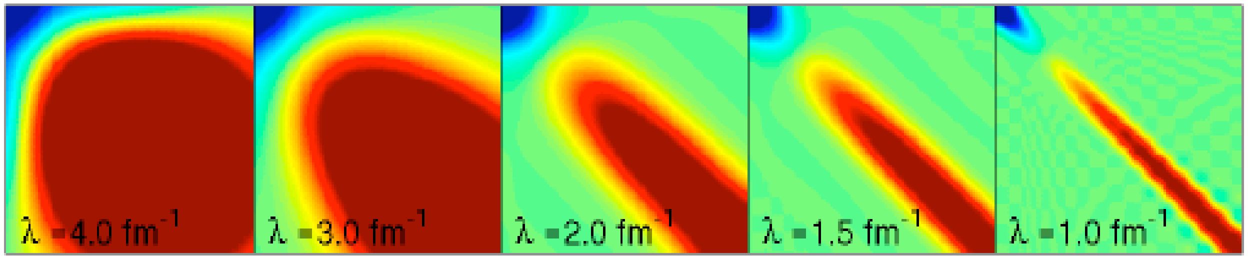

In-medium similarity renormalization group for nuclei and nuclear matter

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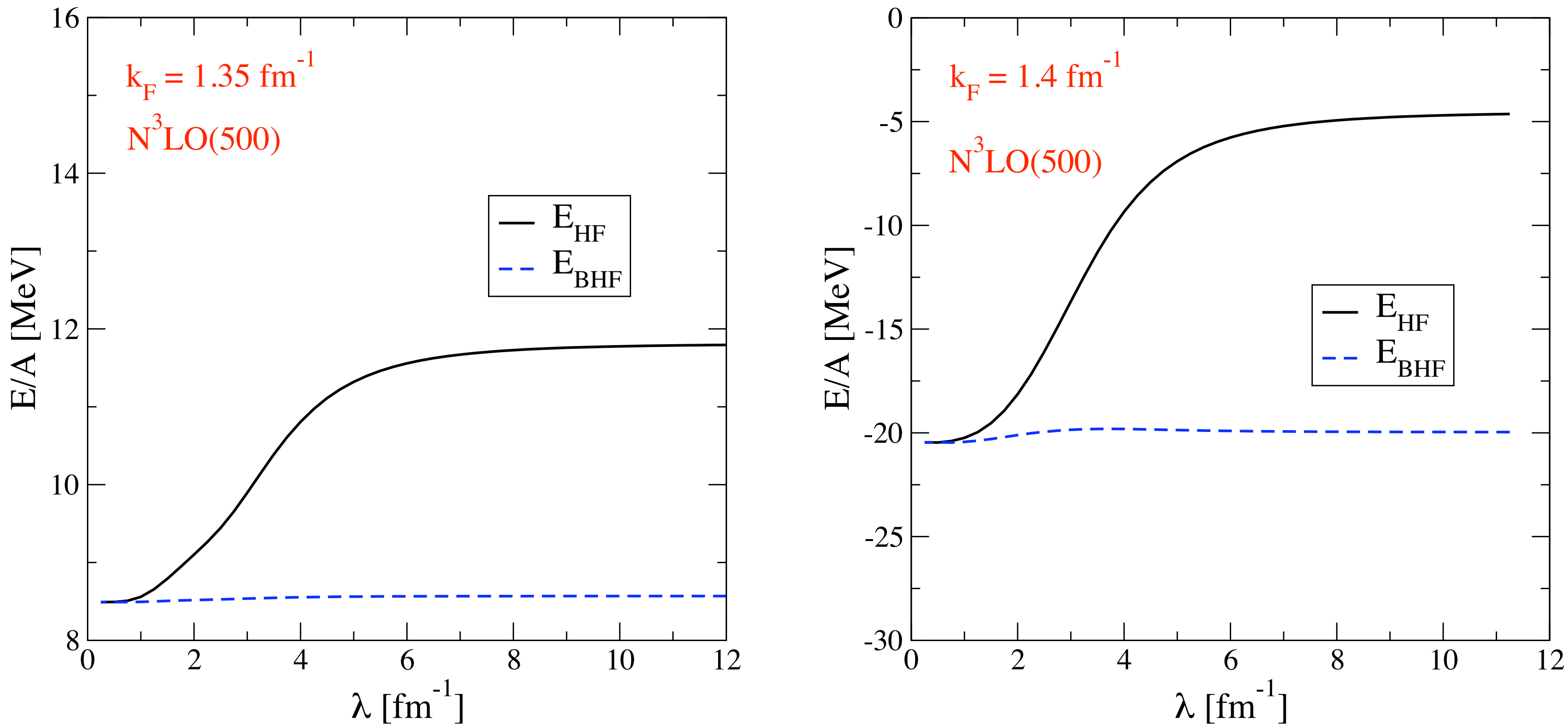
MICHIGAN STATE UNIVERSITY

NSCL

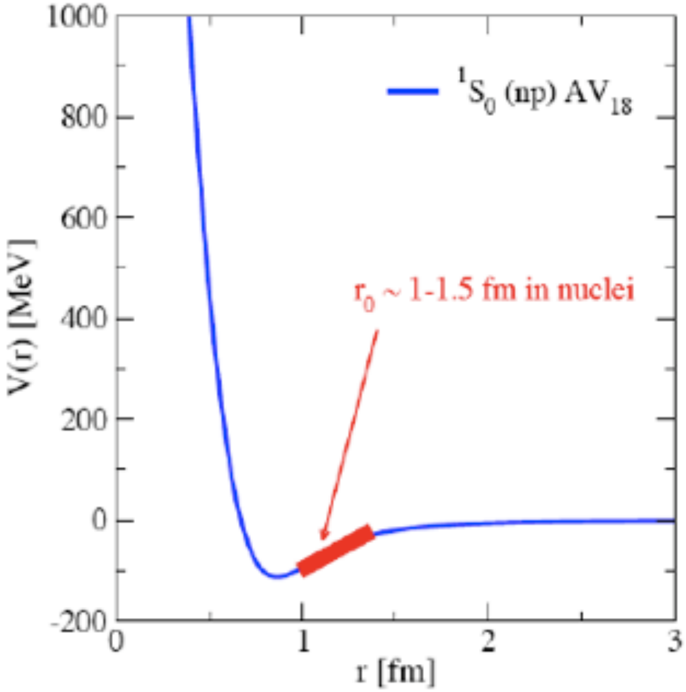
Punch lines of my talk

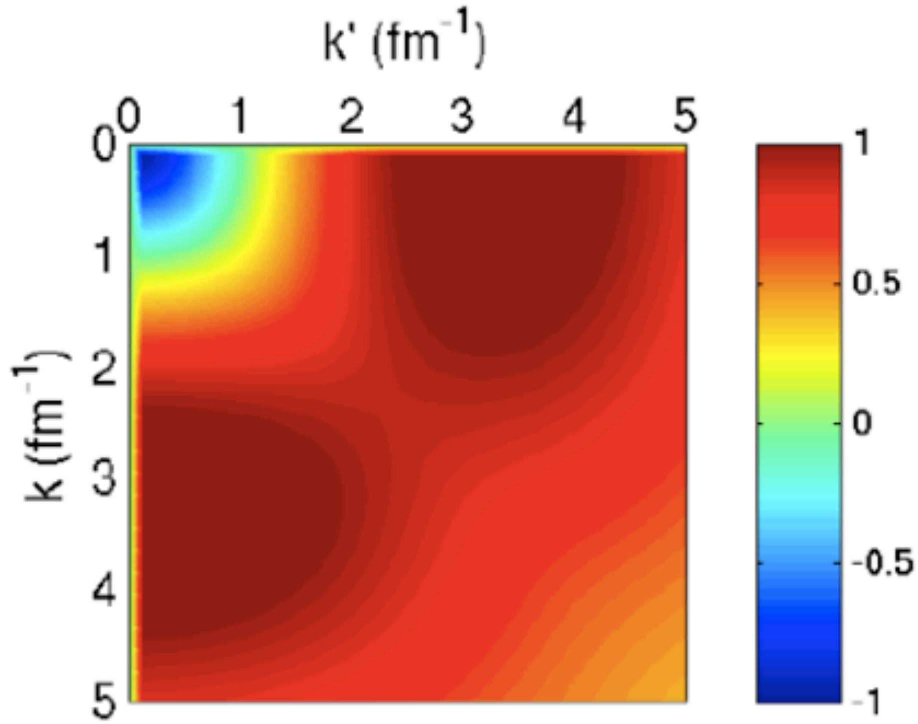
The in-medium SRG (thru the magic of normal-ordering) suggests the following intriguing possibilities:

- RG evolution of 3-body (and higher) interactions **with NN machinery**
- Microscopic basis for phenomenological SM and Landau Fermi Liquid Theory
- new ab-initio method in and of itself?



"Resolution-Dependent" Sources of Non-perturbative Physics





- strong core repulsion
- strong tensor forces

} scheme-dependent, U.V. *details* (e.g., resolution scale Λ)

- deuteron pole
- pairing instability

} scheme-independent (physical, independent of UV details)

Strong coupling of low- and high-k <=> Strong SRC's

Renormalization methods to **decouple** high-k

- Bloch-Horowitz
  - simple (either "integral" or RGE form) ☺
  - energy-dependent (issues with size extensivity, c.f. Brandow et al.) ☹
- Lee-Suzuki
  - energy independent ☺
  - complicated "integral" or RGE form (need to diagonalize a-body problem for a-body H_{eff}) ☹
- Similarity RG
  - energy independent ☺
  - simple RGE form (**never** diagonalize a-body problem) ☺

The Similarity Renormalization Group

[Wegner, Glazek and Wilson]

- Unitary transformation on an initial H = T + V

$$H_s = U(s) H U^\dagger(s) \equiv T + V_s$$

s = continuous flow parameter

- Differentiating with respect to s:

$$\frac{dH_s}{ds} = [\eta(s), H_s] \qquad \text{with} \qquad \eta(s) \equiv \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

- Engineer η(s) to do different things as s -> ∞

$$\eta(s) = [\mathcal{G}_s, H_s]$$

$$\mathcal{G}_s = T \;\Rightarrow\; H_s \text{ driven towards the diagonal in k} - \text{space}$$

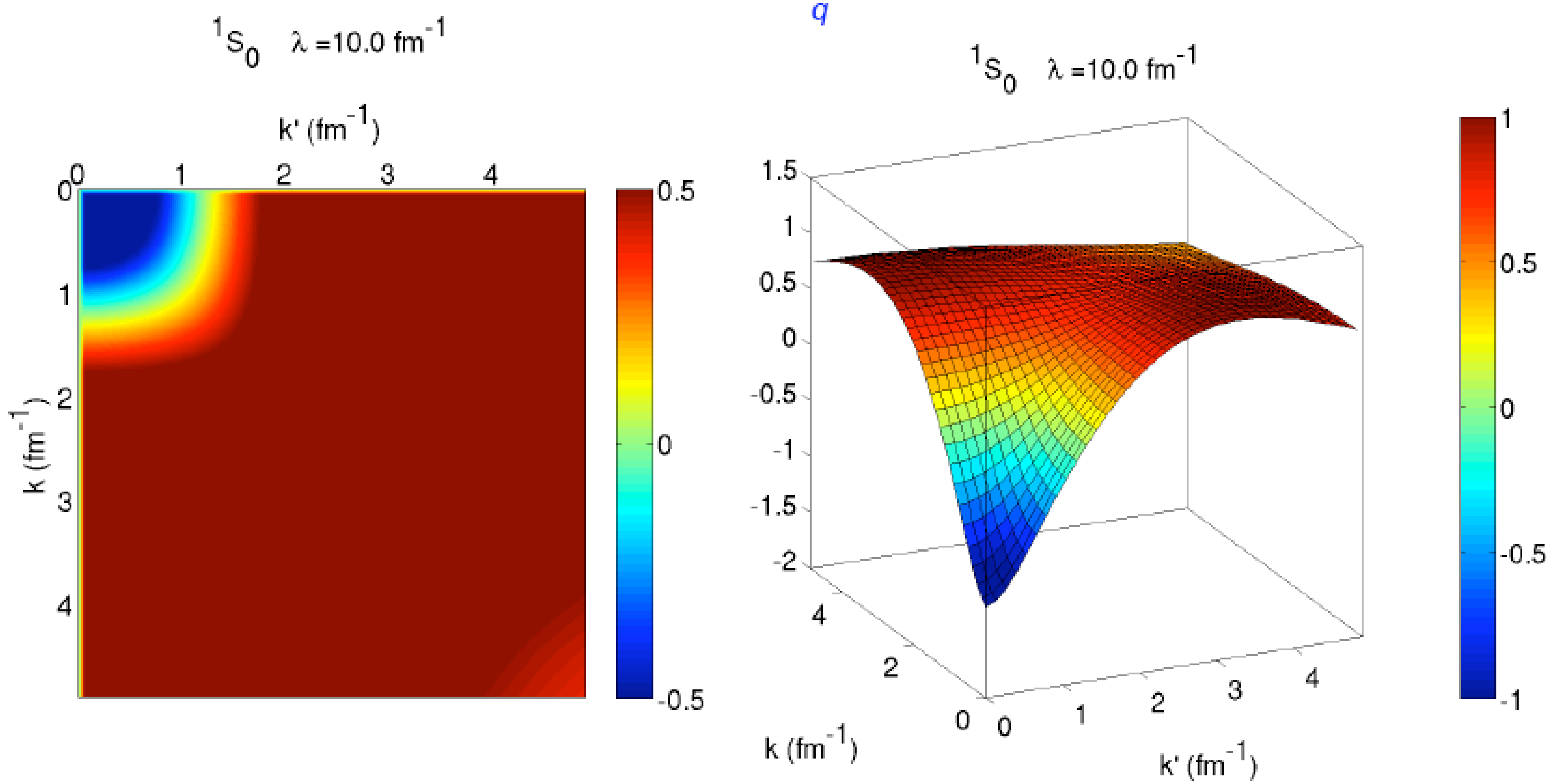
$$\mathcal{G}_s = PH_sP + QH_sQ \;\Rightarrow\; H_s \text{ driven towards block diagonal form}$$

⋮

SRG evolved NN interactions with η = [T,H]

- In each partial wave with $\epsilon_k = \hbar^2 k^2 / M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

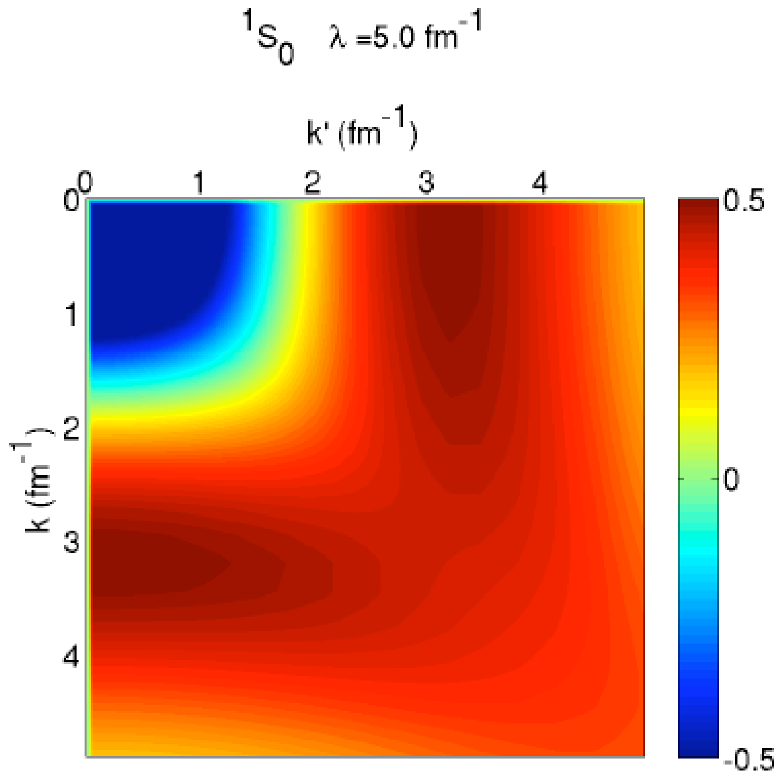


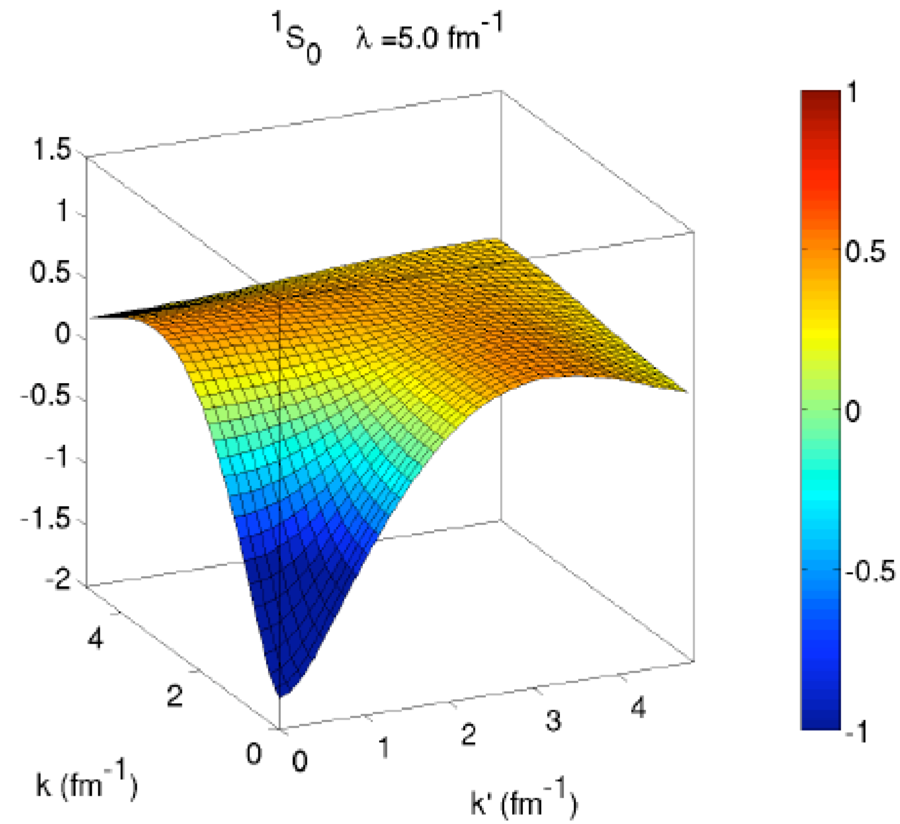
λ = 10.0 fm^{-1}

SRG evolved NN interactions with $\eta = [T,H]$

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



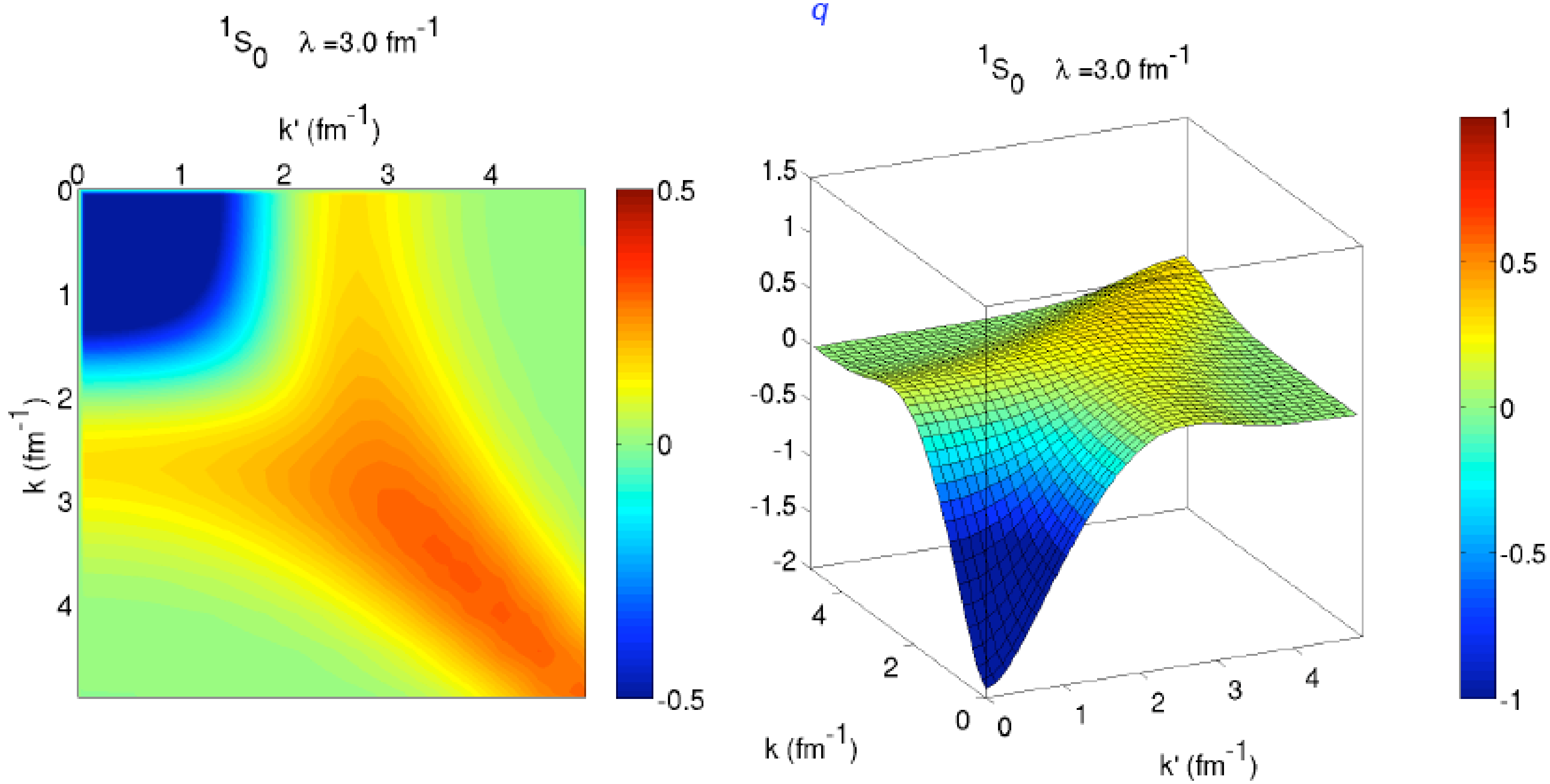


$\lambda = 5.0$ fm^{-1}

SRG evolved NN interactions with η = [T,H]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

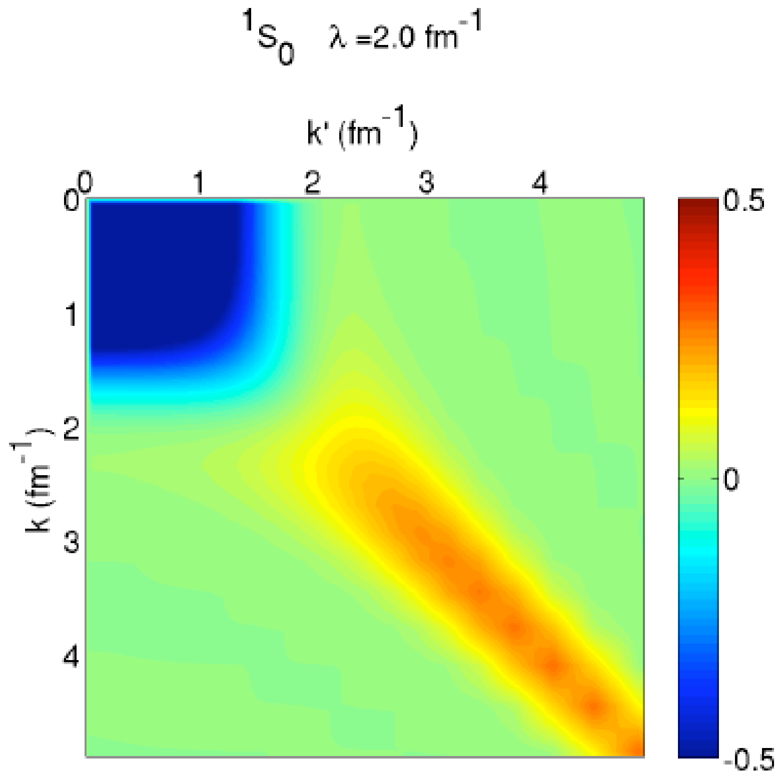


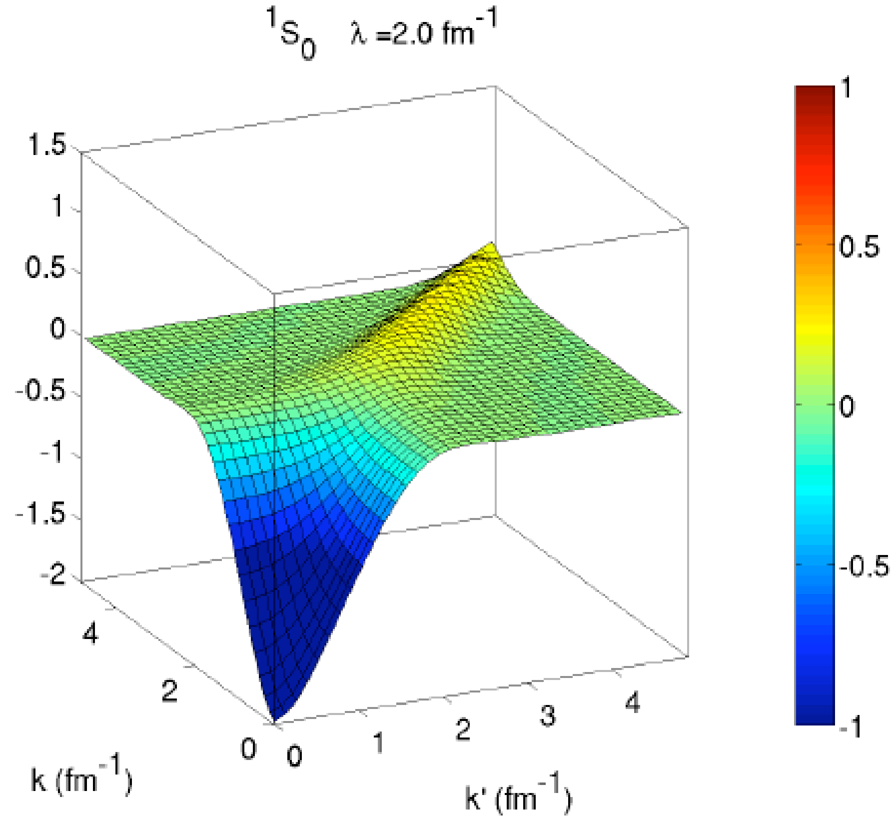
λ = 3.0 fm^{-1}

SRG evolved NN interactions with $\eta = [T,H]$

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

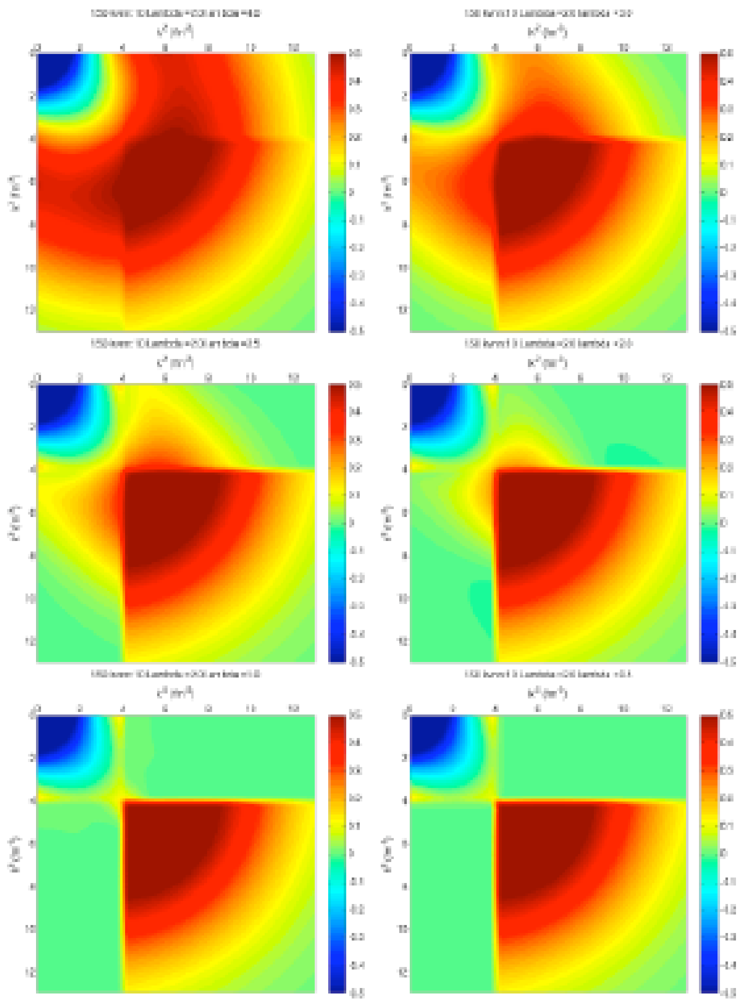




$\lambda = 2.0\ \mathrm{fm}^{-1}$

Flexibility of SRG methods

Can reproduce "conventional" Lee-Suzuki Block-Diagonalization

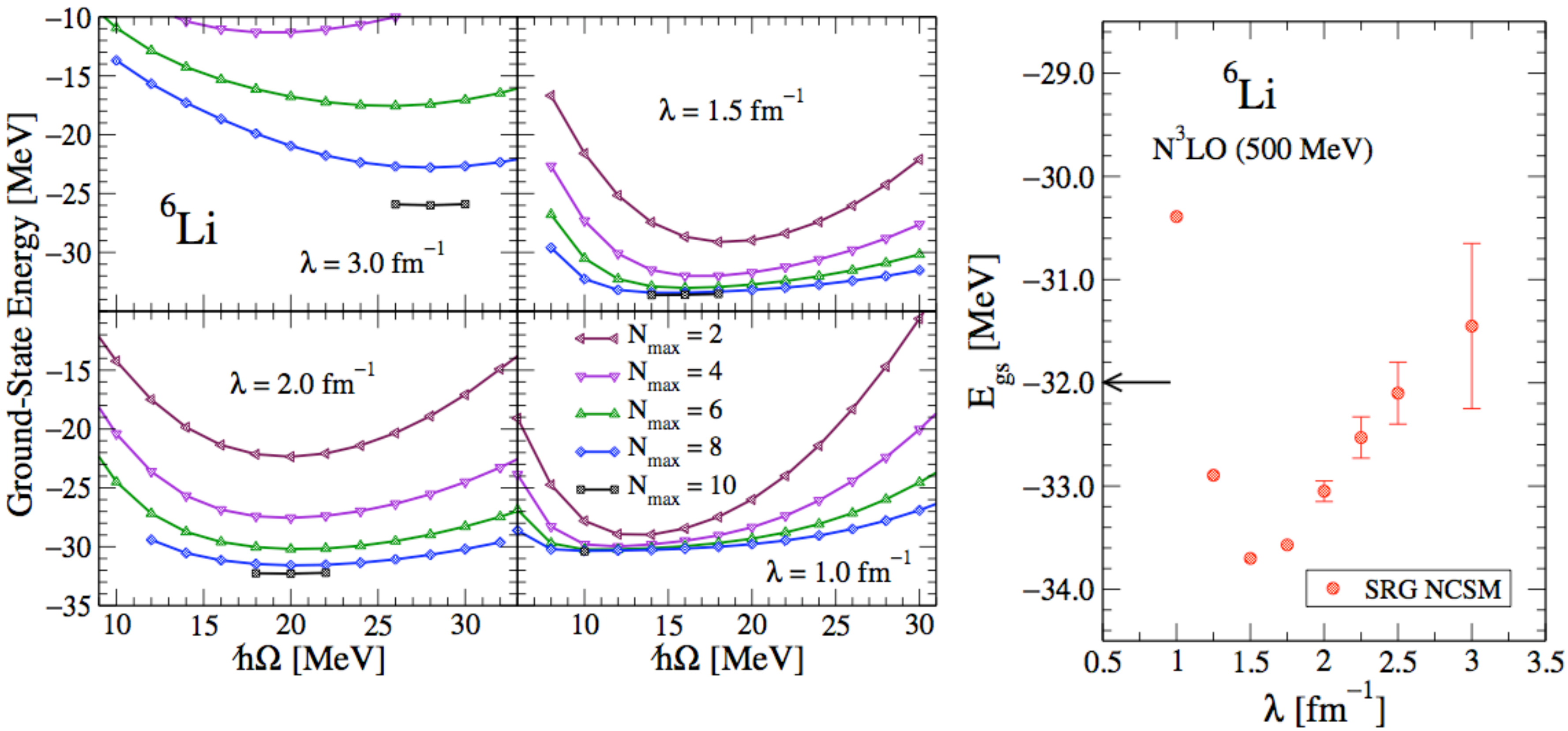


Lee-Suzuki-esque
P & Q space V_{eff}

$$\eta = [PHP + QHQ, H]$$

Key Difference: In SRG approach one never has to **diagonalize** a cluster problem.

Full Configuration Interaction (FCI) Calculations



Decoupling high-k and low-k => accelerated convergence, more perturbative

BUT note...

λ-dependent results (omitted induced 3…A-body forces)

SRG Evolution and Many-Body Forces

- Many-body interactions induced during the flow

$$\frac{dV_s}{ds} = \left[\left[\sum a^\dagger a, \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\right], \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\right] = \cdots + \sum \underbrace{a^\dagger a^\dagger a^\dagger aaa}_{\text{3-body!}} + \cdots$$

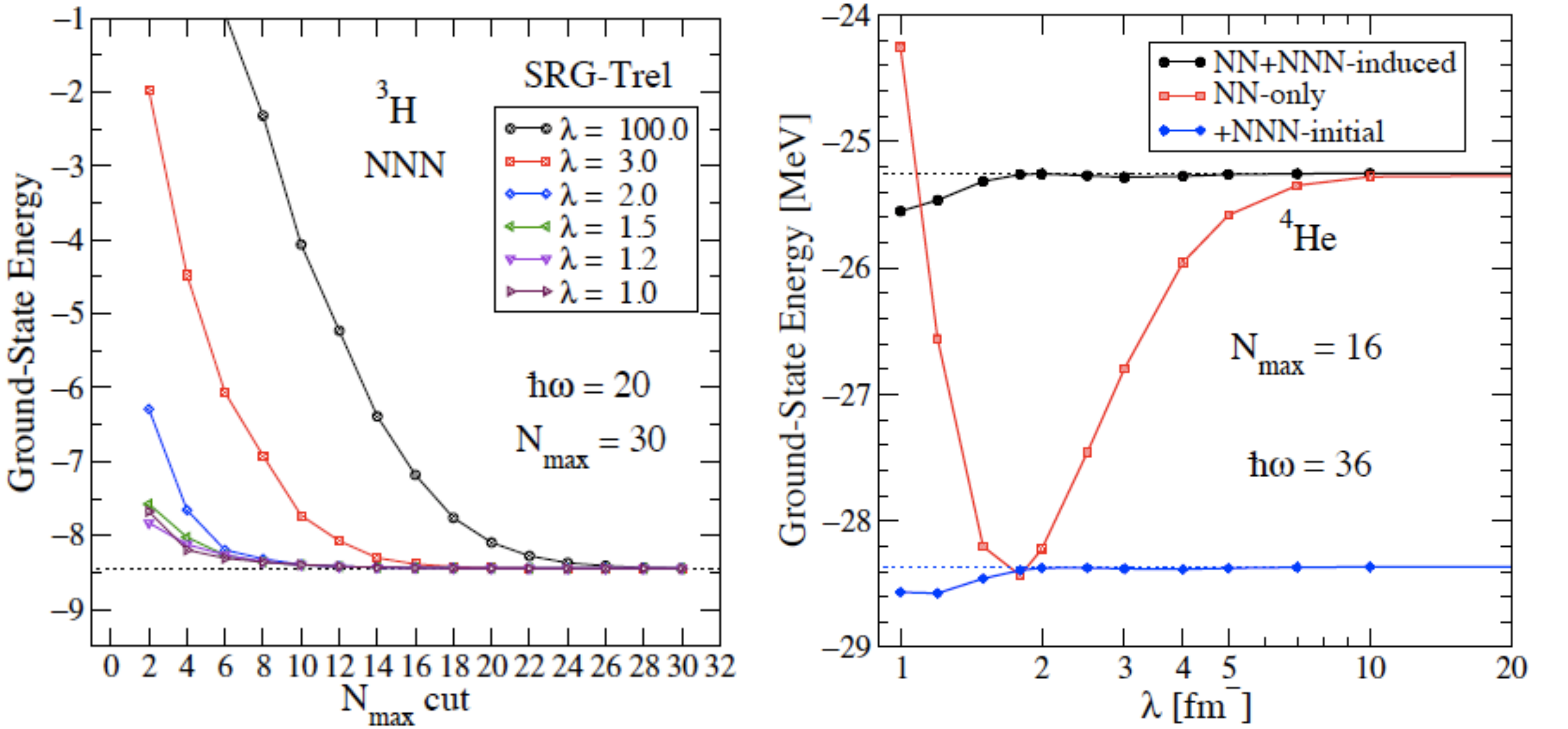
In principle up to A-body operators generated

- Is this a problem?

- Not if "induced" terms are of natural size
- λ-dependence => tool to assess truncation errors
- They're there to begin with anyway. Might as well have them be soft and develop the SRG machinery to evolve them.

Recent developments in 3N SRG evolution

(Jurgenson, Furnstahl, Navratil)



It works! λ-independent ^{3}H, softer convergence

Induced 4N are ≈ 0 for λ ≥ 2 fm^{-1}...

Free space versus in-medium SRG

Free space SRG: $V(\lambda)_{2N}$ fixed in 2N system

$V(\lambda)_{3N}$ fixed in 3N system

⋮

$V(\lambda)_{aN}$ fixed in aN system

Use $T + V(\lambda)_{2N} + V(\lambda)_{3N} + \ldots + V(\lambda)_{aN}$ in A-body system

In-medium SRG: evolution done at finite density (i.e., directly in A-body system). Different mass regions => require different SRG evolutions

inconvenience outweighed (?) by simplifications allowed by normal-ordering

Normal Ordered Hamiltonians

Pick a reference state Φ (e.g., HF) and apply Wick's theorem to 2nd-quantized Hamiltonian

$$
\begin{aligned}
A_i A_j A_k A_l \cdots A_m \;=\; & N(A_i A_j A_k A_l \cdots A_m) \\
+\; & N\Big((\overbrace{A_i A_j} A_k A_l \cdots A_m) + \text{all other single contractions} \Big) \\
+\; & N\Big((\overbrace{A_i A_j} \overbrace{A_k A_l} \cdots A_m) + \text{all other double contractions} \Big) \\
& \vdots \\
+\; & N\Big((\text{all fully contracted terms} \Big)
\end{aligned}
$$

$$\overbrace{a_i^\dagger a_j} = \delta_{ij}\theta(\epsilon_F - \epsilon_i) \qquad \overbrace{a_i a_j^\dagger} = \delta_{ij}\theta(\epsilon_i - \epsilon_F)$$

$$\langle \Phi | N(\cdots) | \Phi \rangle = 0$$

Normal Ordered Hamiltonians

Exactly re-cast H as:

$$H = E_{vac} + \sum f_i N(a_i^\dagger a_i) + \frac{1}{4}\sum \Gamma_{ijkl} N(a_i^\dagger a_j^\dagger a_l a_k) + \frac{1}{36}\sum W_{ijklmn} N(a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l)$$

$$\begin{aligned}
E_{vac} &= \langle\Phi|H|\Phi\rangle \\
f_i &= t_{ii} + \sum_h \langle ih|V_2|ih\rangle\, n_h + \frac{1}{2}\sum_{hh'} \langle ihh'|V_3|ihh'\rangle\, n_h n_{h'} \\
\Gamma_{ijkl} &= \langle ij|V_2|kl\rangle + \sum_h \langle ijh|V_3|klh\rangle\, n_h \\
W_{ijklmn} &= \langle ijk|V_3|lmn\rangle
\end{aligned}$$

$$n_i \equiv \theta(\epsilon_F - f_i)$$

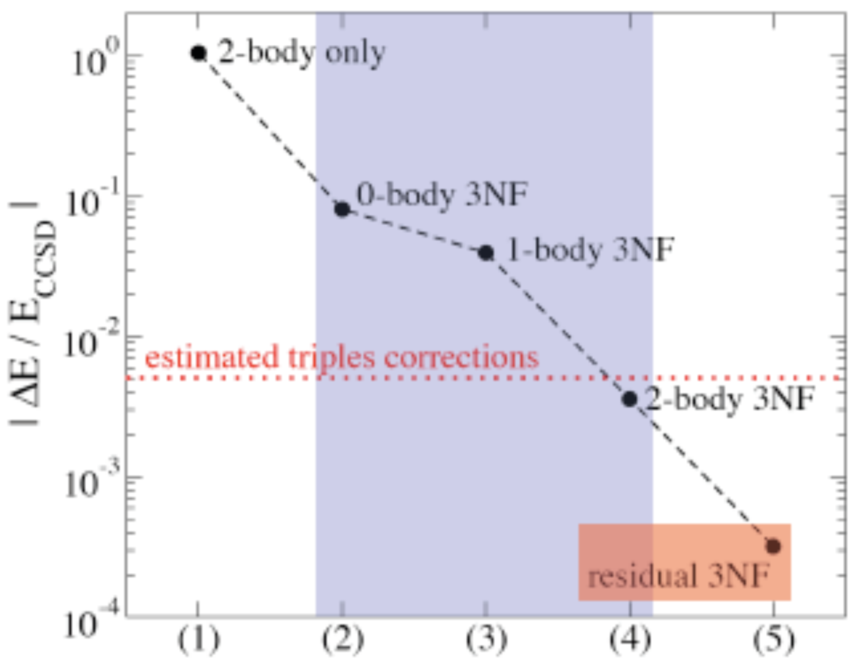
0-, 1-, 2-body terms contain some 3NF effects thru density dependence => Efficient truncation scheme?

Normal Ordering Truncations

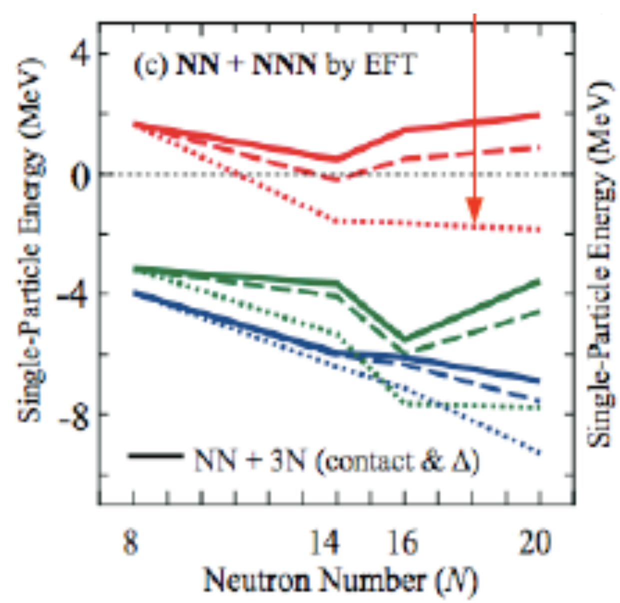
$$H = E_{vac} + \sum f_i :a_i^\dagger a_i: + \frac{1}{4}\sum \Gamma_{ijkl} :a_i^\dagger a_j^\dagger a_l a_k: + \frac{1}{36}\sum W_{ijklmn} :a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l:$$

Good truncation for CC (closed shell) and making contact to phenomenological SM monopole corrections

(Hagen et al.)



Schwenk et al.



Other operators? Resolution dependence of truncations? Open shell systems and multi-reference N-ordering?

In-medium SRG for Infinite NM

- Normal order H w.r.t. non-interacting fermi sea
- Choose SRG generator to eliminate "energy off-diagonal" pieces

$$\eta = [\hat{f} + \hat{\Gamma}_d, \hat{\Gamma}] = [\hat{f}, \hat{\Gamma}] + \mathcal{O}(\frac{1}{A}) \Longrightarrow \lim_{s \to \infty} \Gamma_{od}(s) = 0$$

$$\langle 12 | \Gamma_{od} | 34 \rangle = 0 \text{ if } \mathrm{f}_{12} = \mathrm{f}_{34}$$

- Truncate flow equations to < 2-body normal-ordered operators
  - dominant parts of induced many-body forces included implicitly

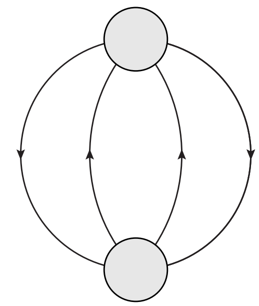
$$H(\infty) = E_{vac}(\infty) + \sum f_i(\infty) N(a_i^\dagger a_i) + \frac{1}{4} \sum [\Gamma_d(\infty)]_{ijkl} N(a_i^\dagger a_j^\dagger a_l a_k)$$

$$\begin{aligned} E_{vac}(\infty) &\rightarrow E_{gs} \\ f_k(\infty) &\rightarrow \epsilon_k \text{ (fully dressed s.p.e.)} \\ \Gamma_d(\infty) &\rightarrow f(k', k) \text{ (Landau q.p. interaction)} \end{aligned}$$

In-medium SRG Equations Infinite Matter

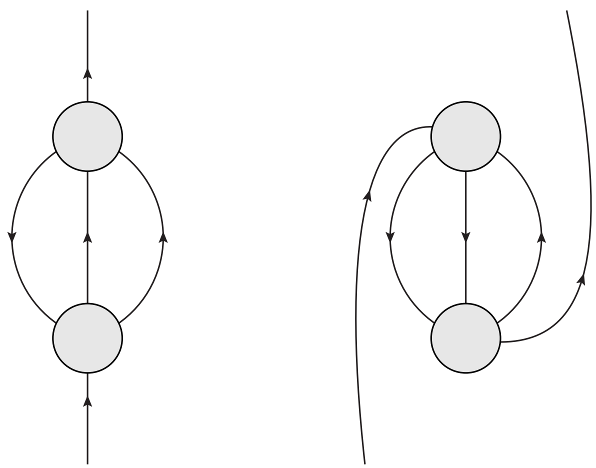
0-body flow

$$\frac{d}{ds}E_{vac} = \frac{1}{2}\sum_{ijkl}(f_{ij} - f_{kl})|\langle ij|\Gamma|kl\rangle|^2 \, n_i n_j \bar{n}_k \bar{n}_l$$



1-body flow

$$\frac{d}{ds}f_a = \sum_{bcd}(f_{ad} - f_{bc})|\langle ad|\Gamma|bc\rangle|^2 \left(\bar{n}_b \bar{n}_c n_d + n_b n_c \bar{n}_d\right)$$

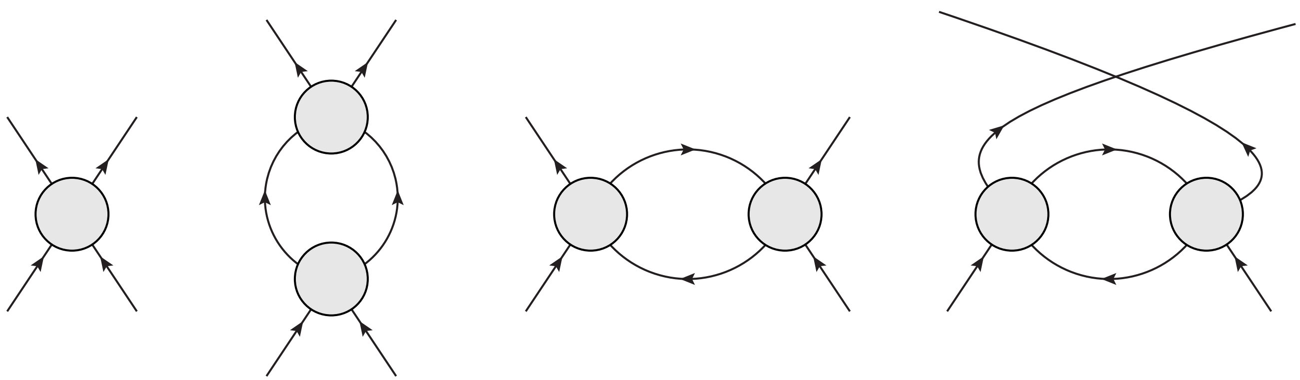


interference of 2p1h 2h1p
self-energy terms

In-medium SRG Equations Infinite Matter

2-body flow

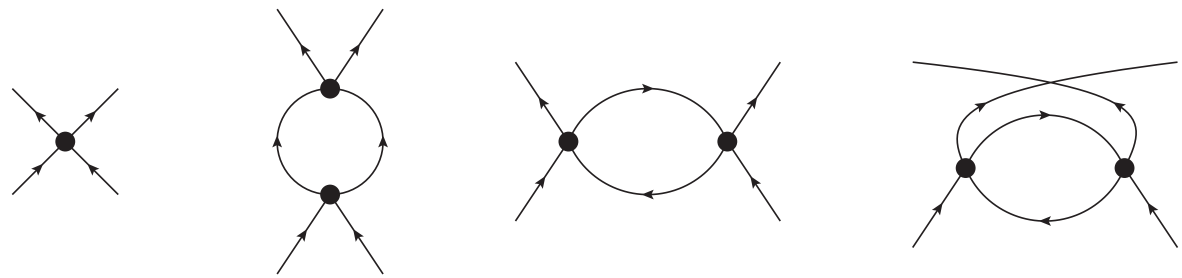
$$
\begin{aligned}
\langle 12|\frac{d\Gamma}{ds}|34\rangle &= -(f_{12}-f_{34})^2\langle 12|\Gamma|34\rangle \\
&+ \frac{1}{2}\sum_{ab}(f_{12}+f_{34}-2f_{ab})\,\langle 12|\Gamma|ab\rangle\langle ab|\Gamma|34\rangle\left(1-n_a-n_b\right) \\
&+ \sum_{ab}\left[(f_{1a}-f_{3b})-(f_{2b}-f_{4a})\right]\langle 1a|\Gamma|3b\rangle\langle b2|\Gamma|a4\rangle(n_a-n_b) \\
&- \sum_{ab}\left[(f_{2a}-f_{3b})-(f_{1b}-f_{4a})\right]\langle 2a|\Gamma|3b\rangle\langle b1|\Gamma|a4\rangle(n_a-n_b)
\end{aligned}
$$



Note the interference between s, t, u channels a-la Parquet theory

SRG is manifestly non-perturbative

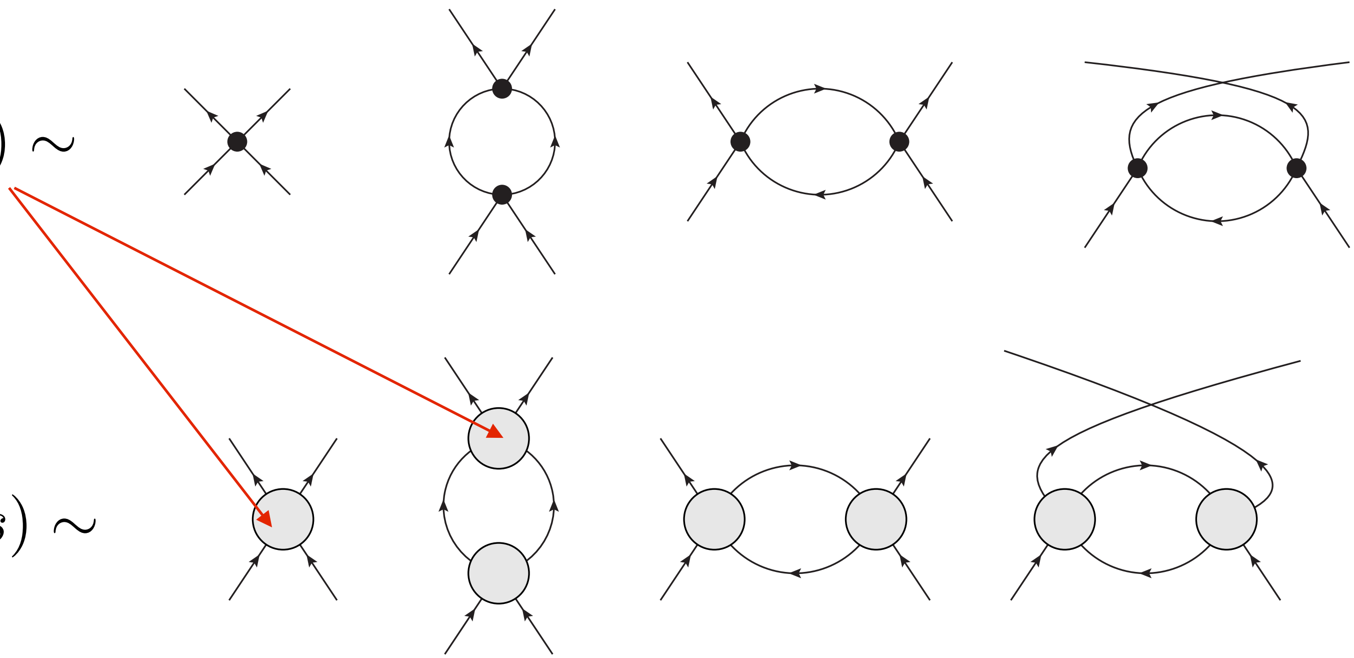
$$\Gamma(\delta s) \sim$$



SRG is manifestly non-perturbative

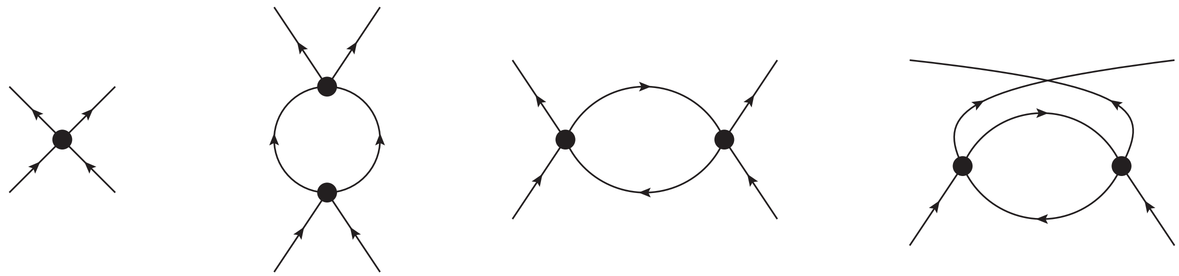
$$\Gamma(\delta s) \sim$$

$$\Gamma(2\delta s) \sim$$

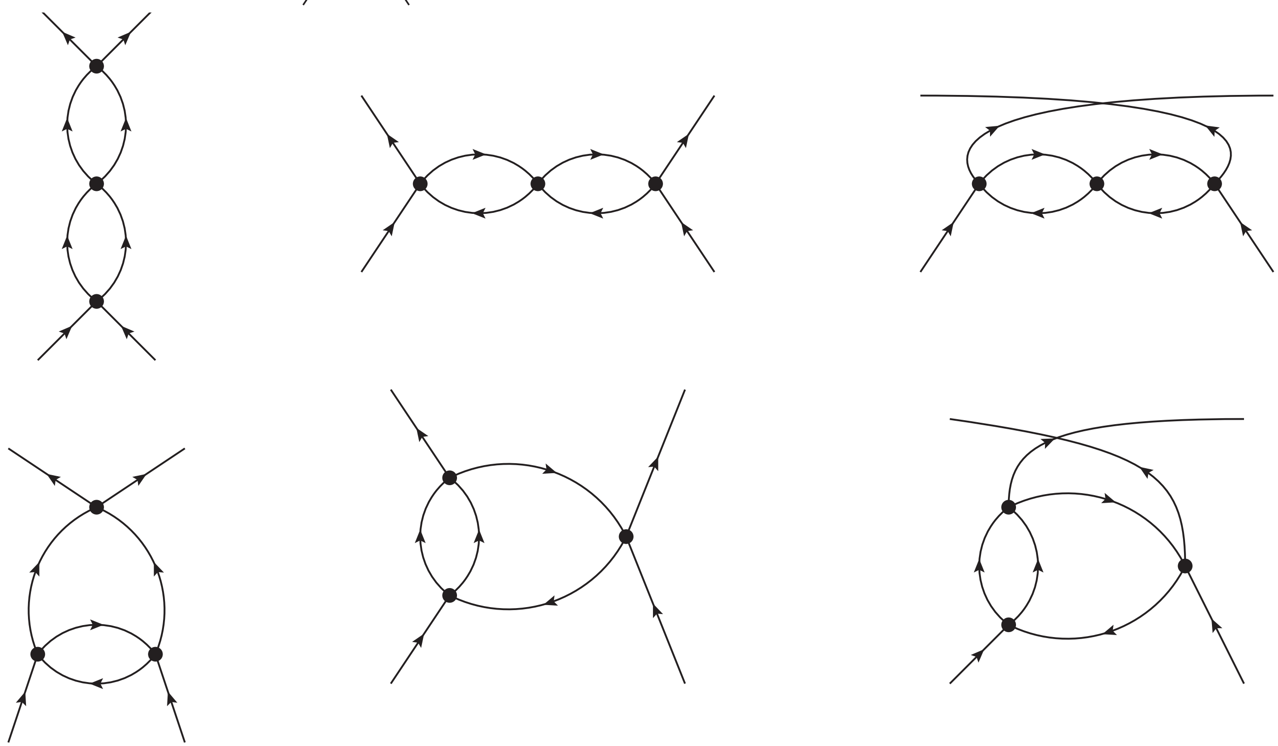


SRG is manifestly non-perturbative

$\Gamma(\delta s) \sim$



$\Gamma(2\delta s) \sim$



+ many more …

Some observations

1) $\frac{d}{ds}\langle H\rangle_0 \leq 0$ for monotonic f_k correlations weakened, HF picks up more binding with increasing s.

2) pp channel + 2 ph channels treated on equal footing

3) Functional derivatives to make contact with induced many-body forces in the original (not normal-ordered) representation

$$\langle ijk|\frac{dV_3}{ds}|lmk\rangle = \frac{\delta}{\delta n_k}\left[\langle ij|\frac{d\Gamma}{ds}|lm\rangle\right]_{\{n=0\}}$$

Etc…

4) no unlinked diagrams (size extensive, etc.)

5) Extension to effective operators immediate

Example: Perturbative content of SRG

- Solve SRG eqn's to 2nd-order the bare coupling

$$E_0(s) \approx E_0(0) + \frac{1}{4}\sum_{1234} n_1 n_2 \bar{n}_3 \bar{n}_4 \frac{|V_{1234}(0)|^2}{f_{12} - f_{34}} \left(1 - e^{-s(f_{12}-f_{34})^2}\right)$$

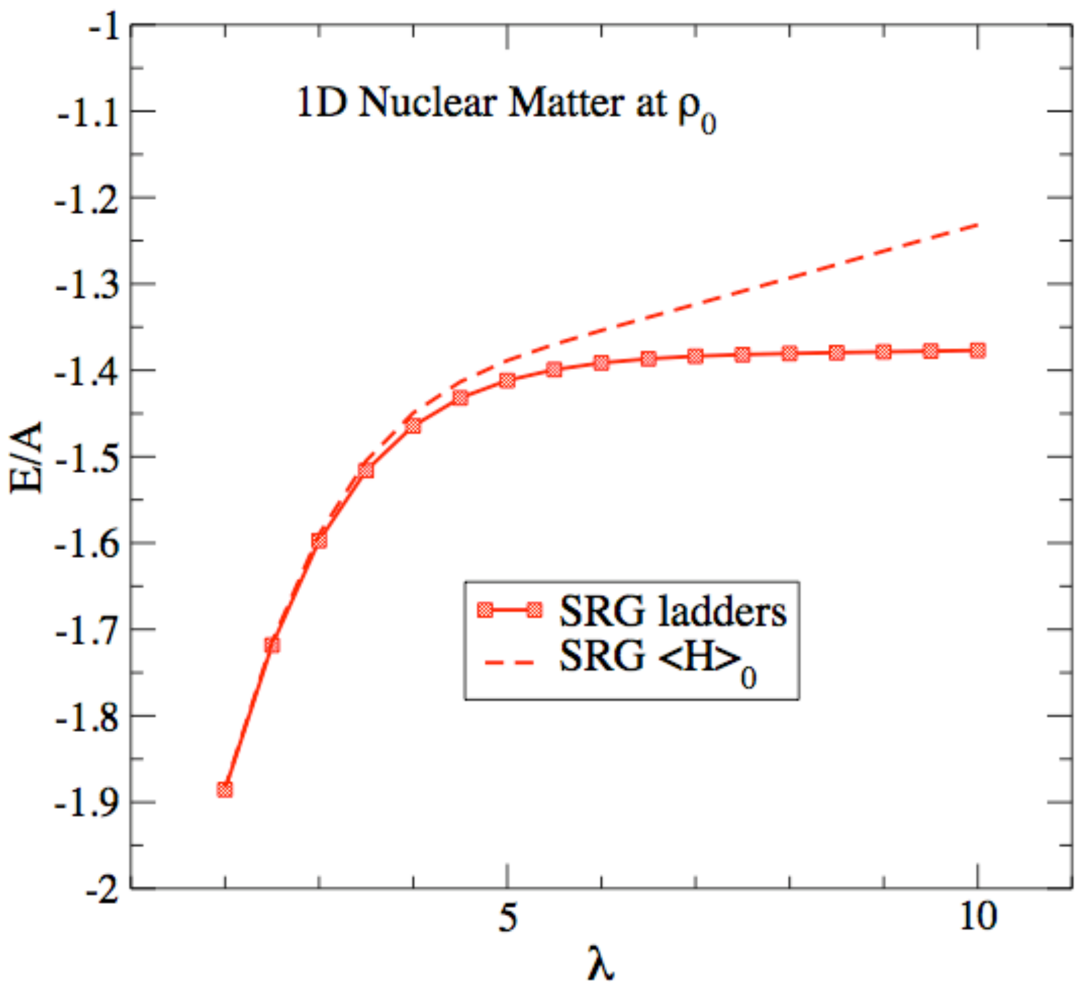
$$E_{corr}(s) \approx \frac{1}{4}\sum_{1234} n_1 n_2 \bar{n}_3 \bar{n}_4 \frac{|V_{1234}(0)|^2}{f_{12} - f_{34}} e^{-s(f_{12}-f_{34})^2}$$

As s increases, contributions shuffled from correlation energy into
The non-interacting VEV contribution (I.e., Hartree-Fock)

Microscopic connection to shell model?
(MF + "weak" A-dependent residual NN interaction)

In-medium SRG in 1D nuclear matter

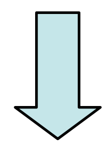
Free-space SRG evolution truncated at NN level



- HF approaches ladder sum at lower λ (weak correlations)

BUT

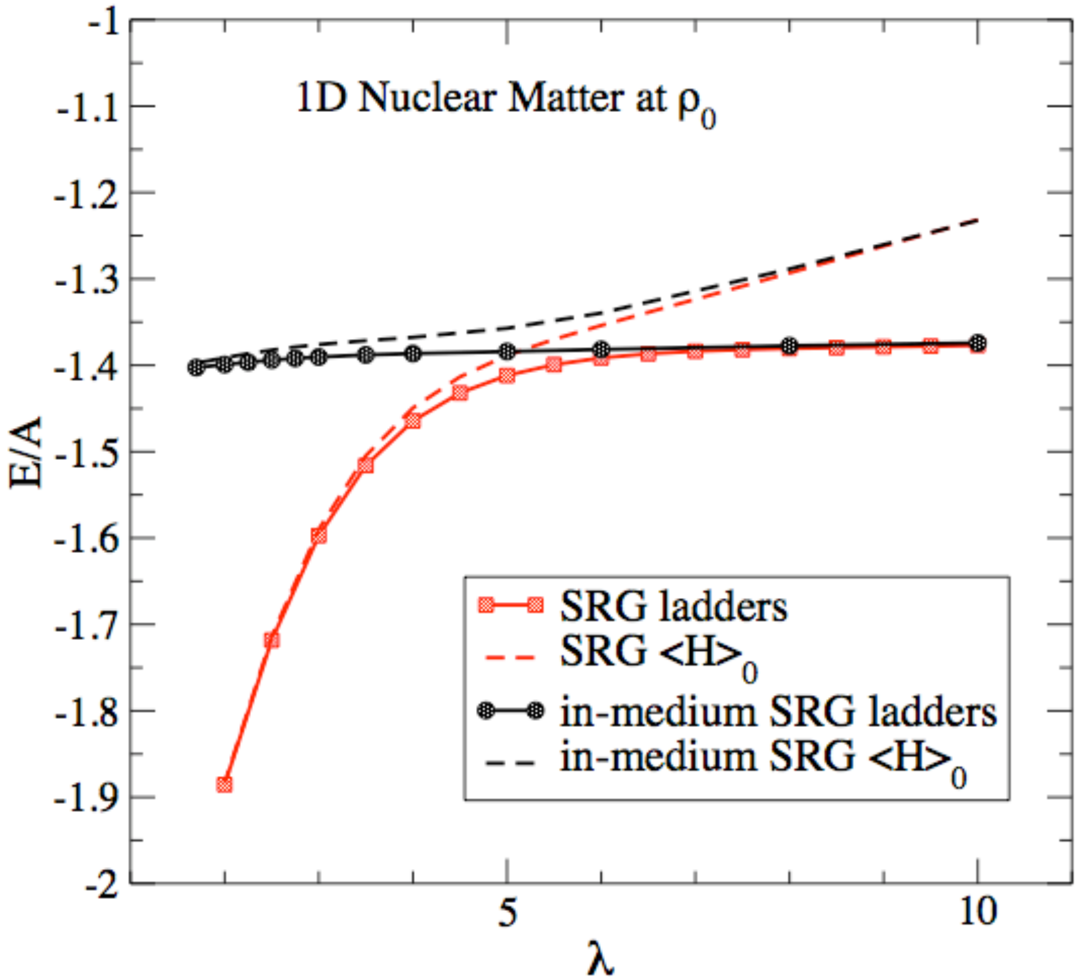
- Strong λ-dependence due to truncation to 2N SRG



- try in-medium SRG w/ normal-ordering to reduce the λ-dependence

In-medium SRG in 1D nuclear matter

In-medium SRG evolution truncated at normal-ordered NN level



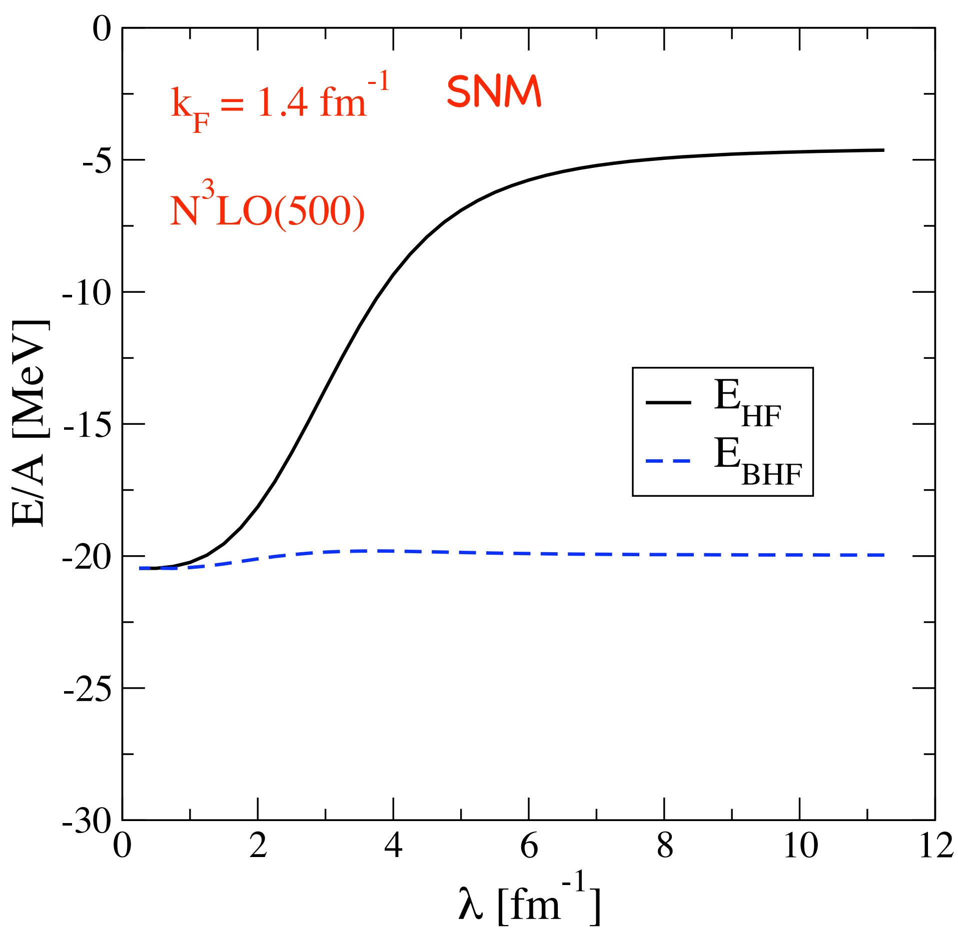
- HF approaches ladder sum at lower λ (weak correlations)

AND

- λ-dependence weak (dominant many-body forces kept by normal ordering).

In-medium SRG in Real 3D nuclear matter

In-medium SRG evolution truncated at normal-ordered NN level



- HF approaches ladder sum at lower λ (weak correlations)

AND

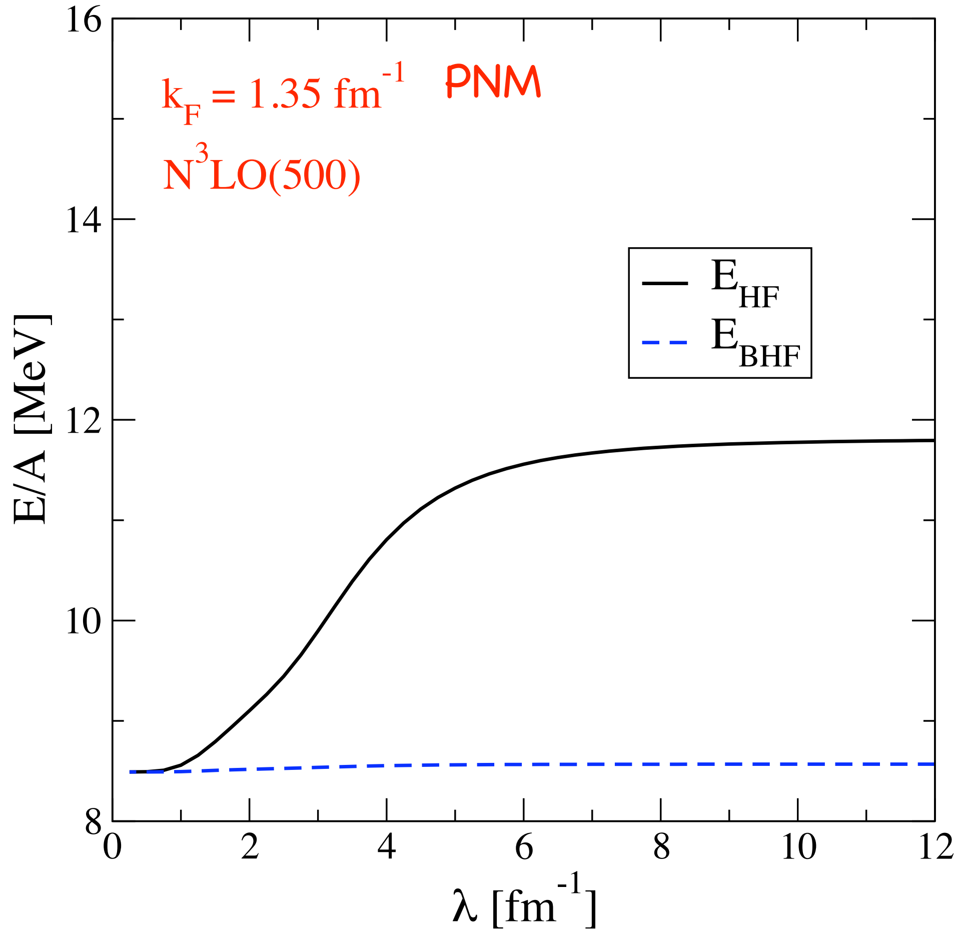
- λ-dependence weak (dominant many-body forces kept by normal ordering).

- soon: ph-channel terms

* Neglected ph channel, angle-averaging, ...

In-medium SRG in Real 3D nuclear matter

In-medium SRG evolution truncated at normal-ordered NN level



- HF approaches ladder sum at lower λ (weak correlations)

AND

- λ-dependence weak (dominant many-body forces kept by normal ordering).

- soon: ph-channel terms

* Neglected ph channel, angle-averaging, ...

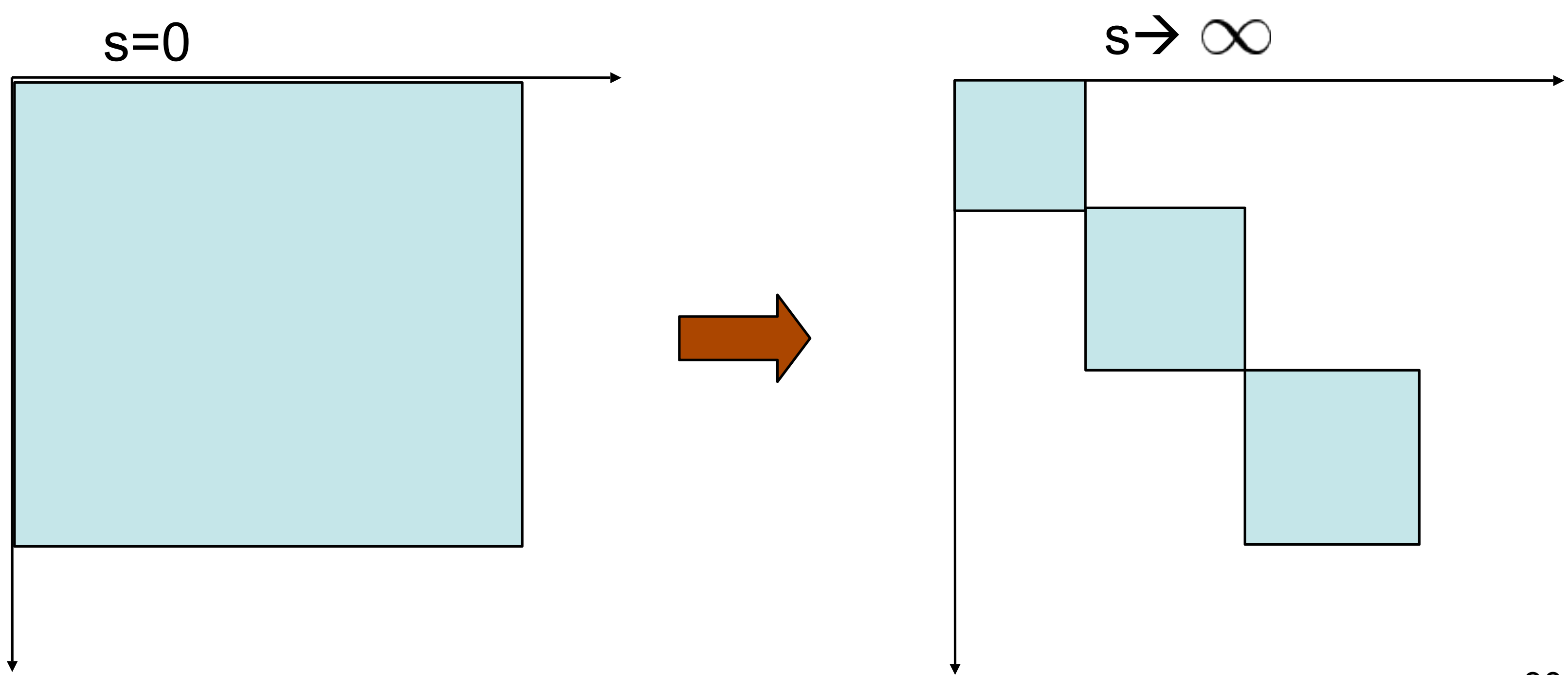
Application to nuclei (K. Tsukiyama, SB, A. Schwenk)

minor annoyance: $$\hat{f} = \sum_i f_i N(a_i^\dagger a_i) \rightarrow \sum_{ij} f_{ij} N(a_i^\dagger a_j)$$

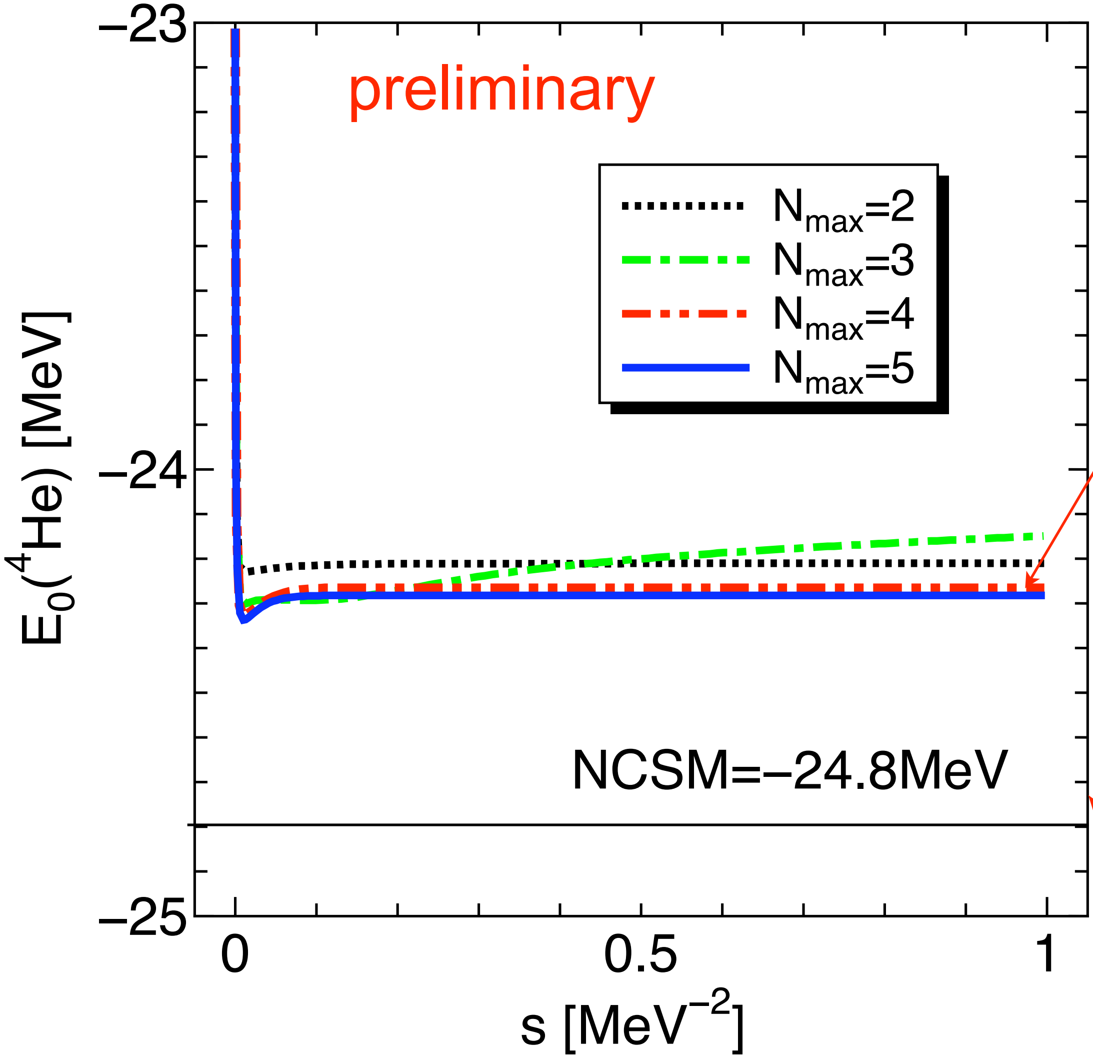
SRG generator: $$\eta = [\hat{f}^d + \hat{\Gamma}^d, \hat{f}^{od} + \hat{\Gamma}^{od}]$$

$$f_{ij}^d = 0 \quad (\epsilon_i \neq \epsilon_j) \qquad \Gamma_{ijkl}^d = 0 \quad (\epsilon_i + \epsilon_j \neq \epsilon_k + \epsilon_l)$$

ϵ_i = HO sp energy



SRG flow of E_{vac} for He-4

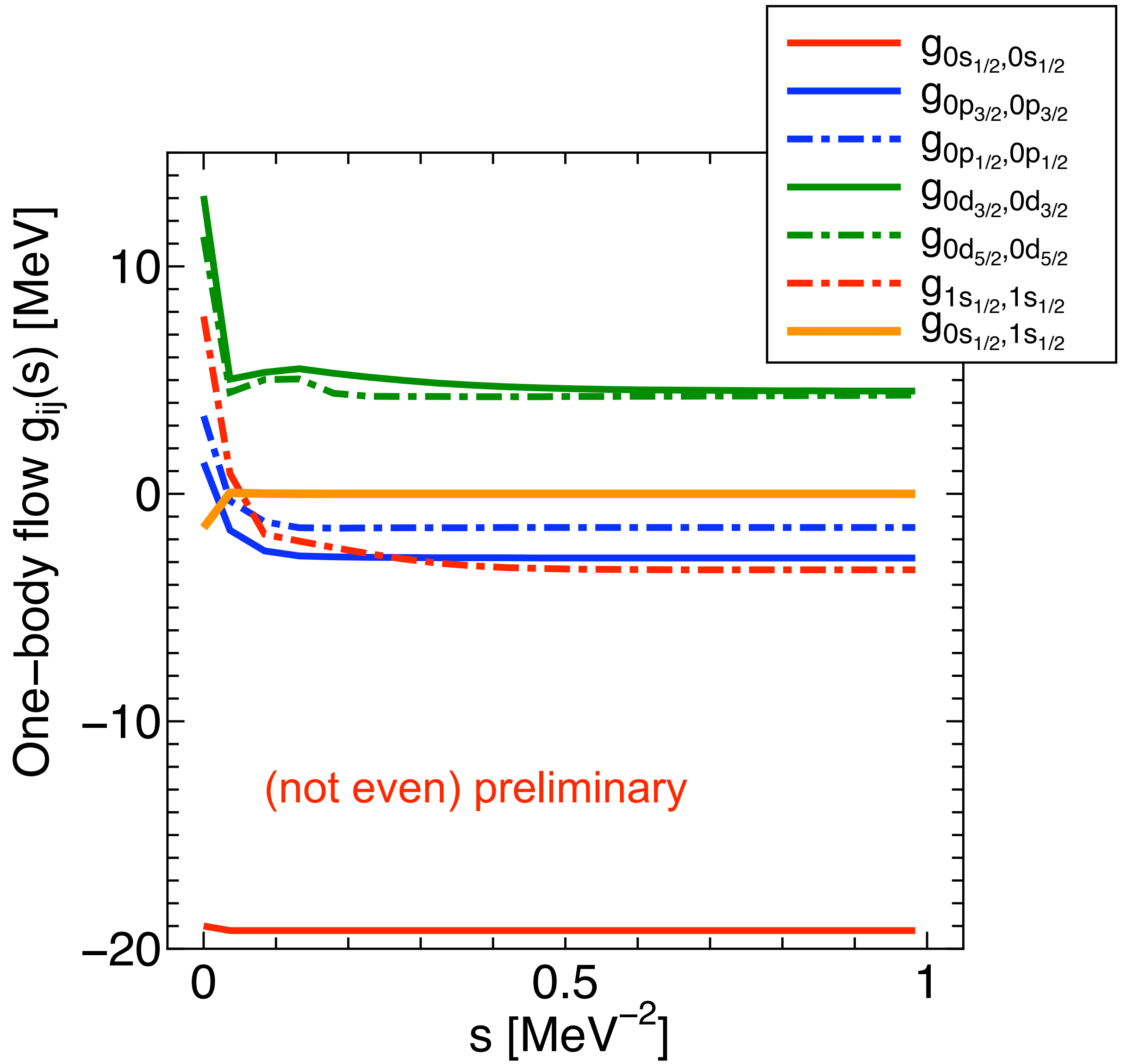


In progress: perturbative corrections from non-zero Γ in the block-diagonal structure

Alternative SRG generators using HF energies to define “diagonal” and “off-diagonal”

O16 and Ca40 next...

SRG flow of 1-body (s.p.e.'s)



Summary of In-Medium SRG w/Normal-Ordering

- 2N formalism includes dominant induced 3,...A-body forces
- Microscopically obtain dominant MF + "weak" residual interaction
- 1st 3D nuclear matter results look pretty good, still need ph terms
- Non-perturbative path to shell model H_{eff} and O_{eff}?
- Ab-initio method to diagonalize medium nuclei?
- First applications to finite nuclei gearing up
- More sophisticated reference state to normal order w.r.t.?
  - ★ quasiparticle vacuum (pairing)
  - ★ multi-reference (open shell systems)