

# Universality in QCD and Halo Nuclei

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Bundesministerium  
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Forschungsgemeinschaft



**Collaborators:** E. Braaten, D. Canham, D. Kang, L. Platter, R. Springer, ...

# Agenda

- Introduction
- Effective Field Theory for large scattering length
- Limit cycle  $\Leftrightarrow$  Efimov effect
- Halo nuclei
- Scattering properties of the  $X(3872)$
- Summary and Outlook

**Review article:** Braaten, HWH, Phys. Rep. **428** (2006) 259

Epelbaum, HWH, Meißner, arXiv:0811.1338 (to appear in Rev. Mod. Phys.)

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## Frequently asked Questions & Answers about EFT

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**Introduction:** To hear some people talk, one might think that one round of the EFT Basic Recipe would cure every ailment known to the human race. Just tap, tap, tap and everyone becomes peaceful—all grievances disappear, all traumas are instantly resolved and no one grows old or dies. That's a worthy thought, of course, but we're not quite there yet.

If EFT had reached such perfection, there would be nothing more to learn and this Question and Answer series would be unnecessary. But because we are always seeking greater proficiency, I am dedicating a significant part of this web site to those Questions and Answers about EFT that will allow us to move forward. The *questions* covered in this section come from both newcomers and experienced practitioners alike. The *answers* are designed to assist EFT users in the "art" of this process which, hopefully, will allow us all to approach perfection—a noble goal indeed. Let's begin.

**QUESTIONS & ANSWERS INDEX**

- [When it doesn't work....???](#)
- [What is the EFT "open hand" policy and what rights and restrictions do I have regarding the EFT materials, workshops, certifications, credentials, organizations and the practice of EFT?](#)
- [What do I say while performing EFT?](#)
- [Are there suggested guidelines for professionals regarding an Informed Consent Statement?](#)
- [What procedures might be taken if a client is undergoing a medical emergency?](#)
- [What do I do when the clients stop tapping even though they have had substantial success with EFT?](#)
- [When EFT relieves pain, does it mask the pain and thus thwart the ability for pain to signal danger?](#)
- [How do I handle Hand-Me-Down EFT?](#)
- [EFT is obvious! So why wasn't it discovered many decades ago?](#)
- [I'm used to affirmations being stated in the positive. Why does the EFT Setup language focus on the negative? What if I used positive statements instead?](#)
- [What's a good way for newcomers to apply EFT to themselves?](#)
- [How do you help a "non-feeling" client?](#)
- [Can EFT cause a client to "feel worse?"](#)
- [Can we do EFT surrogately or through intention? If so do we need permission?](#)

"I've never found a more user-friendly, dependable and precise technique that could produce such profound change in such a short time."

Kim English

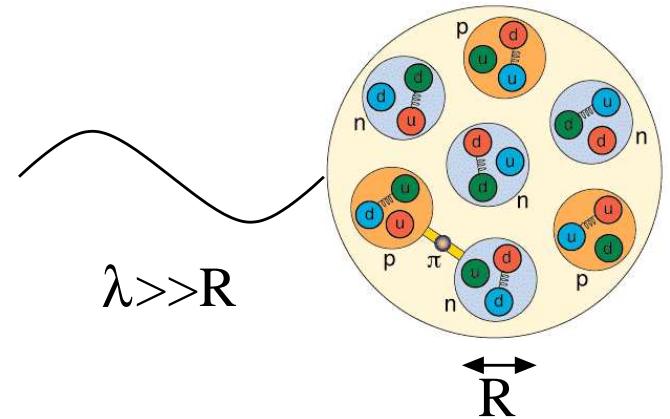
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thanks to D. Phillips

# Effective Field Theory

- Separation of scales:  
 $1/k = \lambda \gg R$
- Limited resolution at low energy:  
→ expand in powers of  $kR$

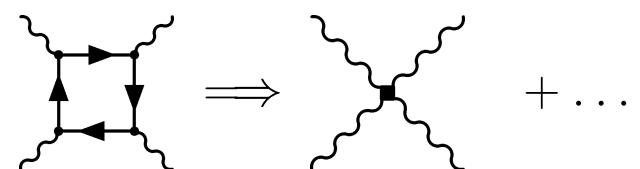
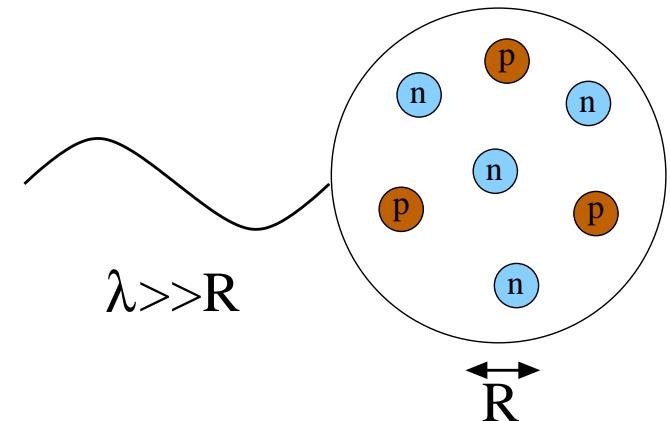


# Effective Field Theory

- Separation of scales:  
 $1/k = \lambda \gg R$
- Limited resolution at low energy:  
 → expand in powers of  $kR$
- Short-distance physics not resolved  
 → capture in low-energy constants using renormalization  
 → include long-range physics explicitly
- Systematic, model independent → error estimates
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

Simpler theory for  $\omega \ll m_e$ :

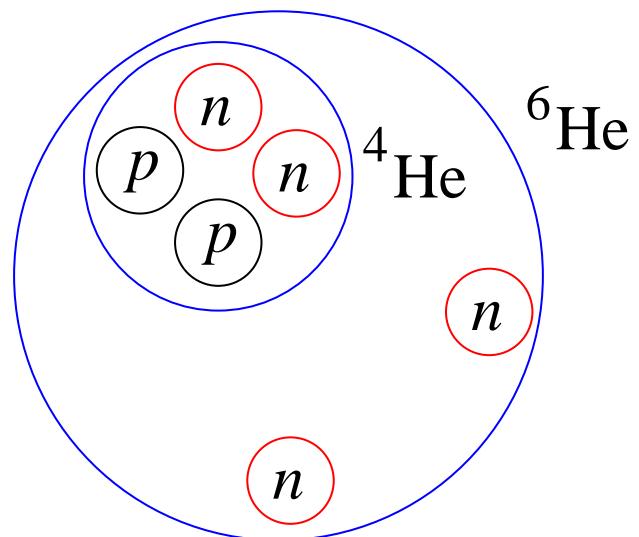
$$\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$$



# EFT for Many-Body Systems

- Few-body systems
  - fix interaction/low-energy constants for many-body calculations
- Few-body physics in many-body systems
  - certain properties of many-body systems are determined by **effective few-body physics**
- Limited resolution at low energy → **cluster structure**

- Example: halo nuclei



# Large Scattering Length

- Strong interaction  $\Rightarrow$  large scattering length  $a$
- Natural expansion parameter:  $\ell/|a|, k\ell, \dots$  ( $\ell \sim r_e, l_{vdW}, \dots$ )

$$a > 0 \implies B_d \approx \frac{\hbar^2}{ma^2}$$

- Atomic physics:
  - ${}^4\text{He}$ :  $a \approx 104 \text{ \AA} \gg r_e \approx 7 \text{ \AA} \sim l_{vdW} \longrightarrow B_d \approx 100 \text{ neV}$
  - Feshbach resonances  $\Rightarrow$  variable scattering length
- Nuclear physics:  $S$ -wave  $NN$ -scattering, halo nuclei,...
  - ${}^1S_0, {}^3S_1$ :  $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2 \text{ MeV}$
  - ${}^6\text{He}$ :  $2n$  separation energy  $\approx 973 \text{ keV}$
- Particle physics:  
Is the  $X(3872)$  a  $D^0 D^{0*}$  molecule? ( $J^{PC} = 1^{++}$ )  
 $m_X - (m_{D^0} + m_{D^{0*}}) = (-0.3 \pm 0.4) \text{ MeV}$

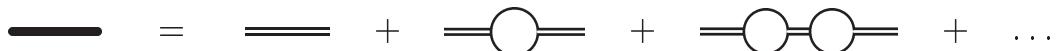
- Effective Lagrangian

(Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

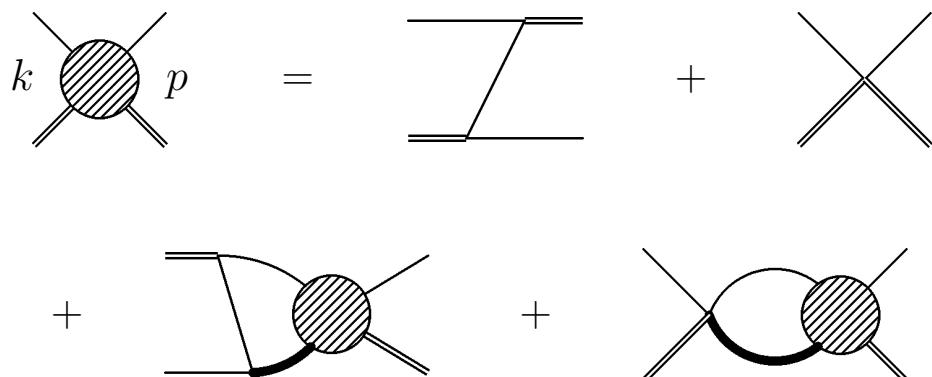
$$\mathcal{L}_d = \psi^\dagger \left( i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$

- 2- and 3-body interaction at leading order  $g_2, g_3$

- 2-body amplitude:



- 3-body equation:  
(S-waves)



- Dimensionless coupling:

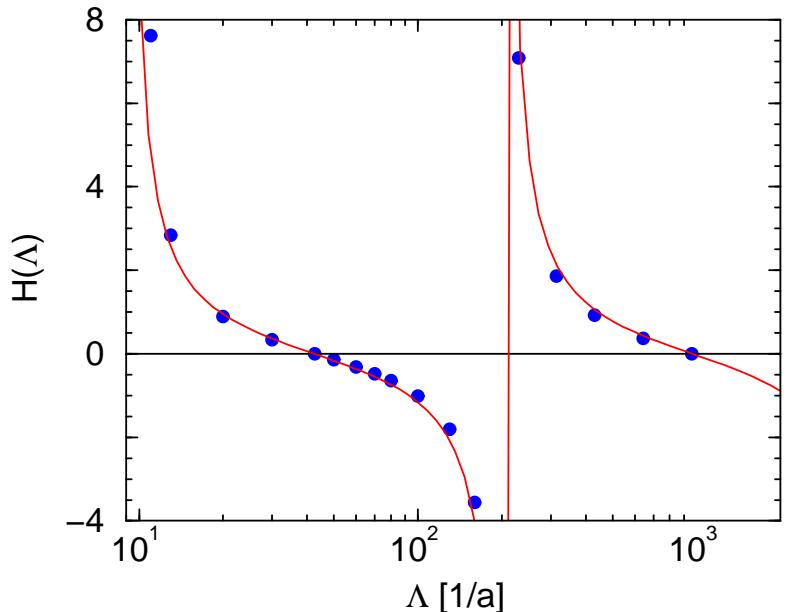
$$\frac{g_3}{9g_2^2} = \frac{H(\Lambda)}{\Lambda^2}$$

# Renormalization

- Demand invariance of observables from  $\Lambda$
- $H(\Lambda)$  periodic: **limit cycle**  

$$\Lambda \rightarrow \Lambda e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(Wilson, 1971)
- Discrete scale invariance



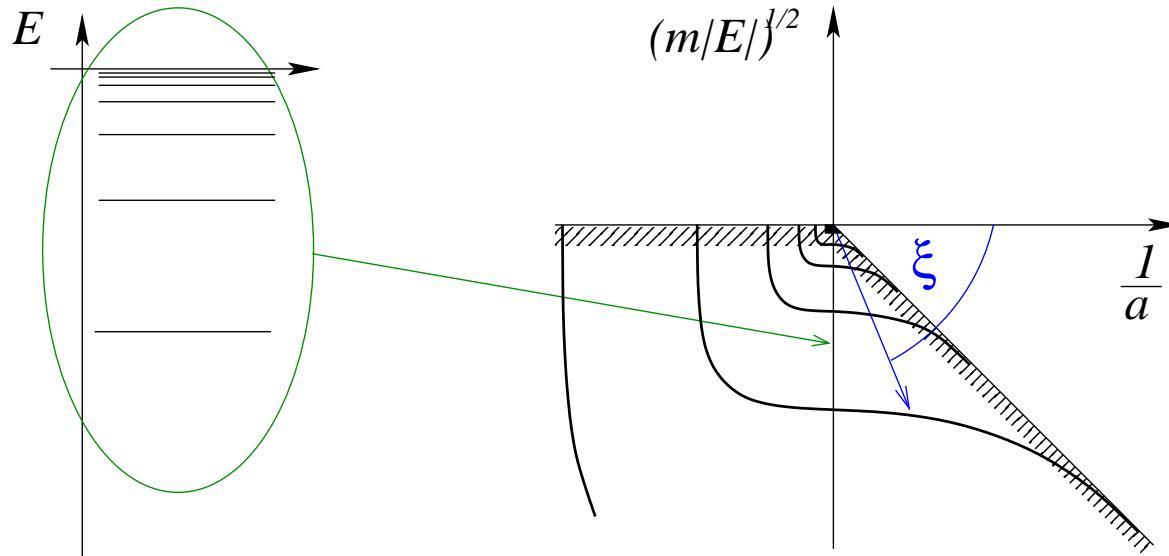
$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}, \quad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Observable Consequences?  
 $\implies$  Efimov effect, log-periodic dependence of observables on scattering length, limit cycle in deformed QCD?...
- Renormalization group approach (Barford, Birse, 2005)

# Limit Cycle: Efimov Effect

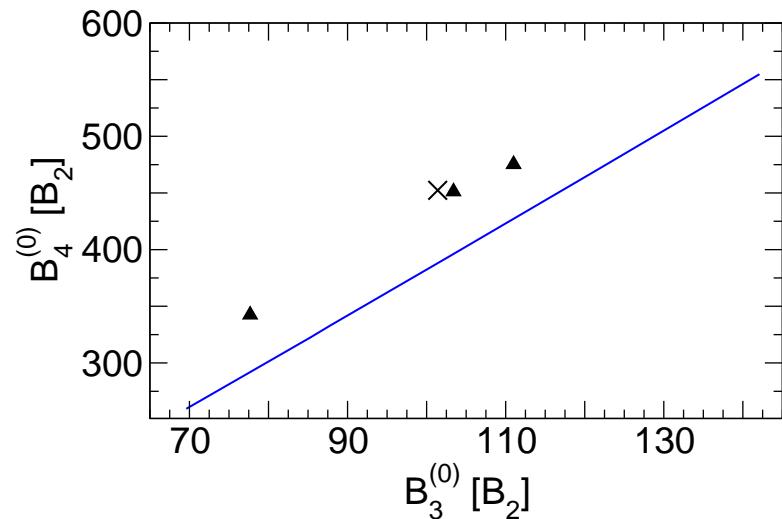
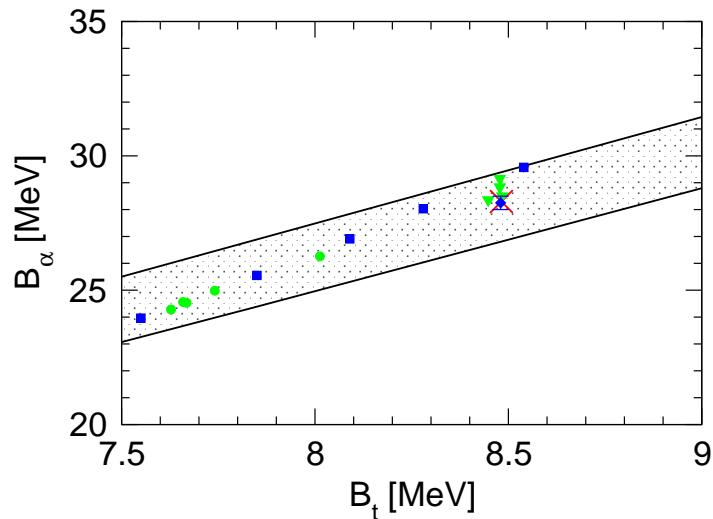
- Universal spectrum of three-body states  
(V. Efimov, Phys. Lett. **33B** (1970) 563)



- Discrete scale invariance for fixed angle  $\xi$
- Geometrical spectrum für  $1/a \rightarrow 0$
- Manifestation in scattering observables
  - ⇒ log-periodic dependence on  $a$
  - ⇒ indirect observation of the Efimov effect

# Universal Correlations

- 2 Parameters at LO  $\Rightarrow$  3-body observables are correlated  
 $\Rightarrow$  Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)  
 $\Rightarrow$  4-body observables are correlated  $\Rightarrow$  Tjon line



- Variation of  $\Lambda_*$  parametrizes correlations
- Nuclear physics:  $\Lambda$  dependence of  $V_{low-k}$  (Bogner et al., 2004)

# More on the 4-Body System

- Universal properties of 4-body system with large  $a$ 
  - Bound state spectrum
  - Scattering Observables
- “Efimov-plot”: 4-body bound state spectrum as function of  $1/a$

$$K = \text{sign}(E) \sqrt{m|E|}$$

$$B_4^{(0)} = 5B_3^{(0)} \quad (1/a \equiv 0)$$

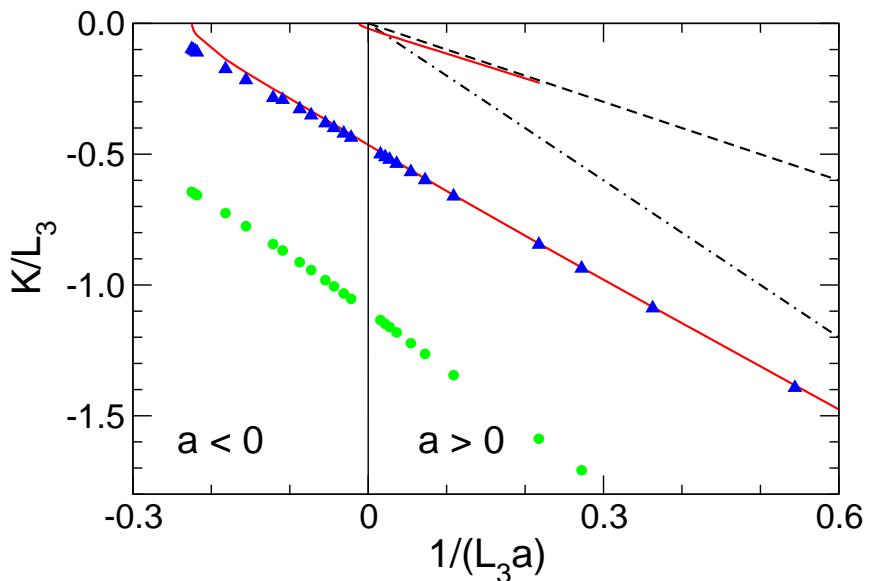
$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)

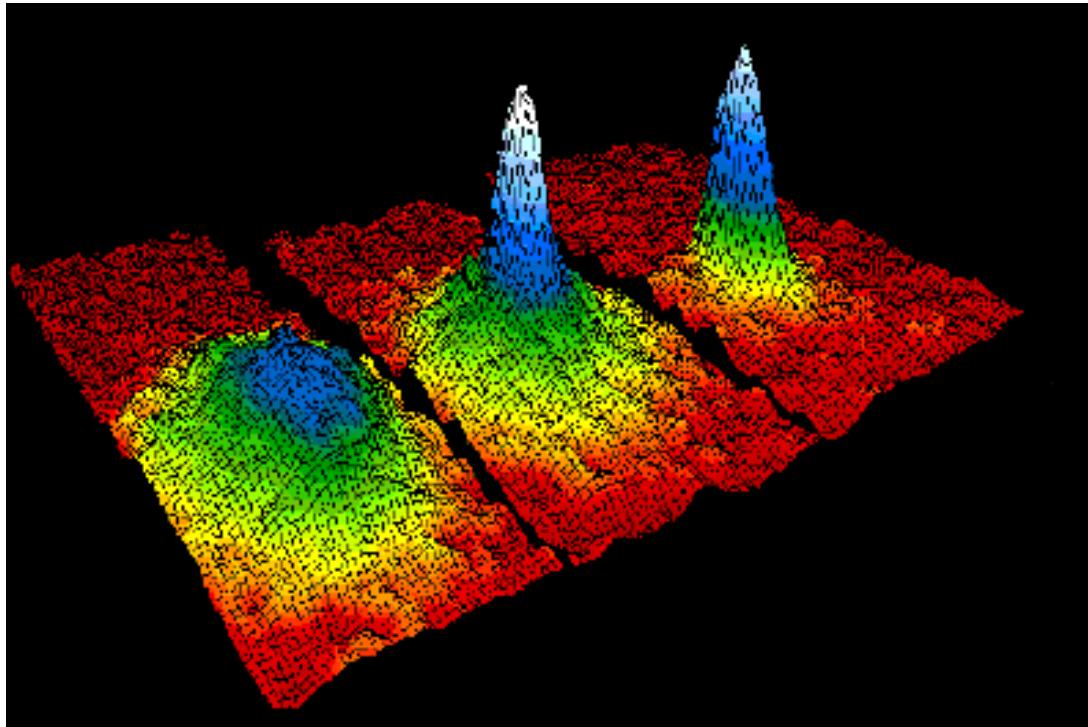
- Signature in Cs loss data

von Stecher, D’Incao, Greene, arXiv:0810.3876

Ferlaino, Knoop, Berninger, Harm, D’Incao, Nägerl, Grimm, arXiv:0903.1276



- Velocity distribution ( $T = 400 \text{ nK}, 200 \text{ nK}, 50 \text{ nK}$ )



(Source: <http://jilawww.colorado.edu/bec/>)

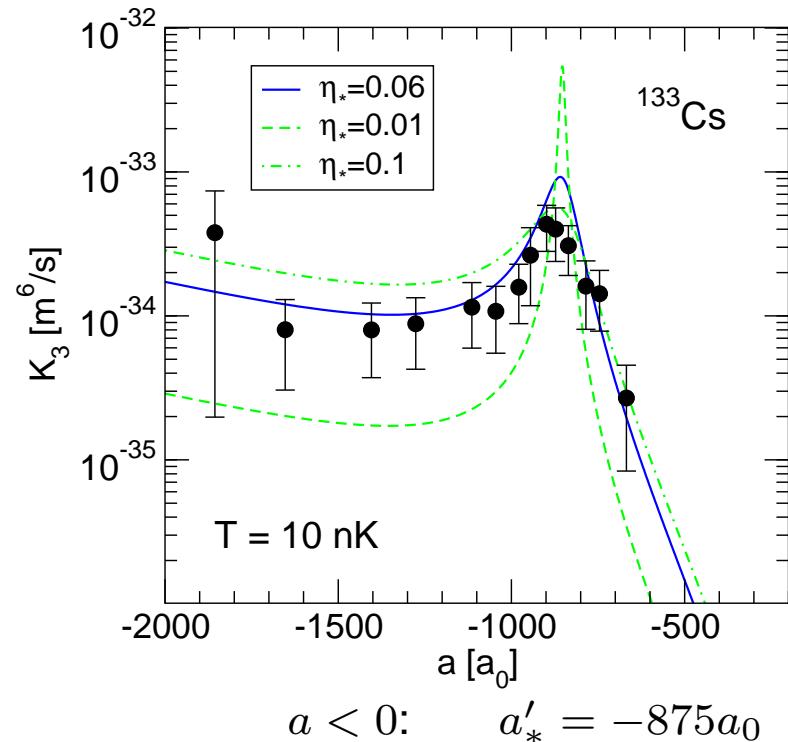
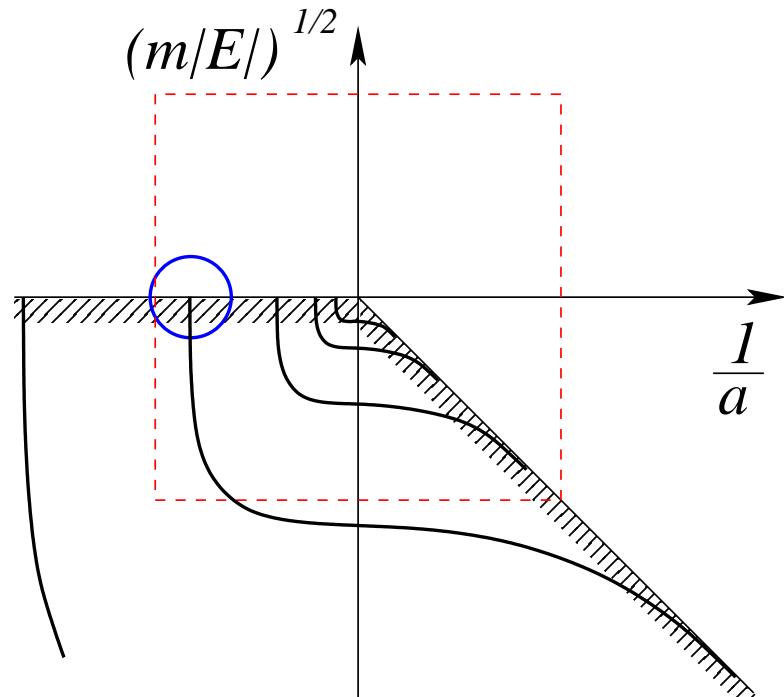
- Variable scattering length via Feshbach resonances

# Efimov States in $^{133}\text{Cs}$

- Experimental evidence for Efimov states in  $^{133}\text{Cs}$

(Kraemer et al. (Innsbruck), Nature **440** (2006) 315 )

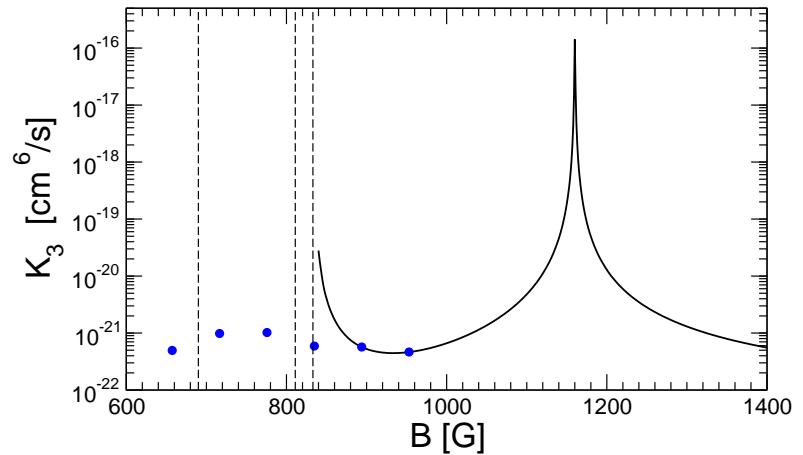
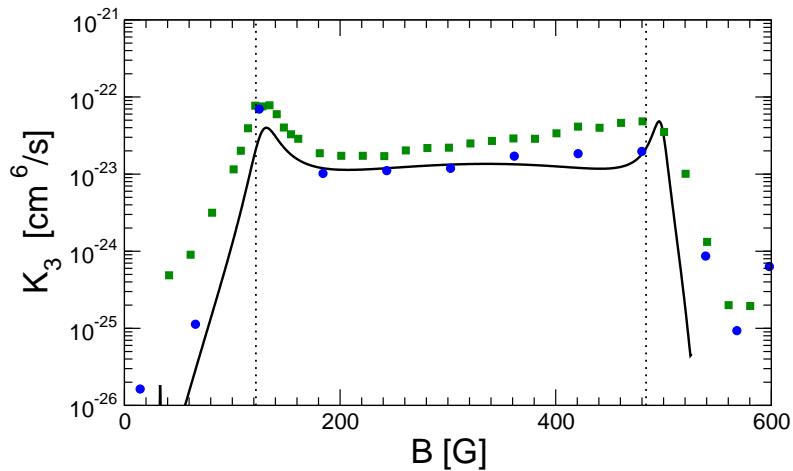
- Identification via 3-body recombination rate



- Treatment of finite temperature (Braaten, HWH, Kang, Platter, 2008)

# Efimov States in ${}^6\text{Li}$

- Efimov effect for fermions  $\Rightarrow \geq 3$  spin states
- Experimental evidence for Efimov states in  ${}^6\text{Li}$ 
  - Ottenstein et al. (Heidelberg), Phys. Rev. Lett. **101** (2008) 203202
  - Huckans et al. (Penn State), arXiv:0810.3288

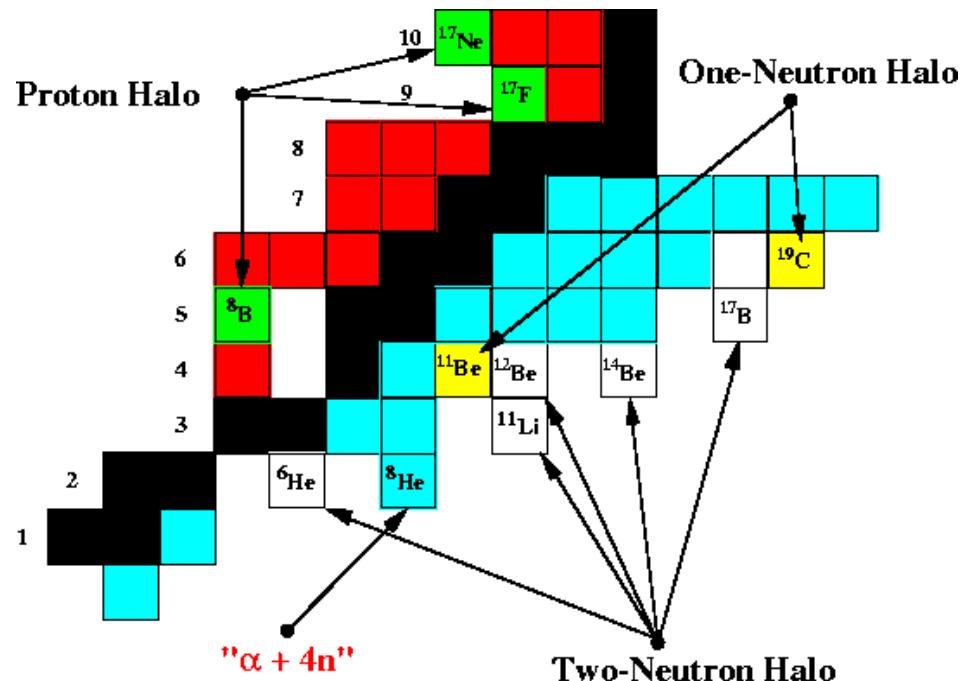


(Braaten, HWH, Kang, Platter, arXiv:0811.3578)

- Systematic normalization error: 70-90%
- Prediction of resonance around  $B \approx 1160$  G
- Other approaches: Schmidt et al., arXiv:0812.1191; Naidon et al., arXiv:0811.4086

# Halo Nuclei

- Low separation energy of valence nucleons:  $B_{valence} \ll B_{core}, E_{ex}$   
 → close to “nucleon drip line” → scale separation → EFT

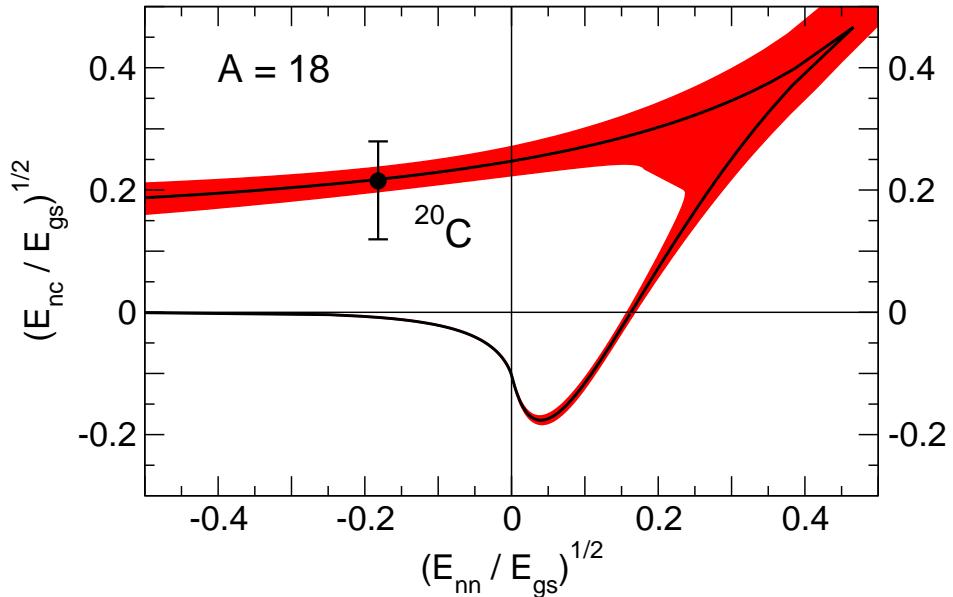
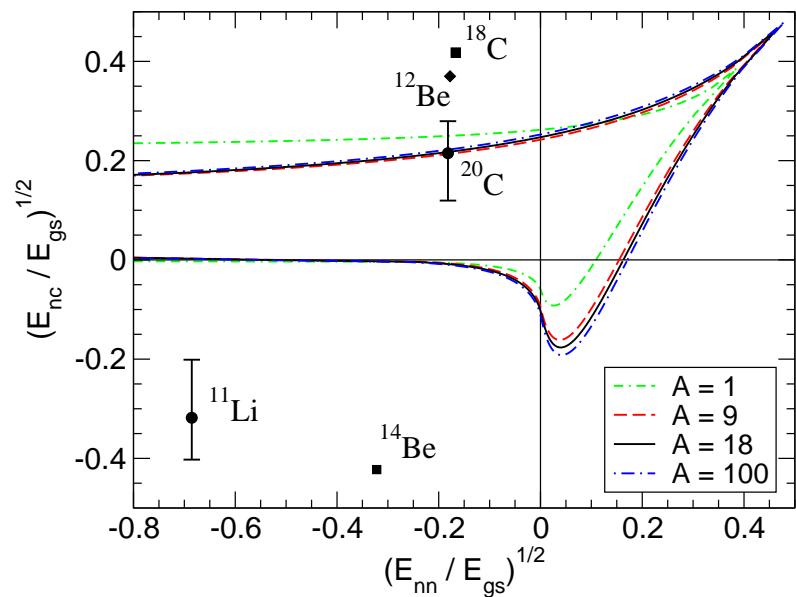


<http://www.nupecc.org>

- EFT for halo nuclei
  - $n\alpha$ -System (“ $^5\text{He}$ ”) (Bedaque, Bertulani, HWH, van Kolck, 2002)
  - $\alpha\alpha$ -System (“ $^8\text{Be}$ ”) (Higa, HWH, van Kolck, 2008)

# 3-Body Halos

- Examples:  $^{14}\text{Be} \longleftrightarrow ^{12}\text{Be} + n + n$ ,  $^{20}\text{C} \longleftrightarrow ^{18}\text{C} + n + n$
- "Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of "core"
- Efimov effect in halo nuclei?  $\Rightarrow$  excited states



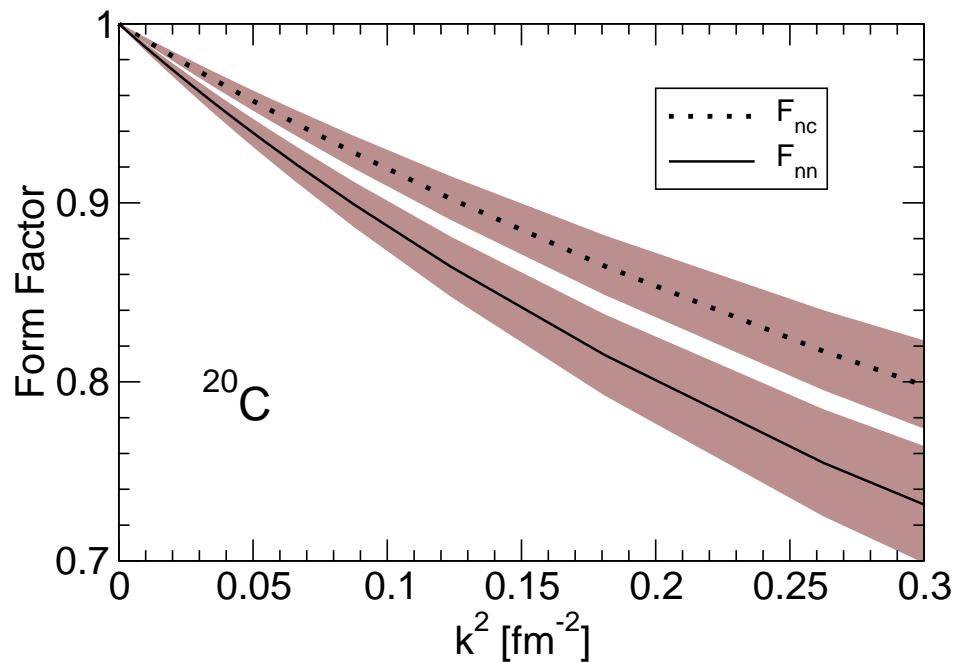
Canham, HWH, Eur. Phys. J. A **37** (2008) 367  
(cf. Amorim, Frederico, Tomio, 1997)

# Form Factors and Radii

- Structure of halo nuclei → matter form factors, radii

$$F(k^2) = \int d^3p d^3q \Psi(\vec{p}, \vec{q}) \Psi(\vec{p}, \vec{q} - \vec{k}) = 1 - \frac{1}{6} k^2 \langle r^2 \rangle + \dots$$

- Candidate:  $^{20}\text{C}$
- Form factors:
  - $F_{nn}$  neutron-neutron
  - $F_{nc}$  neutron-core
  - $F_n$  neutron-cm
  - $F_c$  core-cm



Canham, HWH, Eur. Phys. J. A 37 (2008) 367

# Form Factors and Radii

- Structure of halo nuclei → matter form factors, radii

nucleus	$B_{nnc}$ [keV]	$B_{nc}$ [keV]	$\sqrt{\langle r_{nn}^2 \rangle}$ [fm]	$\sqrt{\langle r_{nc}^2 \rangle}$ [fm]
$^{14}\text{Be}$	1120	-200.0	$4.1 \pm 0.5$	$3.5 \pm 0.5$
$^{20}\text{C}$	3506	161	$2.8 \pm 0.3$	$2.4 \pm 0.3$
	3506	530	$3.0 \pm 0.7$	$2.5 \pm 0.6$
	3506	60	$2.8 \pm 0.2$	$2.3 \pm 0.2$
$^{20}\text{C}^*$	$65 \pm 6.8$	60	$42 \pm 3$	$38 \pm 3$

Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment:  $^{14}\text{Be} \rightarrow \sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0) \text{ fm}$   
 (Marques et al., Phys. Rev. C **64** (2001) 061301)

# Exotic Charmonium Mesons

- Many new  $c\bar{c}$ -mesons at B-factories:  $X, Y, Z$ 
  - Challenge for understanding of QCD
  - Large scattering length physics important
- Example:  $X(3872)$  (Belle, CDF, BaBar, D0)

$$m_X = (3871.55 \pm 0.20) \text{ MeV} \quad \Gamma < 2.3 \text{ MeV} \quad J^{PC} = 1^{++}$$

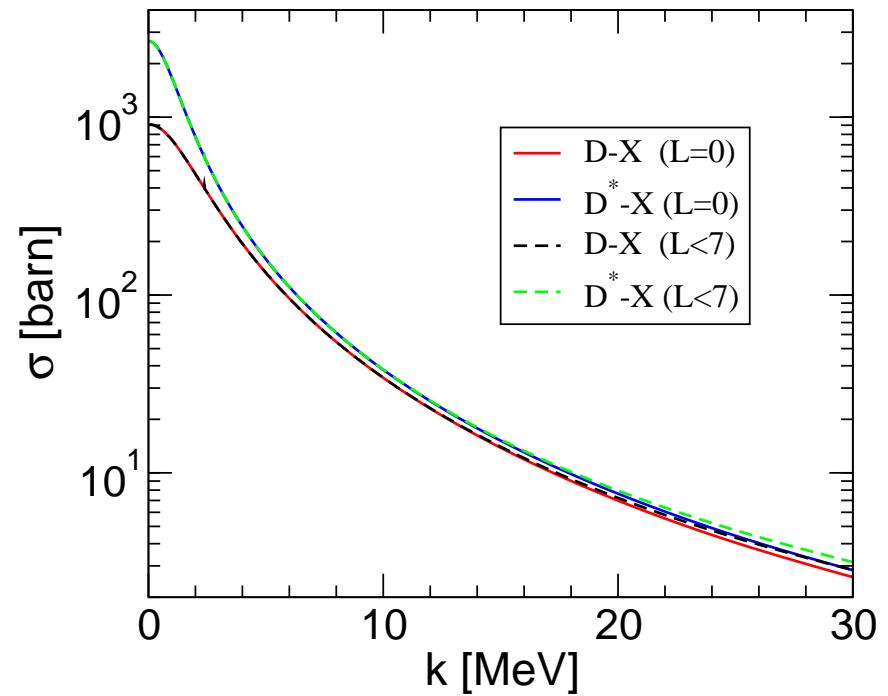
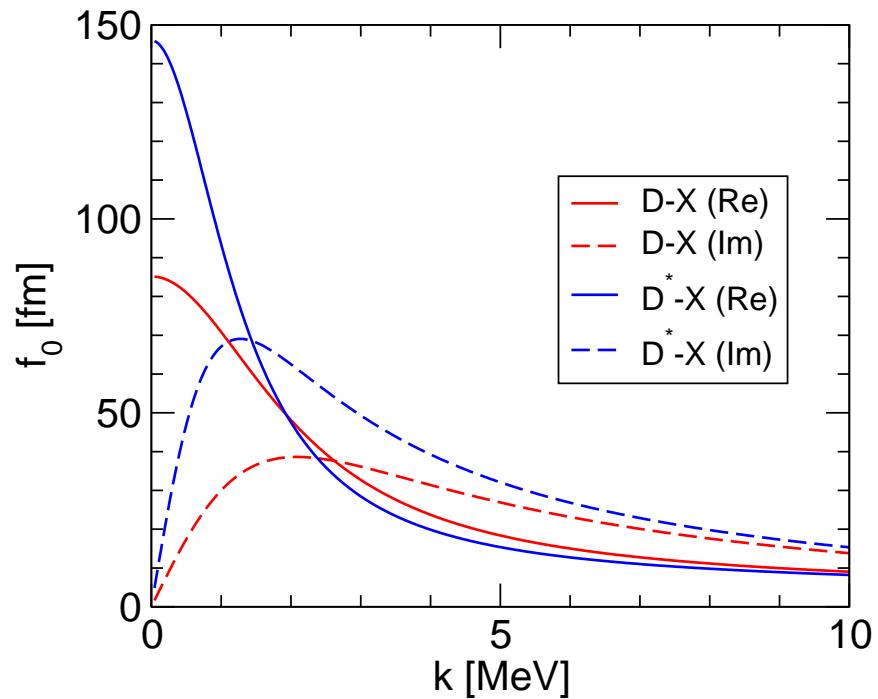
- No ordinary  $c\bar{c}$ -state
  - Decays violate isospin
  - Measured mass depends on decay channel
- Nature of  $X(3872)$ ?
  - $D^0 D^{0*}$ -molecule? (cf. Tornquist, 1991)
  - Tetraquark
  - Charmonium Hybrid
  - ...

# Nature of $X(3872)$

- Nature of  $X(3872)$  not finally resolved
- Assumption:  $X(3872)$  is weakly-bound  $D^0$ - $D^{0*}$ -molecule
  - $\Rightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0D^{0*}\rangle)/\sqrt{2}$ ,  $B_X = (0.26 \pm 0.41) \text{ MeV}$
  - $\Rightarrow$  universal properties (cf. Braaten et al., 2003-2008, ...)
    - Explains isospin violation in decays of  $X(3872)$   
 $\Rightarrow$  superposition of  $I = 1$  and  $I = 0$
    - Different masses due to different line shapes in decay channels
- Large scattering length determines interaction of  $X(3872)$  with  $D^0$  and  $D^{0*}$ 
  - $\Rightarrow$  no Efimov effect  $\Rightarrow$  no  $X$ - $D$ - and  $X$ - $D^*$ -molecule
  - $\Rightarrow$  parameter-free prediction of  $X$ - $D$ -,  $X$ - $D^*$ -scattering
- EFT with explicit pions: Fleming, Kusunoki, Mehen, van Kolck, 2007

# $X$ - $D$ - and $X$ - $D^*$ -Scattering

- Predictions for scattering amplitude/cross section



Canham, HWH, Springer, in preparation

- Rare events at B factories ( $B \rightarrow X, \bar{B} \rightarrow D, D^*$ )
- Final state interaction of  $D, D^*$  mesons in  $B_c$ -decays (e.g. LHCb)

# Summary and Outlook

- EFT  $\Rightarrow$  (effective) few-body properties of many-body systems
- EFT for systems with large scattering length
  - Limit cycle in three-body system  $\Leftrightarrow$  Efimov effect
  - Universal correlations (Phillips, Tjon line,...)
- Applications in atomic, nuclear, and particle physics
  - cold atoms close to Feshbach resonance
  - halo nuclei
  - scattering properties of the  $X(3872)$
- Future: more exciting applications
  - Halo nuclei: Structure, Reactions, ...
  - Three-nucleon system on the lattice: finite  $V$ , limit cycle, ...
  - Cold atoms:  $N \geq 4$ , 2d-systems, ...
  - ...