Some recent work on EFT

David Kaplan

Institute for Nuclear Theory

with:

Silas Beane **Aleksi Vuorinen**

and work in progress with M. Endres, J.-W. Lee, A. Nicholson

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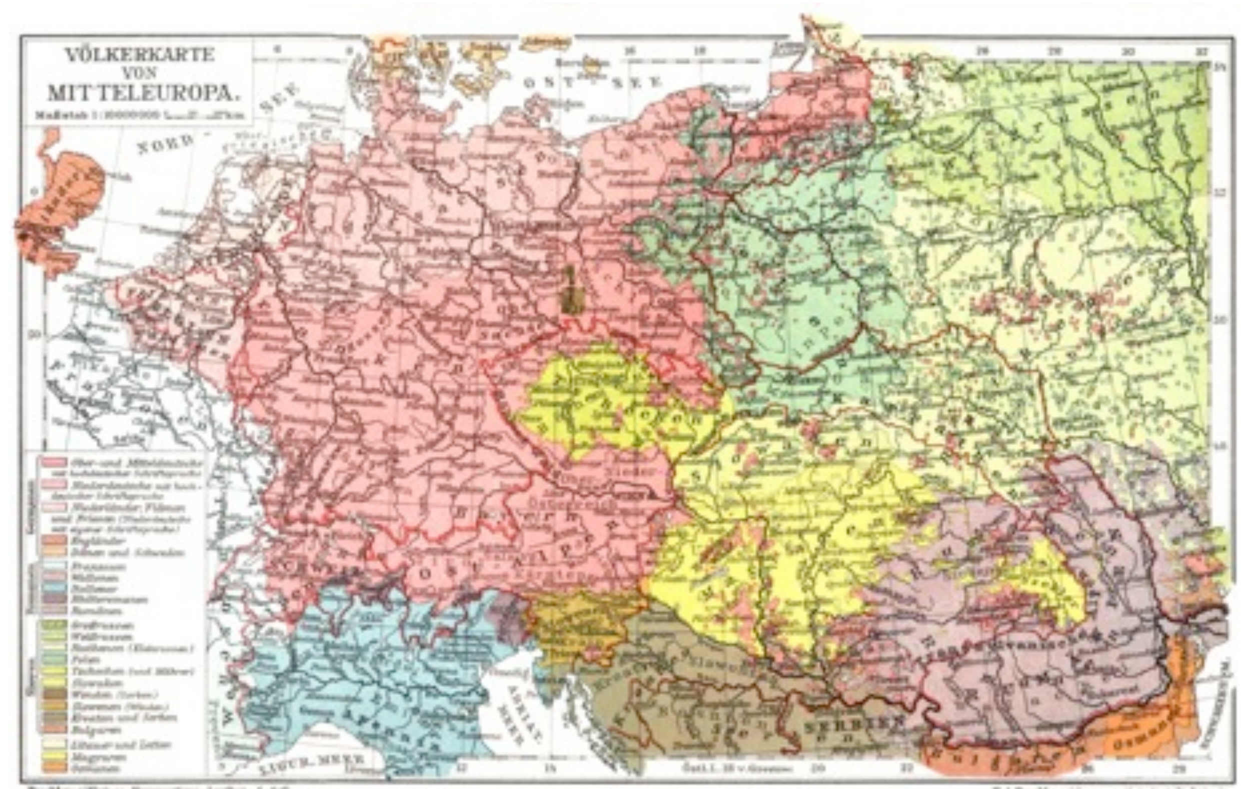

Undisclosed location in Mitteleuropa

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S. Weinberg, 1990:

Effective field theory for nuclear physics?

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To be "effective":

- Hierarchy of length scales
- A power counting scheme: order the expansion, estimate errors

Quintessential examples of EFT:

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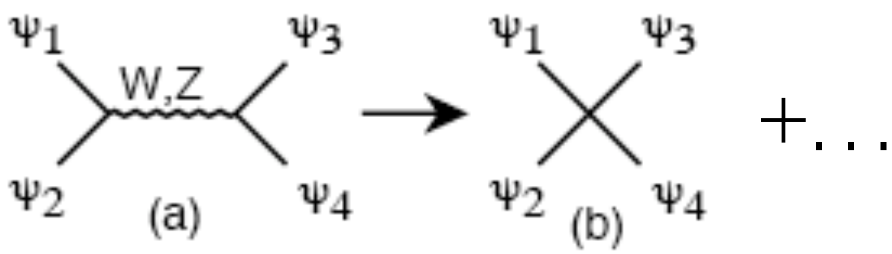
Fermi's theory of weak interactions



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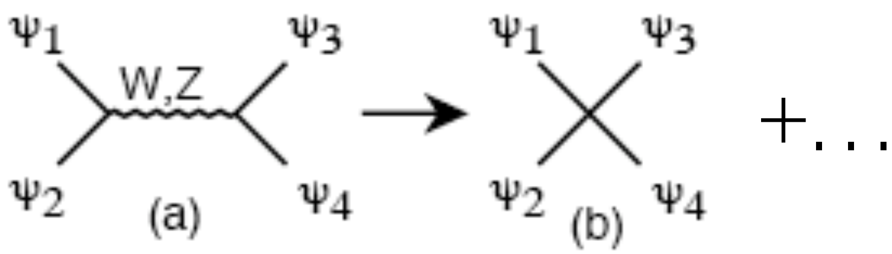
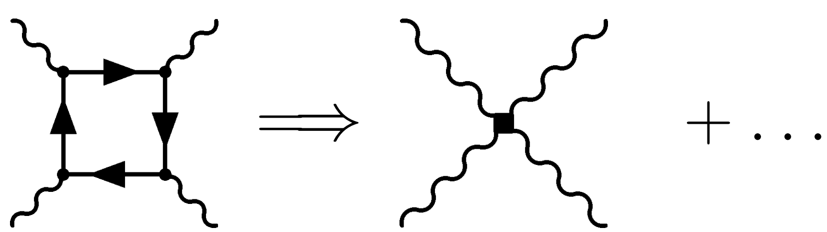


Euler-Heisenberg light-by-light scattering



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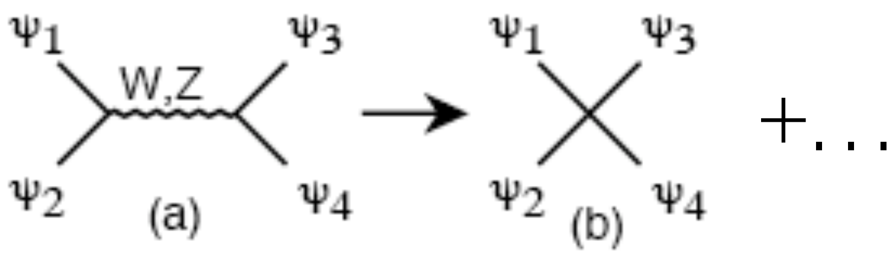

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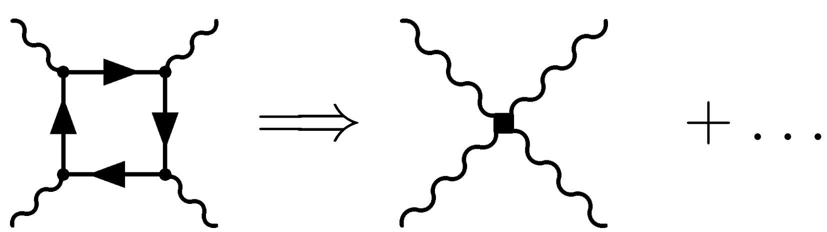
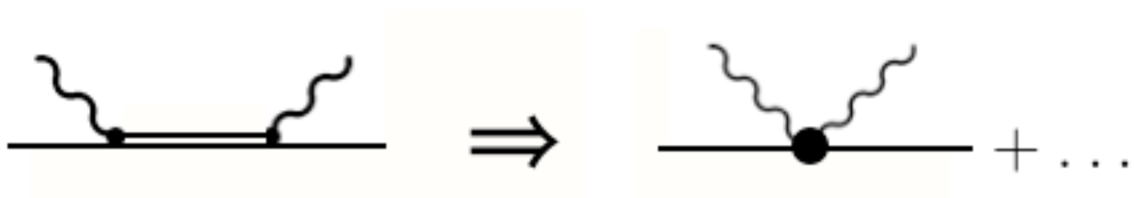


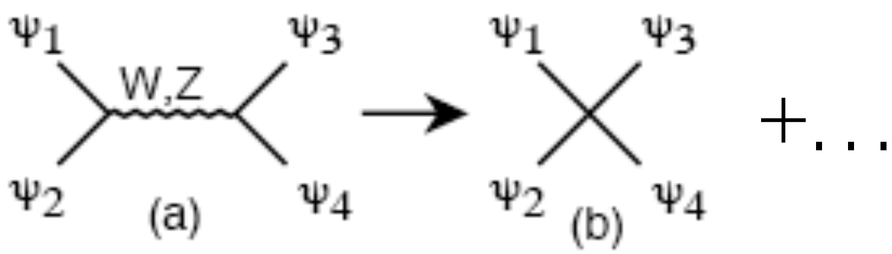
Rayleigh light-by atom scattering



Quintessential examples of EFT:

All *weak* interactions at low energy

Fermi's theory of weak interactions



$$\sigma_{\nu\nu\to\nu\nu} \propto \frac{E_{\nu\nu}^2}{M_Z^4}$$

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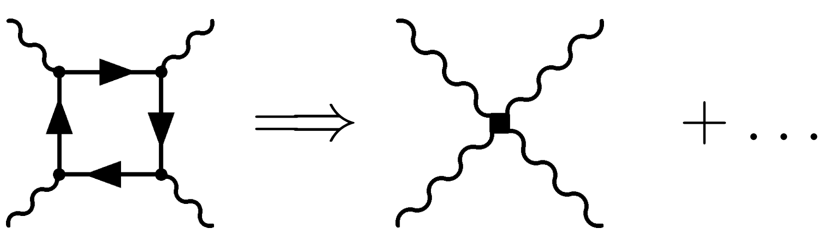
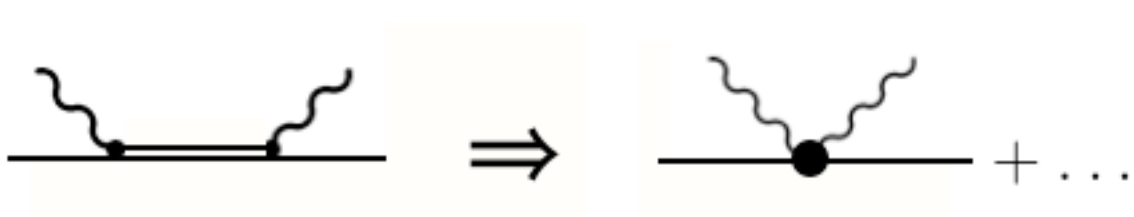


$$\sigma_{\gamma\gamma\to\gamma\gamma} \propto \frac{E_{\gamma\gamma}^6}{m_e^8}$$

Rayleigh light-by atom scattering



$$\sigma_{\gamma A\to\gamma A} \propto \frac{E_{\gamma}^4}{(r_A)^{-6}}$$

Above examples:

$\sigma \sim (\text{Energy})^{p}/(\text{mass scale})^{p+2}$

$p = 2 \times \dim[\text{op}]-10$

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For NN scattering, expect mass scales to be set by:

$m_\pi = 140$ MeV, $f_\pi=93$ MeV, $m_{\rho,\omega} \sim 770$ MeV ...

What do we see?

NN scattering lengths *much longer* than pion Compton wavelength:

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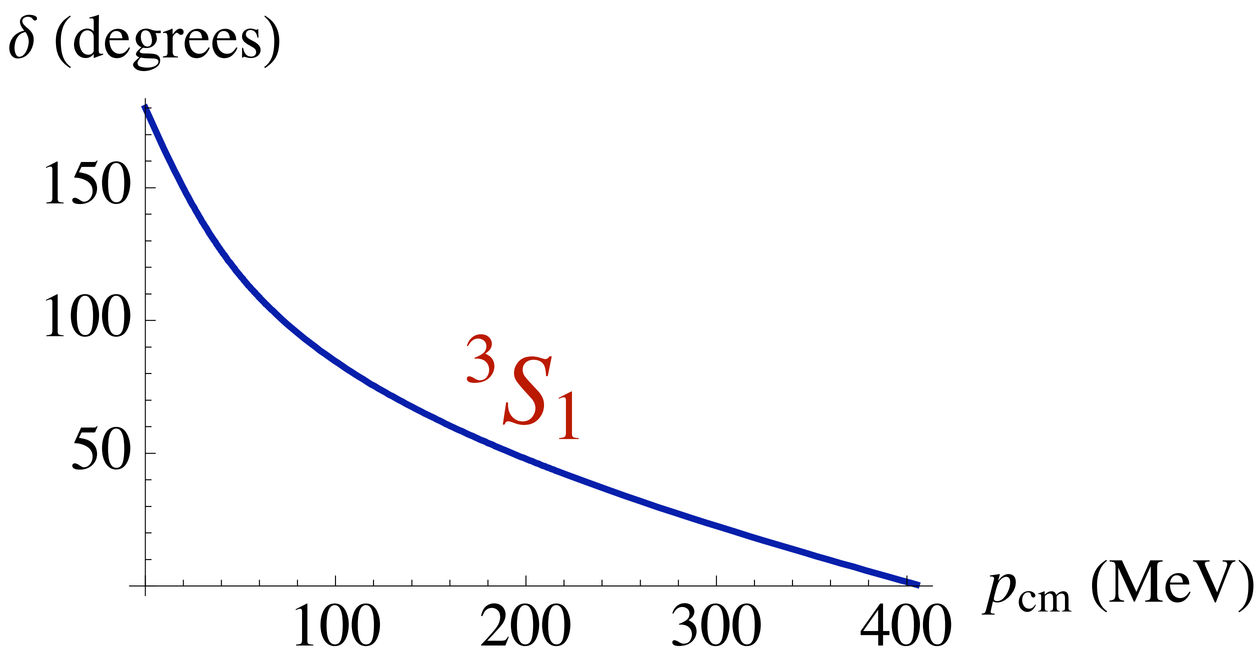

$$a_{^3S_1} = \frac{1}{45\ \mathrm{MeV}}$$

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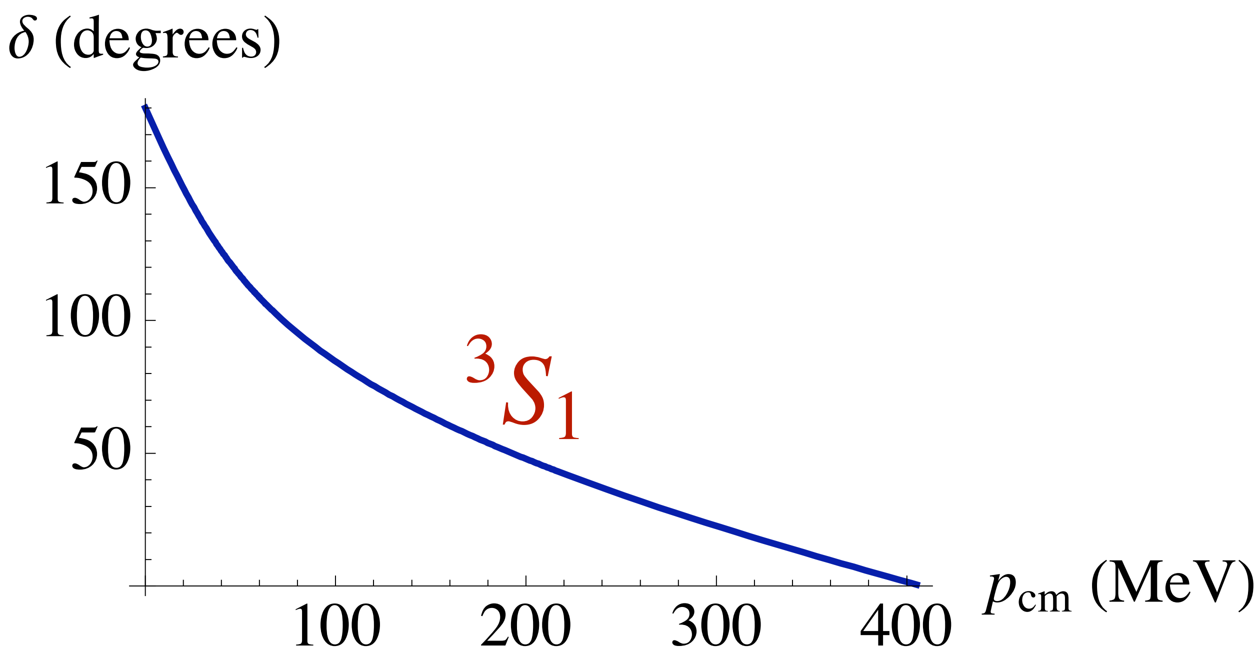

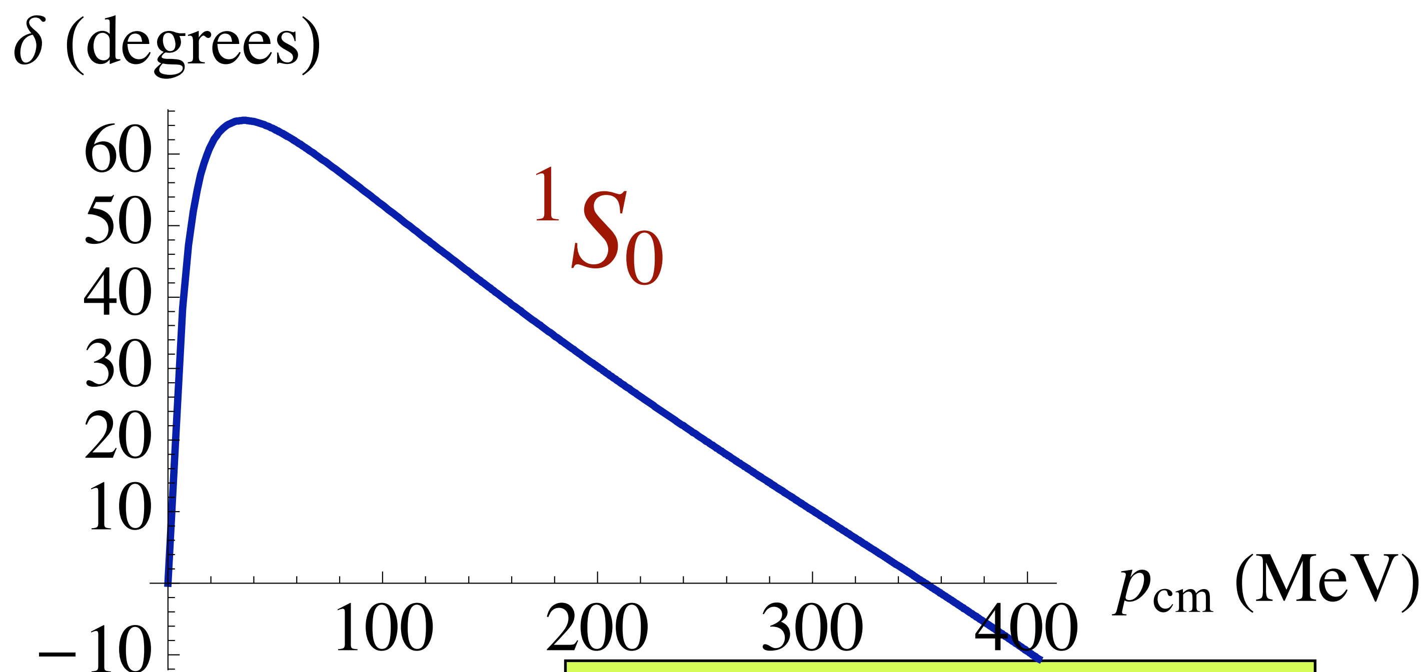

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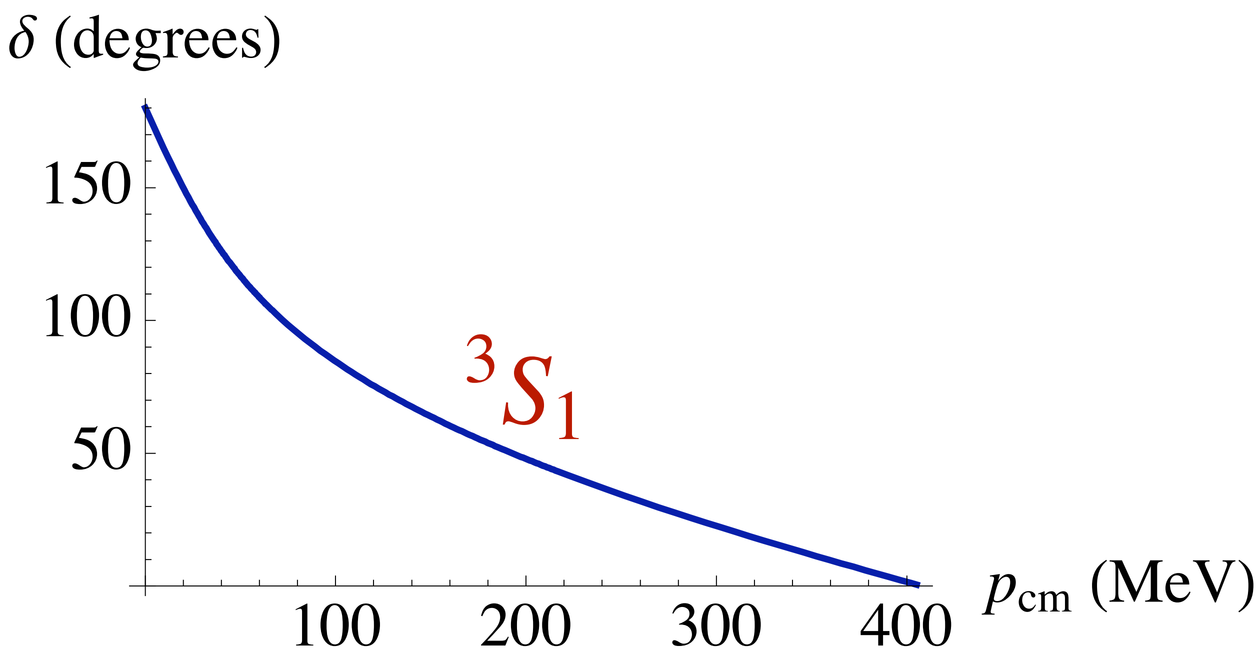

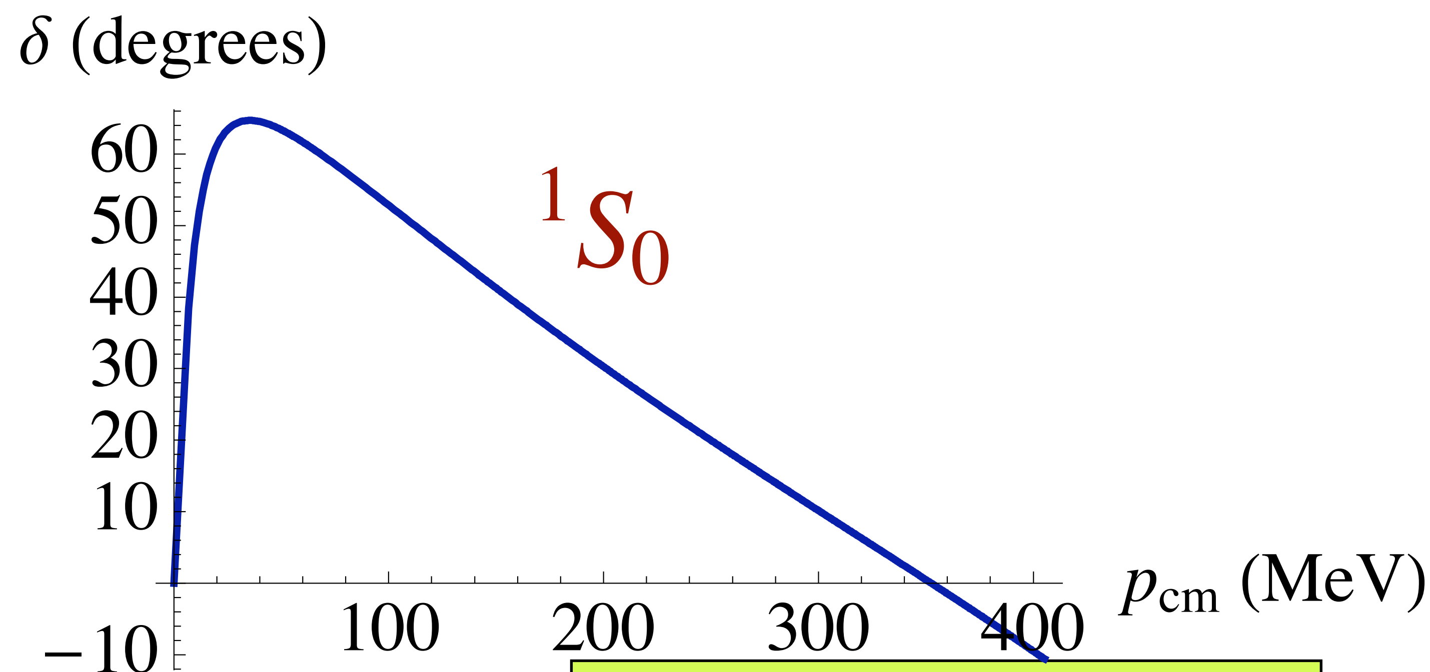

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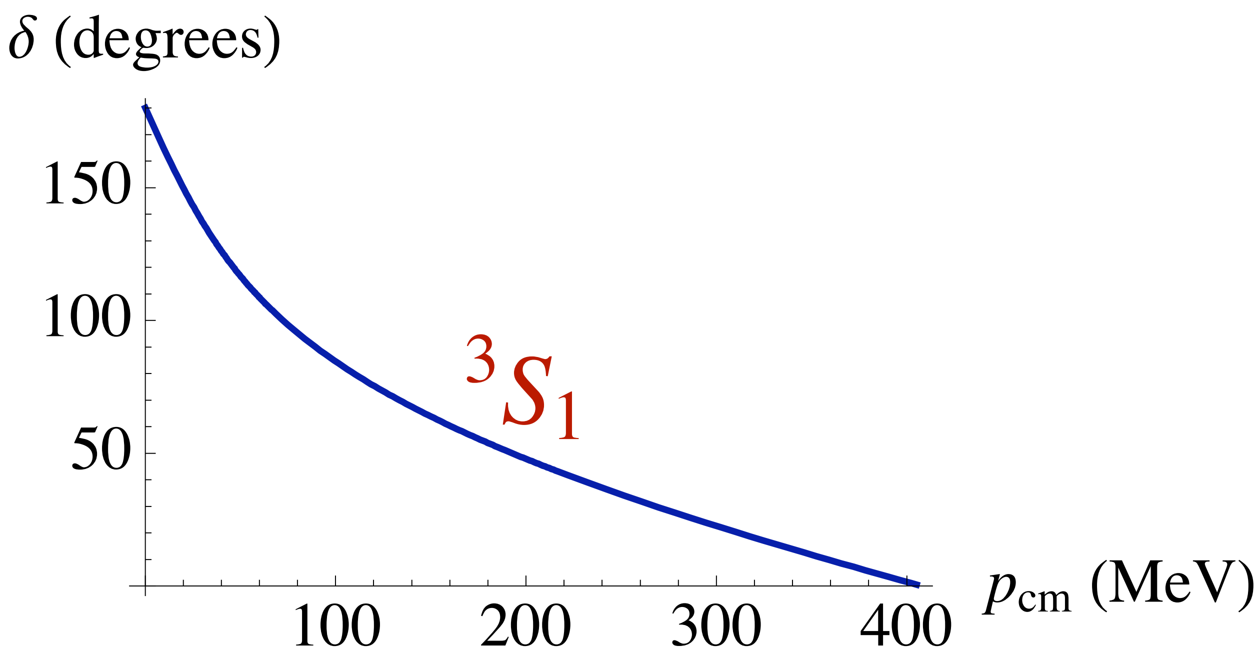

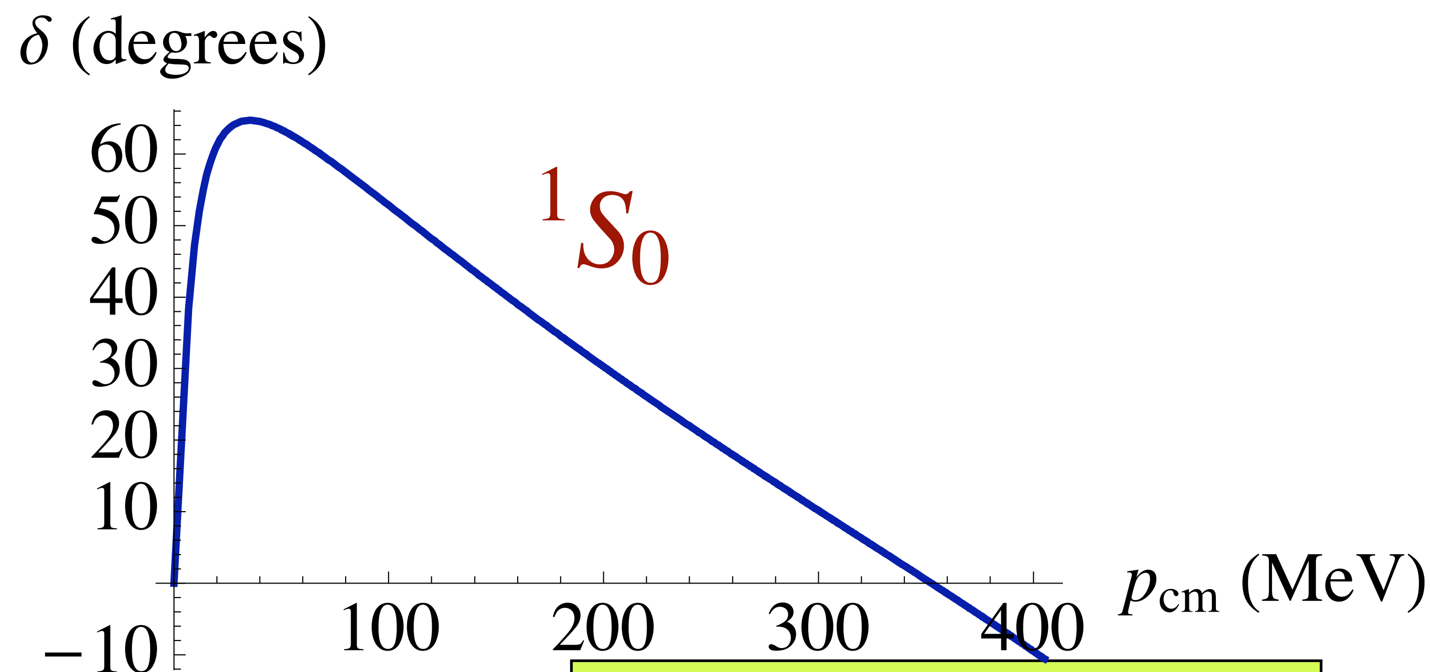

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Not like νν scattering in Fermi theory!

Weinberg: *don't* perform EFT expansion of the scattering amplitude.

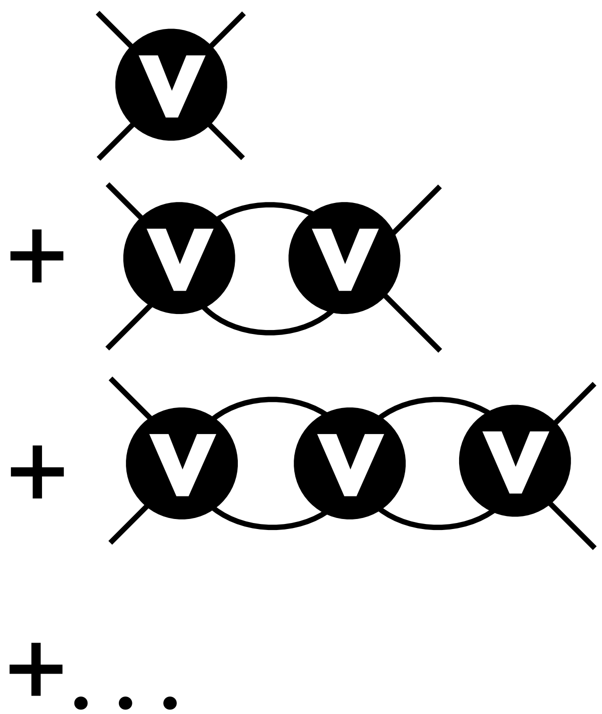
Instead:

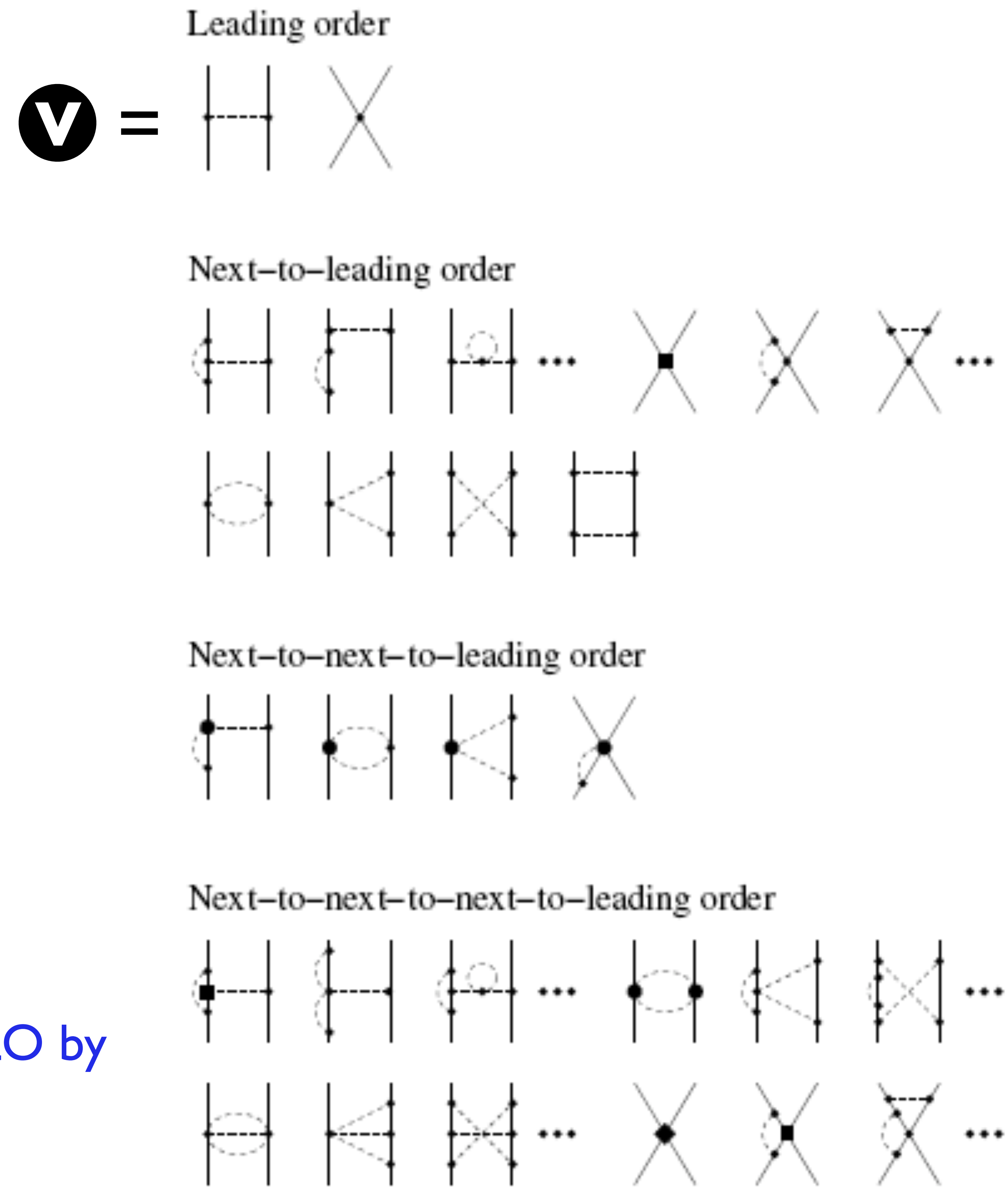
(i) expand NN potential in EFT

(ii) solve Lippmann-Schwinger eq. exactly $\Leftarrow$ *nonperturbative*

Weinberg method:

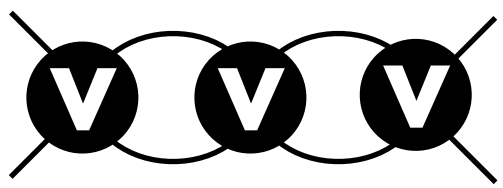
- Expand NN potential in chiral perturbation theory
- Sum up:





Procedure implemented to NNNLO by Epelbaum et al.

Can fit the data well...but there's a problem:



Expand V to order n in the EFT expansion

➥ iterating V requires counterterms at higher order in expansion

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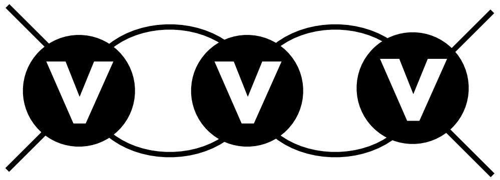


Expand V to order n in the EFT expansion

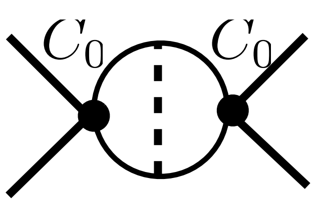
➥ iterating V requires counterterms at higher order in expansion

- *cannot remove regulator*
- *answer is sensitive to short-distance physics*
- *end result only modest improvement over conventional potential models?*

1. Inconsistent power counting: can't renormalize & remove cutoff

Singular potential — Higher order singularities

Simple example:



LO diagram has log divergence for NLO operator:

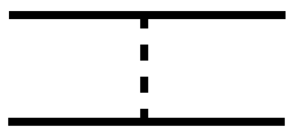
quark mass matrix

$$D_2(N^\dagger M_q N)(N^\dagger N)$$

Without counterterm, can't sensibly compute quark mass dependence of deuteron BE, for example.

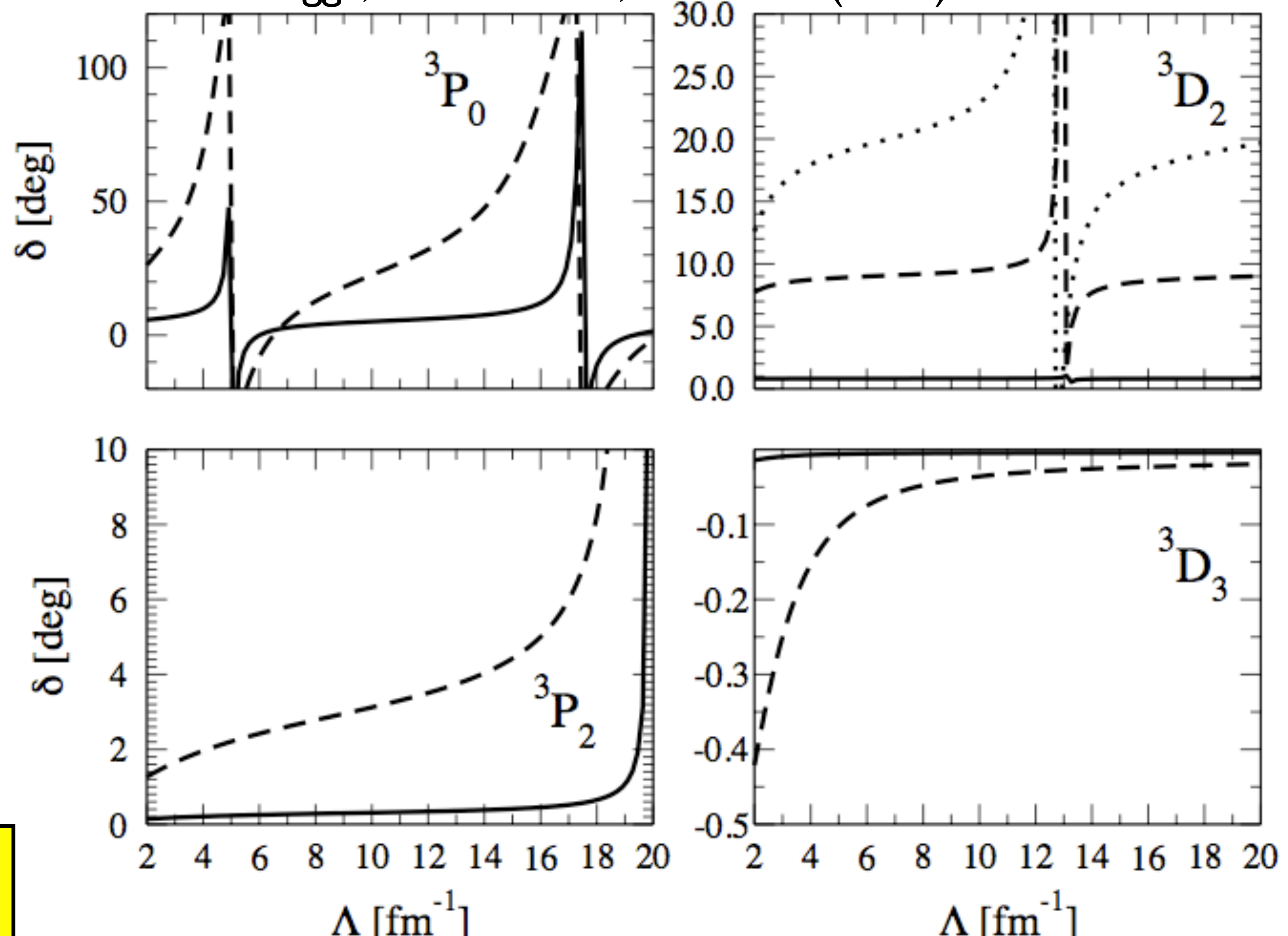
Weinberg approach, with pions

2. Singular tensor potential



$$V(r) \sim -\frac{1}{r^3}$$ in some spin-triplet channels

Nogga, Timmermans, van Kolck (2005)



- Divergences at LO in every attractive tensor channel
- Counterterms only at higher order
- Requires a cutoff that cannot be removed
- RG analysis in Birse (2006, 2007)

——— T_{lab}=10 MeV

- - - - - - - T_{lab}=50 MeV

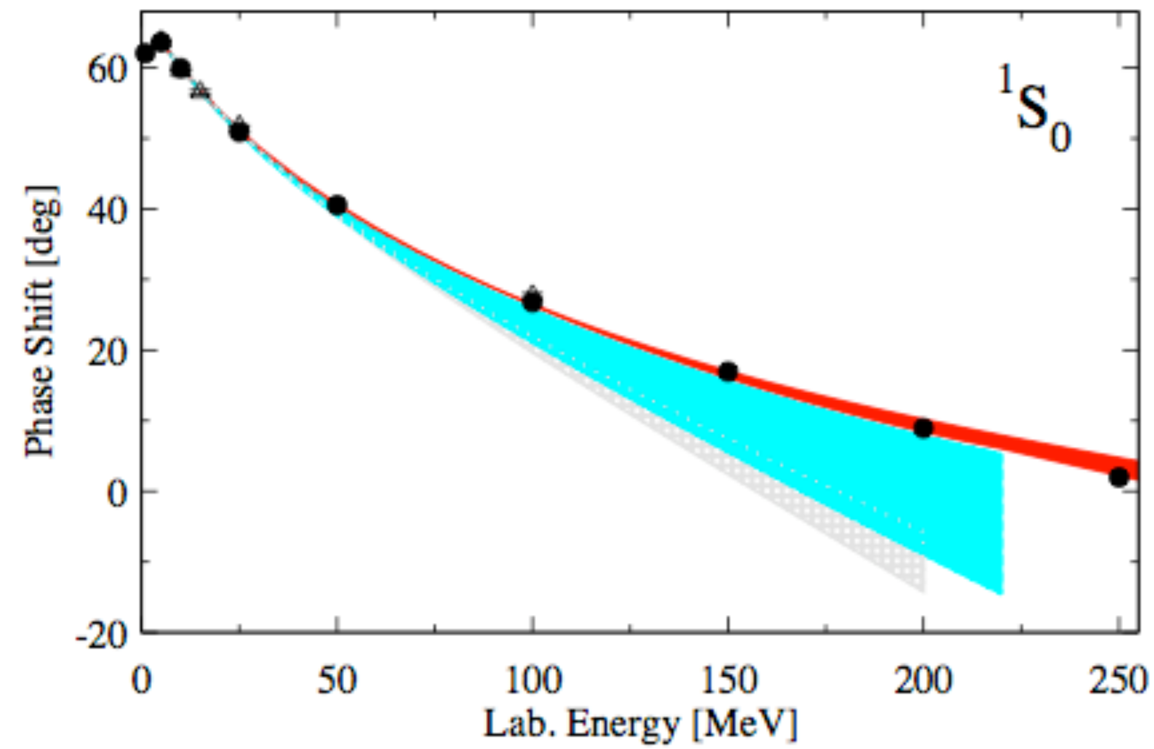
$^1S_0$ NN phase shift, Weinberg approach

Epelbaum, Gloeckle, Meissner 2005

NLO

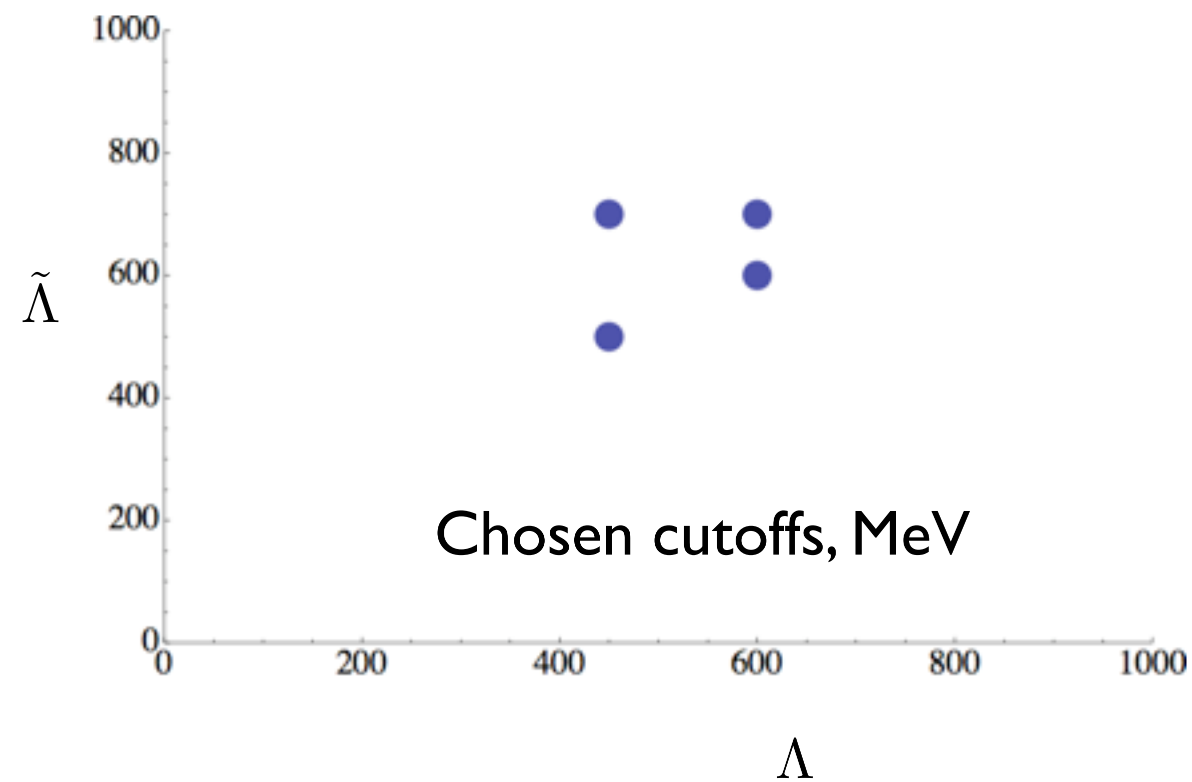
NNLO

NNNLO



At NNNLO:

- Five fit parameters;
- Two different momentum cutoffs



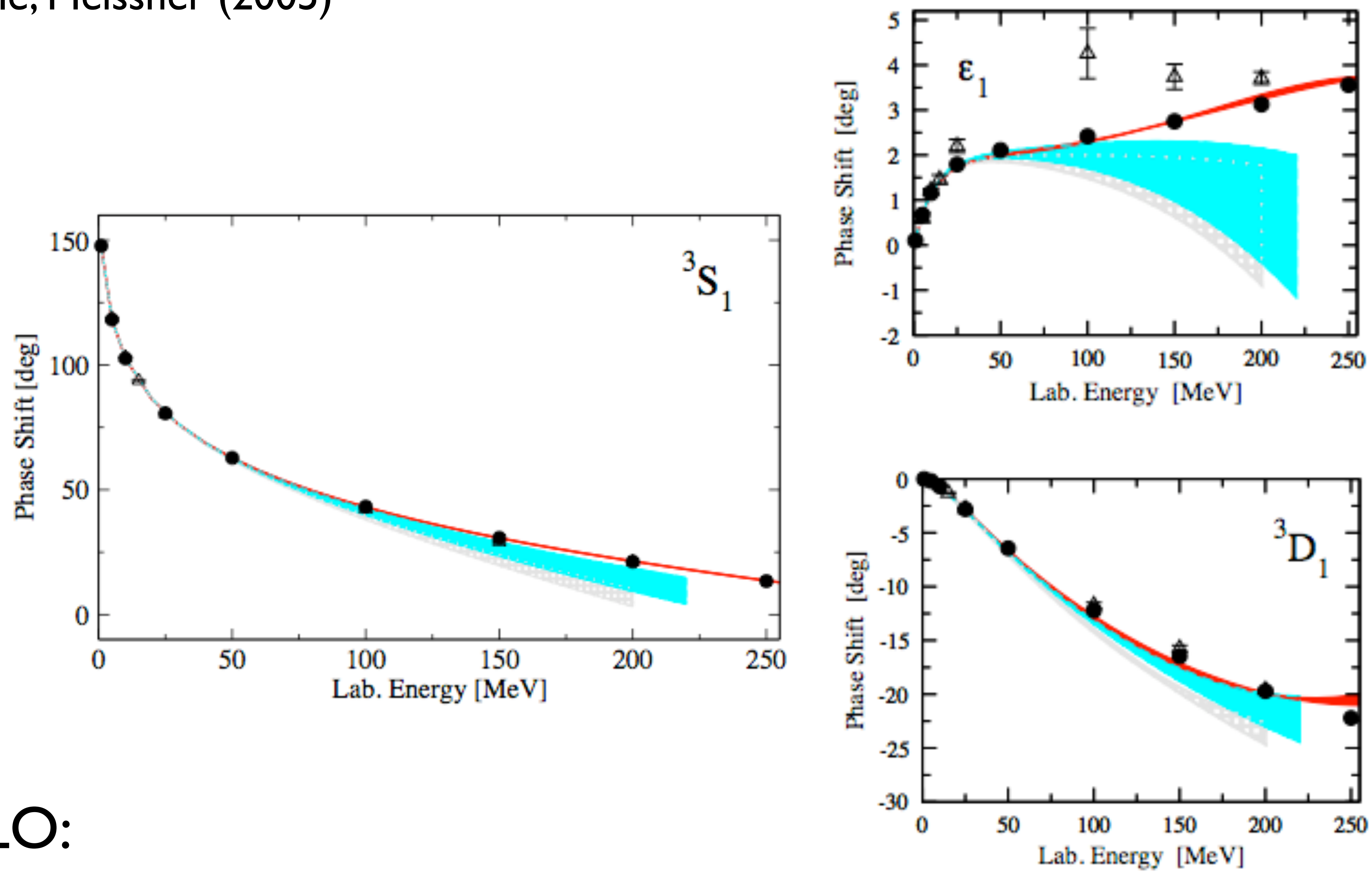
$^3S_1$-$^3D_1$ NN phase shifts, Weinberg approach

Epelbaum, Gloeckle, Meissner (2005)

NLO

NNLO

NNNLO



At NNNLO:

- Eight+ fit parameters;
- Two momentum cutoffs

A recurring theme of this program:

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We are using the Weinberg EFT

You can't torture nuclear physics like that!

A recurring theme of this program:

Not torture: enhanced investigation

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Unjustifiable! Ineffective!

A recurring theme of this program:

It yields reliable intelligence

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A recurring theme of this program:

It yields reliable intelligence

Never!

A recurring theme of this program:

ENDS JUSTIFY
THE MEANS

Never!

Always the same outcome:

Always the same outcome:

KSW approach:

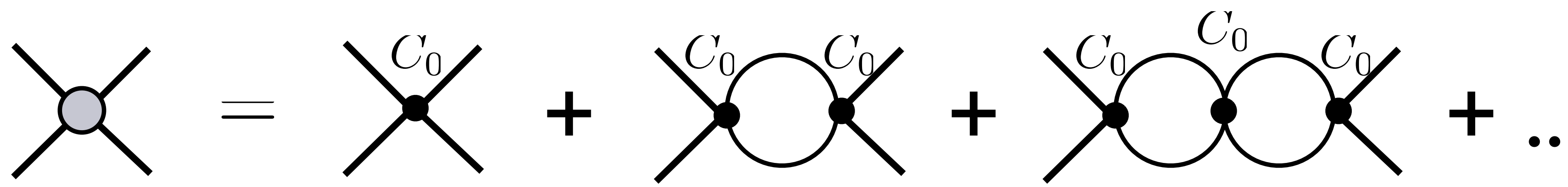
D.K., Savage, Wise (1998)

$$\mathcal{L} = -\,C_0\,(N^\dagger N)^2 - \;C_2\,(N^\dagger \nabla^2 N)(N^\dagger N) + h.c. + \ldots$$

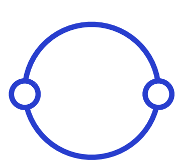
C_0 interaction corresponds to a $\delta^3(r)$ potential: singular! C_0 runs...

Consider summing C_0 interaction to all orders:

$i\mathcal{A}(p) =$



The bubble:



$$= \left(\frac{\mu}{2}\right)^{4-D} \int \frac{d^{D-1}}{(2\pi)^{D-1}} \frac{1}{E - \frac{|\mathbf{q}|^2}{M} + i\epsilon} \xrightarrow{PDS} -\frac{M}{4\pi}(\mu + ip)$$

Find:

$$\mathcal{A}(p) = -\frac{4\pi}{M}\frac{1}{\underline{p\cot\delta(p)} + ip} \simeq -\frac{4\pi}{M}\frac{1}{\underline{\frac{4\pi}{MC_0} + \mu} + ip}$$

Summing C_0 term to all orders = first term in effective range expansion:

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RG scale

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RG scale

scattering length

$$C_0(\mu) = -\frac{4\pi}{M}\frac{1}{\mu + \frac{1}{a}}$$

Define a dimensionless coupling: $$\widehat{C}_0 \equiv -\frac{M\mu}{4\pi}C_0 = \frac{\mu}{\mu + \frac{1}{a}}$$

The beta function is then given by: $$\widehat{\beta} = \mu\frac{\partial \widehat{C}_0}{\partial \mu} = -\widehat{C}_0\left(\widehat{C}_0 - 1\right)$$

KSW approach, pionless EFT

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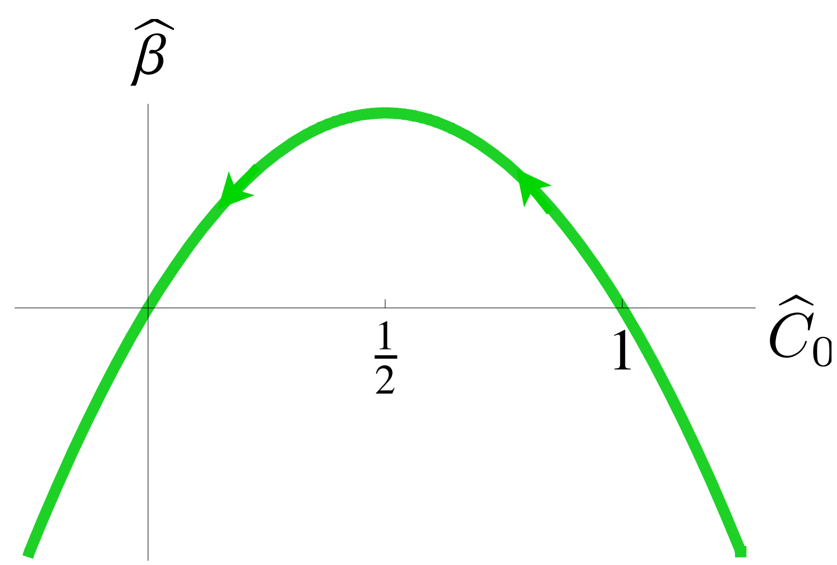

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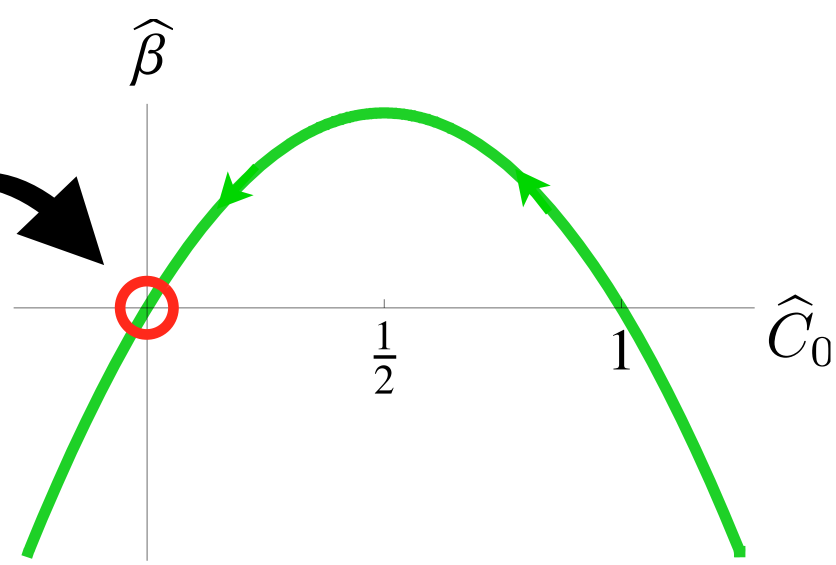

Trivial IR fixed point
...like Fermi theory!
...**not** like nuclear physics!

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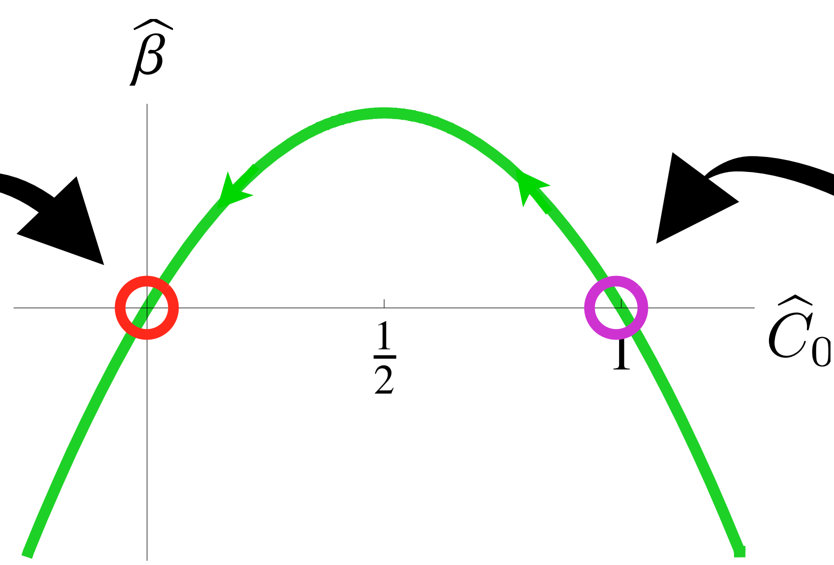
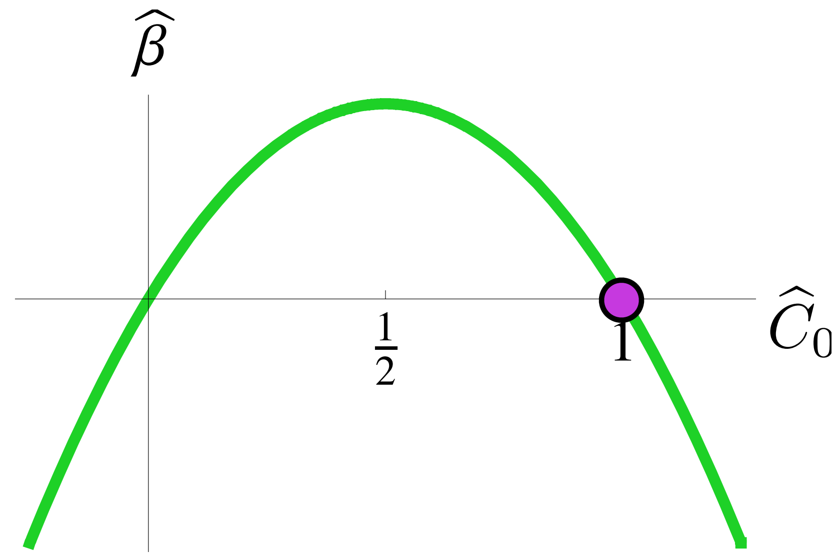


Nontrivial UV fixed point
...very much like nuclear physics!
Infinite a!

KSW expansion: expand about the nontrivial fixed point
= a conformal theory with infinite scattering length:



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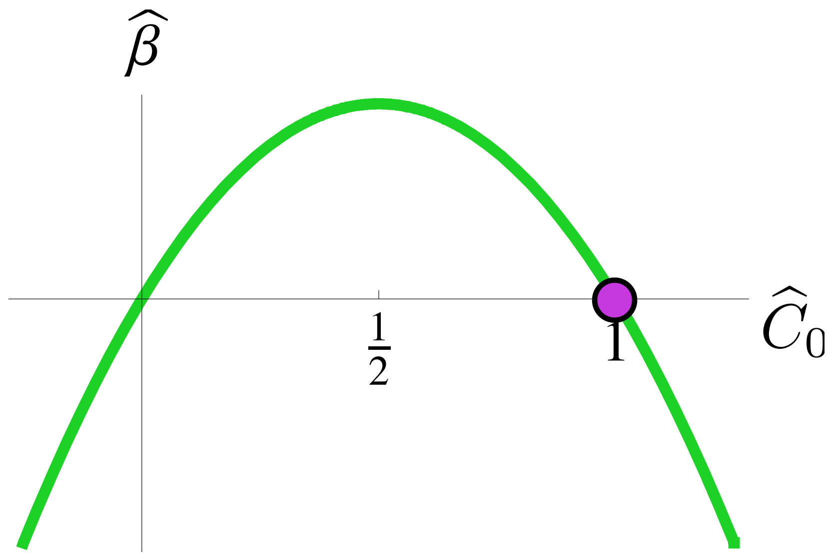

Simple power counting in pionless theory: expand amplitude in powers of "*Q*":

$$
\begin{array}{rcl}
\mu,\ p,\ \frac{1}{a} & \sim & Q \\
M & \sim & 1 \\
C_0 & \sim & Q^{-1} \\
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\ldots & &
\end{array}
$$

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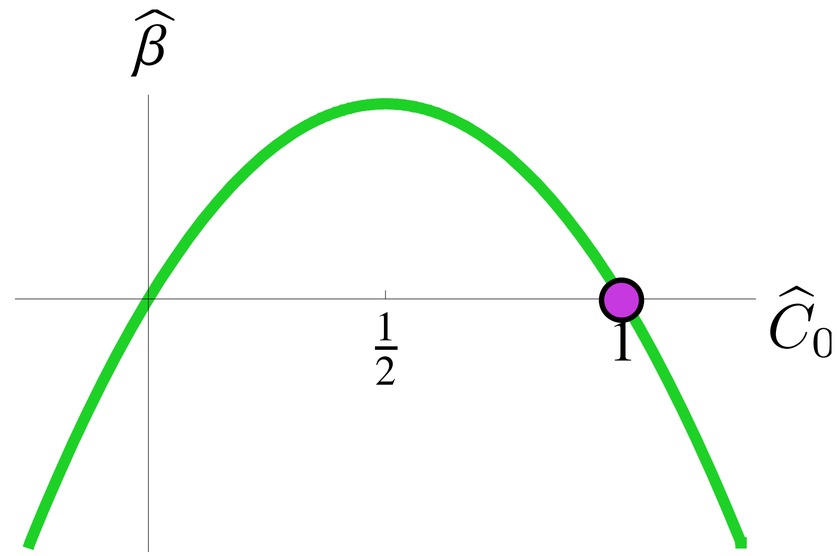

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Distance from fixed point

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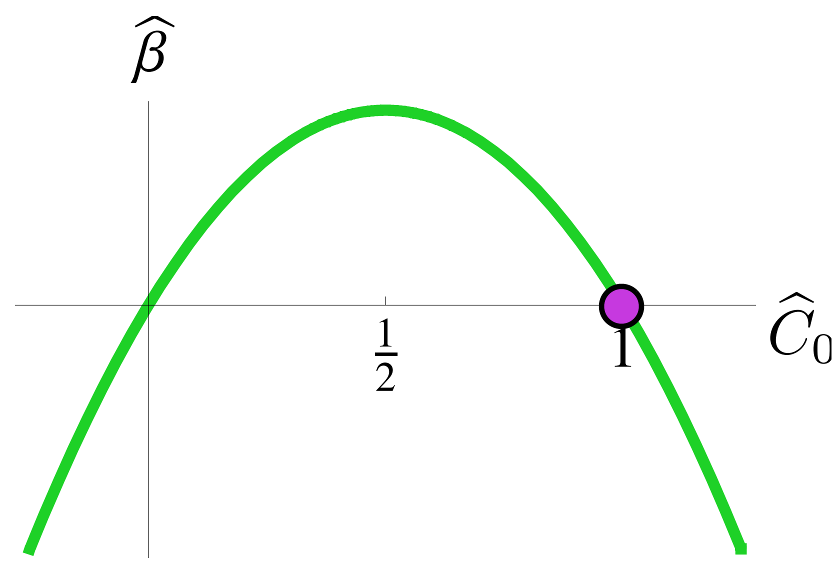

Simple power counting in pionless theory: expand amplitude in powers of "Q":

$$\boxed{C_0(\mu) = -\frac{4\pi}{M}\frac{1}{\mu + \frac{1}{a}}}$$

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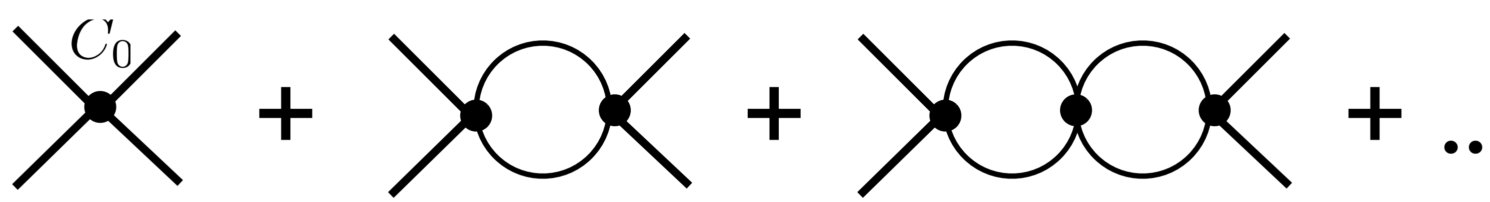
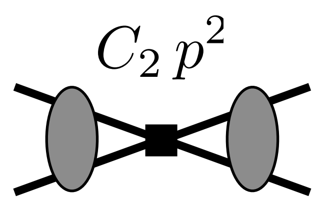
Distance from fixed point

KSW expansion of the *amplitude* $\Rightarrow$ renormalizable:

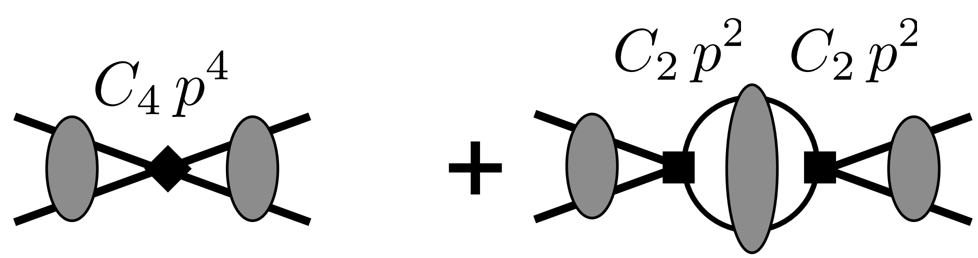
Q^{-1} : $\quad i\mathcal{A}_{-1} =$

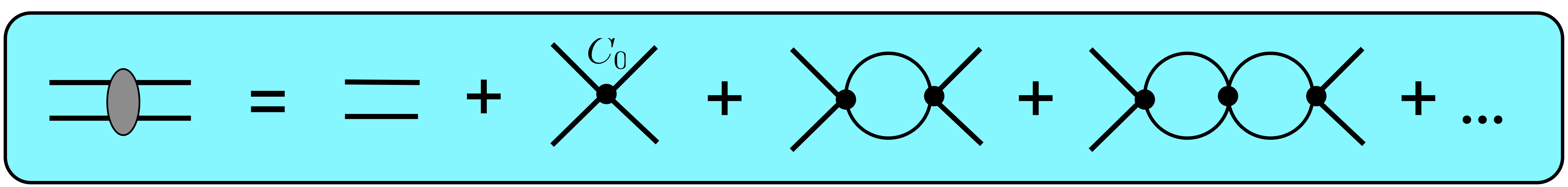


Q^{0} : $\quad i\mathcal{A}_{0} =$



Q^{1} : $\quad i\mathcal{A}_{1} =$



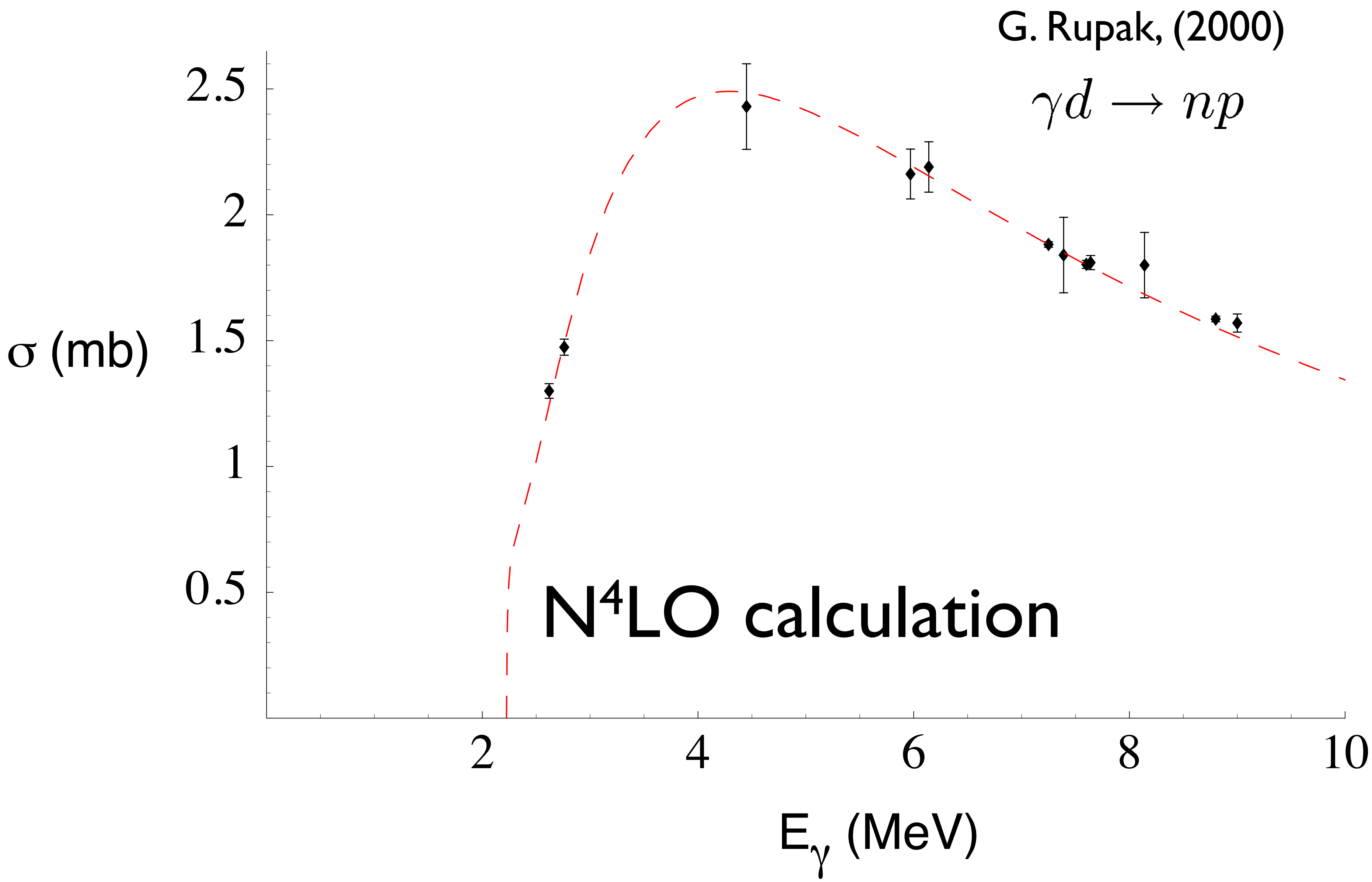


Applications (few-body, low momentum):

- Radiative breakup of the deuteron (Big Bang)
- Neutrino breakup of the deuteron (SNO)
- N d scattering
- Solar fusion processes
- ...

CAN ATTAIN 1% ACCURACY

KSW approach, pionless EFT



- Key: local operators for 2-body EM current

Weak current processes:

A new parameter appears in deuteron physics: 2-body axial charge

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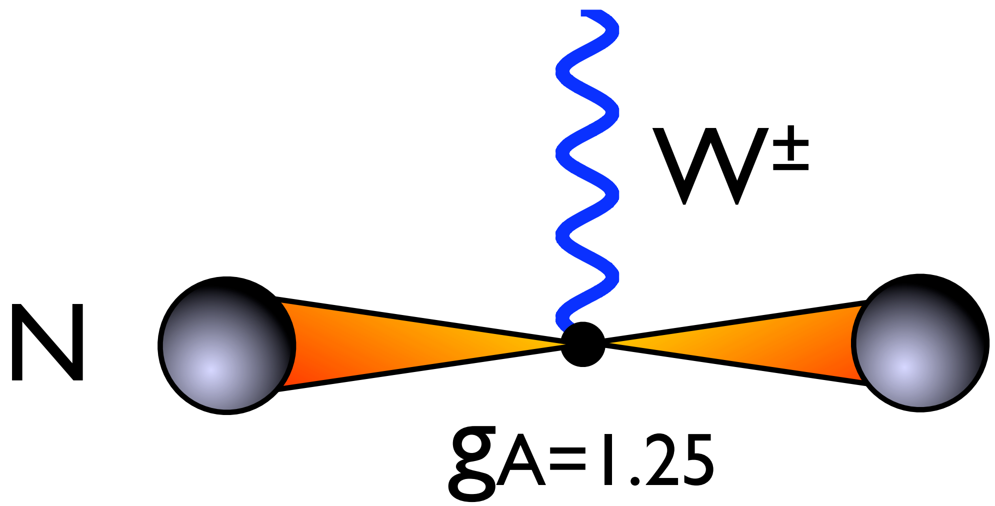

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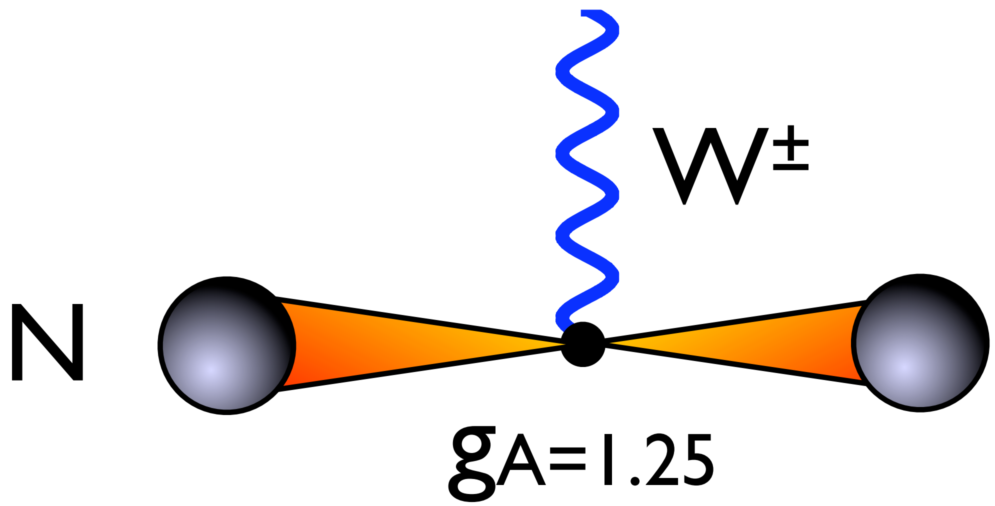

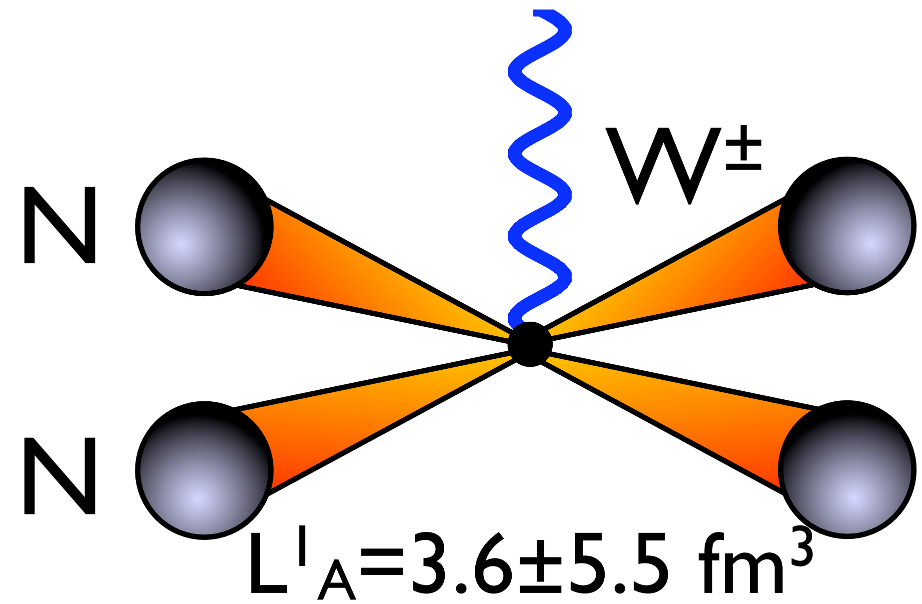

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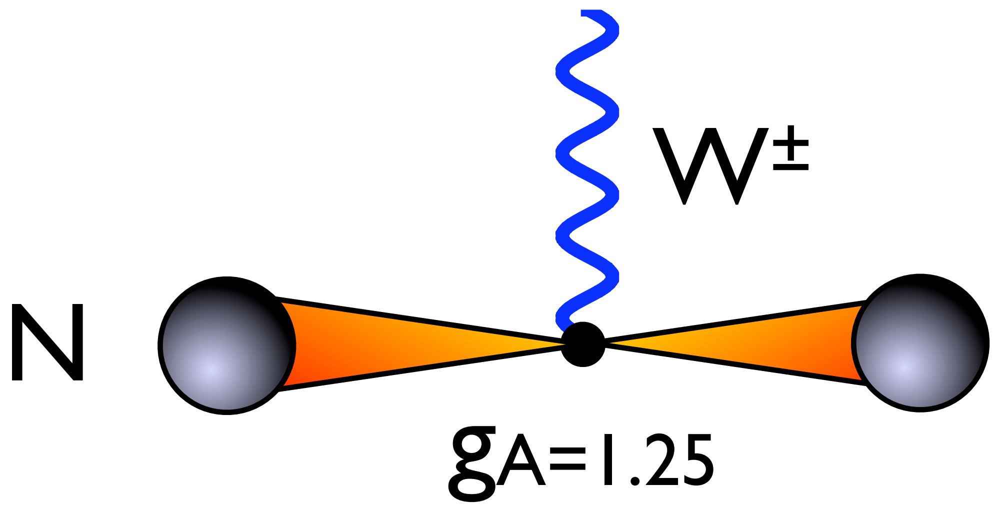

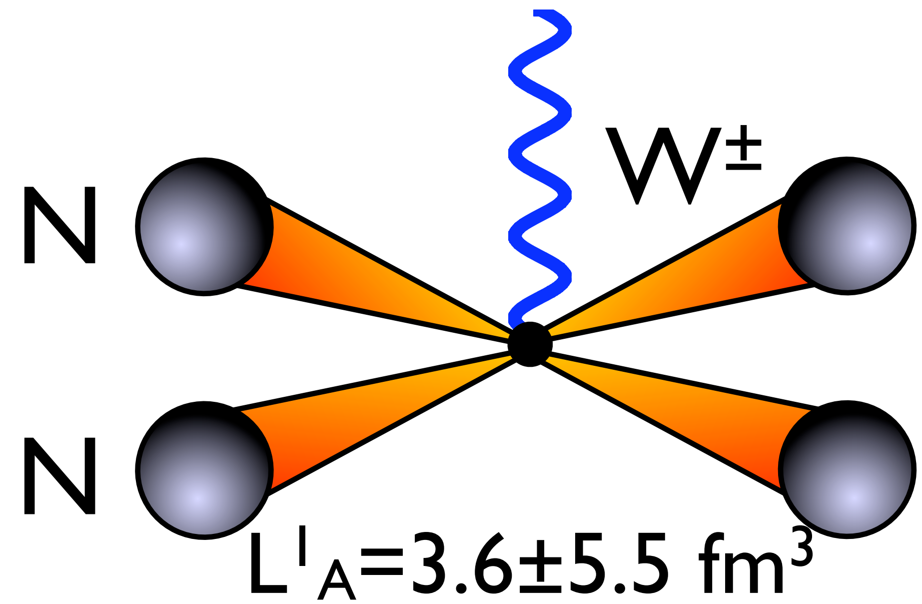

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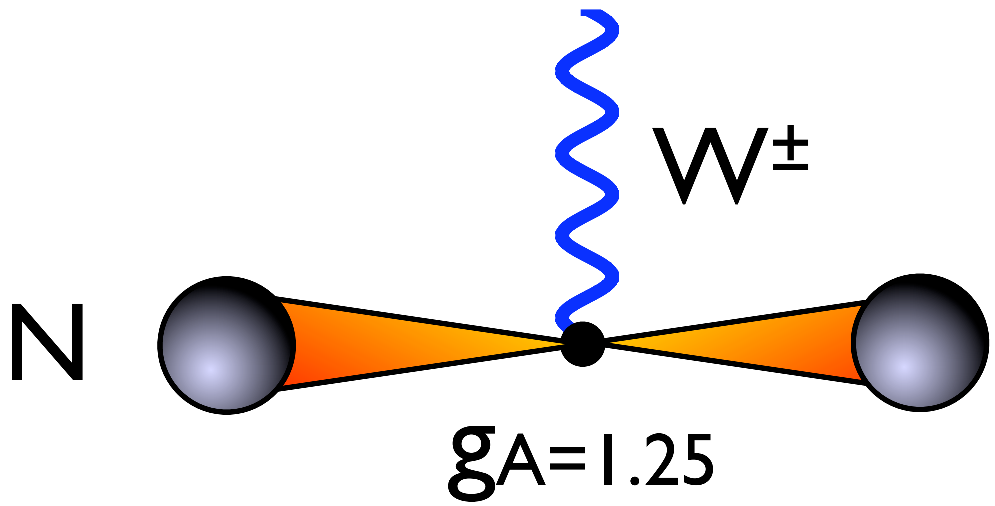

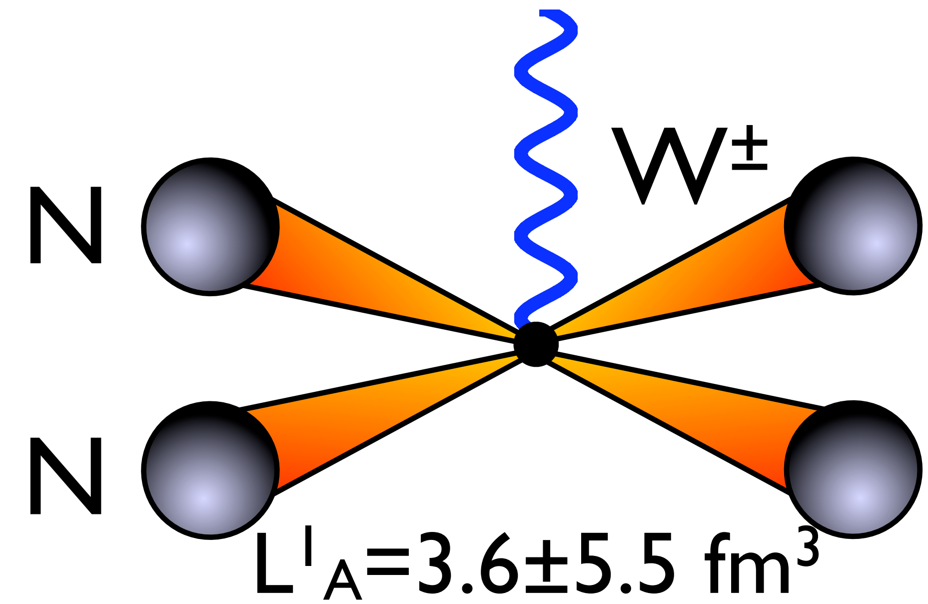

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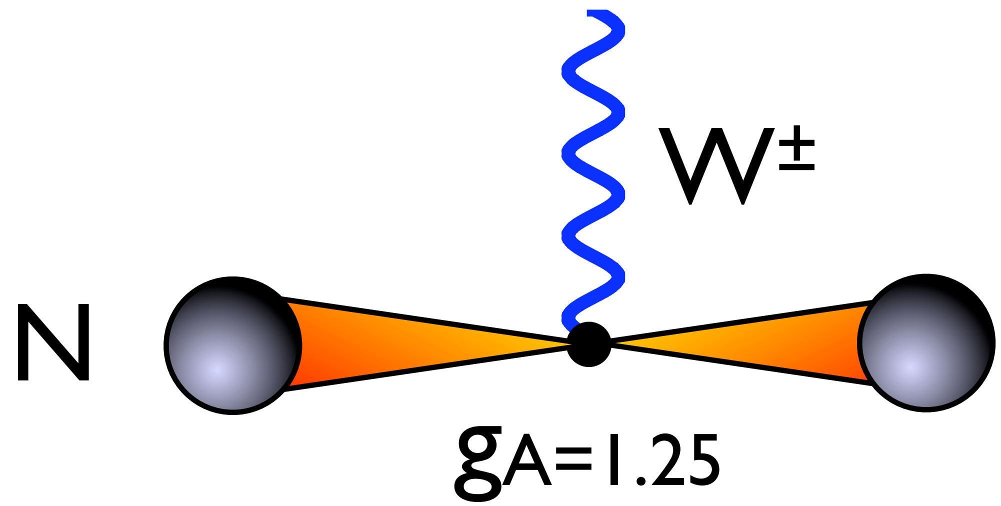

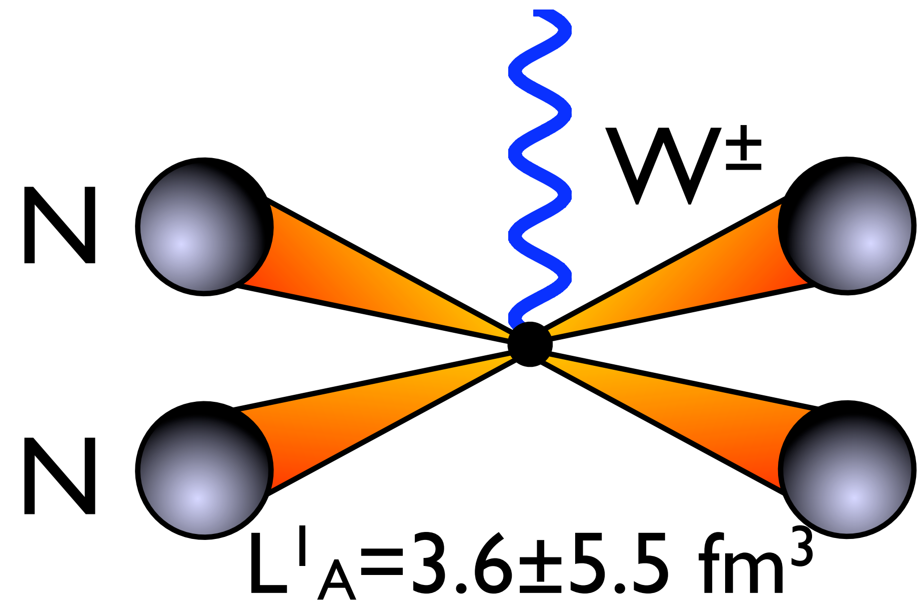

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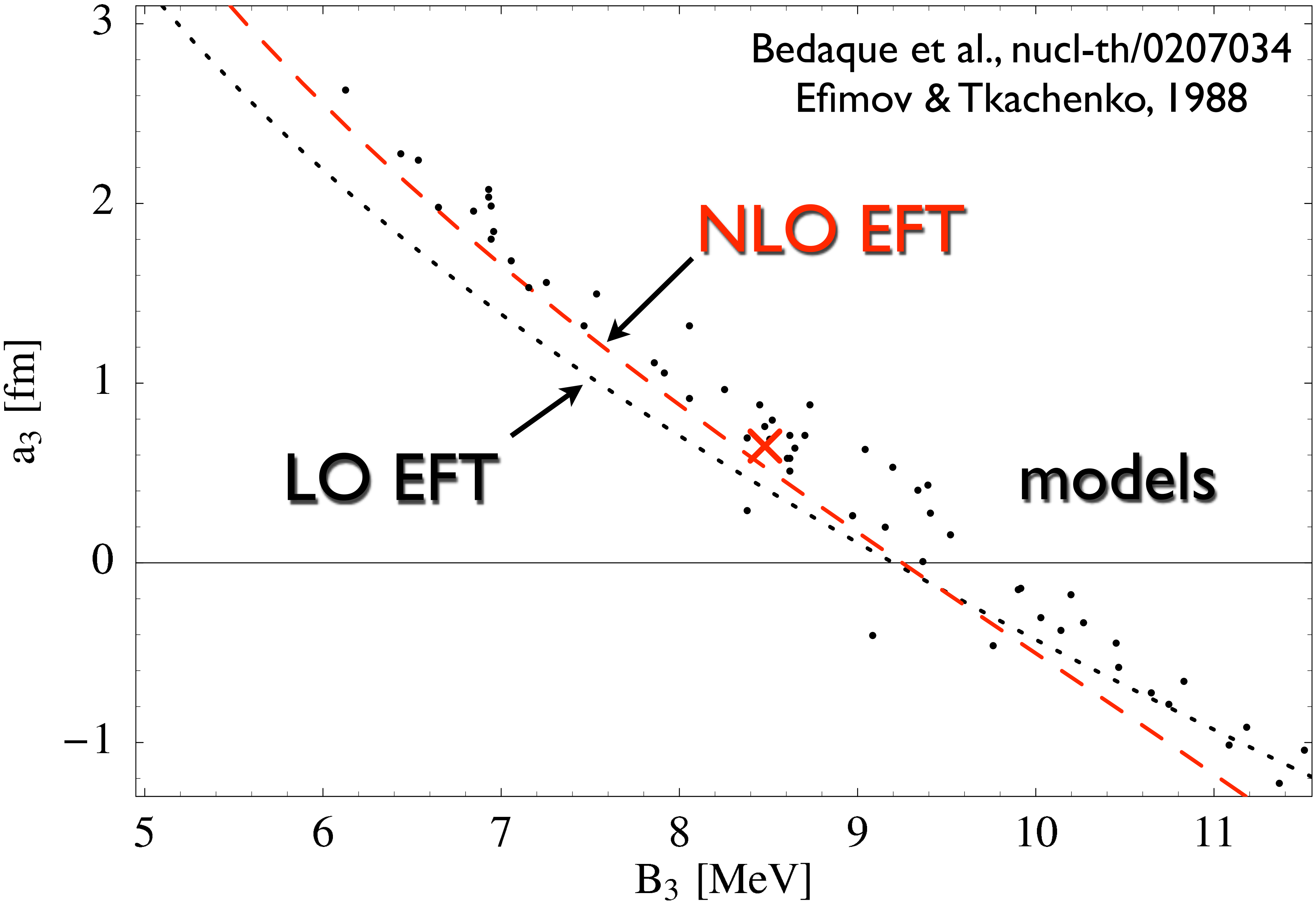
- pp fusion in the sun, N^{5}LO: measurement of $L^1{}_A$ gives fusion rate to ~1% (Butler, Chen 2001)

3-body physics

Phillips line: ${}^3H$ binding energy - Nd scattering length correlation



Phillips line: 3-body interaction

4-body physics

4-body physics

Tjon line

Correlation between 3- and 4-body binding energies

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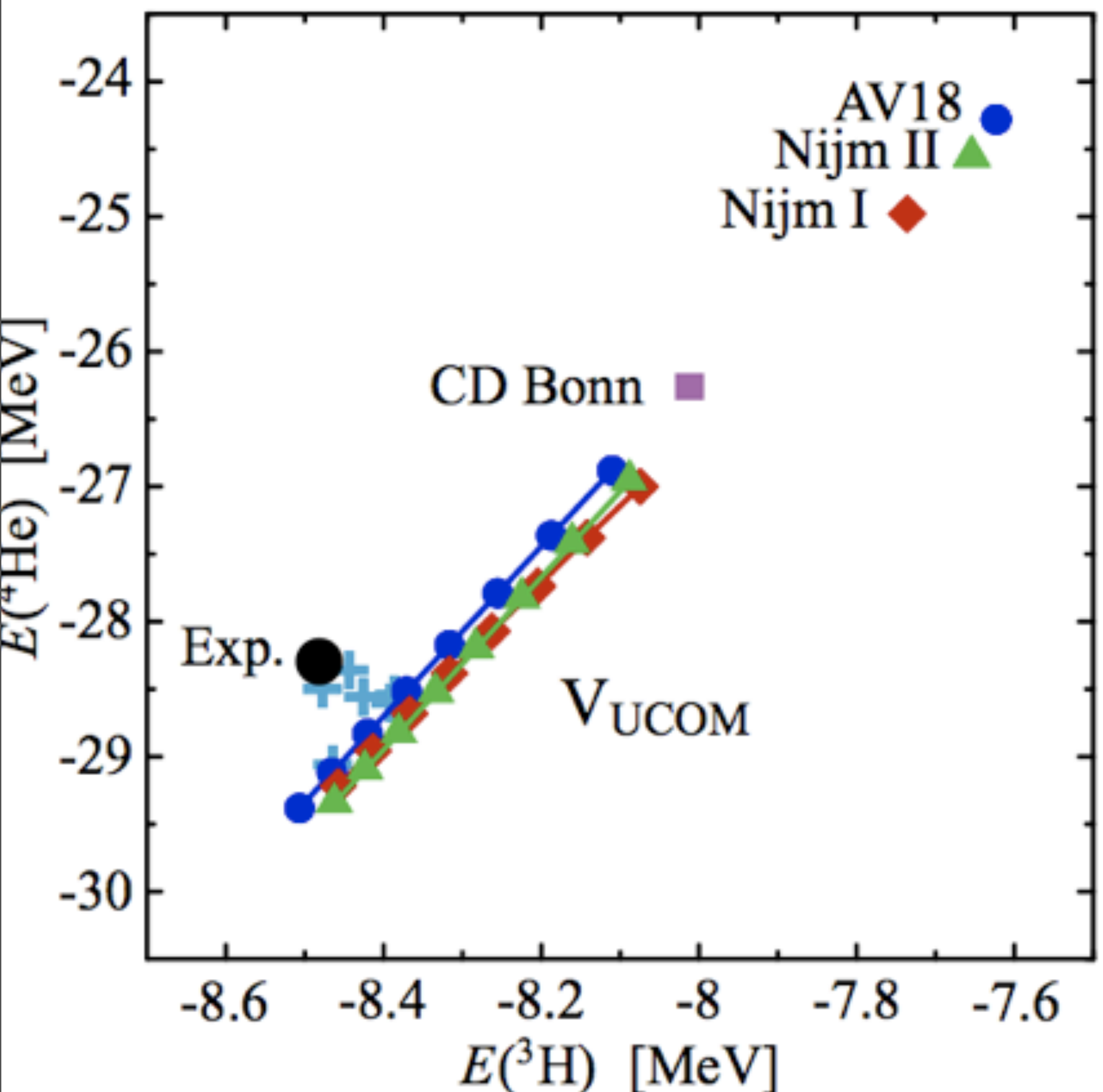
Correlation between 3- and 4-body binding energies

Roth et al., nucl-th/0505080



POTENTIAL MODELS

4-body physics

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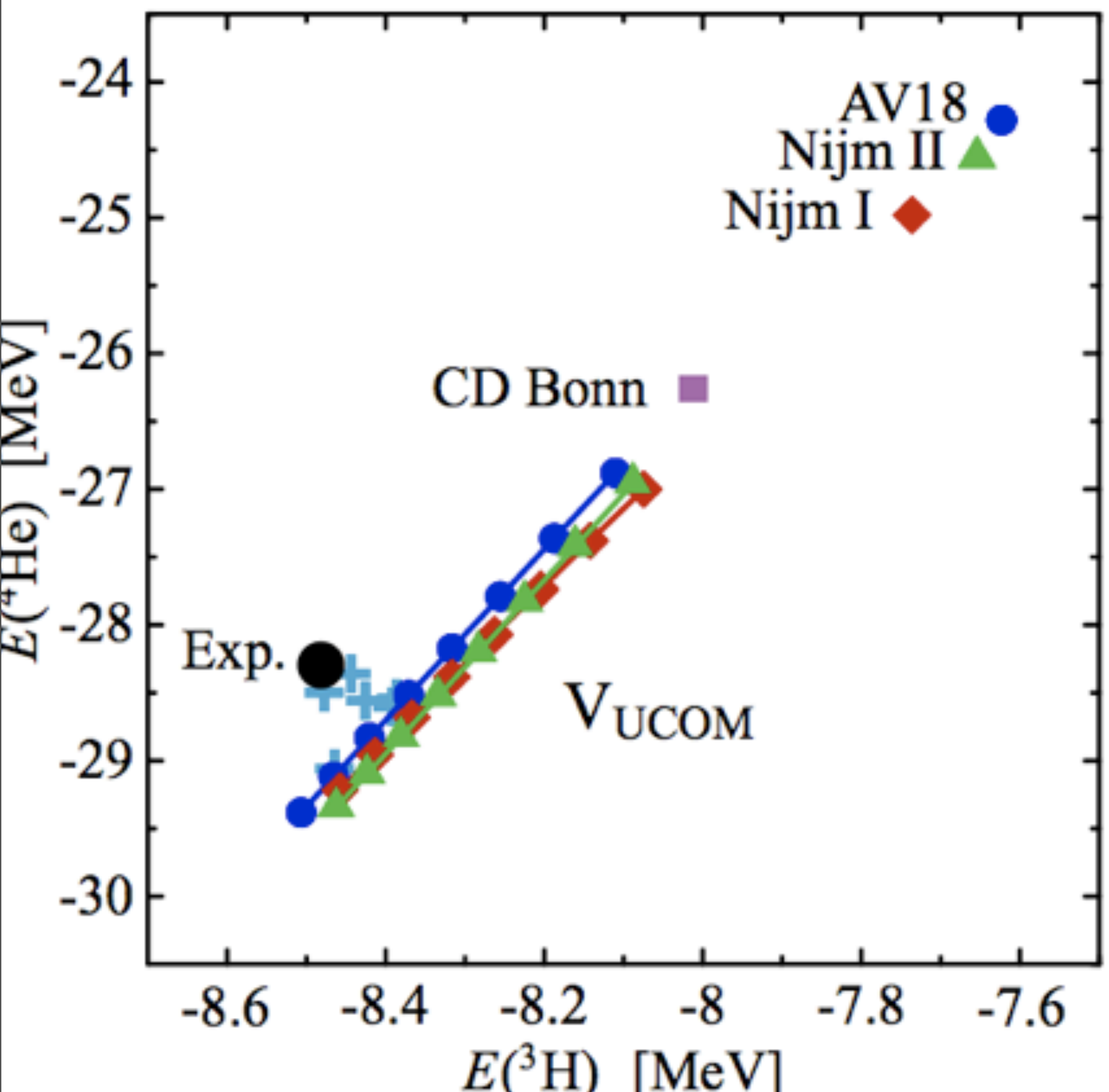
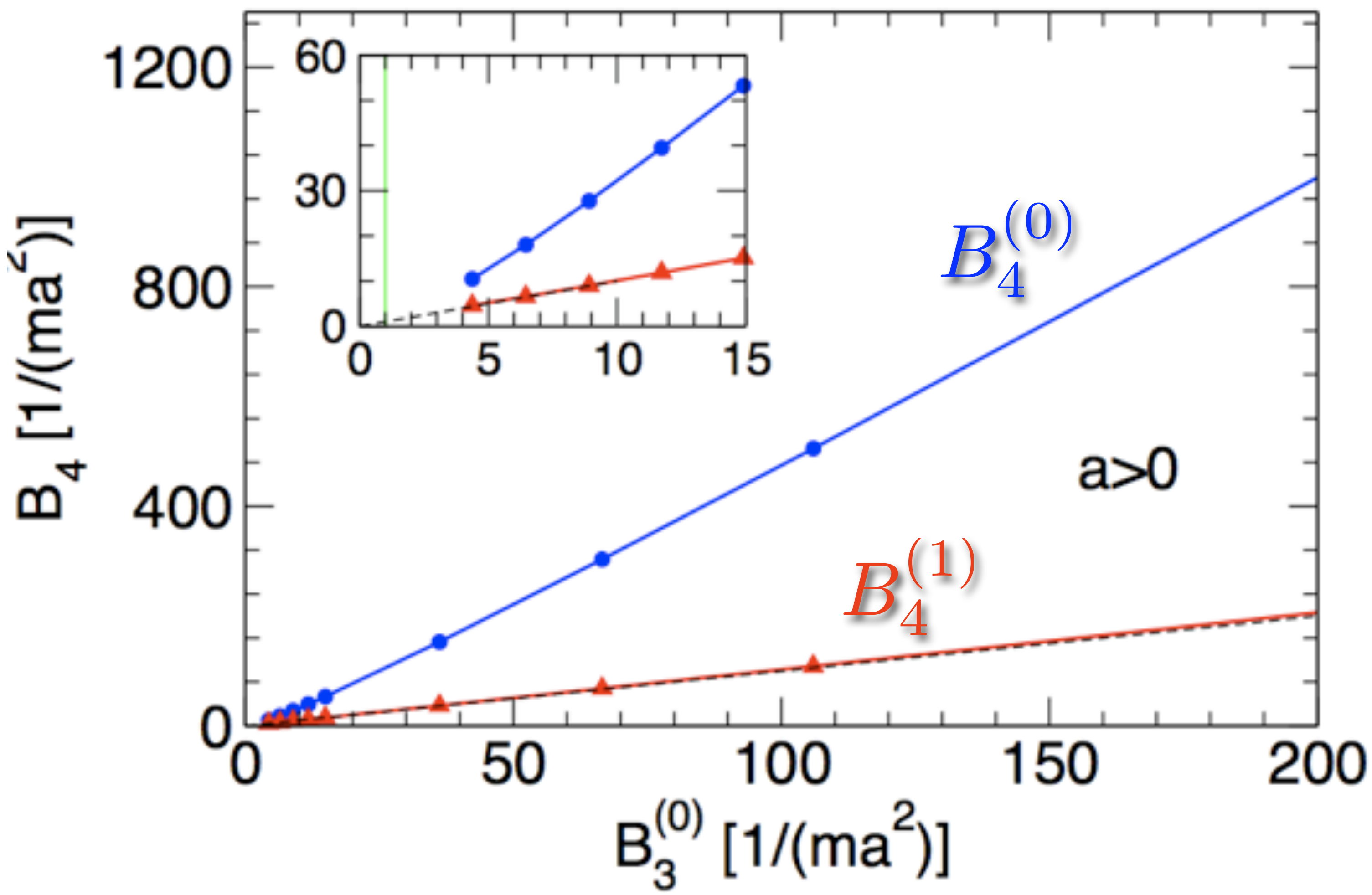


POTENTIAL MODELS

Hammer & Platter, nucl-th/0610105



EFT (large a fermions)

KSW approach, pionless EFT

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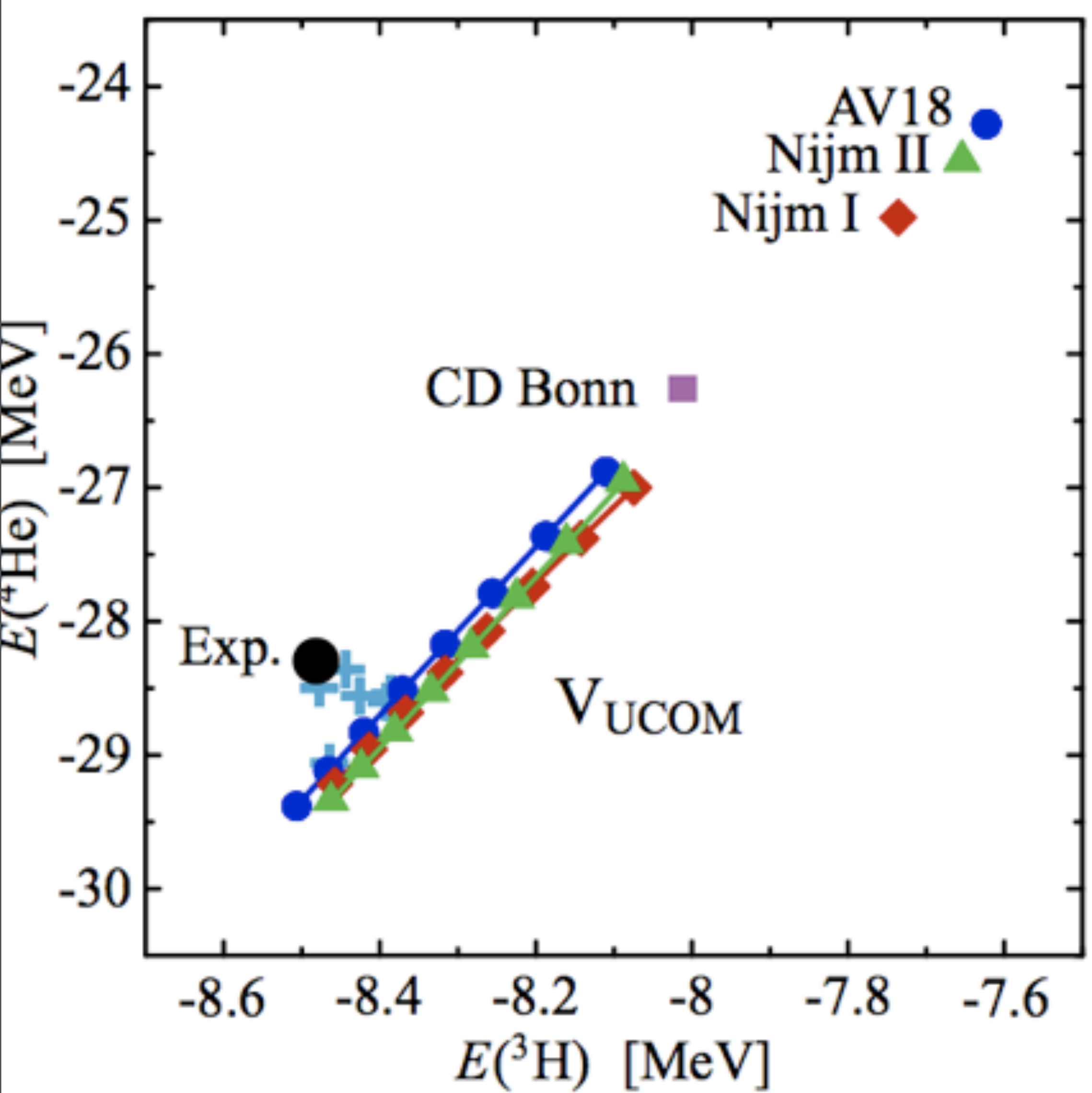
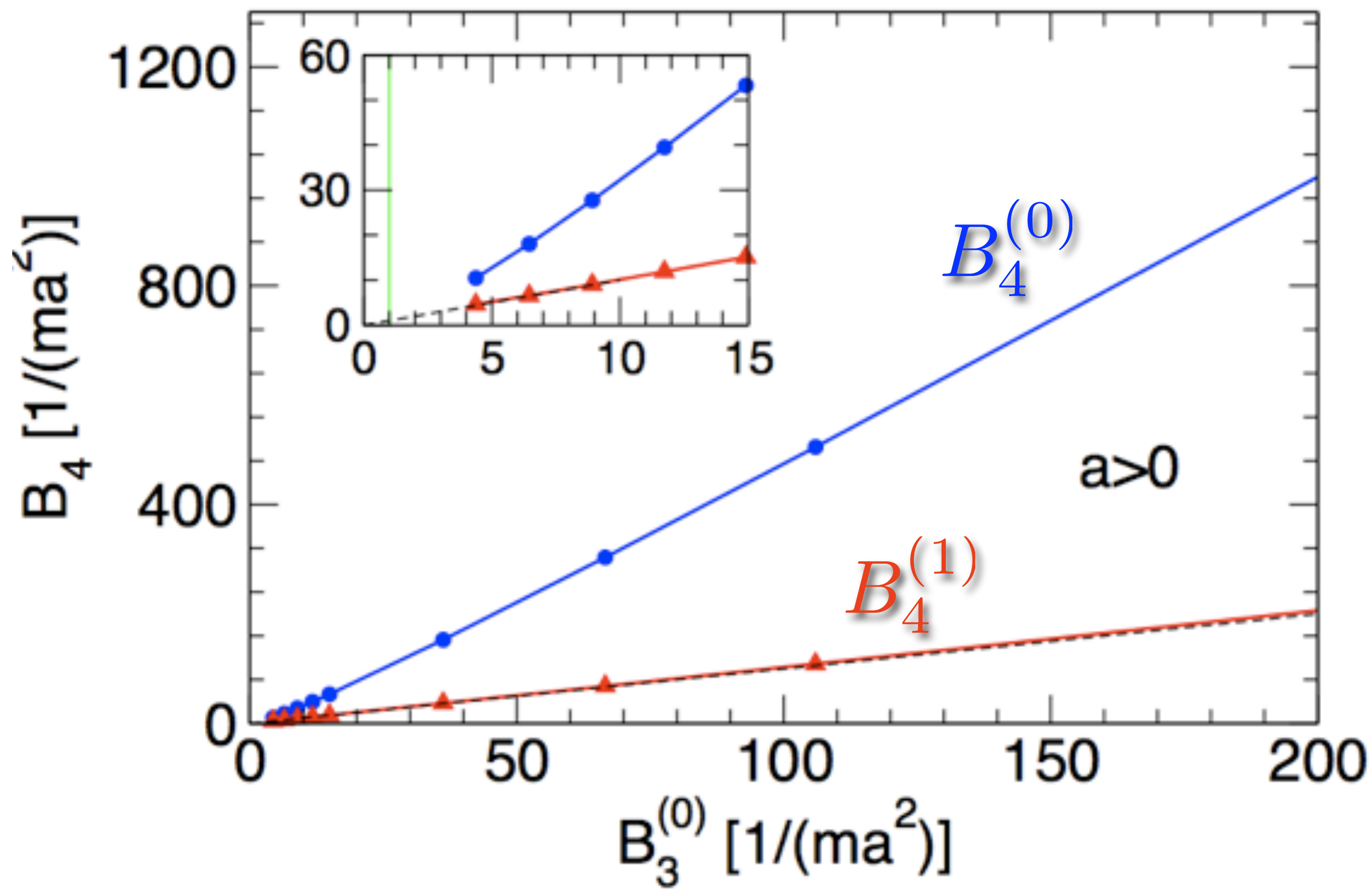


POTENTIAL MODELS

Tjon line due to 3-body interaction

Hammer & Platter, nucl-th/0610105

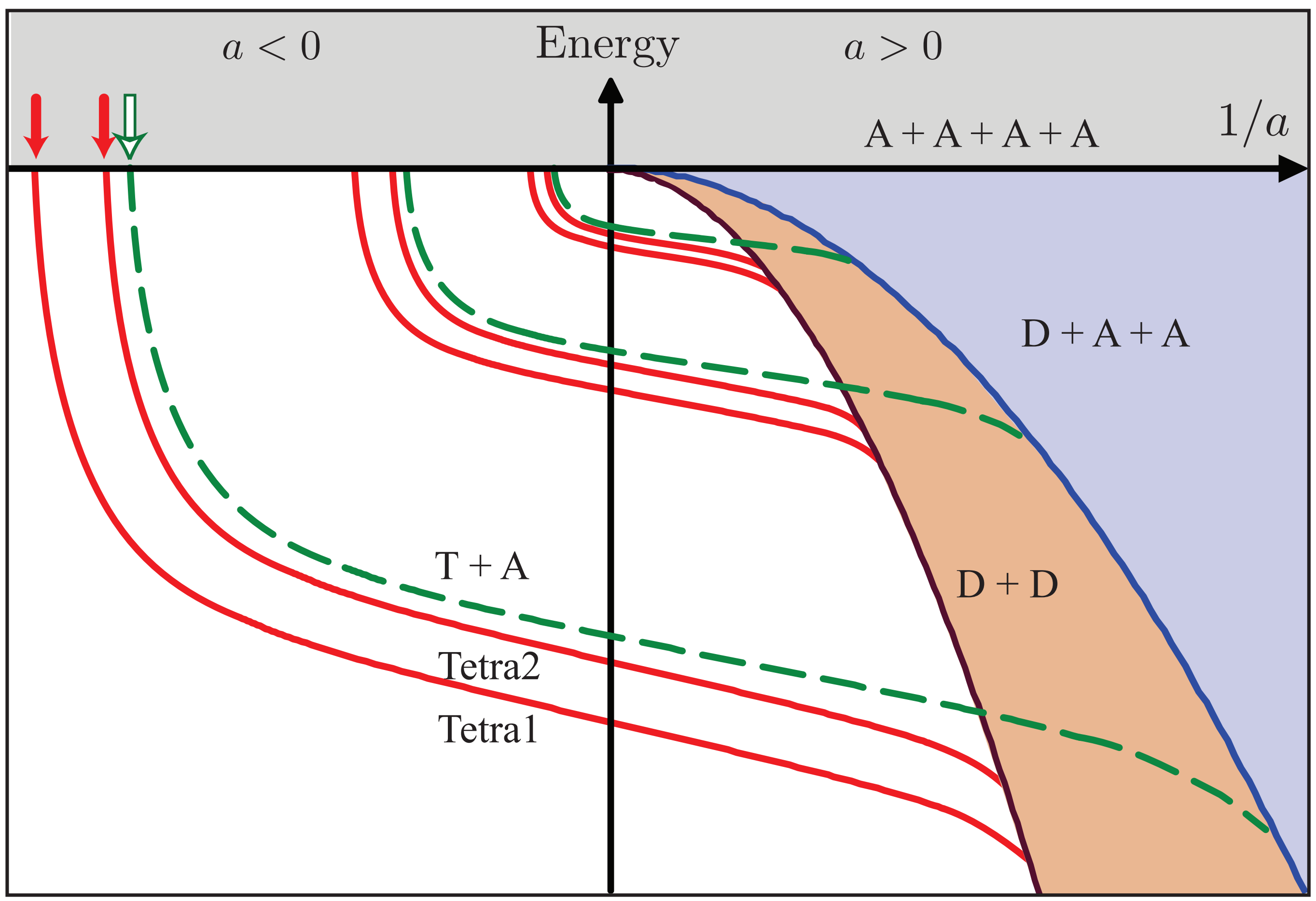


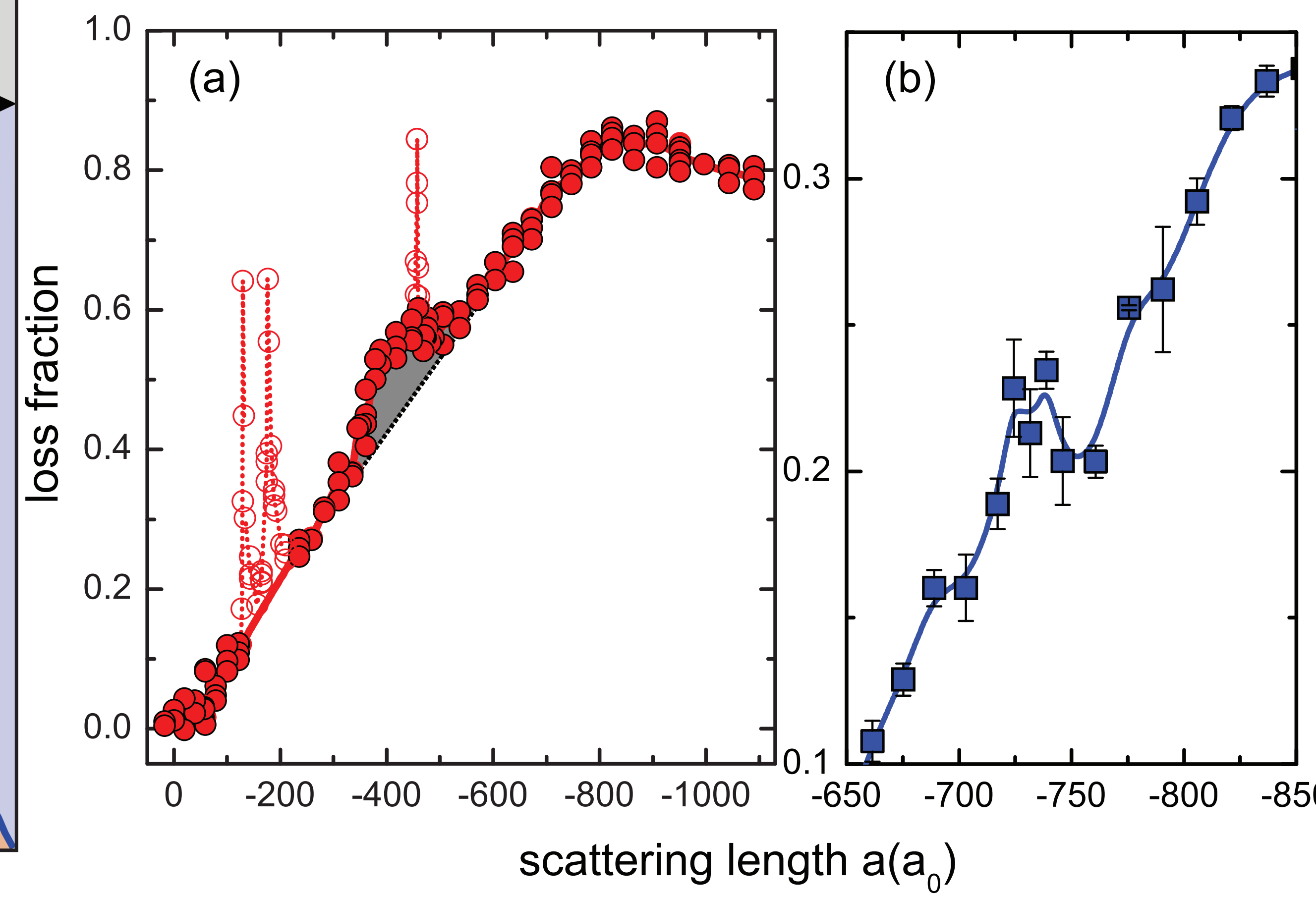
EFT (large a fermions)

Evidence for EFT prediction of "tetramers" in trapped atoms

Evidence for universal four-body states tied to an Efimov trimer

F. Ferlaino,[1] S. Knoop,[1] M. Berninger,[1] W. Harm,[1] J. P. D'Incao,[2,3] H.-C. Nägerl,[1] and R. Grimm[1,2]





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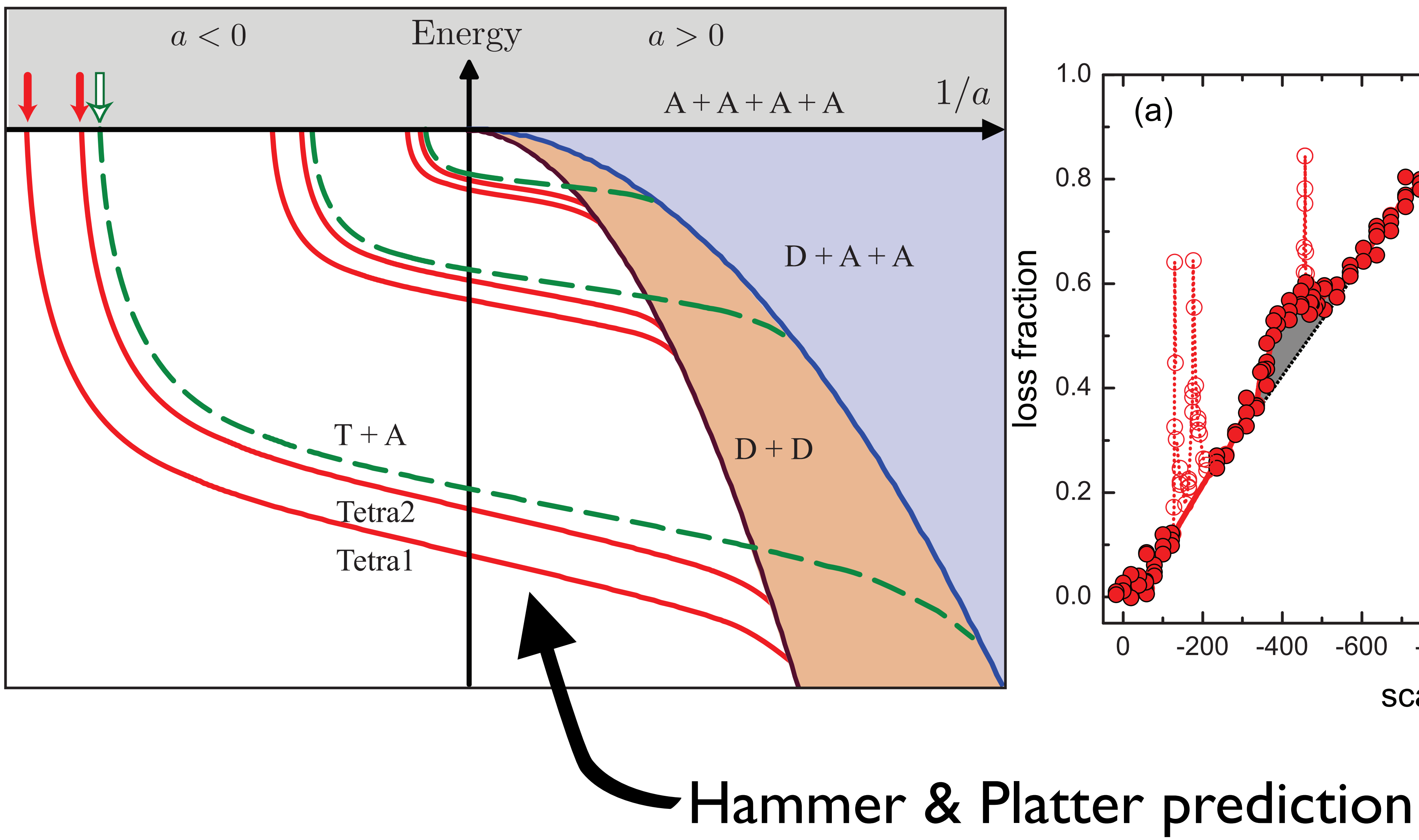
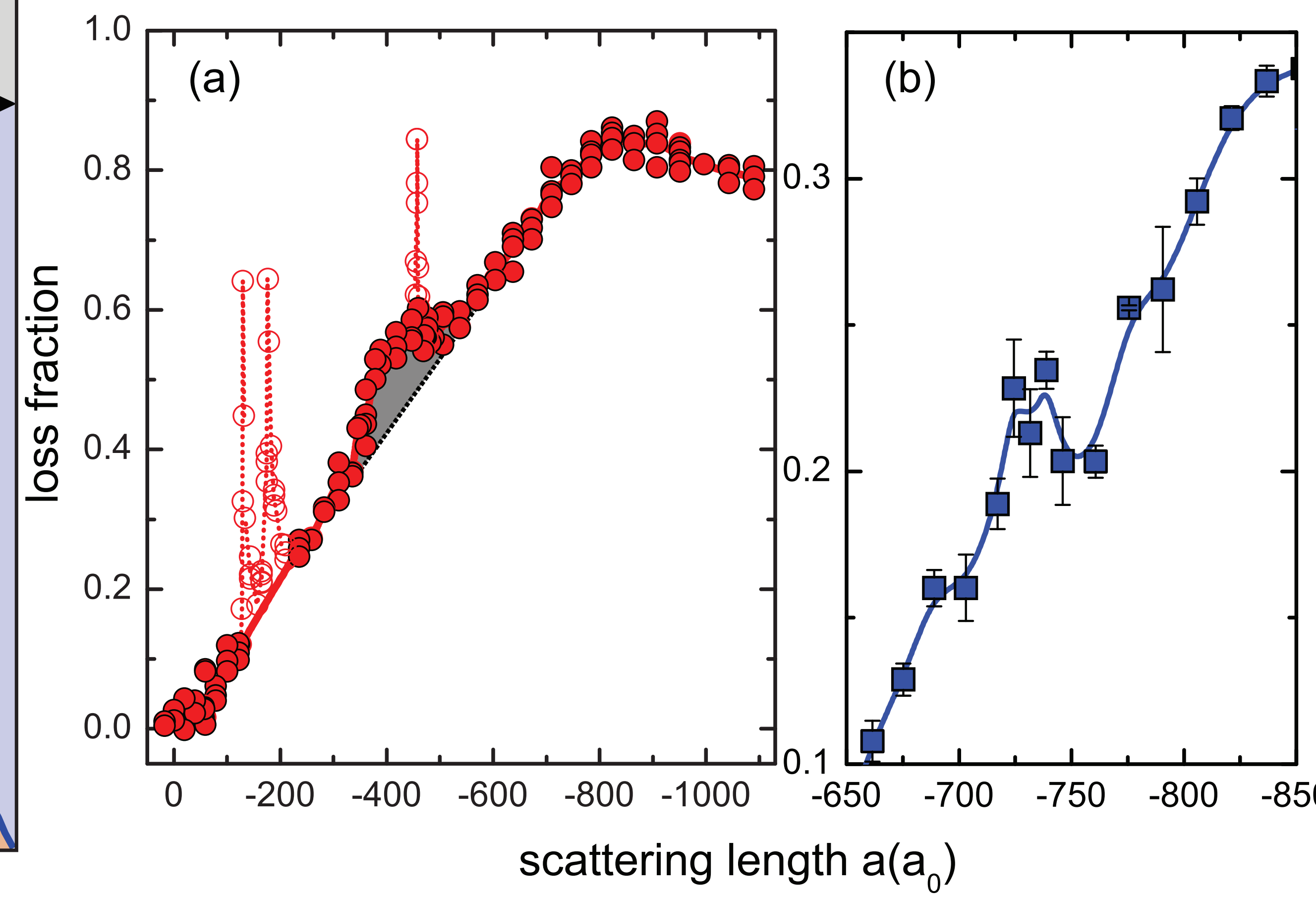




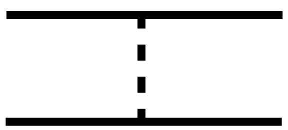
Limitations of the pionless nuclear EFT

- Can't treat scattering at $\mathbf{p \gtrsim m_\pi/2 = 70\ MeV}$
- Can't treat nuclei heavier than $^3H/^3He$

nucleus	BE/nucleon	p/nucleon
Deuteron	1.1 MeV	45 MeV
Tritium	2.8 MeV	73 MeV
^4He	7.0 MeV	115 MeV
^{56}Fe	8.8 MeV	128 MeV

...so include the pions.
Chiral perturbation theory: $\mathbf{m_\pi \sim p}$

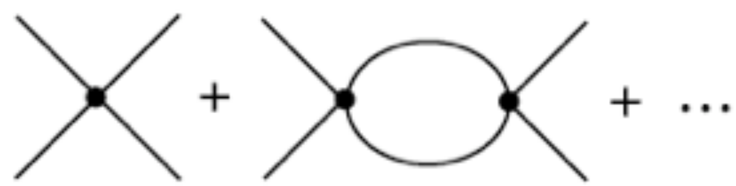
KSW approach, with pions



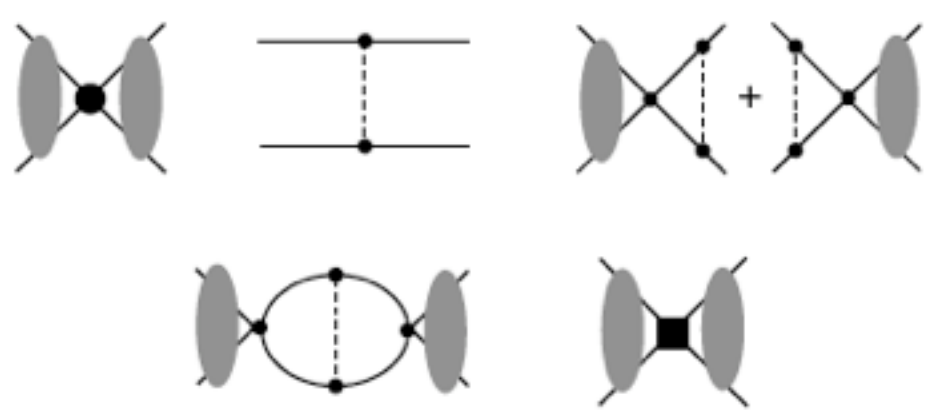
$$= i\frac{g_A^2}{4f_\pi^2}\;\frac{(\mathbf{q}\cdot\sigma_1)(\mathbf{q}\cdot\sigma_2)(\tau_1\cdot\tau_2)}{|\mathbf{q}|^2+m_\pi^2}$$

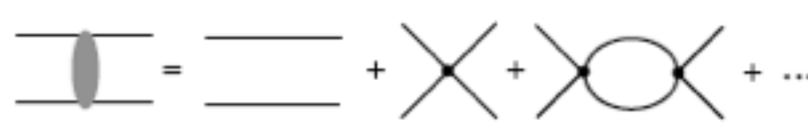
One pion exchange: $O(Q^0)$ in power counting

LO :



NLO :





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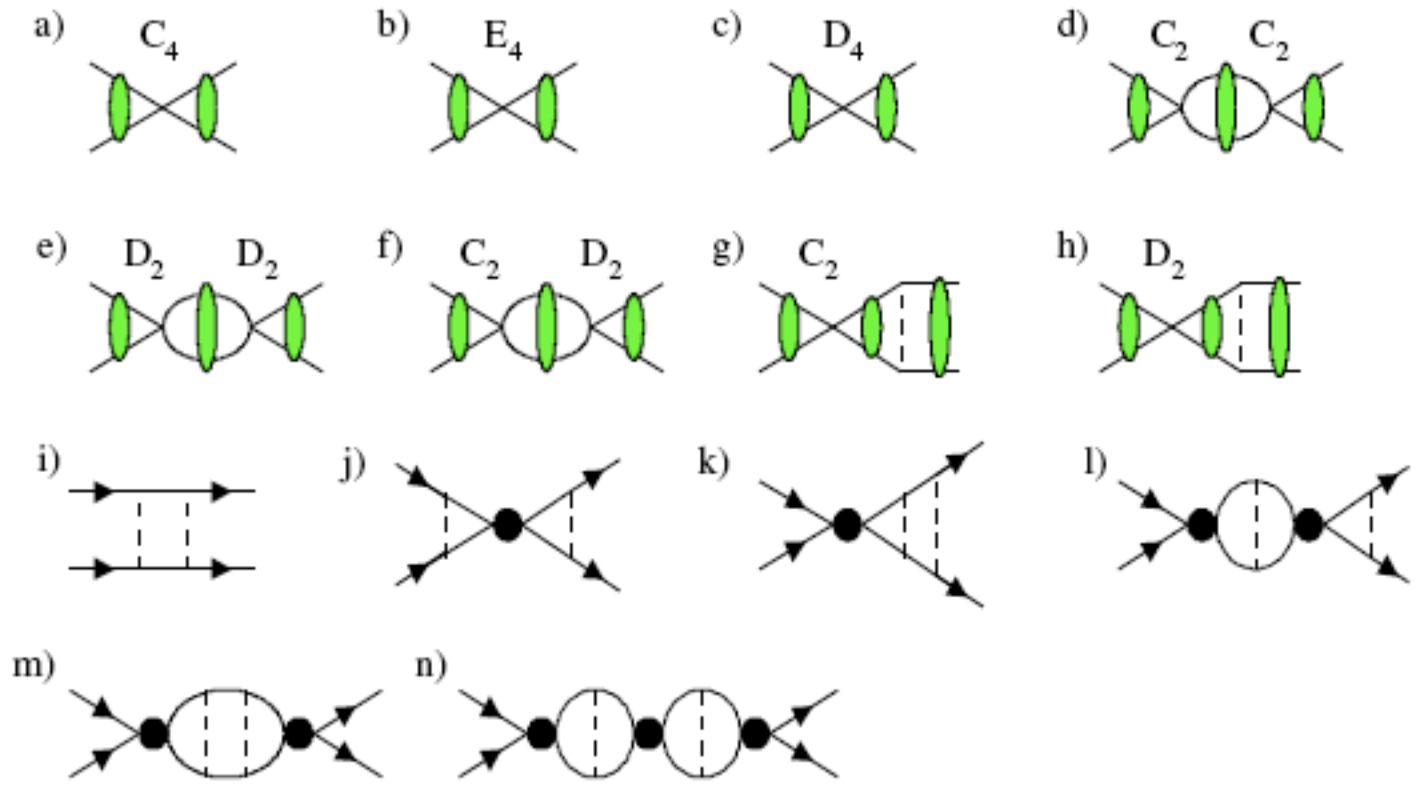
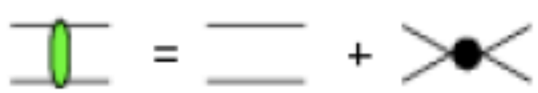
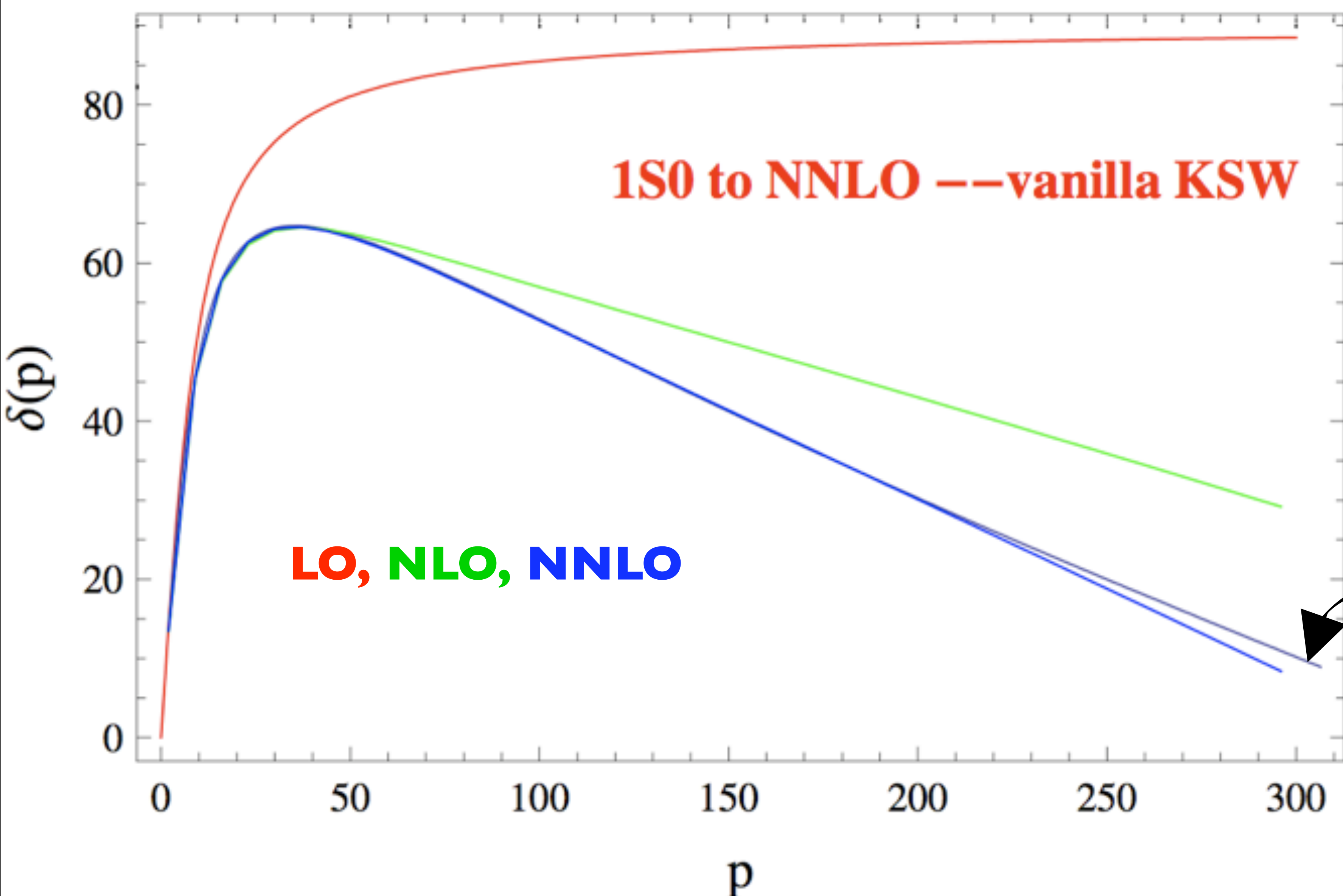




Fleming, Mehen, Stewart (1999)

Fleming, Mehen, Stewart (1999)

Works well for $^1S_0$

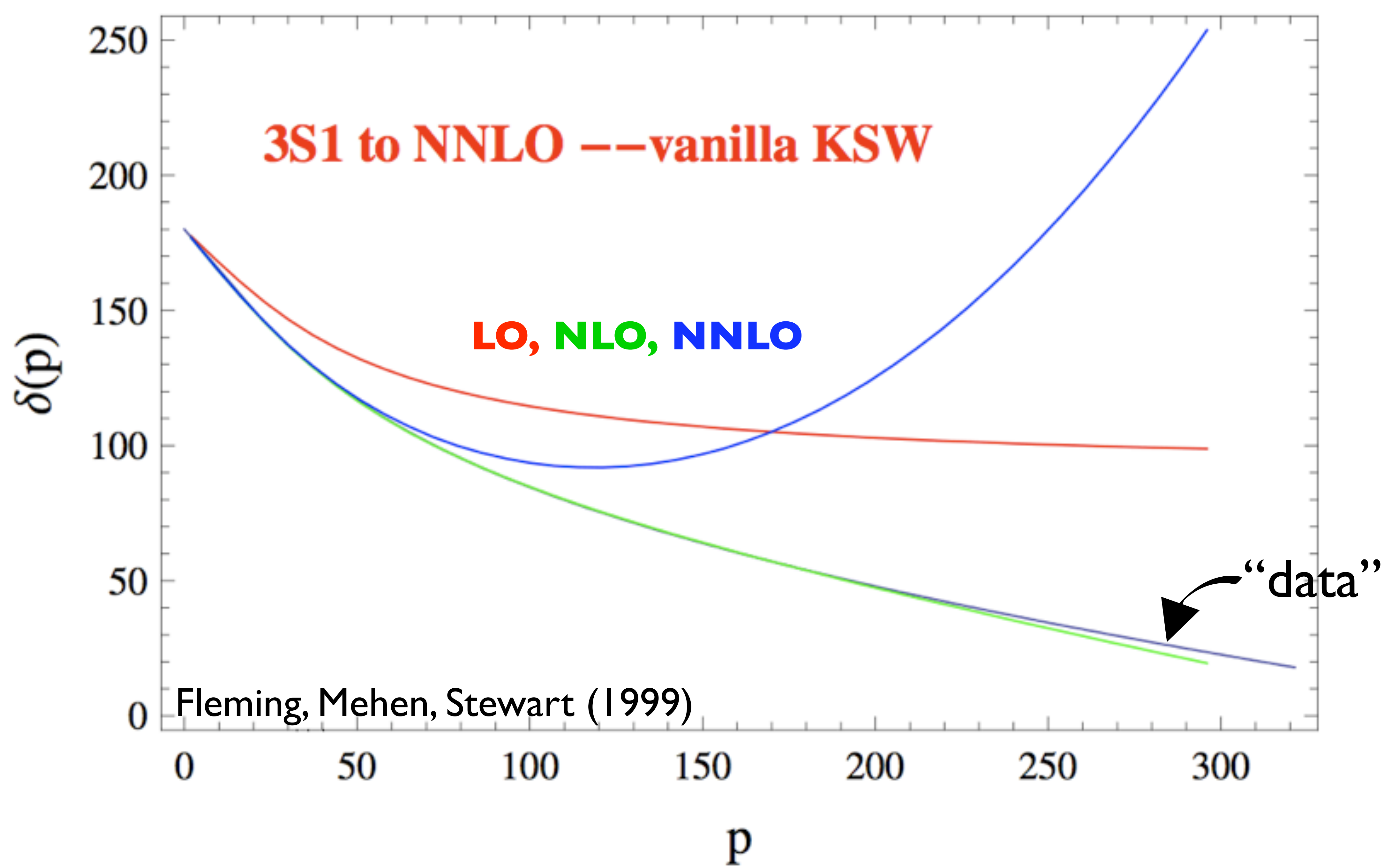


"data"

- Renormalized
- 2 parameter fit
- Analytic

Fleming, Mehen, Stewart (1999)

Fails miserably for $^3S_1$ channel (and $^3D_1$)



THE MORAL:

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The KSW expansion is theoretically virtuous

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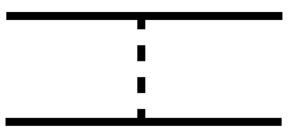
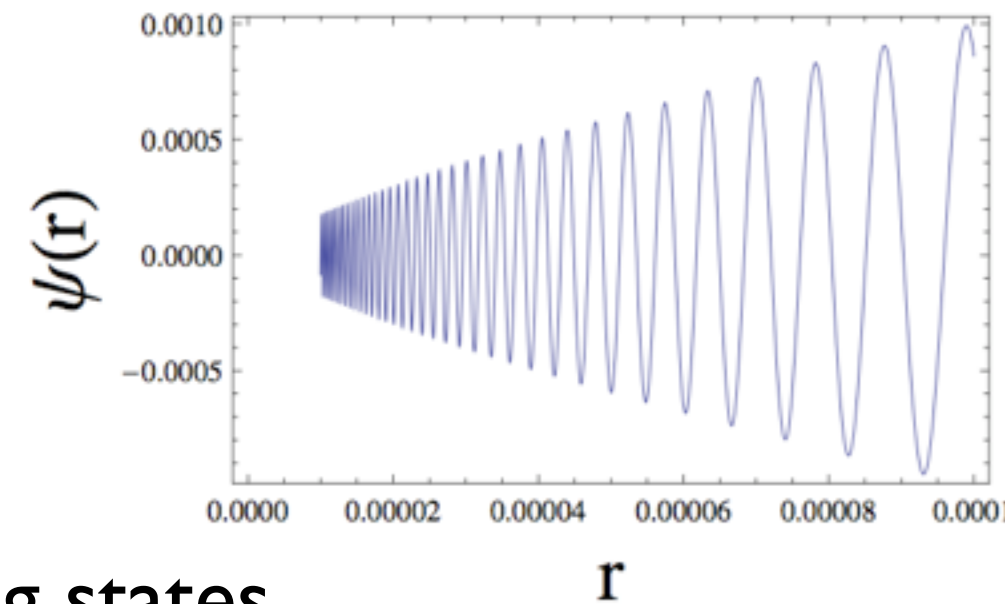
The KSW expansion is theoretically virtuous

Virtue's rewards aren't always in this world

Problem with convergence of KSW is linked to attractive tensor force



$$V(r) \sim -\frac{1}{r^3}$$



- No ground state; pathological scattering states
- Can't be "fixed" with contact interactions
- Perturbation theory in pion exchange is going to break down

...But the problem is "fake":
should be able to eliminate $1/r^3$ in $V(r)$ at small r in favor of contact interactions

S. Beane, D.K., A. Vuorinen (2008):

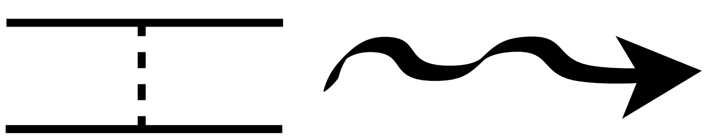
Follow KSW expansion, but modify pion propagator:

$$G_\pi(q,m) = i\frac{g_A^2}{4f_\pi^2}\frac{(\mathbf{q}\cdot\boldsymbol{\sigma}_1)(\mathbf{q}\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{\mathbf{q}^2+m^2}$$ pion, I=J=1, mass m

$$G_{(1,0)}(q,\lambda) = i\frac{g_A^2}{4f_\pi^2}\frac{\lambda^2}{\mathbf{q}^2+\lambda^2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$ I=1, J=0, mass λ

$1/r^3$ cancels for S=1

S=0 unchanged

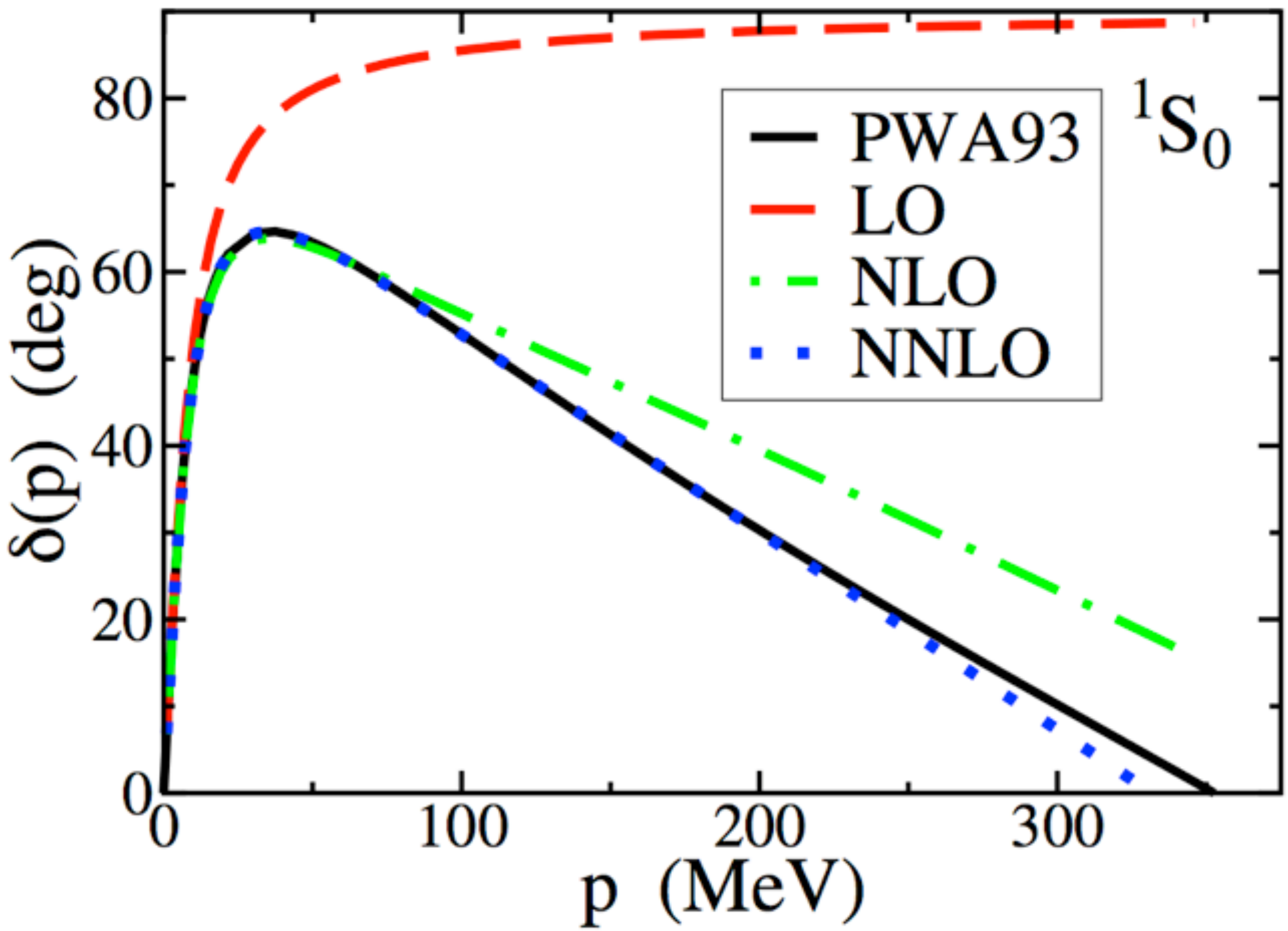


$$G_\pi(q,m_\pi) - G_\pi(q,\lambda) + G_{(1,0)}(q,\lambda)$$

Power counting: KSW + $\lambda \sim O(Q)$

S. Beane, D.K., A. Vuorinen (2008)

$^1S_0$ (and all S=0 channels) unchanged...same as KSW



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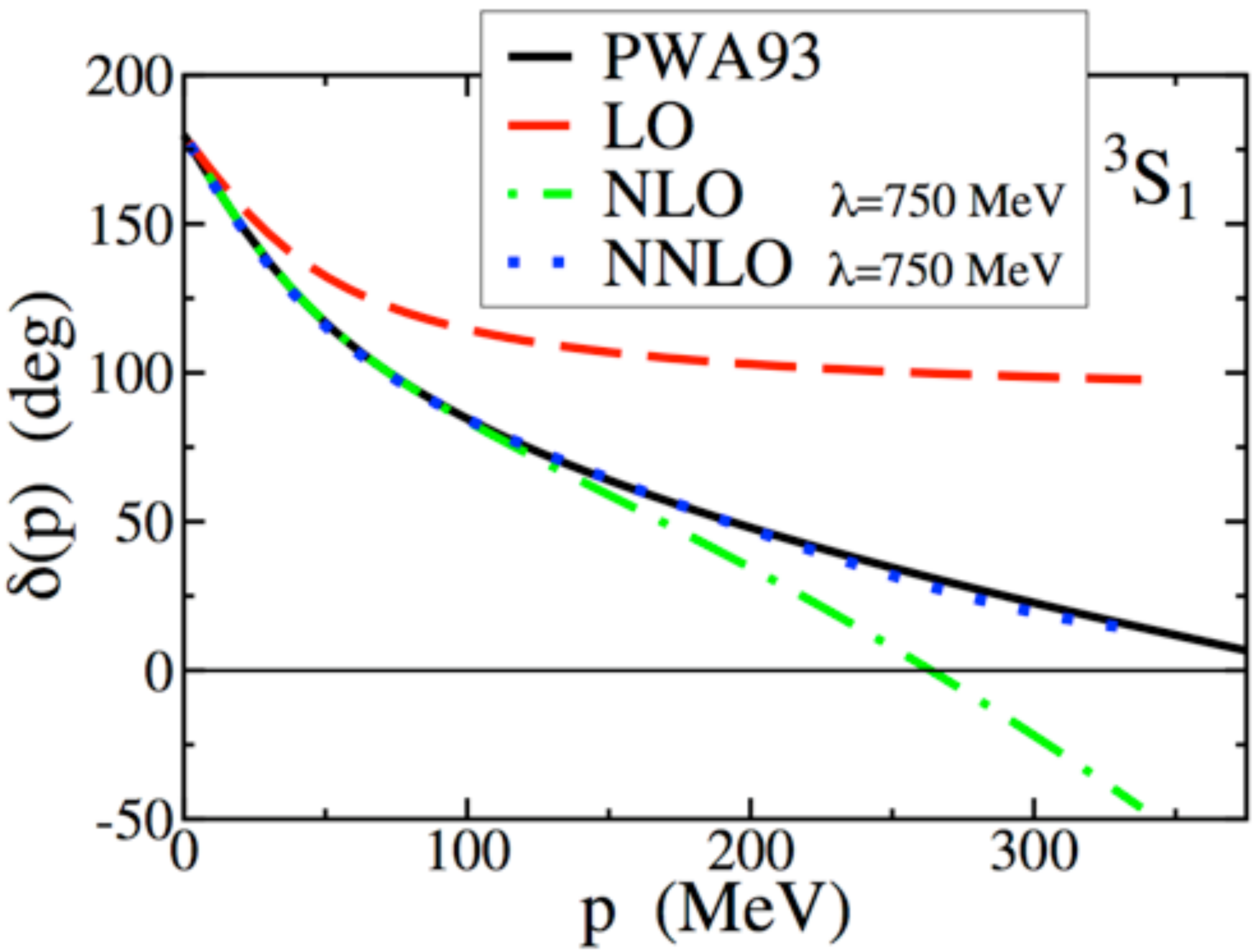
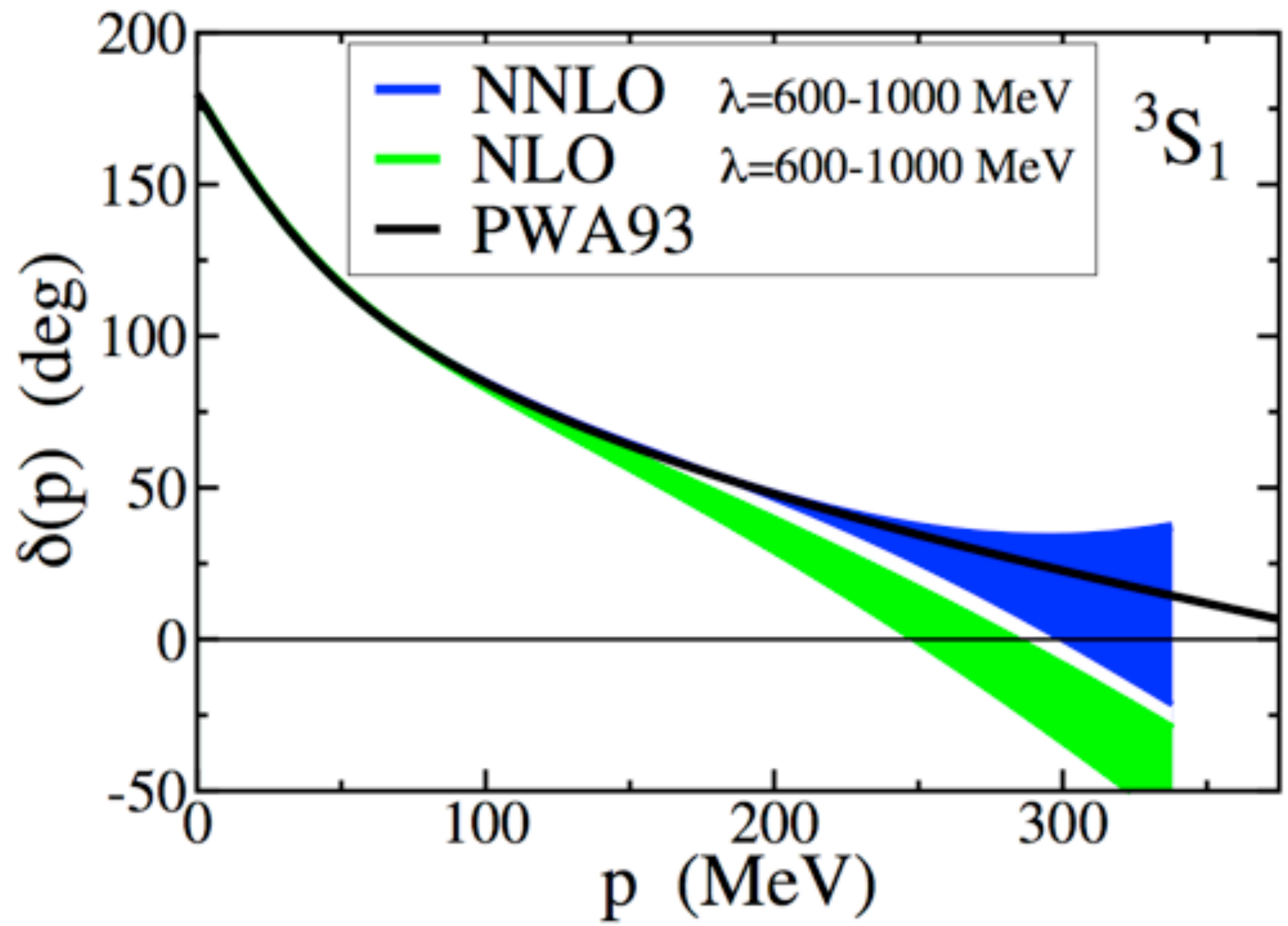


$^3S_1$ greatly improved

Sensitivity to λ



Meaning of λ ?

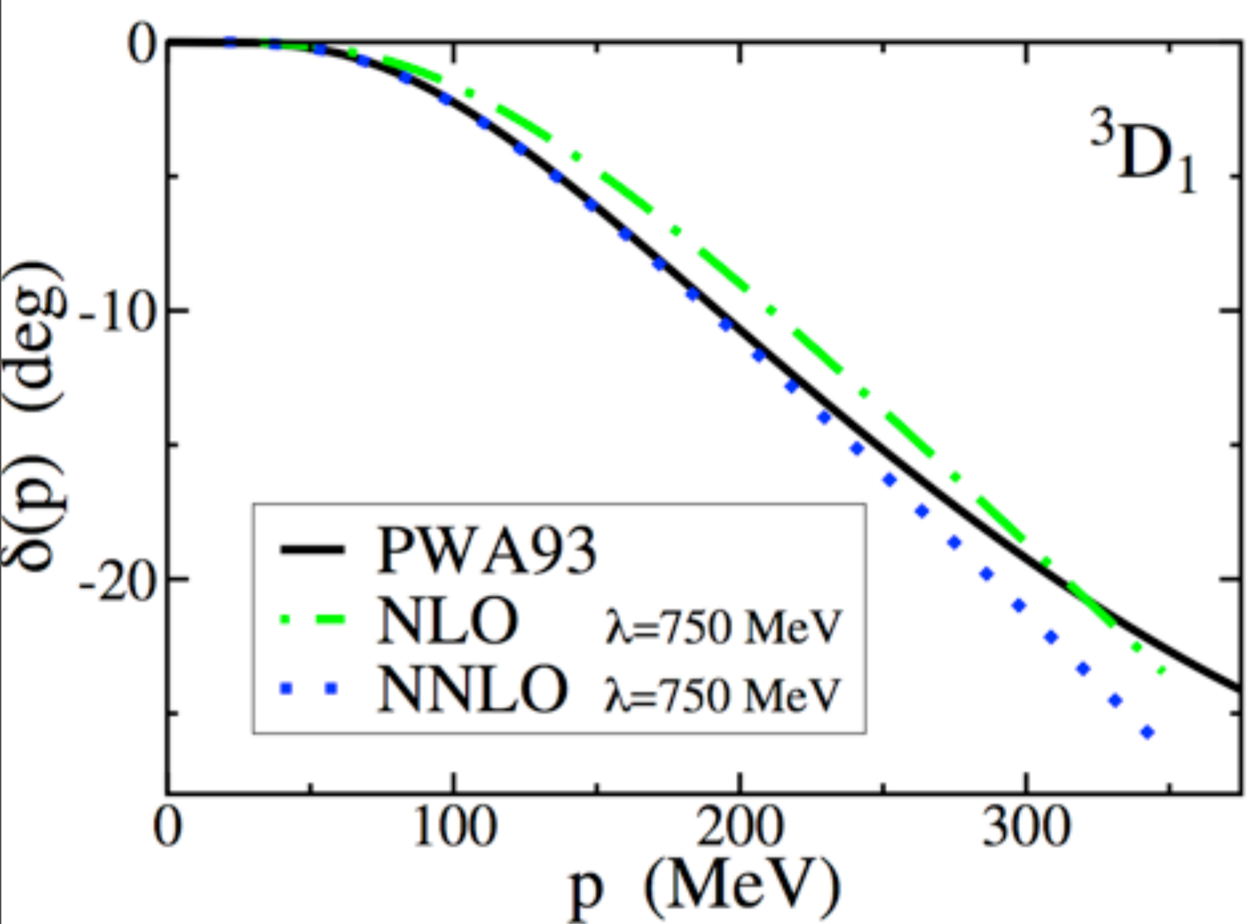
Like μ in PQCD:

- unphysical
- controls resummation of log divergences into coupling constant
- controls convergence of perturbative expansion

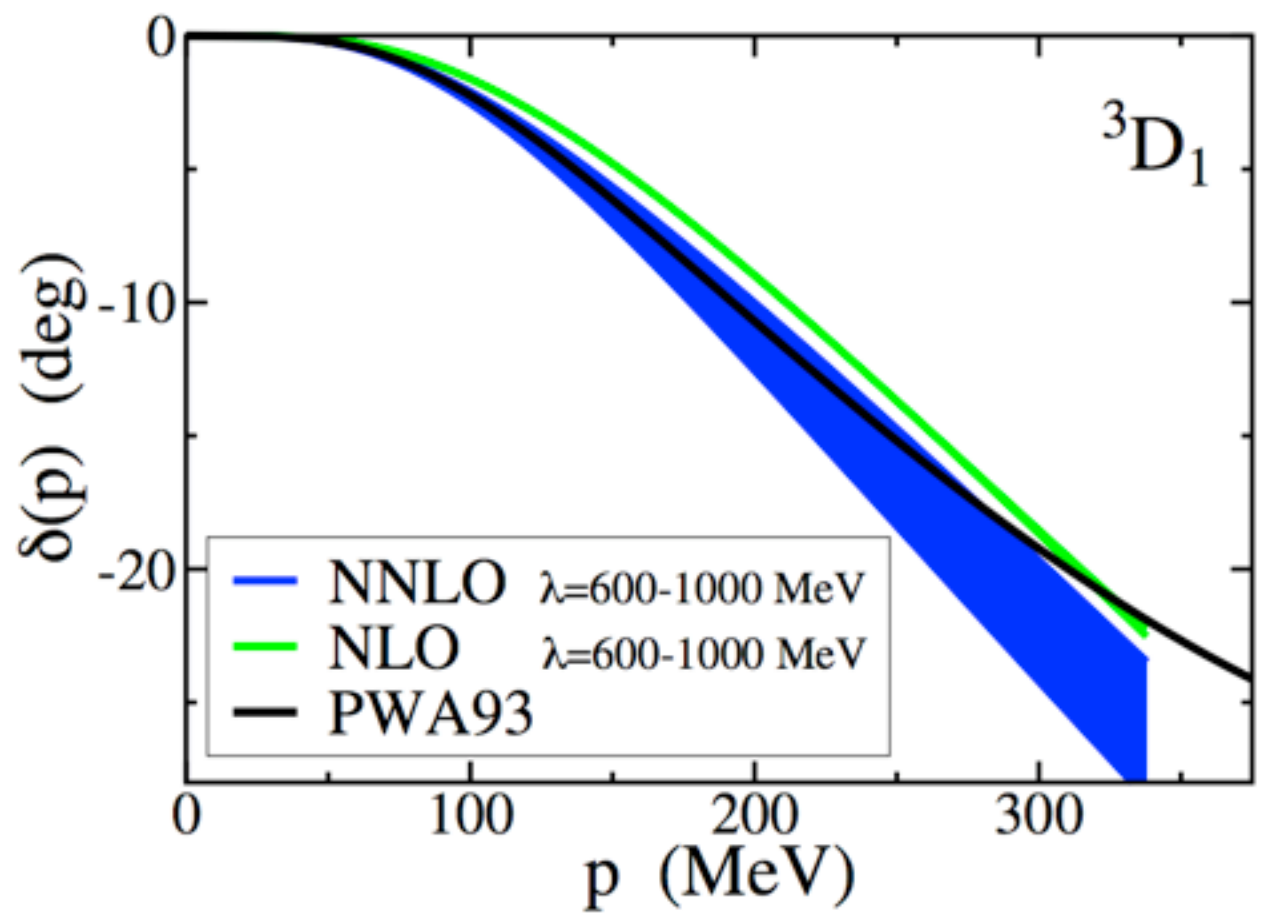
λ is:

- unphysical
- controls resummation $1/r^3$ effects into contact interactions
- controls convergence of perturbative expansion

$\lambda \to \infty$ yields the (poorly converging) KSW expansion



$^3D_1$
PWA93
NLO λ=750 MeV
NNLO λ=750 MeV
δ(p) (deg)
p (MeV)



$^3D_1$
NNLO λ=600-1000 MeV
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PWA93
δ(p) (deg)
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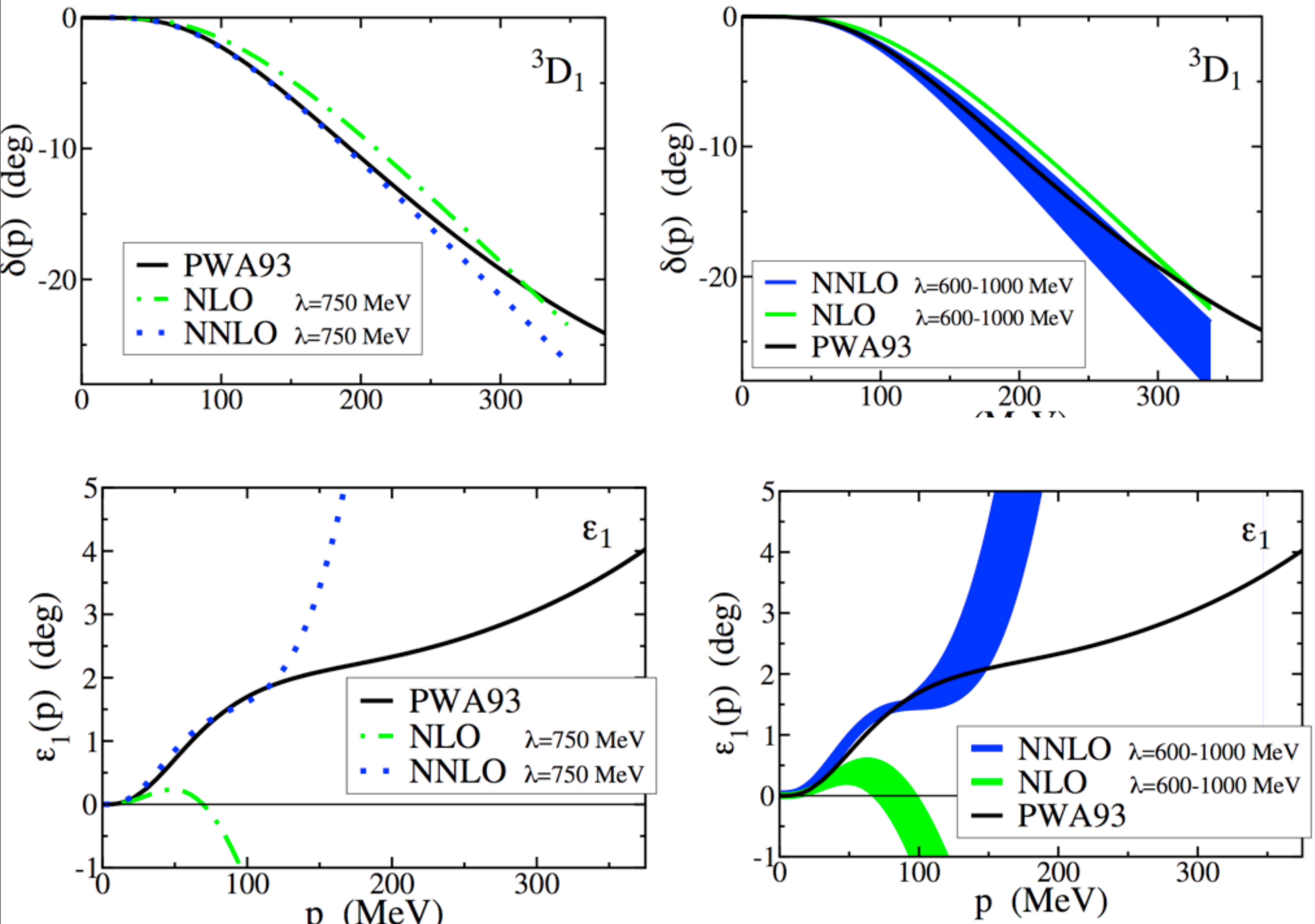

$^3D_1$
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- Looks pretty good in $^1S_0$, $^3S_1$, $^3D_1$
- Looks pretty bad in ε_1...just an accidentally small angle?
- what about other partial waves?

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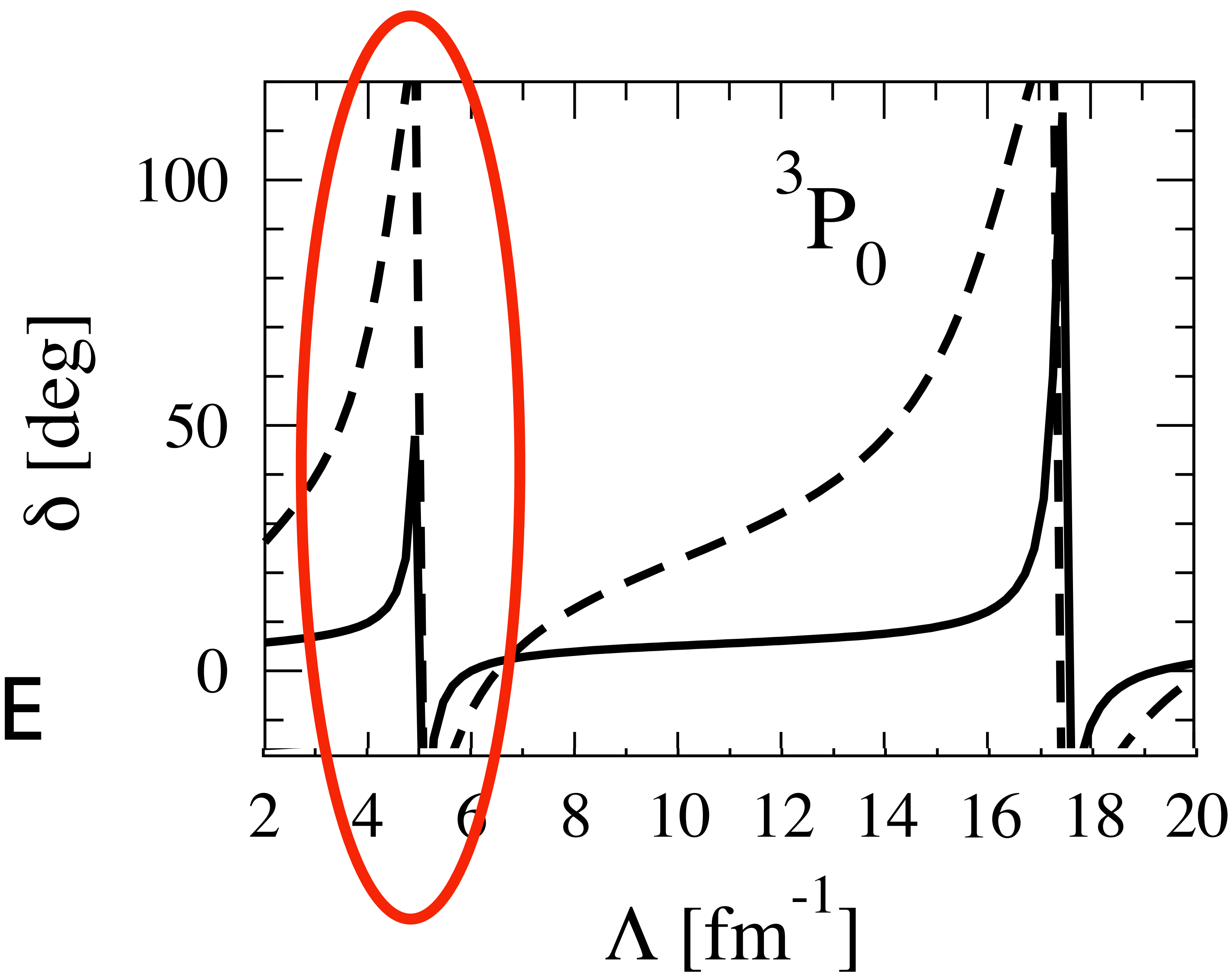
- Looks pretty bad in ε_1...just an accidentally small angle?
- what about other partial waves?

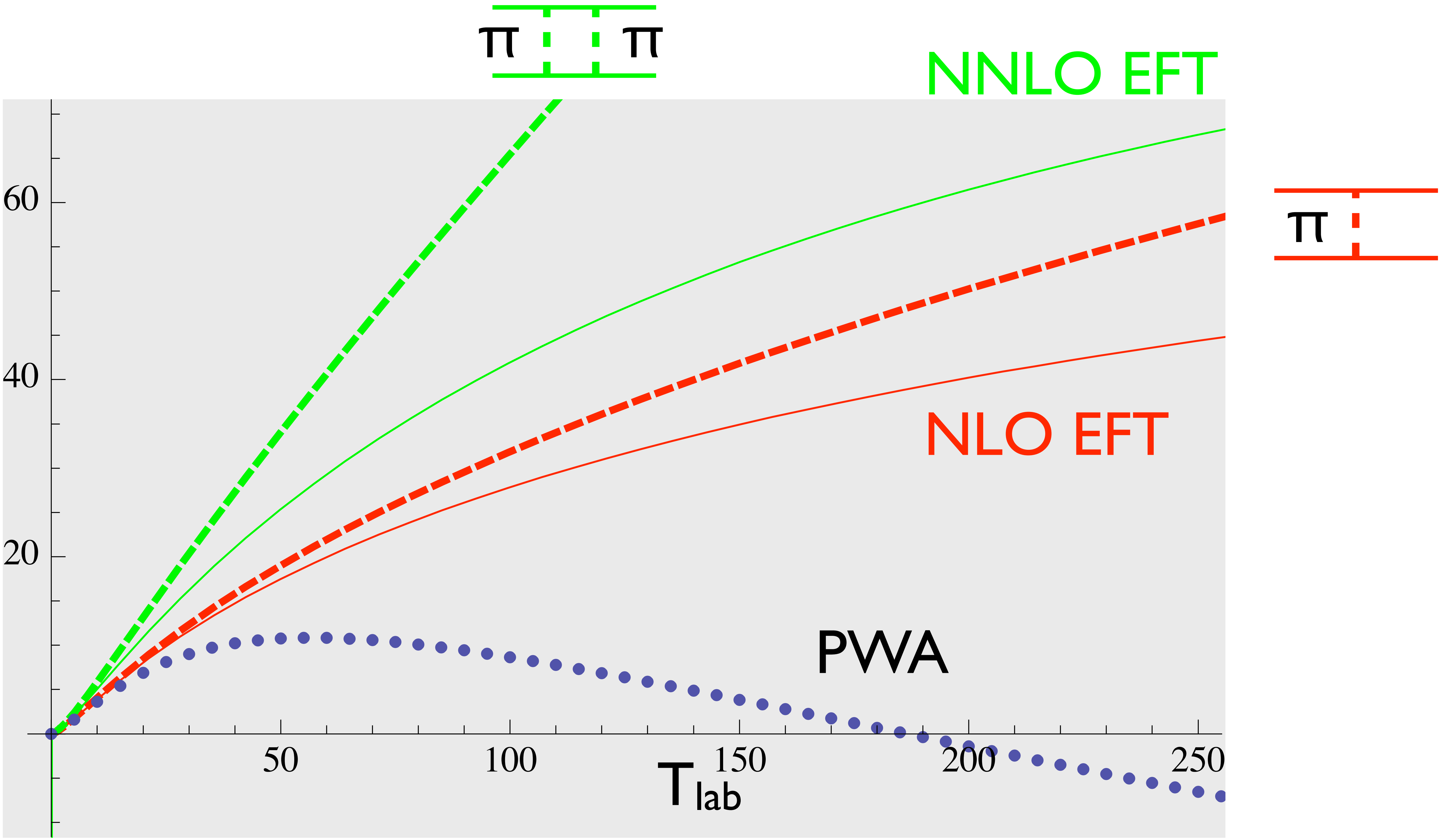
$^3P_0$ may be the biggest challenge:

Recall the Nogga, Timmermans, Van Kolck result using Weinberg counting:



$^3P_0$ *almost* has a bound state in the OPE potential

PRELIMINARY: $^3P_0$ to N^2LO -- Not so good.



Check N^3LO (includes 1st contact interaction)?

Lots to do to see if expansion will converge:

- Higher partial waves (eg, ${}^3D_2$)
- N^3LO amplitudes
- Electromagnetic & weak 2-nucleon processes
- 3-body physics

EFT has had an impact:

- high-precision, low energy few-body physics

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- bridge between lattice QCD and nuclear structure

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    - No QCD-like sign problem?

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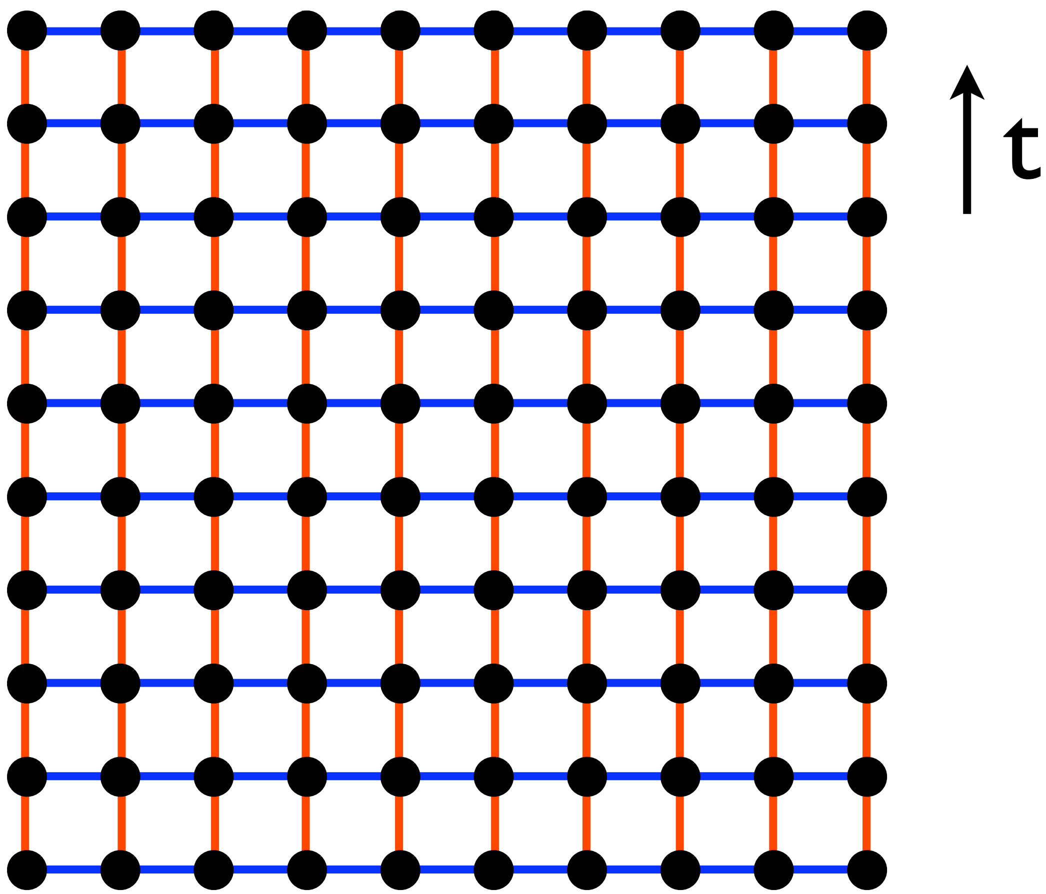
EFT making a big impact?

- new, more efficient way to do N-body physics via **lattice EFT**?
    - No QCD-like sign problem?
    - No QCD-like signal/noise problem?
    - Path integrals avoid big Hilbert space

Lattice EFT (in progress; Endres, Lee, Nicholson, DK)



$$S = \sum_{\mathbf{n}} \left[\bar{\psi}_{\mathbf{n}} (K\psi)_{\mathbf{n}} + \frac{1}{2} m^2 \phi_{\mathbf{n}}^2 \right]$$

$$(K\psi)_{\mathbf{n}} = (\psi_{\mathbf{n}} - \psi_{\mathbf{n}-\hat{\mathbf{t}}}) - \frac{1}{2M} (\nabla^2 \psi)_{\mathbf{n}} - \phi_{\mathbf{n}} \psi_{\mathbf{n}-\hat{\mathbf{t}}}$$

mu=0 version of Chen, Kaplan (2003)

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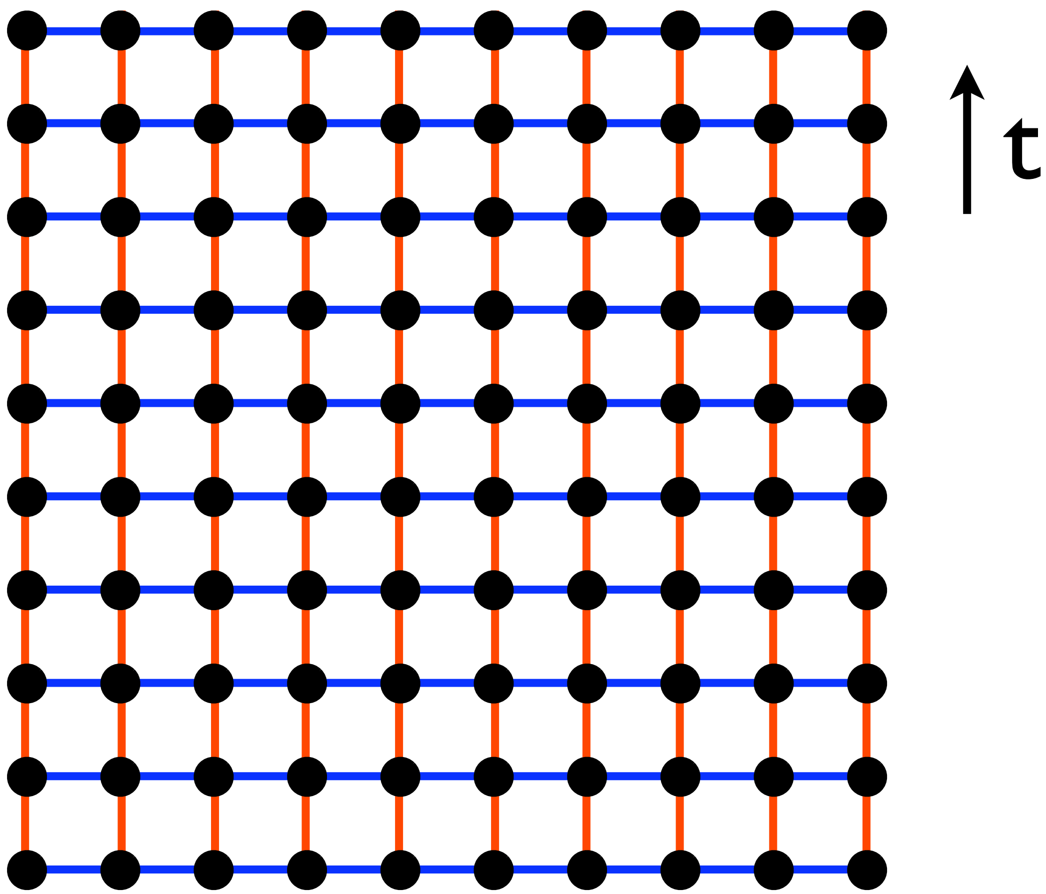

- Field φ generates the C_0 interaction

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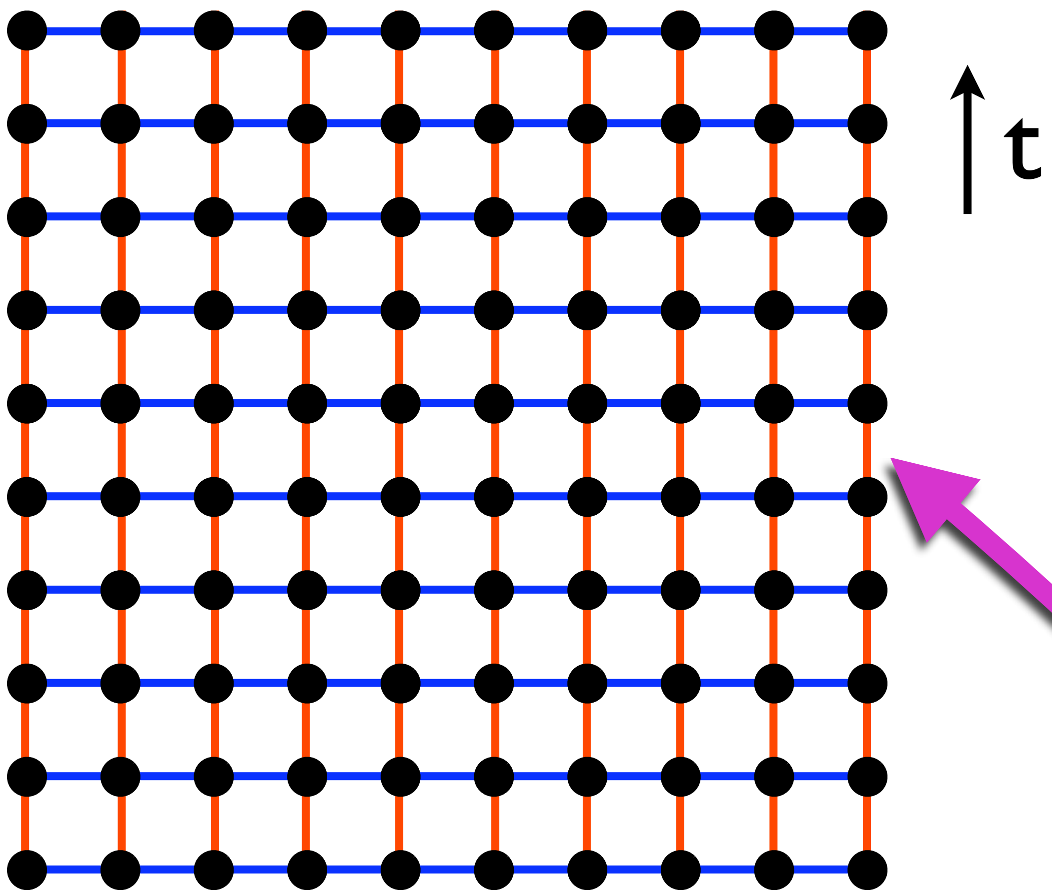

- Field φ generates the C_0 interaction
- φ only lives on <u>time</u> <u>links</u>

$$S = \sum_{\mathbf{n}} \left[\bar{\psi}_{\mathbf{n}} (K\psi)_{\mathbf{n}} + \frac{1}{2} m^2 \phi_{\mathbf{n}}^2 \right]$$

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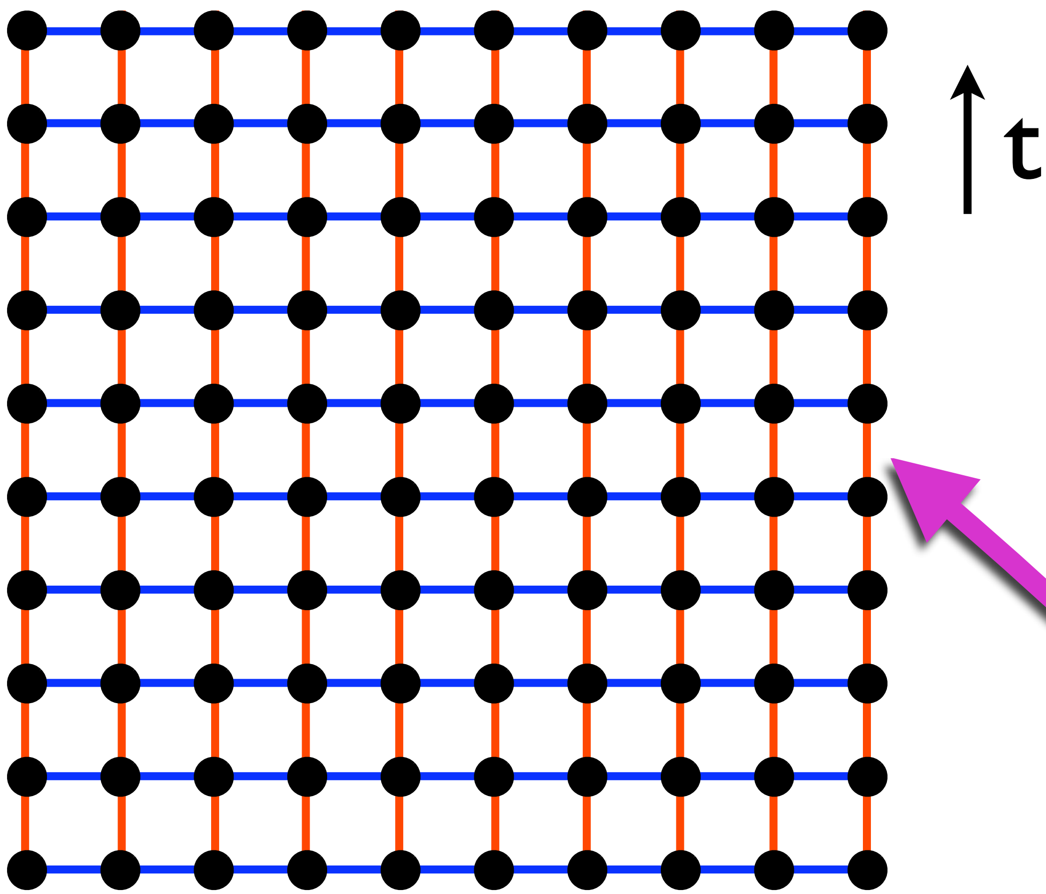

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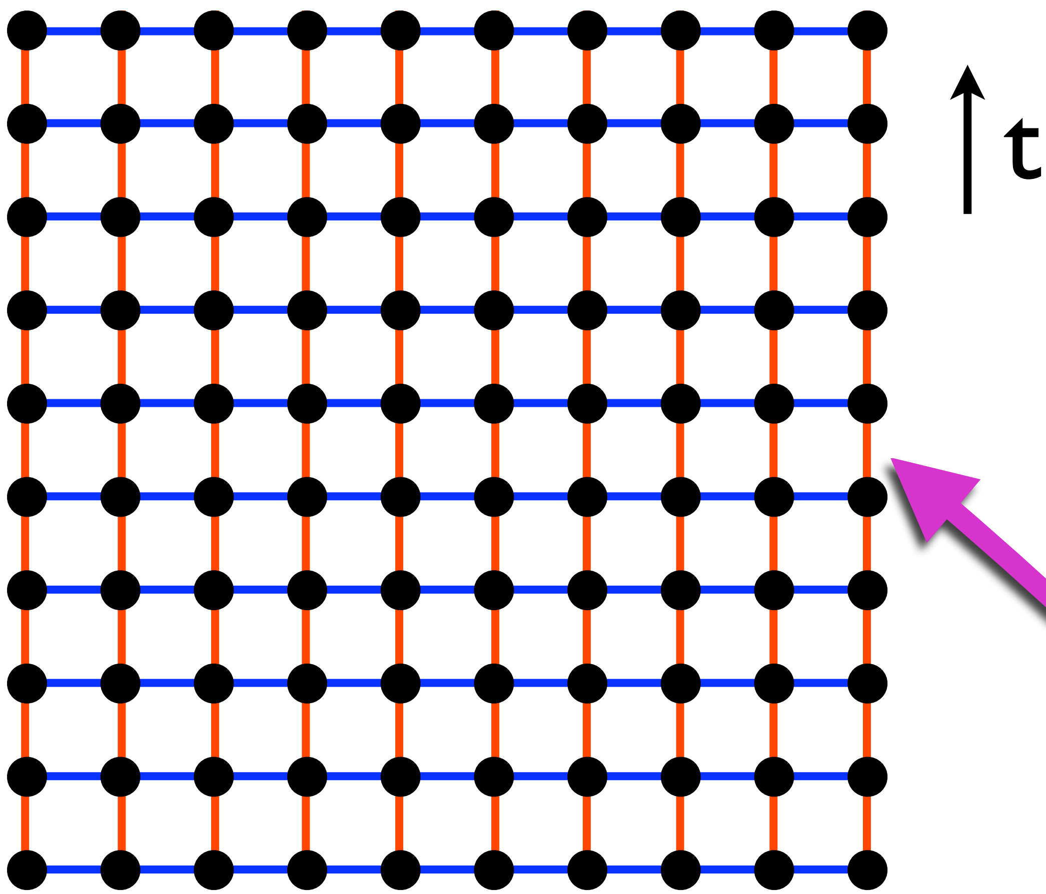

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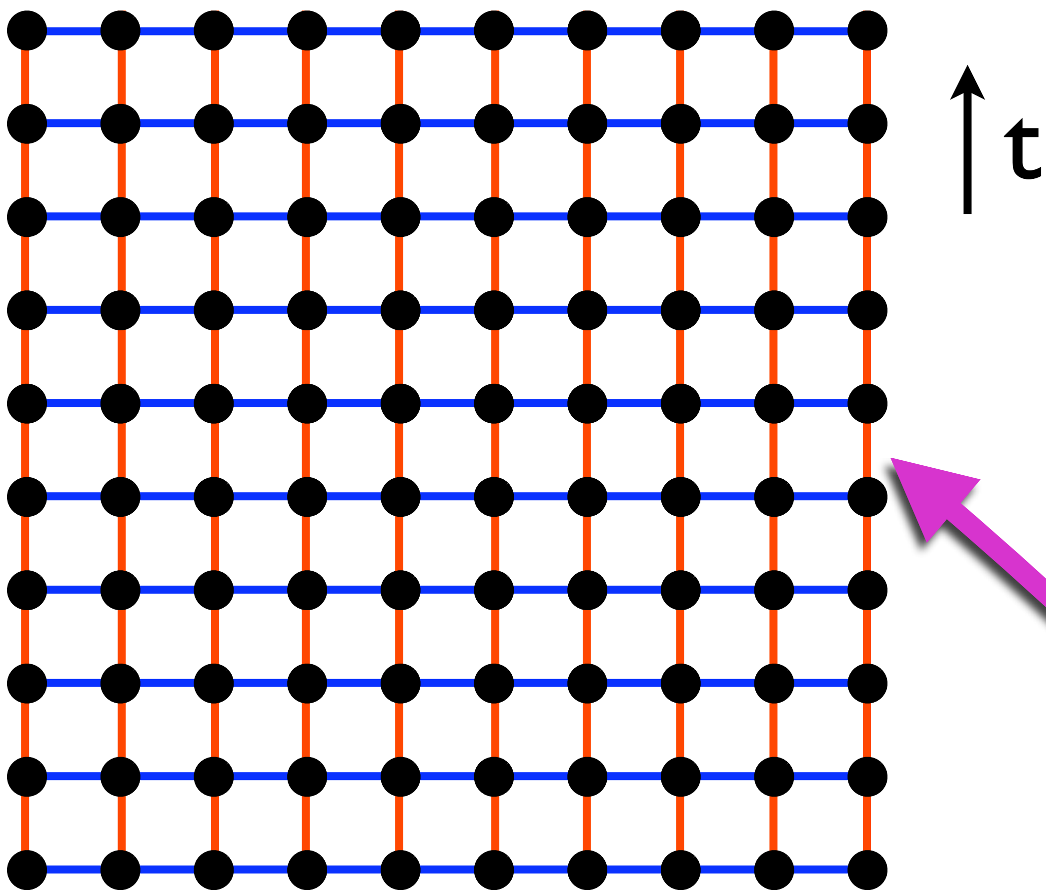

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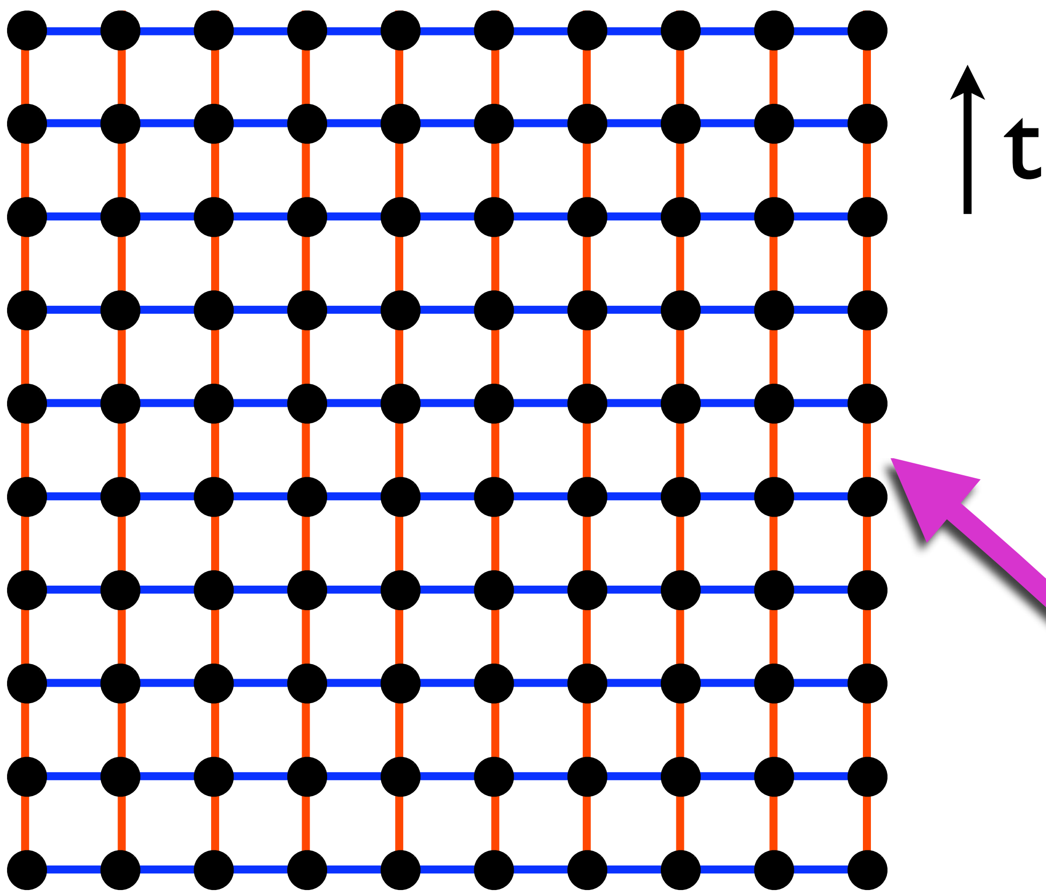

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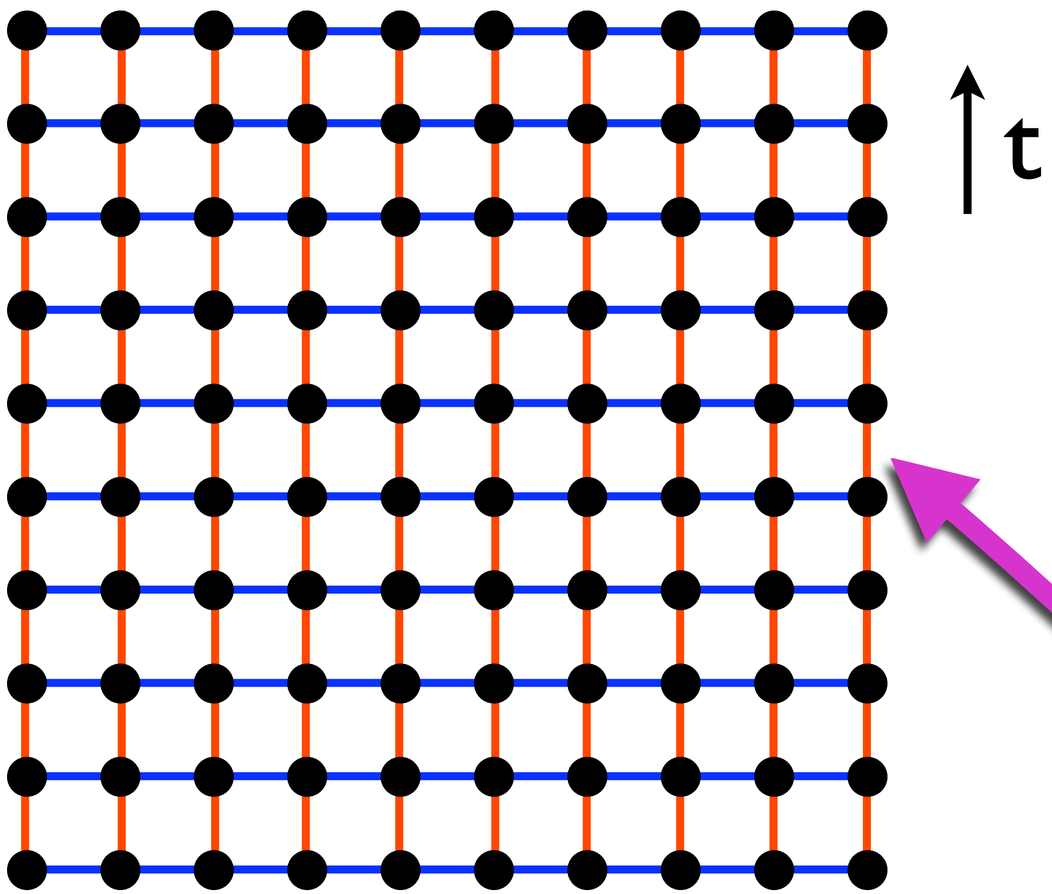

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- Compute correlation functions (like Savage)

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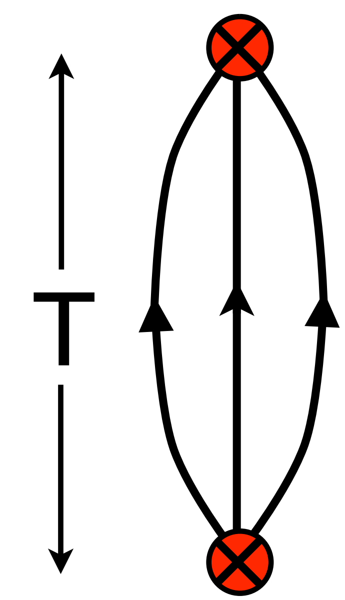
ii. Measure correlation function C(A): 3 quarks propagating for time T



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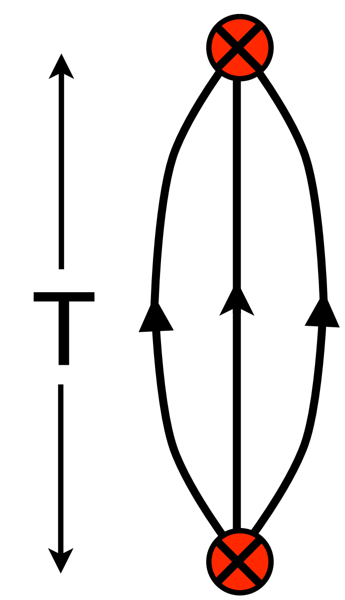

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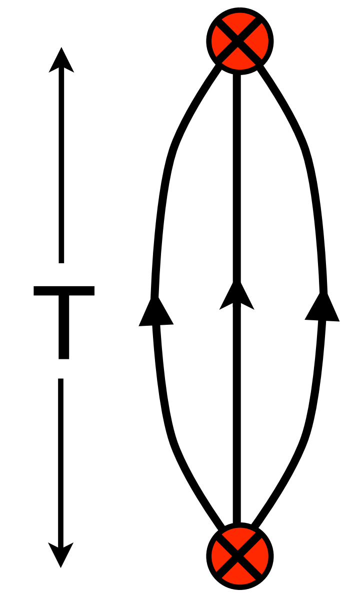

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But what is the signal/noise??

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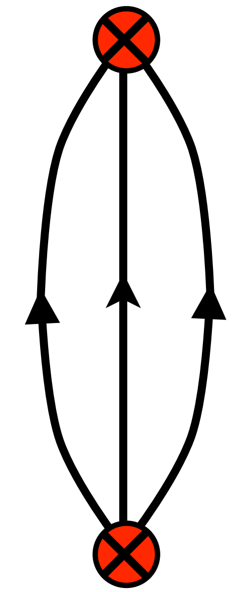

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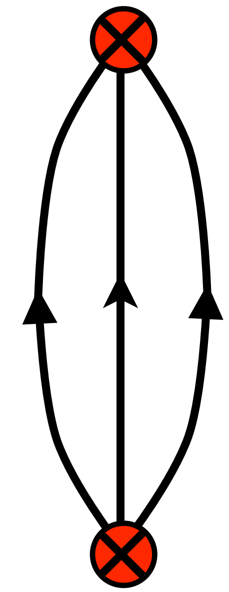

Dispersion in measurement:

anti-quarks
quarks

$$\sigma = \langle C^\dagger C \rangle = \frac{1}{N} \sum_{\{A\}} C^\dagger(A) C(A) \propto e^{-3\boxed{m_\pi}T} + \dots$$

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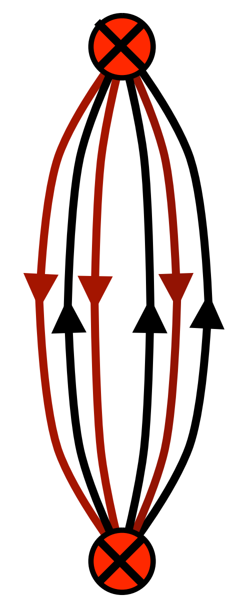

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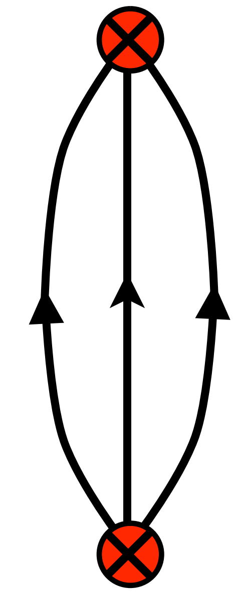

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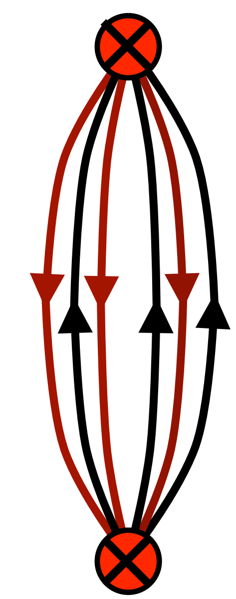

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($M \sim 7m_\pi$! big suppression)

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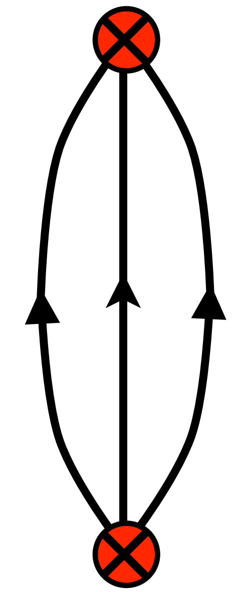

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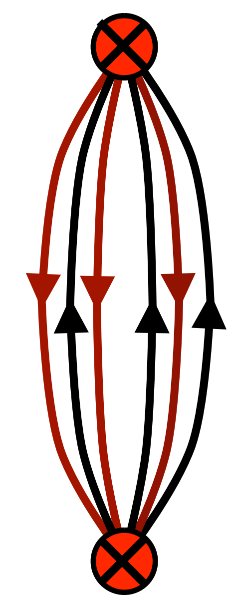

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For A nucleons: exponentially hard

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In a single given gauge field configuration, the quarks are confused!

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The problem: lightest state per quark $\neq$ baryons

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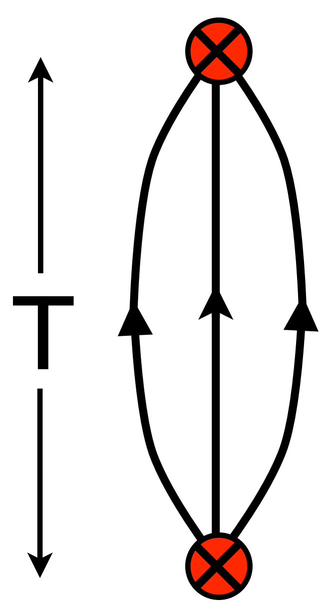

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Currently can study ~12 pions in a box using LQCD

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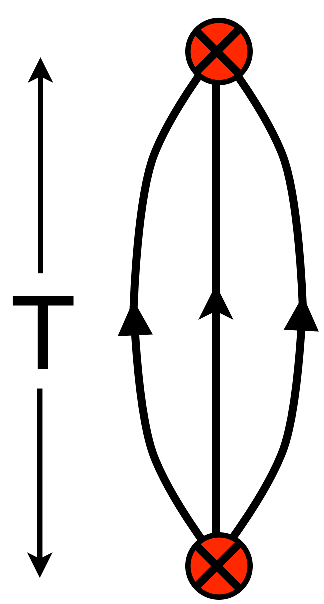

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A new approach currently being explored

Going beyond 2 species (spin states) of unitary fermions: *some tricks*

- Implementation of nonperturbative 3-body interaction via discrete HS field:

$$\frac{1}{3}\sum_{\omega} e^{\omega(n^\dagger n + p^\dagger p)} = e^{(n^\dagger n + p^\dagger p)^3} \qquad \omega = \{1,\ e^{2\pi i/3},\ e^{4\pi i/3}\}$$

2 flavors, 2 spins

- Other tricks for inserting pions, C_n coefficients perturbatively w/o tears for implementing KSW expansion

Can this new lattice EFT offer a different (and for some things, *better*) way to do N-body nuclear physics?

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Stay tuned!