Nuclear matter and pairing in finite nuclei from low-momentum interactions

Kai Hebeler (TRIUMF)

Effective Field Theories and the Many-Body Problem

INT, Seattle, April 9, 2009

TRIUMF

Outline

1. Importance of nuclear matter results
2. Chiral 3N interactions as density-dependent two-body interactions
3. Results for neutron matter from chiral low-momentum interactions
4. Effective-mass approximations in DFT for calculations of pairing gaps
5. Conclusions and outlook

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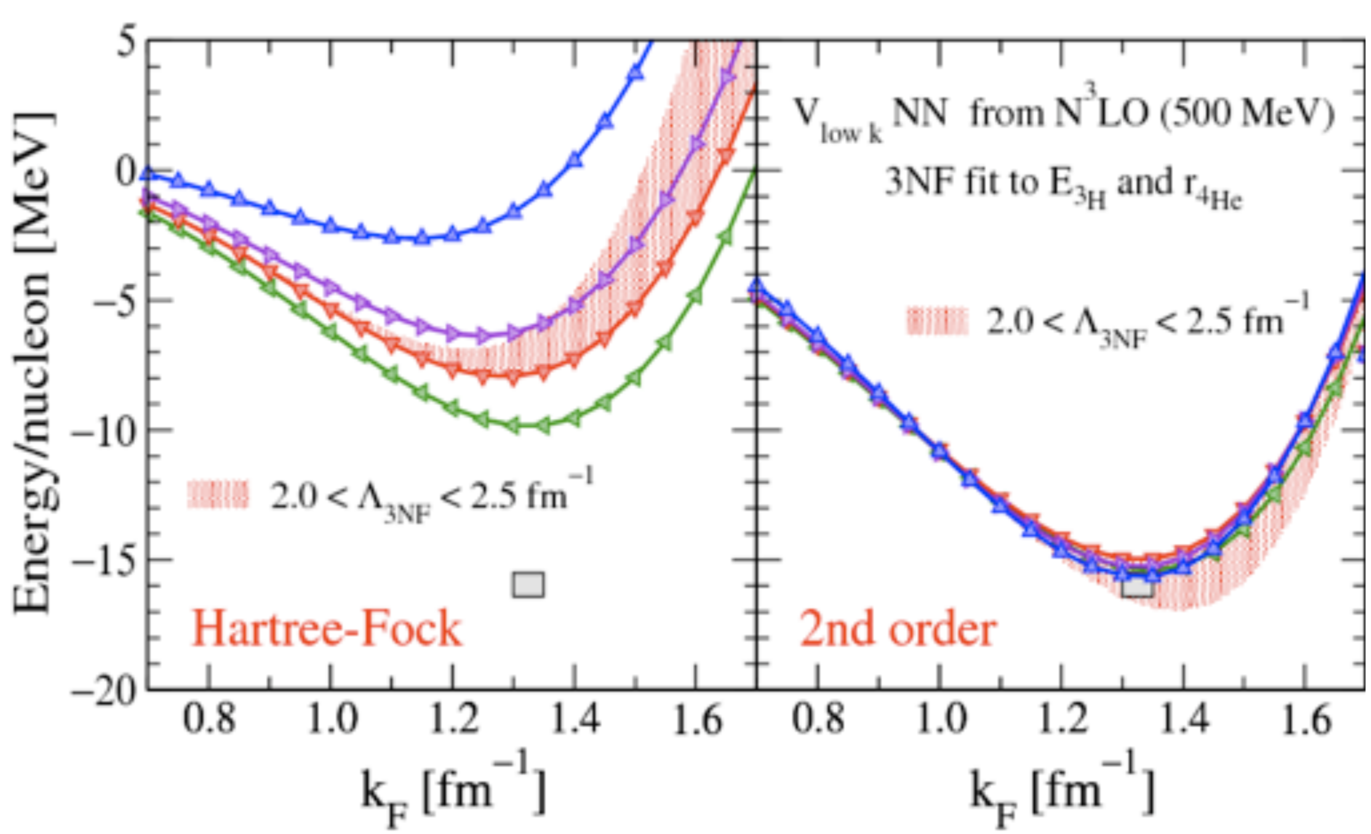
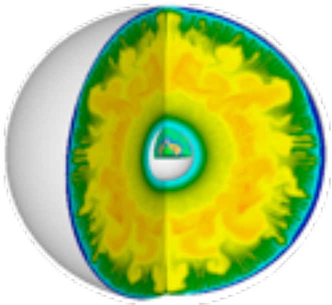
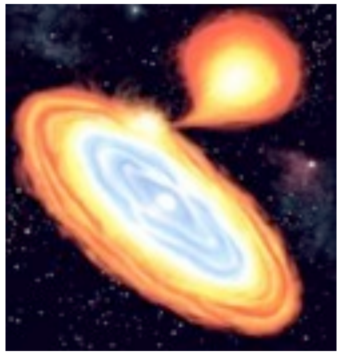
Importance of nuclear matter results

- for the extremes of astrophysics: neutron stars, supernovae, neutrino interactions with nuclear matter
- constraining energy-density functionals, next-generation Skyrme functionals
- my focus: development of reliable and efficient methods to include 3N forces in microscopic many-body calculations of nuclear matter and finite nuclei.

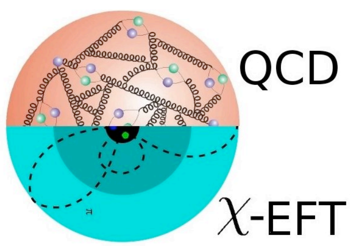


Bogner, Furnstahl, Nogga, Schwenk (2009)





Energy-density functionals for finite nuclei



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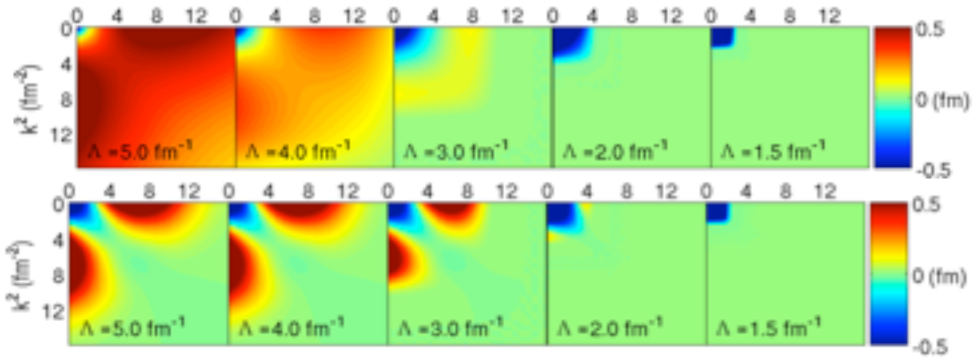
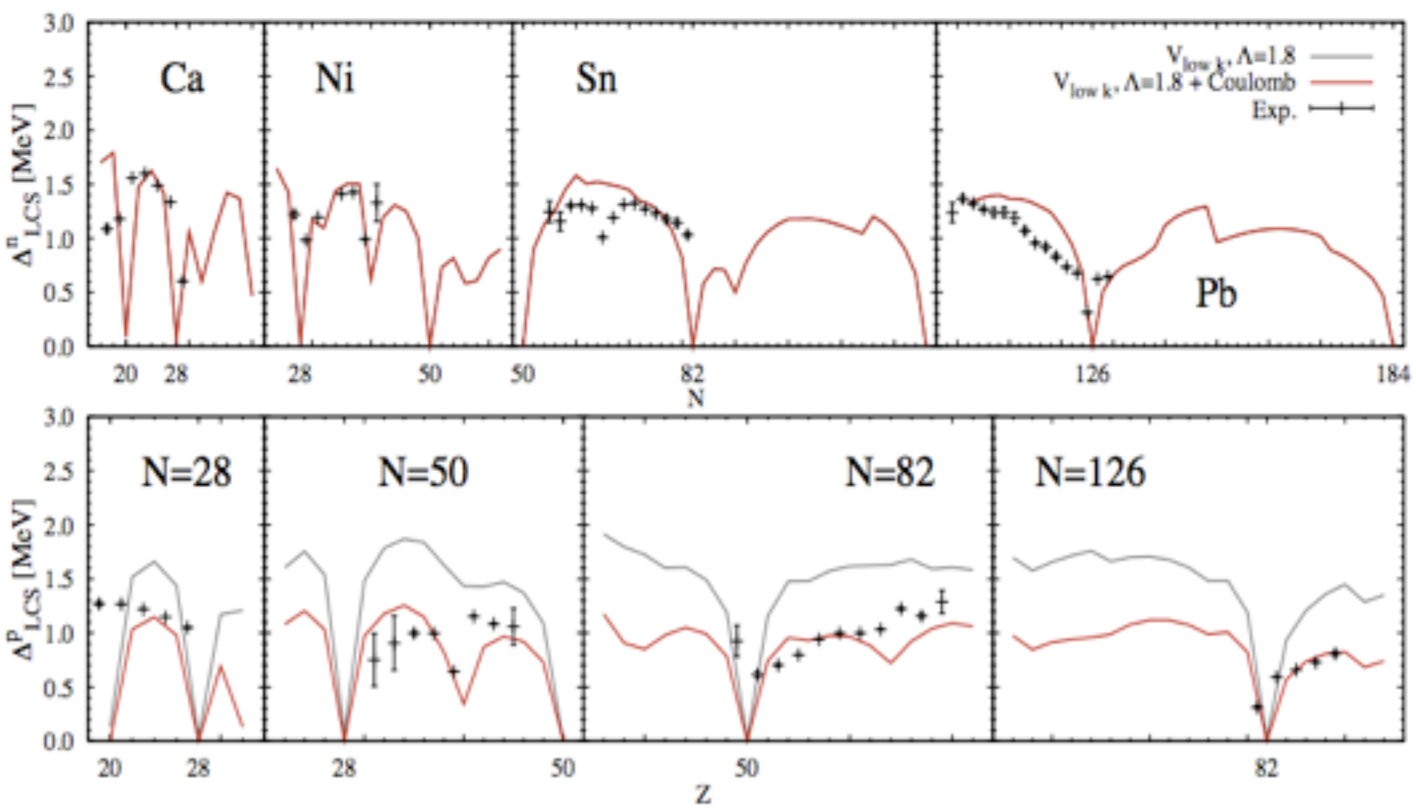


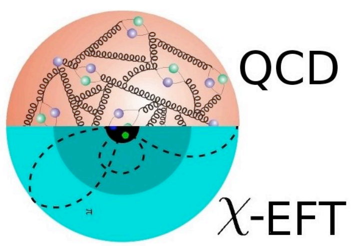
→ microscopic constraints for EDFs

Pairing gaps in semi-magic nuclei



Lesinski, Duguet (2008): talk by T. Lesinski

Energy-density functionals for finite nuclei



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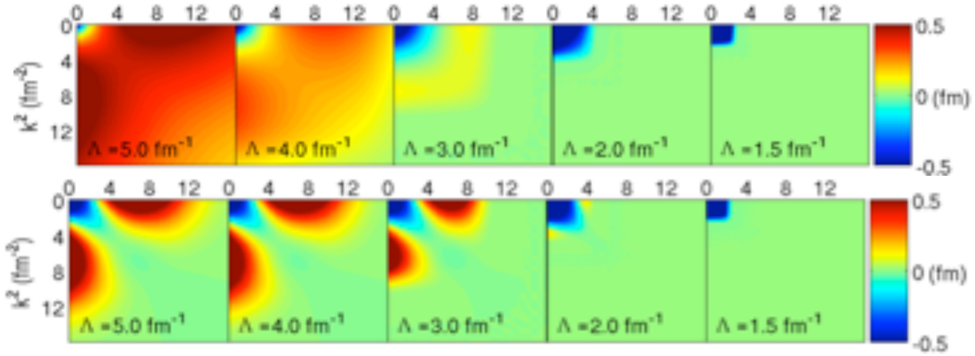

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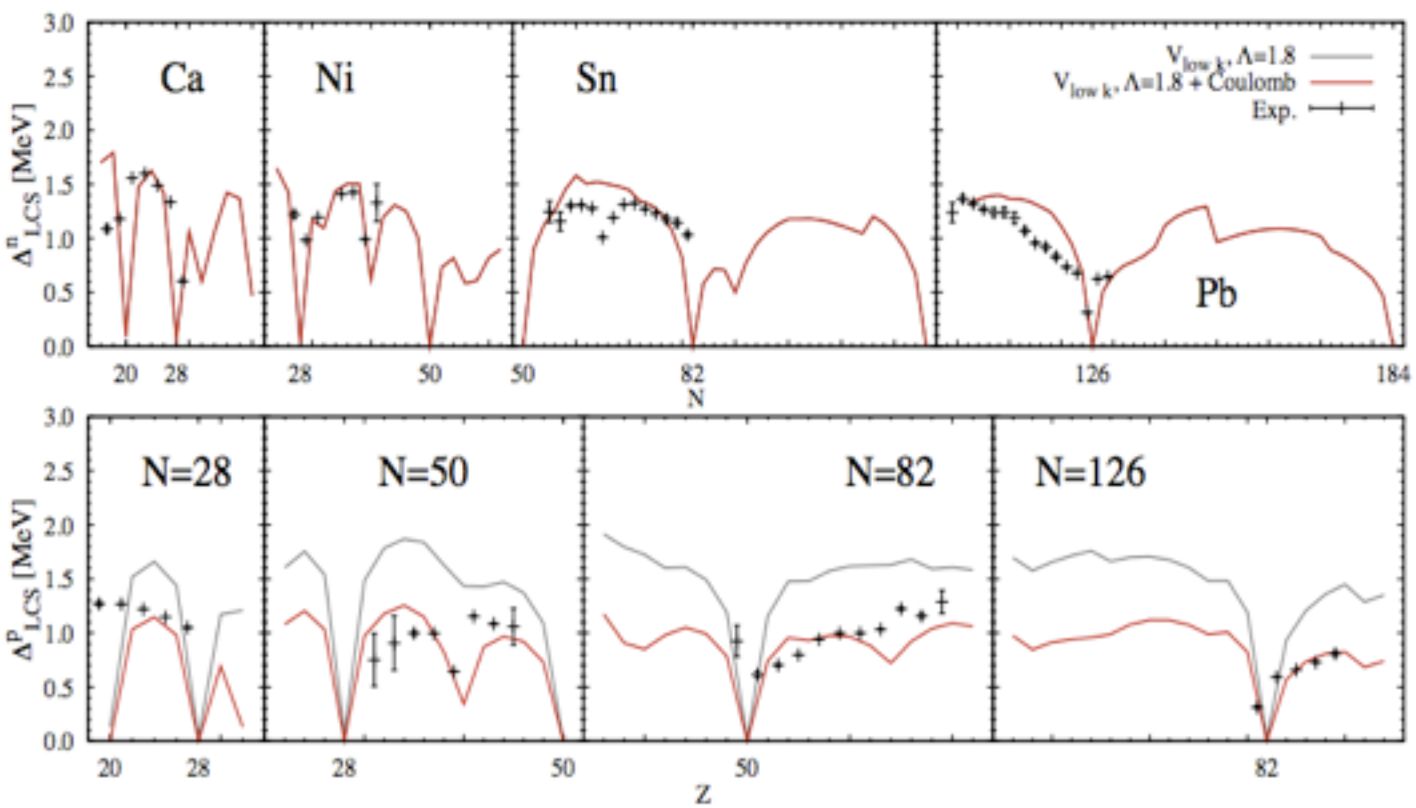

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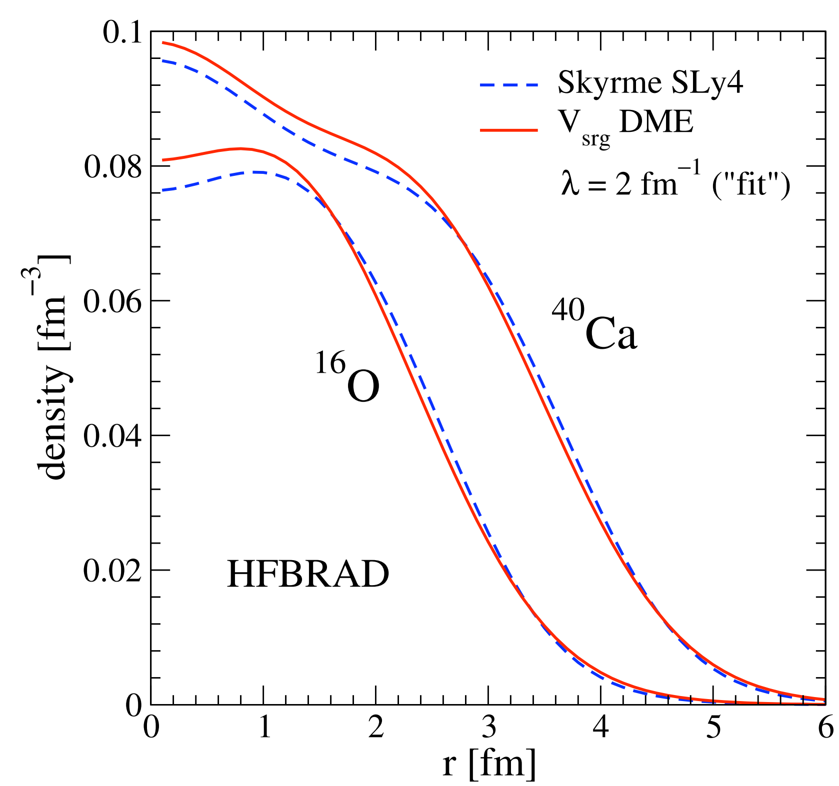
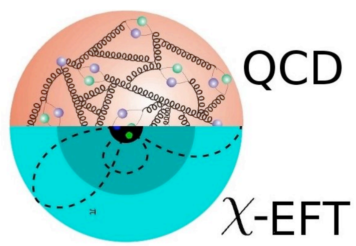
Density Matrix expansions for low-momentum interactions



Bogner, Furnstahl, Platter (2008)

Energy-density functionals for finite nuclei



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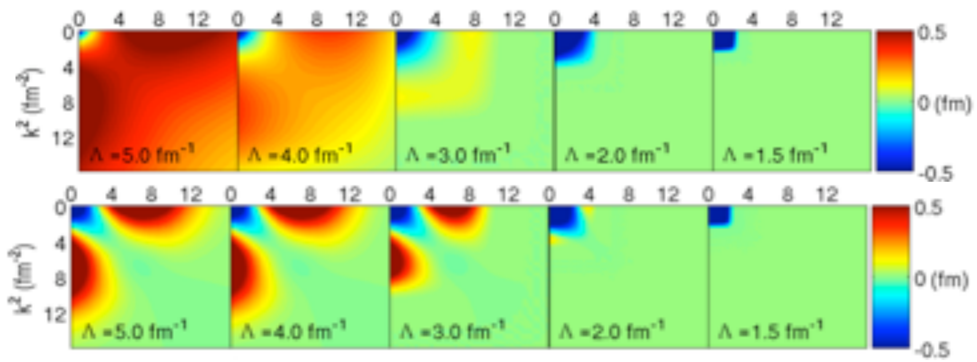

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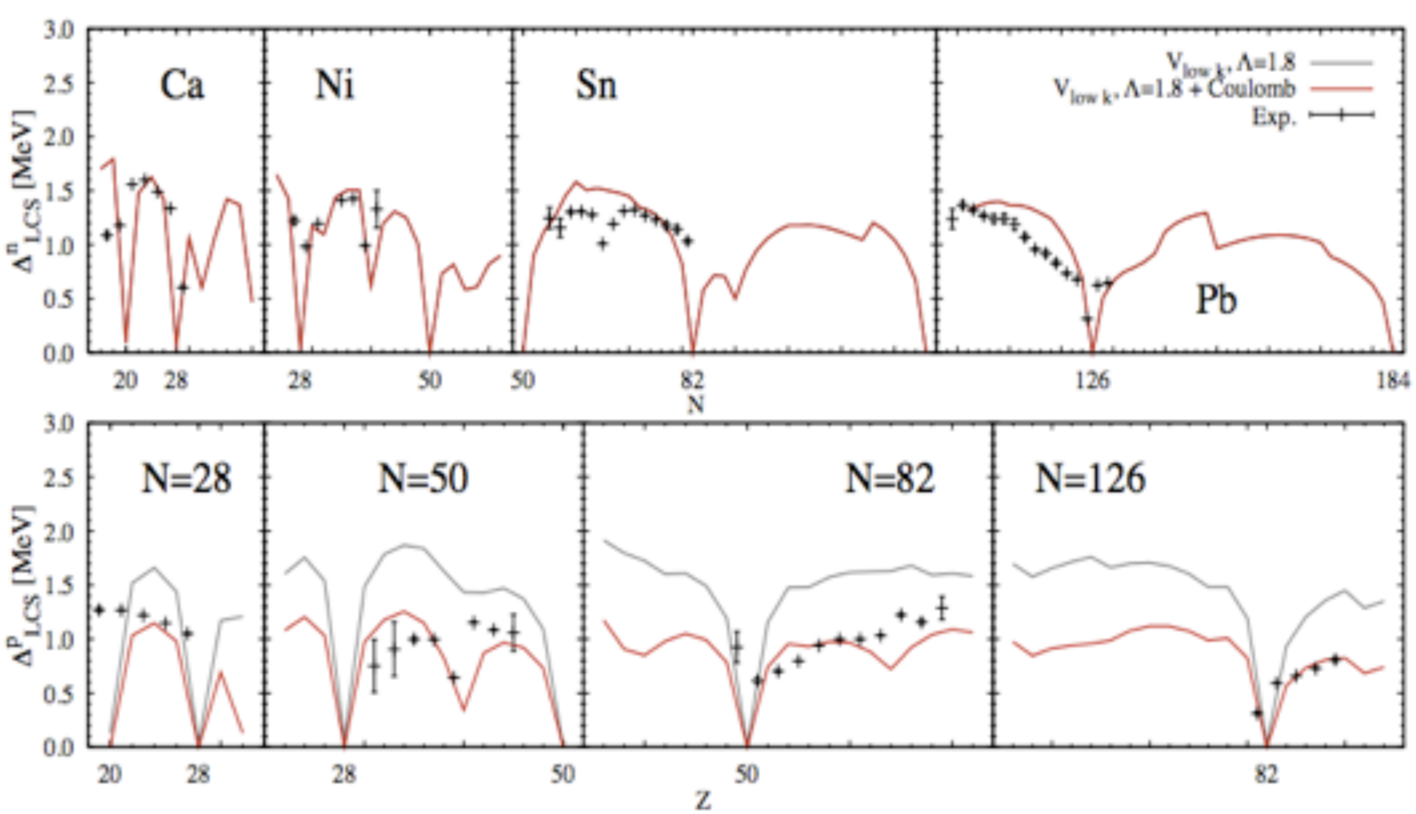

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Density Matrix expansions for low-momentum interactions

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Universal nuclear energy density functional

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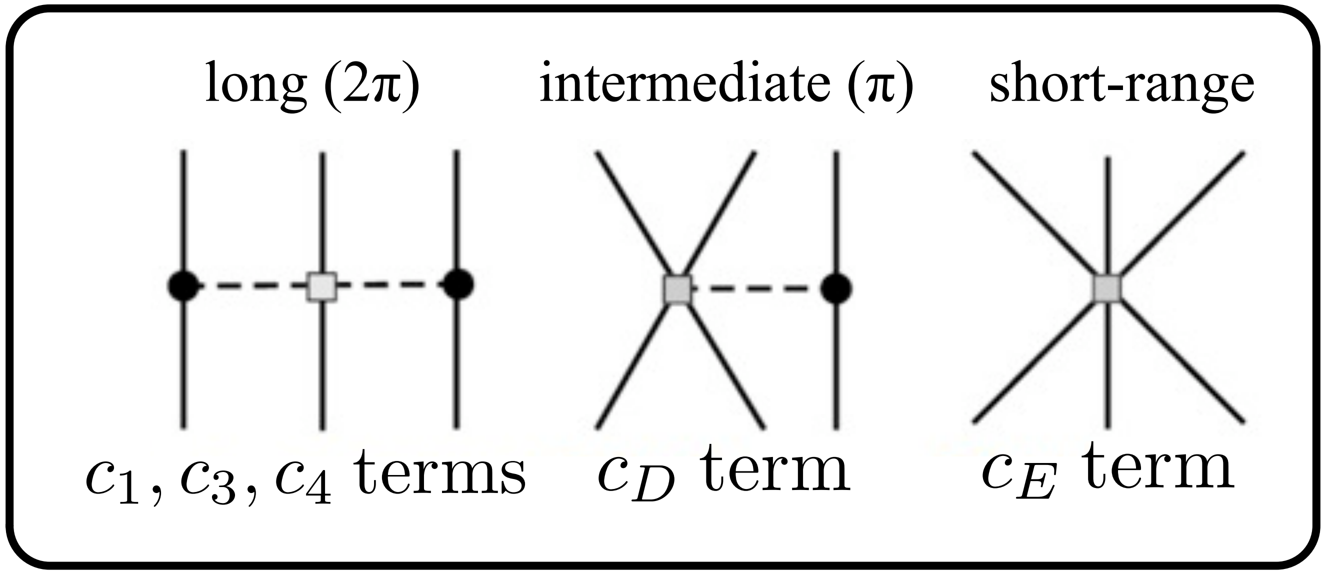
Chiral EFT 3N interactions

3N interactions crucial for saturation of nuclear matter
and correct LS splitting in nuclei

use in the following 3N force to N^2LO:



van Kolck (1994), Epelbaum et al. (2002)

large uncertainties at present: $c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.5}_{-1.0}\,,\ c_4 = 3.5^{+0.5}_{-0.2}$

Meissner et al. ,
Machleidt

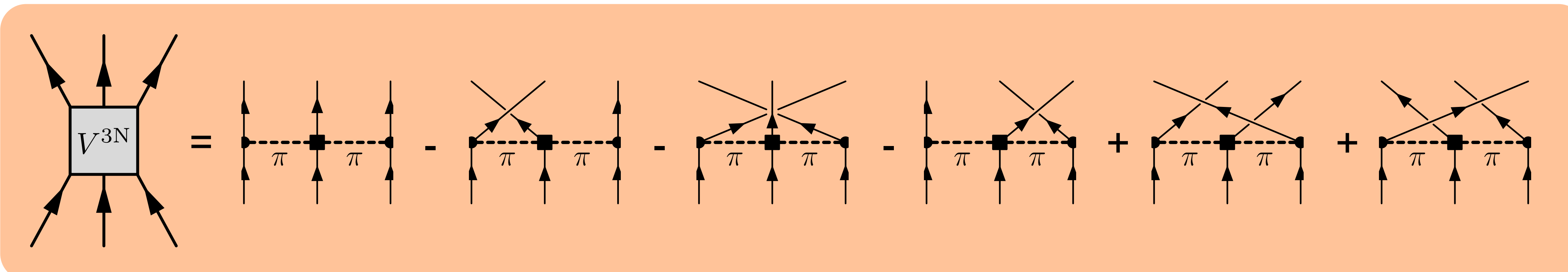
significant reduction of size of couplings in N^3LO

3N interactions perturbative for $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$ Nogga, Bogner, Schwenk (2004)

Chiral 3N interaction as density-dependent two-body interaction

antisymmetrized 3N interaction (at N²LO) in neutron matter:

$$V^{3\mathrm{N}} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \sum_{i\neq j\neq k} A_{ijk} \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_j \cdot \mathbf{q}_j)}{(\mathbf{q}_i^2 + m_\pi^2)(\mathbf{q}_j^2 + m_\pi^2)} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\mathbf{q}_i \cdot \mathbf{q}_j\right]$$



c_4 , c_D and c_E terms vanish in neutron matter

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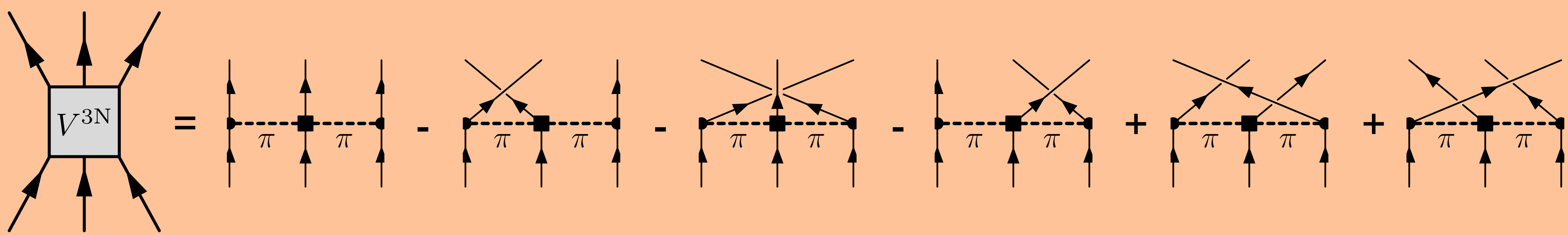
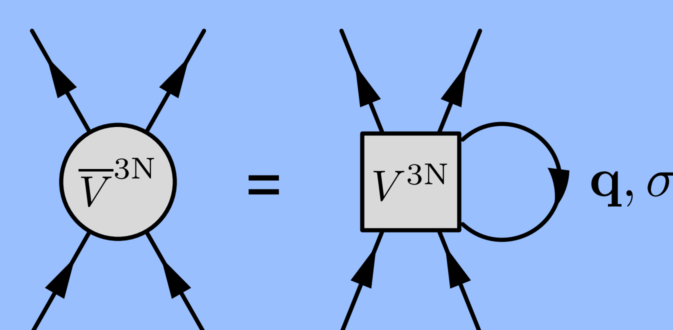


c_4 , c_D and c_E terms vanish in neutron matter

Basic idea: One particle only feels the average density

$$\overline{V}^{3\mathrm{N}} = \sum_{\mathbf{q},\sigma} V^{3\mathrm{N}} n(k_{\mathrm{F}}-q)$$



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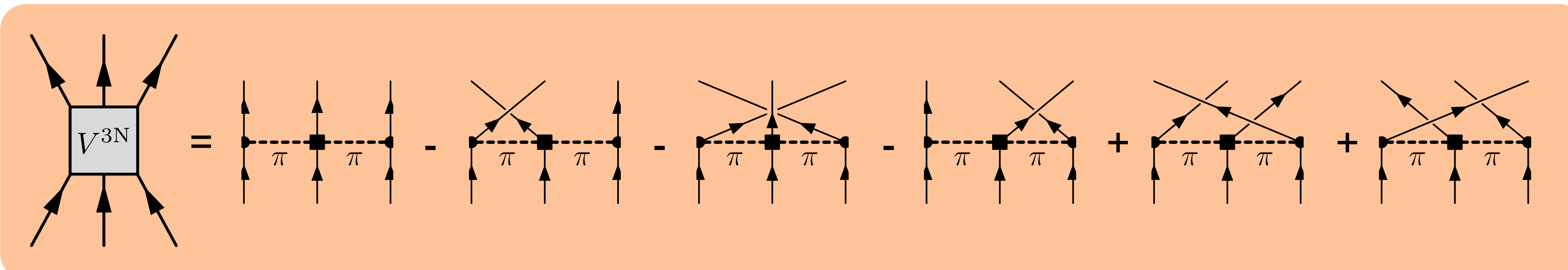


c_4 , c_D and c_E terms vanish in neutron matter

Calculation of $\overline{V}^{3\mathrm{N}}$ in SNM in progress.
Here all 3N diagrams are contributing.

Operator form of $\overline{V}^{3\mathrm{N}}$

general momentum dependence : $\overline{V}^{3\mathrm{N}} = \overline{V}^{3\mathrm{N}}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

P-dependence only weak, evaluate for $\mathbf{P} = 0$:

$$\overline{V}^{3\mathrm{N}}_{P=0} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} A(\mathbf{k}, \mathbf{k}') + \frac{2c_3}{f_\pi^2} B(\mathbf{k}, \mathbf{k}')\right]$$

Operator form of $\overline{V}^{3N}$

general momentum dependence : $\overline{V}^{3N} = \overline{V}^{3N}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

P-dependence only weak, evaluate for $\mathbf{P} = 0$:

$$\overline{V}^{3N}_{P=0} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} A(\mathbf{k}, \mathbf{k}') + \frac{2c_3}{f_\pi^2} B(\mathbf{k}, \mathbf{k}')\right]$$

$$B(\mathbf{k}, \mathbf{k}') =$$

$$-\frac{1}{3}\left\{\frac{\rho(k,k')(\mathbf{k}+\mathbf{k}')^4}{((\mathbf{k}+\mathbf{k}')^2+m_\pi^2)^2} + 2B_1^s(\mathbf{k},\mathbf{k}') - B_1^s(\mathbf{k},-\mathbf{k}') - (B_2^s(\mathbf{k},\mathbf{k}') + B_2^s(\mathbf{k}',\mathbf{k}))\right\}$$

$$+\frac{1}{3}(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\left\{\frac{2}{3}\frac{\rho(k,k')(\mathbf{k}-\mathbf{k}')^4}{((\mathbf{k}-\mathbf{k}')^2+m_\pi^2)^2} + \frac{1}{3}\frac{\rho(k,k')(\mathbf{k}+\mathbf{k}')^4}{((\mathbf{k}+\mathbf{k}')^2+m_\pi^2)^2}\right.$$

$$\left. + B_1^s(\mathbf{k},-\mathbf{k}') - \frac{1}{3}\left[B_2^s(\mathbf{k},\mathbf{k}') + B_2^s(\mathbf{k}',\mathbf{k})\right] - \frac{2}{3}\left[B_2^s(\mathbf{k},-\mathbf{k}') + B_2^s(\mathbf{k}',-\mathbf{k})\right]\right\}$$

$$+\frac{2}{3}\left[\frac{\rho(k,k')(\mathbf{k}+\mathbf{k}')^2 S_{12}(\mathbf{k}+\mathbf{k}')}{((\mathbf{k}+\mathbf{k}')^2+m_\pi^2)^2} - \frac{\rho(k,k')(\mathbf{k}-\mathbf{k}')^2 S_{12}(\mathbf{k}-\mathbf{k}')}{((\mathbf{k}-\mathbf{k}')^2+m_\pi^2)^2}\right]$$

$$+\frac{2}{3}\sigma^a\sigma'^b\left[B_{ab}^t(\mathbf{k},\mathbf{k}') - B_{ab}^t(\mathbf{k},-\mathbf{k}') + B_{ab}^t(\mathbf{k}',\mathbf{k}) - B_{ab}^t(\mathbf{k}',-\mathbf{k})\right]$$

$$+\frac{1}{3}i\,(\sigma^a + \sigma'^a)\left[B_a^v(\mathbf{k},\mathbf{k}') - B_a^v(\mathbf{k},-\mathbf{k}')\right]$$

Operator form of $\overline{V}^{3\mathrm{N}}$

general momentum dependence : $\overline{V}^{3\mathrm{N}} = \overline{V}^{3\mathrm{N}}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

P-dependence only weak, evaluate for $\mathbf{P} = 0$:

$$\overline{V}^{3\mathrm{N}}_{P=0} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} A(\mathbf{k}, \mathbf{k}') + \frac{2c_3}{f_\pi^2} B(\mathbf{k}, \mathbf{k}')\right]$$

$$B_1^s(\mathbf{k}, \mathbf{k}')$$

$$\begin{aligned} B_1^s(\mathbf{k}_1, \mathbf{k}_2) \quad = \quad & \int \frac{d^3\mathbf{q}}{(2\pi)^3} n(q) f_R(\Lambda_{3\mathrm{N}}, q, k_1) f_R(\Lambda_{3\mathrm{N}}, q, k_2) \\ & \cdot \frac{((\mathbf{k}_1 + \mathbf{q}) \cdot (\mathbf{k}_2 + \mathbf{q}))^2}{((\mathbf{k}_1 + \mathbf{q})^2 + m_\pi^2)((\mathbf{k}_2 + \mathbf{q})^2 + m_\pi^2)} \end{aligned}$$

Operator form of $\overline{V}^{3N}$

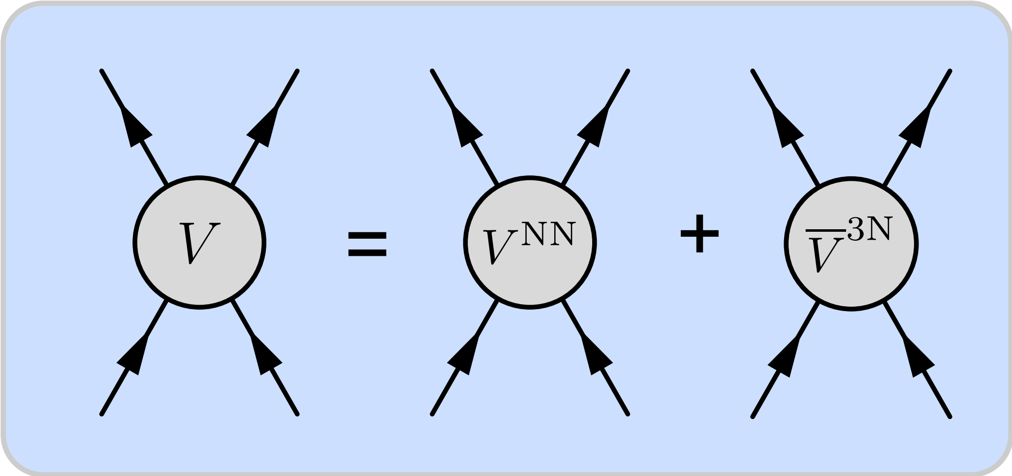
general momentum dependence : $\overline{V}^{3N} = \overline{V}^{3N}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

P-dependence only weak, evaluate for $\mathbf{P} = 0$:

$$\overline{V}^{3N}_{P=0} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} A(\mathbf{k}, \mathbf{k}') + \frac{2c_3}{f_\pi^2} B(\mathbf{k}, \mathbf{k}')\right]$$

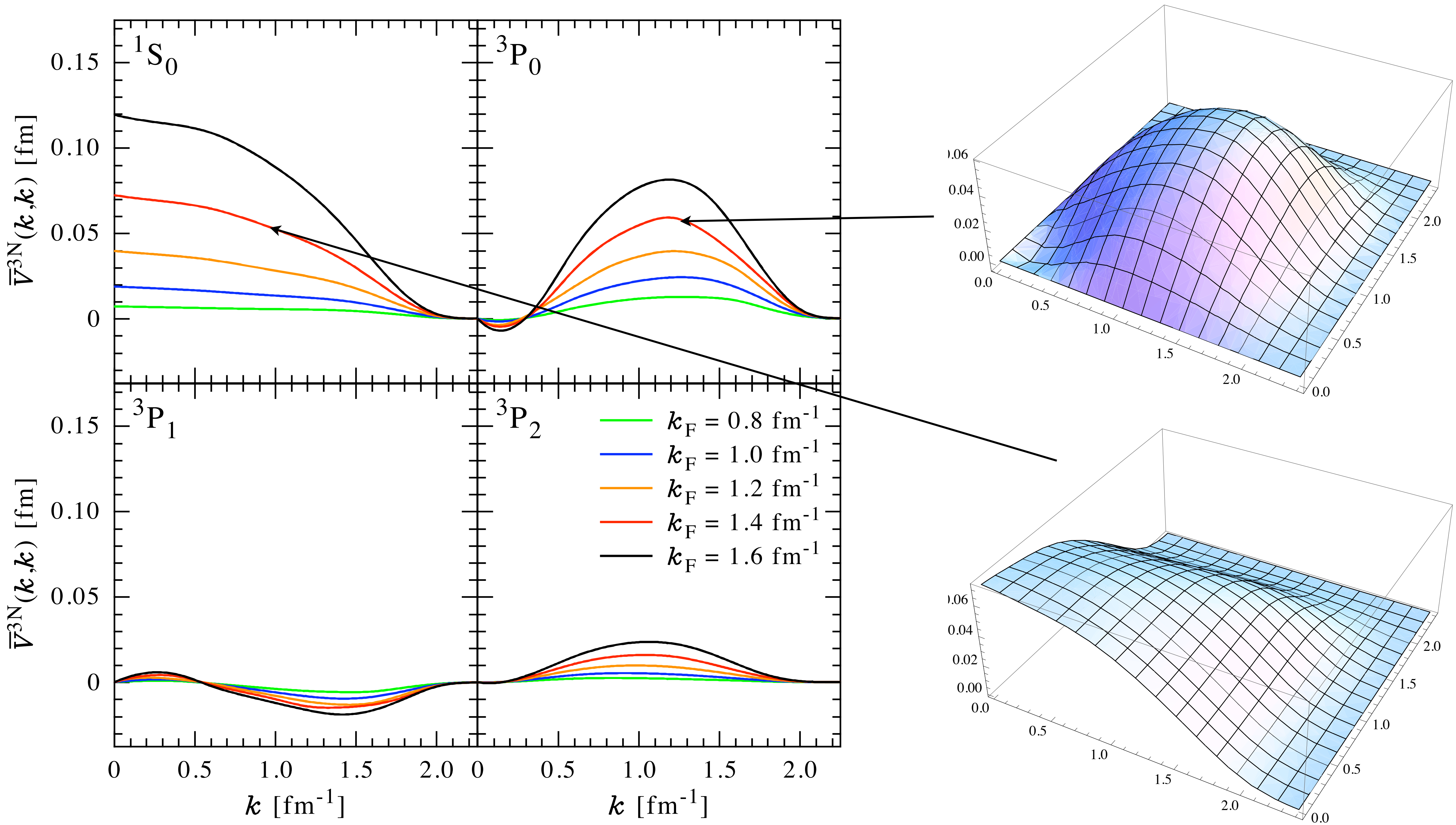
$\overline{V}^{3N}$ provides additive corrections to the free-space NN interaction:

$$V = V^{NN} + \overline{V}^{3N}$$



- neglect P-dependence in the following, set P=0
- in fixed-P approximation $\overline{V}^{3N}$ matrix elements have the same form like genuine free-space NN matrix elements
- straightforward to incorporate in existing many-body schemes

Partial wave matrix elements $(\Lambda_{3N} = 2.0\,\mathrm{fm}^{-1})$



- non-trivial density dependence
- $\overline{V}^{3N}(k, k'; {}^1S_0) \sim k_F^4 \sim \rho^{4/3}$ for $k, k' \lesssim 1.0\,\mathrm{fm}^{-1}$
- strong non-central components

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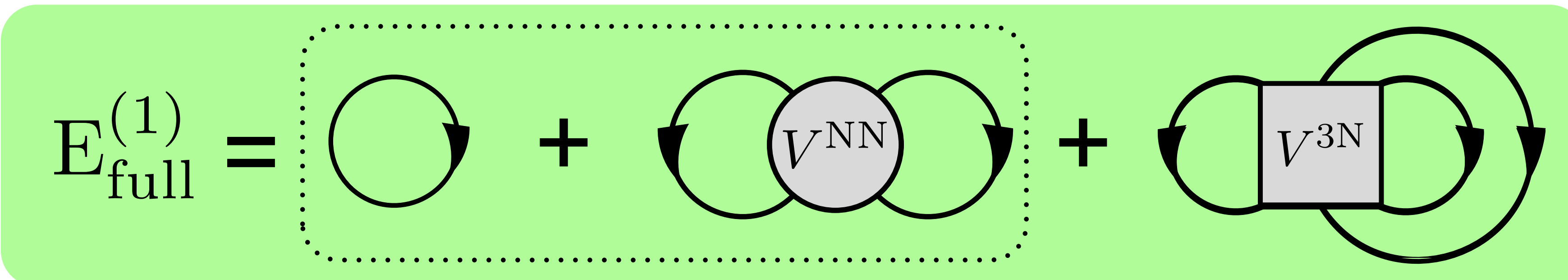
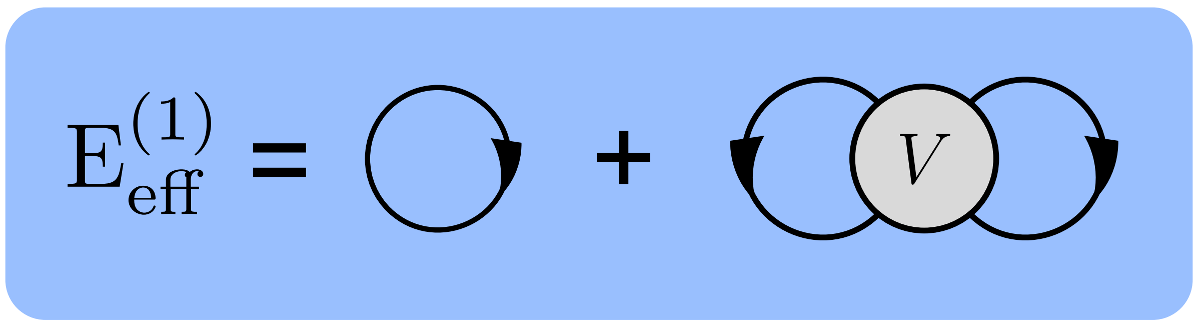
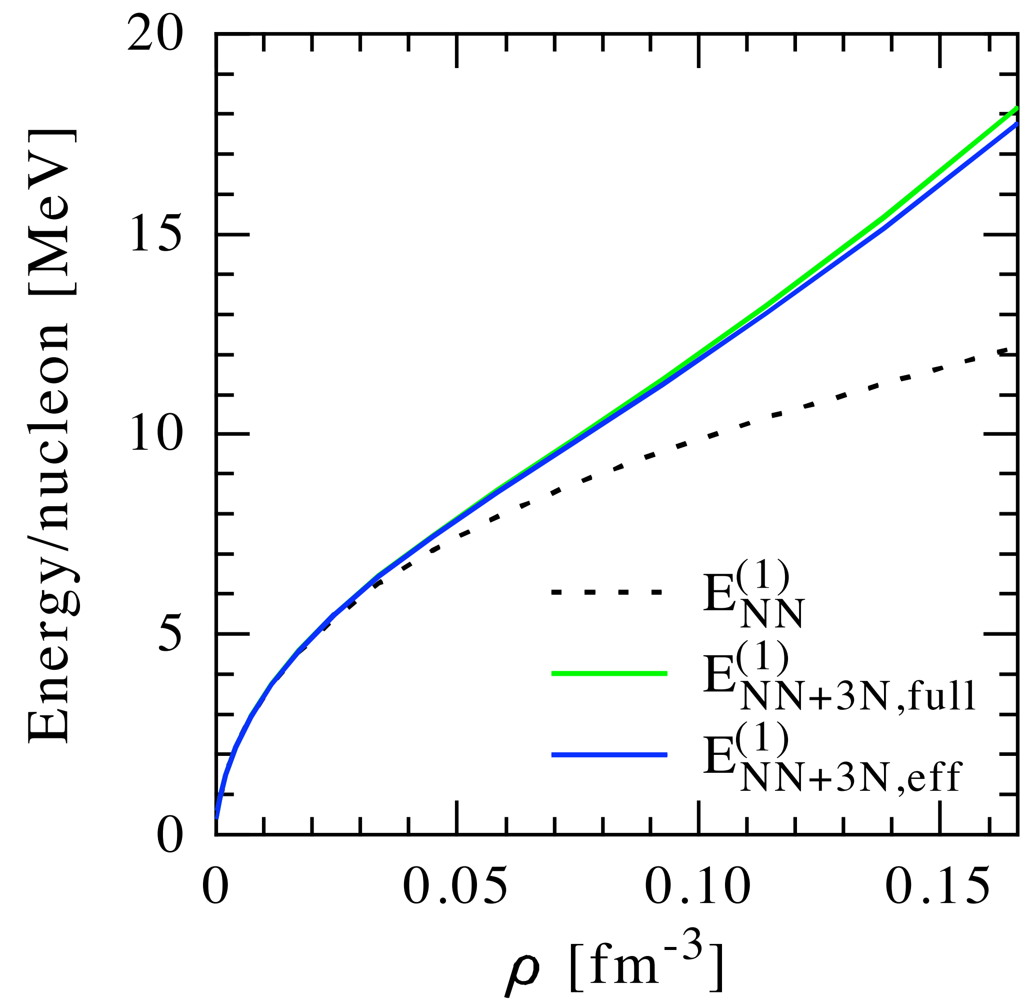
Neutron matter results:
EOS (first order), Test of fixed-P approximation



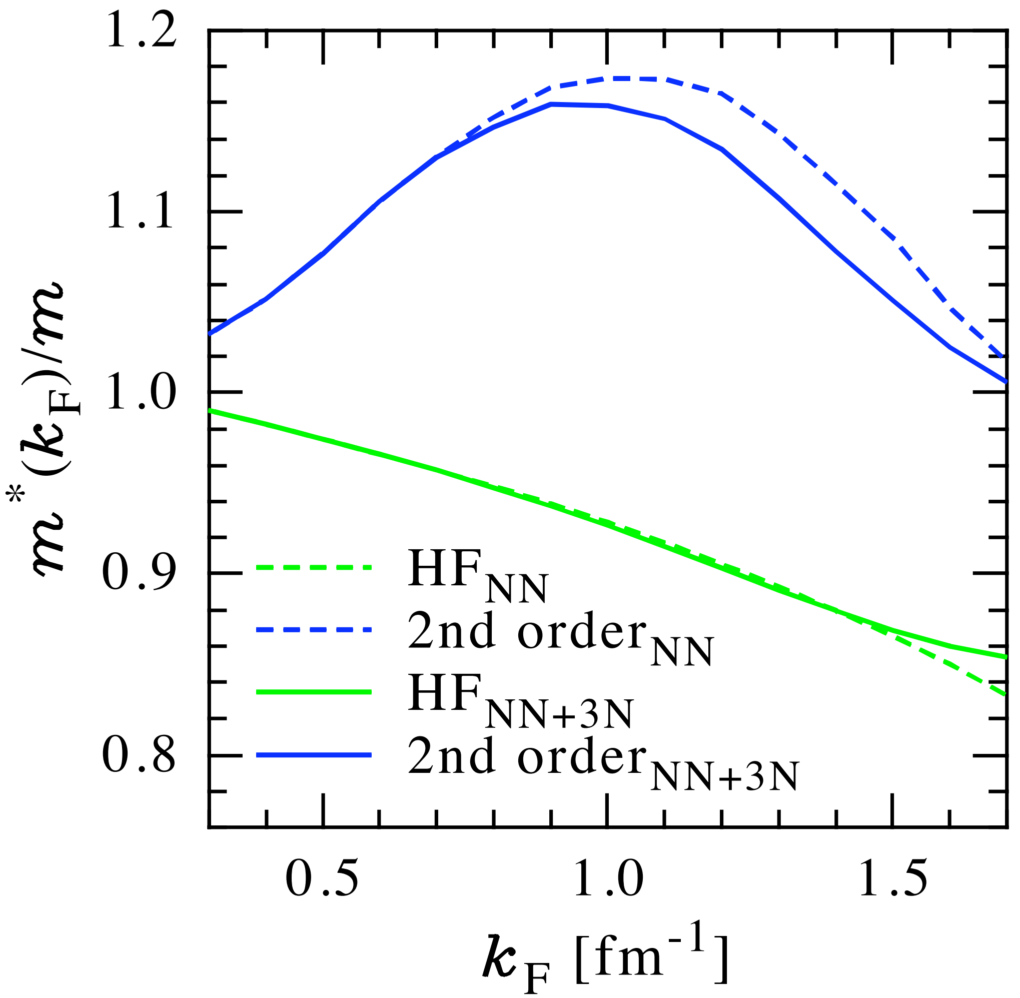


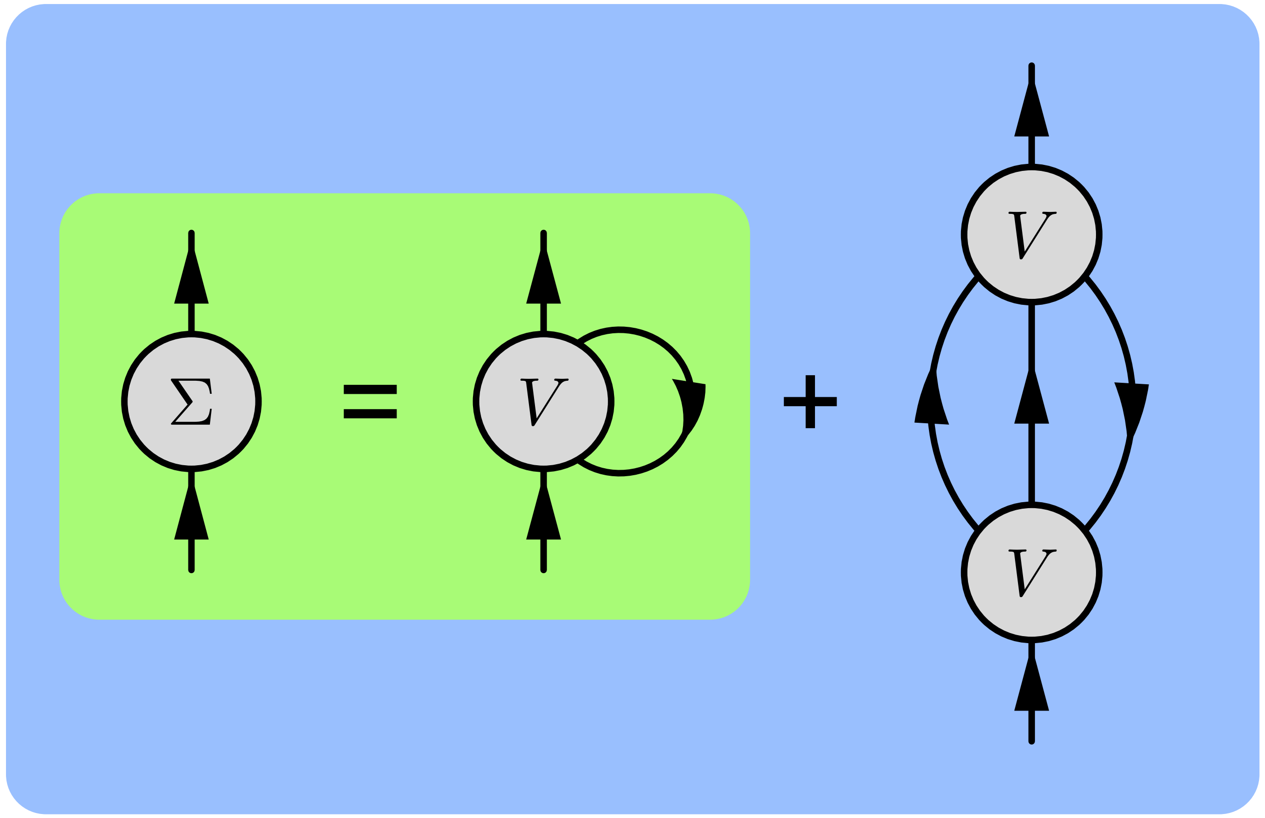
relative difference of
3N contributions only ~3%



P-independent effective NN interaction is a very good approximation!

Neutron matter results: single-particle properties

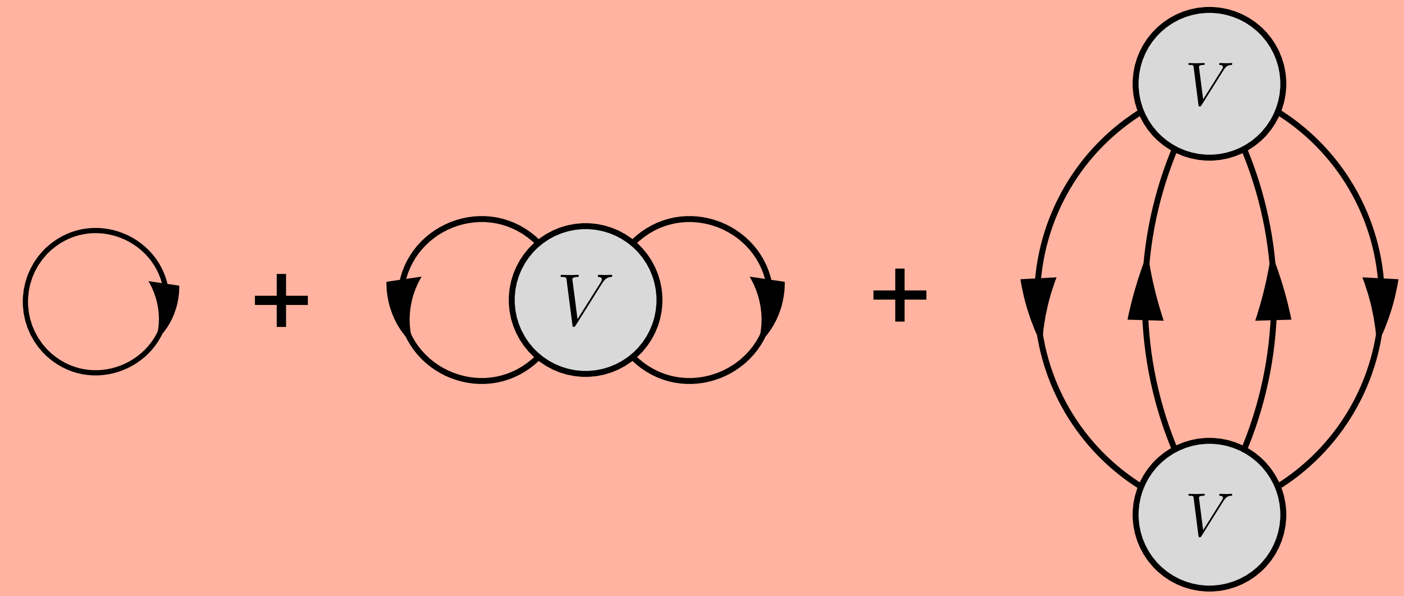




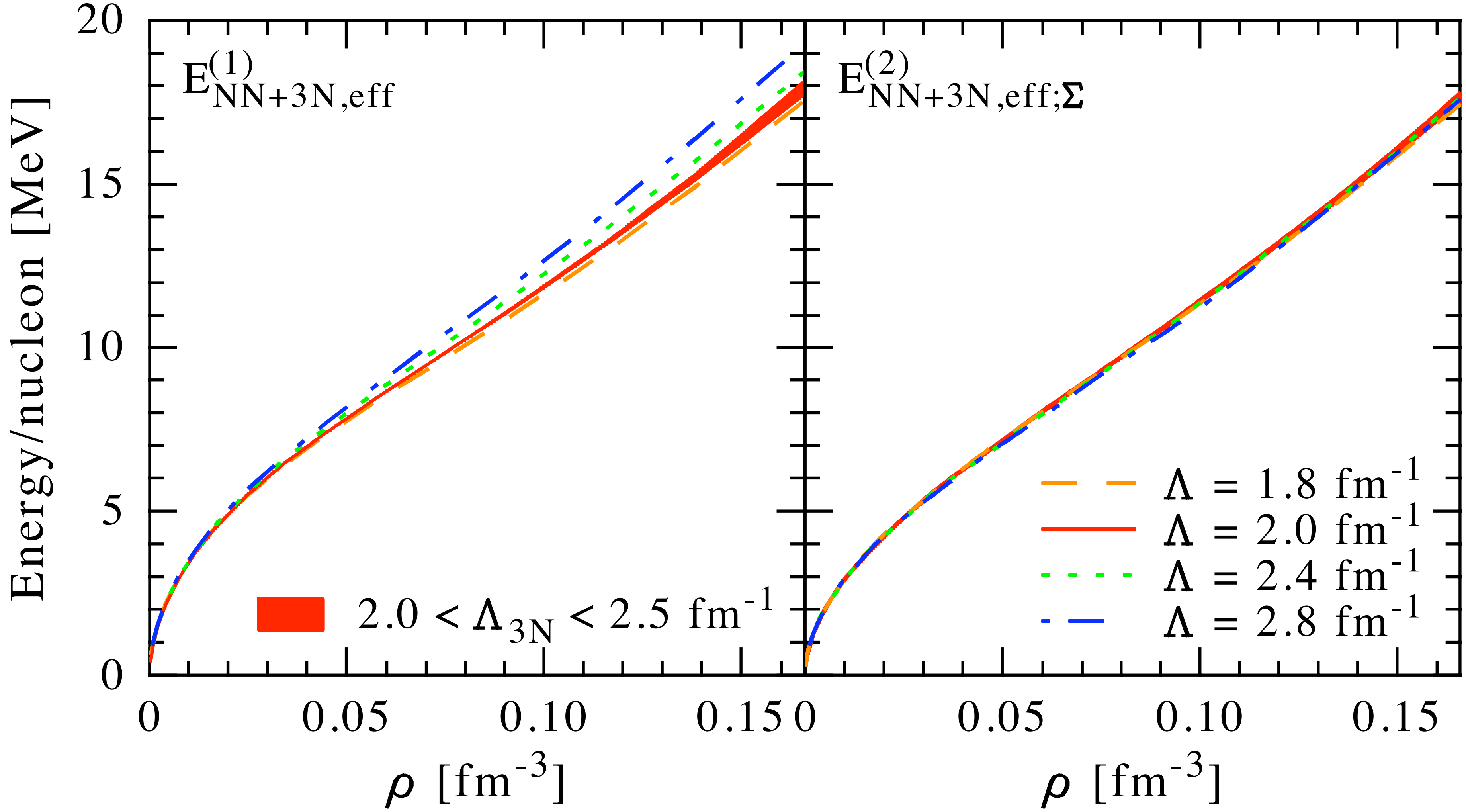
- 3N effects very small at HF level
- enhancement of effective-mass at 2nd order due to e-mass
- slight suppression of effective-mass at 2nd order due to 3N force

Neutron matter results:
EOS (second order)

$E_{\mathrm{eff}}^{(2)} =$

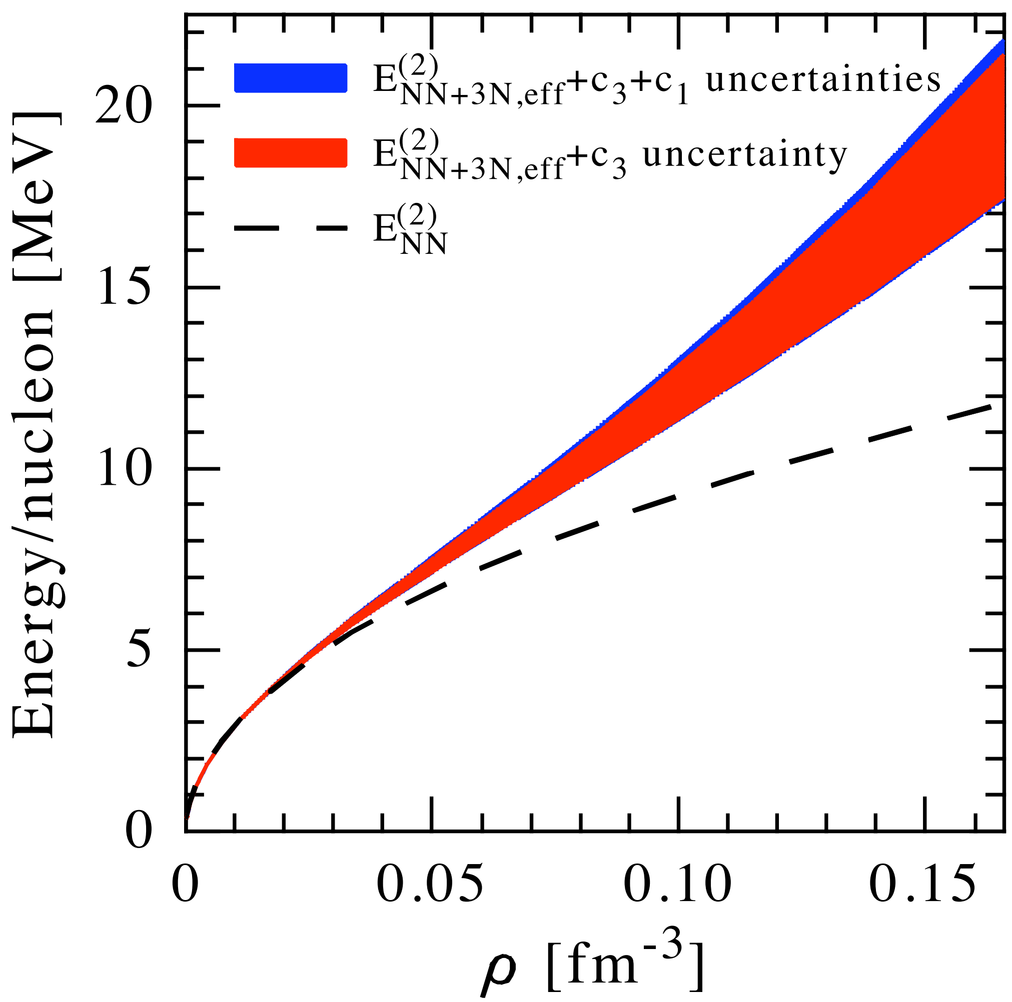


Neutron matter results: EOS (second order)



- result practically cutoff independent at 2nd order
- self-energy effects small
- system seems perturbative for low-momentum interactions
- improvement of recent SNM calculation in progress (exact treatment of double exchange terms and self-consistent self-energy to 2nd order)

Neutron matter results:
Uncertainties due to coupling constants



$$c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.5}_{-1.0}\,,\ \ c_4 = 3.5^{+0.5}_{-0.2}$$

Energy very sensitive to c_3 variations

Compare uncertainty to nuclear phenomenology.

Neutron matter results: Symmetry energy

use:

$$E(\rho, \alpha = 1) = -a_V + \frac{K_0}{18\rho_0^2}(\rho - \rho_0)^2 + S_2(\rho)$$

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_0^2}(\rho - \rho_0)$$

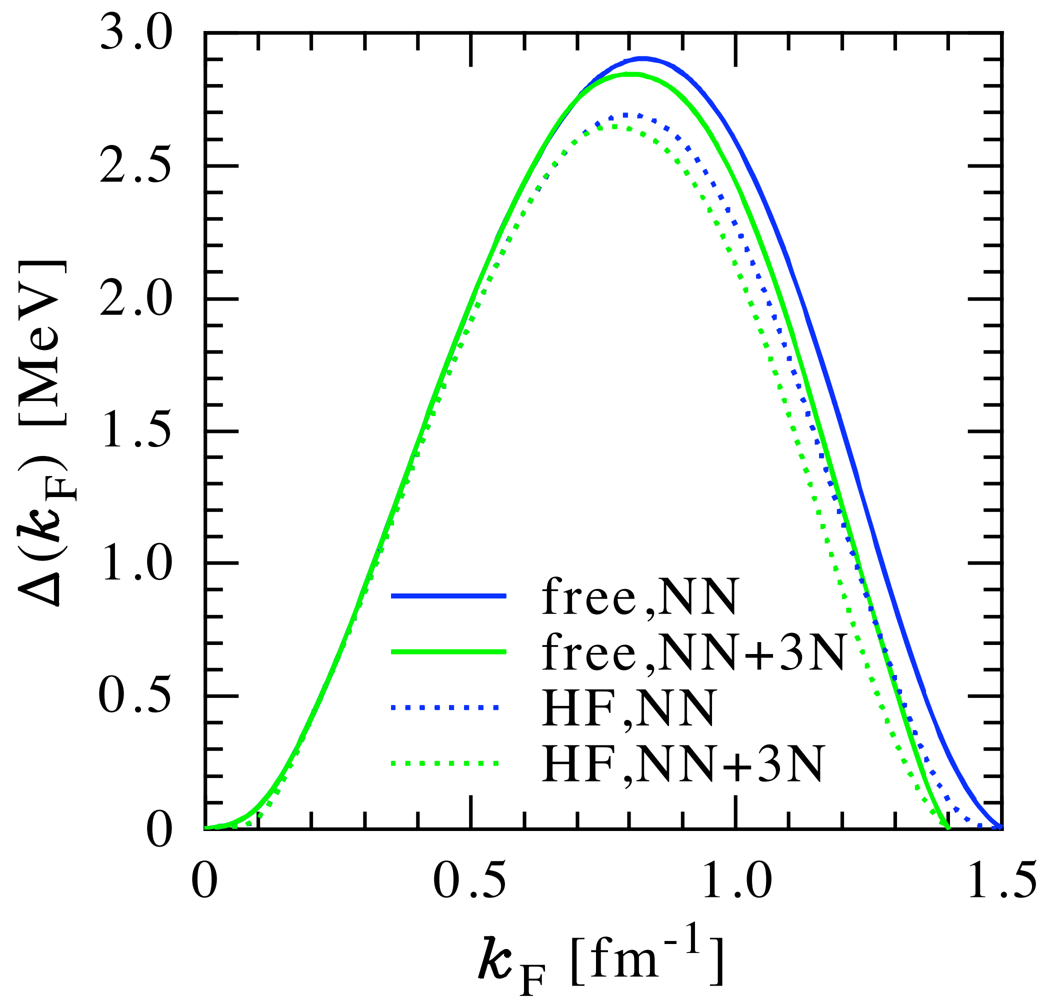
K_0 [MeV]	c_1 [GeV^{-1}]	c_3 [GeV^{-1}]	a_4 [MeV]	p_0 [MeV fm^{-3}]
190	-0.81	-3.2	32.9	2.8
190	-0.81	-5.7	36.5	3.9
240	-0.81	-3.2	32.9	2.9
240	-0.81	-5.7	36.5	4.0

given the experimental constraint $a_4 = 30 \pm 4\,\mathrm{MeV}$

smaller absolute values of c_3 are preferred from our results

Neutron matter results: 3N effects on pairing gaps

$^1S_0$ pairing gap:



- only small overall effects in neutron matter
- below $k_\mathrm{F} \approx 0.6\,\mathrm{fm}^{-1}$ 3N effects negligible
- expect larger effects in SNM due to larger matrix elements of $\overline{V}^{\mathrm{3N}}_{\mathrm{SNM}}$

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Consistency of effective-mass approximations in DFT calculations of pairing gaps using microscopic interactions

Basic problem:

Current EDFs parametrize normal self-energy effects in terms of momentum-independent effective-mass approximations

Approximation generally justified?

Inconsistencies in different EDF calculations of pairing gaps in finite nuclei.
The only difference is the **resolution scale** of the pairing interaction.

talk by T. Lesinski

Strategy:

Perform analogous calculations microscopically and consistently to first order in INM at different cutoffs.

Expansion scheme depends on the cutoff scale!

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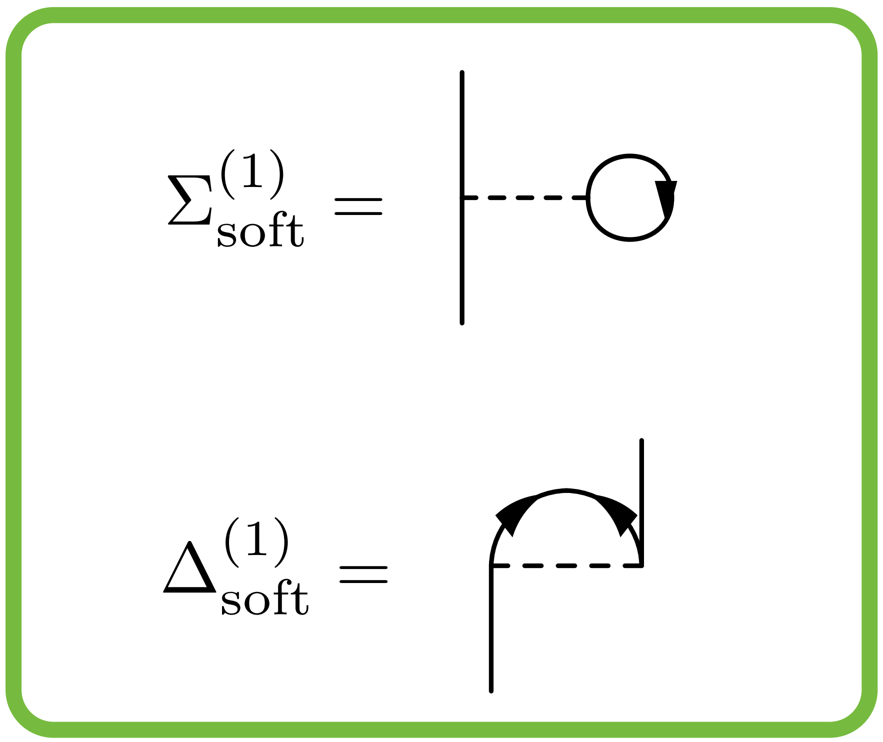
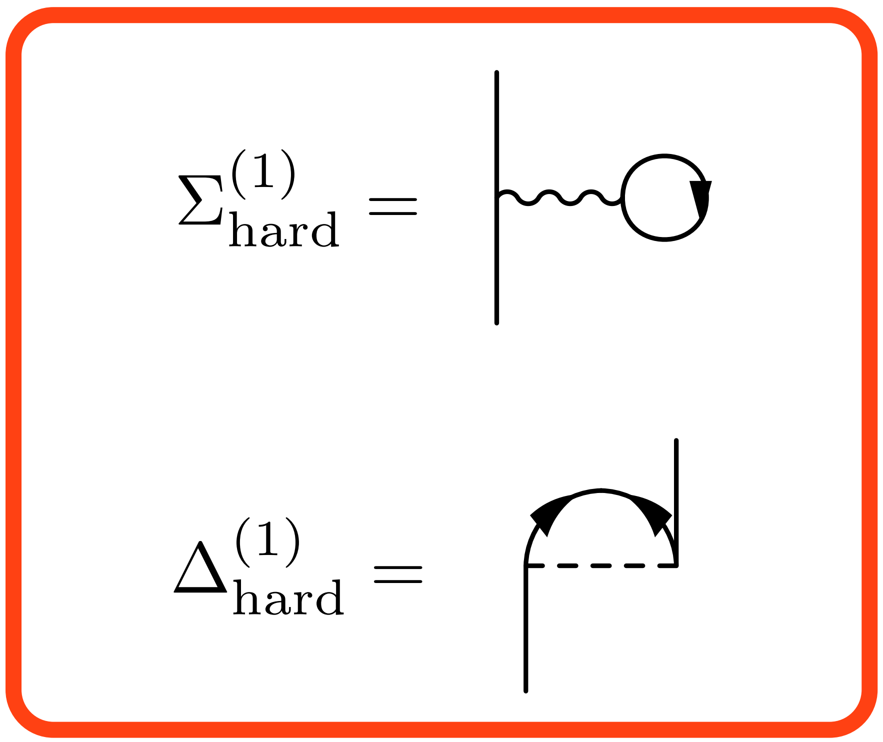
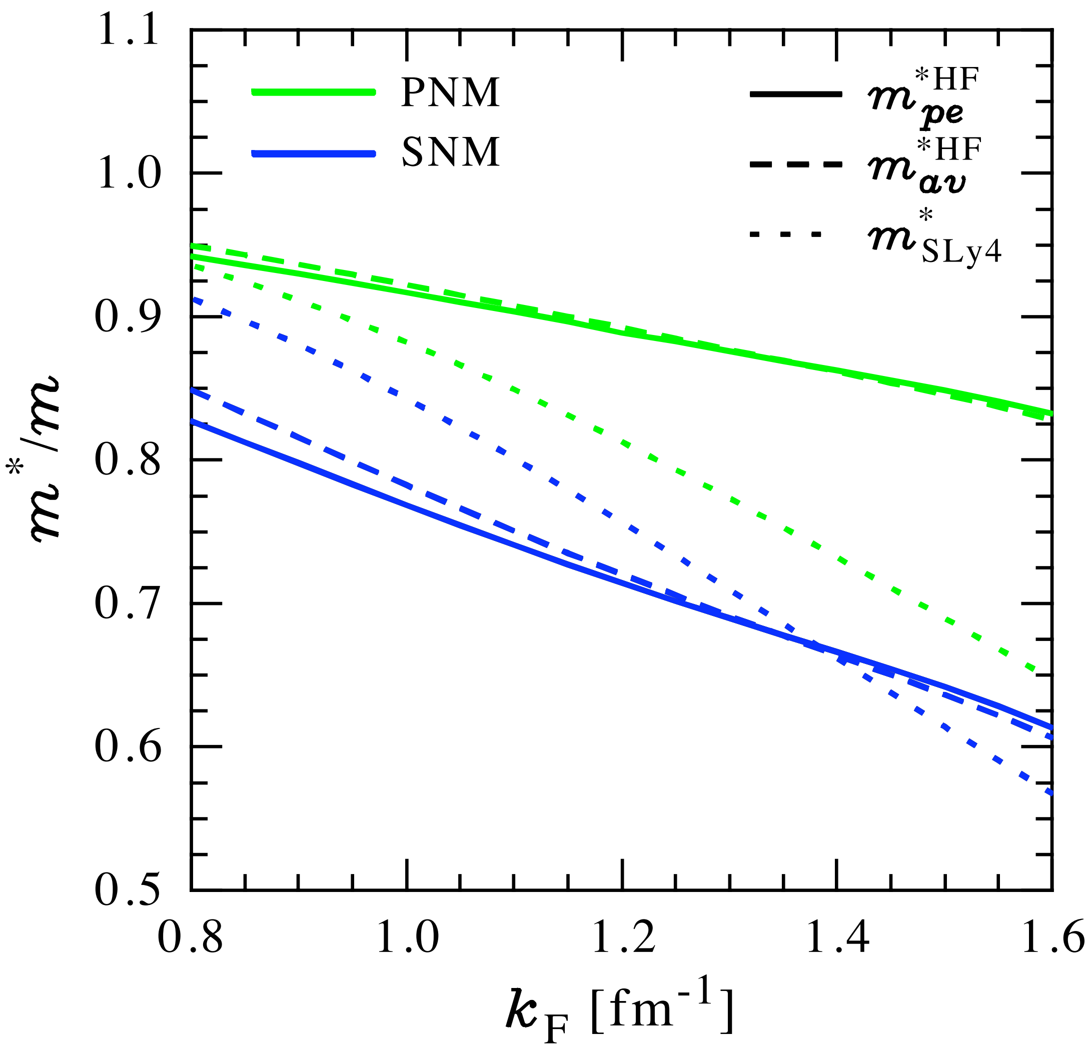


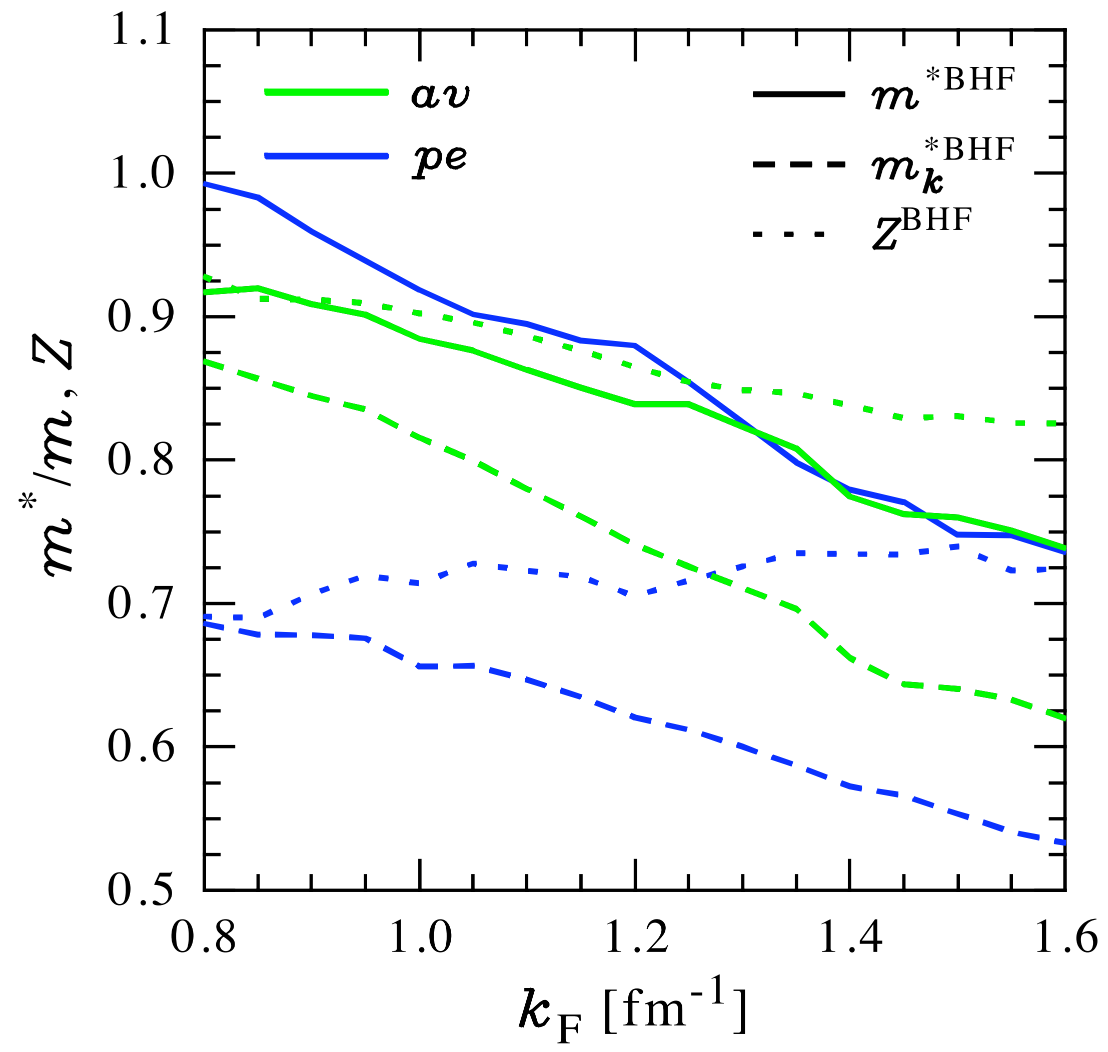


Effective mass results

Probe sensitivity to two different self-energy approximation schemes

$$X_{pe}(k_{\mathrm{F}}) = X(p = k_{\mathrm{F}}) \qquad X_{av}(k_{\mathrm{F}}) = \frac{\int f(q,\Lambda)\, q^2 dq\, X(q,k_{\mathrm{F}})\, \bar{u}_q\, \bar{v}_q}{\int f(q,\Lambda)\, q^2 dq\, \bar{u}_q\, \bar{v}_q}$$





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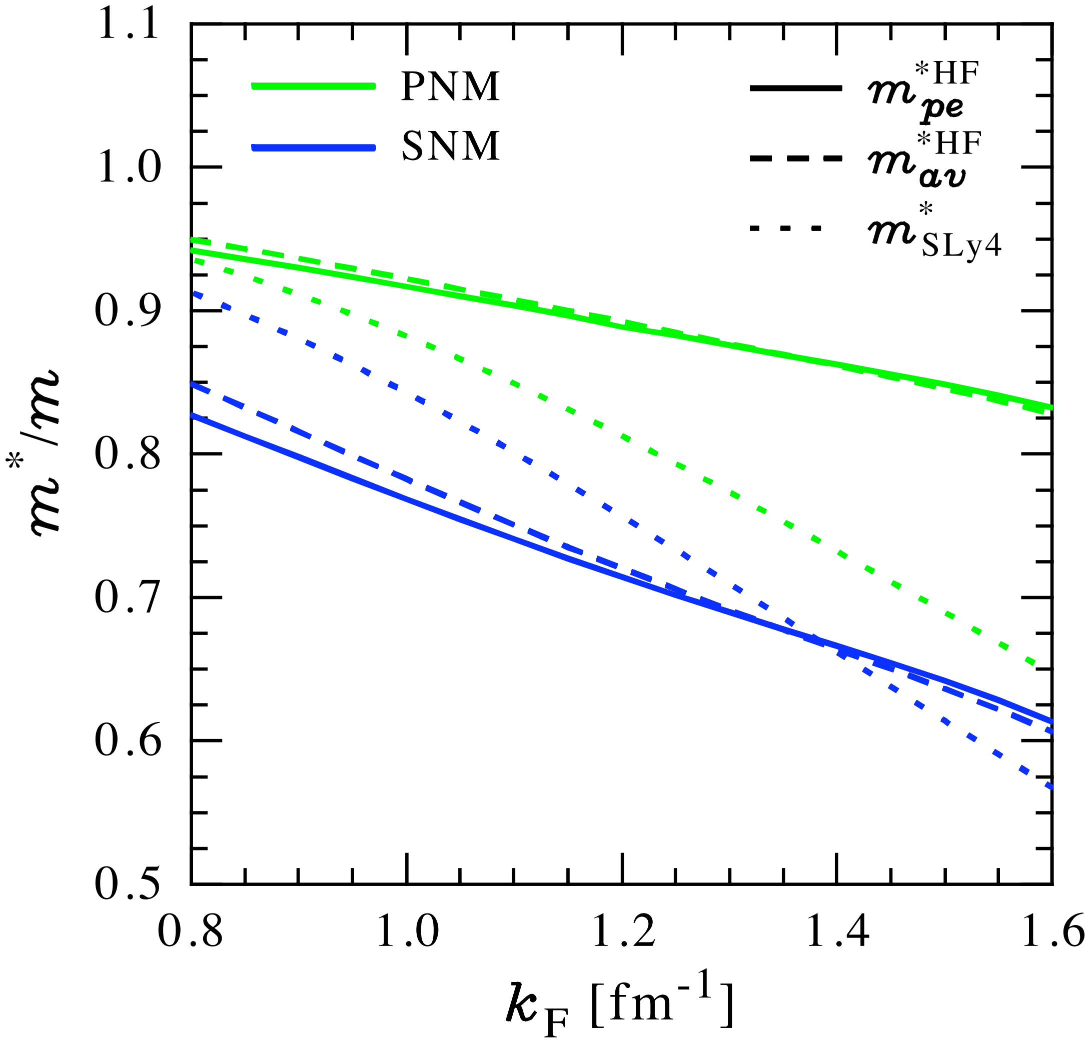
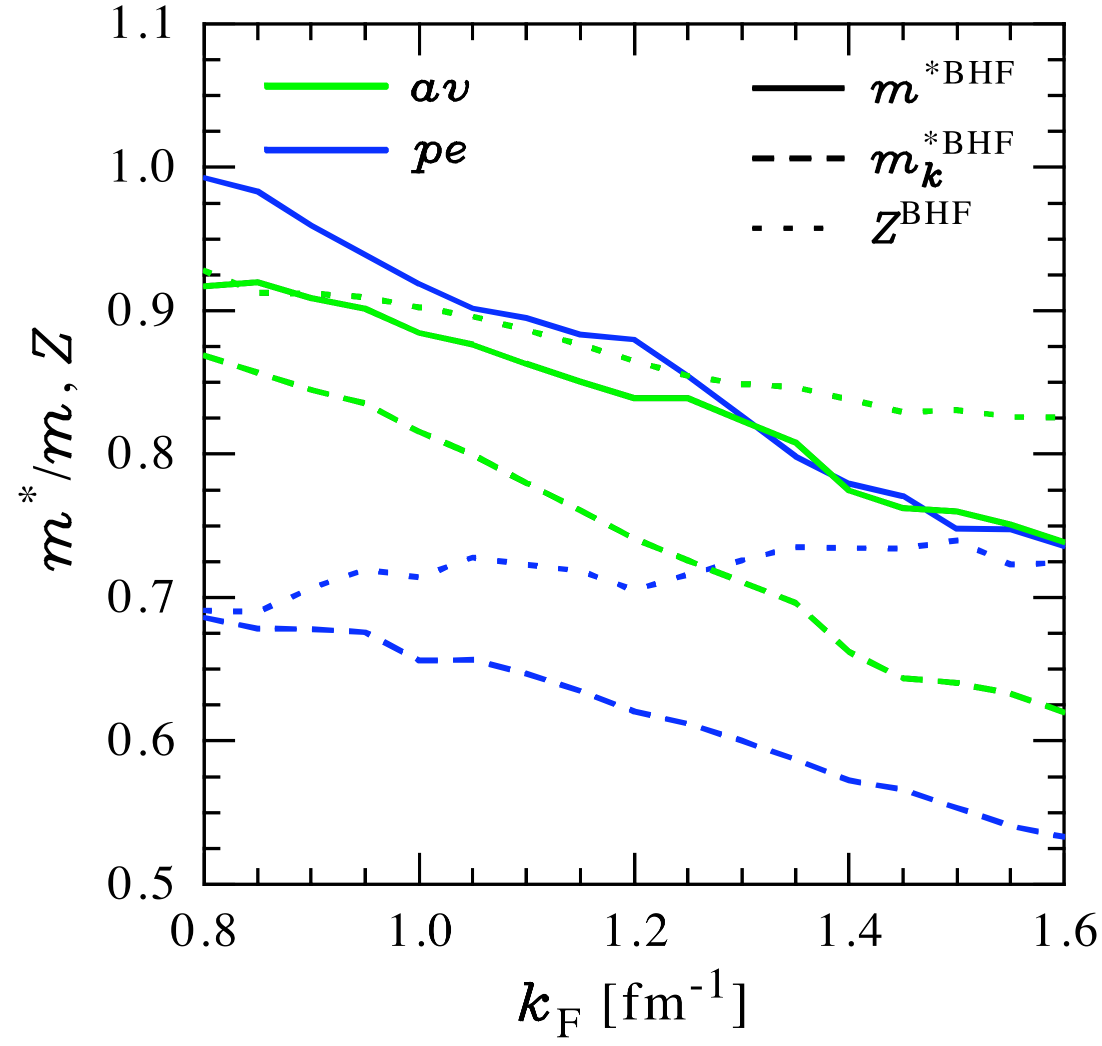
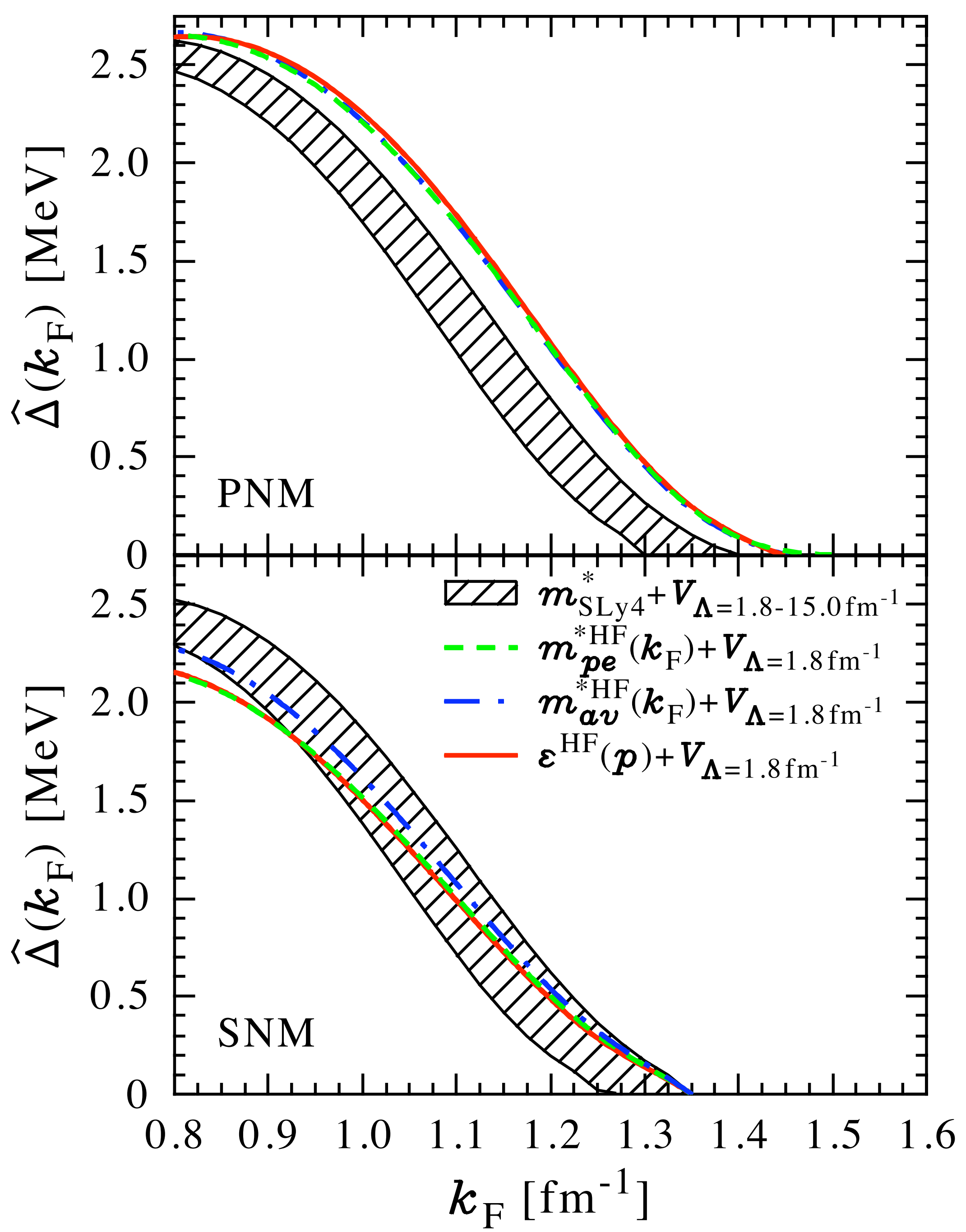
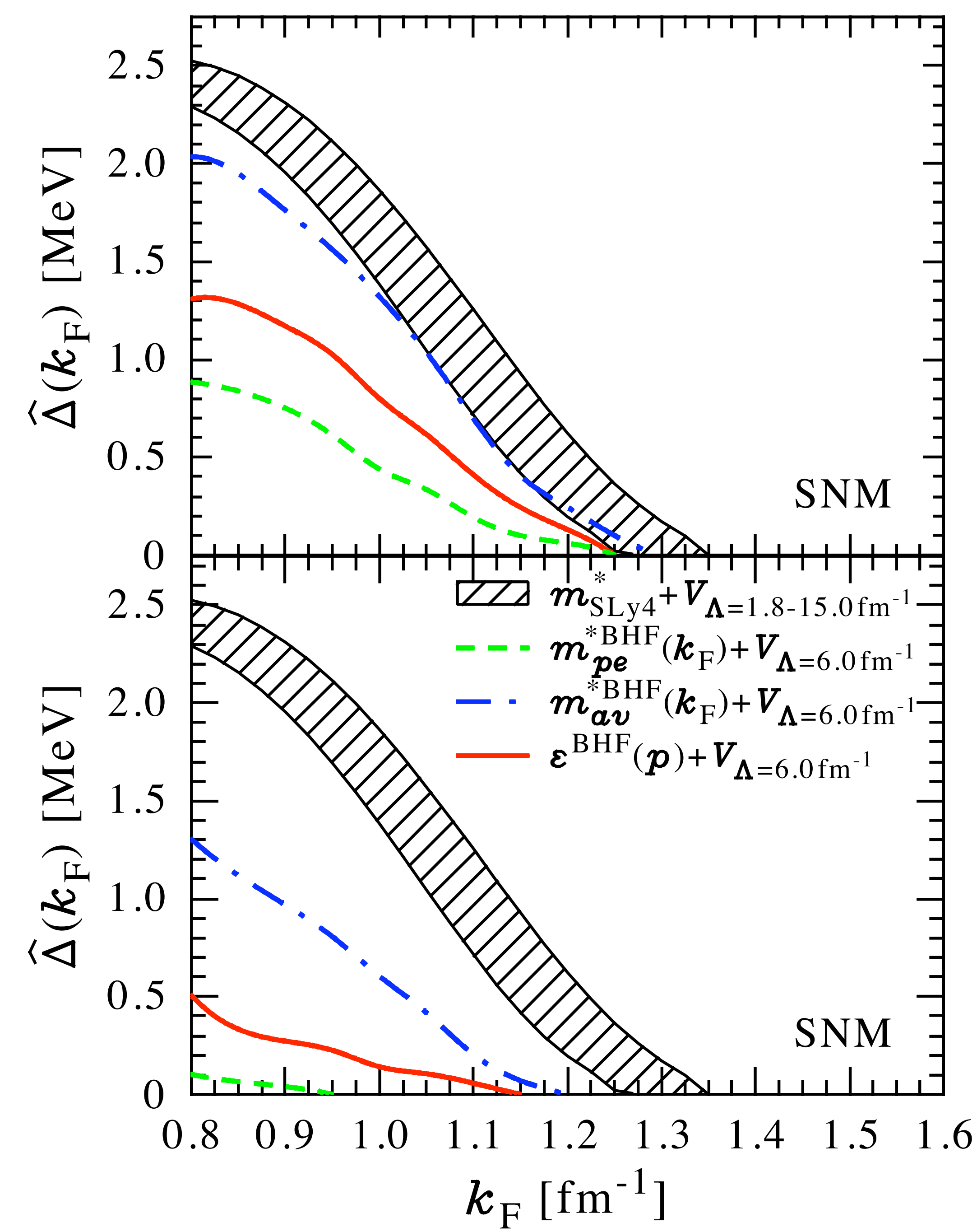




Pairing gap results





- gaps for soft interactions insensitive to self-energy approximation schemes
- gaps for hard interactions depend strongly on the approximation scheme

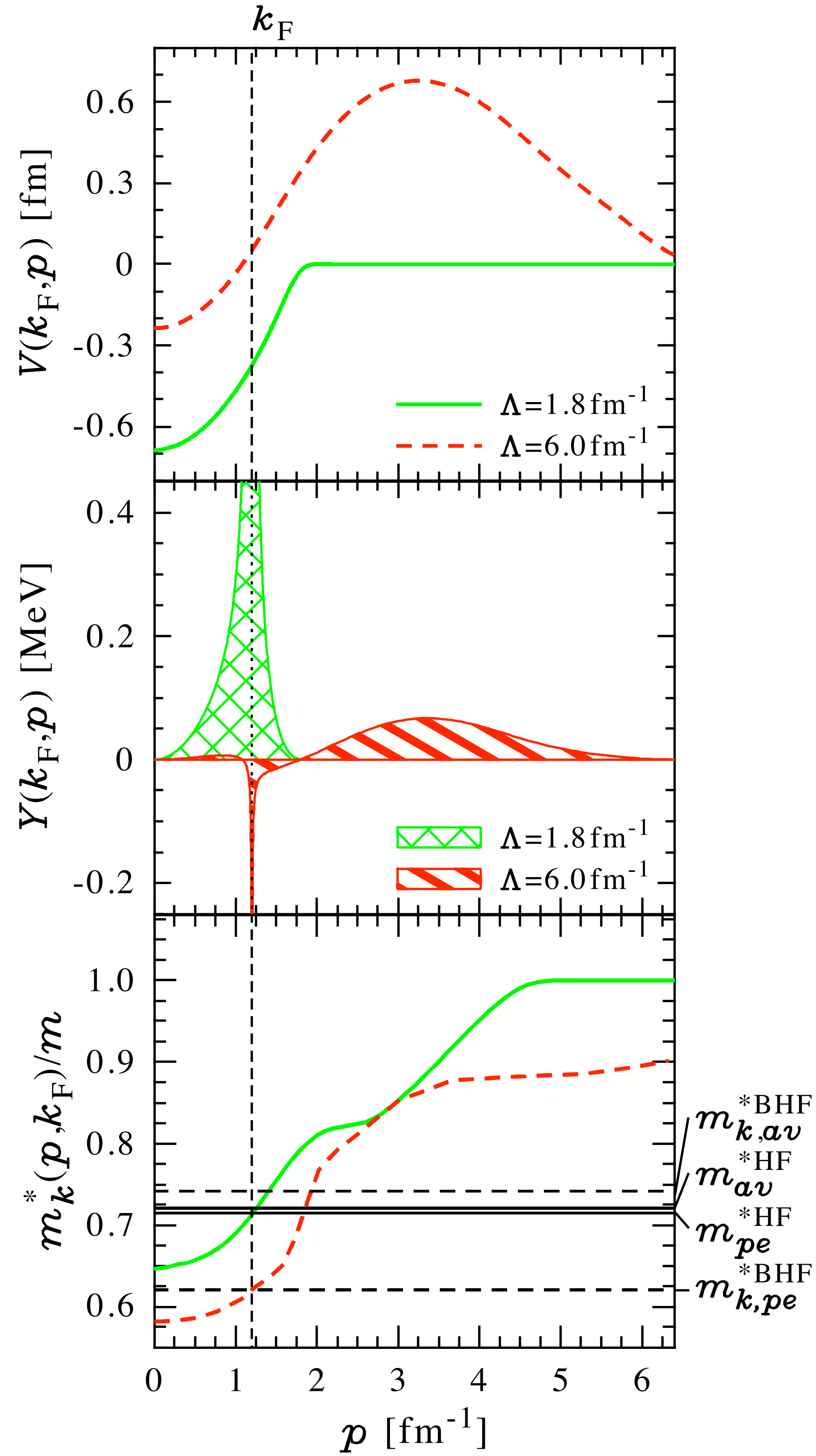
Analysis of results

$$\Delta(k_{\mathrm{F}}) = \int dq\, Y(k_{\mathrm{F}}, q)$$

gap is generated

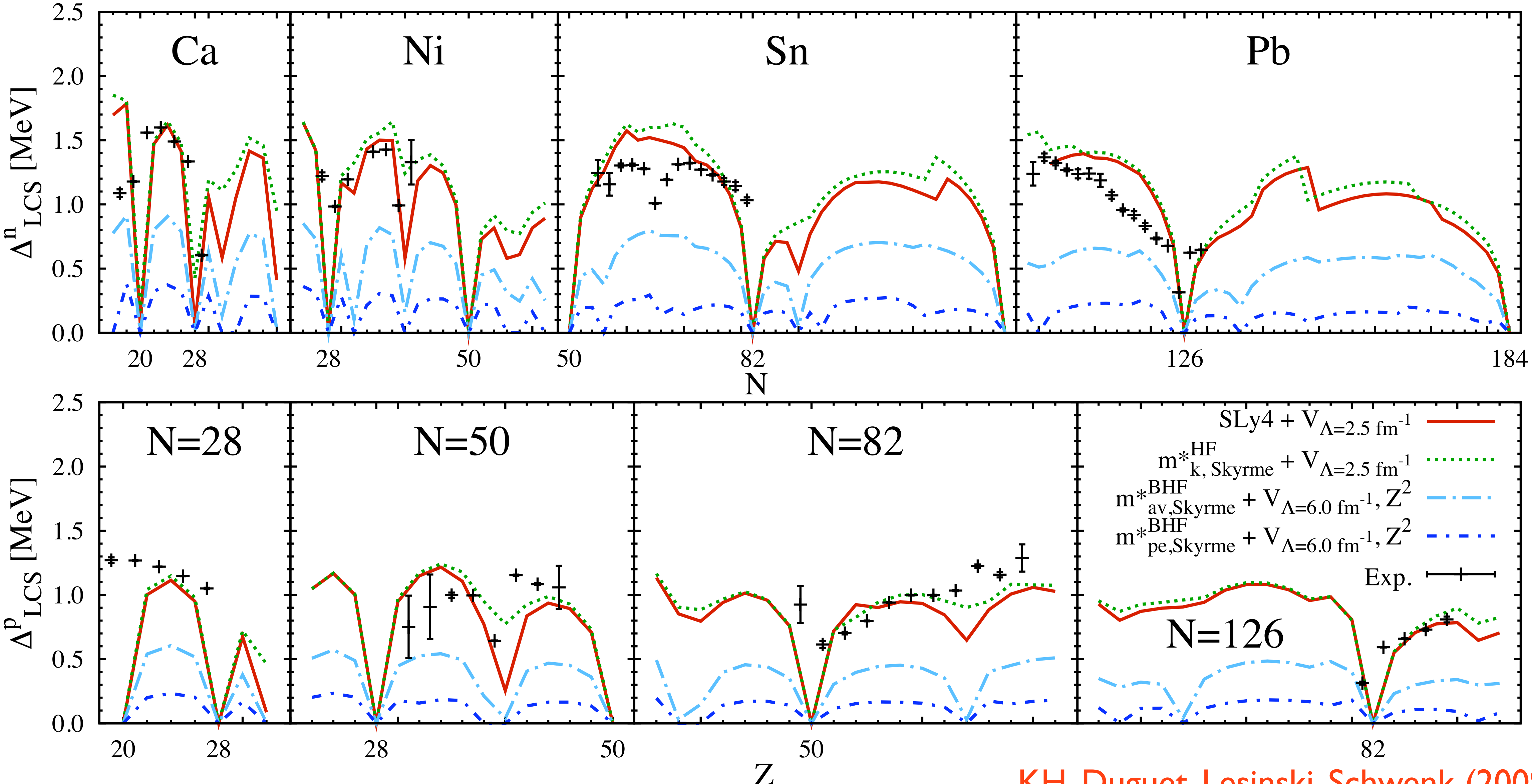
- for low cutoffs from low momentum modes
- for hard cutoffs from high momentum modes

due to coupling of low with high modes in V_{NN}



Momentum-independent effective-mass approximations for the computation of pairing gaps only reliable for soft interactions. Large sensitivity for hard interactions!

Resulting pairing gaps in semi-magic nuclei



KH, Duguet, Lesinski, Schwenk (2009)

more details tomorrow in talk by T. Lesinski!

Outline

1. Importance of nuclear matter results
2. Chiral 3N interactions as density-dependent two-body interactions
3. Results for neutron matter from chiral low-momentum interactions
4. Effective-mass approximations in DFT for calculations of pairing gaps
5. Conclusions and outlook

Conclusions and Outlook

• self-consistent calculation of the EOS of PNM to second order from low-momentum chiral NN (N3LO, Machleidt) and 3N (N2LO) interactions

• microscopic derivation of density dependent effective NN interaction from 3N interaction in the zero P-approximation

• effective NN interaction efficient to use and accounts for 3N effects in many-body systems in a microscopic way

• self-energy approximations used in current EDFs for calculations of pairing gaps only reliable for low-momentum interactions

• extension of effective NN interaction to SNM and asymmetric NM

• application of $\overline{V}^{3\mathrm{N}}$ to finite nuclei (CC, construction/constraints of non-empirical EDFs, density-matrix expansions, ...)