An extended liquid drop approach

Symmetry energy and neutron skin

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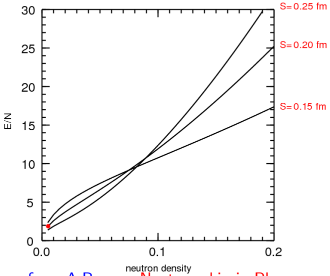
Intro

Issues

1. Symmetry energy
   - exp value $S(\rho = \rho_0) \approx 31 \pm 3$ MeV
     not known very well even for $\rho < \rho_0$
   - relevance: properties of neutron stars
     and cooling
2. Neutron distribution
   - Results depend on parametrization
     of Skyrme forces
   - relevance: fundamental symmetries
     in a nuclear environment
     Atomic pv: weak charge
     as test of standard model at low Q



from A.Brown: Neutron skin in Pb

correlated with density dependence of

$S^{NM}(\rho)$

Nuclear structure and Atomic parity violation

$$H = \frac{G}{2\sqrt{2}} \int d^3r[-N\rho_n(r) + Z(1 - 4\sin^2\theta_W)\rho_p(r)]\psi^\dagger\gamma_5\psi$$

$$< \psi_s|\gamma_5|\psi_p >= iC_Z^{sp}\mathcal{N}[u_1^s(r)u_2^p(r) - u_2^s(r)u_1^p(r)]/r^2$$

factorize m.e.: $H_{if} \sim \frac{G_F}{2\sqrt{2}} C_{if}\mathcal{N}Q_W$ $\quad\gamma = \sqrt{1-(\alpha Z)^2}$

electr overlap for point nucleus $\quad \mathcal{N} = \psi_s^\dagger(0)\gamma_5\psi_p(0) \sim R_p^{2\gamma-2}$

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Nucleus: $Q_w = -Nq_n + Zq_p(1-4\sin^2\theta_W) + \Delta Q_{new}$

q_i : convolution of electr. wf with finite nucleus; expand in $(Z\alpha)^n$

$q_p = 1 - 0.26(Z\alpha)^2 + ...$

$q_n = 1 - \frac{3}{70}(Z\alpha)^2(1+5(\frac{R_n}{R_p})^2) + ...$ (+ corrections to sharp radius)

why large Z? since $H_{pv} \sim Z^3$ enhancement (+rel. corr)

aim: for Ra (Z=88), for 0.1% measurement (to go after ΔQ_{new})

need: $R_p \approx 1\%$, $(R_n - R_p)/R_p \approx 25\%$ (at present no direct exp info)

Let us see whether we can meet these requirements

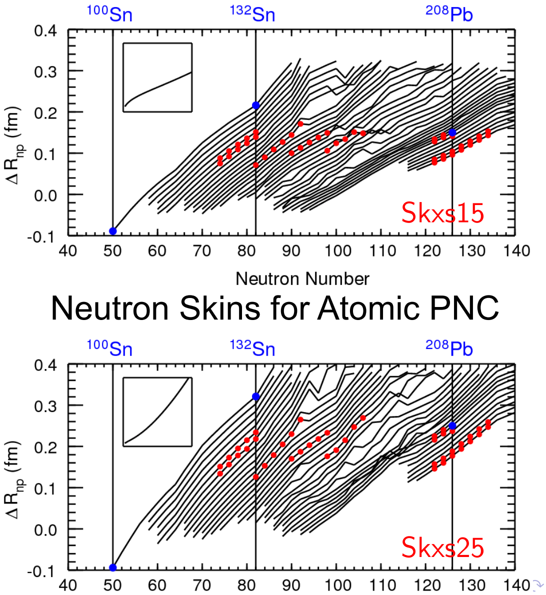
present status of skin

from Brown et al PRC76,2009
based upon Skyrme EDF

depends on Skxs(15,20,25)
idea: PREX@Jlab will fix that

Irregularities due to high spin intruder orbitals ($i13/2$)

Does not fit exp R_c very well (except ^{208}Pb)



our choice

- No microscopic approach available yet that works for small A and n.m.
- Mean field does well for isoscalar properties but isovector parameters are not well determined: ρ, no pions,....

  EFT: pions, but no ρ
- Need simple model to correlate data and extrapolate:

  use minimally extended LDM approach

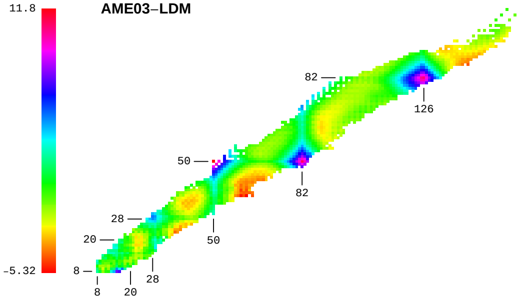
content

- extended LDM;
  surface symmetry energy
  shell corrections
- symmetry energy for $A \to \infty$
- charge radii
- neutron skin
- atomic parity violation: nuclear corrections to weak charge measurements

Conventional Bethe-von Weizsäcker (liquid drop) formula

$$E_A = -a_B A + a_{surf} A^{2/3} + S_{vol}(N-Z)^2/A + a_C \frac{Z^2}{A^{1/3}} + E_{pair}$$

- Incomplete form of symmetry energy
  Need to introduce volume and surface symmetry energy
- Coulomb need to be refined ($R_c(N,Z) \neq r_0 A^{1/3}$)
- shell corrections need to be added



Extended Liquid Drop Model

- Improved BW: $E_{sym} = E(vol, surf)$

  Decompose asymmetry $N - Z = N_s - Z_s + N_v - Z_v$ surface+ volume

  $$E^A_{vol} = a_B A + S_{vol}\frac{(N_v - Z_v)^2}{A}$$
  $$E^A_{surf} = E^0_{surf} + S_{surf}(N_s - Z_s)^2/A^{2/3}$$

  minimize under fixed $N - Z$:

  $$\frac{N_s - Z_s}{N - Z} = \frac{1}{1 + y^{-1}A^{1/3}} \qquad y \equiv S_v/S_S$$

  $$E_A = -a_B A + a_{surf} A^{2/3} + \frac{S_v}{1 + yA^{-1/3}}(N - Z)^2/A + ...$$

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  $$E_A = -a_B A + a_{surf}A^{2/3} + \frac{S_v}{1 + yA^{-1/3}}(N - Z)^2/A + ...$$

- LDM yields also relation between skin and S_s, S_v $(N_s - Z_s \ll A)$

  $$\frac{N}{N_v} = (\frac{R_n}{R_0})^3 \rightarrow \frac{R_n - R_0}{R_0} \approx \frac{N_s}{3N}, \qquad \frac{R_p - R_0}{R_0} \approx \frac{Z_s}{3Z},$$

  $$\frac{R_n - R_p}{R} = \frac{A(N_s - Z_s)}{6NZ} \approx \frac{A}{6NZ}\frac{N - Z - a_c Z A^{2/3}/S_v}{1 + A^{1/3}/y}$$

  Coulomb term: for $N = Z$ $R_p > R_n$

Danielewicz, NPA 727(2003)233; Steiner et al, Phys. Rep. 411,325
differ in the choice of condition $N_s + Z_s = 0$, or $Z_s = 0$

shell corrections

several methods have been proposed, e.g. in microscopic mass model of Duflo-Zuker rms dev ≈ 500 keV, 14-28 parameters

simplest idea: count number of valence particles (holes) with respect to closed shells

↘ $n_v, \ z_v$

$$E_{sh}(N,Z) = a_1(n_v + z_v) + a_2(n_v + z_v)^2 + a_3 n_v . z_v + \ldots$$

however, violates Pauli, midshell cusp

improve: monopole force in single j-shell with degeneracy $D_j = 2j + 1$

$$E_{pair}(n_v) = \frac{g}{D} n_v (D - n_v + 2) \equiv g' n_v . \bar{n}_v + g n_v$$

seniority $v = 0$

$\bar{n}_v \equiv D - n_v$ =number of holes

↘ absorb in core ($g n_v$)

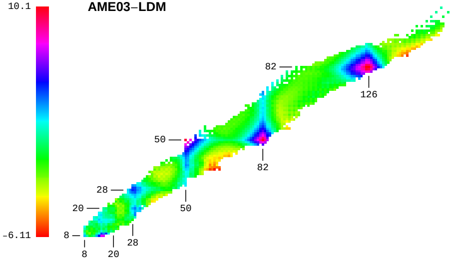
$$E_{\rm shell}(N,Z) = a_1 S_2 + a_2 (S_2)^2 + a_3 S_3 + a_{\rm np} S_{\rm np}$$

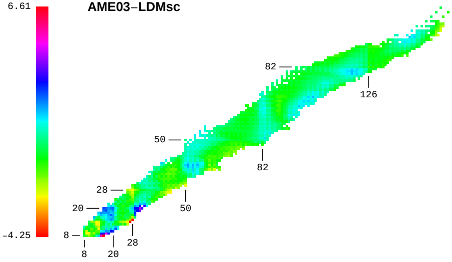
$$\begin{aligned} S_2 &= \frac{n_{\rm v}\bar{n}_{\rm v}}{D_n} + \frac{z_{\rm v}\bar{z}_{\rm v}}{D_z}, \\ S_3 &= \frac{n_{\rm v}\bar{n}_{\rm v}(n_{\rm v} - \bar{n}_{\rm v})}{D_n} + \frac{z_{\rm v}\bar{z}_{\rm v}(z_{\rm v} - \bar{z}_{\rm v})}{D_z}, \\ S_{\rm np} &= \frac{n_{\rm v}\bar{n}_{\rm v} z_{\rm v}\bar{z}_{\rm v}}{D_n D_z}, \end{aligned}$$

with $\bar{n}_{\rm v} \equiv D_n - n_{\rm v}$

Magic numbers: 6, 14, 28, 50, 82, 126, 184

similar to terms in microscopic mass formula of Duflo and Zuker
they refer to S_3 as "monopole drift" (changes sign midshell)



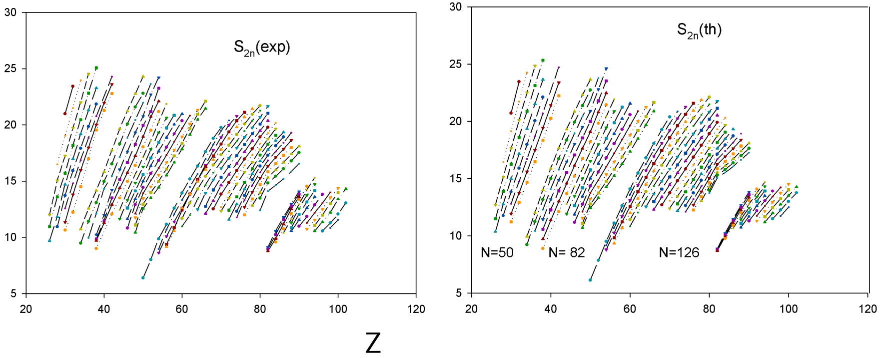


rms deviation 2.4 MeV

0.8 MeV (mostly due to light A)

Illustration: neutron separation energies

Often convenient to consider relative quantities,
e.g. neutron separation energies $S_{2n} = E(N,Z) - E(N-2,Z)$ vs Z



also δ_{2n} is reproduced rather well (talk of Paul Heenen)
relevance: extrapolation, e.g. is $^{70}_{20}Ca_{50}$ stable??

LDM: $E \approx -a_B A + a_{surf} A^{2/3} + \frac{(N-Z)^2}{A} S_A + a_C \frac{Z(Z-1)}{A^{1/3}} + E_{pair} + E_{shell}$

$\downarrow$

$\frac{S_v}{1+yA^{-1/3}}, \quad y = S_v/S_s$

aim: determine S_v, y in isolation of other LDM parameters

Consider isovector chemical pot. $\mu_a = \frac{1}{2}(\mu_n - \mu_p) = \frac{1}{2}(\frac{dE}{dN} - \frac{dE}{dZ})$
$= \frac{1}{4}[B(N-1,Z) - B(N,Z-1) + B(N,Z+1) + B(N+1,Z)]$

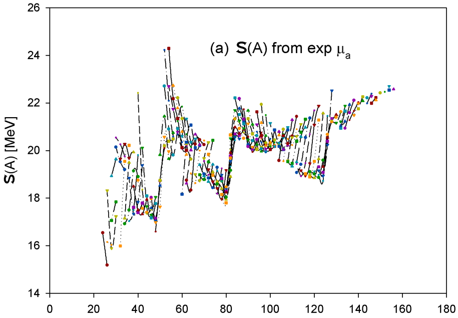
Symmetrize removal and addition

from $E(N,Z)$: $\quad \mu_a(N,Z) = \frac{N-Z}{A} S_A + a_c \frac{Z}{A^{1/3}} + \delta E_{shell}$

invert $(N \neq Z)$ $S_A = \frac{A}{N-Z}(\mu_a - \delta E_C - \delta E_{shell})$ independent of a_B, a_{surf}

Symmetry energy from exp n-p sep energies

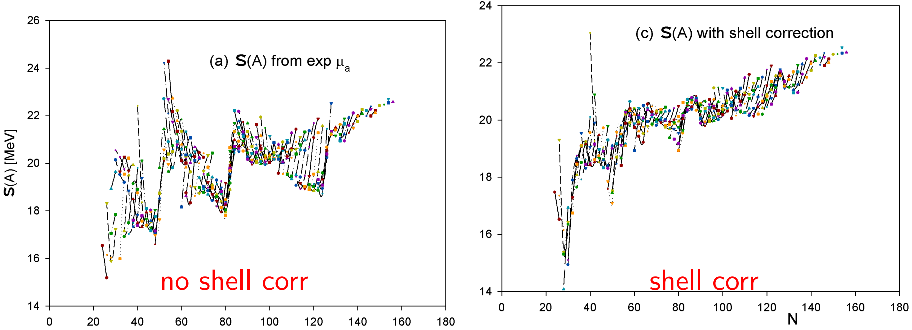
$$S_A = \frac{A}{N-Z}(\mu_a + \delta E_C)$$



Note: the regularity of shell effects (up/down sloping)

Symmetry energy from exp n-p sep energies

$$S_A = \frac{A}{N-Z}(\mu_a + \delta E_C) \qquad S_A = \frac{A}{N-Z}(\mu_a + \delta E_C + \delta E_{shell})$$



Note: the regularity of shell effects (up/down sloping)

particle-particle: $E_{sh} \sim n_v + z_v - [(n_v+1) + (z_v-1)] = 0$

particle-hole (or h-p) : $E_{sh} \sim \pm(n_v - z_v - [(n_z+1) - (z_v-1)]) = \pm 2$

problem at $N = 40$, $Z = 38$ (midshell in 28-50)

$S_A(N,Z) = \frac{S_v}{1+yA^{-1/3}}$, $\quad y = S_v/S_s$

fit in isolation of other parameters

Leads to 2-par fit: $S_A^{-1} = \frac{1+yA^{-1/3}}{S_v}$

extrapolation to $A \to \infty$

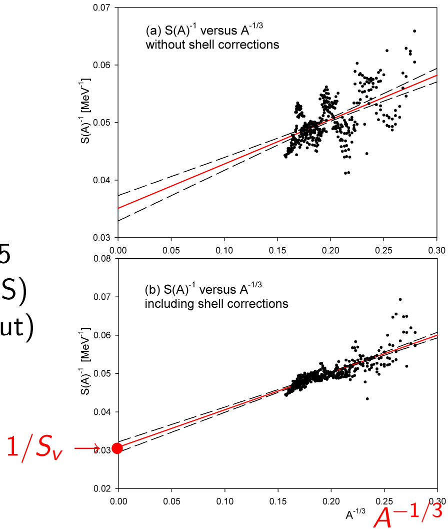
fit values S_v= 32.5 MeV $\quad y = 2.95$

compare with Danielewicz (using IAS)

$S_v = 29 \pm 2$, $\quad y = 2.4 \pm 0.4$ (without)

$S_v = 32.8 \quad$ y=2.8 with shell corr

Note correlation between S_v and y



Symmetry energy $S(\rho)$

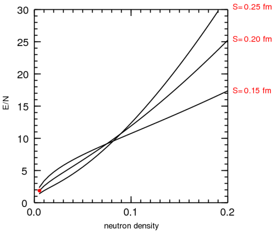
Can be converted to nuclear matter $S(\rho)$:
$S(\rho_0) \equiv S_v$ and S_s related to some $S(\rho < \rho_0)$

Using Thomas-Fermi: $E_a = \frac{\mu_a^2}{4} \int dr \frac{\rho(r)}{S}$
$S_v/S_s = \frac{3}{r_0} \int dr \frac{\rho(r)}{\rho_0} (\frac{S_v}{S(\rho)} - 1)$

simplest case: take $S(\rho) = S_v.(\rho/\rho_0)^\gamma$

note $\gamma = 1 \rightarrow S(\rho) = \text{constant} \rightarrow S_s = \infty$

we find $\gamma \approx 0.7 \pm 0.1$
Danielewicz $\gamma = 0.65 \pm 0.1$ soft EOS



radii

express in terms of iso-scalar/vector

$$R_i(N,Z) = R_0(N,Z) \pm \frac{N-Z}{2A} R_1(N,Z) \quad \text{(i=n,p)}$$

$R_{0,1}$ depend only weakly on $N - Z$

mass radius: $R_0 = r_0 A^{1/3} + a A^{-2/3} + c\frac{(N-Z)^2}{A^2}$, in practice $c \approx 0$

isovecor radius $R_1 = b$

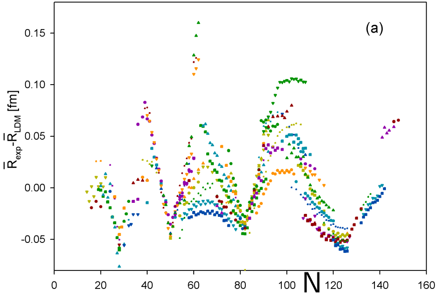
by charge symmetry: $R_p(N,Z)$ determines R_n

however there are Coulomb effects

1. in case of sharp radius: $\frac{\delta R_c}{R_0} = -\frac{a_c}{144 S_v} \frac{A^{8/3}}{NZ(1+y^{-1}A^{1/3})}$
   i.e. for $N = Z$ $\quad R_p > R_n$
2. polarization correction (if R is converted to rms radius): since $\rho_p(r) \neq \rho_n(r)$ in interior

charge radii, shell corrections

Closed shells: Strong binding ↔ small R
near closed shells: decrease of binding, increase of R
midshell: deformation: increase of binding, increase R



rms dev= 0.036 fm

charge radii, shell corrections

Closed shells: Strong binding $\leftrightarrow$ small R

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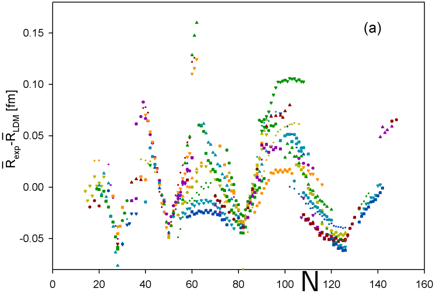

rms dev= 0.036 fm

Try: $\delta R_{shell}/R = a_2(n_v\bar{n}_v + z_v\bar{z}_v) + a_{pn}(n_v\bar{n}_v.z_v\bar{z}_v)$

$R_c(N,Z) = R_0(A) + \frac{N-Z}{A}R_1 + \delta R_c + \delta R_{shell}$

charge radii, shell corrections

Closed shells: Strong binding ↔ small R

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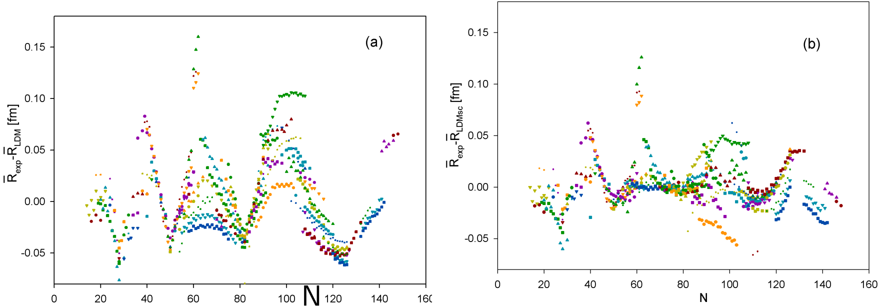


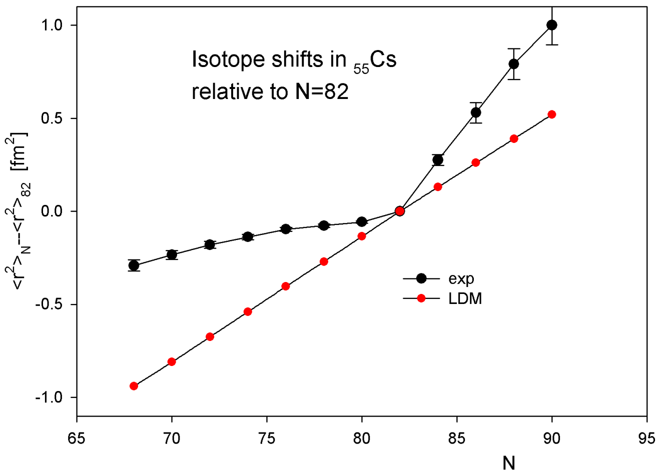
rms dev= 0.036 fm 0.021 fm

Try: $\delta R_{shell}/R = a_2(n_v\bar{n}_v + z_v\bar{z}_v) + a_{pn}(n_v\bar{n}_v.z_v\bar{z}_v)$

$R_c(N,Z) = R_0(A) + \frac{N-Z}{A}R_1 + \delta R_c + \delta R_{shell}$

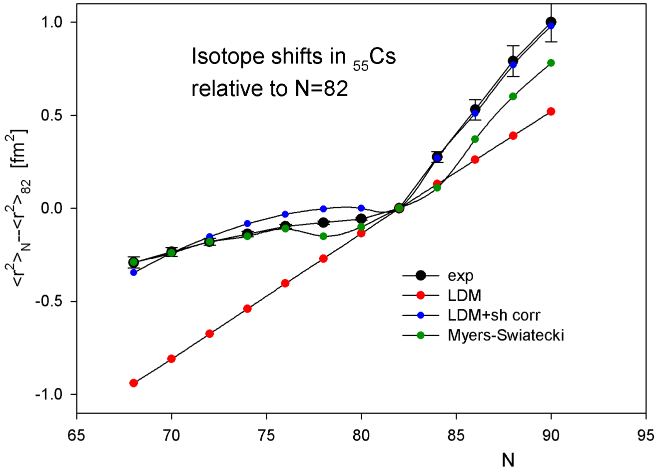
some data (Angeli) have large error bars (mix of stat, syst, theor errors)

isotope shifts for Ce



isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=82}$

isotope shifts for Ce



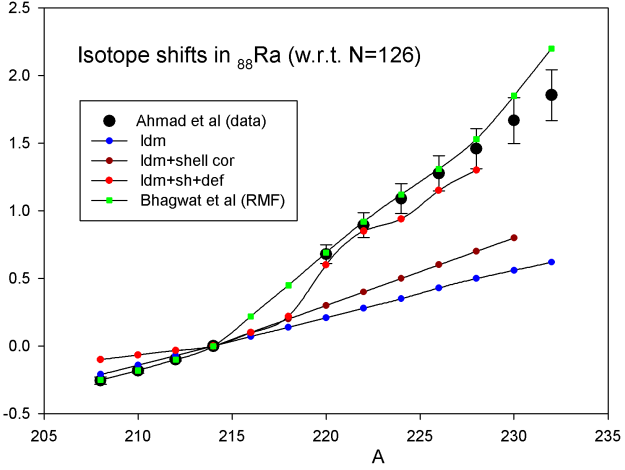
isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=82}$

charge radii, deformation

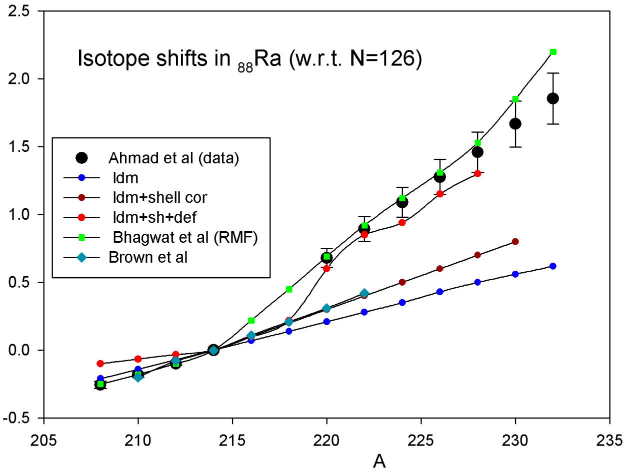
Away from closed shells with say $n_v, \quad z_v$
deformation of the ground state occurs:
increase of binding, increase of radius

$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta))$$
then $R^2 = R^2_{spher}(1 + \frac{5}{4\pi}\beta_2^2 + ..)$

Simplest: take β_2 from exp BE(2) or Q-moments
We are working on a dynamic approach using a collective H to describe excited states.


Isotope shifts in $_{88}$Ra (w.r.t. N=126)
Ahmad et al (data)
ldm
ldm+shell cor
ldm+sh+def
Bhagwat et al (RMF)
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
205
210
215
220
225
230
235
A

isotope shifts for Ra



Deformation effect is substantial

neutron skin

options:

1. from isovector term in R: $\Delta R = \frac{N-Z}{A} R_1 + \delta R_C$
   provides global trend
   $\downarrow$
   in ^{208}Pb reduction of skin by 30%

neutron skin

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1. from isovector term in R: $\Delta R = \frac{N-Z}{A} R_1 + \delta R_C$
   provides global trend
   $\downarrow$
   in ^{208}Pb reduction of skin by 30%

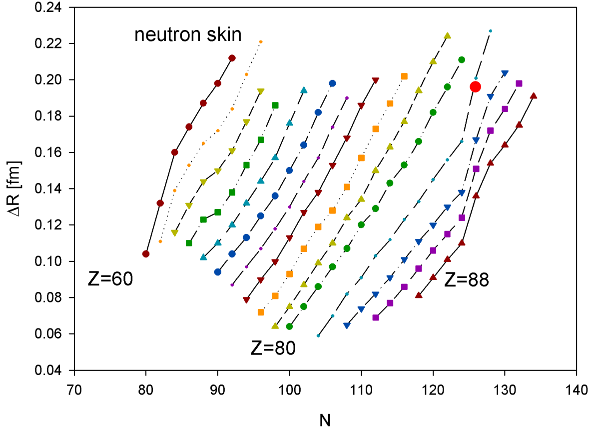
2. from exp μ_a
   (i) $\frac{R_n - R_p}{R} = \frac{A(N_S - Z_S)}{6NZ} \approx \frac{A}{6NZ} \frac{N - Z - a_c Z A^{2/3}/S_v}{1 + A^{1/3}/y}$

   (ii) $\mu_a(N, Z) = \frac{2(N-Z)}{A} \frac{S_v}{1 + yA^{-1/3}} - \frac{5a_c}{6} \frac{Z}{A^{1/3}}$

   from (i)+(ii)
   $\frac{\Delta \bar{R}}{\bar{R}_0} = \frac{\mu_{\mathrm{a}}}{12 S_{\mathrm{s}}} \frac{A^{5/3}}{NZ} + \frac{5 a_{\mathrm{c}}}{72 S_{\mathrm{s}}} \frac{A^{4/3}}{N}$

   μ_a from exp, shell effects implicitly included

Results for ΔR



^{208}Pb exp: $\Delta R = 0.20 \pm 0.04 \pm 0.05$ fm (anti-protonic atoms)

present: method(1): 0.19 fm; method(2): 0.20 fm

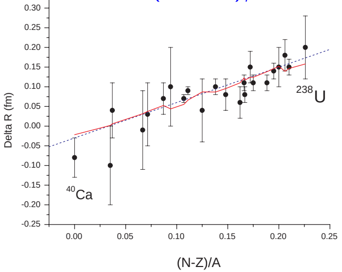
Brown: 0.15- 0.25 fm, Piekarewicz 0.22 fm

$\Delta R(^{222}\text{Ra-}^{210}\text{Ra})$ present 0.12 fm Brown 0.084 fm

neutron skin $\Delta R = R_n - R_p$

- present info on skin

$- - -$ $0.09(N-Z)/A - 0.03$



anti-protonic atoms: model dependent
(Brown et al, PRC76,034305)
e.g. ^{208}Pb: $\Delta R = 0.20 \pm 0.04 \pm 0.05$ fm
also restricted to stable nuclei

- Can one rely on calculations of R_p and ΔR??
  ^{208}Pb: predictions vary $\Delta R = 0.10...0.30$ fm
- Can one determine neutron skin in atomic PNC?
  - PREX@Jlab : pv electon scattering on Pb
  - isotopic chain measurements?

use isotopic chain

Two strategies

- measure neutron skin using standard model as a tool

  ratio $\mathcal{R} = \frac{E'_{PNC}}{E_{PNC}} = \frac{Q'_W}{Q_W}(\frac{R'_p}{R_p})^{2\gamma-2}$

  $\sim \frac{N'}{N}(\frac{R'_p}{R_p})^{2\gamma-2}(1 + f_n(\frac{R_n}{R_p}) - f_n(\frac{R'_n}{R'_p})) \qquad f(x) \approx (\alpha Z)^2(1+5x^2)$

  compare neutron-rich with neutron-depleted isotope

isotopic chains

use isotopic chain

Two strategies

- measure neutron skin using standard model as a tool

  ratio $\mathcal{R} = \frac{E'_{PNC}}{E_{PNC}} = \frac{Q'_W}{Q_W}(\frac{R'_p}{R_p})^{2\gamma-2}$

  $\sim \frac{N'}{N}(\frac{R'_p}{R_p})^{2\gamma-2}(1 + f_n(\frac{R_n}{R_p}) - f_n(\frac{R'_n}{R'_p})) \quad f(x) \approx (\alpha Z)^2(1 + 5x^2)$

  compare neutron-rich with neutron-depleted isotope

- get rid of skin

  suppose new physics $\tilde{Q}_W = Nh_0 + Zg_p + Ng_n$

  sensitivity to new physics $F = g_p/h_0 = (\frac{\mathcal{R}}{\mathcal{R}_0} - 1)\frac{NN'}{Z\Delta N}$

  Skin uncertainty: $\delta F \sim \frac{NN'}{Z\Delta N}\delta(f_n(x) - f_n(x')) \sim \delta[\frac{R'^2_n}{R'^2_p} - \frac{R^2_n}{R^2_p}]$

  Idea : neutron skins of different N are correlated

Summary

- Extended LDM for masses and radii with simple shell corrections
- Symmetry energy $S_v = 30 \pm 2\text{MeV}$, $S_v/S_s = 3.0 \pm 0.3$
- Radii R_c can be described by similar shell corrections; for a quantitative fit need deformation
- neutron skin predicted from n-p separation energies, no strong shell effects
- apv can be used to determine neutron skin or weak charge