Nuclear clusters in Halo EFT

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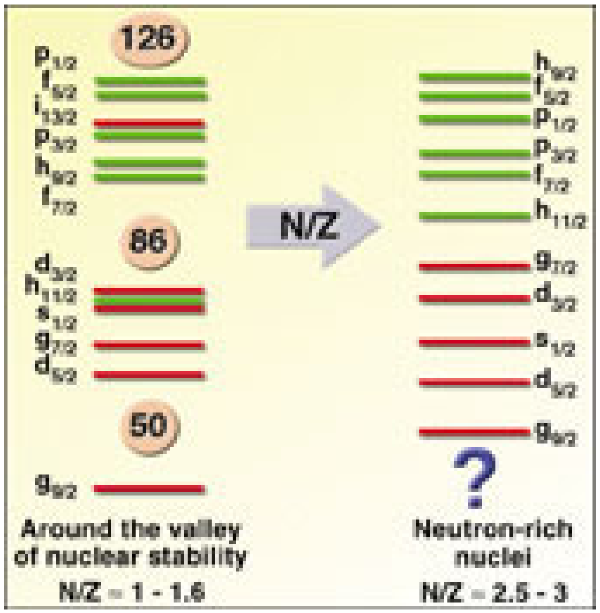
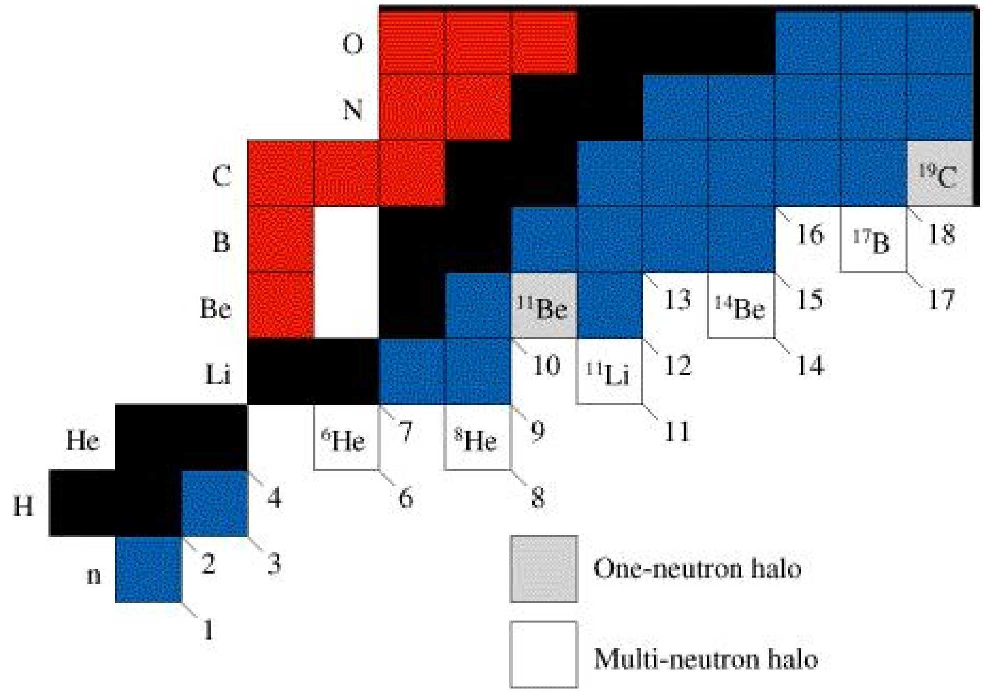
Outline

- Motivation
- EFT with contact interactions
  - ⋆ universality
- halo/cluster EFT
  - ⋆ Coulomb interactions
  - ⋆ $\alpha\alpha$ scattering
  - ⋆ $N\alpha$ scattering
- Summary and outlook



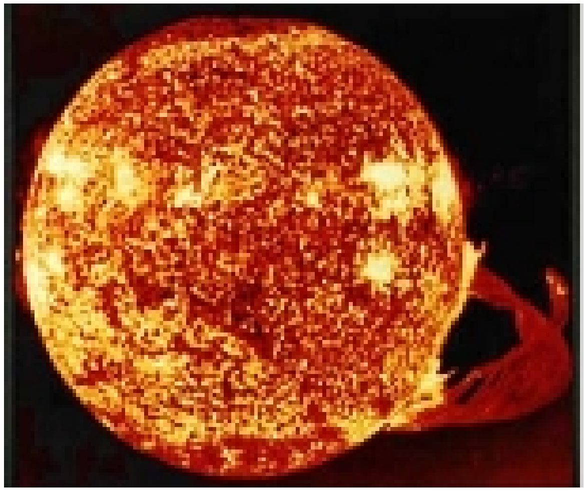
http://fy.chalmers.se/subatom/halo/halo.html



http://www.ornl.gov/info/ornlreview/v34_2_01/search.htm

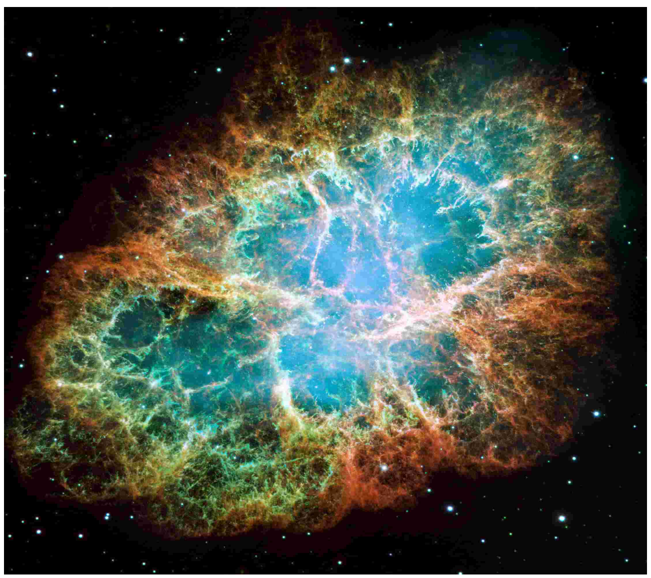
- $N/Z \gg 1$: challenge for shell model
- few nucleon systems: formation of halo systems

⋆ ^{11}Be , ^{19}C, ^{11}Li , ^{6}He , ^{14}Be, ^{8}He, ^{8}B, ^{17}Ne, ...



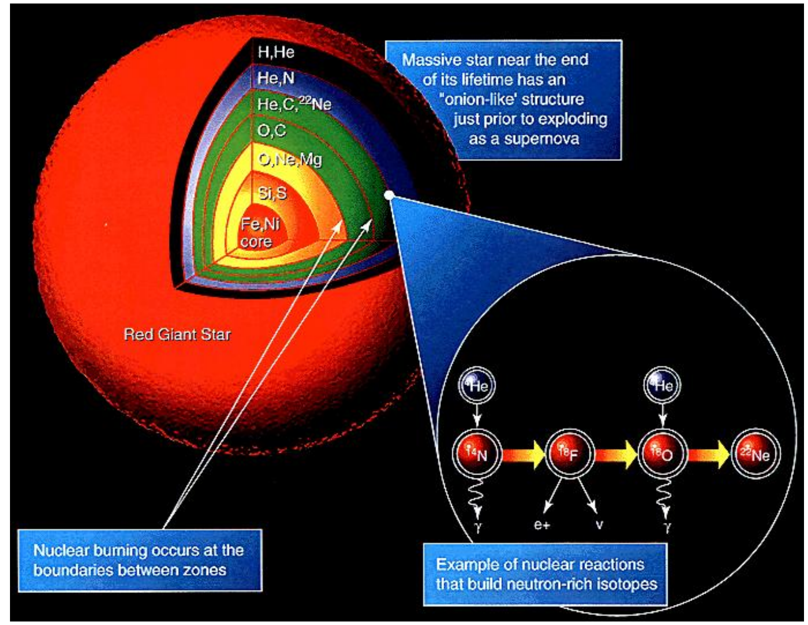
$p + {}^{7}\text{Be} \rightarrow {}^{8}\text{B} + \gamma$

solar neutrinos



$^{9}\text{Be}(\frac{1}{2}_{+}) + \alpha \rightarrow {}^{12}\text{C} + \text{n}$

core-colapse supernovae



$^{8}\text{Be} + \alpha \rightarrow {}^{12}\text{C} + \gamma$

red giants

EFT with short-range interactions

$k \sim 1/a \sim M_{lo}\,, \quad 1/R \sim M_{hi}$

- 2-body: shallow bound state ($E_2 = \hbar^2/ma^2 + \cdots$), scaling limit @ LO
  RG flow towards a non-trivial fixed point (Birse *et al.*, ...)
  $|a| \to \infty$: unitary limit $\Rightarrow$ no scales (NR-conformal invariance)

- 3-body: correct renormalization requires a 3-body interaction at LO
  $\Rightarrow$ its functional dependence exhibits a limit cycle

- new paradigm in understanding the Thomas collapse and the Efimov effect

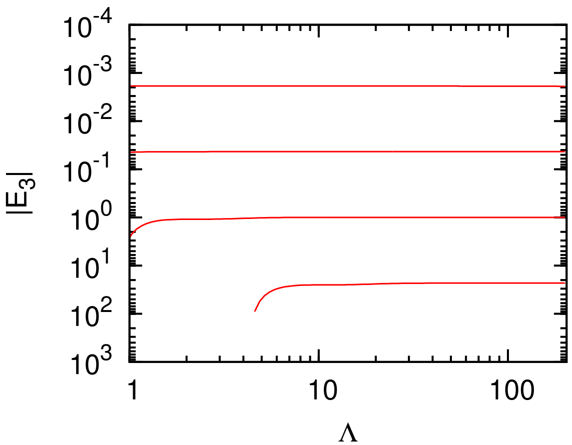
Thomas: V_2 range $\to 0$, E_2 fixed
$|E_3| \to \infty$

Efimov: $|a| \to \infty$, large n
$E_3^{(n+1)}/E_3^{(n)} \to e^{-2\pi/s_0}$
$s_0 \approx 1.00624$

⋆ Efimov, Amado and Nobel, Adhikari *et al.*, Minlos and Fadeev, Frederico *et al.*, Fedorov *et al.*, ...

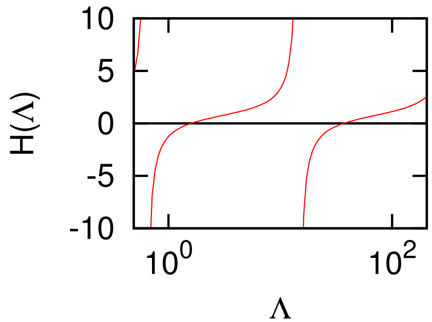
EFT and limit cycles: universality

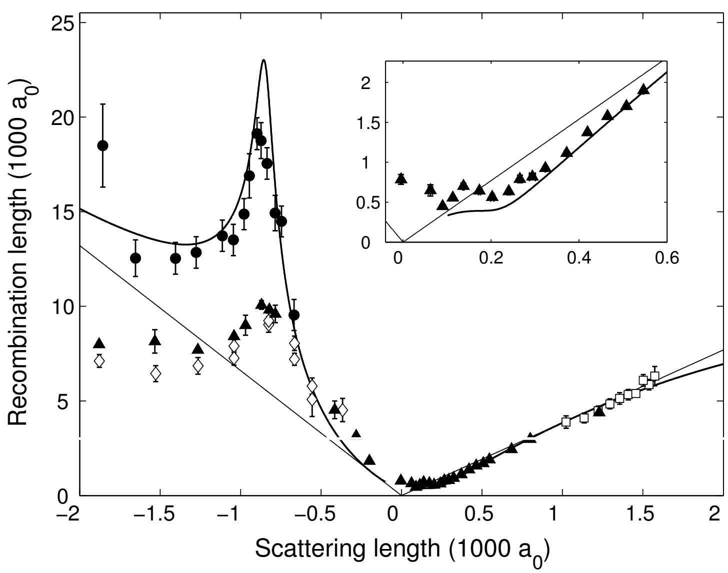
(H.-W. Hammer and R.H., Eur. J. Phys. A 37, 193)



$$\Leftrightarrow \quad \frac{\mathrm{doll}^{(n+1)}}{\mathrm{doll}^{(n)}} \approx 0.77 \approx e^{-2\pi/s_0}$$

with $s_0 \approx 23.95$





◇ Recombination length in ^{133}Cs atoms

(T. Krämer *et al.*, Nature 440, 315)

- evidence for Efimov states
- universal functions provided by EFT

(E. Braaten and H.-W. Hammer, Phys. Rept. 428, 259)

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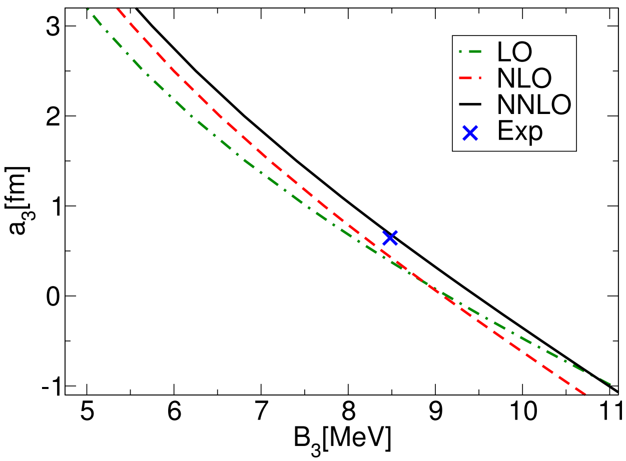
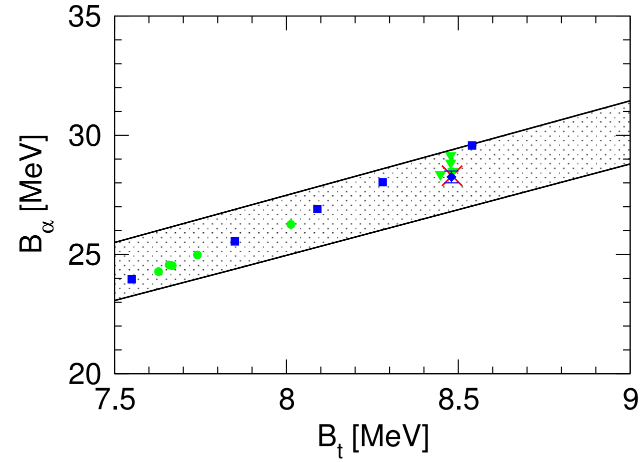
EFT and limit cycles in nuclear systems



(L. Platter, Phys. Rev. C 74, 037001)

- Phillips line: B_t vs. $a_{nd}^{1/2}$
  V. Efimov and E.G. Tkachenko (1985),
  P.F. Bedaque *et al.* (2000), L. Platter (2006)

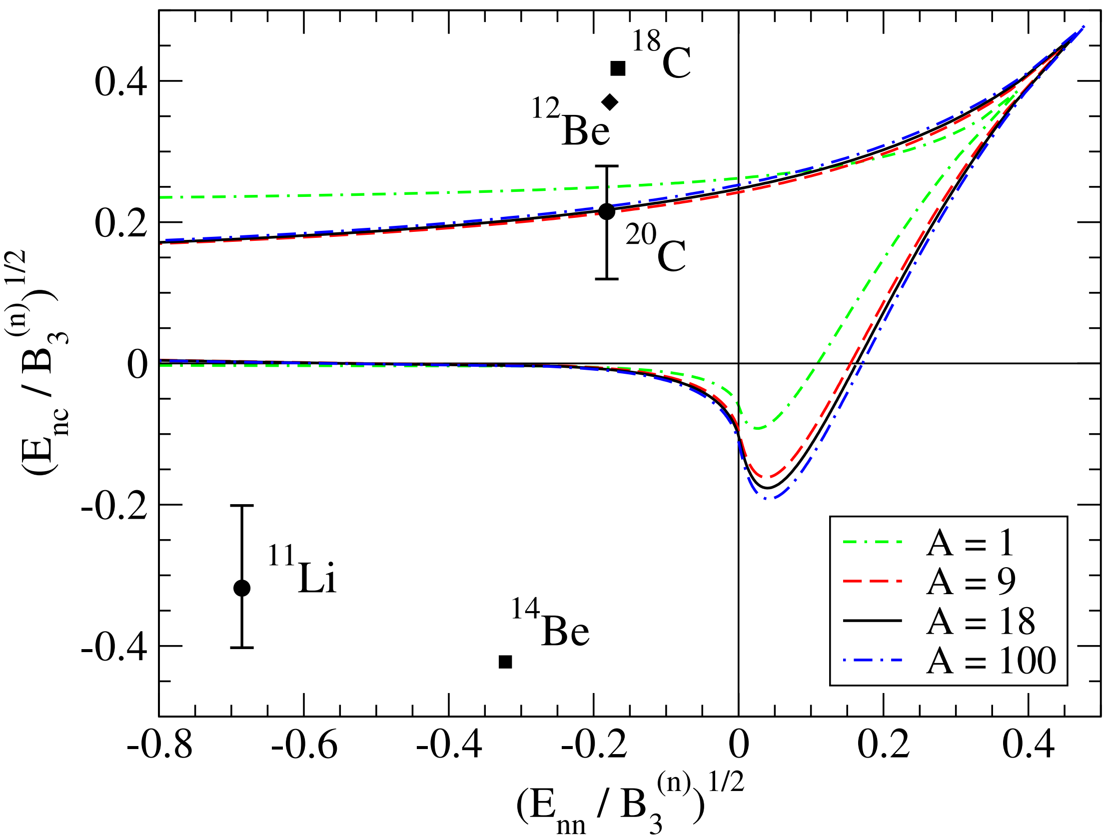
(Phys. Lett. B 607, 254)



- Tjon line: B_t vs. B_α
  L. Platter, H.-W. Hammer,
  U.-G. Meißner (2005)

- Friar *et al.*: B_t vs. $\sqrt{\langle r_c^2 \rangle_t}$ (L. Platter and H.-W. Hammer, 2007)
- EM/W processes Chen and Savage, Rupak, Sadeghi *et al.*, Kong and Ravndal, Ando *et al.*, Phillips *et al.*, ...

Efimov states in nuclei?



- Canham and Hammer, Eur. Phys. J. A 37, 367 (2008)
- Amorim *et al.*, Phys. Rev. C 56, R2378 (1997)

halo/cluster EFT: separation of scales

- excitation of each cluster $\sqrt{m_c E_c^*} \sim M_{hi} \ (\gtrsim m_\pi)$
- binding of the valence nucleons (clusters) $\sim M_{lo} \ll M_{hi}$
- extension of the core—treated in *perturbation theory*
- power-counting: modified to account for other effects (resonance/Coulomb)
- expansion around the resonance: rearrangement of the perturbative series, improved convergence
- Coulomb interactions

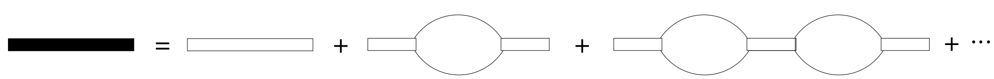
halo/cluster EFT: $k \ll m_\pi, \sqrt{m_c E_c^*} \sim M_{hi}$

Physical quantities: $k, 1/a_0 \sim M_{lo}, \qquad r_0 \sim M_{hi}^{-1}, \mathcal{P} \sim M_{hi}^{-3}, \ldots$

$$T_l = -\frac{2\pi}{\mu} \frac{k^{2l}(2l+1)}{k^{2l+1}(\cot\delta_l - i)} P_l(\cos\theta)$$

$$k^{2l+1}\cot\delta_l \approx -1/a_l + \frac{r_l}{2}k^2 + \frac{\mathcal{P}_l}{4}k^4 + \cdots$$

$$\mathcal{L} = \phi^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{4\mu} \right] \phi + \sigma\, d^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{8\mu} - \Delta \right] d + g \left[d^\dagger \phi\phi + (\phi\phi)^\dagger d \right] + \cdots ,$$



$$\Delta \sim M_{lo} \quad \rightarrow \quad iD_d^{(0)} = \frac{i\sigma}{-\Delta + i\epsilon} \sim \frac{1}{M_{lo}} \qquad (NN)$$

$$\Delta \sim M_{lo}^2/\mu \quad \rightarrow \quad iD_d^{(0)} = \frac{i\sigma}{q_0 - \boldsymbol{q}^2/8\mu - \Delta + i\epsilon} \sim \frac{\mu}{M_{lo}^2} \qquad (\alpha\alpha)$$

- non-perturbative Coulomb (Kong and Ravndal, NPA 665, 137)

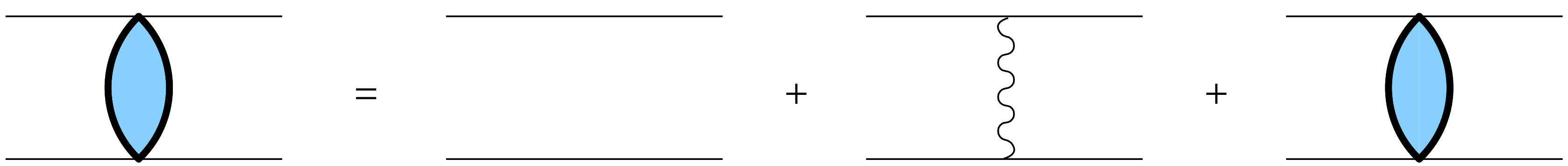
Coulomb wave functions: $|\boldsymbol{k}\rangle \rightarrow |\chi_k^{(\pm)}\rangle$

$$\langle \boldsymbol{r}|\chi_k^{(\pm)}\rangle \equiv \chi_k^{(\pm)}(\boldsymbol{r}) = e^{-\frac{\eta\pi}{2}}\,\Gamma(1 \pm i\eta)\, M(\mp i\eta, 1; \pm ikr - i\boldsymbol{k}\cdot\boldsymbol{r})\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

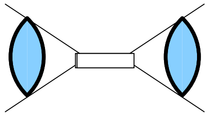
$$F_l \sim \sin(kr - l\pi/2 - \eta \ln 2kr + \sigma_l)\,, \qquad G_l \sim \cos(kr - l\pi/2 - \eta \ln 2kr + \sigma_l)$$

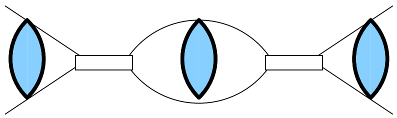
$$\sigma_l = \arg\Gamma(1 + l + i\eta)\,, \qquad \eta = Z^2 \alpha_{em}\mu/k$$

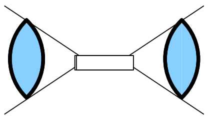
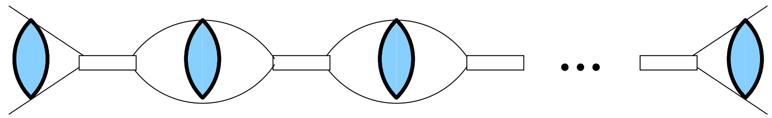
$$G_C^{(\pm)}(E) = \frac{1}{E - \hat{H}_0 - \hat{V}_C \pm i\epsilon} = 2\mu \int \frac{d^3q}{(2\pi)^3} \frac{|\chi_q^{(\pm)}\rangle\langle\chi_q^{(\pm)}|}{2\mu E - \boldsymbol{q}^2 \pm i\epsilon}$$



$$T_{CS} = \langle \chi_{k'}^{-} | \, \hat{V}_S \, | \chi_k^{+} \rangle + \langle \chi_{k'}^{-} | \, \hat{V}_S \, G_C^{+} \, \hat{V}_S \, | \chi_k^{+} \rangle + \cdots$$

$T_{CS}^{(0)} =$  $= C(E) \, \chi_{k'}^{(-)\,*}(0) \, \chi_k^{(+)}(0) = C(E) \, C_\eta^{(0)\,2} \, e^{2i\sigma_0} \,,$

$T_{CS}^{(1)} =$  $= C(E) \, C_\eta^{(0)\,2} \, e^{2i\sigma_0} \, C(E) \, J_0(E) \,,$

$T_{CS} =$  $+ \cdots +$ 

$$= C_\eta^{(0)\,2} \, \frac{C(E) \, e^{2i\sigma_0}}{1 - C(E) \, J_0(E)} \,,$$

$$J_0(E) = 2\mu \int \frac{d^3q}{(2\pi)^3} \, \frac{\chi_q^{(+)}(0) \chi_q^{(+)\,*}(0)}{k^2 - \boldsymbol{q}^2 \pm i\epsilon} = 2\mu \int \frac{d^3q}{(2\pi)^3} \, \frac{2\pi\eta_q}{e^{2\pi\eta_q} - 1} \, \frac{1}{k^2 - q^2 + i\epsilon}$$

$(\eta_q = 1/a_B q = Z^2 \alpha_{em} \mu / q)$

$\alpha\alpha$ scattering

- $0+$ resonance (^{8}Be g.s.):

  $E_R^{\rm LAB} = 184.15 \pm 0.07$ keV , $\Gamma_R^{\rm LAB} = 11.14 \pm 0.50$ eV

  $M_{lo} \approx \sqrt{\mu E_R^{\rm LAB}} \sim 20$ MeV, $\quad M_{hi} \sim m_\pi \sim 140$ MeV

- power-counting: $E_{LAB} \leq 3.0$ MeV

- scattering: Afzal *et.al.* (1969)

  ⋆ $E_{LAB} \leq 3.0$ MeV : data from Heydenburg and Temmer (1956)
  ⋆ ERE parameters from Russell *et.al.* (1956), Rasche (1967):
  $a_0 = (-1.65 \pm 0.17) \times 10^3$ fm, $\quad r_0 = 1.084 \pm 0.011$ fm,
  $\mathcal{P}_0 = -1.76 \pm 0.22$ fm^3

$$T_{CS} = C_\eta^{(0)\,2}\,\frac{C(E)\,e^{2i\sigma_0}}{1 - C(E)\,J_0(E)} = -\frac{2\pi}{\mu}\,\frac{C_\eta^{(0)\,2}\,e^{2i\sigma_0}}{-\frac{1}{a_0} + \frac{r_0}{2}\,k^2 - i\epsilon + \frac{2\pi}{\mu}\,J_0(E)}$$

$$= -\frac{2\pi}{\mu}\,\frac{C_\eta^{(0)\,2}\,e^{2i\sigma_0}}{-\frac{1}{a_0^c} + \frac{r_0}{2}\,k^2 - \frac{2}{a_B}\,H(\eta)}\,,$$

$$a_B = \frac{1}{Z^2\alpha_{em}\mu} \sim \frac{1}{M_{hi}}$$

$$H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \ln(i\eta) \Rightarrow \begin{cases} \overset{\eta\ll 1}{\longrightarrow}\ \frac{a_B}{2}\,ik \\ \overset{\eta\gg 1}{\longrightarrow}\ \frac{1}{12}\,(a_B\,k)^2 + \frac{1}{120}\,(a_B\,k)^4 \end{cases}$$

- without Coulomb: conformal invariance in ^{8}Be , Efimov state in ^{12}C at LO (RH, H.-W. Hammer, van Kolck, Nucl. Phys. A 809, 171)
- with Coulomb: ^{8}Be and ^{12}C 0+ states remain close to threshold

fine-tuning puzzle

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{hi}^2}{\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(natural)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{hi}M_{lo}}{\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(fine-tuned like } NN\text{)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{lo}^2}{\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(fine-tuned to get } E_R\text{)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{lo}^3}{M_{hi}\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(fine-tuned to get } \Gamma_R\text{)}$$

$\sim$ factor of 1000!!!

(Oberhummer *et al.*, Science 289, 88; RH, H.-W. Hammer, van Kolck, 2008)

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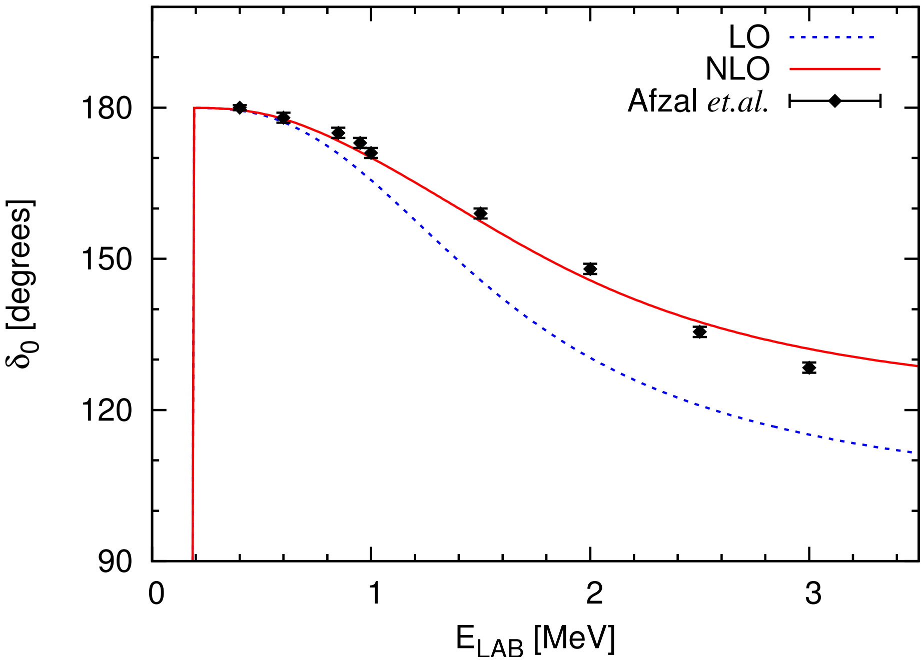
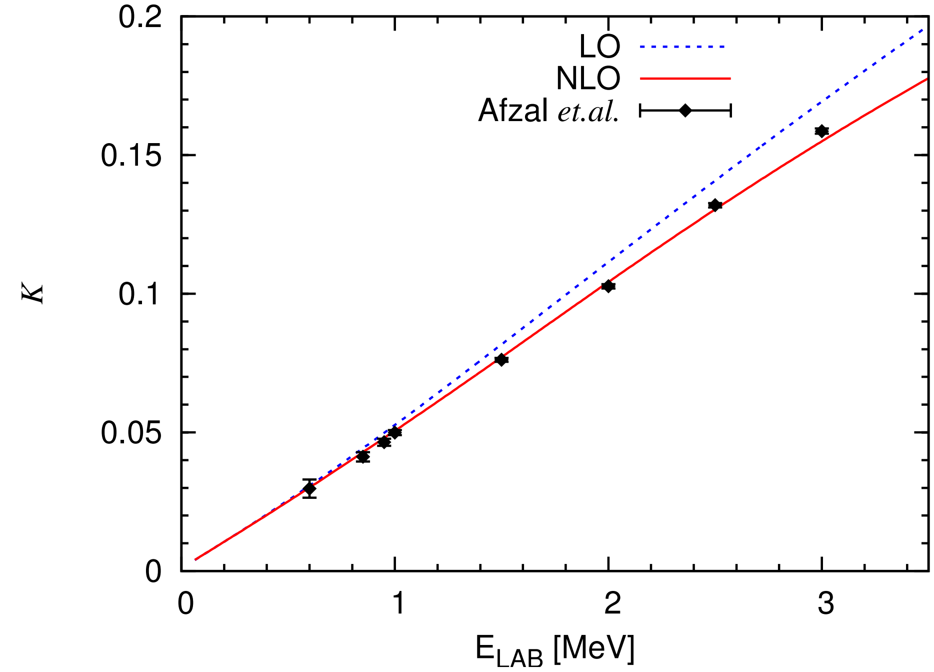
(RH, Hammer, van Kolck, 2008)





	a_0 (10^3 fm)	r_0 (fm)	$\mathcal{P}_0$ (fm^3)
LO	-1.80	1.083	—
NLO	-1.92 ± 0.09	1.098 ± 0.005	-1.46 ± 0.08
Rasche	-1.65 ± 0.17	1.084 ± 0.011	-1.76 ± 0.22

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$p\alpha$ scattering: $S_{1/2}$, $P_{3/2}$, $P_{1/2}$

$$\mathcal{L}_{\rm LO} = \phi^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2m_\alpha}\right]\phi+N^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2m_N}\right]N$$

$$+\eta_{1+}\, t^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2(m_\alpha+m_N)}-\Delta_{1+}\right]t$$

$$+\frac{g_{1+}}{2}\left\{t^\dagger\vec{S}^\dagger\cdot\left[N\vec{\nabla}\phi-(\vec{\nabla}N)\phi\right]+\text{H.c.}-r\left[t^\dagger\vec{S}^\dagger\cdot\vec{\nabla}(N\phi)+\text{H.c.}\right]\right\}$$

$$\mathcal{L}_{\rm NLO} = \eta_{0+}\, s^\dagger\left[-\Delta_{0+}\right]s+g_{0+}\left[s^\dagger N\phi+\phi^\dagger N^\dagger s\right]+g'_{1+}\, t^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2(m_\alpha+m_N)}\right]^2 t$$

(Bertulani, Hammer, van Kolck, NPA 712, 37;

Bedaque, Hammer, van Kolck, PLB 569, 159)

P-wave: $$k^3 \cot\delta_1 = -1/a_1 + r_1/2\, k^2 + \mathcal{P}_1/4\, k^4 + \cdots$$

$$\langle \boldsymbol{p}'|\, \hat{V}_S\, |\boldsymbol{p}\rangle = \frac{\eta_{1+}\, g_{1+}^2\, (S_j S_k^\dagger)_{\beta\alpha}}{q_0 - \boldsymbol{q}^2/2(m_\phi + m_N) - \Delta_{1+} + i\varepsilon}\, p_j'\, p_k$$

Amplitude:

$$T_{CS}^{\mathrm{LO}} = -C_\eta^{(1)\,2}\, e^{2i\sigma_1}\, \frac{2\pi}{\mu}\, \frac{k^2\left(2\cos\theta + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\sin\theta\right)}{-\frac{1}{a_{1+}} + \frac{r_{1+}}{2}\, k^2 + i\epsilon + \frac{6\pi}{\mu}\, J_1(E)}$$

$$= -\frac{2\pi}{\mu}\left[F(k,\theta) + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}G(k,\theta)\right]$$

$$J_1 = -\frac{\mu}{3\pi a_B}\left(k^2 + \frac{1}{a_B^2}\right)\left[H(\eta) + \frac{1}{D-4} - \ln\left(\frac{a_B\kappa\sqrt{\pi}}{2}\right) - 1 + \frac{3}{2}C_E\right] - \frac{4\pi\mu}{3a_B^3}\,\zeta'(-2)$$

$$\left[a_B = 1/(Z_1 Z_2 \alpha_{em}\mu)\right]$$

$p\alpha$ scattering

- $T_{\mathrm{LO}} = T_{\mathrm{LO},S_{1/2}} + T_{\mathrm{LO},P_{3/2}}$
- $T_{\mathrm{NLO}} = T_{\mathrm{NLO},S_{1/2}} + T_{\mathrm{NLO},P_{3/2}}$
- $T_{\mathrm{N^2LO}} = T_{\mathrm{N^2LO},S_{1/2}} + T_{\mathrm{N^2LO},P_{3/2}} + T_{\mathrm{LO},P_{1/2}}$

$$F_C(k,\theta) = -\frac{\eta}{2k}\,\csc^2\theta/2\,\exp\left[i\eta\ln(\csc^2\theta/2) + 2i\sigma_0\right]$$

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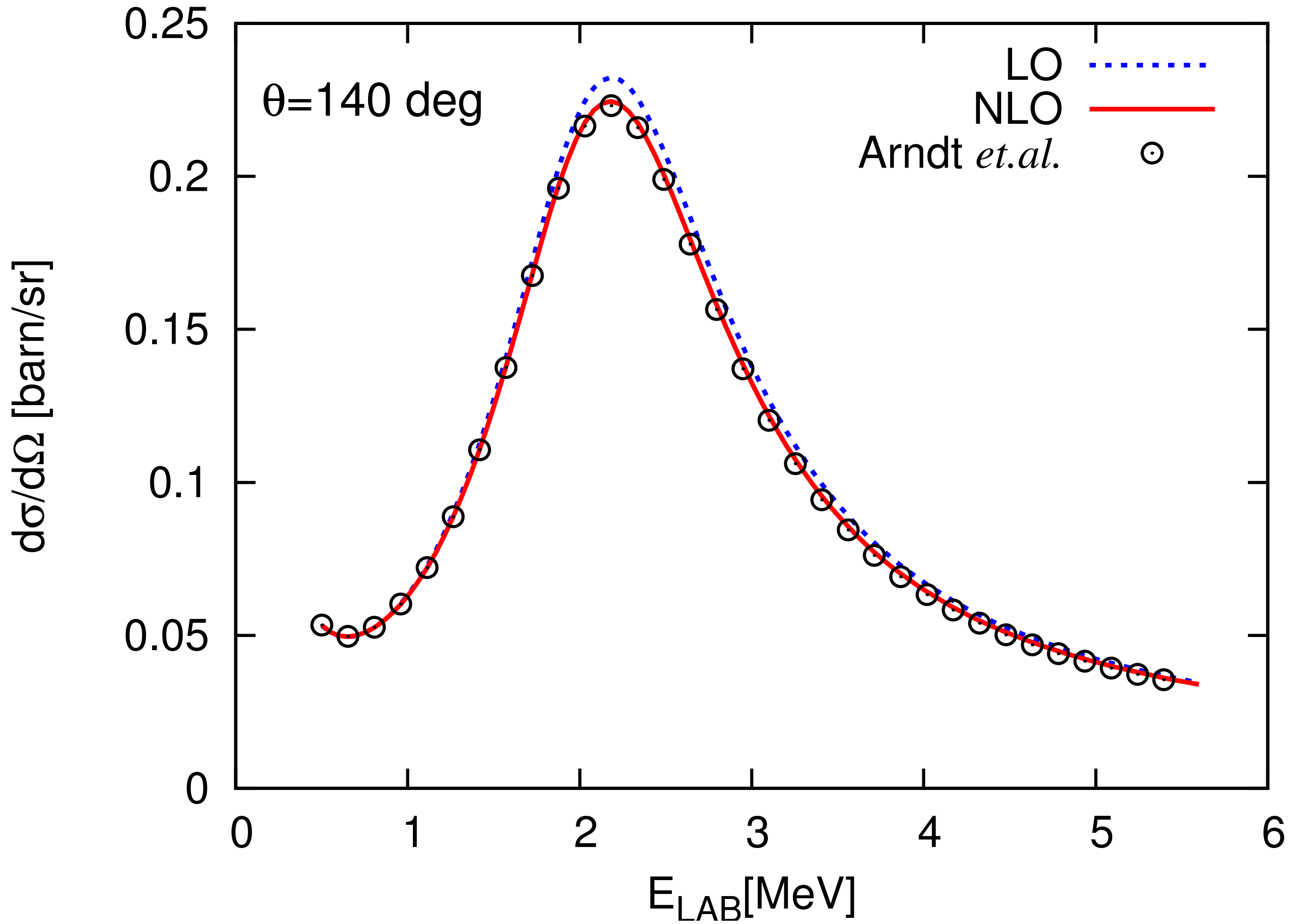
(RH, Bertulani, van Kolck, in progress)

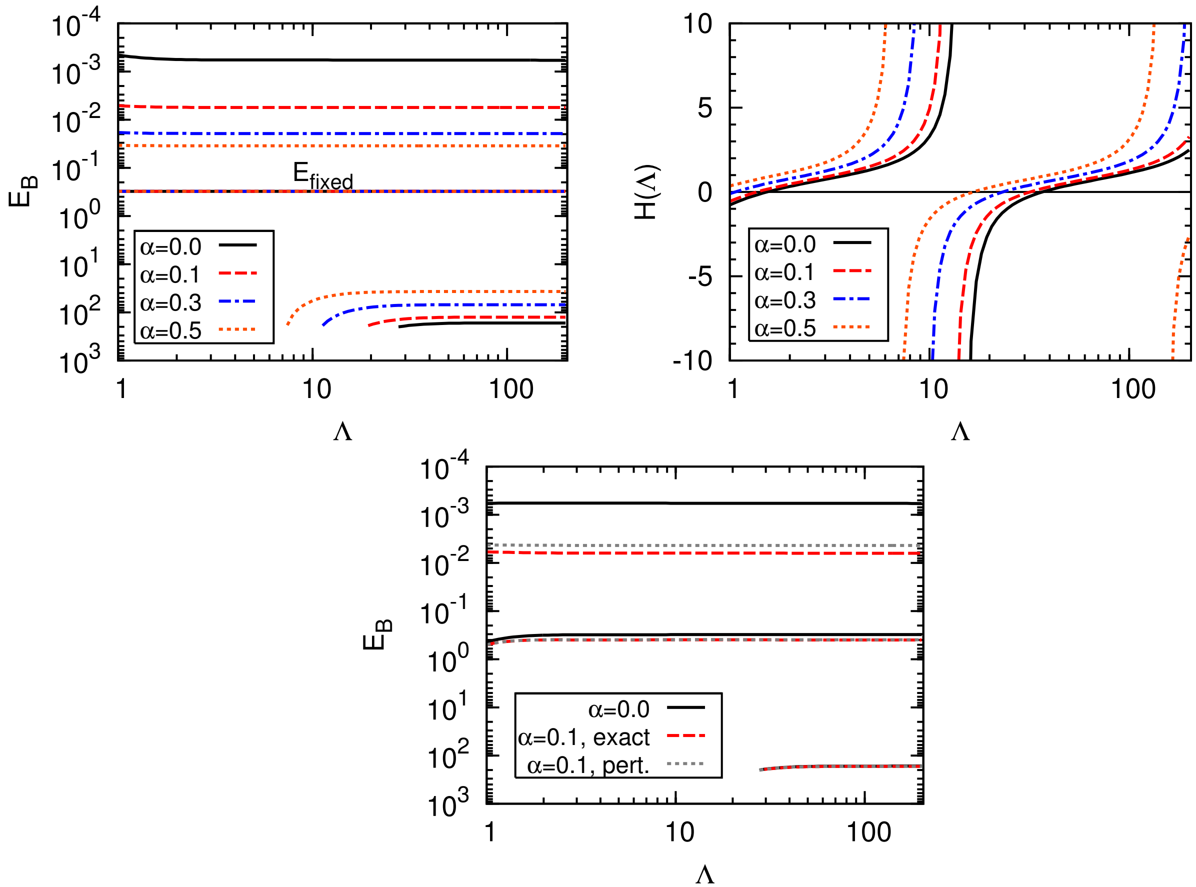


c/r^2+Coulomb: warm-up for 3α

- 3-body problem with large a $\sim$ 1D Schrödinger Eq. with $V(R) = 1/R^2$
- limit cycle for $c < -1/4$ $\Leftrightarrow$ Efimov spectrum
  (Beane *et al.*, Bawin and Coon, Braaten and Phillips, ...)
- counterterm: log-periodic function of the cutoff
- Counterterm parameter Λ_*: iteration of quantum corrections
  (dimensional transmutation)

$1/r^2$+Coulomb: warm-up for 3α

(H.-W. Hammer, RH, Eur. J. Phys. A 37, 193)



Summary

- Halo nuclei, cluster systems: promising area for halo/cluster EFT
- universality $\Leftrightarrow$ limit cycles
- $\alpha\alpha$ scattering
  - ⋆ Coulomb turned off $\Rightarrow$ conformal invariance @LO, Efimov spectrum in ^{12}C
  - ⋆ incredible amount of fine-tuning
  - ⋆ LO (parameter-free) works only at very low energies, NLO improves description up to $E_{LAB} \approx 3$ MeV
  - ⋆ extraction of the ERE parameters with improved errorbars
- p-α scattering: good description of the $P_{3/2}$ resonance
- future: 3α, p-^{7}Be, Borromean halos, heavier nuclei, ...

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