Universality in QCD and Halo Nuclei

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Agenda

- Introduction
- Effective Field Theory for large scattering length
- Limit cycle $\Leftrightarrow$ Efimov effect
- Halo nuclei
- Scattering properties of the $X(3872)$
- Summary and Outlook

Review article: Braaten, HWH, Phys. Rep. **428** (2006) 259

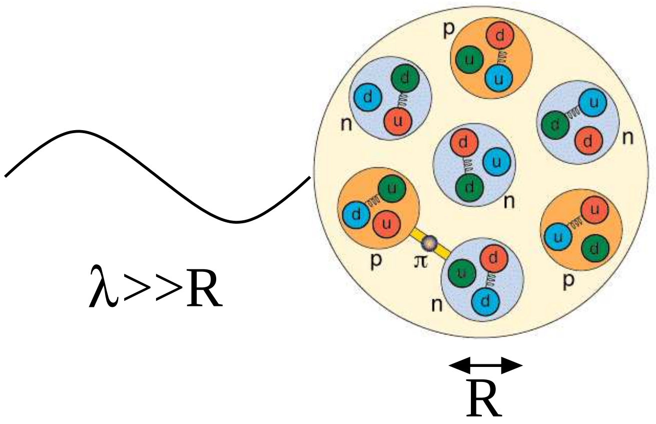
Epelbaum, HWH, Meißner, arXiv:0811.1338 (to appear in Rev. Mod. Phys.)

EFT

thanks to D. Phillips

Effective Field Theory

- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



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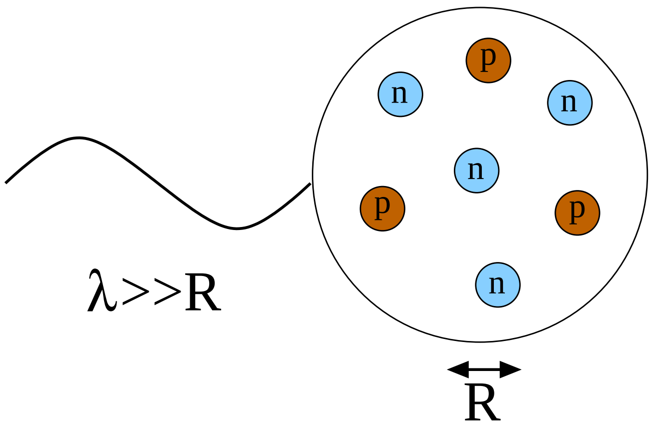
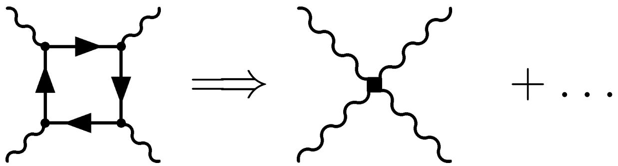


- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\rightarrow$ error estimates
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

  Simpler theory for $\omega \ll m_e$:

  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$



EFT for Many-Body Systems

- Few-body systems

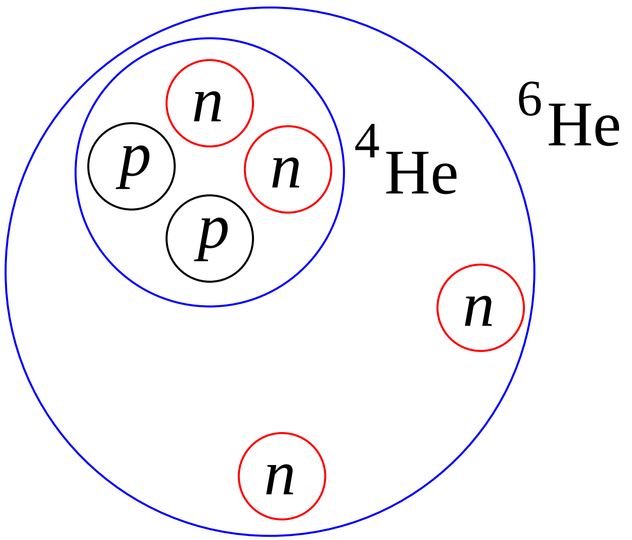
  $\longrightarrow$ fix interaction/low-energy constants for many-body calculations

- Few-body physics in many-body systems

  $\longrightarrow$ certain properties of many-body systems are determined by effective few-body physics

- Limited resolution at low energy $\longrightarrow$ cluster structure

- Example: halo nuclei



Large Scattering Length

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- Strong interaction $\Longrightarrow$ large scattering length a
- Natural expansion parameter: $\ell/|a|$, $k\ell$,... $(\ell \sim r_e, l_{vdW}, \ldots)$

$$a > 0 \quad \Longrightarrow \quad B_d \approx \frac{\hbar^2}{ma^2}$$

- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ variable scattering length
- Nuclear physics: S-wave NN-scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{6}He: $2n$ separation energy $\approx$ 973 keV
- Particle physics:
  Is the $X(3872)$ a $D^0 D^{0*}$ molecule? $(J^{PC} = 1^{++})$

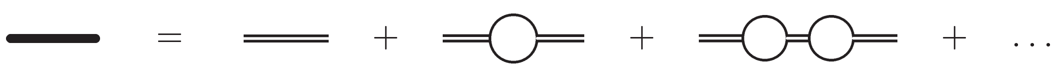
$$m_X - (m_{D^0} + m_{D^{0*}}) = (-0.3 \pm 0.4) \text{ MeV}$$

EFT for large a

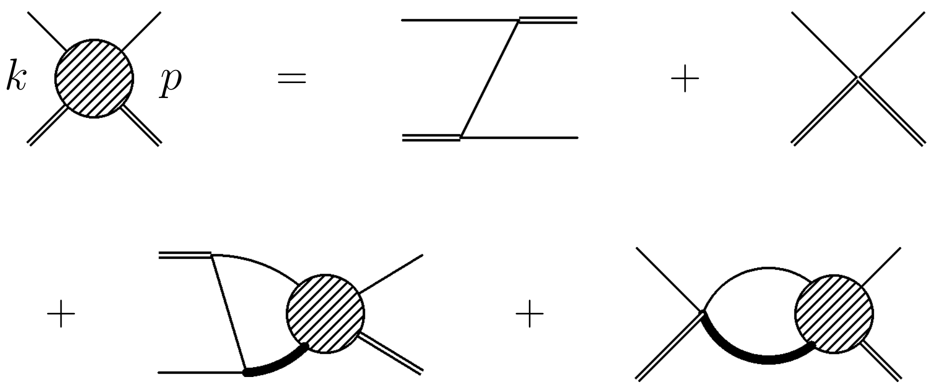
- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$

- 2- and 3-body interaction at leading order g_2, g_3
- 2-body amplitude:



- 3-body equation:
  (S-waves)



- Dimensionless coupling: $\dfrac{g_3}{9g_2^2} = \dfrac{H(\Lambda)}{\Lambda^2}$

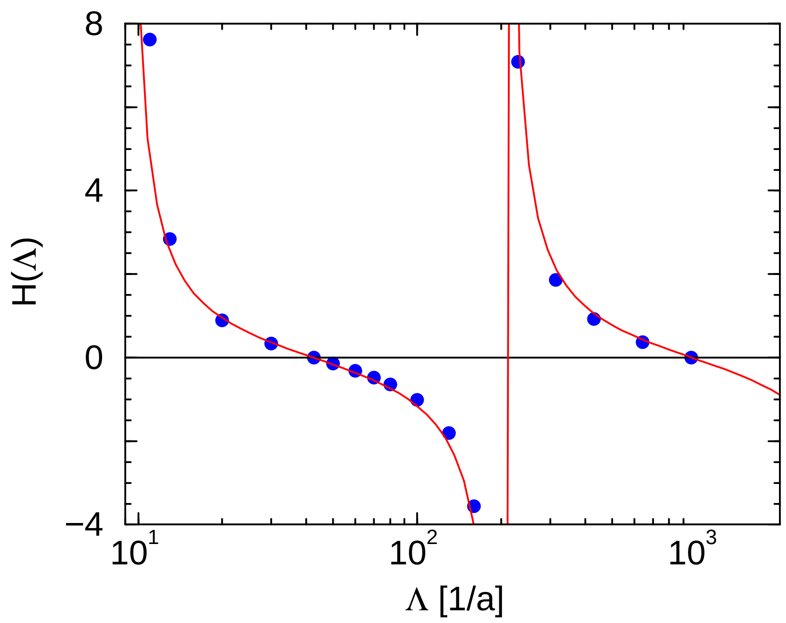
Renormalization

- Demand invariance of observables from Λ
- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(Wilson, 1971)

- Discrete scale invariance



$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

(Bedaque, HWH, van Kolck, 1999)

- Observable Consequences?

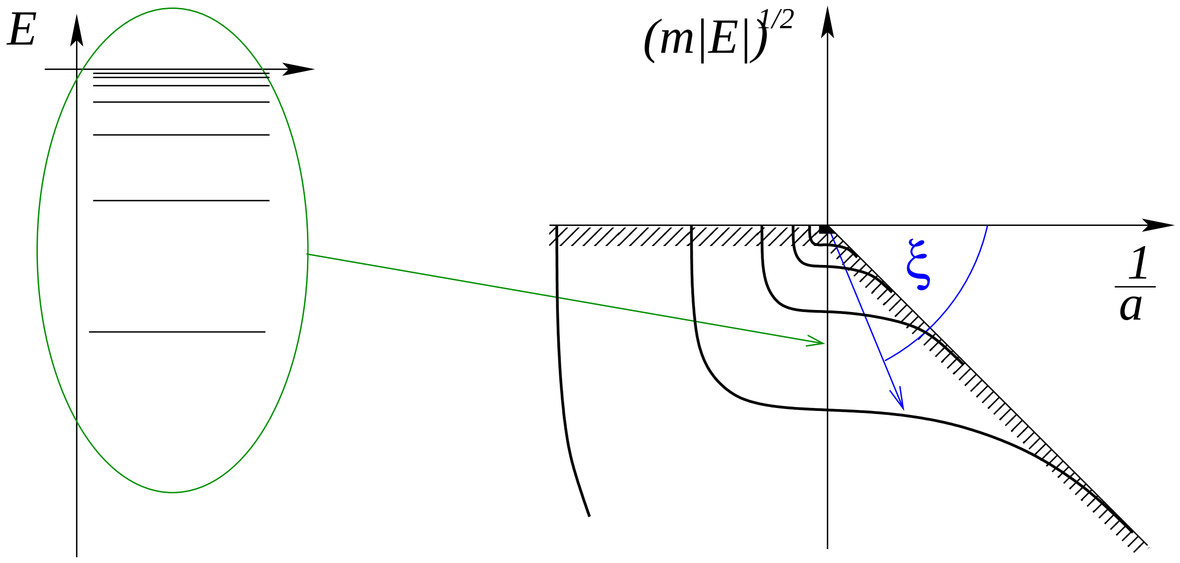
$\Longrightarrow$ Efimov effect, log-periodic dependence of observables on scattering length, limit cycle in deformed QCD?,...

- Renormalization group approach (Barford, Birse, 2005)

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states

  (V. Efimov, Phys. Lett. **33B** (1970) 563)



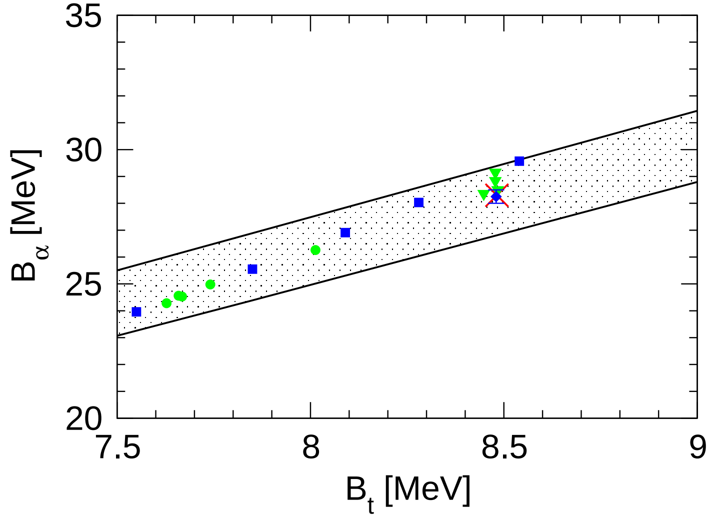
  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum für $1/a \to 0$

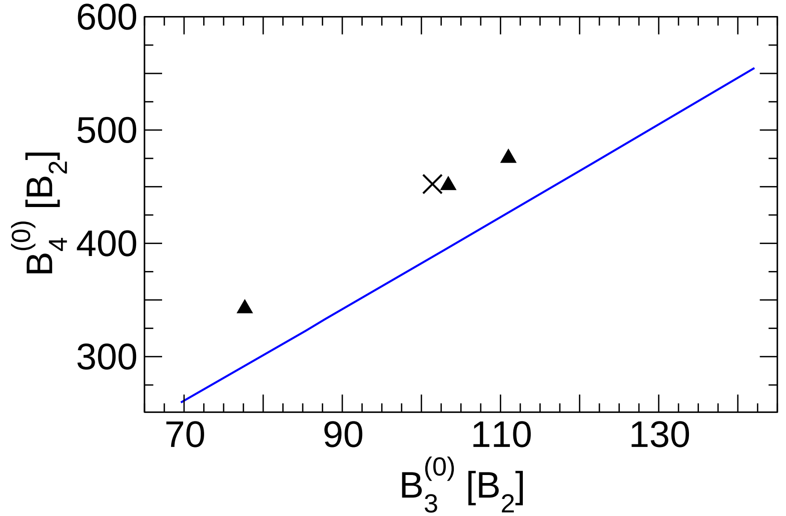
- Manifestation in scattering observables

  $\Longrightarrow$ log-periodic dependence on a
  $\Longrightarrow$ indirect observation of the Efimov effect

Universal Correlations

- 2 Parameters at LO $\Rightarrow$ 3-body observables are correlated
  $\Longrightarrow$ Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line





- Variation of Λ_* parametrizes correlations
- Nuclear physics: Λ dependence of V_{low-k} (Bogner et al., 2004)

More on the 4-Body System

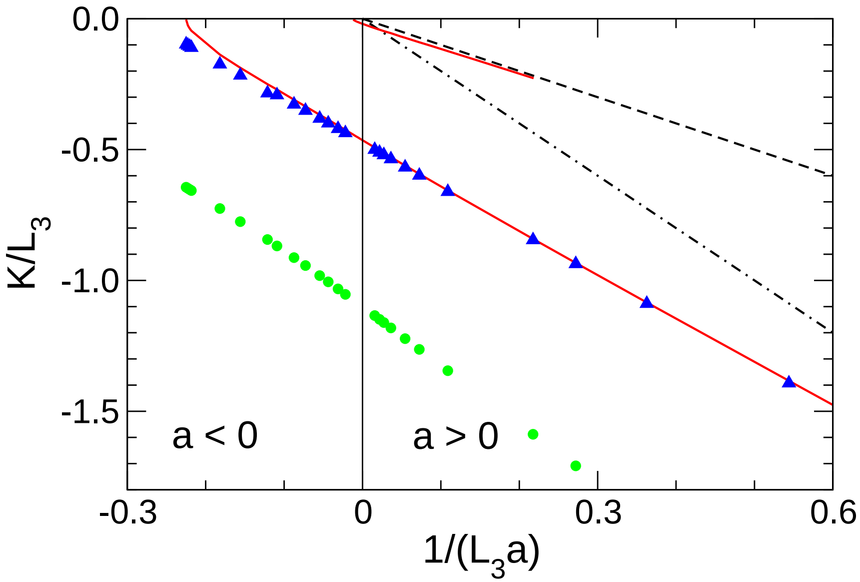
- Universal properties of 4-body system with large a
  - Bound state spectrum
  - Scattering Observables
- "Efimov-plot": 4-body bound state spectrum as function of $1/a$

$$K = \text{sign}(E)\sqrt{m|E|}$$

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$

$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



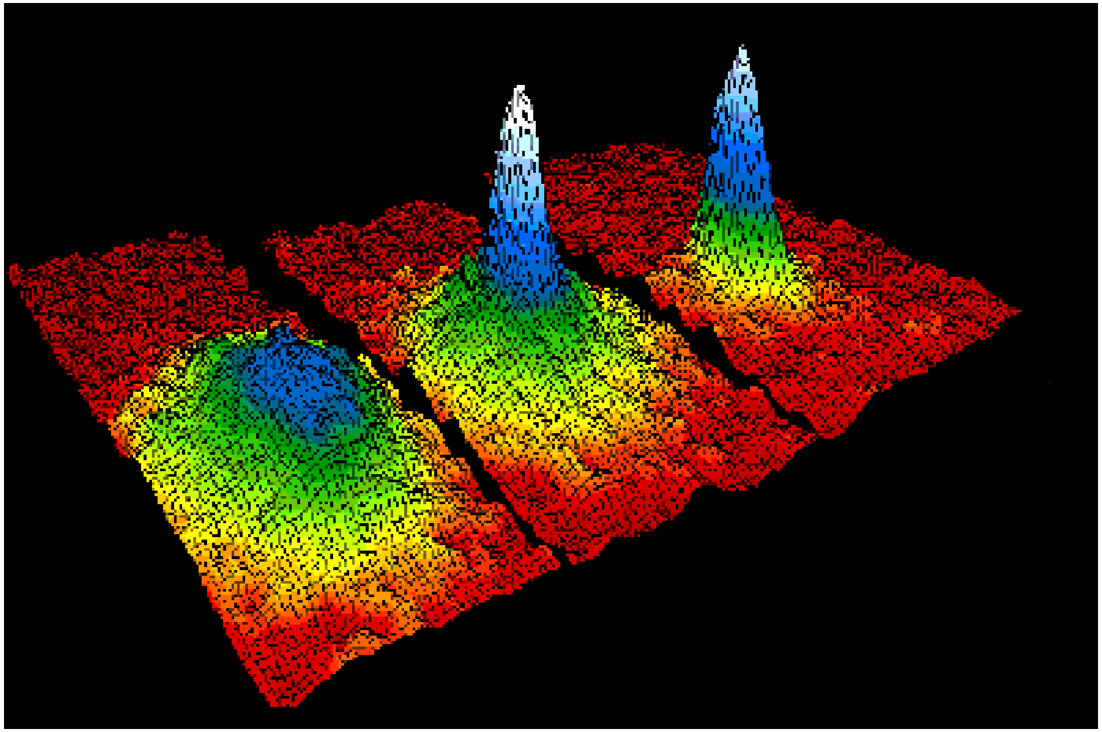
- Signature in Cs loss data

von Stecher, D'Incao, Greene, arXiv:0810.3876

Ferlaino, Knoop, Berninger, Harm, D'Incao, Nägerl, Grimm, arXiv:0903.1276

Cold Atoms/BEC

- Velocity distribution ($T =$ 400 nK, 200 nK, 50 nK)

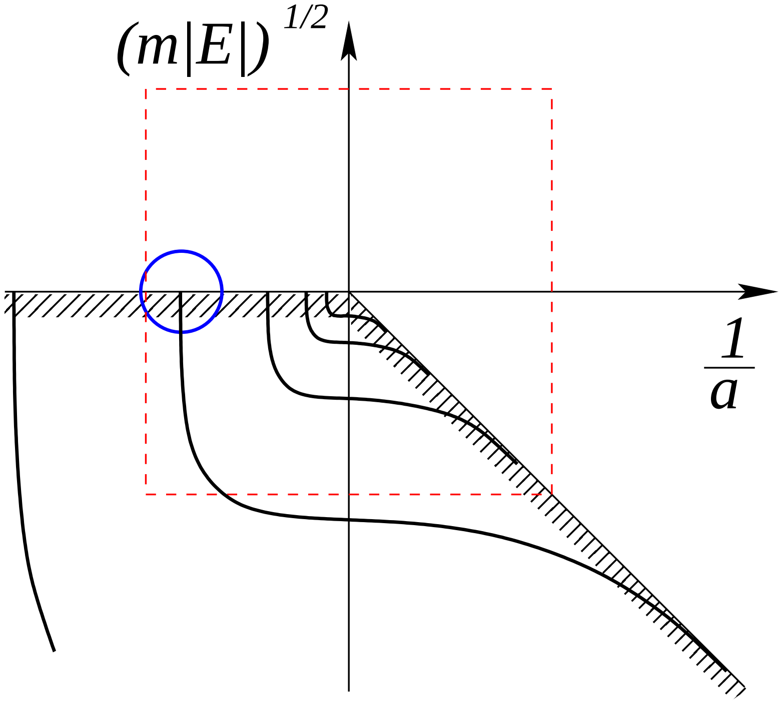


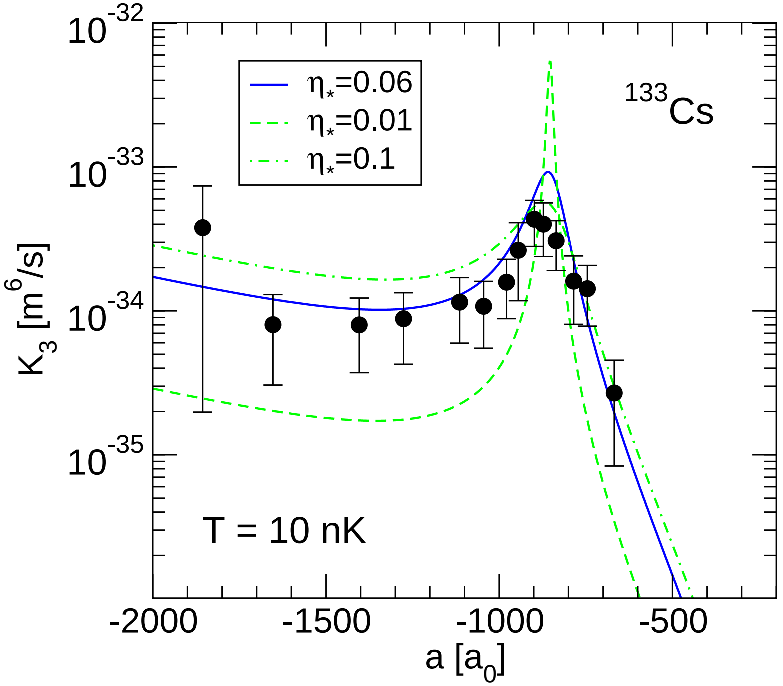
(Source: http://jilawww.colorado.edu/bec/)

- Variable scattering length via Feshbach resonances

Efimov States in ^{133}Cs

- Experimental evidence for Efimov states in ^{133}Cs
  (Kraemer et al. (Innsbruck), Nature **440** (2006) 315)
  - Identification via 3-body recombination rate





$a < 0$: $\quad a'_* = -875 a_0$

- Treatment of finite temperature (Braaten, HWH, Kang, Platter, 2008)

Efimov States in ^{6}Li

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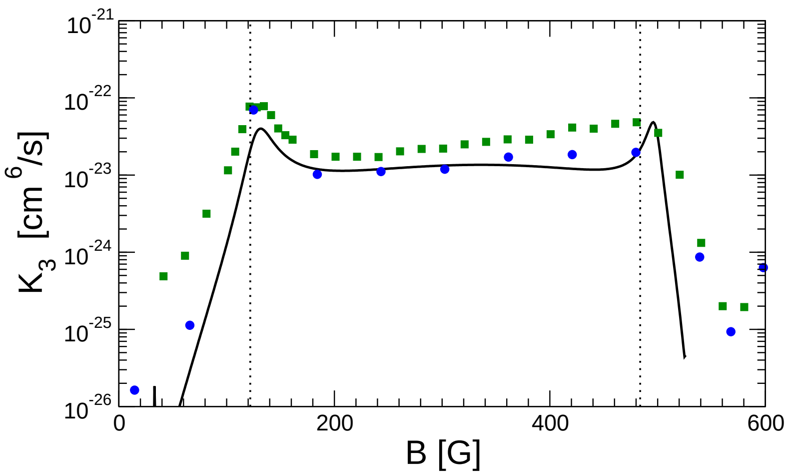
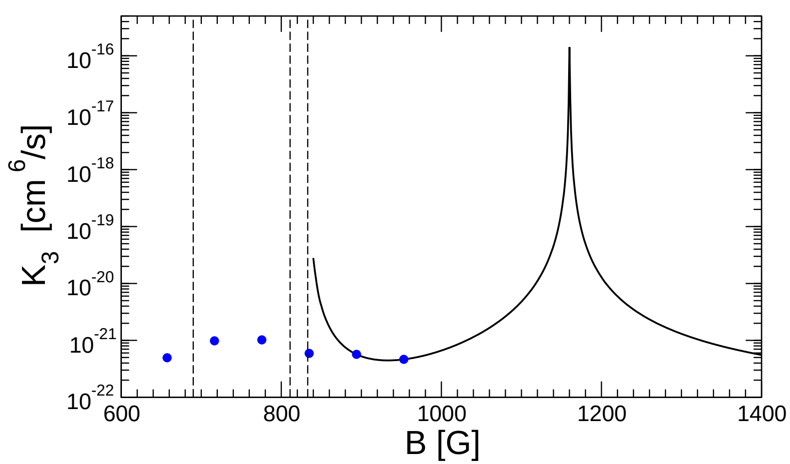
- Efimov effect for fermions $\Rightarrow \geq 3$ spin states
- Experimetal evidence for Efimov states in ^{6}Li
  - ■ Ottenstein et al. (Heidelberg), Phys. Rev. Lett. **101** (2008) 203202
  - ● Huckans et al. (Penn State), arXiv:0810.3288





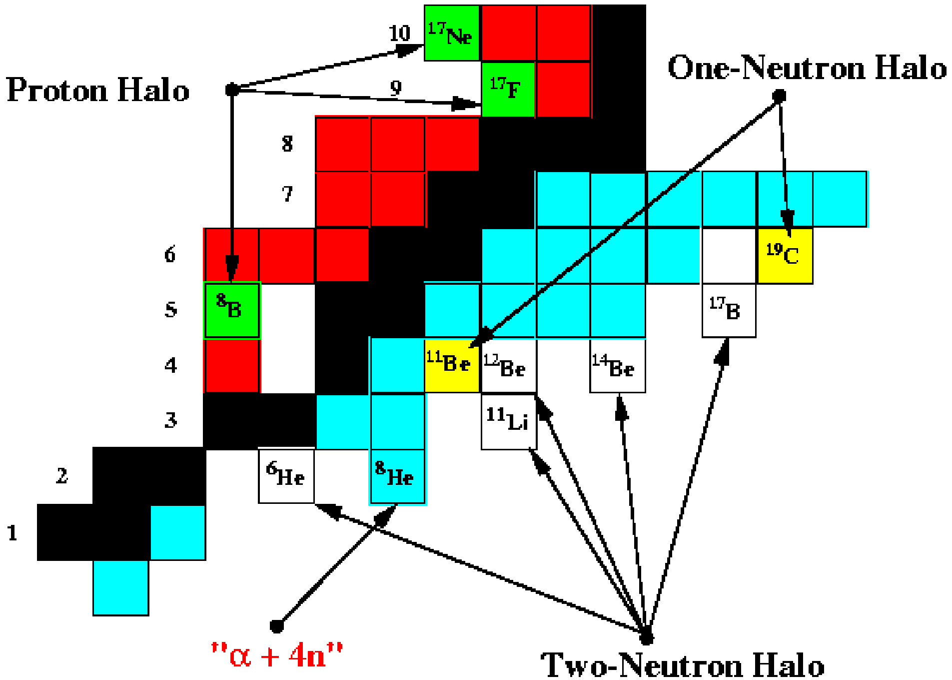
(Braaten, HWH, Kang, Platter, arXiv:0811.3578)

- Systematic normalization error: 70-90%
- Prediction of resonance around $B \approx 1160$ G
- Other approaches: Schmidt et al., arXiv:0812.1191; Naidon et al., arXiv:0811:4086

Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to “nucleon drip line” $\longrightarrow$ scale separation $\longrightarrow$ EFT

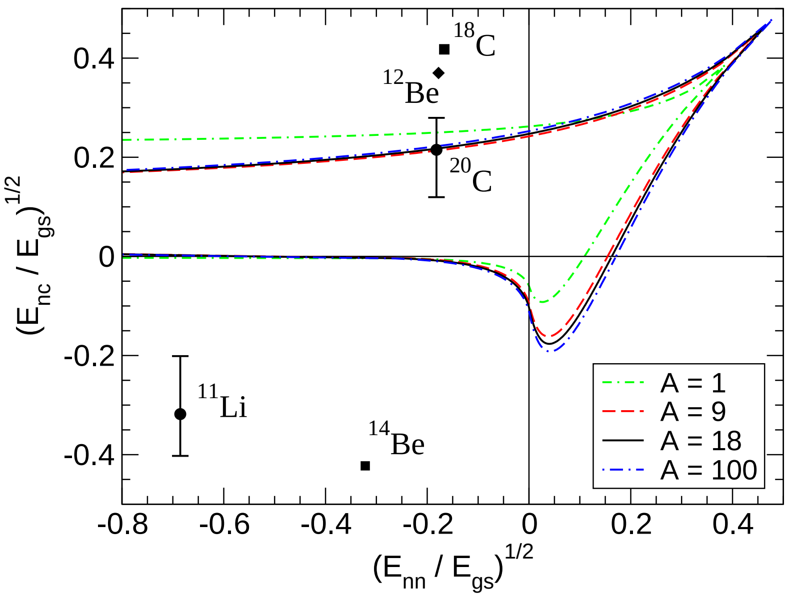


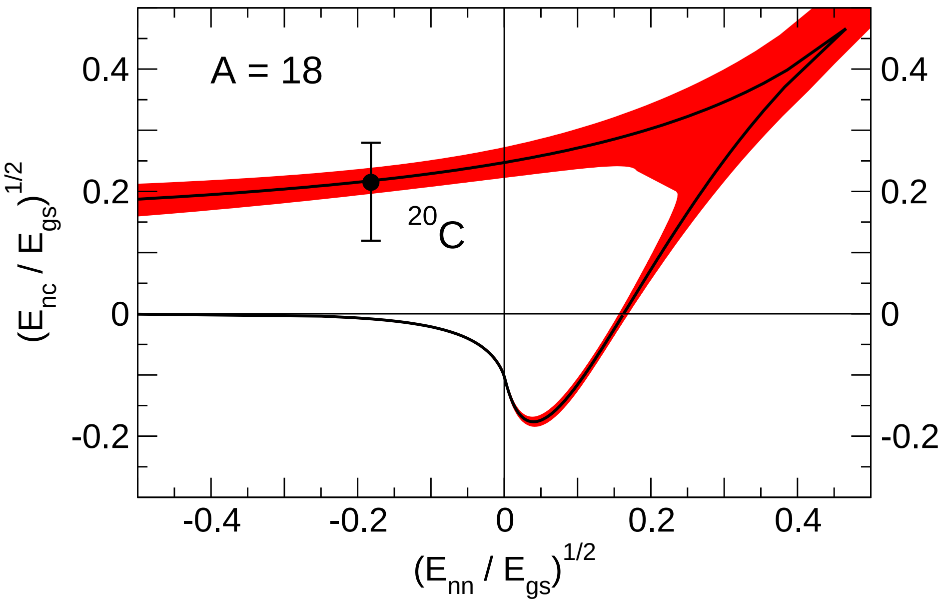
http://www.nupecc.org

- EFT for halo nuclei
  - $n\alpha$-System (“^{5}He”) (Bedaque, Bertulani, HWH, van Kolck, 2002)
  - $\alpha\alpha$-System (“^{8}Be”) (Higa, HWH, van Kolck, 2008)

3-Body Halos

- Examples: $^{14}\mathrm{Be} \longleftrightarrow {}^{12}\mathrm{Be}$ +n +n, $^{20}\mathrm{C} \longleftrightarrow {}^{18}\mathrm{C}$ +n +n
- “Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of “core”
- Efimov effect in halo nuclei? $\Rightarrow$ excited states





Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Amorim, Frederico, Tomio, 1997)

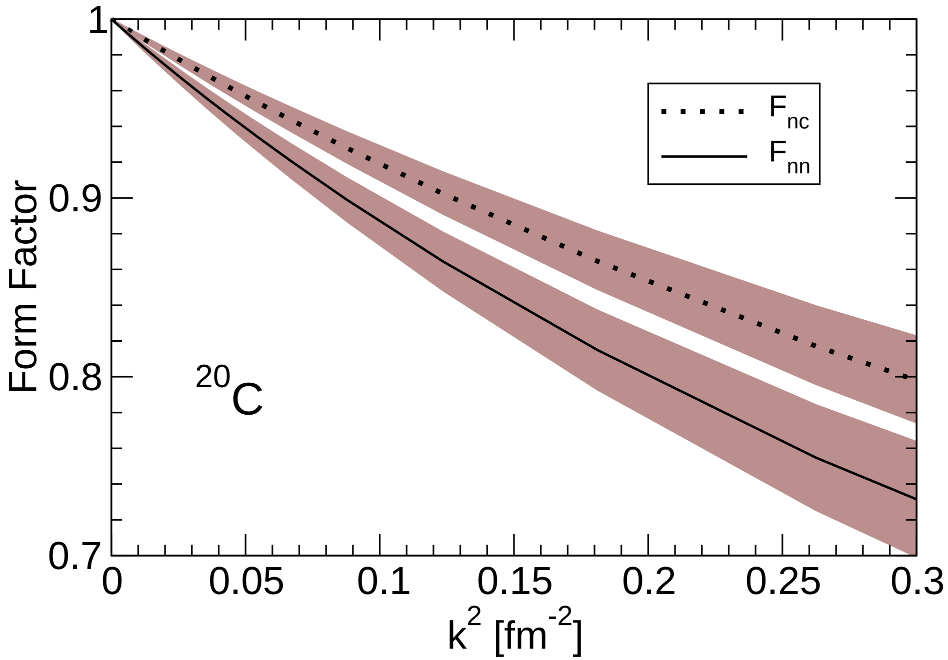
Form Factors and Radii

- Structure of halo nuclei $\longrightarrow$ matter form factors, radii

$$F(k^2) = \int d^3p\, d^3q\, \Psi(\vec{p}, \vec{q})\Psi(\vec{p}, \vec{q} - \vec{k}) = 1 - \frac{1}{6}k^2\langle r^2\rangle + \ldots$$

- Candidate: ^{20}C
- Form factors:

  F_{nn} neutron-neutron
  F_{nc} neutron-core
  F_{n} neutron-cm
  F_{c} core-cm



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Form Factors and Radii

- Structure of halo nuclei $\rightarrow$ matter form factors, radii

nucleus	B_{nnc} [keV]	B_{nc} [keV]	$\sqrt{\langle r^2_{nn} \rangle}$ [fm]	$\sqrt{\langle r^2_{nc} \rangle}$ [fm]
^{14}Be	1120	-200.0	4.1 ± 0.5	3.5 ± 0.5
^{20}C	3506	161	2.8 ± 0.3	2.4 ± 0.3
	3506	530	3.0 ± 0.7	2.5 ± 0.6
	3506	60	2.8 ± 0.2	2.3 ± 0.2
^{20}C*	65 ± 6.8	60	42 ± 3	38 ± 3

Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: ^{14}Be $\rightarrow \sqrt{\langle r^2_{nn} \rangle} = (5.4 \pm 1.0)$ fm

  (Marques et al., Phys. Rev. C **64** (2001) 061301)

Exotic Charmonium Mesons

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- Many new $c\bar{c}$-mesons at B-factories: X, Y, Z
  - Challenge for understanding of QCD
  - Large scattering length physics important
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)

$m_X = (3871.55 \pm 0.20)$ MeV $\Gamma < 2.3$ MeV $J^{PC} = 1^{++}$

- No ordinary $c\bar{c}$-state
  - Decays violate isospin
  - Measured mass depends on decay channel
- Nature of $X(3872)$?
  - $D^0 D^{0*}$-molecule? (cf. Tornquist, 1991)
  - Tetraquark
  - Charmonium Hybrid
  - ...

Nature of $X(3872)$

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- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound D^0-D^{0*}-molecule

  $\Longrightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}\,, \qquad B_X = (0.26 \pm 0.41)$ MeV

  $\Longrightarrow$ universal properties (cf. Braaten et al., 2003-2008, ...)

  - Explains isospin violation in decays of $X(3872)$ $\Rightarrow$ superposition of $I = 1$ and $I = 0$
  - Different masses due to different line shapes in decay channels
- Large scattering length determines interaction of $X(3872)$ with D^0 and D^{0*}

  $\Longrightarrow$ no Efimov effect $\Rightarrow$ no X-D- and X-D^*-molecule

  $\Longrightarrow$ parameter-free prediction of X-D-, X-D^*-scattering
- EFT with explicit pions: Fleming, Kusunoki, Mehen, van Kolck, 2007

X-D- and X-D^*-Scattering

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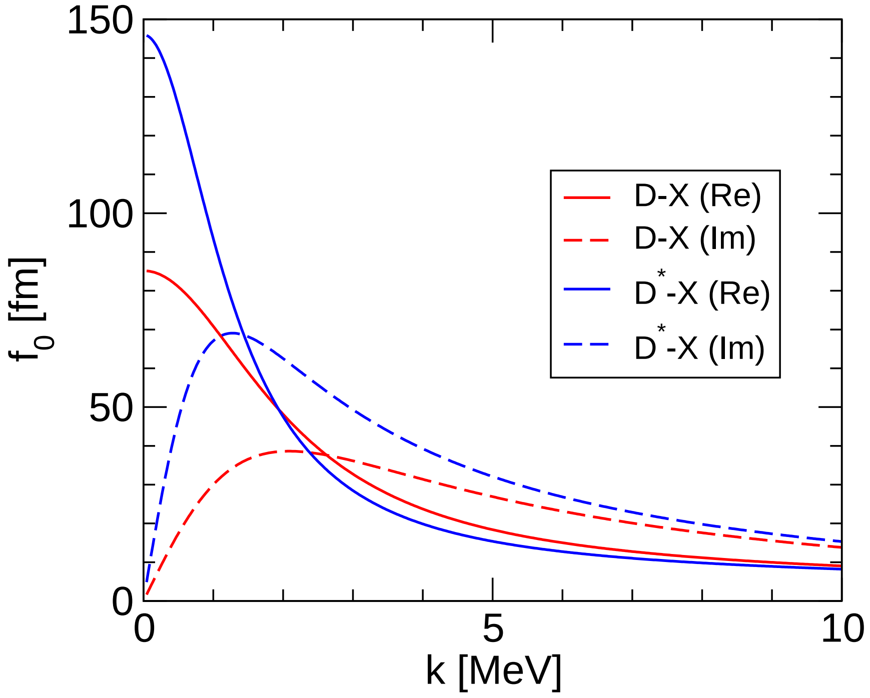
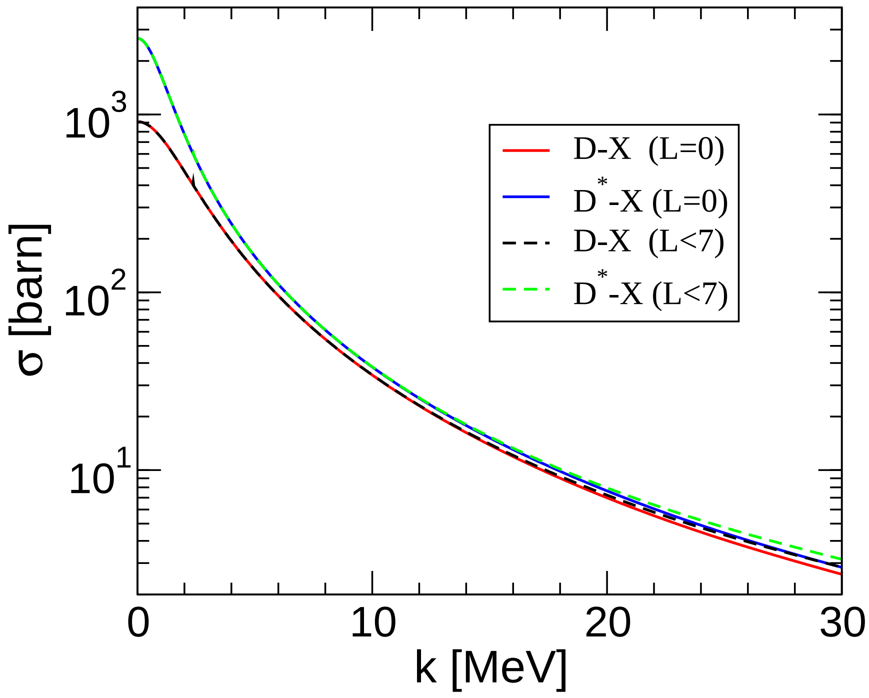
- Predictions for scattering amplitude/cross section





Canham, HWH, Springer, in preparation

- Rare events at B factories ($B \to X$, $\bar{B} \to D, D^*$)
- Final state interaction of D, D^* mesons in B_c-decays (e.g. LHCb)

Summary and Outlook

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- EFT $\Rightarrow$ (effective) few-body properties of many-body systems
- EFT for systems with large scattering length
  - Limit cycle in three-body system $\Leftrightarrow$ Efimov effect
  - Universal correlations (Phillips, Tjon line,...)
- Applications in atomic, nuclear, and particle physics
  - cold atoms close to Feshbach resonance
  - halo nuclei
  - scattering properties of the $X(3872)$
- Future: more exciting applications
  - Halo nuclei: Structure, Reactions, ...
  - Three-nucleon system on the lattice: finite V, limit cycle, ...
  - Cold atoms: $N \geq 4$, 2d-systems, ...
  - ...