The New World Out There: Beyond the Two-Body Interaction in Many-Body Systems

- **Nuclear Models**
- **Choice of Interaction**
  - The 2-body interaction problem
  - QCD derived nuclear interaction to the rescue
- **Are A-body interactions important?**
  - Extended pairing interaction
  - Application to heavy nuclei
- Q&A **about C_D-C_E curve, Λ, and the NNN interaction terms…**

Vesselin Gueorguiev

UC Merced

Simple Models, when applicable, Help Understand Complex Systems

- **Liquid Drop Model** (key: nuclear surface $R(\theta,\varphi)=R_0(1+\alpha^{\lambda\mu}Y_{\lambda\mu}(\theta,\varphi))$
- **Fermi Gas Model** (key: particle statistics)
- **Energy-Density Functional** (key: framework reformulation)
- **Independent Particles in a Mean Field**
  - **Harmonic Oscillator, Square Well** (key: exactly solvable)
  - **Nilsson Model** (key: nuclear deformation & spin-orbit interaction)
  - **Wood-Saxon** (key: finite range, diffuse surface, spin-orbit term)
- **Microscopic Shell Models** - "Deriving the Mean Field''
  - **Self-consistent Mean Field** (Hartree-Fock)
  - **Algebraic Models** (key: exactly solvable -- Lie algebra based)
  - **Spherical Shell-Model with Realistic Interactions** (key: matrix elements are adjusted to reproduce the available nuclear structure data)
  - ***Ab-initio No-Core Shell Model*** (key: *highly accurate interactions fitted to nucleon-nucleon data*, *properties of A-body systems are derived from the few-body interactions*)
  - *Other methods for few- and many-body systems (Monte-Carlo methods, Coupled-clusters, cluster models …)*

Nuclear Shell-Model Hamiltonian

$$H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{i,j,k,l} V_{ijkl} a_i^+ a_j^+ a_k a_l$$

where a_i^+ and a_i are fermion creation and annihilation operators,

ε_i and V_{ijkl} are real and $V_{ijkl} = V_{klij} = -V_{jikl} = -V_{ijlk}$

- Spherical shell-model basis states are eigenstates of the one-body part of the Hamiltonian - **single-particle states**.

- The two-body part of the Hamiltonian H is dominated by the **quadrupole-quadrupole interaction** Q·Q ~ C_2 of SU(3).

- SU(3) basis states - **collective states** - are eigenstates of H for degenerate single particle energies **ε** and a pure Q·Q interaction.

pre-XXI century NN-Potentials

Usual NN-potentials are combination of:

Central scalar potential + spin-orbit (LS) + tensor force S_{ij}

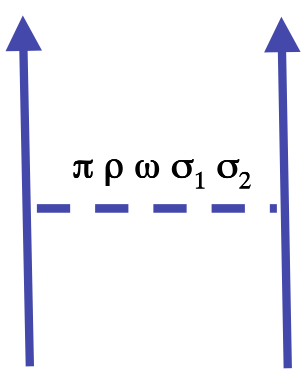
Argonne V18 potential, Phys. Rev. C 51, 38 (1995), $v_{ij} = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p,$

$$O_{ij}^{p=1,8} = 1, \tau_i \cdot \tau_j, \sigma_i \cdot \sigma_j, (\tau_i \cdot \tau_j)(\sigma_i \cdot \sigma_j), S_{ij}, S_{ij}(\tau_i \cdot \tau_j), L \cdot S, L \cdot S(\tau_i \cdot \tau_j)$$

$$O_{ij}^{p=9,14} = L^2, L^2(\tau_i \cdot \tau_j), L^2(\sigma_i \cdot \sigma_j), L^2(\tau_i \cdot \tau_j)(\sigma_i \cdot \sigma_j)), (L \cdot S)^2, (L \cdot S)^2(\tau_i \cdot \tau_j)$$

$$O_{ij}^{p=15,18} = T_{ij}, (\sigma_i \cdot \sigma_j) T_{ij}, S_{ij} T_{ij}, (\tau_{iz} + \tau_{jz}), \text{ with isospin breaking } T_{ij} = 3\tau_{iz}\tau_{jz} - (\tau_i \cdot \tau_j)$$

- Common phenomenological choices for $v(\mathbf{r}_{ij})$ are of Yukawa or Gaussian form.
- **Some Modern interaction are giving up locality, e. g. CD-Bonn is non-local. In general, however, the form of $v(\mathbf{r}_{ij})$ ($v(\mathbf{p}_{ij})$) is derived from chiral perturbation theory or meson exchange theory:**



$$L_{N\pi^i N} = -g_{\pi^i} \bar{\psi}_N (i\gamma^5 \tau_i \varphi_{\pi^i}) \psi_N$$

$$v_5(r) = T_{m_\pi}(r) = \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{\exp(-m_\pi r)}{m_\pi r}$$

The 2-body to A-body problem!

2<A<5 Nuclear Systems

The highly accurate modern NN-interactions fail for 2<A<5

TABLE V. Two- and three-nucleon bound-state properties. (Deuteron binding energy B_d; asymptotic S state A_S; asymptotic D/S state η; deuteron radius r_d; quadrupole moment Q; D-state probability P_D; triton binding energy B_t.)

	N^3LO[a]	CD-Bonn [10]	AV18 [22]	Empirical[b]
Deuteron				
B_d(MeV)	2.224575	2.224575	2.224575	2.224575(9)
A_S($fm^{-1/2}$)	0.8843	0.8846	0.8850	0.8846(9)
η	0.0256	0.0256	0.0250	0.0256(4)
r_d(fm)	1.978[c]	1.970[c]	1.971[c]	1.97535(85)
Q(fm^2)	0.285[d]	0.280[d]	0.280[d]	0.2859(3)
P_D(%)	4.51	4.85	5.76	
Triton				
B_t(MeV)[e]	7.855	8.00	7.62	8.48

[a] This work.
[b] See Table XVIII of Ref. [10] for references.
[c] With meson-exchange currents (MEC) and relativistic corrections (RC) [42].
[d] Including MEC and RC in the amount of 0.010 fm^2.
[e] As obtained in a charge-dependent 34-channel Faddeev calculation applying only $2N$ forces.

Entem& Machleidt, Phys. Rev. C **68**, R041001(2003)

TABLE V. Binding energies of light nuclei (in MeV) for different Hamiltonians as computed by the variational Monte Carlo (VMC) method, with comparisons to relevant Green's function Monte Carlo (GFMC) results of Ref. [7] and Faddeev-Yakubovsky (FY) results of Ref. [18]. We also show FY results for the Nijmegen potential at the bottom. Different versions of the Tucson-Melbourne (TM) force are characterized by the cutoff parameter Λ shown in parentheses.

Hamiltonian	Method	^3H	^3He	^4He
	Experiment	-8.482	-7.718	-28.295
AV18	VMC	−7.50(1)	−6.77(1)	−23.70(2)
	GFMC	−7.61(1)	−6.87(1)	−24.07(4)
	FY	−7.623	−6.924	−24.28
AV18pq	VMC	−7.50(1)	−6.77(1)	−23.79(2)
AV8′	VMC	−7.65(1)	−7.01(1)	−24.69(2)
	GFMC	−7.76(1)	−7.12(1)	−25.14(2)
AV18/UIX	VMC	−8.29(1)	−7.53(1)	−27.58(2)
	GFMC	−8.46(1)	−7.71(1)	−28.33(2)
	FY	−8.478	−7.760	−28.50
AV18pq/UIX	VMC	−8.22(1)	−7.47(1)	−27.21(2)
AV8′/UIX	VMC	−8.51(1)	−7.86(1)	−29.05(2)
	GFMC	−8.68(1)	−8.03(1)	−29.82(2)
AV18/TM′(4.756)	VMC	−8.26(1)	−7.50(1)	−27.51(2)
	FY	−8.444	−7.728	−28.36
AV18pq/TM′(4.756)	VMC	−8.22(1)	−7.46(1)	−27.42(2)
AV8′/TM′(4.756)	VMC	−8.44(1)	−7.79(1)	−28.83(2)
Nijm I	FY	−7.736	−7.085	−24.98
Nijm II	FY	−7.654	−7.012	−24.56
Nijm I/TM(5.035)	FY	−8.392	−7.720	−28.60
Nijm II/TM(4.975)	FY	−8.386	−7.720	−28.54

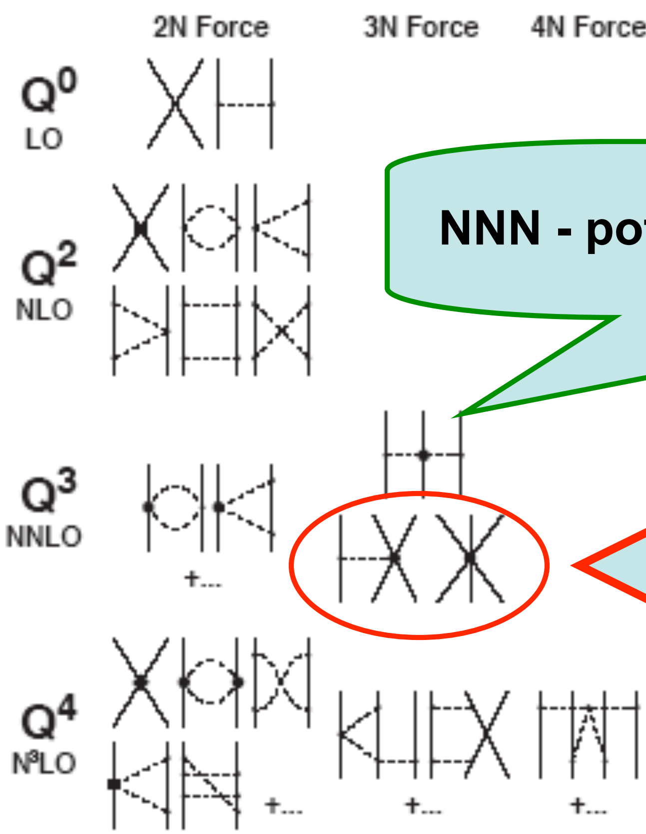
Wiringa *et. al,* Phys. Rev. C **68**,054006 (2003)

Nucleon Interaction from QCD
(Chiral Perturbation Theory)

Chiral perturbation theory (χPT) allows for controlled power series expansion

Expansion parameter : $\left(\frac{Q}{\Lambda_\chi}\right)^{\upsilon}$, Q – momentum transfer,

$\Lambda_\chi \approx 1\ GeV$, χ - symmetry breaking scale



NNN - potentials such as Tucson-Melbourne & Urbana

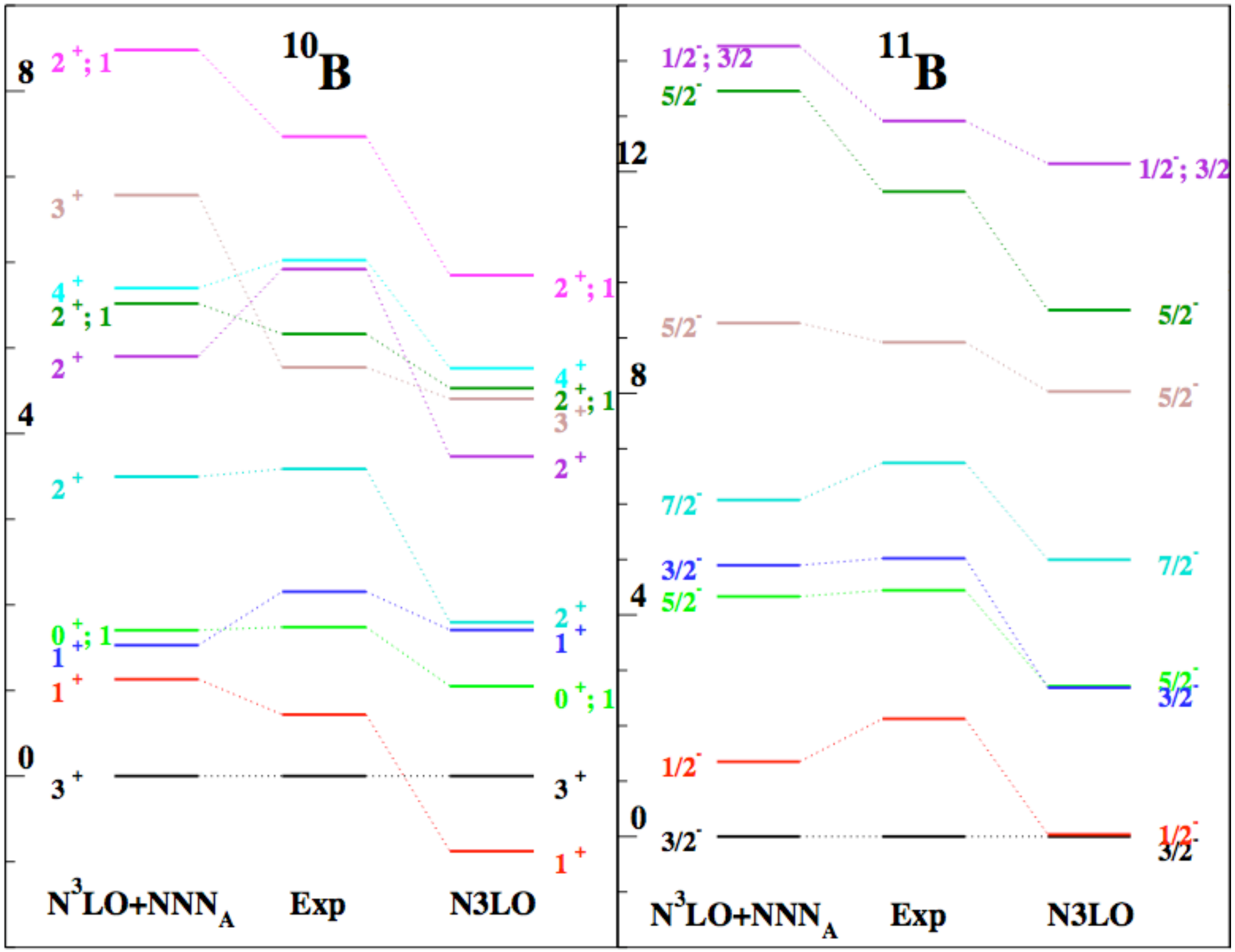
Terms suggested within the Chiral Perturbation Theory

R. Machleidt, D. R. Entem, nucl-th/0503025

Regularization is essential, which is quite obvious within the Harmonic Oscillator wave function basis.

Would the NNN interaction be sufficient to describe nuclear structure?

NN versus NN+NNN in ^{10}B & ^{11}B

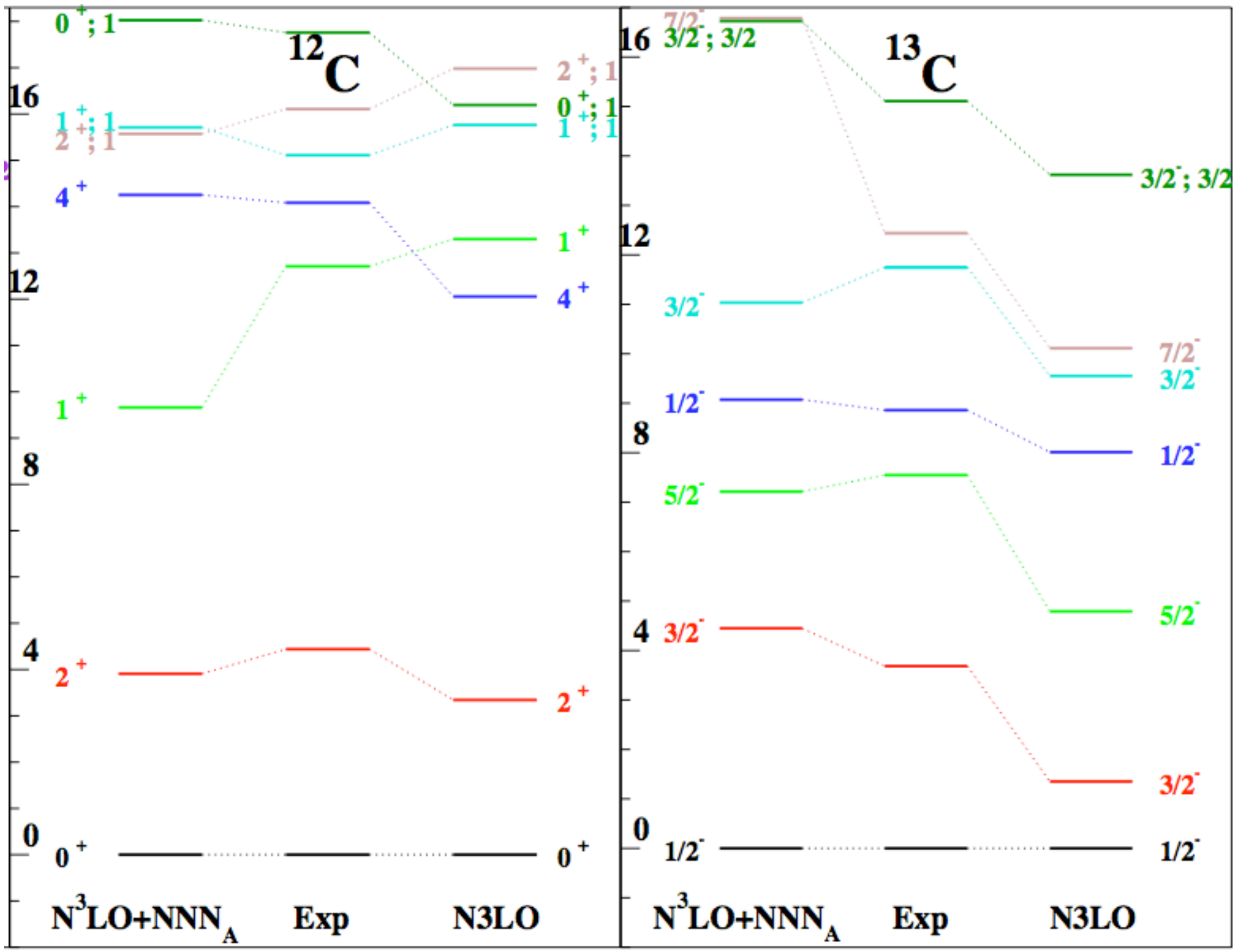


Nucleus/property	Exp	N^{3}LO+NNN$_A$	N^{3}LO
$^{10}B:\ \|E(3^+,0)\|$ [MeV]	64.751	64.027	55.613
r_p [fm]	2.30(12)	2.168	2.224
$Q(3^+_1,0)$ [e fm^2]	+8.472(56)	6.104	6.665
B(E2;$1^+_1 0 \to 3^+_1 0$)	4.13(6)	0.356	4.003
B(E2;$1^+_2 0 \to 3^+_1 0$)	1.71(0.26)	2.771	N/A
B(GT;$2^+_1 1 \to 3^+_1 0$)	0.083(3)	0.061	N/A
B(GT;$2^+_2 1 \to 3^+_1 0$)	0.95(13)	1.559	N/A
$rms(Exp-Th)$ [MeV]	-	0.875	1.333
$^{11}B:\ \|E(\frac{3}{2}{}^-_1\,\frac{1}{2})\|$ [MeV]	76.205	76.704	67.293
$r_p(\frac{3}{2}{}^-_1\,\frac{1}{2})$ [fm]	2.24(12)	2.141	2.196
$Q(\frac{3}{2}{}^-_1\,\frac{1}{2})$ [e fm^2]	+4.065(26)	3.085	2.989
$\mu(\frac{3}{2}{}^-_1\,\frac{1}{2})[\mu_N]$	+2.689	2.040	2.597
$\|E(\frac{1}{2}{}^-_1\,\frac{1}{2})\|$ [MeV]	74.080	75.353	67.254
$\mu(\frac{1}{2}{}^-_1\,\frac{1}{2})[\mu_N]$	-	-0.350	-0.490
B(E2;$\frac{3}{2}{}^-_1\,\frac{1}{2} \to \frac{1}{2}{}^-_1\,\frac{1}{2}$)	2.6(4)	1.507	0.750
B(GT;$\frac{3}{2}{}^-_1\,\frac{1}{2} \to \frac{3}{2}{}^-_1\,\frac{1}{2}$)	0.345	0.222	0.663
B(GT;$\frac{3}{2}{}^-_1\,\frac{1}{2} \to \frac{1}{2}{}^-_1\,\frac{1}{2}$)	0.440	0.435	0.841
B(GT;$\frac{3}{2}{}^-_1\,\frac{1}{2} \to \frac{5}{2}{}^-_1\,\frac{1}{2}$)	0.526	0.479	0.394
B(GT;$\frac{3}{2}{}^-_1\,\frac{1}{2} \to \frac{3}{2}{}^-_2\,\frac{1}{2}$)	0.525	0.727	0.574
B(GT;$\frac{3}{2}{}^-_1\,\frac{1}{2} \to \frac{5}{2}{}^-_2\,\frac{1}{2}$)	0.461	0.899	0.236
$rms(Exp-Th)$ [MeV]	-	0.950	1.765

Petr Navratil, V. G. Gueorguiev, J. P. Vary, W. E. Ormand, and A. Nogga,

Phys. Rev. Lett. **99**, 042501 (2007), (nucl-th-0701038).

NN versus NN+NNN in ^{12}C & ^{13}C

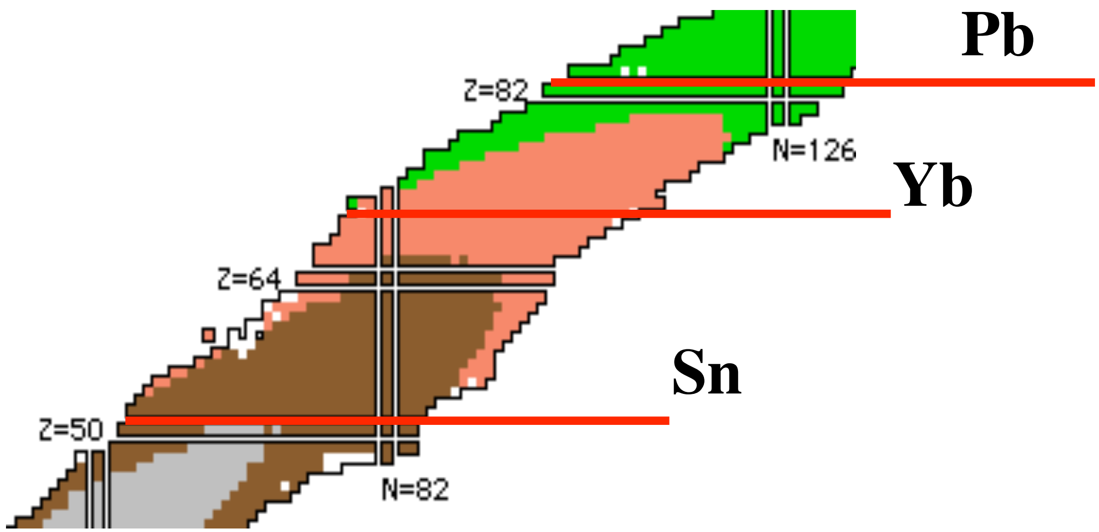


Nucleus/property	Exp	$N^3LO+NNN_A$	N^3LO
$^{12}C:\ \lvert E_{gs}\rvert$ [MeV]	92.162	94.259	83.517
r_p [fm]	2.35(2)	2.187	2.205
Q_{2^+} [e fm^2]	+6(3)	4.324	4.818
$B(E2;2^+0 \to 0^+0)$	7.59(42)	4.260	5.231
$B(M1;1^+0 \to 0^+0)$	0.0145(21)	0.005	0.003
$B(M1;1^+1 \to 0^+0)$	0.951(20)	0.952	0.379
$B(E2;2^+1 \to 0^+0)$	0.65(13)	0.480	0.306
$rms(Exp-Th)$ [MeV]	-	2.095	1.237
$^{13}C:\ \lvert E(\frac{1}{2}^-_1\frac{1}{2})\rvert$ [MeV]	97.108	102.210	90.310
$r_p(\frac{1}{2}^-_1\frac{1}{2})$ [fm]	2.29(3)	2.150	2.195
$\mu(\frac{1}{2}^-_1\frac{1}{2})[\mu_N]$	+0.702	0.349	0.862
$B(E2;\frac{3}{2}^-_1\frac{1}{2} \to \frac{1}{2}^-_1\frac{1}{2})$	6.4(15)	2.444	4.584
$B(M1;\frac{3}{2}^-_1\frac{1}{2} \to \frac{1}{2}^-_1\frac{1}{2})$	0.70(7)	0.598	1.148
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{1}{2}^-_1\frac{1}{2})$	-	0.075	0.328
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{3}{2}^-_1\frac{1}{2})$	-	1.321	2.155
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{1}{2}^-_2\frac{1}{2})$	-	0.764	0.263
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{3}{2}^-_2\frac{1}{2})$	-	1.219	0.221
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{3}{2}^-_3\frac{1}{2})$	-	0.010	0.096
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{1}{2}^-_3\frac{1}{2})$	-	0.051	0.049
$B(GT;\frac{1}{2}^-_1\frac{1}{2} \to \frac{3}{2}^-_1\frac{3}{2})$	-	0.408	0.151
$rms(Exp-Th)$ [MeV]	-	1.947	2.089

Petr Navratil, V. G. Gueorguiev, J. P. Vary, W. E. Ormand, and A. Nogga,

Phys. Rev. Lett. **99**, 042501 (2007), (nucl-th-0701038).

What about the A-body interaction in heavy nuclei?

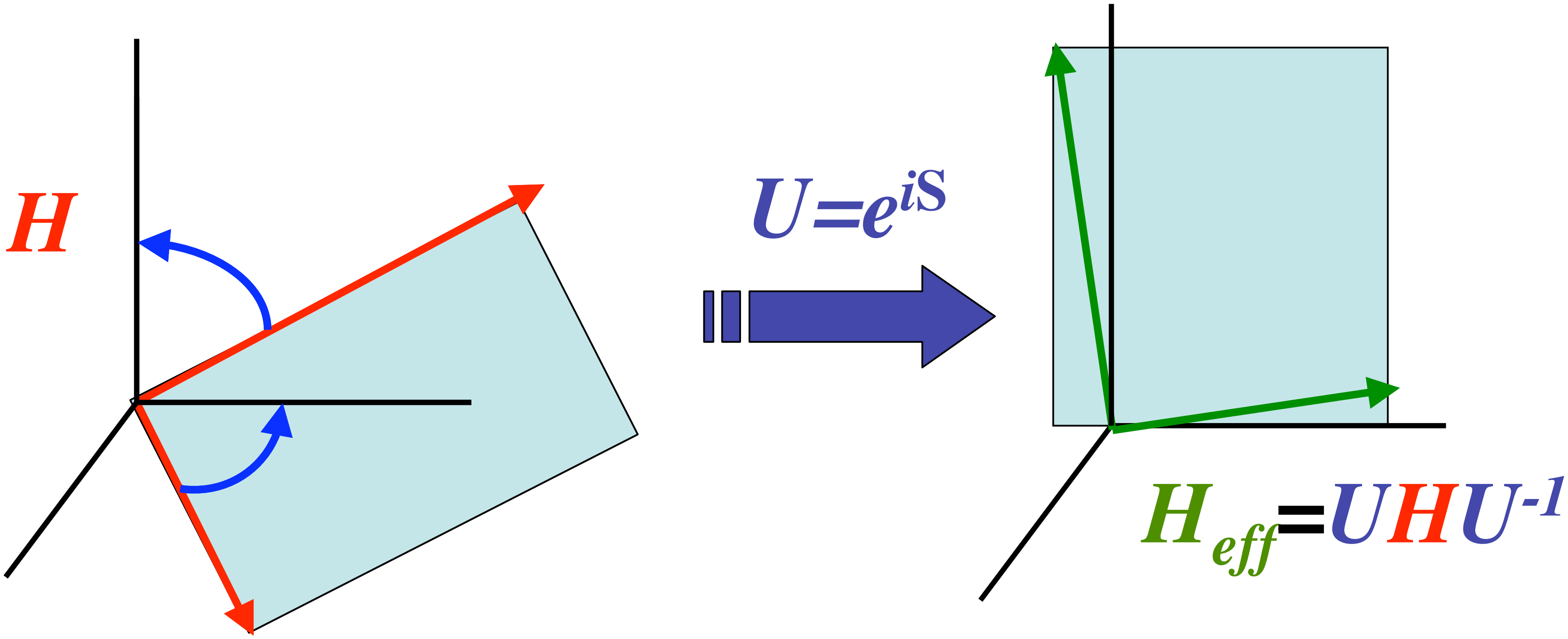


V. G. Gueorguiev, Feng Pan and J. P. Draayer, (**nucl-th-0403055**)

“Application of the extended pairing model to heavy isotopes”,

The European Physical Journal A, Vol. **25** No. Supplement 1 (September 2005) p.515.

Effective Interactions in a Finite Model Space



$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j}^{A} V_{ij}(q) \quad \Longrightarrow \quad H_{eff} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i_1<i_2}^{A} V_{i_1 i_2}(q) + \ldots + \sum_{i_1<\ldots<i_A}^{A} V_{i_1 \ldots i_A}(q)$$

$U = e^{iS}$

1 and 2-body interaction terms

1, 2, 3 … up to A-body interaction terms

Effective Hamiltonian in Second Quantized Form

$$H_{eff} = \sum_{\alpha} \varepsilon_{\alpha} a_{\alpha}^{+} a_{\alpha} + \cdots + \sum_{k=1}^{A} \frac{1}{(k!)^2} \sum_{\substack{\alpha_1 \neq \ldots \neq \alpha_k \\ \alpha_{k+1} \neq \ldots \neq \alpha_{2k}}} V_{\alpha_1 \ldots \alpha_{2k}} a_{\alpha_1}^{+} \ldots a_{\alpha_k}^{+} a_{\alpha_{k+1}} \ldots a_{\alpha_{2k}}$$

- Calculating **3-body effective interactions** is difficult, however doable, and **essential in understanding the structure of light nuclei**! P. Navratil and W. E. Ormand, Phys. Rev. C **68**, 034305(2003).

- In principle, **A-body effective Hamiltonians** can be calculated from realistic interactions, but their applications **are yet ahead**!

- **Need for exactly solvable models to understand possible implications of the A-body effective interactions**!

Extended Pairing Problem

The Standard Pairing Problem

$$\hat{H} = \sum_j \varepsilon_j n_j - \sum_{jj'} c_{jj'} A_j^+ A_{j'}^-,$$

$$A_j^+ = \sum_{m>0} (-)^{j-m} a_{jm}^+ a_{j-m}^+ .$$

Exactly solvable cases:

- **Constant pairing** $c_{jj'}=G$ with **non-degenerate single particle energies** $\varepsilon_j \neq \varepsilon_{j'}$ R. W. Richardson, *Phys. Lett.* **5,** 82 (1963).
- **Separable pairing** $c_{jj'}=c_j c_{j'}$ with **degenerate single particle energies** $\varepsilon_j = \varepsilon_{j'}$ F. Pan, J. P. Draayer, W. E. Ormand, *Phys. Lett.* B **422,** 1 (1998).
- Arbitrary Simple Lie algebra based **Richardson-Gaudin models** with **non-degenerate single particle energies**, including proton-neutron pairing, J. Dukelsky, V.G. Gueorguiev, P. Van Isacker (PRL96,072503(2006)).

Algebraic Models

- **Standard pairing** is actually an **SU(2) RG-model**.
- **Proton-neutron T=1 pairing** is **SO(5) RG-model**.
- One can **"easily" solve** such RG-models exactly.
- Possible **applications**:
  - **Nuclear physics**,
  - Condensed matter (**superconductivity**),
  - High energy physics (**pairing in quarks**).
- Generalized Richardson-Gaudin models are new interesting set of **exactly solvable algebraic models**.

rank	A_n *su*(n+1)	B_n *so*(2n+1)	C_n *sp*(2n)	D_n *so*(2n)
1	***su*(2) pairing**	*so*(3)~*su*(2)	***sp*(2) ~*su*(2)**	*so*(2) ~*u*(1)
2	*su*(3) Elliott	***so*(5) pn-pairing**	***sp*(4) ~*so*(5)**	*so*(4) ~*su*(2)⊕*su*(2)
3	*su*(4) Wigner	*so(7)⊂so(8) FDSM*	*sp*(6) Rowe & Rosensteel	*so*(6)~*su*(4)
4	*su*(5)	*so(9)*	*sp*(8)	*so*(8) Evans, FDSM

T=1 Proton-Neutron Pairing

SO(5) RG-model

- **Nucleon pairs:** $b_{-1}^{+}=n^{+}\overline{n}^{+},\ b_{+1}^{+}=p^{+}\overline{p}^{+},\ b_{0}^{+}=(n^{+}\overline{p}^{+}+p^{+}\overline{n}^{+})/\sqrt{2}$
- **$u(1)xsu_T(2)$ algebra:**
$$T_{+1}=(p^{+}n+\overline{p}^{+}\overline{n})/\sqrt{2},\ T_{-1}=(n^{+}p+\overline{n}^{+}\overline{p})/\sqrt{2}$$
$$T_{0}=(p^{+}p+\overline{p}^{+}\overline{p})/2-(n^{+}n+\overline{n}^{+}\overline{n})/2$$
$$N=p^{+}p+\overline{p}^{+}\overline{p}+n^{+}n+\overline{n}^{+}\overline{n}$$
- **Pairing Hamiltonian:**
$$H=\sum_{\rho=p,n}\sum_{jm}\varepsilon_{j}(\rho_{jm}^{+}\rho_{jm}+\overline{\rho}_{jm}^{+}\overline{\rho}_{jm})-g\sum_{\mu=-1,0,1}\sum_{j,m}b_{\mu,jm}^{+}b_{\mu,jm}$$
- **Generalized Richarsdon equations:**

$$\frac{1}{g}=\sum_{i}\frac{\Omega^{i}}{z_{i}-e_{\alpha}}+2\sum_{\beta(\neq\alpha)}^{N}\frac{1}{e_{\alpha}-e_{\beta}}+\sum_{\gamma=1}^{N-T}\frac{1}{w_{\gamma}-e_{\alpha}},$$

$$0=\sum_{\alpha=1}^{N}\frac{1}{e_{\alpha}-w_{\gamma}}+\sum_{\delta(\neq\gamma)}^{N-T}\frac{1}{w_{\gamma}-w_{\delta}},\quad E=\sum_{\alpha=1}^{N}e_{\alpha}$$

Extended Pairing Model

$$H_{eff} = \sum_{\alpha} \varepsilon_{\alpha} a_{\alpha}^{+} a_{\alpha} + \cdots + \sum_{k=1}^{A} \frac{1}{(k!)^2} \sum_{\substack{\alpha_1 \neq \ldots \neq \alpha_k \\ \alpha_{k+1} \neq \ldots \neq \alpha_{2k}}} V_{\alpha_1 \ldots \alpha_{2k}} a_{\alpha_1}^{+} \ldots a_{\alpha_k}^{+} a_{\alpha_{k+1}} \ldots a_{\alpha_{2k}}$$

- ***Nilsson single particle energies*** ε_{jm} play the role of ***1-body effective Hamiltonian*** that takes into account the nuclear deformation due to the Q·Q interaction.

- ***Simplifying assumption***: ***equal coupling between different configurations:***

$$V_{\alpha_1\alpha_2} = V_{\alpha_1\alpha_2\alpha_3\alpha_4} = V_{\alpha_1 \ldots \alpha_{2A}} = G$$

- **The RESULT is *Exactly solvable Hamiltonian*:**

F. Pan, V. G. Gueorguiev, J. P. Draayer, Phys. Rev. Lett. ***92****, 112503 (2004).*

$$\hat{H} = \sum_{i} \varepsilon_i n_i - G \sum_{ij} b_i^{+} b_j - G \sum_{k=2}^{A} \frac{1}{(k!)^2} \sum_{i_1 \neq \ldots \neq i_{2k}} b_{i_1}^{+} \ldots b_{i_k}^{+} b_{i_{k+1}} \ldots b_{i_{2k}}, \quad b_i^{+} = a_{i\uparrow}^{+} a_{i\downarrow}^{+}$$

Bethe Ansatz

Wavefunction:

$$\left|k,\varsigma;j_1\ldots j_m\right\rangle \propto \sum_{i_1<\cdots<i_k} C^{(\varsigma)}_{i_1 i_2\ldots i_k} b^+_{i_1}\ldots b^+_{i_k}\left|j_1,j_2\ldots j_m\right\rangle$$

$$C^{(\varsigma)}_{i_1\ldots i_k} = \frac{1}{z^{(\varsigma)} - 2\sum_{n=1}^{k}\varepsilon_{i_n}}$$

$$b_i\left|j_1\ldots j_m\right\rangle = 0 \text{ if } i \neq j_s$$

$$b_i^+\left|j_1\ldots j_m\right\rangle = 0 \text{ if } i = j_s$$

Bethe Ansatz Equation:

$$\sum_{i_1<\cdots<i_k} \frac{G}{2\sum_{n=1}^{k}\varepsilon_{i_n} - z^{(\varsigma)}} = 1$$

Eigenenergy:

$$E_k^{(\varsigma)} = z^{(\varsigma)} - G(k-1)$$

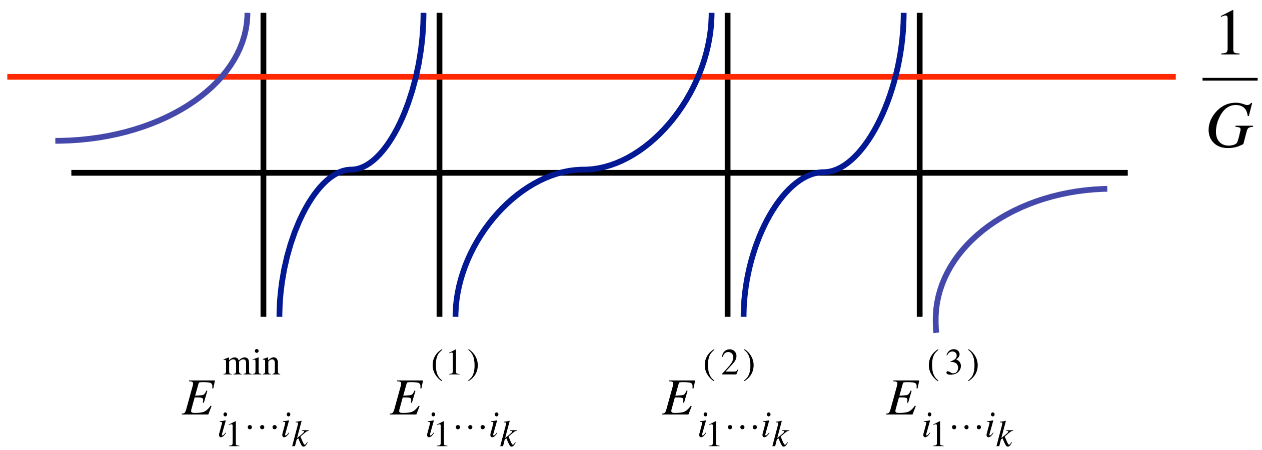
Solving the Equations

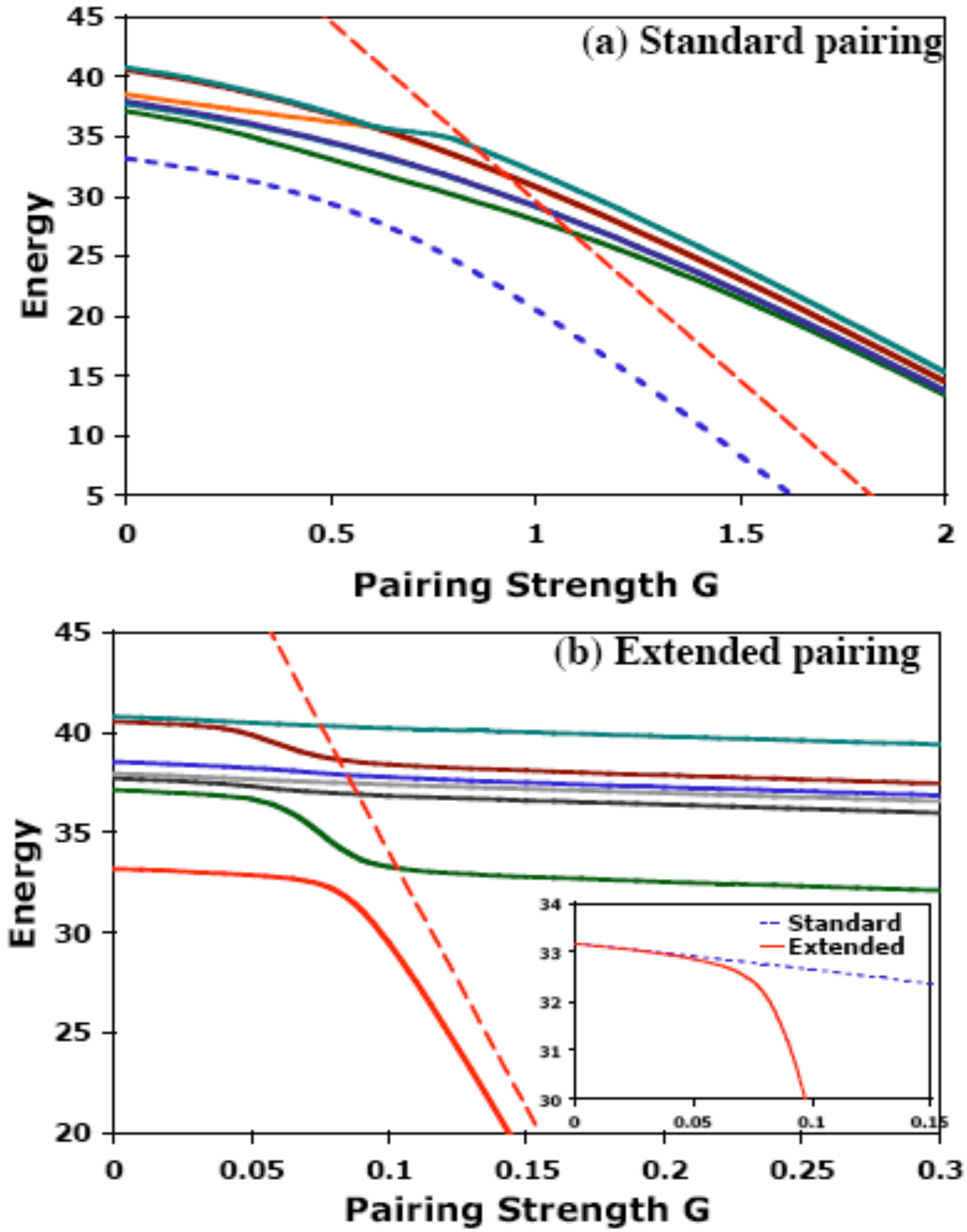
$$E_k^{(\varsigma)} = z^{(\varsigma)} - G(k-1)$$

$$1 = \sum_{i_1 < \cdots < i_k} \frac{G}{E_{i_1 \cdots i_k} - z^{(\varsigma)}}$$

$$E_{i_1 \cdots i_k} = 2\sum_{n=1}^{k} \varepsilon_{i_n}$$

$$\boxed{\frac{k-1}{z^{(\varsigma)} - E_k^{(\varsigma)}} = \sum_{i_1 < \cdots < i_k} \frac{1}{E_{i_1 \cdots i_k} - z^{(\varsigma)}}, \quad G = \frac{z^{(\varsigma)} - E_k^{(\varsigma)}}{k-1}}$$





5 pairs in

10 levels

e_1=1.179

e_2=2.650

e_3=3.162

e_4=4.588

e_5=5.006

e_6=6.969

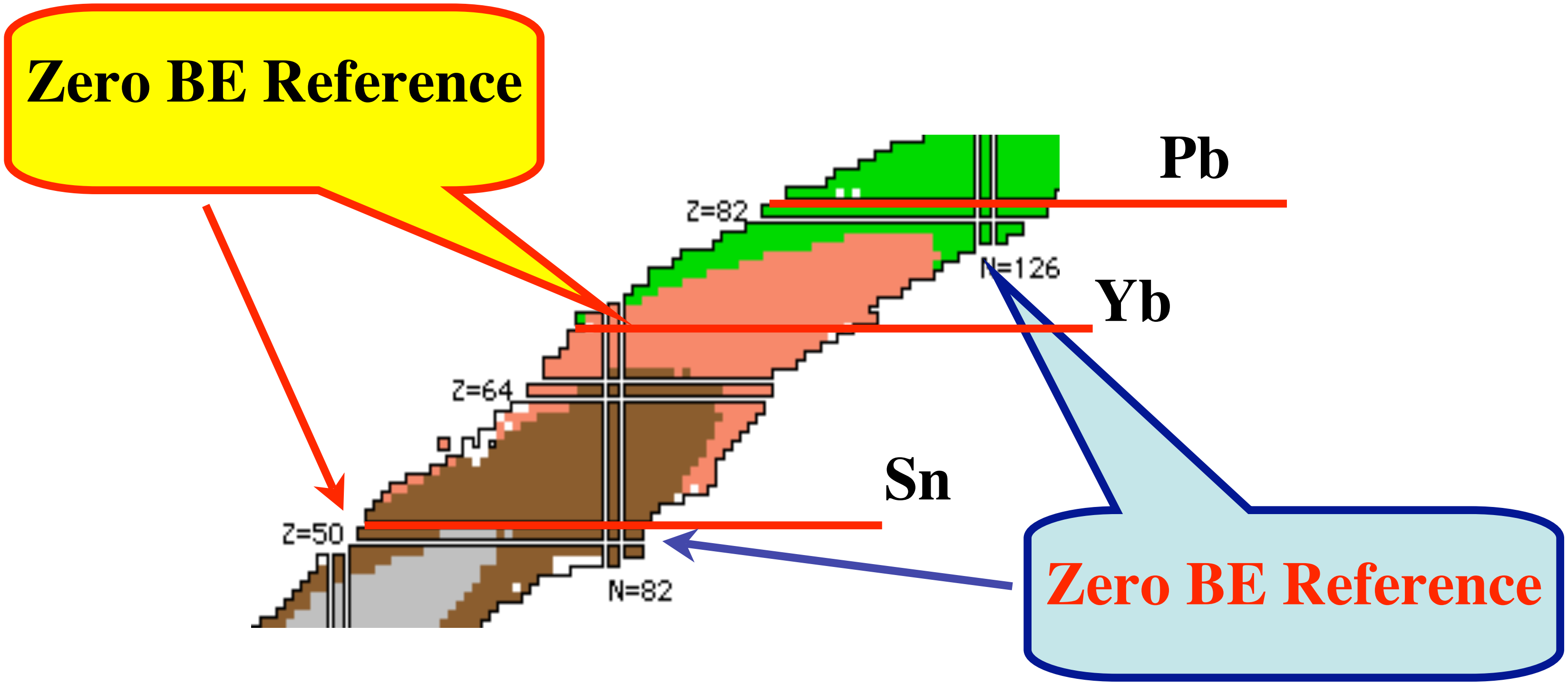
e_7=7.262

e_8=8.687

e_9=9.899

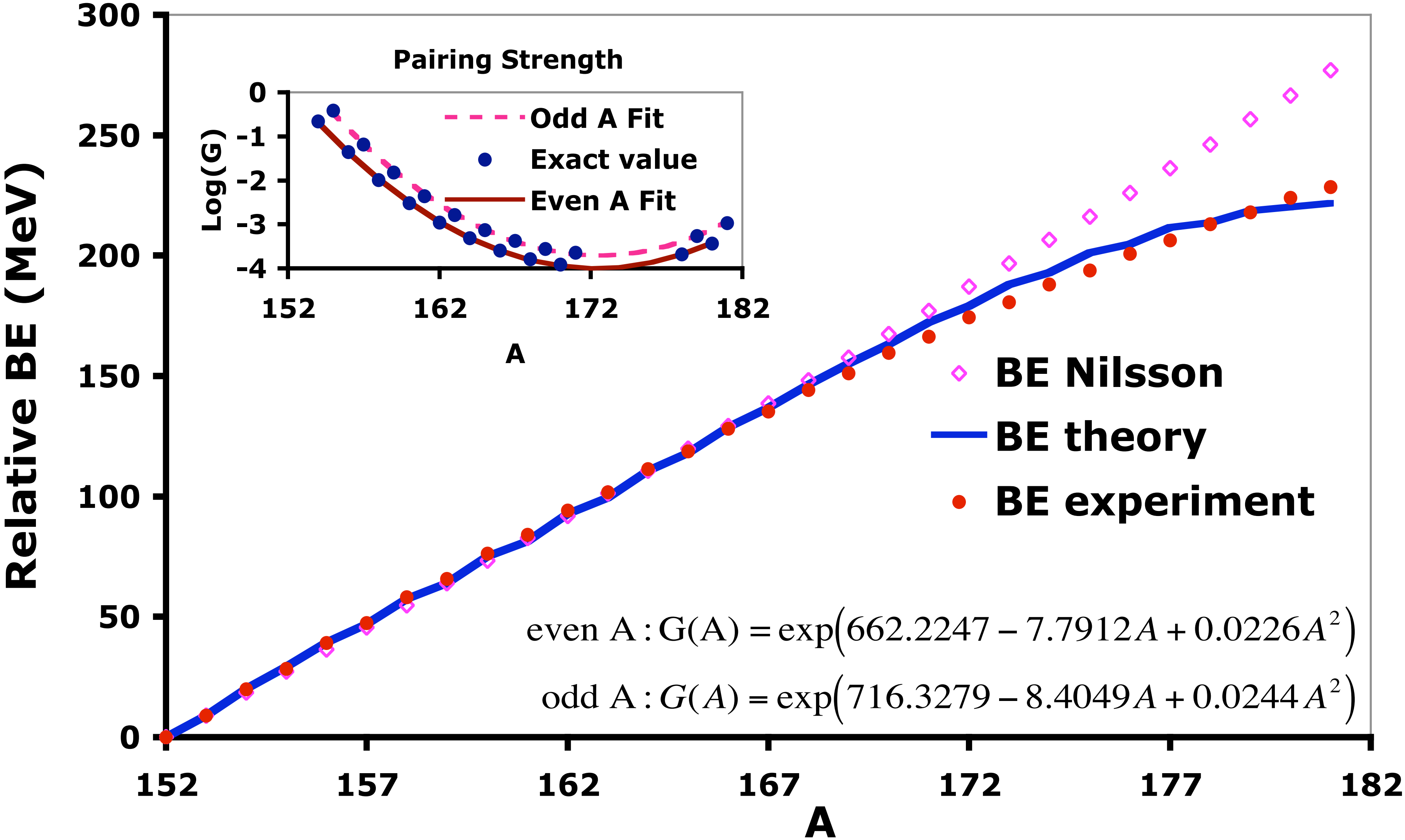
e_{10}=10.20

Extended Pairing for Nuclei

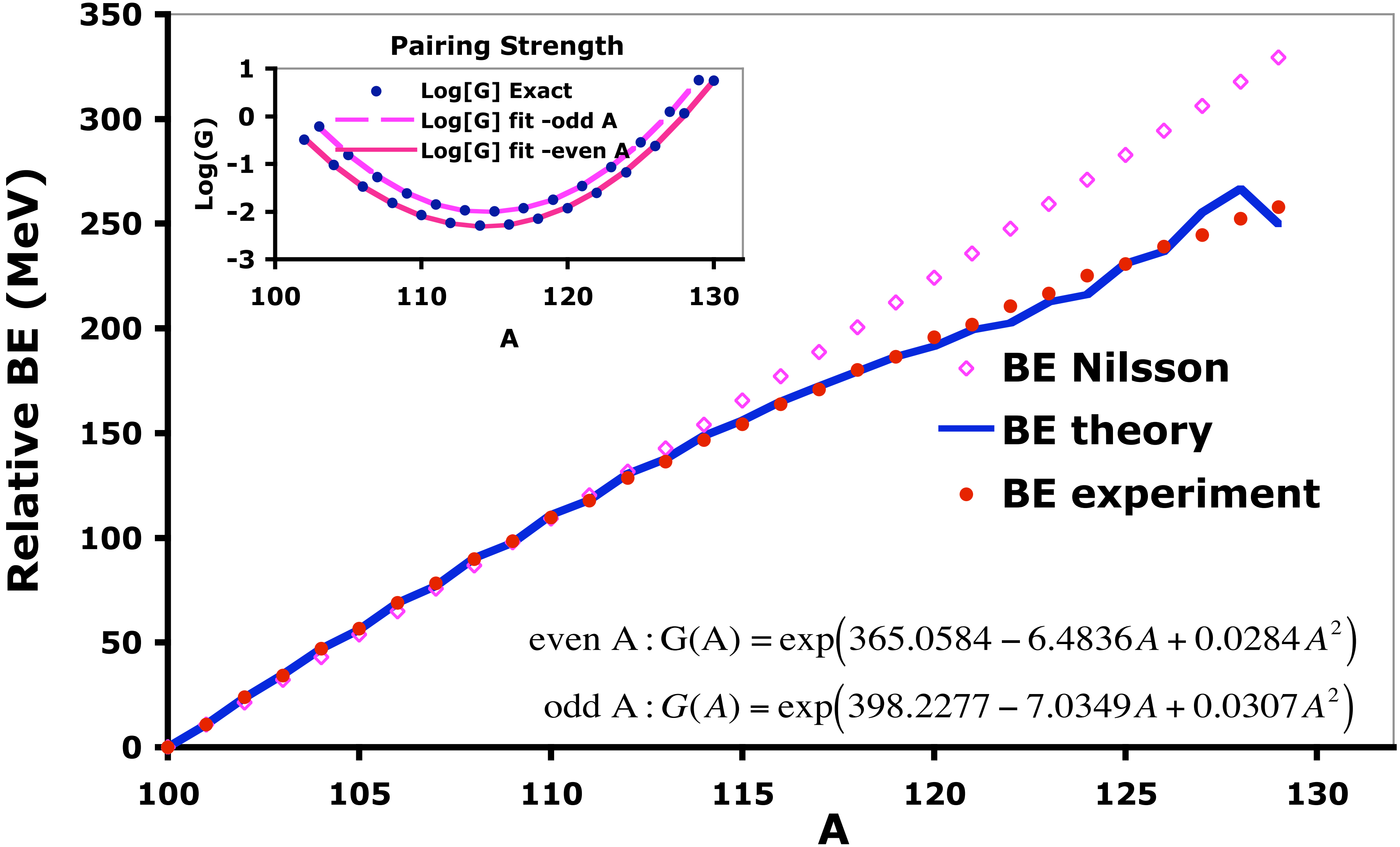


- Nilsson levels using nuclear deformation *(Audi & Wapstra (1995)).*
- Pauli blocking for odd A nuclei.
- Set the scale of the single particle energies from near closed shell system… (Nilsson BE is 3/4 E filling, *Ring & Schuck*)

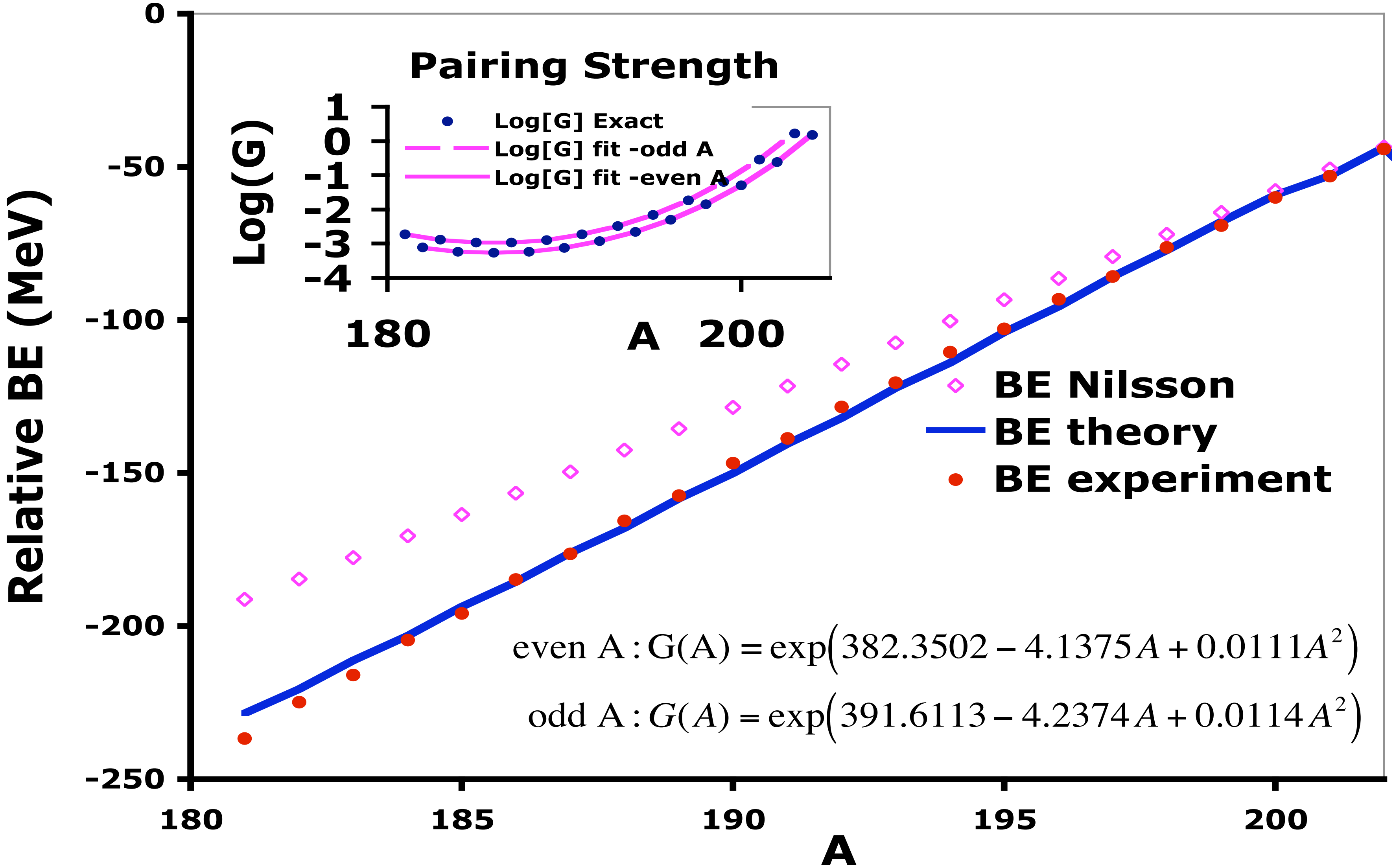
Binding Energy of the Yb Isotopes



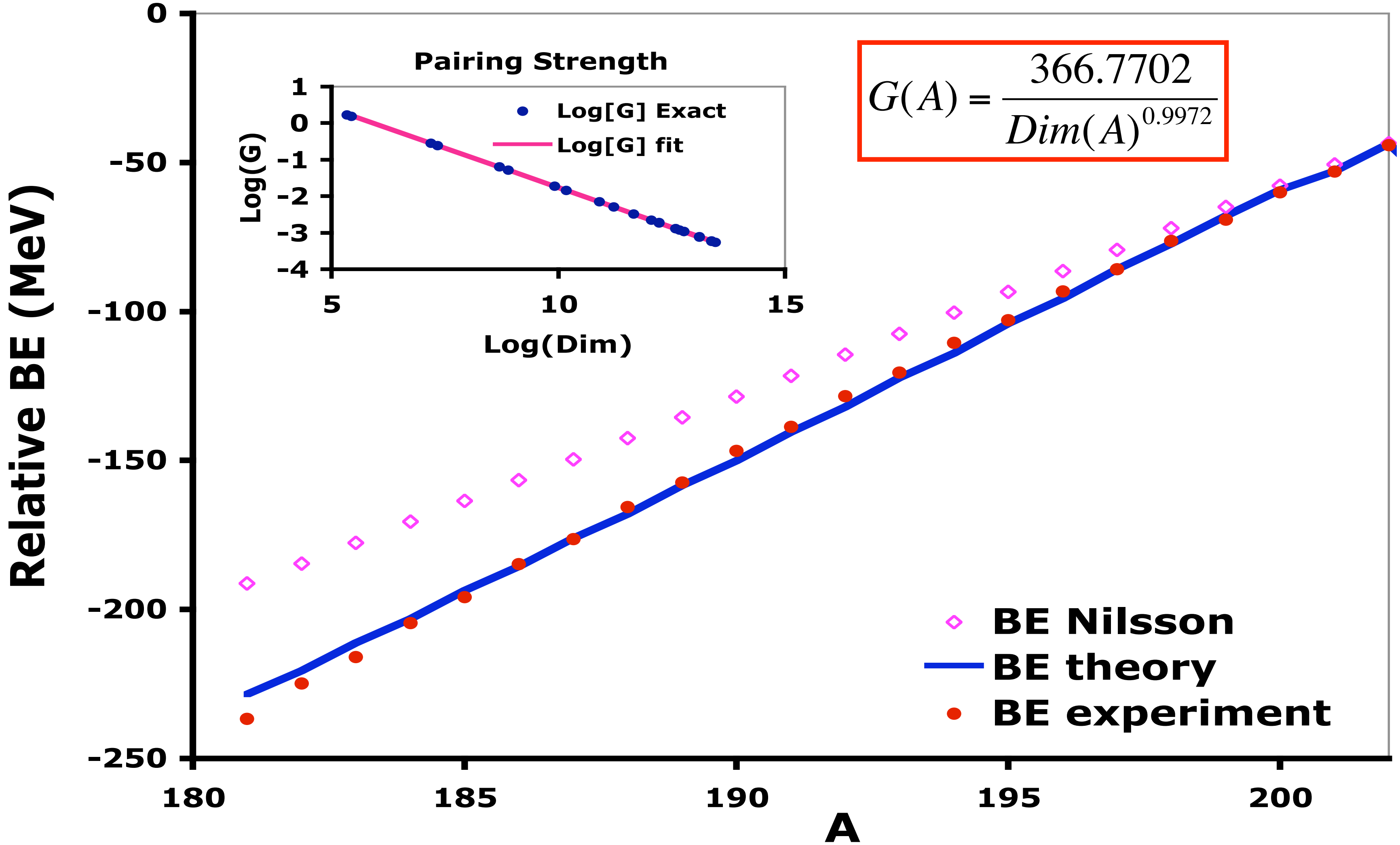
Binding Energy of the Sn Isotopes



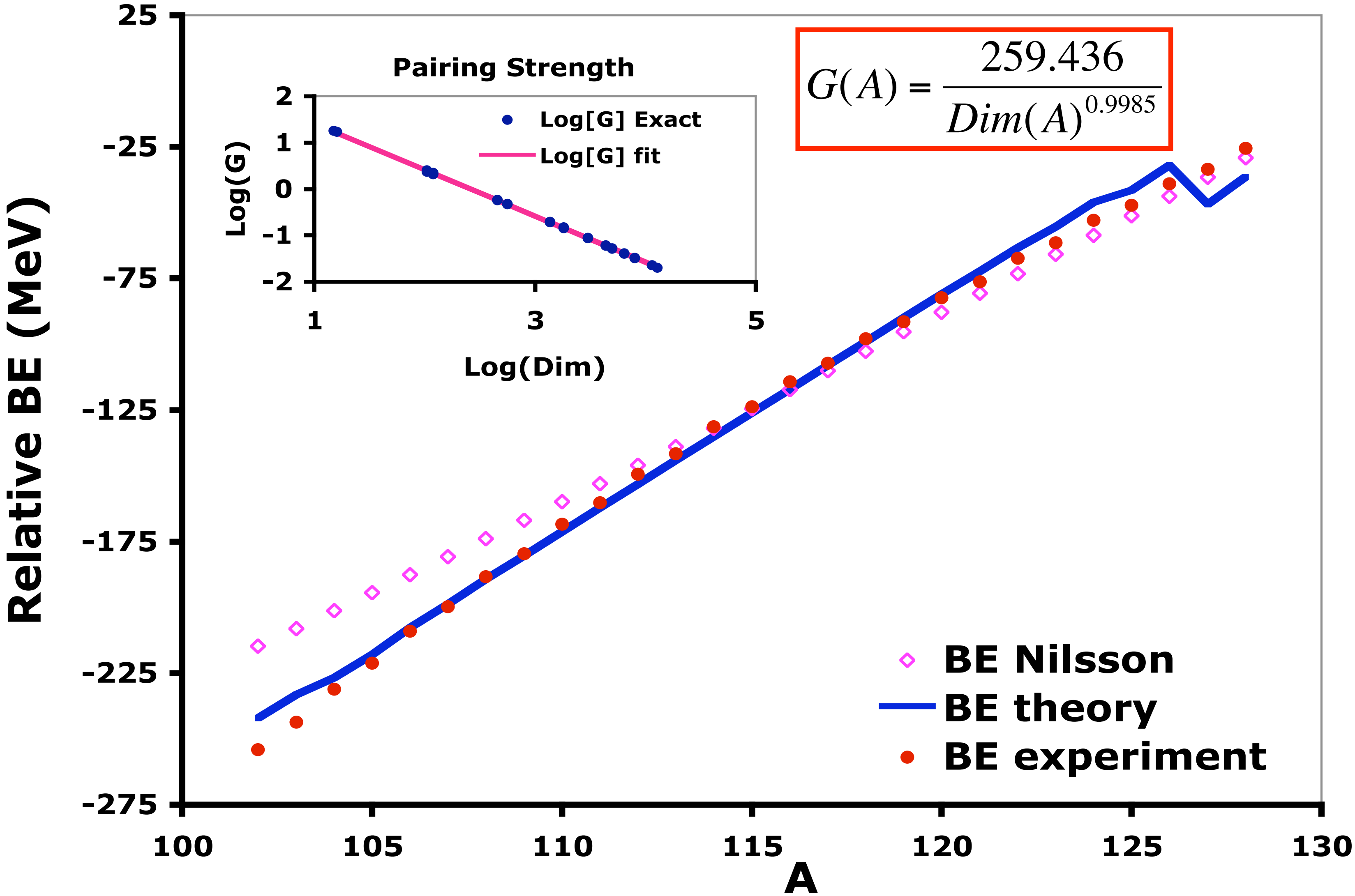
Binding Energy of the Pb Isotopes



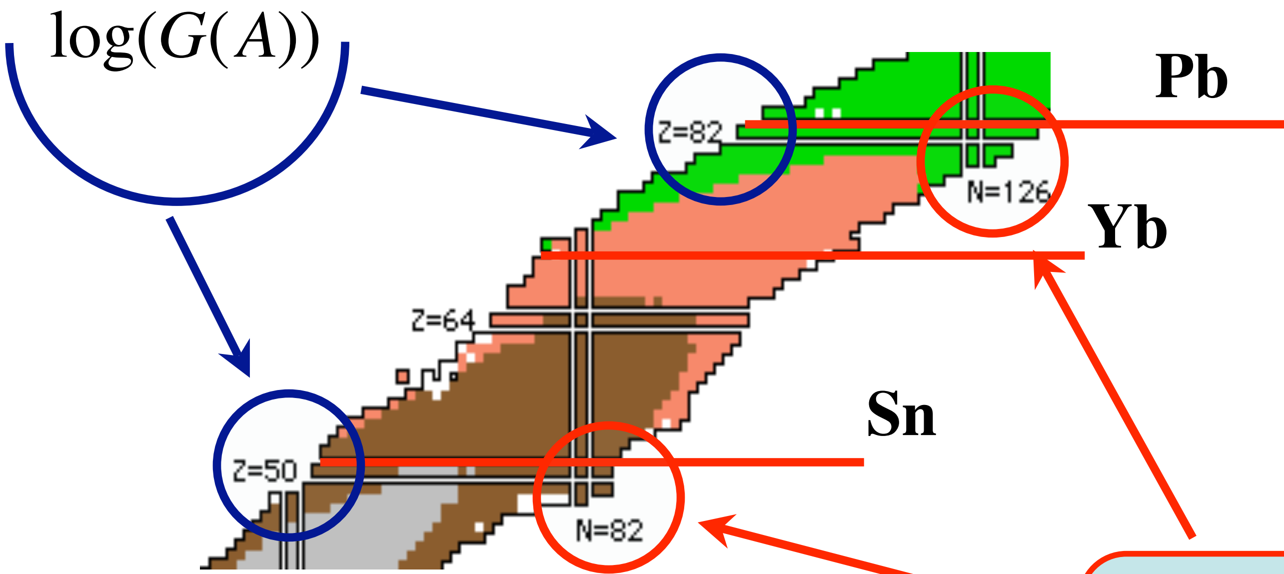
Binding Energy of the Pb Isotopes



Binding Energy of the Sn Isotopes



Extended Pairing



$$G(A) \sim \frac{\alpha}{Dim(A)}$$

- Has nice and interesting systematic behavior!
- The Extended Pairing model gives reasonable results…
- Many-body interactions beyond two- & three- body!

A-body interaction in nuclei

- Start getting used to the idea that we may need A-body interactions to understand nuclear properties across the nuclear chart…

- Write your codes with A-body interactions in mind!

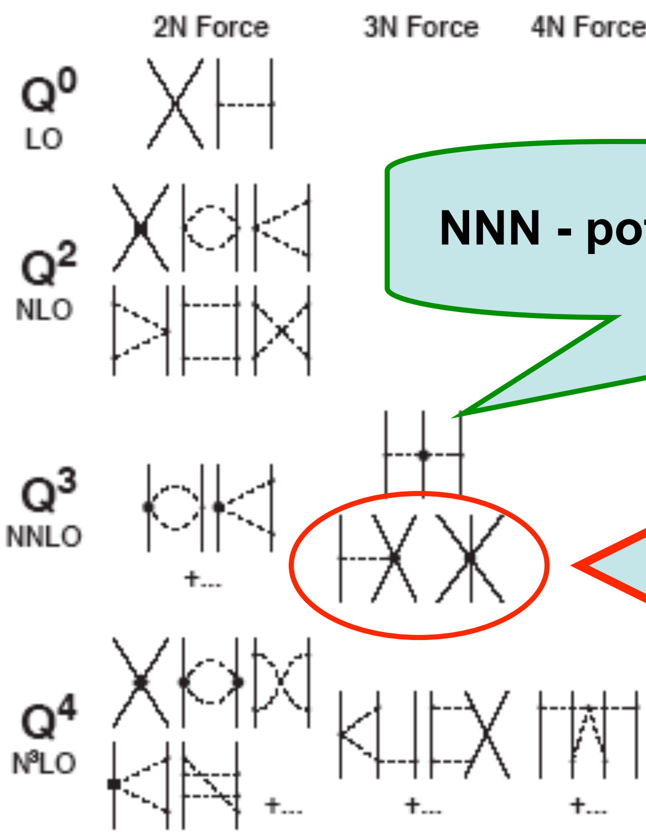
Q&A **about C_D-C_E curve, Λ, and the NNN interaction terms…**

Nucleon Interaction from QCD
(Chiral Perturbation Theory)

Chiral perturbation theory (χPT) allows for controlled power series expansion

Expansion parameter : $\left(\frac{Q}{\Lambda_\chi}\right)^{\nu}$, Q – momentum transfer,

$\Lambda_\chi \approx 1\ GeV$, χ - symmetry breaking scale

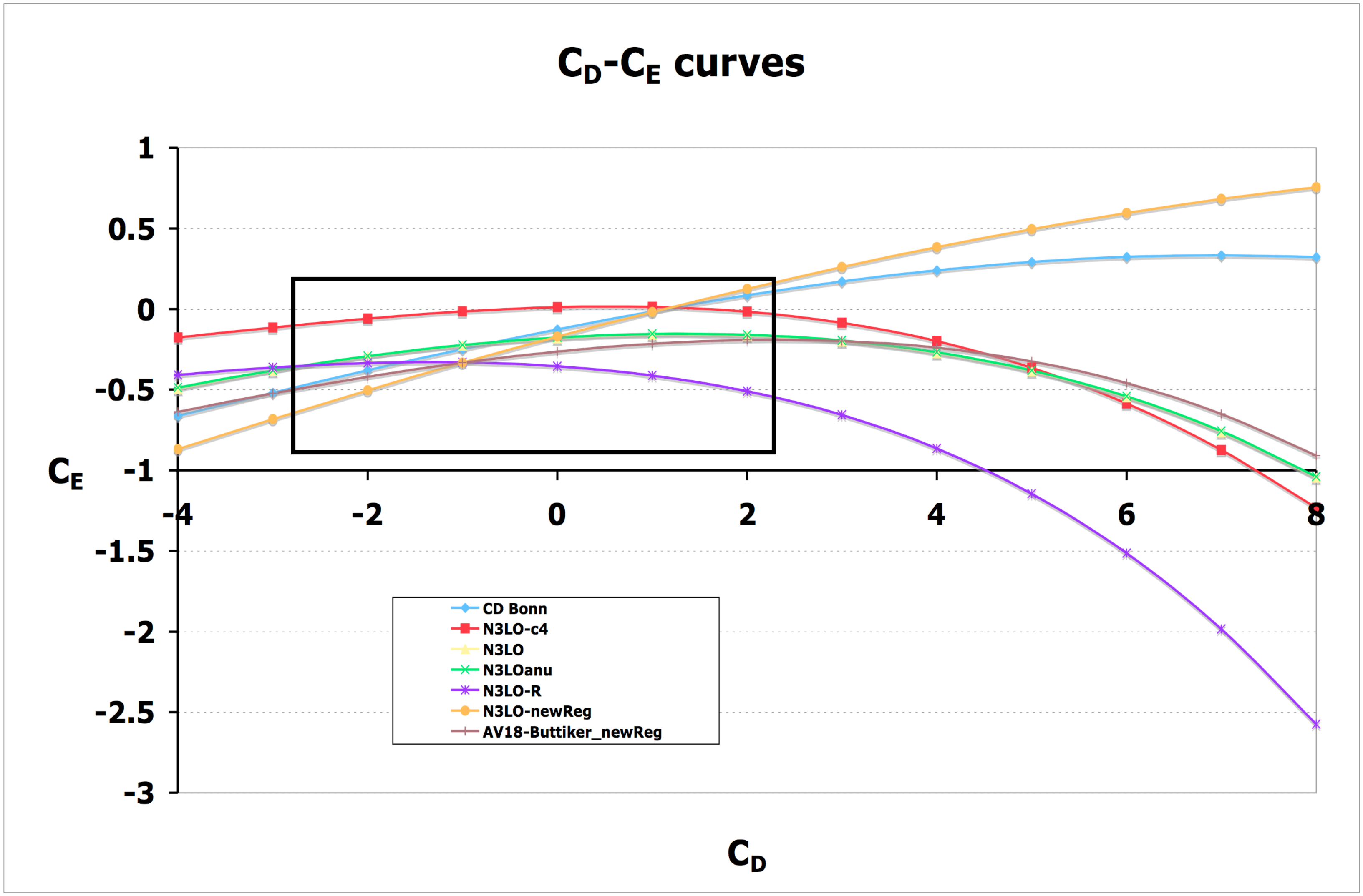


NNN - potentials such as Tucson-Melbourne & Urbana

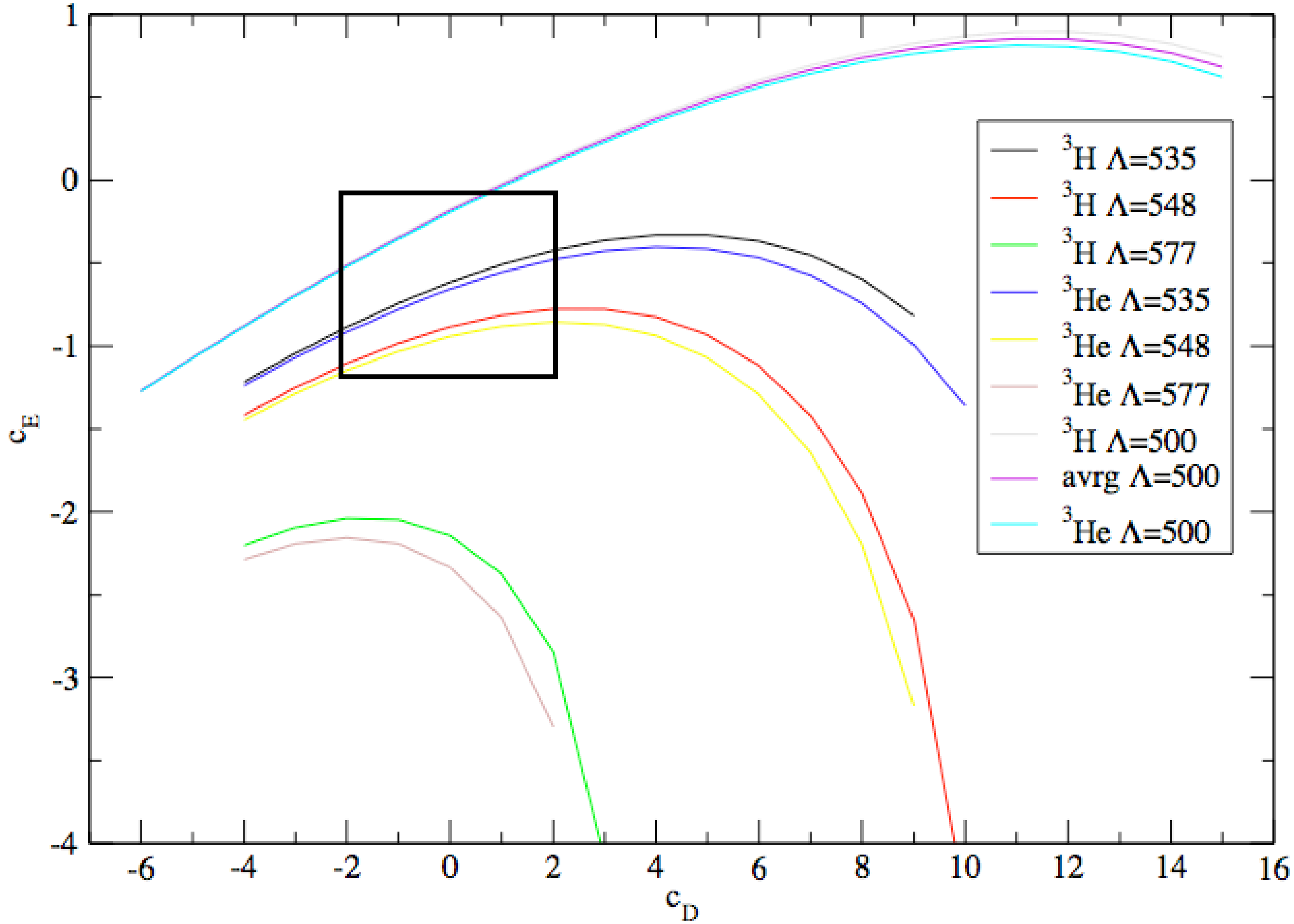
Terms suggested within the Chiral Perturbation Theory

R. Machleidt, D. R. Entem, nucl-th/0503025

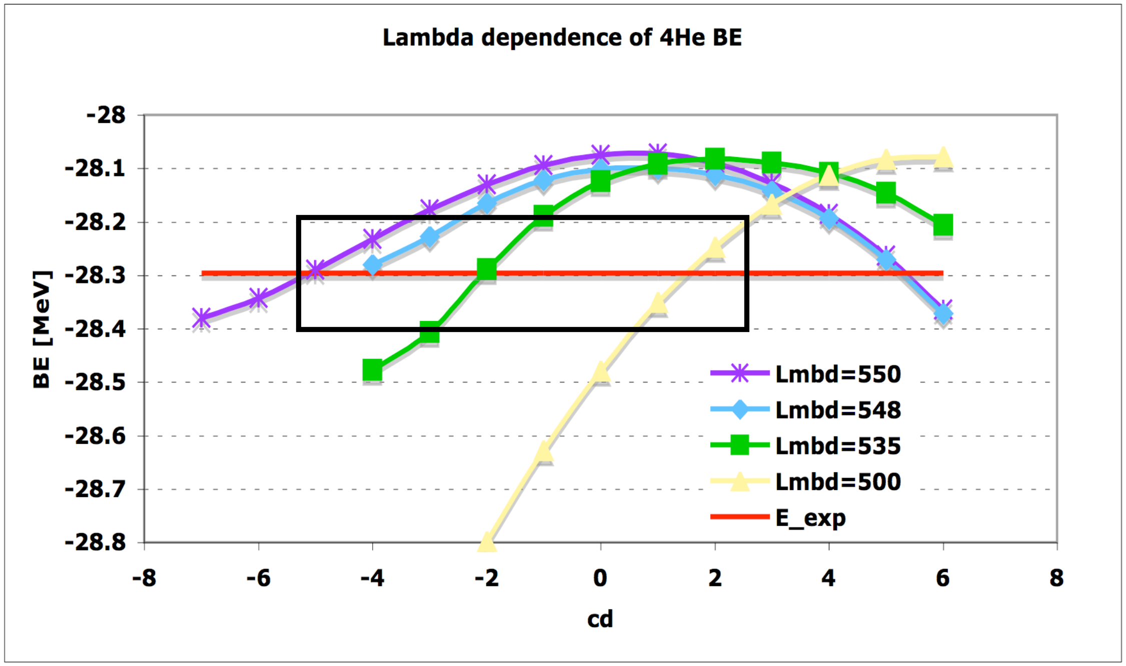
Regularization is essential, which is quite obvious within the Harmonic Oscillator wave function basis.

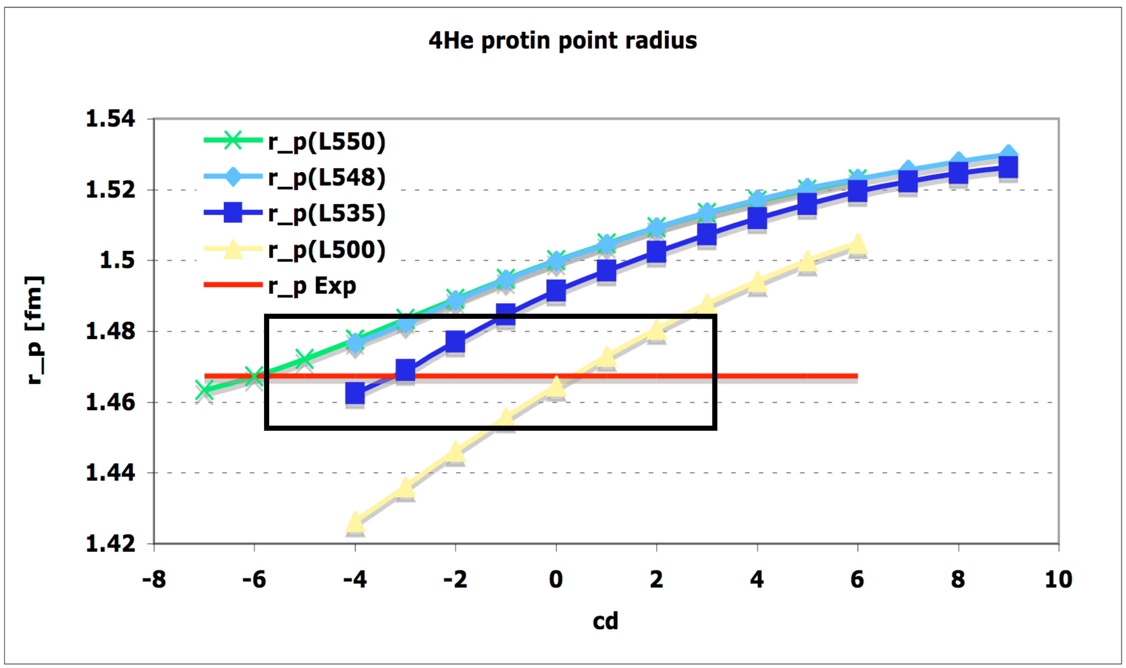


C_D-C_E curves
1
0.5
0
-0.5
C_E
-1
-1.5
-2
-2.5
-3
-4
-2
0
2
4
6
8
C_D
CD Bonn
N3LO-c4
N3LO
N3LOanu
N3LO-R
N3LO-newReg
AV18-Buttiker_newReg

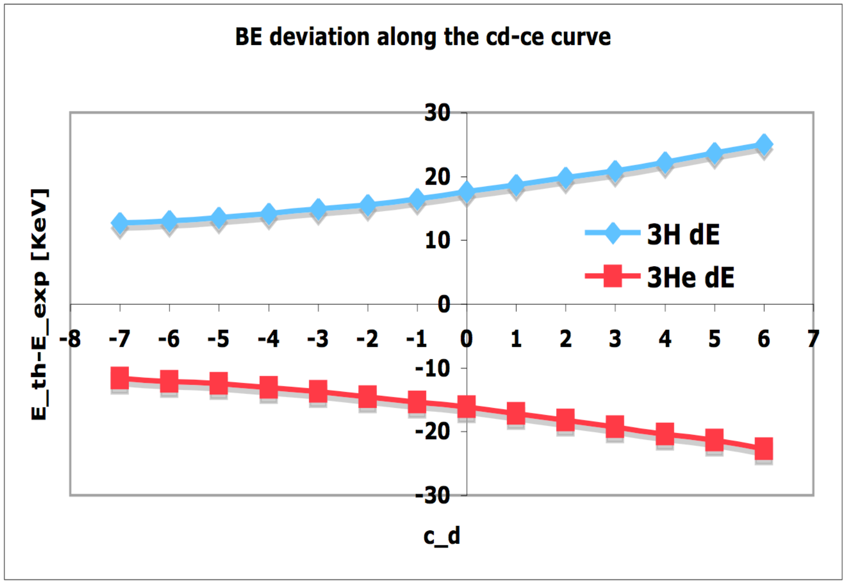


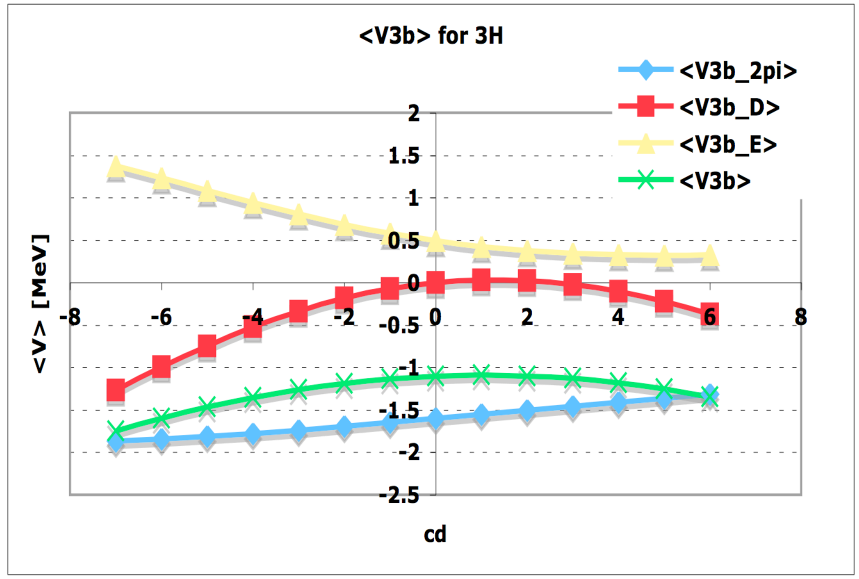
1
0
-1
-2
-3
-4
-6
-4
-2
0
2
4
6
8
10
12
14
16
c_E
c_D
3H Λ=535
3H Λ=548
3H Λ=577
3He Λ=535
3He Λ=548
3He Λ=577
3H Λ=500
avrg Λ=500
3He Λ=500

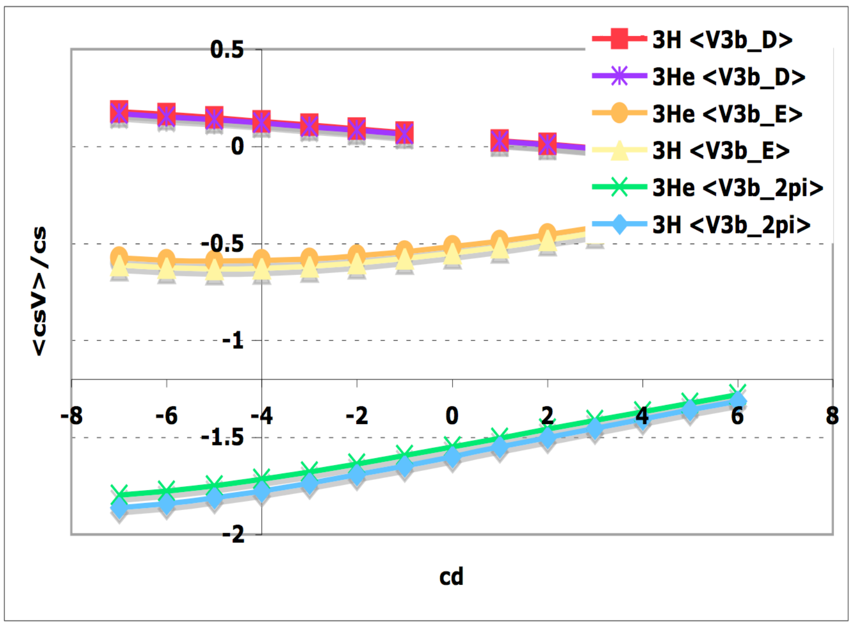

Lambda dependence of 4He BE
BE [MeV]
cd
Lmbd=550
Lmbd=548
Lmbd=535
Lmbd=500
E_exp
-28
-28.1
-28.2
-28.3
-28.4
-28.5
-28.6
-28.7
-28.8
-8
-6
-4
-2
0
2
4
6
8

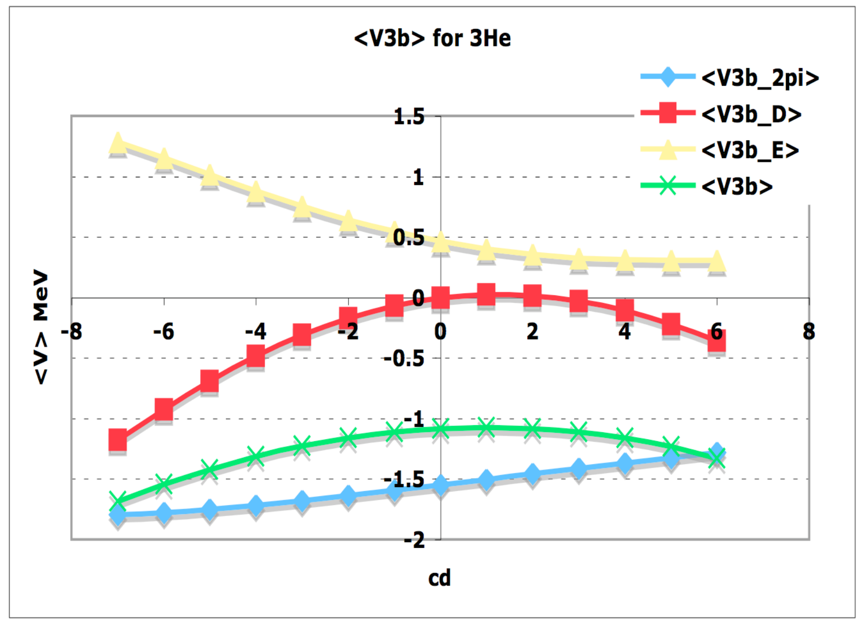

4He protin point radius
r_p [fm]
cd
r_p(L550)
r_p(L548)
r_p(L535)
r_p(L500)
r_p Exp
1.54
1.52
1.5
1.48
1.46
1.44
1.42
-8
-6
-4
-2
0
2
4
6
8
10

Lambda=550 MeV

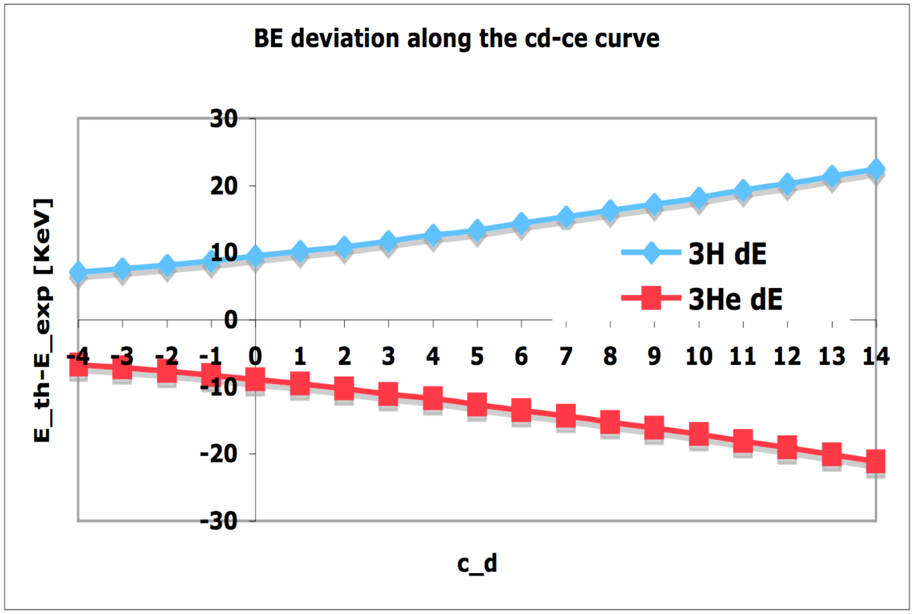


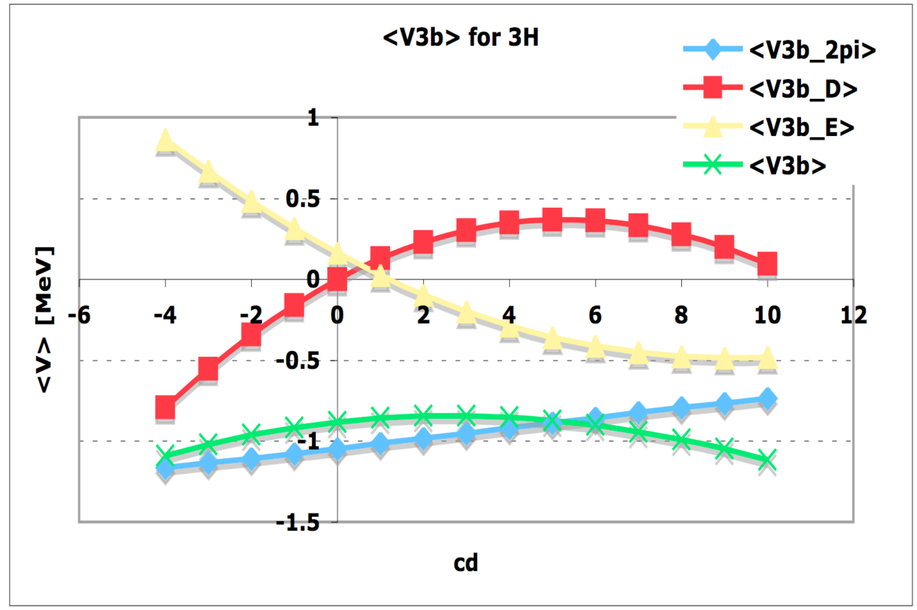


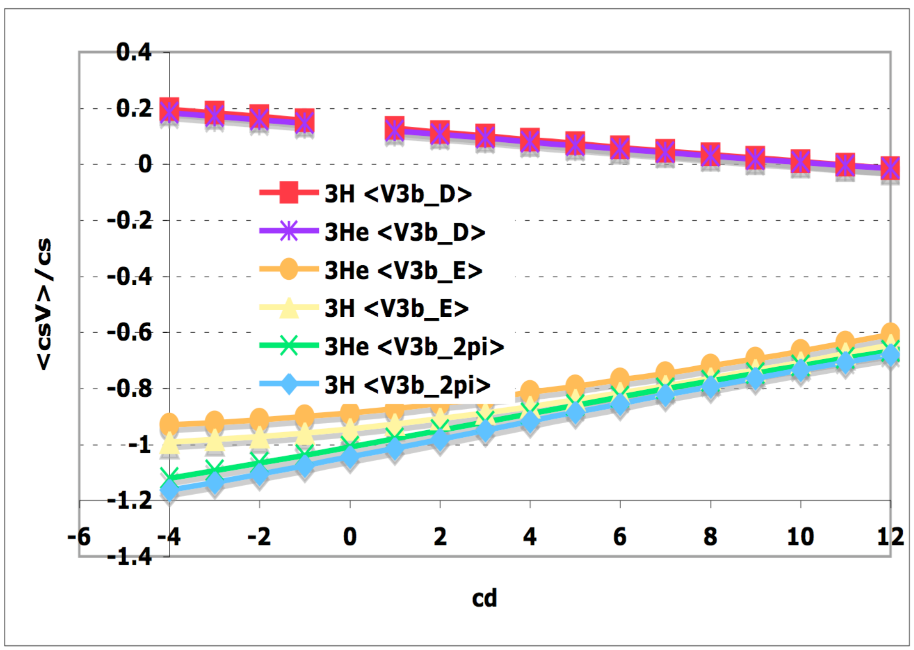


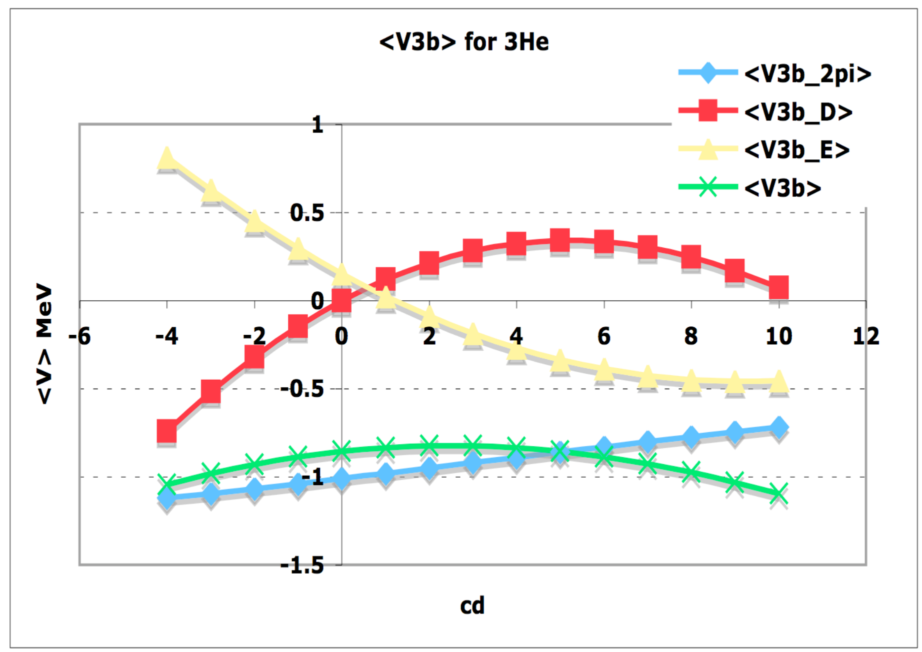


Lambda=500 MeV



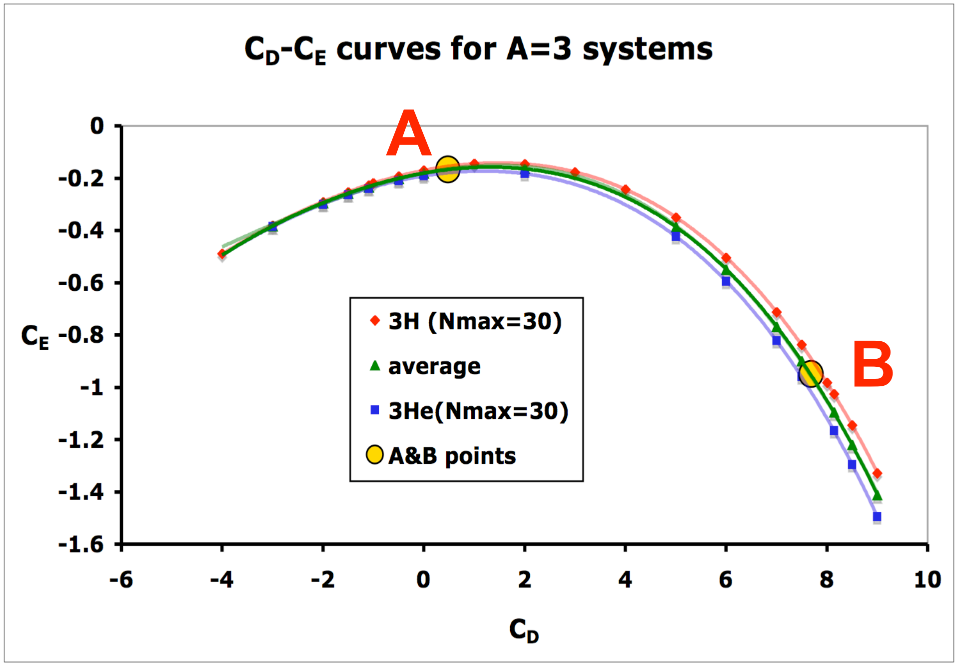


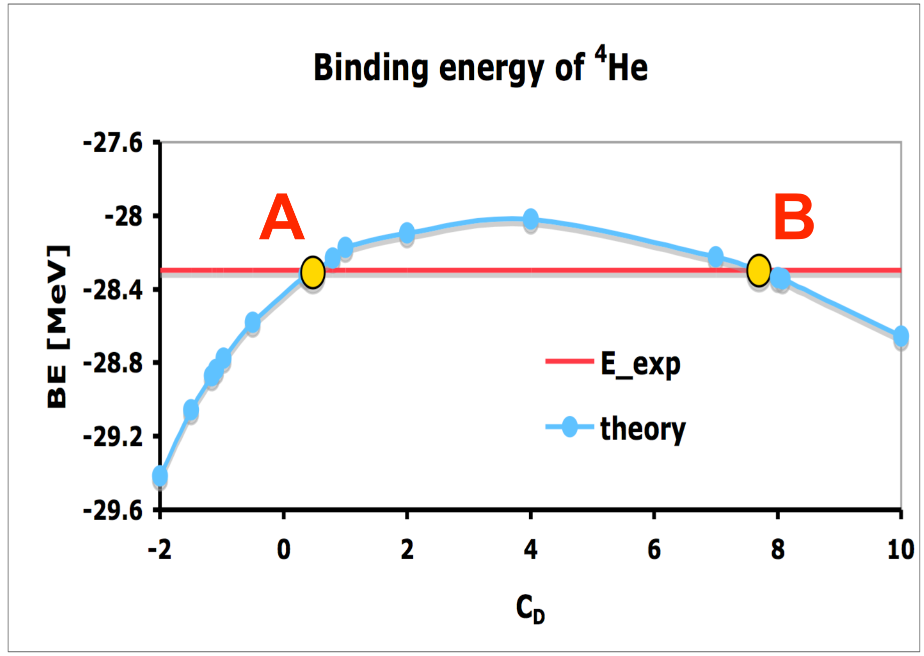




^{3}H, ^{3}He, and ^{4}He Binding Energy

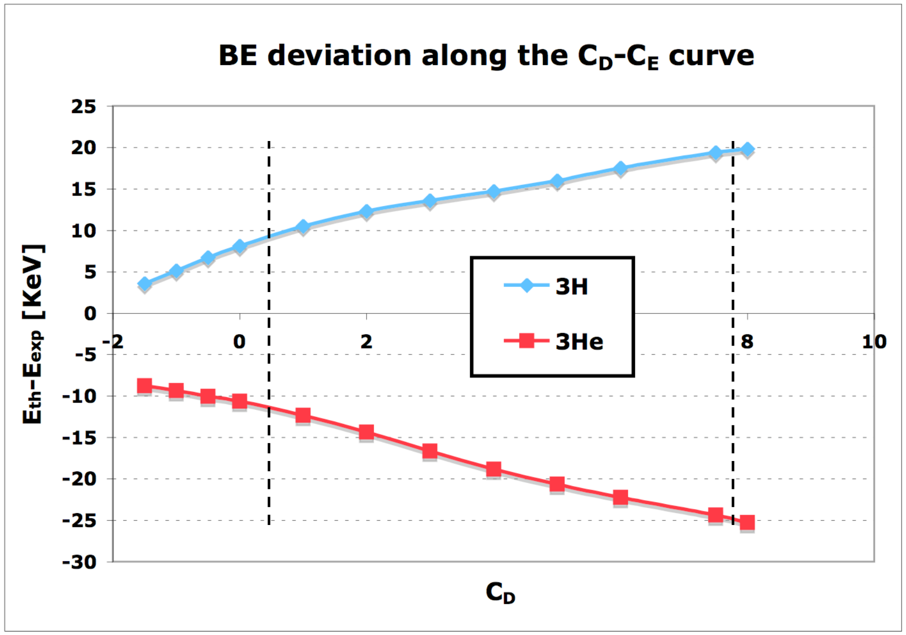
First values for $\{C_D,C_E\}$=({-1.11,-0.66}, {8.14,-2.02}) by A. Nogga et al, Nucl. Phys. A737, 236 (2004) in momentum space with $\mathrm{Exp}[-((p^2+q^2)/\Lambda^2)^2]$ cut-off function. We keep the same regulator for the contact terms but use $\mathrm{Exp}[-((Q^2)/\Lambda^2)^2]$ cut-off function for the $V_{2\pi}$ and work in coordinate space.

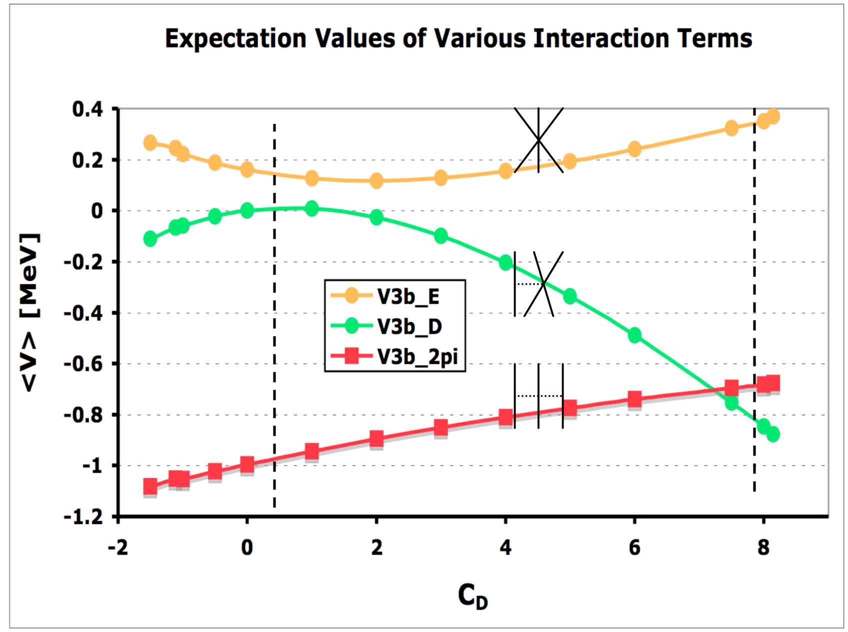




Along the appropriate curve, all C_D-C_E points reproduce BE of ^{3}H & ^{3}He within 0.1 keV. Our $\{C_D,C_E\}$ points for N3LO were deduced in N_{max}=16 model space for ^{4}He: $\{0.476,-0.167\}_A$ and $\{7.686,-0.949\}_{B.}$

Along the C_D-C_E Curve (average)





BE deviation is within the accuracy of the kinetic energy for equal mass nucleons $m_n=m_p=2\mu_{pn}$.

$$\delta T = T\frac{\delta m}{m} \approx 26keV(\approx 37\times 0.7\times 10^{-3})$$

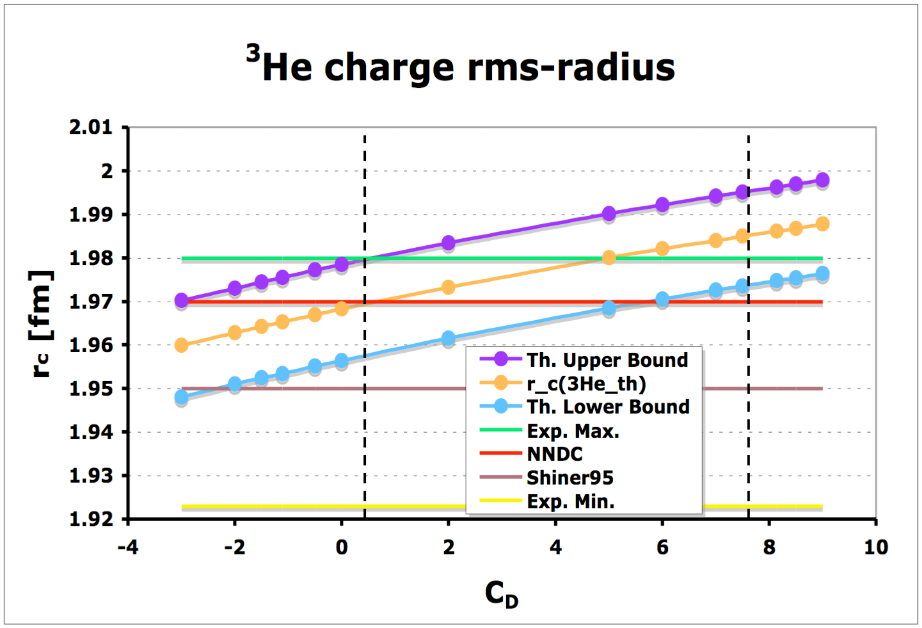
3N-contact $\langle V_E \rangle > 0$ (less binding)

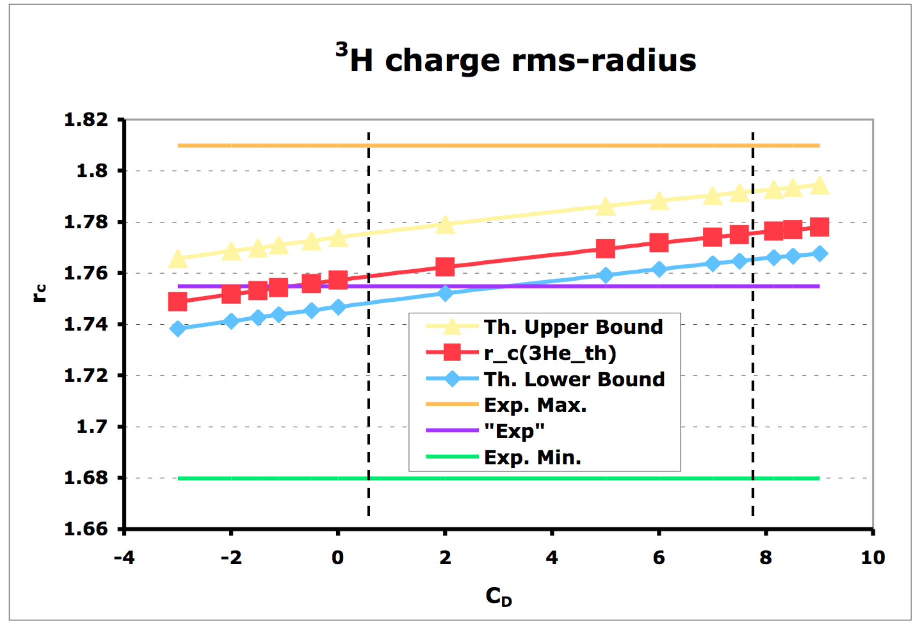
2N contact 1π–exchange, $\langle V_D \rangle$ {-,+,-}

2π–exchange $\langle V_{2\pi} \rangle < 0$ (more binding)

T=3/2 channel has been neglected in the calculations.

^3H and ^3He Charge Radii



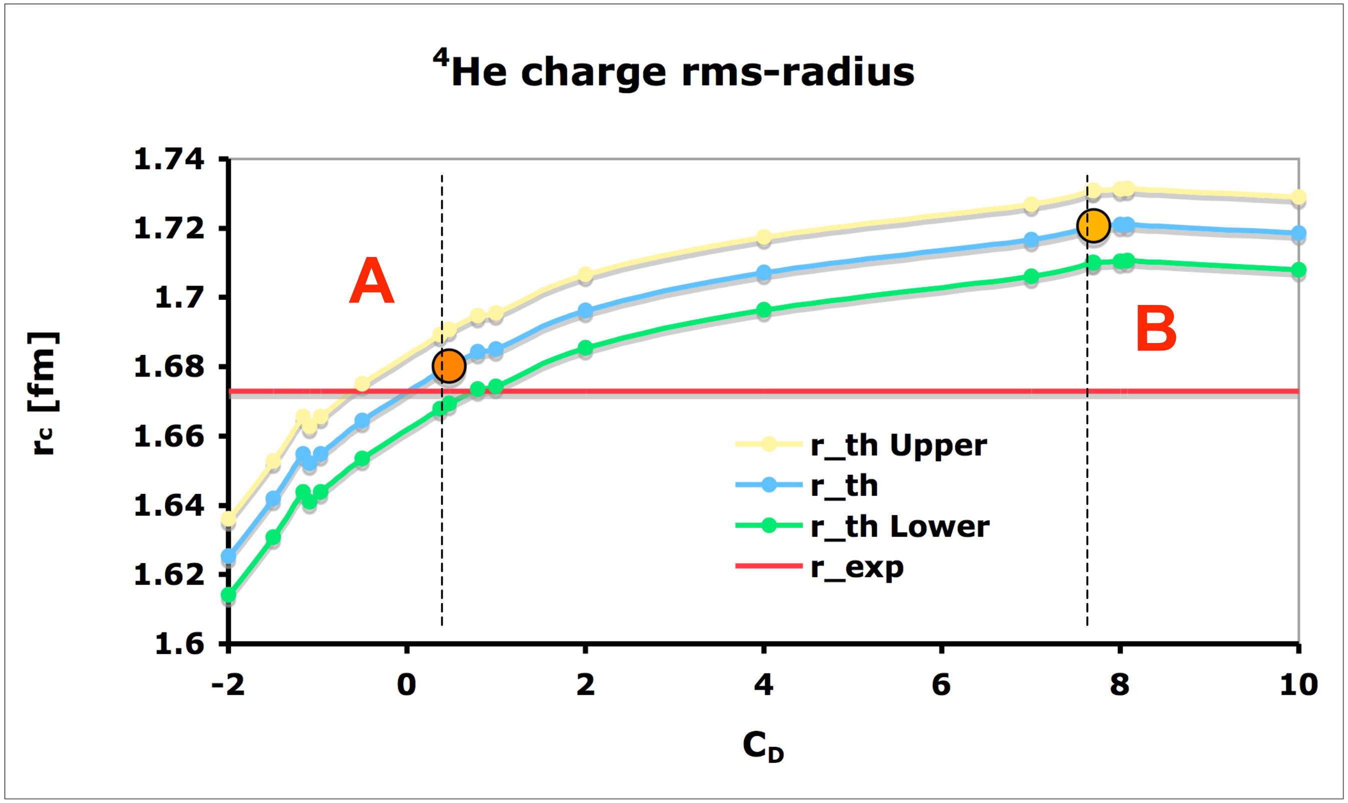


$$R_{ch}^{\ 2} = r_p^{\ 2} + R_p^{\ 2} + \left(\frac{N}{Z}\right) R_n^{\ 2}$$

R_p={**0.854**, **0.875**, **0.895**} [fm]
R^2_n={-0.12,-0.11} [fm^2]

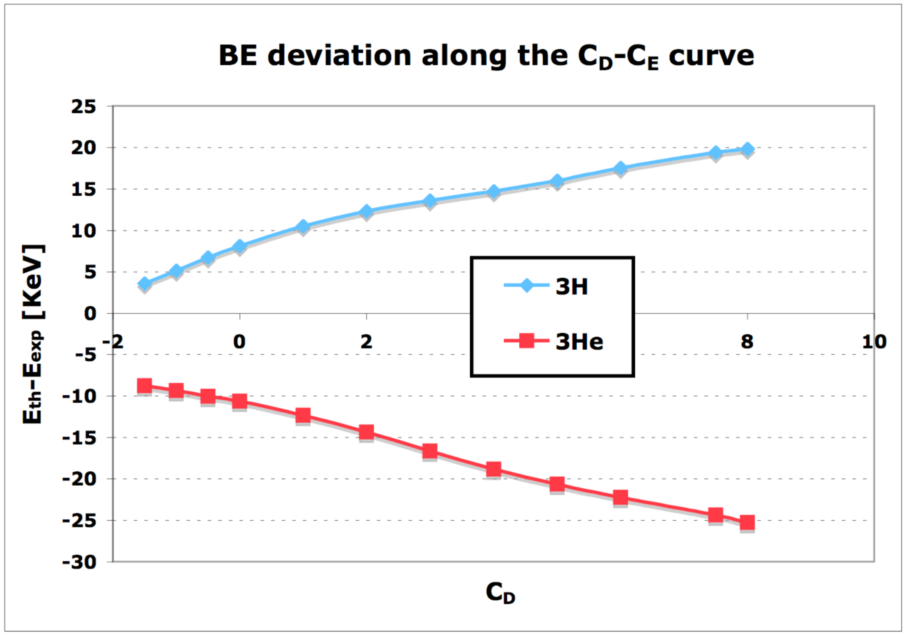
Relating the theoretical point proton radius r_p to the experimental charge radius R_{ch} is very sensitive to the chosen proton charge radius R_p.

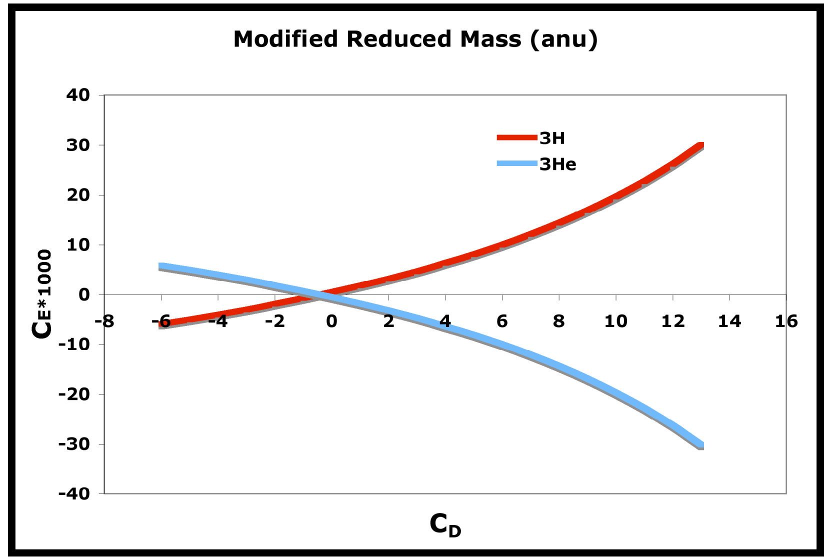
^{4}He Charge Radius



➢ **Point A** gives definitely **better charge radius for ^{4}He**.

Along the C_D-C_E Curve (average)





$$m=(Zm_p+(A-Z)m_n)/A$$

BE deviation is within the accuracy of the kinetic energy for equal mass nucleons $m_n=m_p$.

$$\delta T = T\frac{\delta m}{m} \approx 26keV(\approx 37\times 0.7\times 10^{-3})$$

PRC60, 044304(1999) G. P. Kamuntavicius, P. Navratil, B. R. Barrett, G. Sapragonaite, and R. K. Kalinauskas

T=3/2 channel has been neglected in the calculations.

ab-initio no core with QCD derived nuclear interaction

- Pushing the limits using the full N^3LO,
- We may need codes with explicit m_p & m_n.
- We will need efficient A-body codes.