Coupled-Cluster theory for Nuclei

Gaute Hagen[1]
Collaborators : D. Dean, M. Hjorth-Jensen, T. Papenbrock and A. Schwenk

[1]Oak Ridge National Laboratory, Physics Division, E-mail: hageng@ornl.gov

INT Workshop
Effective Field Theories and the Many-Body Problem
April 7, 2009

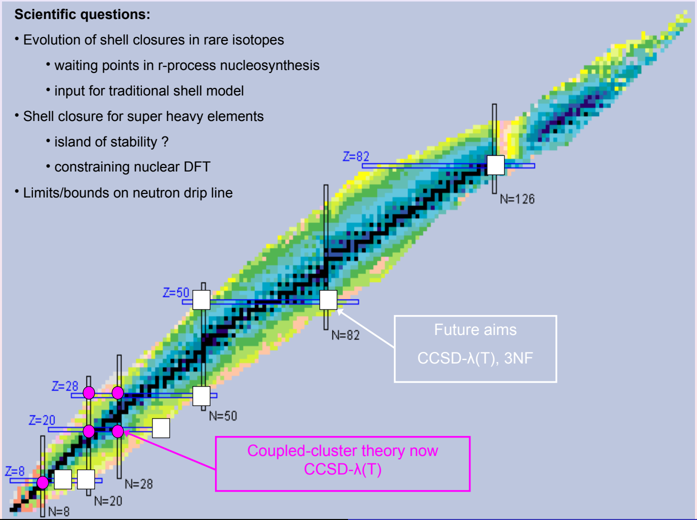
Outline

1. Status and goals in microscopic nuclear structure approaches
2. Coupled Cluster approach to nuclear structure
3. Coupled-Cluster in J-coupled scheme
   - CCSD and ΛCCSD(T) results with "bare" chiral interactions applied to $^{16-28}$O, ^{40}Ca, ^{48}Ca and ^{48}Ni
4. Coupled Cluster for open quantum systems
   - CCSD calculation of Helium chain
   - Charge radii and densities in ^{4}He and ^{8}He
5. Conclusion and Perspectives

Ab-initio approaches to light and medium mass nuclei

Scientific questions:

- Evolution of shell closures in rare isotopes
  - waiting points in r-process nucleosynthesis
  - input for traditional shell model
- Shell closure for super heavy elements
  - island of stability ?
  - constraining nuclear DFT
- Limits/bounds on neutron drip line



N-N force from Chiral perturbation theory

"If you want more accuracy, you have to use more theory (more orders)"

Effective Lagrangian → obeys QCD symmetries (spin, isospin, chiral symmetry breaking)

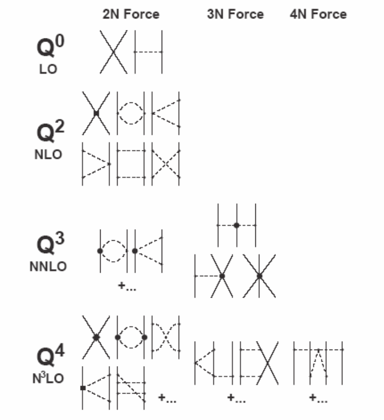
Lagrangian
→ infinite sum of Feynman diagrams.

Expand in $O(Q/\Lambda_{QCD})$

Weinberg, Ordonez, Ray, van Kolck

NN amplitude uniquely determined by two classes of contributions: contact terms and pion exchange diagrams.

24 paramters (rather than 40 from meson theory) to describe 2400 data points with



Coupled Cluster Theory

Exponential Ansatz for Ψ

$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle, \quad \hat{T} = \hat{T}_1 + \hat{T}_2 + \ldots + \hat{T}_A$$

$$\hat{T}_1 = \sum_{i,a} t_i^a \hat{a}_a^\dagger \hat{a}_i, \; \hat{T}_2 = \frac{1}{2} \sum_{i<j,a<b} t_{ij}^{ab} \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_j \hat{a}_i.$$

Coupled Cluster Equations

$$\Delta E = \langle\Phi_0|\left(H_N exp(T)\right)_C|\Phi_0\rangle$$

$$0 = \langle\Phi_p|\left(H_N exp(T)\right)_C|\Phi_0\rangle$$

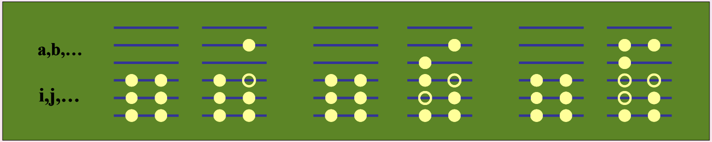
$$\bar{H} = \left(H_N exp(T)\right)_C$$

1. Coupled Cluster Theory is fully microscopic .
2. Coupled Cluster is size extensive. No unlinked diagrams enters, and error scales linearly with number of particles.
3. Low computational cost (CCSD scales as $n_o^2 n_u^4$).
4. Capable of systematic improvements.
5. Amenable to parallel computing.

Coupled Cluster in pictures

$$|\Psi\rangle = e^{T^{(A)}}|\Phi\rangle, \quad T^{(A)} = \sum_{k=1}^{m_A} T_k$$

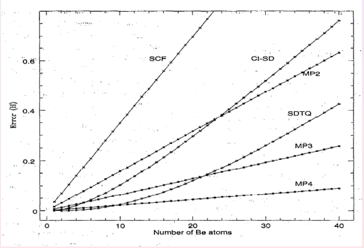
$$T_1 = \sum_{\substack{i\\a}} t_i^a |\Phi_i^a\rangle, \quad T_2 = \sum_{\substack{i>j\\a>b}} t_{ij}^{ab} |\Phi_{ij}^{ab}\rangle, \quad T_3 = \sum_{\substack{i>j>k\\a>b>c}} t_{ijk}^{abc} |\Phi_{ijk}^{abc}\rangle$$



Size Matters! Role of size-extensivity

Goldstone's linked cluster theorem (1955)

Formal diagrammatic proof of Brueckner's conjecture that perturbation theory is size extensive. Only linked diagrams contribute to the energy of a (closed shell) nucleus. **Unlinked diagrams do not scale with mass number A and the sum of all unlinked diagrams is zero.**



- **Size extensive theories:** Many-body perturbation theory, Full Configuration Interaction (FCI) and Coupled-cluster theory
- **Non-size extensive theories:** Particle-hole truncated shell-model (CISD, CISDT...)

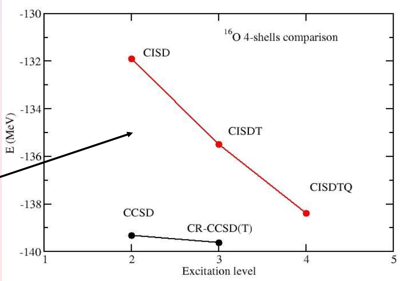
Comparison with Shell Model/Configuration Interaction

In Shell Model approach a linear excitation operator is used instead of an exponential. $\Psi = (1 + B_1 + B_2 + \ldots)\Phi_0$

- Any particle-hole truncation introduces unlinked diagrams, and it is therefore not size extensive.
- Dimension increases dramatically with number of active particles.

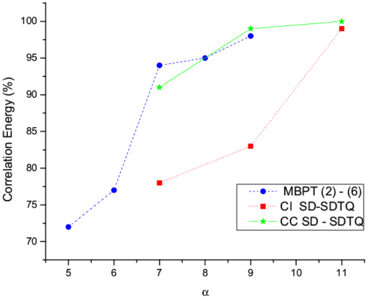
Comparison of CC with CI at given excitation level.

Nuclear Example (Kowalski et al PRL 2004).



Comparison with Shell Model/Configuration Interaction

Performance of theories for the correlation energy in small molecules. To facilitate comparisons, the ordinate gives the size-scaling parameter of the approximation, $\alpha = \alpha_n + \alpha_N + \alpha_{it}$ in the computational cost function $n^{\alpha_n} N^{\alpha_N} N_{it}^{\alpha_{it}}$.



Relationship between shell model and CC amplitudes

$$B_1 = T_1$$

$$B_2 = T_2 + \frac{1}{2} T_1^2$$

$$B_3 = T_3 + T_2 T_1 + \frac{1}{6} T_1^3$$

$$B_4 = T_4 + T_3 T_1 + \frac{1}{2} T_2^2 + \frac{1}{2} T_2 T_1^2 + \frac{1}{24} T_1^4$$

$$\ldots$$

CCSD

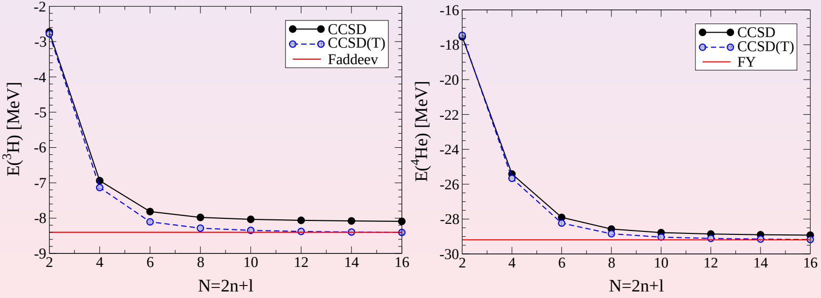
CCSDT

Disconnected quadruples

Connected quadruples

Coupled Cluster meets benchmarks of ^{3}H and ^{4}He!

CCSD(T) and Faddeev (-Yakubovsky) results for ^{3}H and ^{4}He using $V_{\mathrm{low}-k}$ from AV18 with $\Lambda = 1.9\mathrm{fm}^{-1}$. **CCSD(T) are within the errors (50 keV) of the Faddeev results!** (G. Hagen et al., Phys. Rev. C 76, 044305 (2007))



Disagreement between CCM and IT-NCSM for ^{40}Ca

	Coupled-Cluster	IT-NCSM
^{16}O	-142.8 (CCSD)	
	-148.2 (CCSD(T))	-137.8 (4p-4h)
^{40}Ca	-491.2 (CCSD)	-461.8 (3p-3h)
	-502.9 (CCSD(T))	-471.0 (4p-4h)

- Coupled-Cluster theory **size extensive**, energy scales correctly with size
- IT-NCSM is **not size extensive**. Can not judge the quality of a calculation of large system from the quality of a small (light) system.

Refs.:

1. Roth and Navratil PRL 99, 092501 (2007)
2. Hagen, Dean, Hjorth-Jensen, Papenbrock, Schwenk PRC 76, 044305 (2007)
3. Dean, Hagen, Hjorth-Jensen, Papenbrock, Schwenk PRL. 101, 119201 (2008) (comment)
4. Roth and Navratil, arxiv:0801.1484 (reply)

Spherical Coupled-Cluster Approach

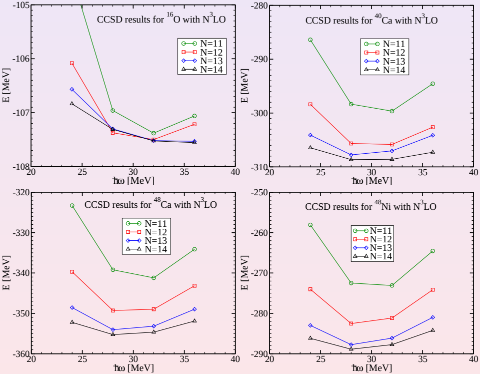
1. Possible for nuclei with closed sub-shell (or cs ± 1)
2. Relatively simple since similarity transformed Hamiltonian is two-body (CCSD) or three-body (CCSDT) at most
3. Enourmous computational reduction: $n_o + n_u \to (n_o + n_u)^{2/3}$ (naive estimate)
   CCSD(T) for ^{40}Ca and ^{48}Ca on a single CPU (now)
   CCSDT for ^{48}Ca on many CPUs (future)
   CCSD(T) for ^{100}Sn and ^{132}Sn with "bare" chiral interactions on many CPUs.

$$\hat{T}_1 = \sum_{j_i, j_a} t_{j_i}^{j_a} (a_{j_a}^\dagger \times \tilde{a}_{j_i})^{(0)} ,$$

$$\hat{T}_2 = \sum_{j_i, j_j, j_a, j_b, J} t_{j_i, j_j}^{j_a, j_b}(J)(a_{j_a}^\dagger \times a_{j_b}^\dagger)^{(J)} \cdot (\tilde{a}_{j_j} \times \tilde{a}_{j_i})^{(J)} .$$

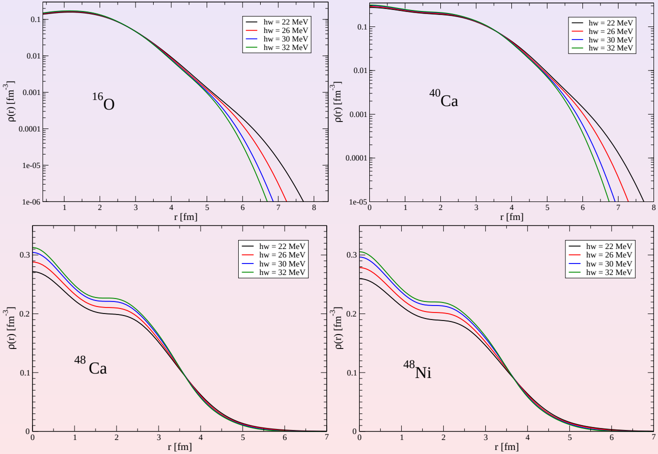
j_i and j_a denote the spin of the occupied and unoccupied subshells.

^{16}O, ^{40}Ca, ^{48}Ca and ^{48}Ni with "bare" chiral interactions



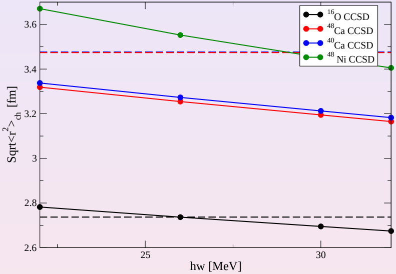
Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions, G. Hagen et. al, Phys. Rev. Lett. 101, 092502 (2008)

^{16}O, ^{40}Ca, ^{48}Ca and ^{48}Ni ground state densities



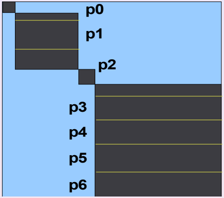
Charge and matter radii/Summary of results

- Mirror nuclei ^{48}Ca and ^{48}Ni differ by 1.38 MeV/A $\longrightarrow$ close to mass table predictions.
- 3NF and triples expected to yield $\sim$ 1MeV/A ?
- Radii and and densities stronger model space dependence.



Nucleus	E/A	V/A	Q	$\Delta E/A$	$< r^2 >_{ch}^{1/2}$	$< r^2 >_{ch}^{1/2}$ (Exp)
^{4}He	-5.99	-22.75	0.90	1.08		1.673(1)
^{16}O	-6.72	-30.69	1.08	1.25	2.72(5)	2.737(8)
^{40}Ca	-7.72	-36.40	1.18	0.84	3.25(9)	3.4764
^{48}Ca	-7.40	-37.97	1.21	1.27	3.24(9)	3.4738
^{48}Ni	-6.02	-36.04	1.20	1.21	3.52(15)	?

Parallelization of coupled-cluster code



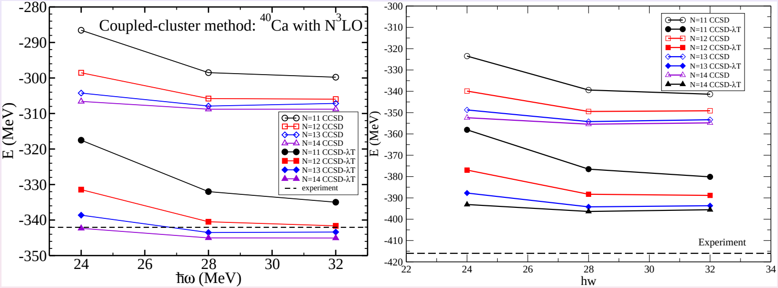
Computational challenges

- Inclusion of three-nucleon force
- Needs coding in current J-coupled scheme
- Generation of matrix elements
- Approximation possible ?

Computational details

- 350 cpu hours per point (N=14)
- N = 14 requires 20Gb of memory
- Triples correction factor 20 more expensive
- MPI code with distributed matrix elements of interaction and intermediates.
- BLAS routines for (some) computationally expensive tasks
- Weak scaling expected

Triples correction to ground state energies

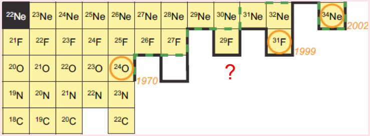


- ~ 400keV/A missing for ^{16}O and ^{48}Ca.
- Interesting isospin behavior of three-body force in Calcium isotopes.

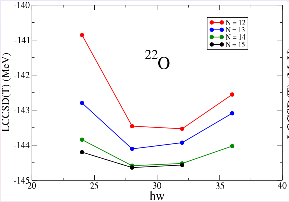
	CCSD		ΛCCSD(T)	
Nucleus	E/A	$\Delta E/A$	E/A	$\Delta E/A$
^{16}O	-6.72	1.25	-7.56	0.41
^{40}Ca	-7.72	0.84	-8.63	-0.08
^{48}Ca	-7.40	1.27	-8.26	0.40

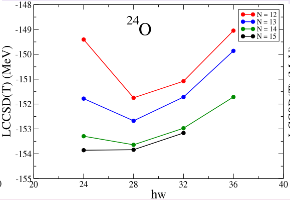
Dripline in Oxygen isotopes

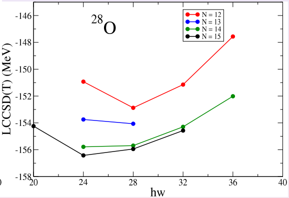
- Evidence of new magic numbers in oxygen isotopes $N = 14$ and $N = 16$.
- Does ^{28}O exist ? ^{28}O is a doubly magic nucleus.
- All shell model calculation in the $s - d$ shell with realistic NN interactions predicts dripline beyond ^{28}O.
- Preliminary results from Otsuka et al. with inclusion of 3NF predicts ^{28}O unstable.
- Adding one more proton binds 6 more neutrons in fluorine isotopes.
- Can Ab-initio theory answer this question ?



^{22}O, ^{24}O, and ^{28}O with N^{3}LO (500 MeV)

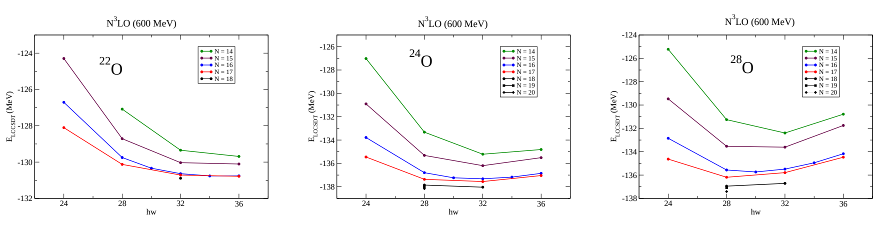






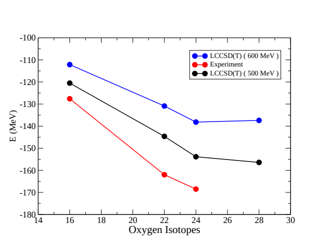
	^{22}O	^{24}O	^{28}O
E_0	50.37	56.19	71.58
ΔE_{CCSD}	-175.79	-190.39	-207.67
$\Delta E_{\mathrm{CCSD}-\lambda\mathrm{T}}$	-19.22	-19.64	-19.85

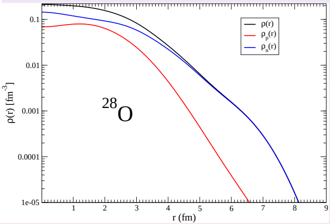
^{22}O, ^{24}O, and ^{28}O with N^{3}LO (600 MeV)



	^{22}O	^{24}O	^{28}O
E_0	46.33	52.94	68.57
ΔE_{CCSD}	-156.51	-168.49	-182.42
$\Delta E_{\mathrm{CCSD}-\lambda\mathrm{T}}$	-20.71	-22.49	-22.86

Is ^{28}O stable ?





- Ab-initio Coupled-Cluster calculations can not rule out the existence of ^{28}O.
- Cutoff variation indicates that three-nucleon forces will play a crucial role in the determination of the neutron dripline.

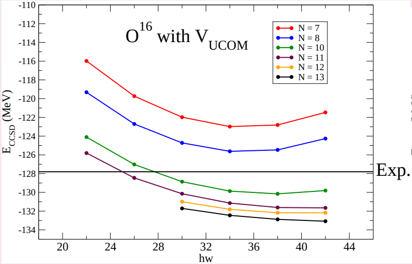
Cutoff dependence and summary of results for Oxygen isotopes

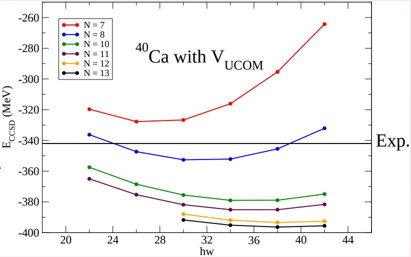
N^3LO (500 MeV)					
	CCSD		ΛCCSD(T)		
Nucleus	E/A	$\Delta E/A$	E/A	$\Delta E/A$	$< r^2 >^{1/2}$
^{16}O	-6.72	1.25	-7.53	0.44	
^{22}O	-5.72	1.64	-6.59	0.77	2.82
^{24}O	-5.58	1.42	-6.42	0.58	2.89
^{28}O	-4.86	?	-5.57	?	3.06

N^3LO (600 MeV)					
	CCSD		ΛCCSD(T)		
Nucleus	E/A	$\Delta E/A$	E/A	$\Delta E/A$	$< r^2 >^{1/2}$
^{16}O	-6.06	1.92	-7.01	0.97	
^{22}O	-5.01	2.36	-5.95	1.45	2.93
^{24}O	-4.84	2.18	-5.76	1.26	3.08
^{28}O	-4.11	?	-4.90	?	3.30

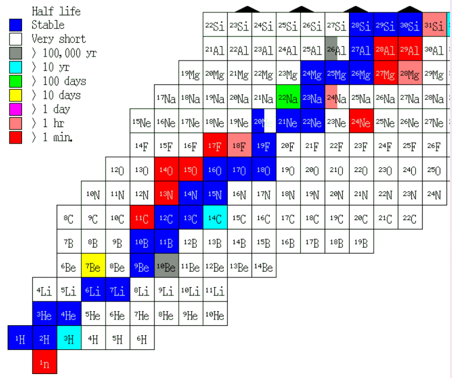
^{16}O, ^{40}Ca with V_{UCOM}

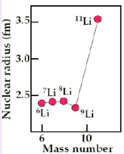
V_{UCOM} overbinds and converge slowly for medium mass nuclei.





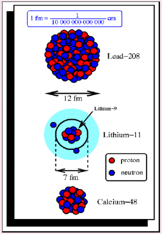
Ab-initio approach weakly bound and unbound nuclear states





I. Tanihata et al.
Phys. Rev. Lett. 55, 2676 (1985)

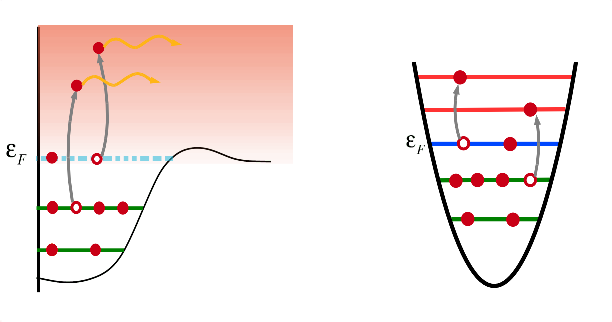
Interaction cross section measurements at Bevalac (790 MeV/u)



Coupled Cluster for open quantum systems

Open Quantum System.
Coupling with continuum taken into account.

Closed Quantum System.
No coupling with external continuum.

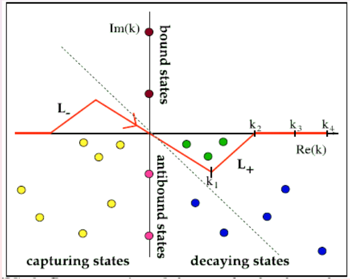


Berggren Single-particle basis

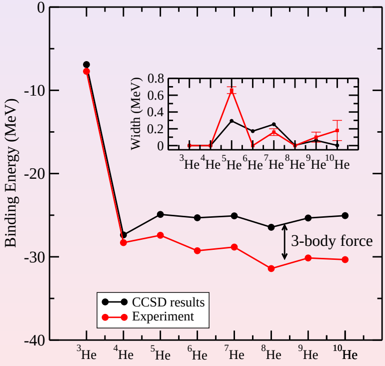
Complex energies requires a generalized completeness relation

$$|\Psi(\mathbf{r},t)|^2 = |\Phi(\mathbf{r})|^2 exp(-\frac{\Gamma}{\hbar}t), \quad E = E_r - i\Gamma/2.$$

$$\mathbf{1} = \sum_{n=b,d} |\psi_l(k_n)\rangle\langle\tilde{\psi}_l(k_n)| + \int_{L^+} dk\, k^2 |\psi_l(k)\rangle\langle\tilde{\psi}_l(k)|.$$



CCSD results for Helium chain using $V_{\mathrm{low}-k}$

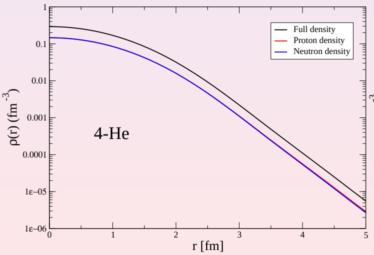


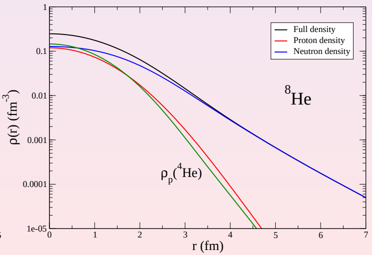
- $V_{\mathrm{low}-k}$ from N3LO with $\Lambda = 1.9\mathrm{fm}^{-1}$.
- G. Hagen et al., Phys. Lett. B 656, 169 (2007). arXiv:nucl-th/0610072.

- First *ab-initio* calculation of decay widths of a whole isotopic chain.
- CCM unique method for dripline nuclei.
- ~ 1000 active orbitals
- Underbinding hints at missing 3NF

^{4}He and ^{8}He density distributions with V-srg

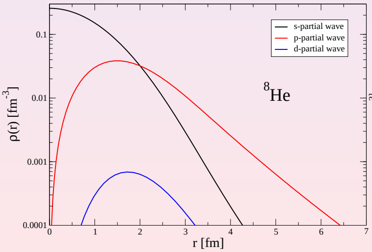
- Single-particle density in ^{4}He and ^{8}He.
- Gamow-Hartree-Fock basis has correct asymptotics.
- N^3LO evolved down to $\lambda = 2.0\mathrm{fm}^{-1}$ from similarity renormalization group theory.

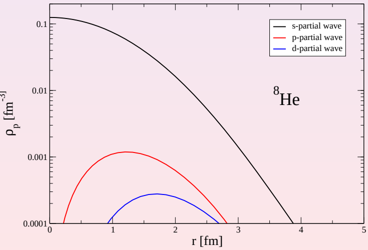




Partial wave decomposition of ^{8}He density

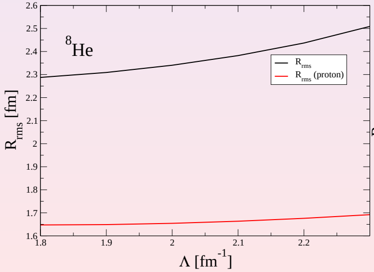
- N^{3}LO evolved down to $\lambda = 2.0\mathrm{fm}^{-1}$ from similarity renormalization group theory.
- Neutron skin in ^{8}He is mainly built from $s-$ and $p-$partial waves. Protons are mainly occupying $s-$ partial waves.

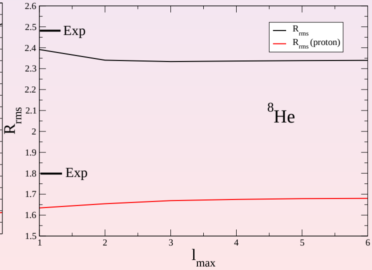




Matter and charge radii of ^{8}He using V-srg

- Λ dependence on ^{8}He charge and matter radii indicates missing 3NF.
- Hamiltonians with two-body renormalized interactions (SRG/low-k) underestimates matter and charge radii.





Conclusion

- Coupled Cluster meets few-body benchmark calculations.
- J-coupled CCSD and ΛCCSD(T) code has been derived and implemented. Coupled cluster approach to medium mass and driplines with bare interactions!
- Derived and implemented Equation of Motion CCM; calculation of density distributions and radii.
- CCM has been succesfully applied to the description of weakly bound and unbound helium isotopes.
- We have a tool to attack the structure and properties of dripline and medium mass nuclei !

Future perspectives

- Revisit Helium chain with 3NF. Spin-orbit splitting in He7 and He9.
- Matter and charge radii of ^{11}Li.
- Excited states and matter densities for dripline nuclei.
- Coupled Cluster approach to nuclear matter.
- Construction of effective interaction for shell-model calculations.
- Coupled-Cluster approach to nuclear reactions; CC-LIT and construction of optical potentials from folding procedures.
- We are developing a J-coupled CCSDT code.
- Ab-initio description of ^{56}Ni, ^{100}Sn and ^{208}Pb within reach!

Coupled-Cluster method and the Center of Mass

1. Our implementation of CCM does not preserve translational invariance.
2. In oscillator basis $\langle H_{\mathrm{CM}} \rangle \sim 200$keV for ^{40}Ca with vlowk and $N = 8$.
   In HF basis energies are expected to be much better than wave functions.

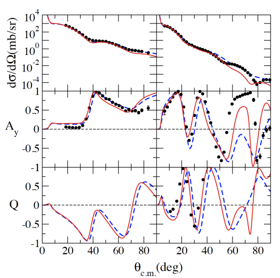
Compare two methods

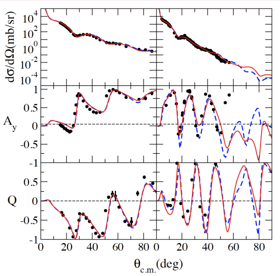
1. Exact diagonalization (CI) of intrinsic Hamiltonian in all A-body states built from this s.p. basis → Variational but breaks translational invariance, result :E_{CI}
2. NCSM in $N\hbar\Omega$ space spanned by this basis → Variational keeps translational invariance, result E_{NCSM}

$E_{\mathrm{EXACT}} \leq E_{CI} \leq E_{NCSM}$

Elastic scattering of protons from ^{16}O and ^{40}Ca

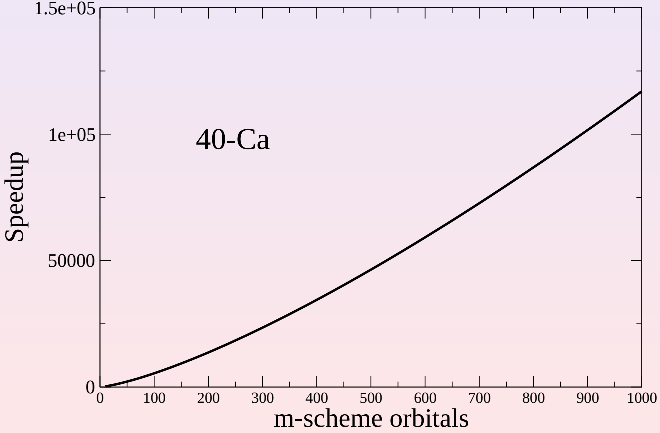
- *g*-folding optical potential model with realistic one-body density matrices from ab-initio Coupled-Cluster calculations. With K. Amos (To be submitted for publication in Phys. Rev. C)
- Elastic scattering of protons from ^{16}O (left figure) and ^{40}Ca (right figure) at 65 MeV and 200 MeV.





Spherical Coupled-Cluster Approach

Speedup of J-coupled CCSD code for ^{40}Ca as compared to m-scheme CCSD code.



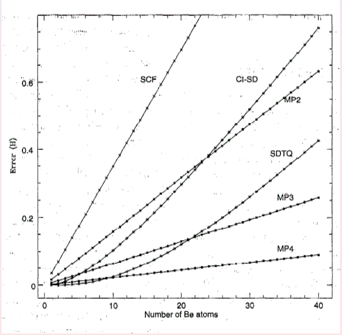
How well does SR-CC describe open-shell nuclei?

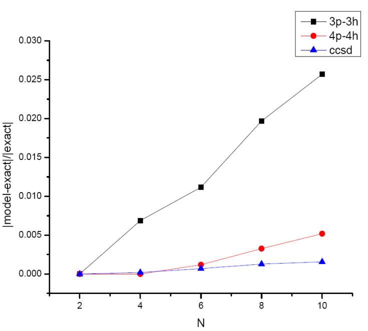
Various Coupled Cluster approaches to the $^{3-6}$He ground states. Single reference Coupled-Cluster methods works!

Method	^{3}He	^{4}He	^{5}He	^{6}He	$\langle J^2 \rangle$, ^{6}He
CCSD	-6.21	-26.19	-21.53	-20.96	0.61
CCSD(T)	-6.40	-26.27	-21.88	-22.60	0.65
CCSDT-1	-6.41	-28.27	-21.89	-22.85	0.29
CCSDT-2	-6.41	-28.26	-21.89	-22.78	0.25
CCSDT-3	-6.42	-26.27	-21.92	-22.90	0.26
CCSDT	-6.45	-26.28	-22.01	-22.52	0.04
FCI	-6.45	-26.3	-22.1	-22.7	0.00

Size Matters! Role of size-extensivity

Disconnected diagrams in truncated shell model/CI models (CISD, CISDT,...) leads to wrong scaling of energy with increasing number of particles.

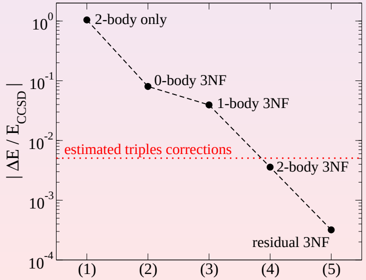




Different contributions to E_{CCSD} from 3NF in ^{4}He

Three-body Hamiltonian in normal ordered form:
(G. Hagen et al., PRC (76) 034302 (2007))

$$\hat{H}_3 = \frac{1}{6}\sum_{ijk}\langle ijk||ijk\rangle + \frac{1}{2}\sum_{ijpq}\langle ijp||ijq\rangle\{\hat{a}^\dagger_p\hat{a}_q\} + \frac{1}{4}\sum_{ipqrs}\langle ipq||irs\rangle\{\hat{a}^\dagger_p\hat{a}^\dagger_q\hat{a}_s\hat{a}_r\} + \hat{h}_3 \ ,$$

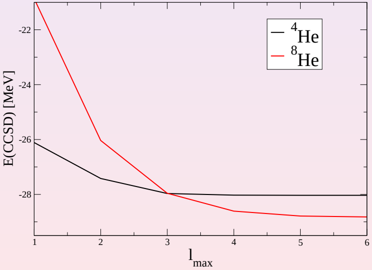


Really good news!

- The "density dependent" terms of 3NF are dominant!
- ϵ from residual 3NF costs $1 - \epsilon$ of work !
- "2-body" machinery can be used.
- **Residual 3NF can be neglected!**

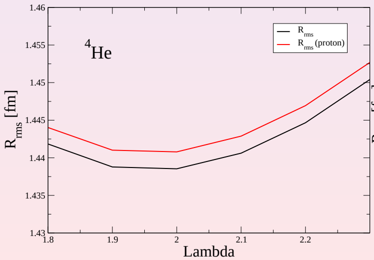
Properties of weakly bound nuclei

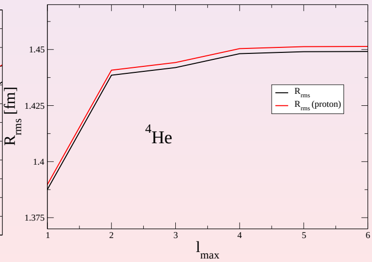
Convergence of ^{4}He and ^{8}He ground state energies with increasing number of partial waves in the basis.



Matter and charge radii of ^{4}He using V-srg

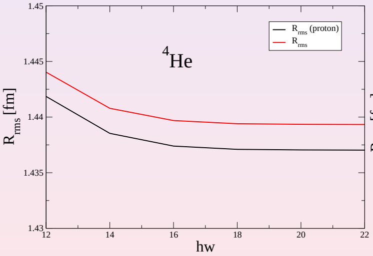
- Λ dependence on ^{4}He charge and matter radii indicates missing 3NF.
- Hamiltonians with two-body renormalized interactions (SRG/low-k) underestimates matter and charge radii.

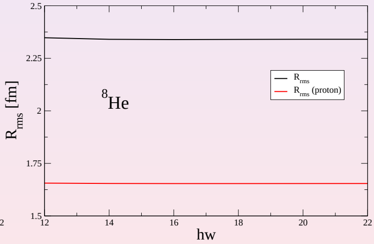




Properties of weakly bound nuclei

$\hbar\omega$ dependence on ^{4}He and ^{8}He charge and matter radii.





Coupled Cluster Theory

Exponential Ansatz for Ψ

$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle, \quad \hat{T} = \hat{T}_1 + \hat{T}_2 + \ldots + \hat{T}_A$$

$$\hat{T}_1 = \sum_{i,a} t_i^a \hat{a}_a^\dagger \hat{a}_i, \; \hat{T}_2 = \frac{1}{2} \sum_{i<j,a<b} t_{ij}^{ab} \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_j \hat{a}_i.$$

Coupled Cluster Equations

$$\Delta E = \langle\Phi_0|\left(H_N exp(T)\right)_C|\Phi_0\rangle$$

$$0 = \langle\Phi_p|\left(H_N exp(T)\right)_C|\Phi_0\rangle$$

$$\bar{H} = \left(H_N exp(T)\right)_C$$

Iterative CCSDT-n approximations to full CCSDT

$$\begin{aligned}
\mathrm{CCSDT}-1 \quad 0 &= \langle\Phi_{ijk}^{abc}|\left(F_N T_3 + H_N T_2\right)_C|\Phi_0\rangle \\
\mathrm{CCSDT}-2 \quad 0 &= \langle\Phi_{ijk}^{abc}|\left(F_N T_3 + H_N T_2 + H_N T_2^2/2\right)_C|\Phi_0\rangle \\
\mathrm{CCSDT}-3 \quad 0 &= \langle\Phi_{ijk}^{abc}|\left(F_N T_3 + H_N \exp(T_1 + T_2)\right)_C|\Phi_0\rangle \\
\mathrm{CCSDT} \quad 0 &= \langle\Phi_{ijk}^{abc}|\left(H_N \exp(T_1 + T_2 + T_3)\right)_C|\Phi_0\rangle
\end{aligned}$$

Coupled Cluster Theory with 3NF

- We have derived and implemented Coupled Cluster equations for three-body Hamiltonians.
- Probe cutoff dependence of $V_{\mathrm{low}-k}$ with three nucleon force in light and medium heavy nuclei.
- Does 3NF provide the necessary repulsion/attraction needed to approach experimental mass values ?
- *“ Coupled-cluster theory for three-body Hamiltonians “*
  G. Hagen et al., PRC (76) 034302 (2007).

N-N force from Chiral perturbation theory

"If you want more accuracy, you have to use more theory (more orders)"

Effective Lagrangian → obeys QCD symmetries (spin, isospin, chiral symmetry breaking)

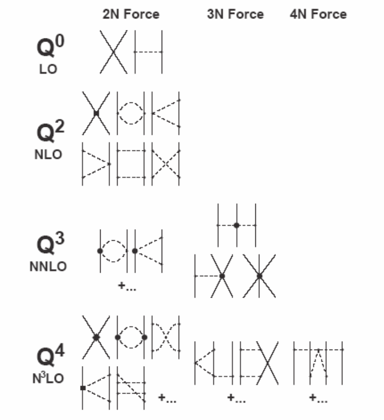
Lagrangian
→ infinite sum of Feynman diagrams.

Expand in $O(Q/\Lambda_{QCD})$

Weinberg, Ordonez, Ray, van Kolck

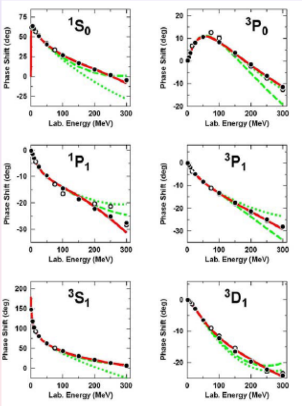
NN amplitude uniquely determined by two classes of contributions: contact terms and pion exchange diagrams.

24 paramters (rather than 40 from meson theory) to describe 2400 data points with

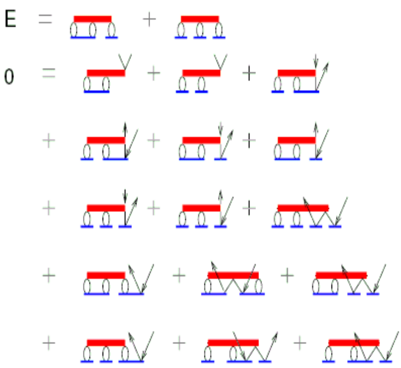


N-N phase shifts order by order in Chiral EFT

N-N phase shifts up to 300 MeV from NLO, N2LO and N3LO interactions.
Red line from Entem&Machleidt, PRC 68, 041001 (2003),
green lines from Epelbaum et. al.,
Eur. Phys. J. A15, 543 (2002).



3NF contribution to the $\hat{T}_1$ cluster equation



Energy and 1p-1h equation as examples.

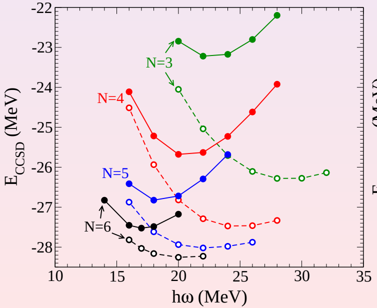
Factorization of diagrams very useful!

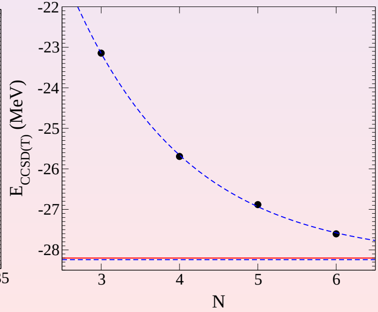
1p-1h: 15 diagrams

2p-2h: 51 diagrams

Coupled Cluster Results for ^{4}He with 3NF

- $V_{\mathrm{low}-k}$ from AV18 with $\Lambda = 1.9\mathrm{fm}^{-1}$.
- 3NF brings in repulsion as expected !
- CCSD and CCSD(T) with 3NF meets Faddeev-Yakubovsky benchmark ! $E_{\mathrm{CCSD(T)}} \approx -28.24$ MeV. F-Y $E = -28.20(5)$MeV.

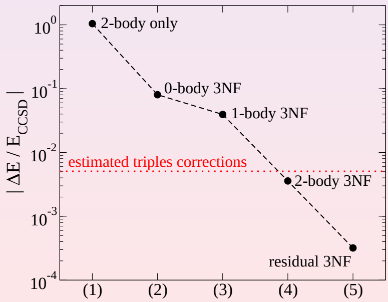




Different contributions to E_{CCSD} from 3NF in ^{4}He

Three-body Hamiltonian in normal ordered form:

$$\hat{H}_3 = \frac{1}{6}\sum_{ijk}\langle ijk||ijk\rangle + \frac{1}{2}\sum_{ijpq}\langle ijp||ijq\rangle\{\hat{a}_p^\dagger\hat{a}_q\} + \frac{1}{4}\sum_{ipqrs}\langle ipq||irs\rangle\{\hat{a}_p^\dagger\hat{a}_q^\dagger\hat{a}_s\hat{a}_r\} + \hat{h}_3\ ,$$



Really good news!

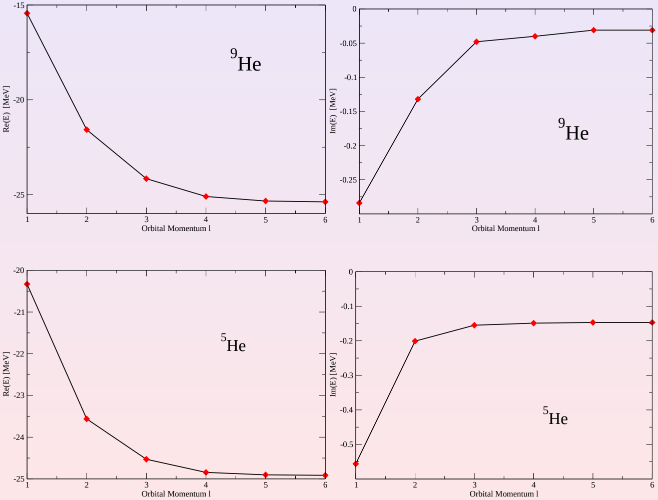
- The "density dependent" terms of 3NF are dominant!
- ϵ from residual 3NF costs $1-\epsilon$ of work !
- "2-body" machinery can be used.
- **Residual three-nucleon force can be neglected!**

CCM vs. exact calculations for open-shell nuclei.

Various Coupled Cluster approaches to the $^{3-6}$He ground states. Single reference Coupled-Cluster methods works!

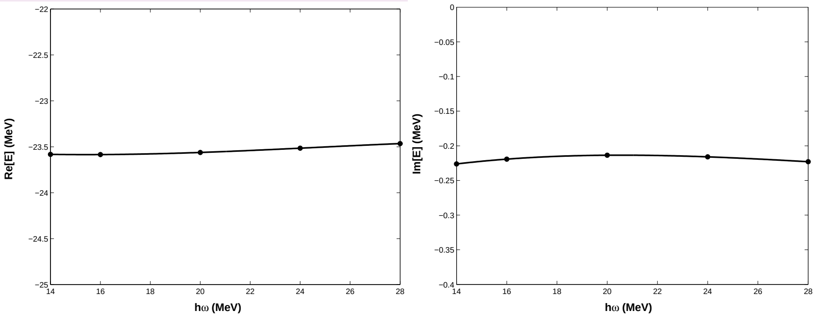
Method	^{3}He	^{4}He	^{5}He	^{6}He	$\langle J \rangle$, ^{6}He
CCSD	-6.21	-26.19	-21.53	-20.96	0.61
CCSD(T)	-6.40	-26.27	-21.88	-22.60	0.65
CCSDT-1	-6.41	-28.27	-21.89	-22.85	0.29
CCSDT-2	-6.41	-28.26	-21.89	-22.78	0.25
CCSDT-3	-6.42	-26.27	-21.92	-22.90	0.26
CCSDT	-6.45	-26.28	-22.01	-22.52	0.04
Exact	-6.45	-26.3	-22.1	-22.7	0.00

Convergence of CCSD results



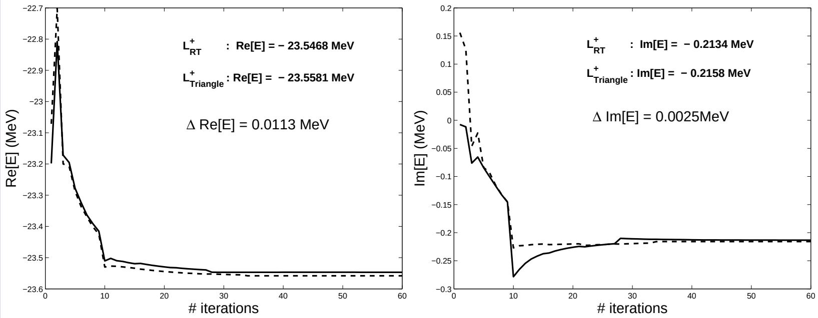
Convergence of CCSD energy with $2n + l \leq 10$ truncation.

- ^{5}He ground state energy starting with oscillator bases given for different $\hbar\omega$ values.
- Weak $\hbar\omega$ dependence, Results are well converged. $\Delta Re[E] \sim 0.1$MeV, $\Delta Im[E] \sim 0.01$MeV



Convergence of CCSD energy.

CCSD convergence of ^{5}He ground state energy for the $s-d$ space (300 orbitals) using $n=20$ discretization points for L^+. The calculation where performed using two very different L^+ contours



Coupled Cluster Results for Helium isotopes with TNF

CC results with $V_{\mathrm{low}-k}$ from N3LO NN-interaction. Rather limited model-space $N = 3$. Only contact term at NN2LO is retained in the three nucleon force. TNF fitted to reproduce binding energy of ^{4}He.