Structure Properties of Hypernuclei with Skyrme force based Energy Density Functional

Shashi K Dhiman

Department of Physics, Himachal Pradesh University, Shimla, India

Outline of talk:

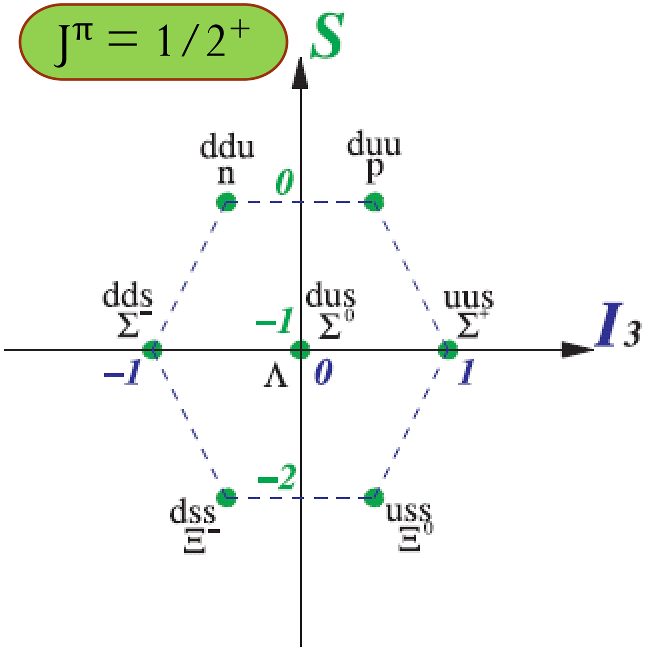
Introduction

Energy density Functional for Hypernuclei

Results (preliminary)

Future studies for Hypernuclei

Hyperons: A probe of nuclear interior



Isospin – Strangeness Plan
"Strangeness" degree of freedom

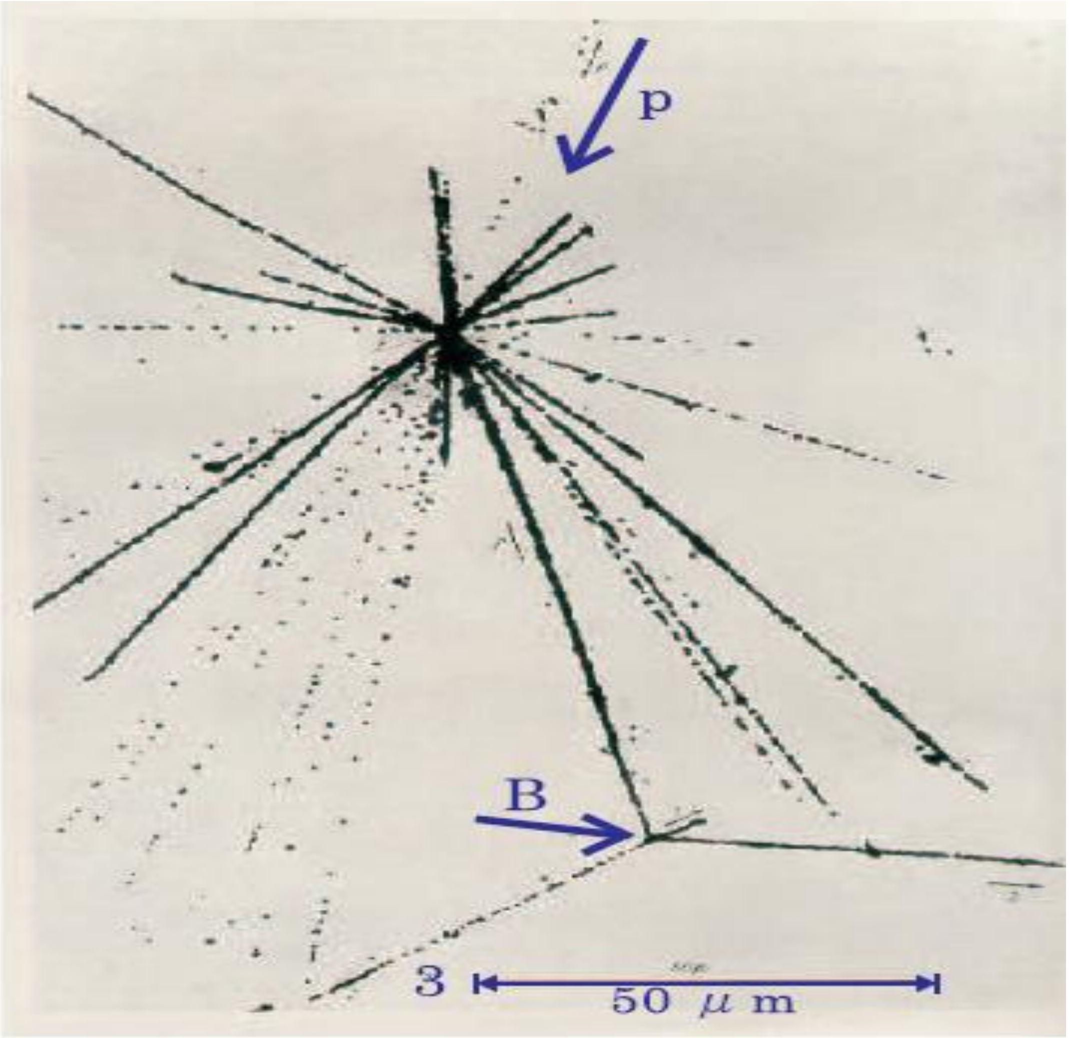
The baryon-baryon interaction provides information where direct or traditional scattering can not.

Hypernucleus is formed by replacing one or two nucleons in the normal nucleus

Hyperon contains at least one strange quark, which makes it different from nucleons in nucleus.
Hyperons bound state have narrow spreading width of less than100keV as compared to nucleons/hole 10 MeV deep.

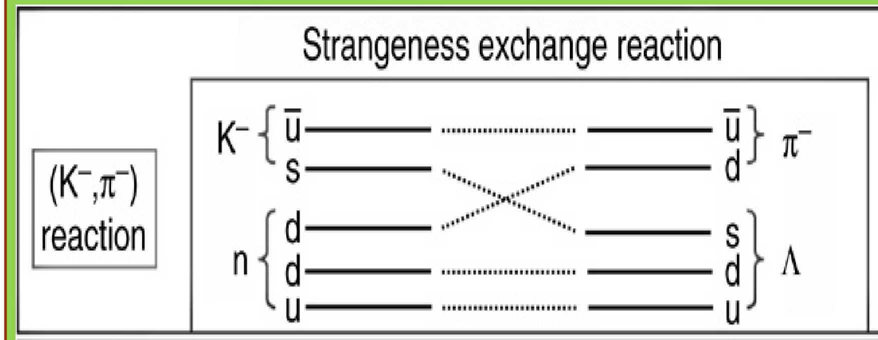
1. **YN interaction is weak than the NN interaction**
2. **YN spin orbit interaction is weak**
3. **a Y with zero isospin can excite only isoscalar p-h modes of the core nucleus**
4. **No exchange term with nucleon is required.**

**First observation of Λ hypernucleus,
Denyez and Pnieski Phil. Mag. 44, (1953) 348**



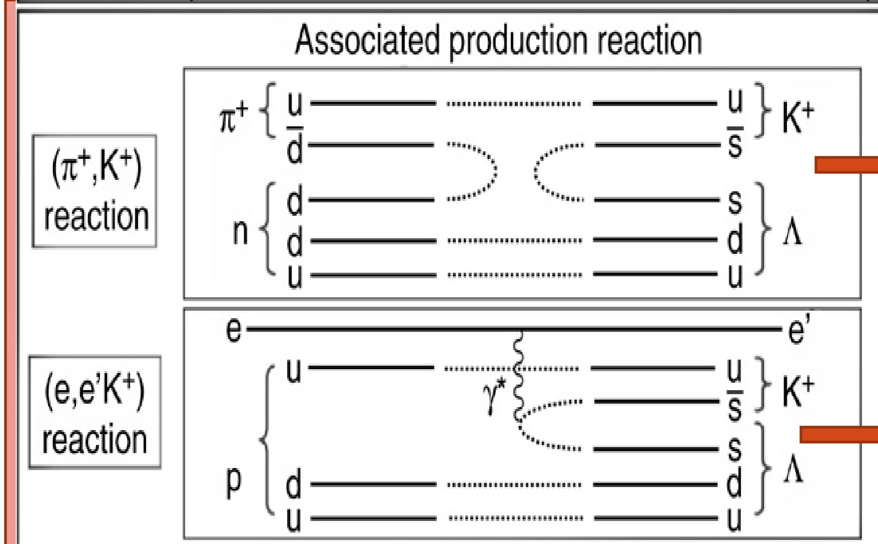

Profesor Marian Danysz (1909-1983)
Profesor Jerzy Pniewski (1913-1989)

Hypernuclei Productions



CERN, BNL, KEK, DaΦne

- Strange quark exchange reactions (K⁻, π⁻) ^{12}B (K⁻, π⁻) $^{12}{}_{\Lambda}$B

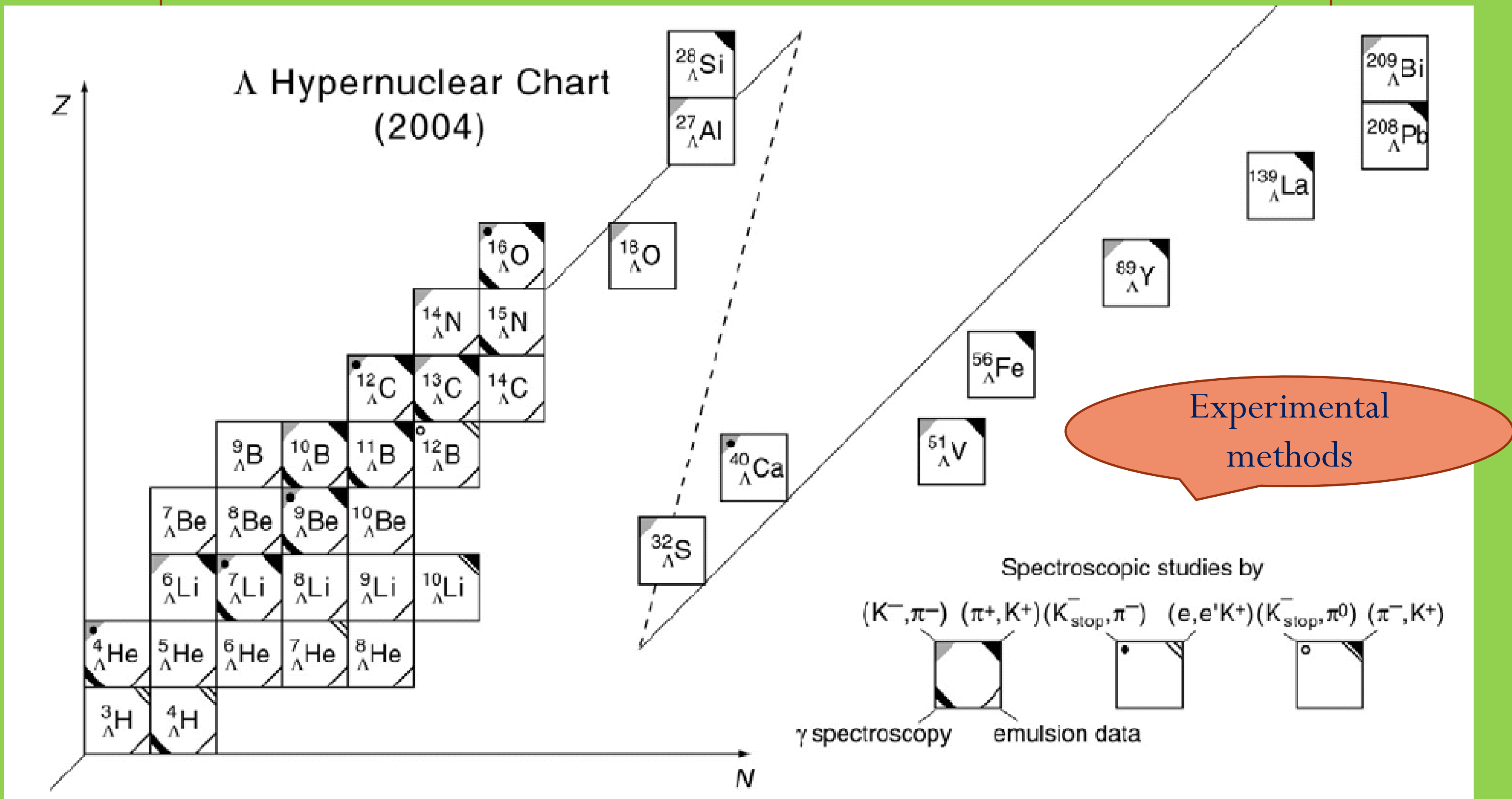


BNL and KEK

Jlab and planned at Mainz

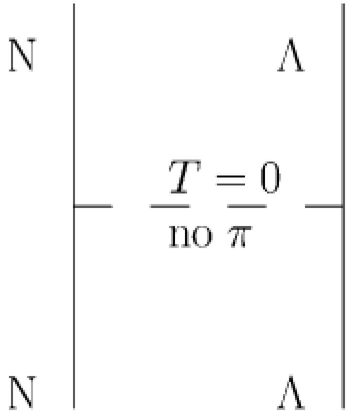
- Electro/photo-production reactions (gamma, K⁺), (e, e' K⁺) ^{12}C(e, e' K⁺) $^{12}{}_{\Lambda}$B

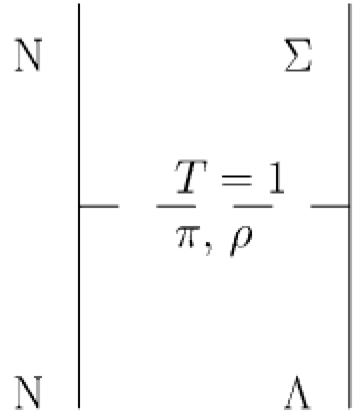
Λ Hypernuclei Chart



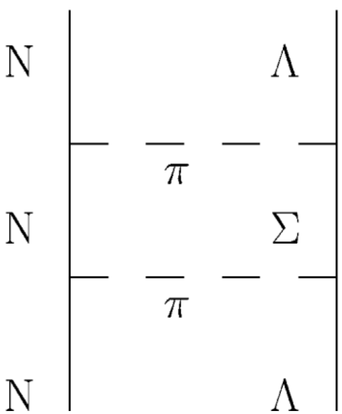
The ΛN (YN) Effective Interaction

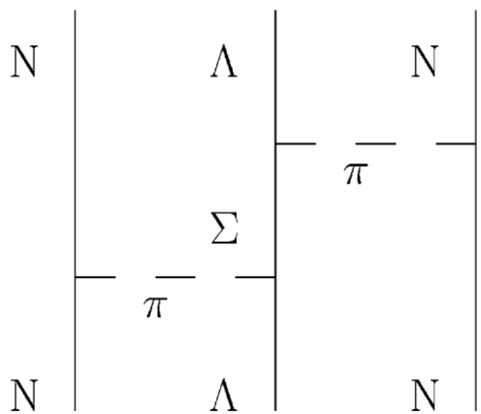
ΛN – ΛN, **no pions**





ΛN – ΣN, **one pions**





ΛN – ΣN, **two pions exchange**

Theoretical Investigations

❖ From QCD point of view hypernuclei lie in the non-perturbative low momentum regime, therefore the lattice QCD calculations should be ideal tool to study the structure of hypernuclei.

❖ The scattering length and effective range for ΛN scattering has been extracted in both QCD and partially-quenched CQD.

Beane, Bedaquw, Parreno, and Savage, Nucl. Phys. A747, 55, 05

❖Hyperon-Nucleon interaction has been calculated with quenched Lattice QCD simulations based on plaquette guage action and the Wilson quark action

Nemura, Ishii, Aoki and Hatsuda ,Phys Letts.B673, 136, 09

Relativistic Mean Field Models have been with empirically adjusted meson-hyperon vertices

Phys Rev C58, 99, Phys Rev C76, 06,

Theoretical Investigations

Shell Model: The experimental information of gamma spectroscopy for hypernuclei, $^{7}_{\Lambda}Li$, $^{9}_{\Lambda}Be$, $^{10,11}B$, $^{12}_{\Lambda}C$ and $^{16}_{\Lambda}O$ with p-shell core nucleus for both Λ and Σ hyperons configurations,

Millener, Lect. Notes Phys. 724: 31, 2007

Quark –Meson Coupling Model: Structure properties has been calculated over the periodic chart (spe's, etc.)

Thomas et al.,Prog. Part. Nucl. Phys. 58, 1-167, 2007

Skyrme Hartree Fock Theory, Rayet. Nucl.Phys A 1981 and subsequently Phys Rev C 55, 2330, 97, Phys Rev C,98 Phys Rev C ,06

Hamiltonian and ΛN Skyrme force

$$H = H_{core\ nucleus} + T_\Lambda + V_{\Lambda N}$$

$^{12}{}_{\Lambda}B = {}^{11}B + \Lambda$

The $V_{\Lambda N}$ interaction can be constructed the Skyrme nucleon-nucleon force as;

$$V_{\Lambda N}(\mathbf{r}_\Lambda, \mathbf{r}_N) = u_0(1 + y_0 P_\sigma)\delta(\mathbf{r}_{N\Lambda}) + \frac{1}{2}u_1(\vec{p}'^2\delta(\mathbf{r}_{N\Lambda}) + \delta(\mathbf{r}_{n\Lambda})\vec{p}^2$$
$$+ u_2\vec{p}'\delta(\mathbf{r}_{N\Lambda}).\vec{p} + iW_0\vec{p}'\delta(\mathbf{r}_{N\Lambda}).(\vec{\sigma}\times\vec{p}).$$

$$\vec{\sigma} = \vec{\sigma}_\Lambda - \vec{\sigma}_N$$

$$P_\sigma = \tfrac{1}{2}(1 + \vec{\sigma}_\Lambda.\vec{\sigma}_N)$$

$$\mathbf{r}_{\Lambda N} = \mathbf{r}_\Lambda - \mathbf{r}_N,\ \vec{p} = (\vec{\nabla}_\Lambda - \vec{\nabla}_N)/2i$$

$$V_{\Lambda NN}(\mathbf{r}_\Lambda, \mathbf{r}_1, \mathbf{r}_2) = \frac{3}{8}u_3(1 + y_3 P_\sigma)\delta(r_\Lambda - r_1)\delta(r_\Lambda - r_2).$$

→ Three-body interactions

Where the Skyrme force parameterization can be obtained self consistently from the G-matrix calculation

$H_{\Lambda N}$ Hamiltonian Density

$$H_{\Lambda N}(r) = \frac{\hbar^2}{2m_\Lambda}\tau_\Lambda + H_0 + H_{eff.} + H_{fin} + H_{den} + H_{s.o.}$$

$$H_0 = u_0(1 + \frac{1}{2}y_0)\rho_N\rho_\Lambda$$

$$H_{eff} = \frac{1}{4}(u_1 + u_2)(\tau_\Lambda\rho_N + \tau_N\rho_\Lambda)$$

$$= \frac{3}{5}(3\pi^2)^{2/3}\frac{1}{4}(u_1 + u_2)\rho_\Lambda\left(\rho_N\rho_\Lambda^{2/3} + \rho_N^{5/3}\right)$$

$$H_{fin} = \frac{1}{4}(3u_1 - u_2)(\nabla\rho_N \cdot \nabla\rho_\Lambda)$$

$$H_{den} = \frac{3}{8}u_3(1 + \frac{1}{2}y_3)\rho_N^{\beta+1}\rho_\Lambda$$

$$H_{s.o.} = \frac{1}{2}W_0^\Lambda(\nabla\rho_N \cdot J_\Lambda + \nabla\rho_\Lambda \cdot J_N)$$

We use the values of $\hbar^2/2m_\Lambda = 17.44054$ MeV fm^2 and $\beta = \frac{1}{3}$

$V_{\Lambda N}$ Parameterizations

- Skyrme parameterizations is determined by reproducing the G-matrix calculation

- The density dependence of G-matrix is originate from ΛN-ΣN coupling, repulsive core singularity and tensor force

- The coupled channel Bethe-Goldstone equation is used to solve the G-matix .

- Then observed data of hypernuclei is reproduced with condition that $V_{\Lambda N}$ = -30MeV at normal nuclear density

- Afterwards the skyrme parameters are determined so as to reproduce the $V^S_\Lambda(p_F)$ in singlet even , $V^T_\Lambda(p_F)$ in triplet even , and $V^e_{\Lambda N}(p_F) = U^S_\Lambda(p_F) + U^T_\Lambda(p_F)$ for u0, u1, u3, and $V^o_{\Lambda N}(p_F)$ for u2,

- Additional the experimental binding energy of $BE(^{13}{}_\Lambda C)$ = 11.69 MeV hypernuclei used for fine tuning of the parameters.

- Julich model for set A, Nijmegen model for set B and Soft core model for set C

$V_{\Lambda N}$ Parameterizations

SET	u_0 (MeV fm^3)	u_1 (MeV fm^5)	u_2 (MeV fm^5)	u_3 (MeV fm$^{3+3\beta}$)	y_0	y_3
A	-476.0	42.0	23.00	1514.10	-0.0452	-0.2800
B	-622.0	116.0	-30.00	1880.30	-0.0172	0.0679
C	-542.5	56.0	8.00	1387.00	-0.1534	0.1074
D	-265.7	97.17	12.83	...	-0.2160	...
E	-176.5	-35.8	44.10	...	...	...

Skyrme Energy Density Functional for Λ Hypernucleus

$$\mathcal{E}_{1\Lambda}^{H} = \mathcal{E}_{NN}(\rho_n, \rho_p, \tau_n, \tau_p, J_n, J_p) + \mathcal{E}_{\Lambda N}(\rho_n, \rho_p, \rho_\Lambda, \tau_\Lambda) + \mathcal{E}_{R}^{\Lambda}(\rho_n, \rho_p, \rho_\Lambda)$$

$$\mathcal{E}_{NN} = \int d^3r H_{NN}(r),$$

$$\mathcal{E}_{\Lambda N} = \int d^3r H_{\Lambda N}(r),$$

$$\rho_q = \sum_{i=1}^{N_q} v_q^i \, |\, \phi_i(r,q)\, |^2$$

$$\tau_q = \sum_{i=1}^{N_q} v_q^i \, |\, \nabla \phi_i(r,q)\, |^2,$$

$$J_q = \sum_{i=1}^{N_q} v_q^i \phi_i^*(r,q)(\nabla \phi_i(r,q) \times \sigma)/i,$$

Skyrme Energy Density Functional for Λ Hypernucleus

$$E_{pair} = -\sum_{q\in p,n} G_q \left[\sum_{\alpha\in q} \sqrt{v_\alpha(1-v_\alpha)} \right]^2$$

$$v_q^2 = \frac{1}{2}\left[1 - \frac{\epsilon_q - \mu_q}{\sqrt{(\epsilon_q-\mu_q)^2 + (\Delta_q)^2}} \right]$$

$$\mathcal{E}_{c.m.} = \frac{< P_{c.m.}^2 >}{2(A-n)m_N + n m_\Lambda}$$

$$< P_{c.m.}^2 > = \sum_\alpha v_\alpha^2 < \alpha\alpha \mid \mathbf{p}^2 \mid \alpha\alpha > - \sum_{\alpha,\beta} v_\alpha v_\beta (v_\alpha v_\beta - u_\alpha u_\beta) < \alpha\beta \mid \mathbf{p}_1 . \mathbf{p}_2 \mid \alpha\beta >$$

$$\mathcal{E}_R^\Lambda = -\frac{1}{2}\int d^3 r \rho_\Lambda (\rho_N^2 + 2\rho_p \rho_n).$$

Solutions of SHF equations by minimization of EDF

$$\left[-\nabla\frac{\hbar^2}{2m_q^*(r)}.\nabla + V_q(r) - iW_q(r).(\nabla\times\sigma)\right]\phi_i(r,q) = \epsilon_q^i\phi_i(r,q)$$

$$V_{\Lambda N}(r) = m_0^\Lambda\rho_N + \frac{3}{5}(3\pi^2)^{2/3}m_1^\Lambda(\rho_N\rho_\Lambda^{2/3} + \rho_N^{5/3}) + m_3^\Lambda\rho_n^{\beta+1}$$

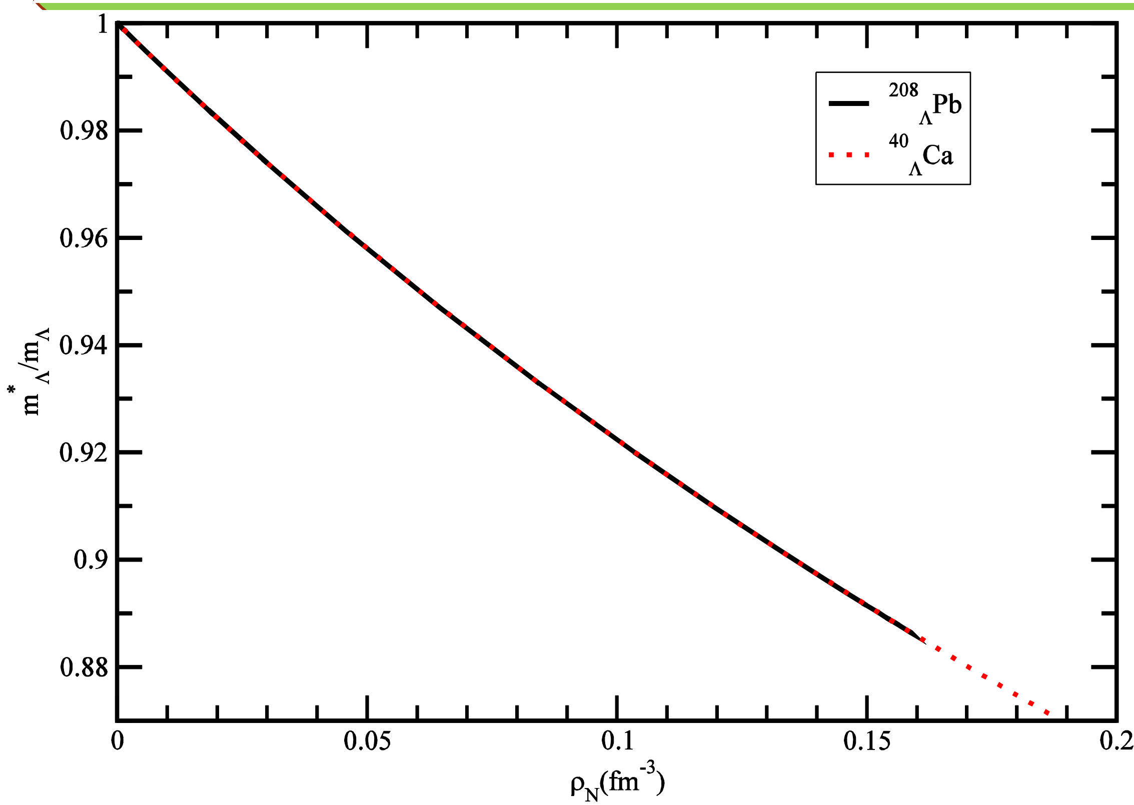
$$m_0 = u_0(1-\tfrac{1}{2}y_0) \text{ and } m_3 = \tfrac{3}{8}u_3(1+\tfrac{1}{2}y_3)$$

The Λ Effective mass

$$\frac{m^*_\Lambda}{m_\Lambda} = [1 + \frac{2m_\Lambda}{\hbar^2} m_1^\Lambda \rho_N]^{-1}$$

$$= 1 - \frac{2m_\Lambda}{\hbar^2} m_1^\Lambda \rho_N + \left(\frac{2m_\Lambda}{\hbar^2} m_1^\Lambda\right)^2 \rho_N^2 - \left(\frac{2m_\Lambda}{\hbar^2} m_1^\Lambda\right)^3 \rho_N^3 + \ldots$$

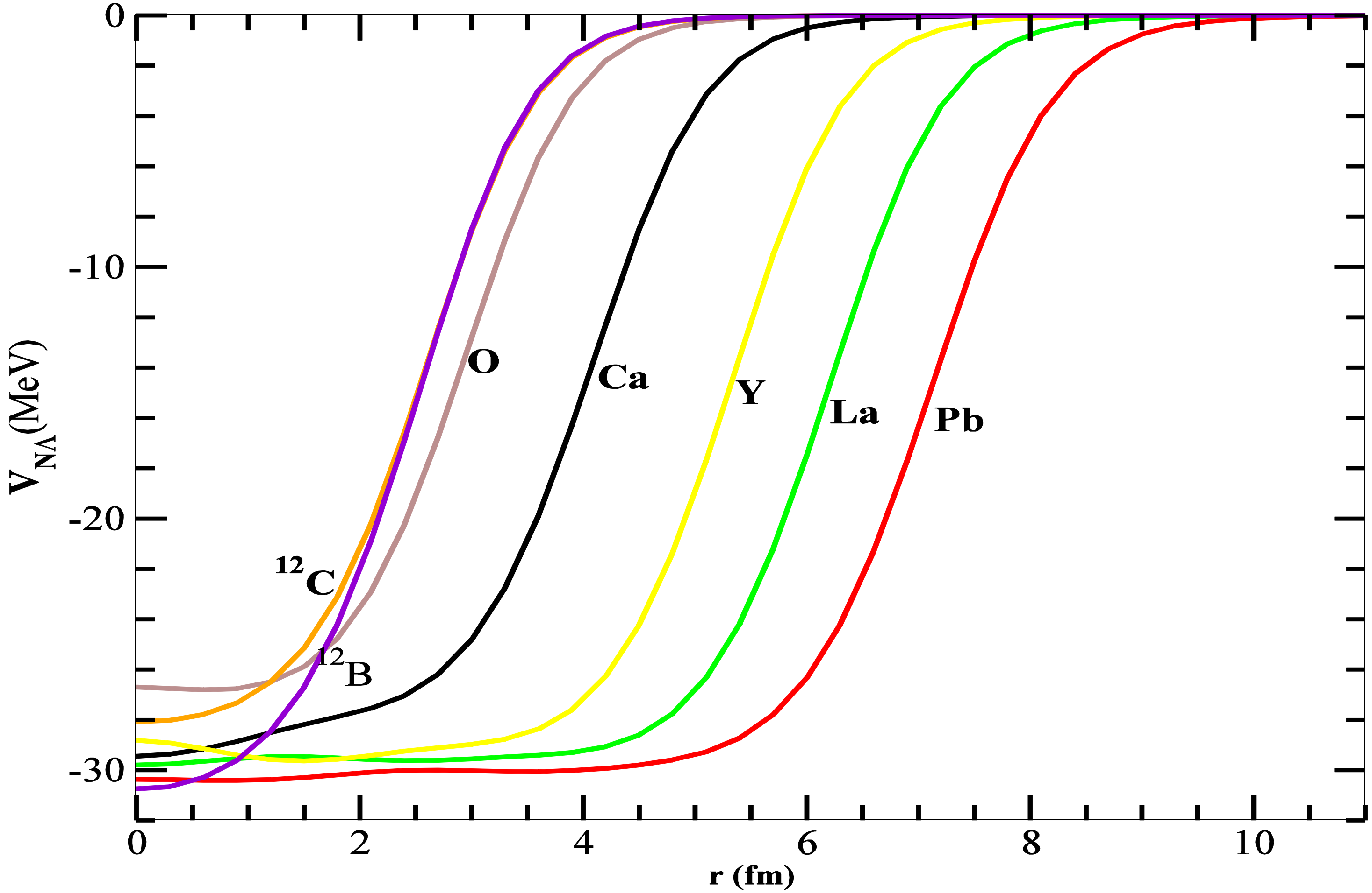
$$m_1^\Lambda = \frac{1}{4}(u_1 + u_2)$$



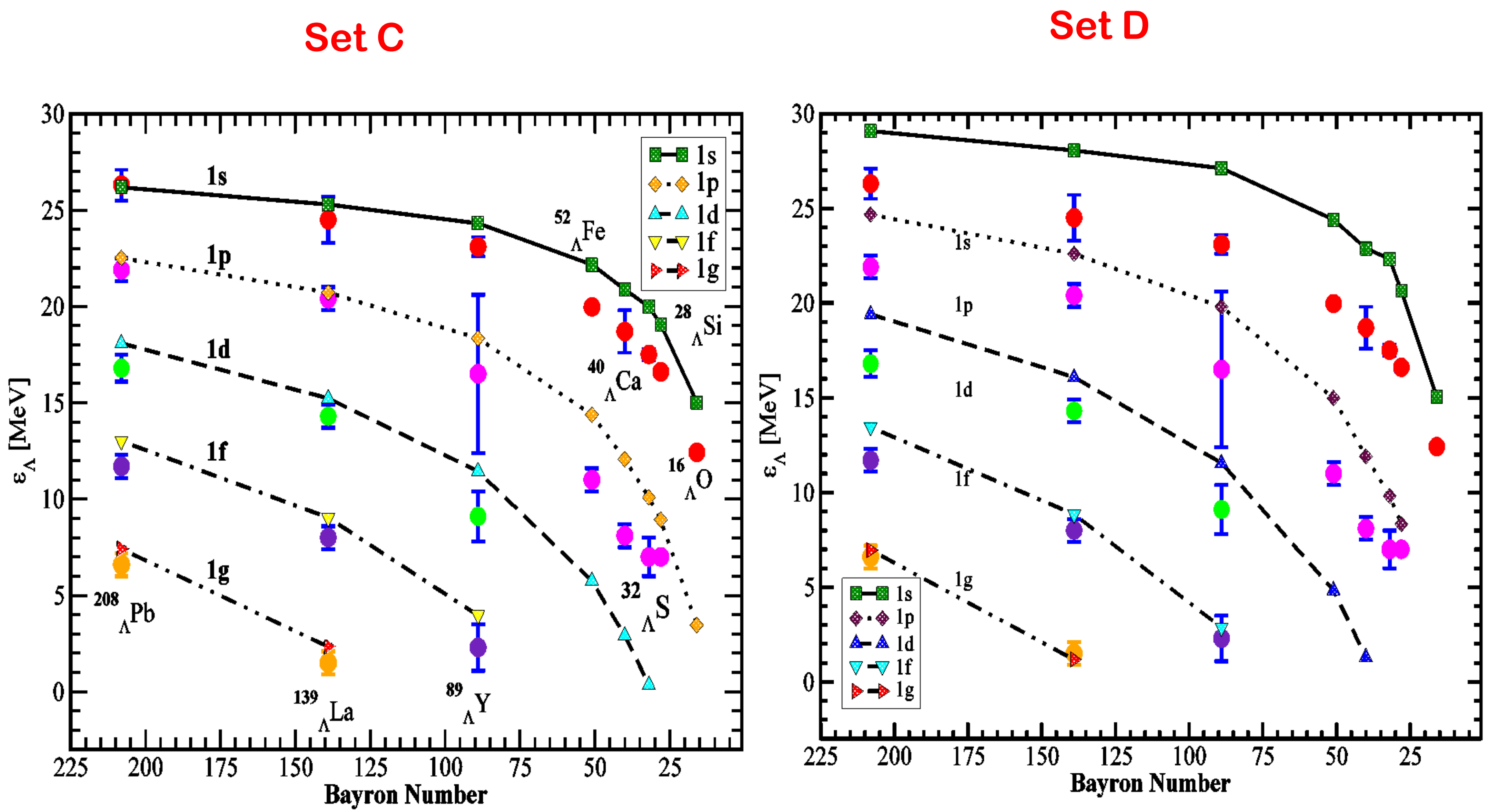
m^*_Λ/m_Λ = 0.885

SHF mean field potential

$$V_{\Lambda N}(r) = m_0^{\Lambda}\rho_N + \frac{3}{5}(3\pi^2)^{2/3} m_1^{\Lambda}(\rho_N \rho_{\Lambda}^{2/3} + \rho_N^{5/3}) + m_3^{\Lambda}\rho_n^{\beta+1}$$

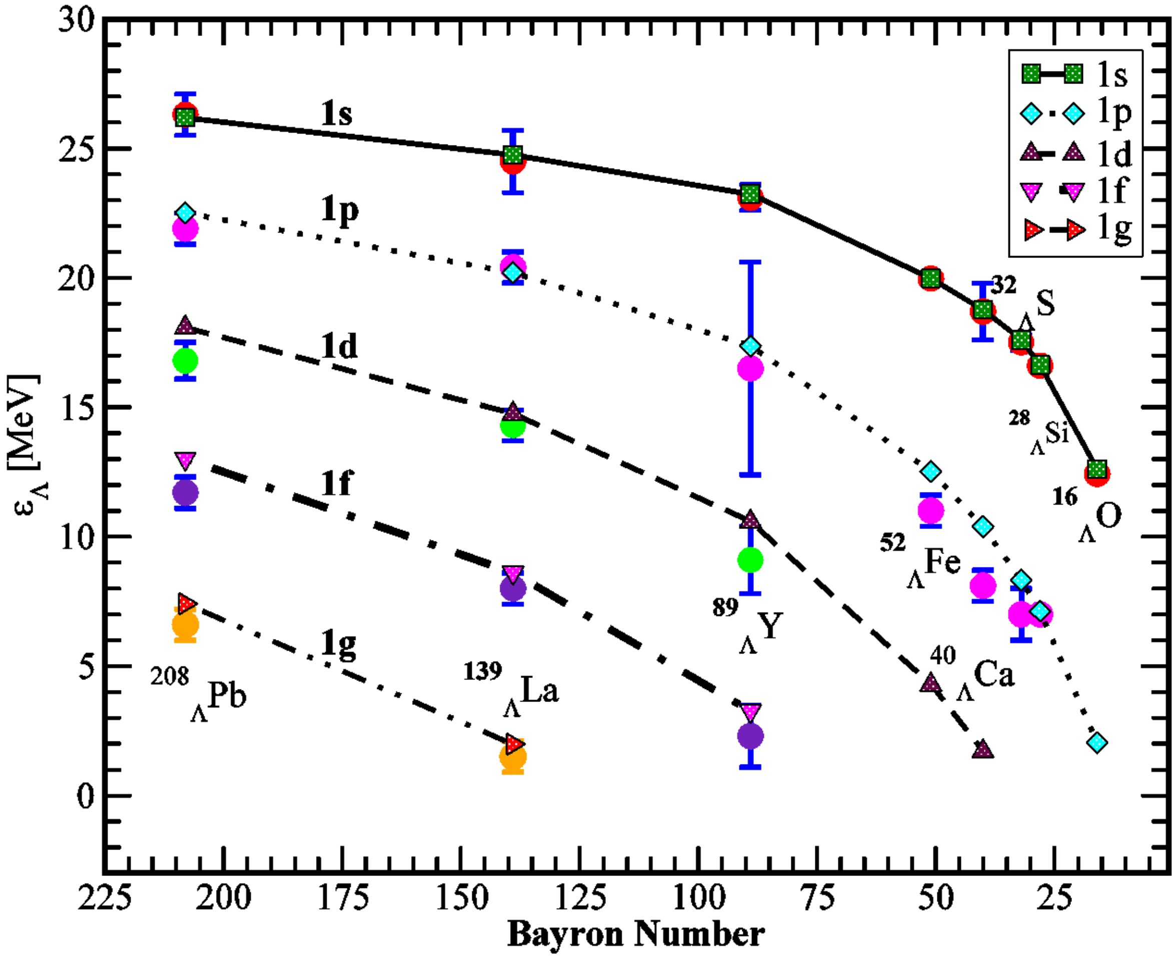


Single particle energies in Hypernuclei



Single particle energies in Hypernuclei

Set C, u_0 parameter of Skyrme force is adjusted to reproduce 1s SPE's.



Separation energies

$S_\Lambda = BE(^{A-1}Z) - BE(^{A}_{\Lambda}Z)$

	S_Λ^{Expt}(MeV)	S_Λ^{Theor} (MeV)	B.E./A (MeV)	$r^\Lambda(s_{1/2})$
$^{28}_{\Lambda}$Si	16.6 ± 0.20 [26]	16.764	8.073	3.16
$^{32}_{\Lambda}$S	17.5 ± 0.30	17.115	8.603	3.27
$^{33}_{\Lambda}$S	17.96 ± 0.00	17.266	8.664	3.29
$^{40}_{\Lambda}$Ca	18.70 ± 1.10	18.686	8.574	3.35
$^{41}_{\Lambda}$Ca	19.24 ± 1.00	18.819	8.585	3.36
$^{51}_{\Lambda}$V	19.97 ± 0.13 [1]	19.814	8.798	3.51
$^{56}_{\Lambda}$Fe	21.00±1.50 [1, 30]	20.209	8.664	3.57
$^{89}_{\Lambda}$Y	23.1 ± 0.50 [1]	23.239	8.732	3.94
$^{139}_{\Lambda}$La	24.5 ± 1.2	24.253	8.384	4.46
$^{208}_{\Lambda}$Pb	26.3 ± 0.80 [1]	25.282	7.889	4.93

Excitation spectra (mirror hypernuclei $^{12}_{\Lambda}B$ and $^{12}_{\Lambda}C$)

TABLE II: The single particle energies calculated with set C parameterizations for ΛN interaction and s-p orbitals energy spacing obtained from $^{12}_{\Lambda}B$ and $^{12}_{\Lambda}C$ hypernuclei excitation spectra compared with recent measurement [22] and shell model calculation [22, 23].

	$^{12}_{\Lambda}B$			$^{12}_{\Lambda}C$		
States	Experiment [22]	EDF	Shell Model [22]	Experiment [22]	EDF	Shell Model [23]
1s	11.70±0.10	11.4135	...	10.76	10.9578	...
1p	0.50±0.10	0.3738	...	0.10	0.3833	...
Δs-p	11.20±0.10	11.0407	11.06	10.66±0.10	10.5745	10.60
$\Delta sp(^{12}_{\Lambda}B)$-$\Delta sp(^{12}_{\Lambda}C)$	0.50±0.20	0.4667	0.46	0.50±0.20	0.4667	0.46

Excitation spectra $^{16}{}_{\Lambda}O$

Hypernucleus	J^{π}	p-h state	Expt.[31] [MeV]	SkHF [MeV]
$^{16}_{\Lambda}O$	0^-	$(1s_{1/2})_{\Lambda}, (1p_{1/2})_n^{-1}$	-0.26	0.0
	1_1^-	$(1s_{1/2})_{\Lambda}, (1p_{1/2})_n^{-1}$	0.0	0.0
	1_2^-	$(1s_{1/2})_{\Lambda}, (1p_{3/2})_n^{-1}$	6.532	6.398
	2_1^-	$(1s_{1/2})_{\Lambda}, (1p_{3/2})_n^{-1}$	6.784	6.398
	0_1^+	$(1p_{1/2})_{\Lambda}, (1p_{1/2})_n^{-1}$	10.570	12.068
	1_1^+	$(1p_{3/2})_{\Lambda}, (1p_{1/2})_n^{-1}$	-	12.930
	2_1^+	$(1p_{3/2})_{\Lambda}, (1p_{1/2})_n^{-1}$	10.610	12.930
	2_2^+	$(1p_{3/2})_{\Lambda}, (1p_{3/2})_n^{-1}$	16.590	17.309
	2_3^+	$(1p_{3/2})_{\Lambda}, (1p_{3/2})_n^{-1}$	16.590	17.309
	0_2^+	$(1p_{3/2})_{\Lambda}, (1p_{3/2})_n^{-1}$	17.140	17.309

Excitation spectra $^{40}_{\Lambda}Ca$

J^π	p-h state	Expt. [MeV]	SkHF [MeV]	B.E./B [MeV]	r_p^Λ [fm]	$r_h^{neutron}$ [fm]
1_1^+	$(1s_{1/2})_\Lambda, (1d_{3/2})_n^{-1}$	0.22	0.00	8.4930	3.39	4.18
2_1^+	$(1s_{1/2})_\Lambda, (1d_{3/2})_n^{-1}$	0.00	0.00			
0^+	$(1s_{1/2})_\Lambda, (2s_{1/2})_n^{-1}$	2.92	2.2388	8.4416	3.39	4.52
1_1^+	$(1s_{1/2})_\Lambda, (2s_{1/2})_n^{-1}$	3.44	2.2388			
3^+	$(1s_{1/2})_\Lambda, (1d_{5/2})_n^{-1}$	6.01	7.1412	7.2338	3.28	4.02
2_2^+	$(1s_{1/2})_\Lambda, (1d_{5/2})_n^{-1}$	6.19	7.1412			
2_1^-	$(1p_{3/2})_\Lambda, (1d_{3/2})_n^{-1}$	7.98	8.2804	8.114	3.37	4.17
3^-	$(1p_{3/2})_\Lambda, (1d_{3/2})_n^{-1}$	8.55	8.2804			
1_1^-	$(1p_{3/2})_\Lambda, (1d_{3/2})_n^{-1}$	8.78	8.2804			
0^-	$(1p_{3/2})_\Lambda, (1d_{3/2})_n^{-1}$	9.54	8.2804			
1_2^-	$(1p_{1/2})_\Lambda, (1d_{3/2})_n^{-1}$	9.27	8.2804			
2_2^-	$(1p_{1/2})_\Lambda, (1d_{3/2})_n^{-1}$	9.53	8.2804			

$^{208}_{\Lambda}Pb$ Excitation spectra

Neutron hole - $1i_{13/2}$

Λ-state	Expt1. [MeV]	Expt2 [MeV]	SkHF [MeV]	r^{Λ} [fm]
$1s_{1/2}$	-3.0	1.6	1.6	4.65
$1p_{3/2}$	4.6	6.8	5.4133	5.20
$1d_{5/2}$	8.24	12.84	6.3851	5.62
$1f_{7/2}$	13.69	18.29	11.2422	5.98
$1g_{9/2}$	17.49	17.49	16.8096	6.31

Double Λ force

$$V_{\Lambda\Lambda}(\mathbf{r}_{\Lambda\Lambda}) = \lambda_0 \delta(\mathbf{r}_{\Lambda\Lambda}) + \frac{1}{2}\lambda_1 \left(\vec{p}'^2 \delta(\mathbf{r}_{\Lambda\Lambda}) + \delta(\mathbf{r}_{\Lambda\Lambda}) \vec{p}^2 \right)$$

$$\mathbf{r}_{\Lambda\Lambda} = \mathbf{r}_{\Lambda_1} - \mathbf{r}_{\Lambda_2}$$

$$V_{\Lambda\Lambda N}(\mathbf{r}_{\Lambda_1}, \mathbf{r}_{\Lambda_2}, \mathbf{r}_N) = \lambda_3 \delta(\mathbf{r}_{\Lambda_1} - \mathbf{r}_N) \rho_N^{\beta} \delta(\mathbf{r}_{\Lambda_2} - \mathbf{r}_N)$$

Energy density Functional for ΛΛ hypernuclei

$$\mathcal{E}_{2\Lambda}^{H} = \mathcal{E}_{1\Lambda}^{H} + \mathcal{E}_{\Lambda\Lambda}$$

$$\mathcal{E}_{\Lambda\Lambda} = \int d^3 r H_{\Lambda\Lambda}(r),$$

$H_{\Lambda\Lambda}$ Hamiltonian density

$$H_{\Lambda\Lambda} = n_0^{\Lambda}\rho_{\Lambda}^2 + n_1^{\Lambda}\rho_{\Lambda}\tau_{\Lambda} + n_2^{\Lambda}\rho_{\Lambda}\nabla^2\rho_{\Lambda} + \frac{1}{4}n_3^{\Lambda}\rho_{\Lambda}^2\rho_N^{\beta}$$

$$n_0^{\Lambda} = \frac{1}{4}\lambda_0,\ n_1^{\Lambda} = \frac{1}{8}(\lambda_1 - 3\lambda_2),\ n_2^{\Lambda} = \frac{3}{32}(\lambda_2 - \lambda_1).$$

$$n_3^{\Lambda} = \frac{1}{4}\lambda_3$$

$$V_{\Lambda\Lambda}(r) = V_{\Lambda N}(r) + n_0^{\Lambda}\rho_{\Lambda} + \frac{3}{5}(3\pi^2)^{2/3}n_1^{\Lambda}\rho_{\Lambda}^{5/3} + \frac{1}{2}n_3^{\Lambda}\rho_{\Lambda}\rho_N^{\beta}$$

Effective mass acquire additional terms as,

$$\frac{m_\Lambda^*}{m_\Lambda} = \left[\left(\frac{m_\Lambda}{m_\Lambda^*}\right)_{s\Lambda} + n_1^\Lambda \rho_\Lambda\right]^{-1}$$

$$= \left(\frac{m_\Lambda}{m_\Lambda^*}\right)_{s\Lambda} - n_1^\Lambda \rho_\Lambda \left(\frac{m_\Lambda}{m_\Lambda^*}\right)_{s\Lambda}^2 + \left(n_1^\Lambda \rho_\Lambda\right)^2 \left(\frac{m_\Lambda}{m_\Lambda^*}\right)_{s\Lambda}^3 - \ldots,$$

Conclusion and Outlook

- Theoretical calculations for Λ hypernuclei across the periodic table are shown with in Skyrme H F theory by using the Skyrme parameterizations from the literature.

- The description of single particle energies require the Lambda Nucleon interaction have density dependence, which arise from the zero range three body interaction in Skyrme HF calculation.

- Plan to parameterize Skyrme ΛN force by the fitting the recent data of single particle energies of hypernuclei over the periodic table.

Suggested Reading:

Hashimoto &Tamura, Progress in Particle and Nuclear Physics 57, 564, 06

Thanks