{J^\pi}_{\mathrm{SS},\lambda} = \mathcal{A}\sum_{j}^{n_k}\left[\frac{1}{R_j}Y_{L_j}(\hat{\vec{R}}_j)\otimes\left[\psi_j^{J_1^{\pi_1}}\otimes\psi_j^{J_2^{\pi_2}}\right]^{S_{c_j}}\right]^J\left(\delta_{\lambda j}F_{L_j}(R_j) + a_{\lambda j}\tilde{G}_{L_j}(R_j) + \underbrace{\sum_m b_{\lambda jm}R_j^{L_j+1}e^{-\omega_{jm}\vec{R}_j^2}}\right)$$

open & distortion channels

Coulomb functions

approximates state in interaction region

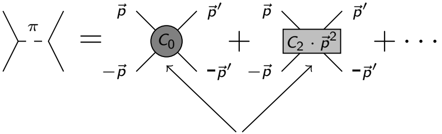


The Effective Field Theory:

- theory breaks down as momenta approach pion mass m_π
- (unnaturally) small deuteron binding energy relative to the triplet $n-p$ effective range
- non-relativistic isospin doublet of Pauli spinors: $N = \begin{pmatrix} p \\ n \end{pmatrix}$

LO: $B_d = \frac{1}{a_t^2 M} \ll \frac{m_\pi^2}{M}$

- Lorentz symmetry at small momentum



truncation based on two estimates:

coupling strength & relative contribution of graphs

naïve dimensional analysis & naturalness

- encode short distance physics
- match to underlying theory or low-energy data

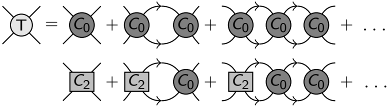
$$\mathcal{L}_{\not\pi} = N^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2M} \right) N + C_0 (N^\dagger N)(N^\dagger N) + C_2 (N^\dagger N)(N^\dagger \partial_i^2 N) + \ldots$$

$\frac{1}{a} = \mathcal{O}(p_{\text{typ}}) \;\Rightarrow$ certain operators require resummation

others can still be treated perturbatively

The two philosophies:

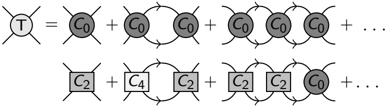
- power counting on the Lagrangean level **and** on the graph level:



LO

NLO

- iterate **effective potential** by solving Schrödinger/Lippmann-Schwinger equation:



LO

"NLO"

In **both** approaches higher order terms are expected to contribute $\mathcal{O}\left[\left(\frac{p_{\text{typ}}}{m_\pi}\right)^{N>n}\right]$ at order n