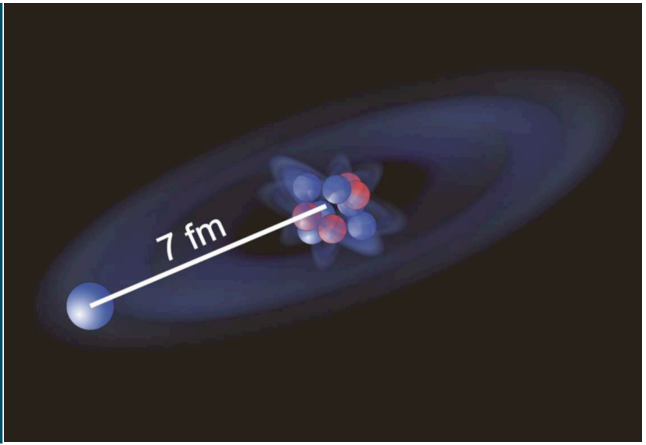
Moon Halo



Nuclear Halo

Halo Nuclei in the Helium Isotope Chain

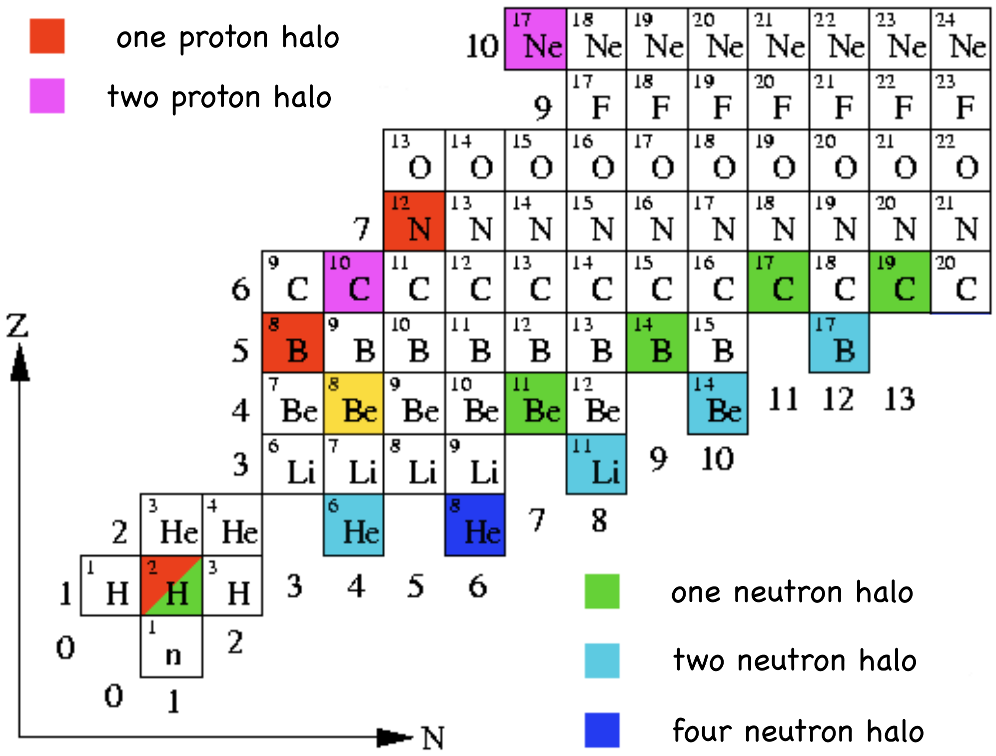
Sonia Bacca, TRIUMF Theory Group

in collaboration with:
Gaute Hagen, Thomas Papenbrock, Achim Schwenk

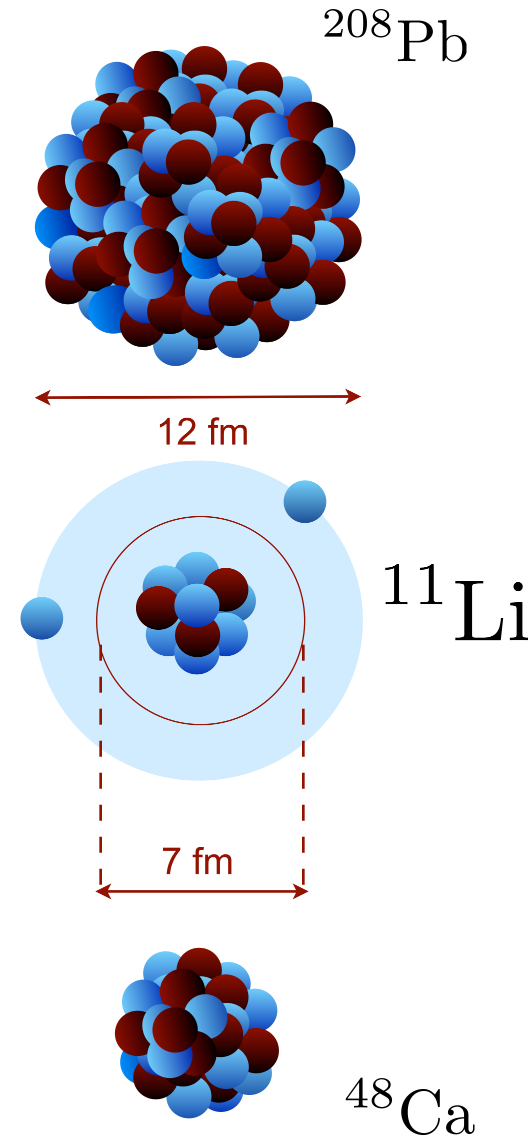
Outline

- Why are halo nuclei interesting?
- Brief summary on experimental advances
- Overview of different theoretical approaches
- Our approach:
  - Use hyper-spherical harmonics for ^{6}He
  - Use coupled cluster theory for ^{8}He
- Results for binding energy
- Results for radii 
- Summary and Outlook

Halo Nuclei



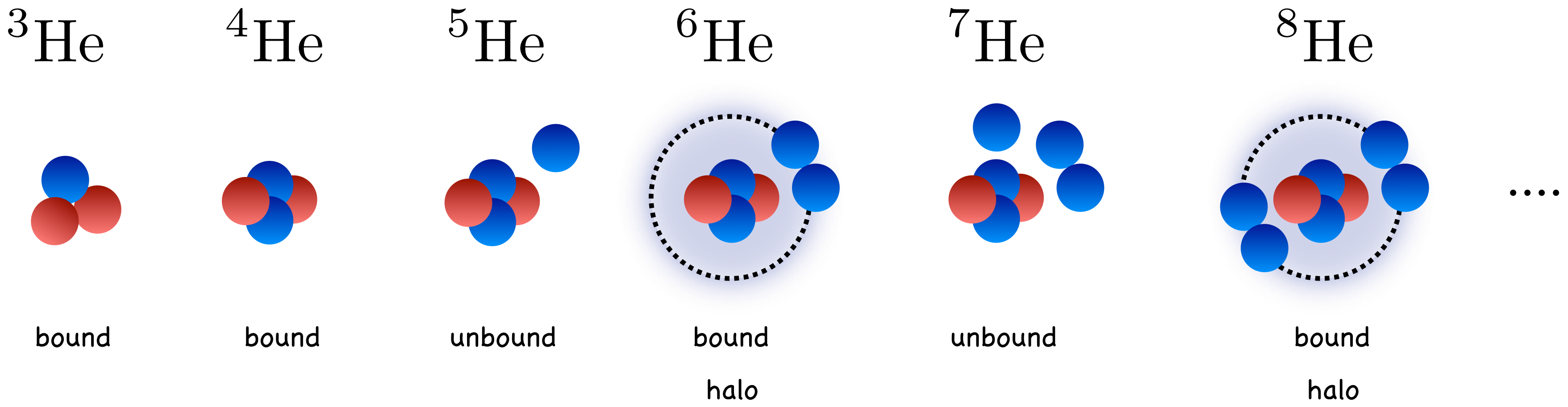
one proton halo
two proton halo
one neutron halo
two neutron halo
four neutron halo
Z
N



^{208}Pb
12 fm
^{11}Li
7 fm
^{48}Ca

The helium isotope chain

Shows many interesting features:



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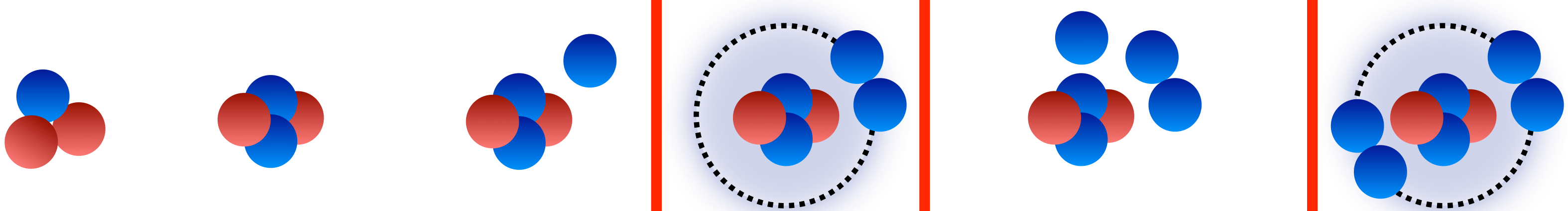

^{3}He	^{4}He	^{5}He	^{6}He	^{7}He	^{8}He	....
bound	bound	unbound	bound	unbound	bound	
			halo		halo	
			Borromean system		Most exotic nucleus "on earth"	
			lives 806 ms		lives 108 ms	

$$\frac{N}{Z} = 3$$

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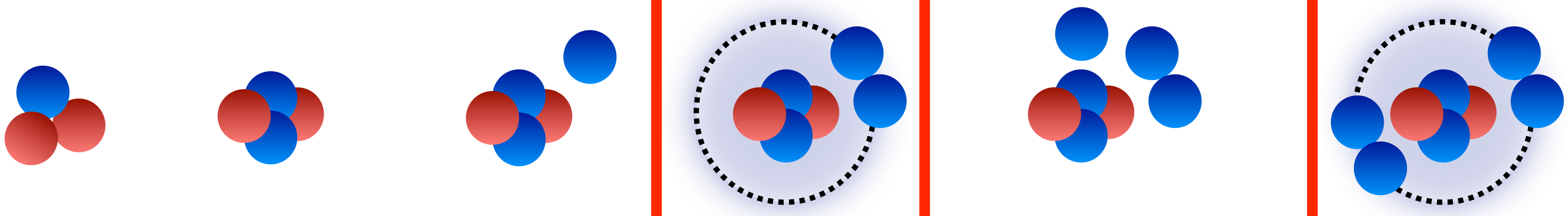


Borromean system

lives 806 ms

Most exotic nucleus "on earth"

$$\frac{N}{Z} = 3$$

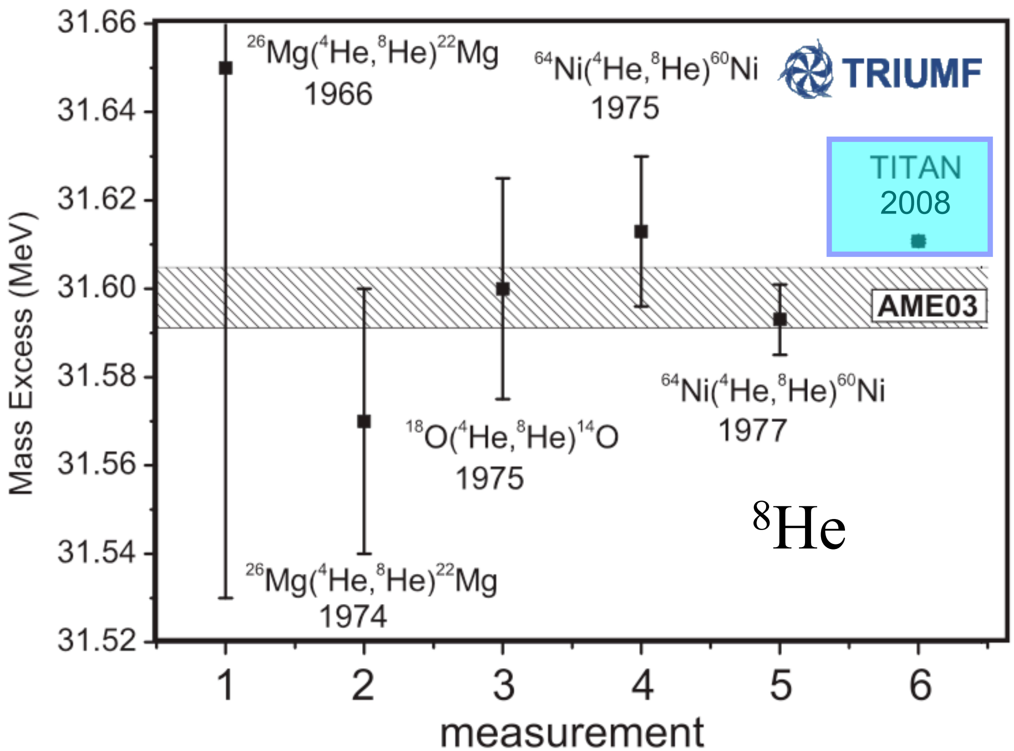
lives 108 ms

Even if they are exotic short lived nuclei, they can be investigated experimentally. From a comparison of theoretical predictions with experiment we can test our knowledge on nuclear forces in the neutron rich region

Halo Nuclei - Experiment

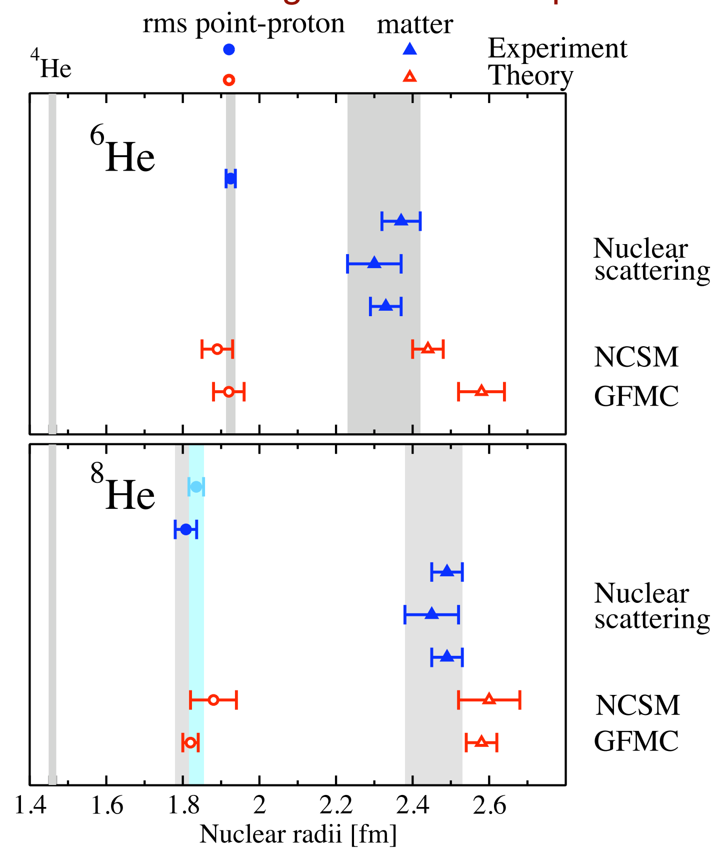
New Era of Precision Measurements for masses and radii

Mass measurement of ^{8}He with the Penning trap



TRIUMF, Ryjkov et al. PRL 101, 012501 (2008)

Measurement of charge radii via isotope shift



ARGONNE, Wang et al. PRL 93, 142501 (2004)
GANIL, Mueller et al. PRL 99, 252501 (2007)

$$\delta\nu_{AA'} = \delta\nu_{A,A'}^{mass} + K\delta\langle r_{\rm ch}^2\rangle_{AA'}$$

$$\langle r_p^2\rangle = \langle r_{ch}^2\rangle - \langle R_p^2\rangle - \frac{3}{4M_p^2} - \frac{N}{Z}\langle R_n^2\rangle$$

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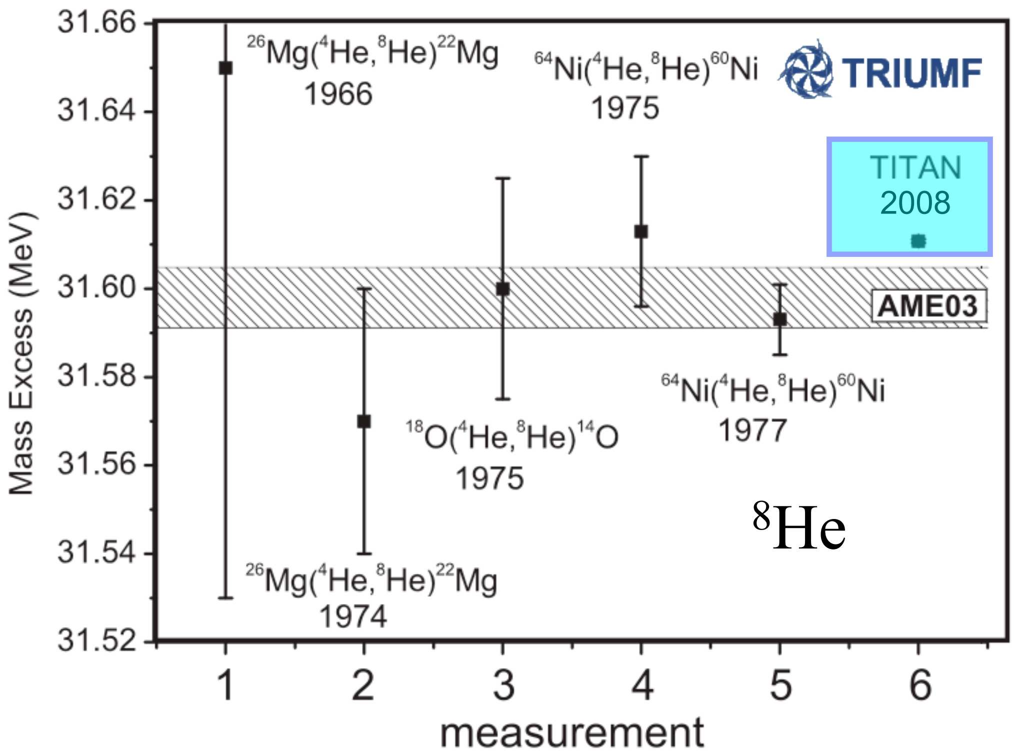

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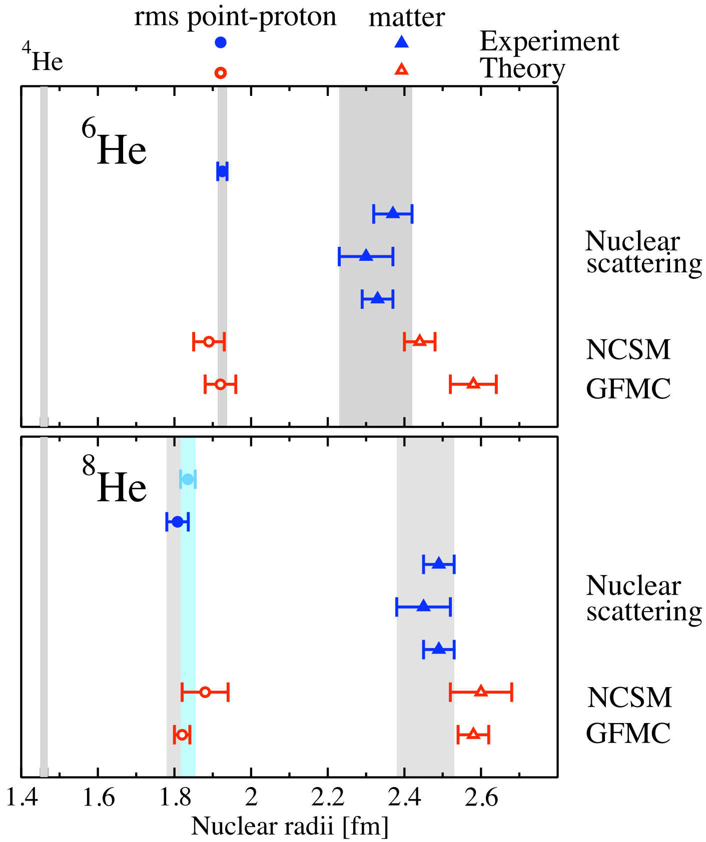

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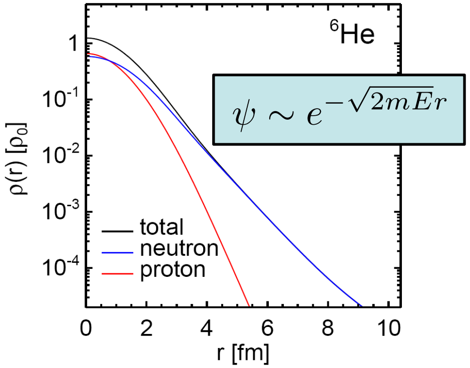
Masses and radii of helium isotopes are important challenges for theory!

Halo Nuclei -Theory

Why are halo nuclei a challenge to theory?

- It is difficult to describe the long extended wave function
- They test nuclear forces at the extremes, where less is known



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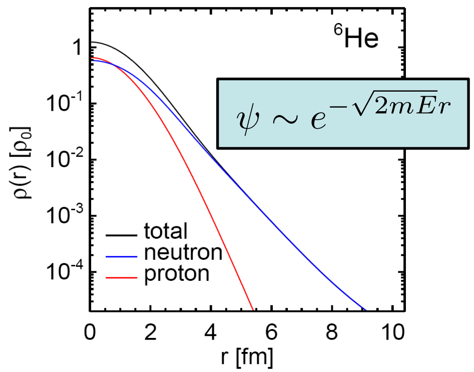

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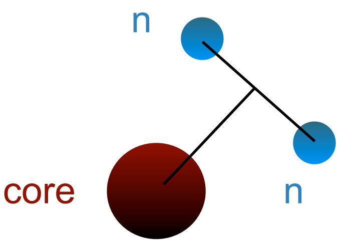
Cluster models:



3-body models with phenomenological interactions

^{6}He, ^{11}Li - borromean systems

can do reactions, Faddeev calculations

but difficult to add core polarizations

Efros, Fedorov, Garrido, Hagino, Bertulani, ...

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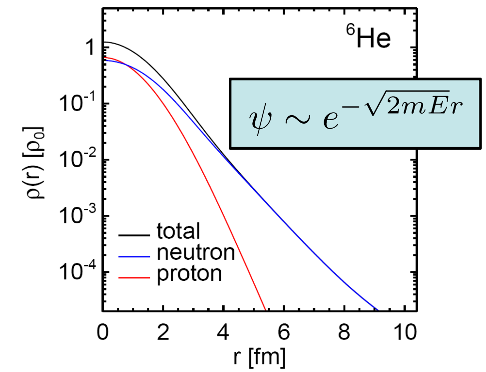

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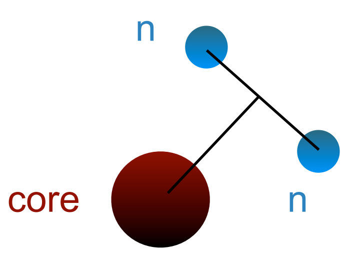

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New: Revived by halo EFT

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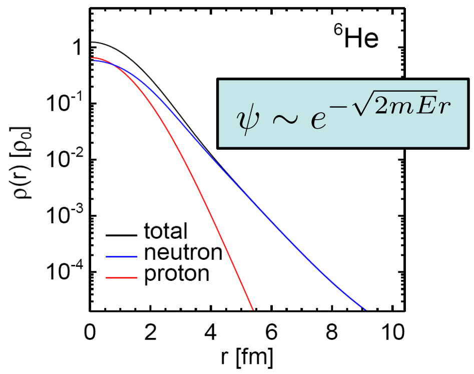
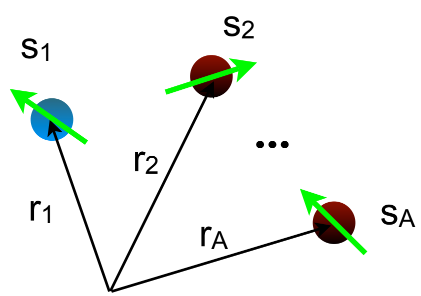


Ab-initio calculations:

treat the nucleus as an A-body problem



full antisymmetrization of the w.f.

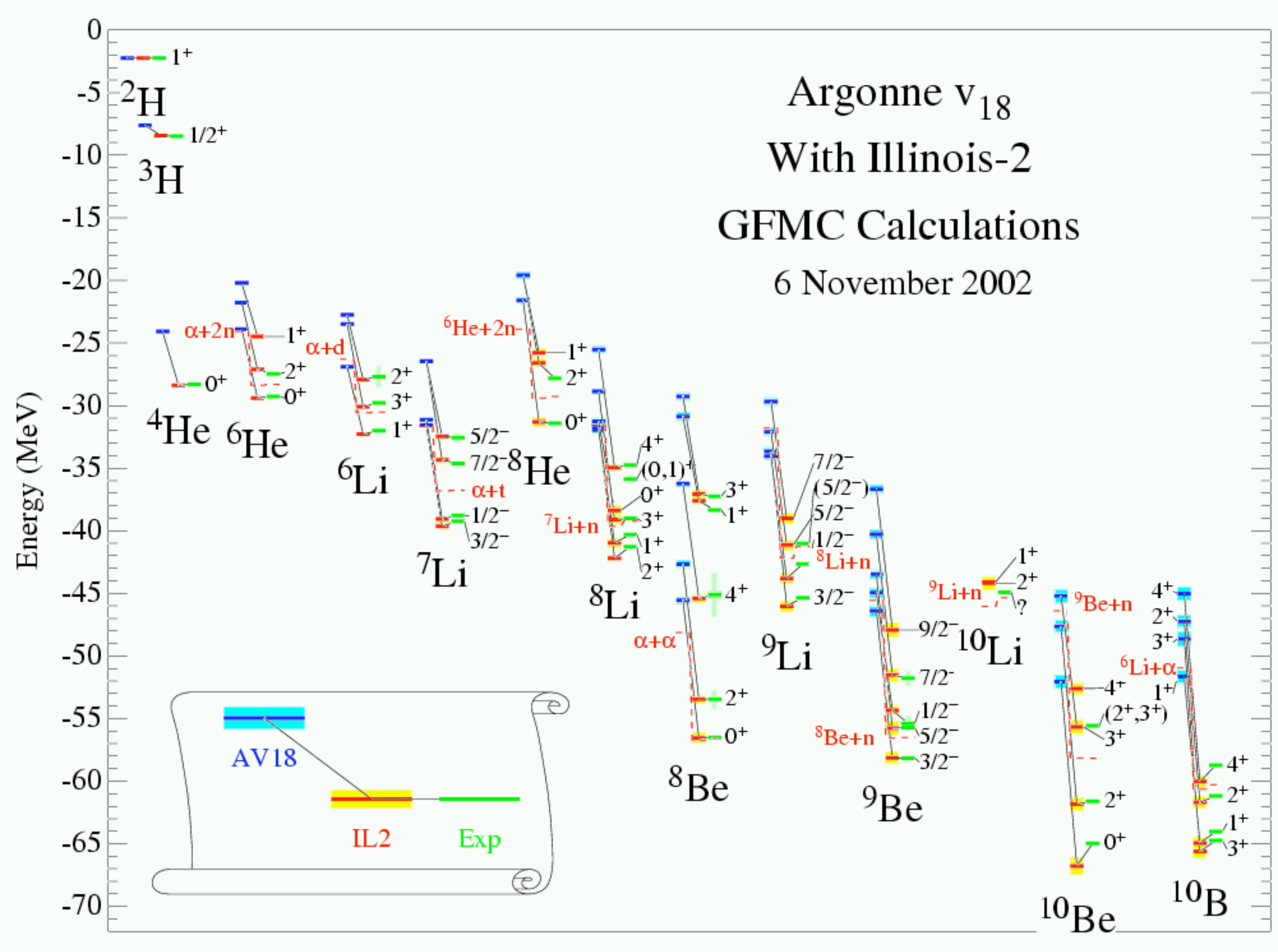
use modern Hamiltonians to predict halo properties

$$H = T + V_{NN} + V_{3N} + \ldots$$

Methods: GFMC, NCSM, CC, HH

Ab-initio Calculations

GFMC Quantum Monte Carlo Method,
Uses local two- and three-nucleon forces → short range phenomenology



Pieper et al. (2002)

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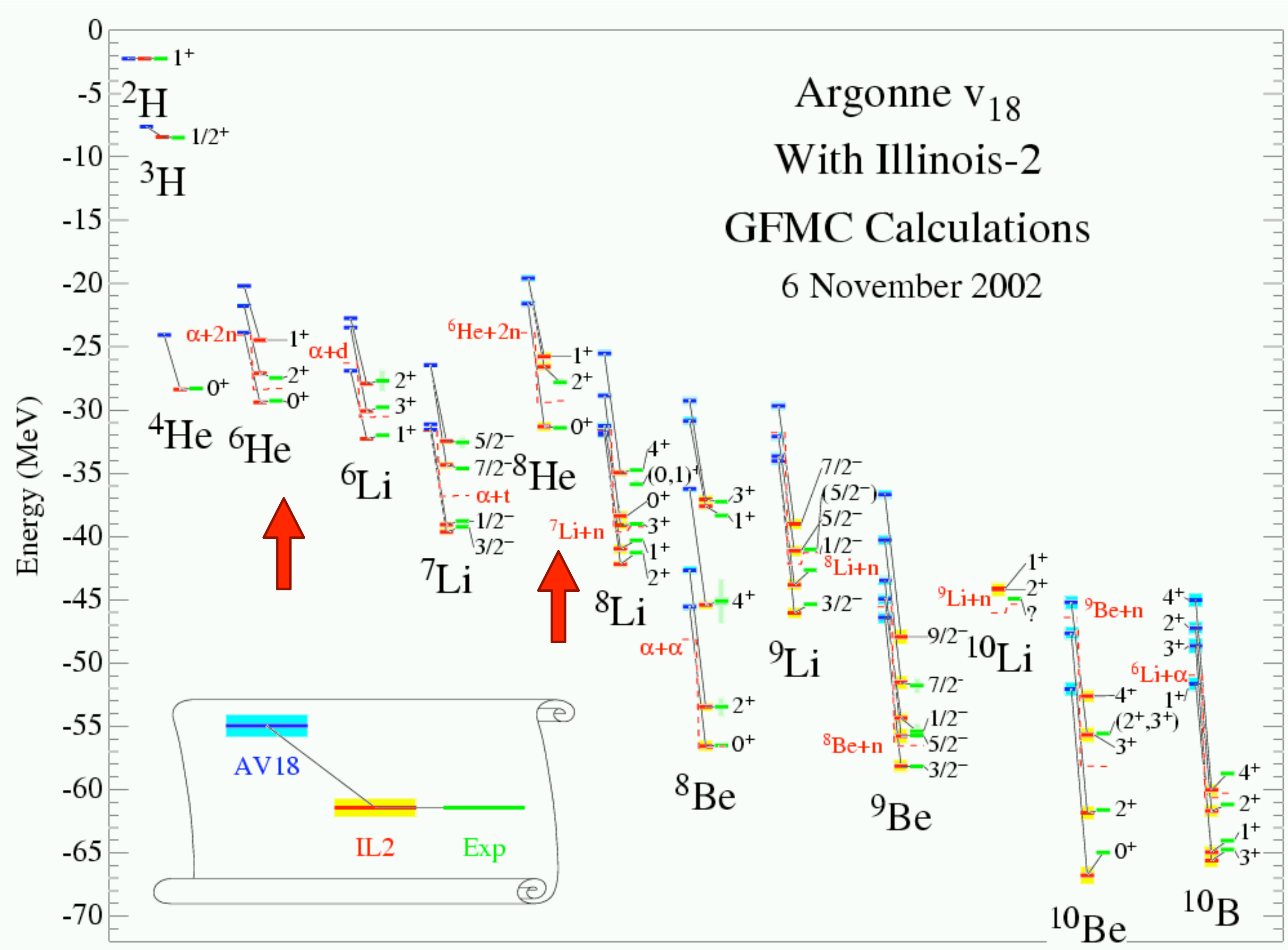

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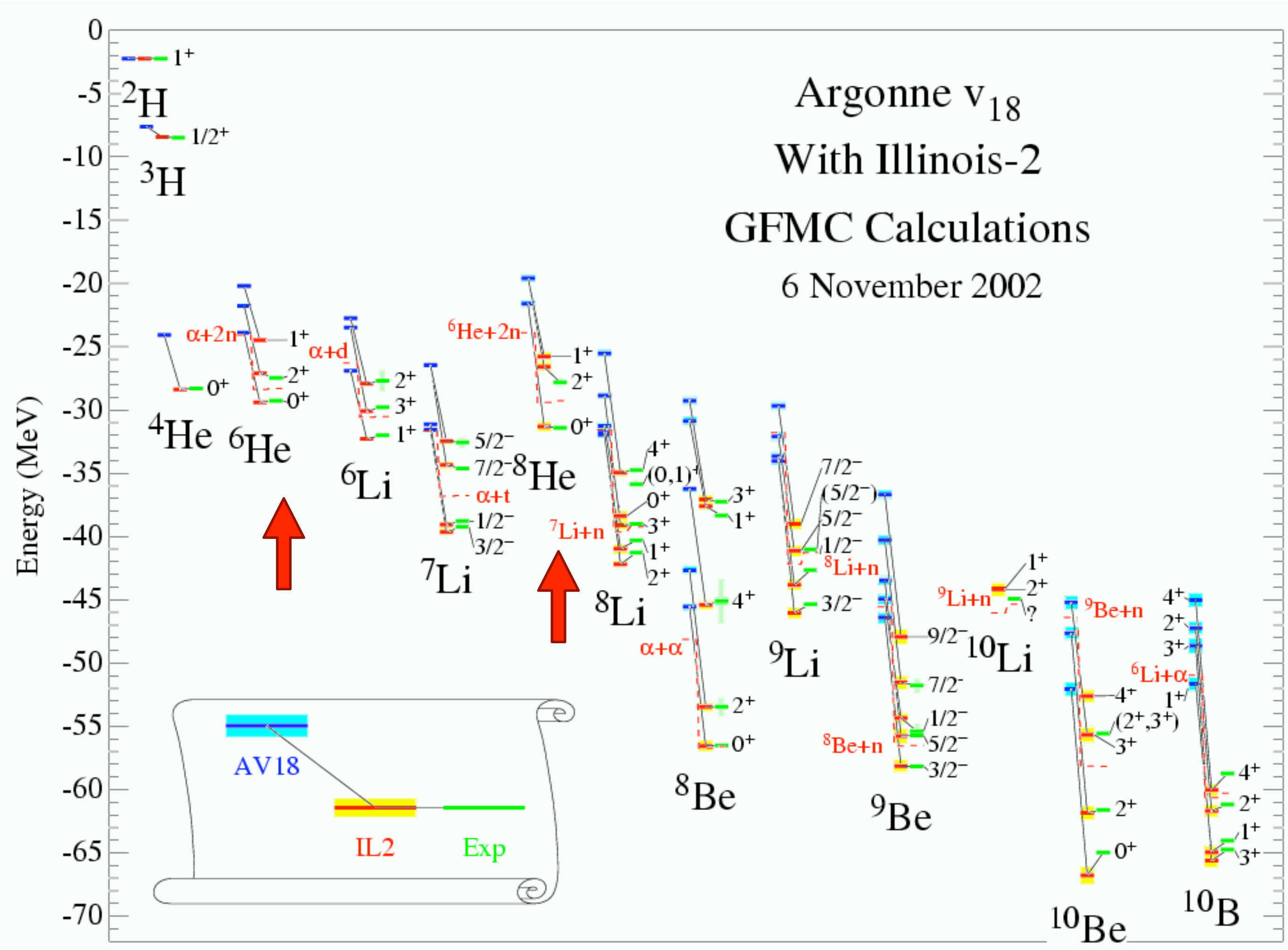

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does not bind the helium halo with respect to 2n emission

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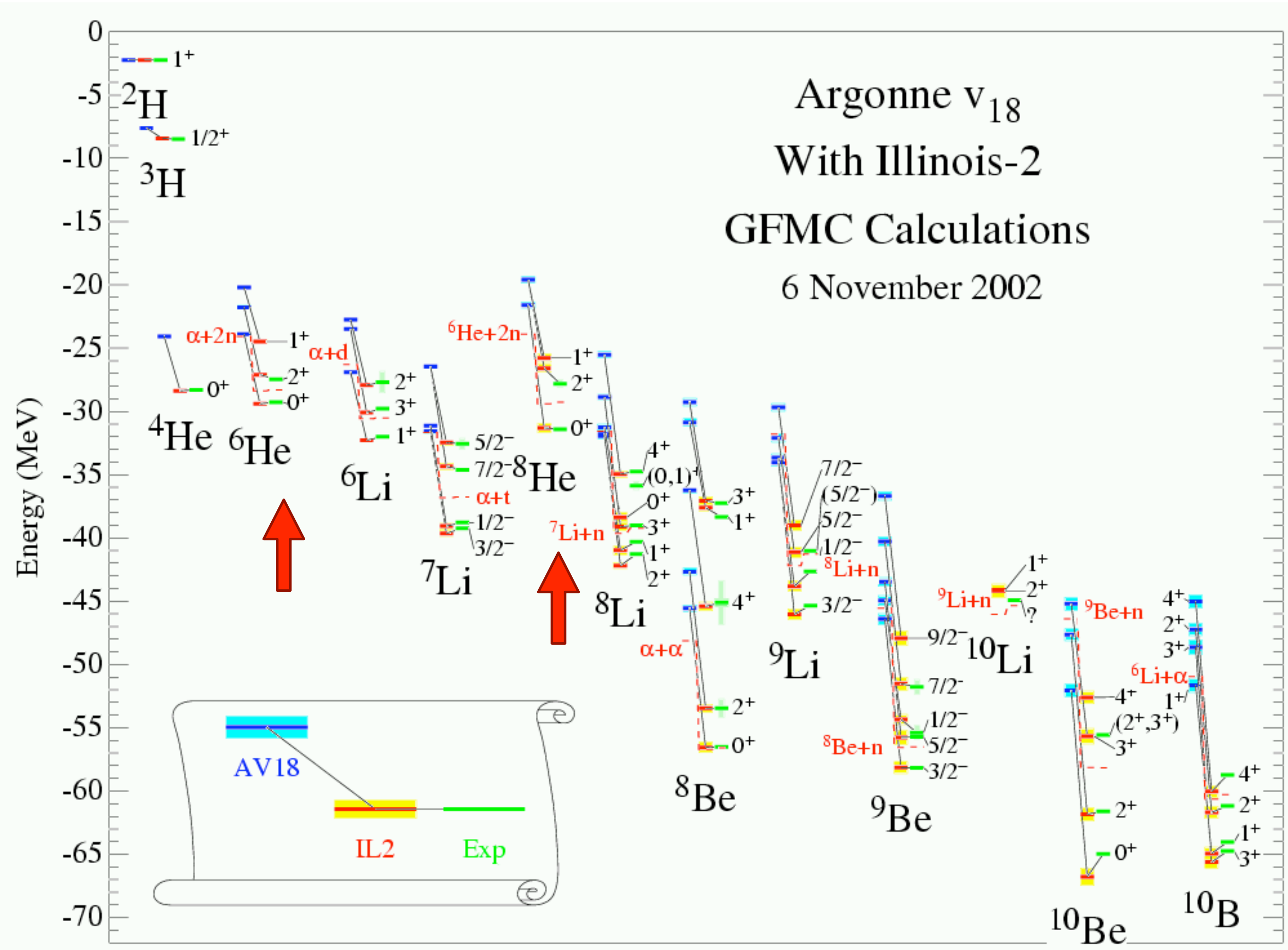

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$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^{R}$$

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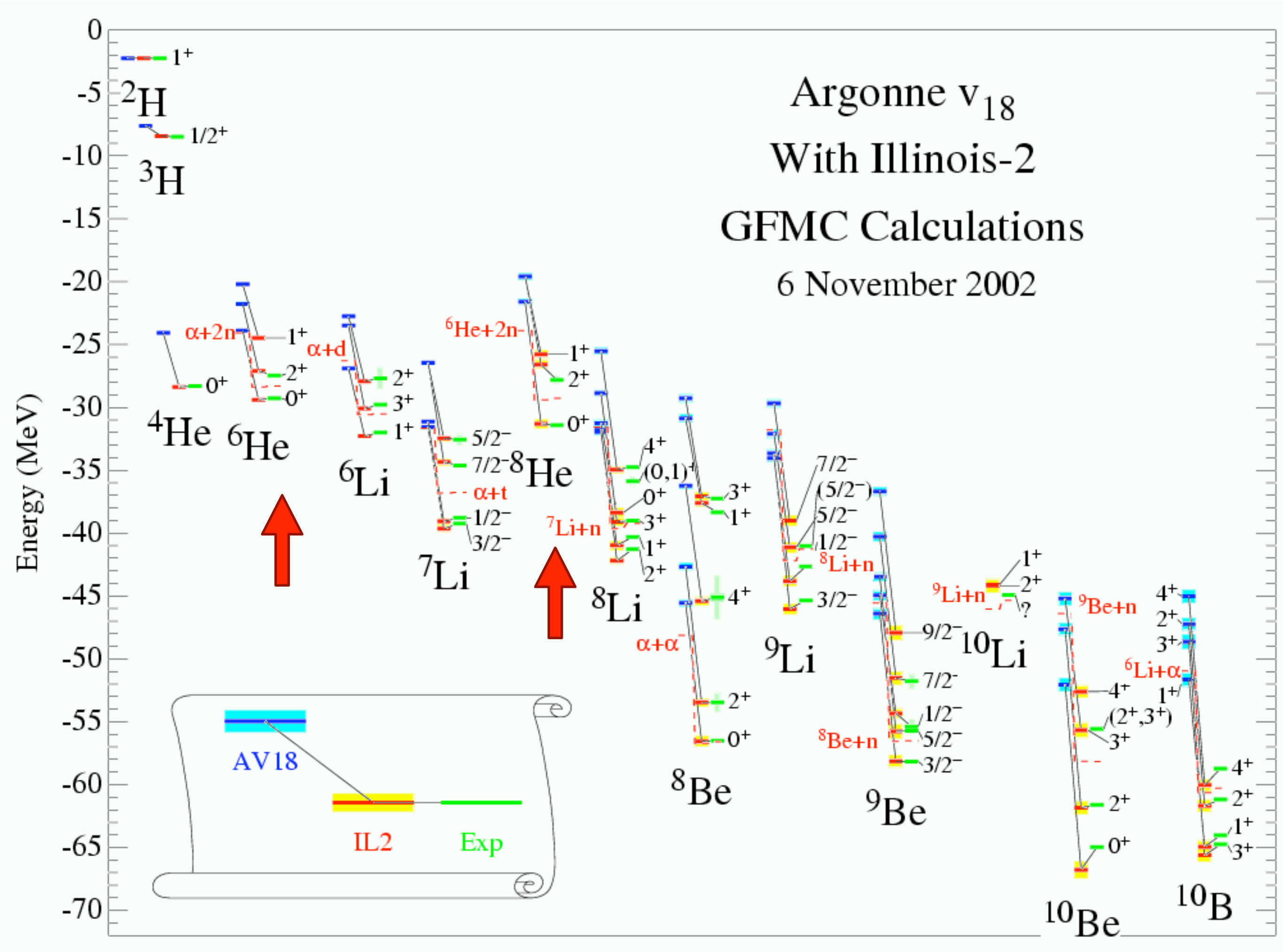

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$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^{R}$$

N.B.: parameters of the IL2 force are obtained from a fit of 17 states of A<9 including the binding energy of ^{6}He and ^{8}He

Ab-initio Calculations

NCSM Diagonalization Method using Harmonic Oscillator Basis $\psi_{nl}(r) \sim e^{-\nu r^2} L_n^{l+1/2}(2\nu r^2) \quad \nu = m\omega/2\hbar$

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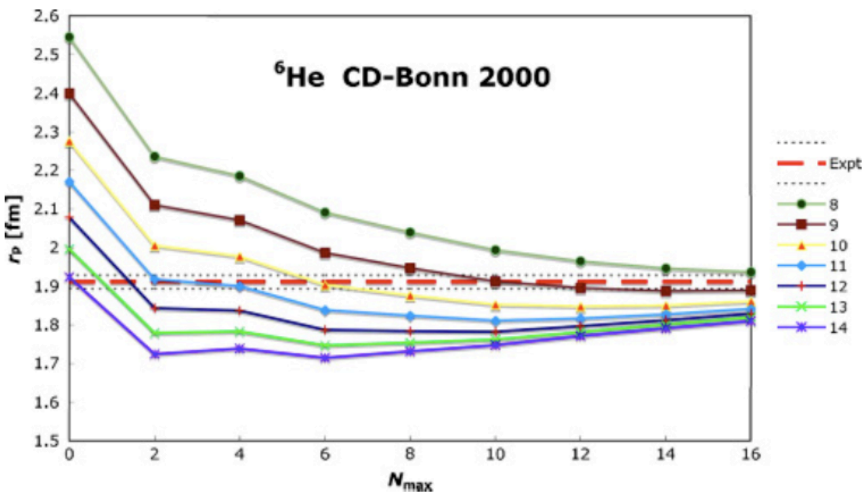
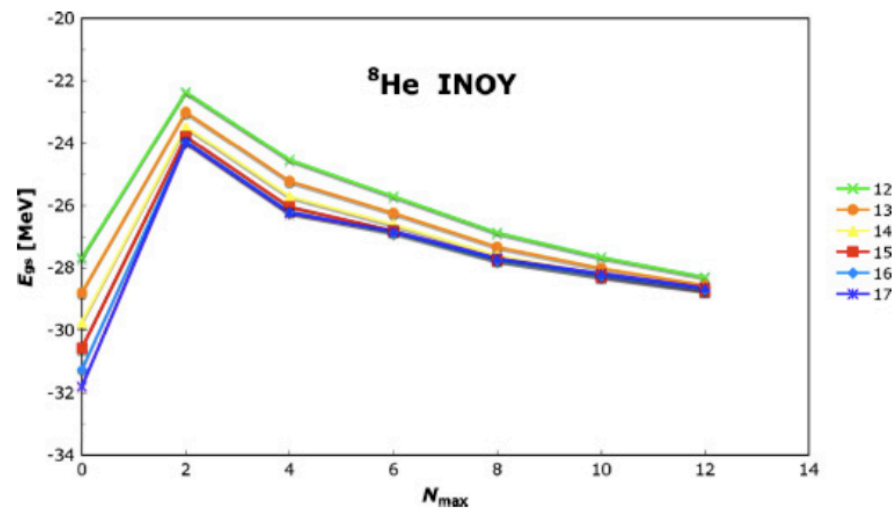
Helium Isotopes

Caurier and Navratil, PRC 73, 021302(R) (2006)





NN only with effective interaction and no effective operator

INOY ➡ short range phenomenology

CD-Bonn ➡ meson exchange theory

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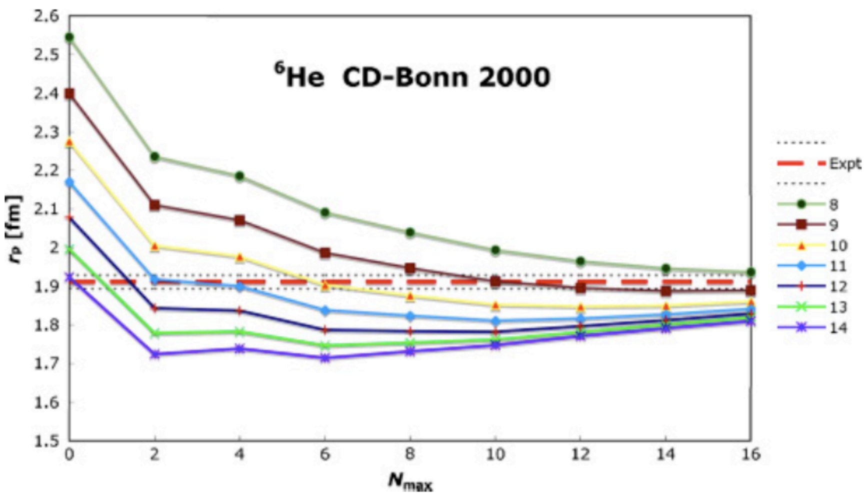

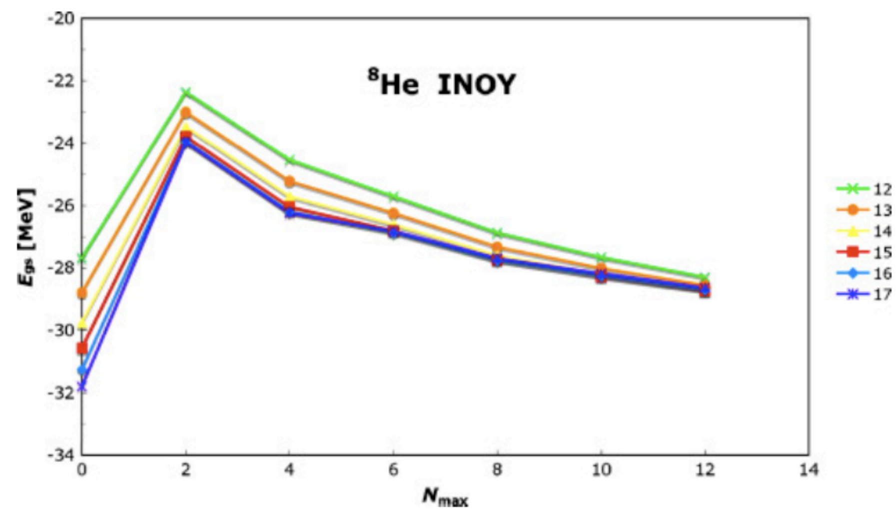

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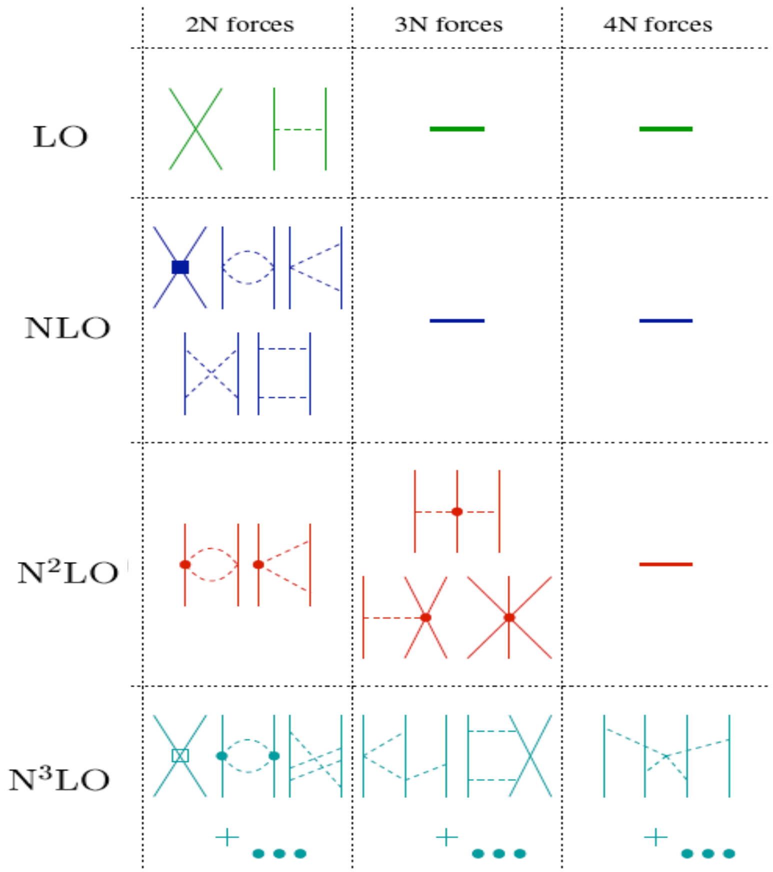
Slow convergence and HO parameter dependence in radius

What is missing?

An ab-initio calculation of helium halo nuclei
from chiral effective field theory potentials



TRIUMF

Ideally we want:

- To use methods that enable to incorporate the correct asymptotic of the w.f. for loosely bound systems
- To obtain convergent calculations, with no dependence on the model space parameters
- To systematically study the cutoff (in)dependence of predicted observables

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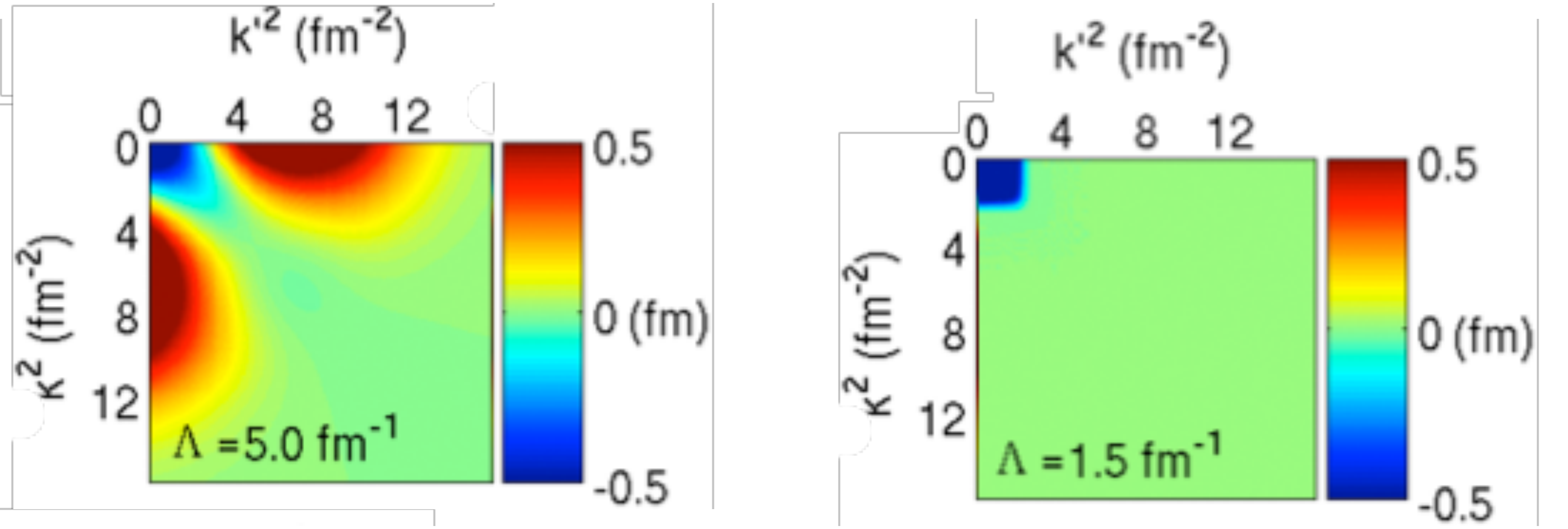
To facilitate convergence we use low-momentum interactions

Low momentum interactions

Effective field theory potentials and low-momentum evolution $V_{\mathrm{low\ k}}$

evolve to lower resolution (cutoffs) by integrating out
high-momenta Bogner, Kuo, Schwenk (2003)
smooth cutoff Bogner, Furnstahl, Ramanan, Schwenk (2007)



$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$

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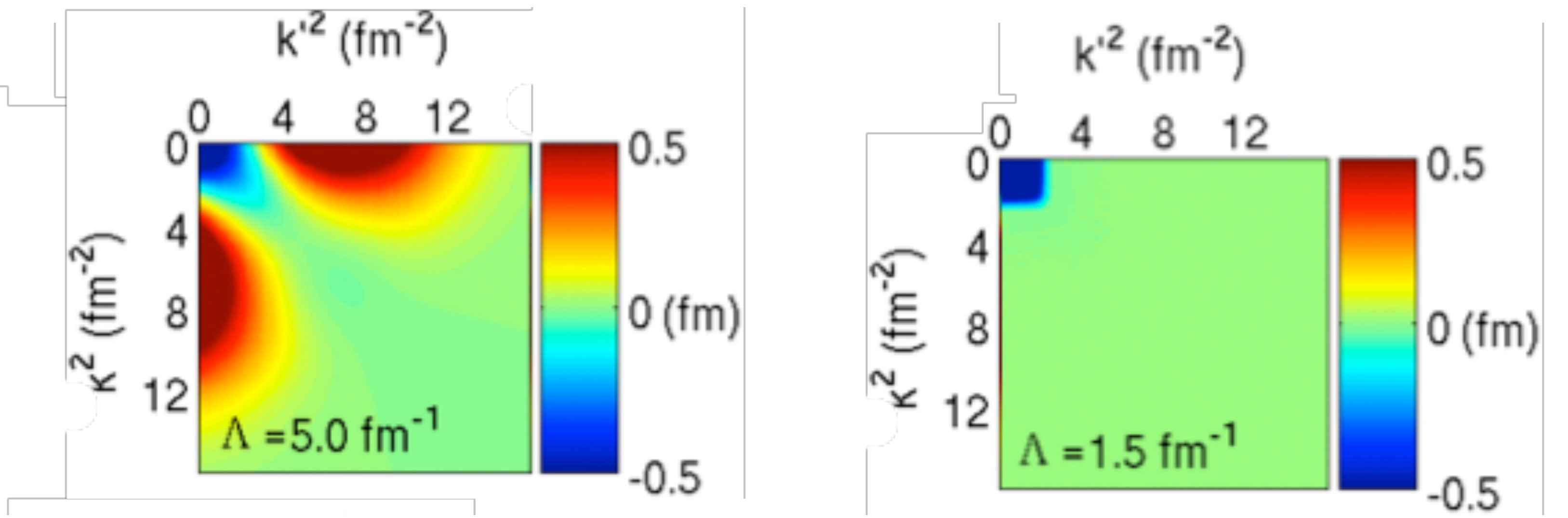

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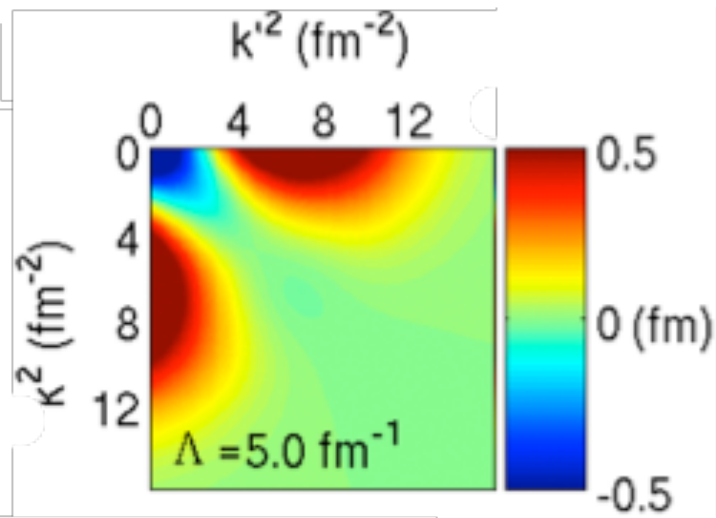
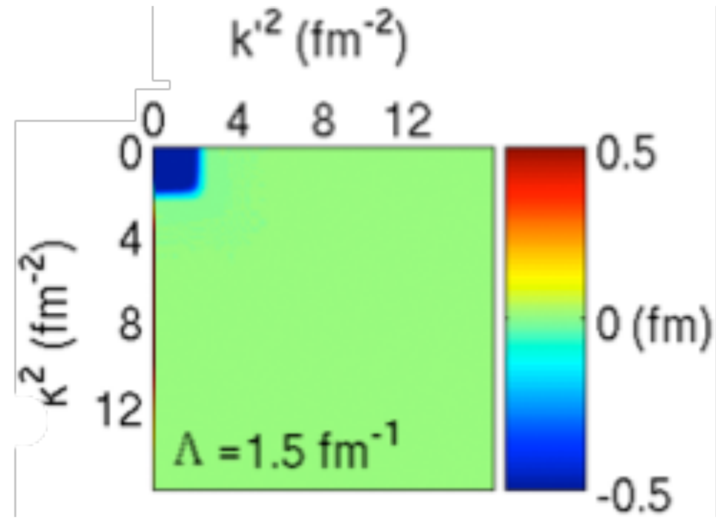


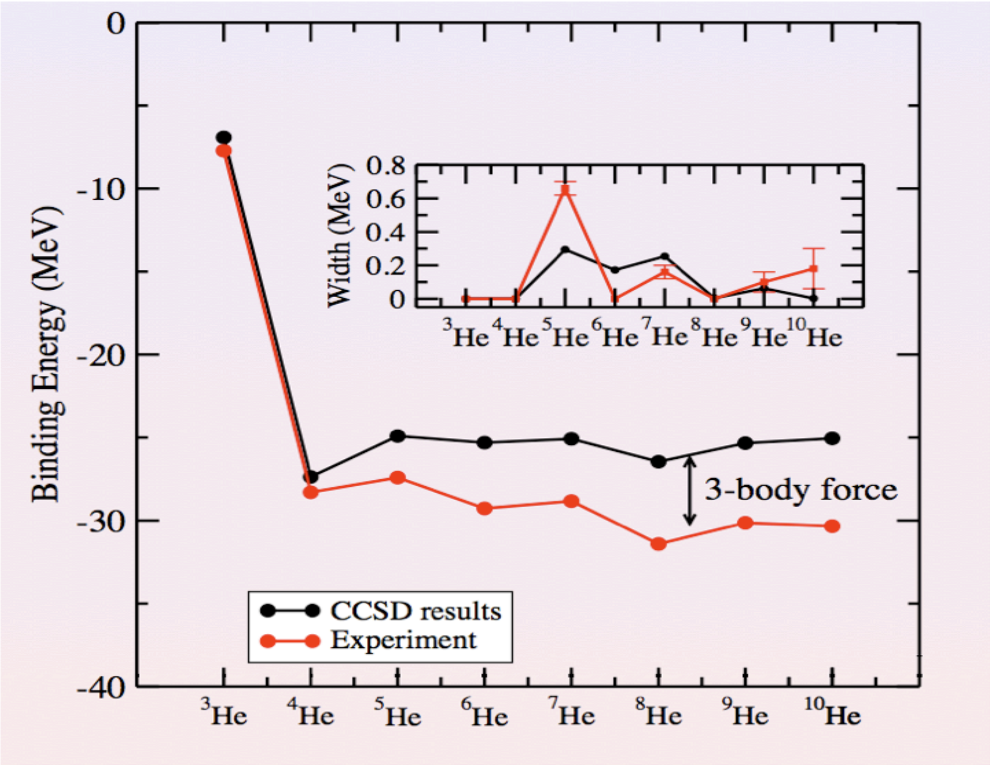


$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$

Variation of the cutoff provides a tool to estimate the effect of 3N forces

What is done already?

- Coupled Cluster calculations for the helium isotope chain



G.Hagen et al., Phys. Lett. B656, 169 (2007)

V_{lowk} sharp cutoff with

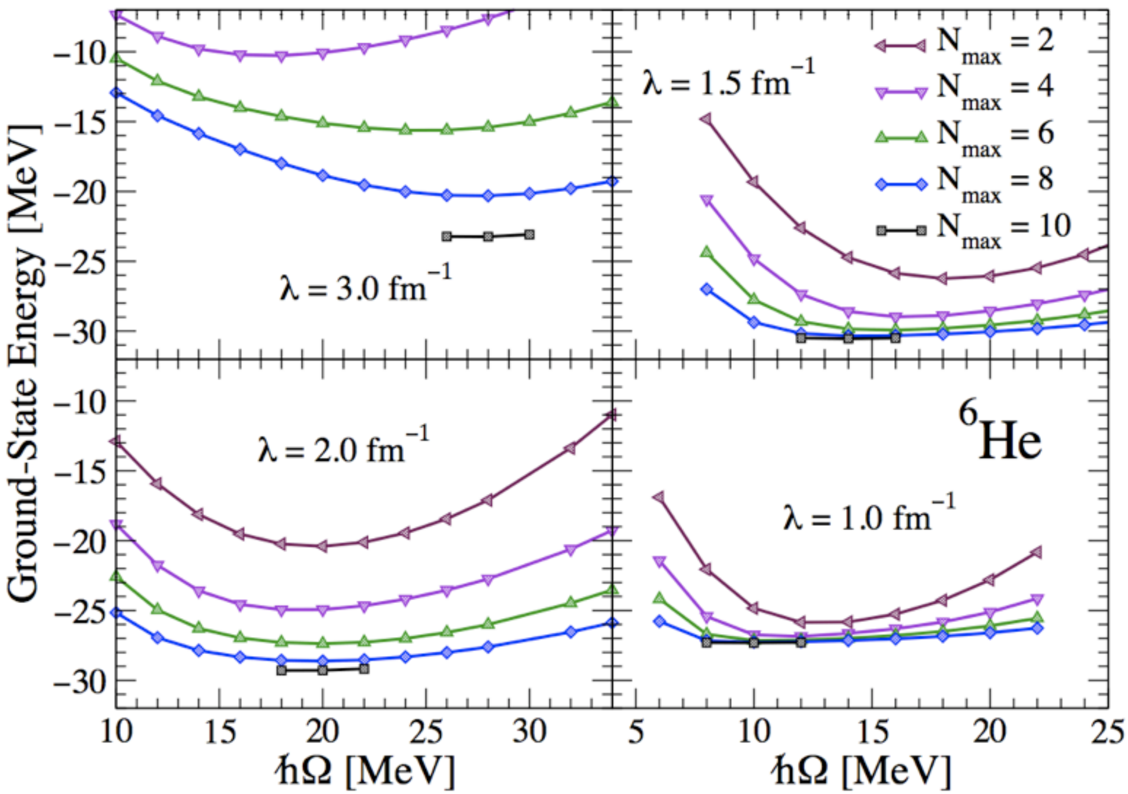
$$\Lambda = 1.9\text{fm}^{-1}$$

as a convenient choice to minimize the net 3NF effect on ^3H and ^4He

least accuracy of coupled cluster in ^6He, open shell

What is done already?

- NCFC calculations for ^{6}He with SRG potentials



S.K.Bogner et al., Nucl.Phys. A801,21 (2008)

Reduced model space used, difficult to extrapolate

running of binding energy in λ due to missing 3NF

What is done already?

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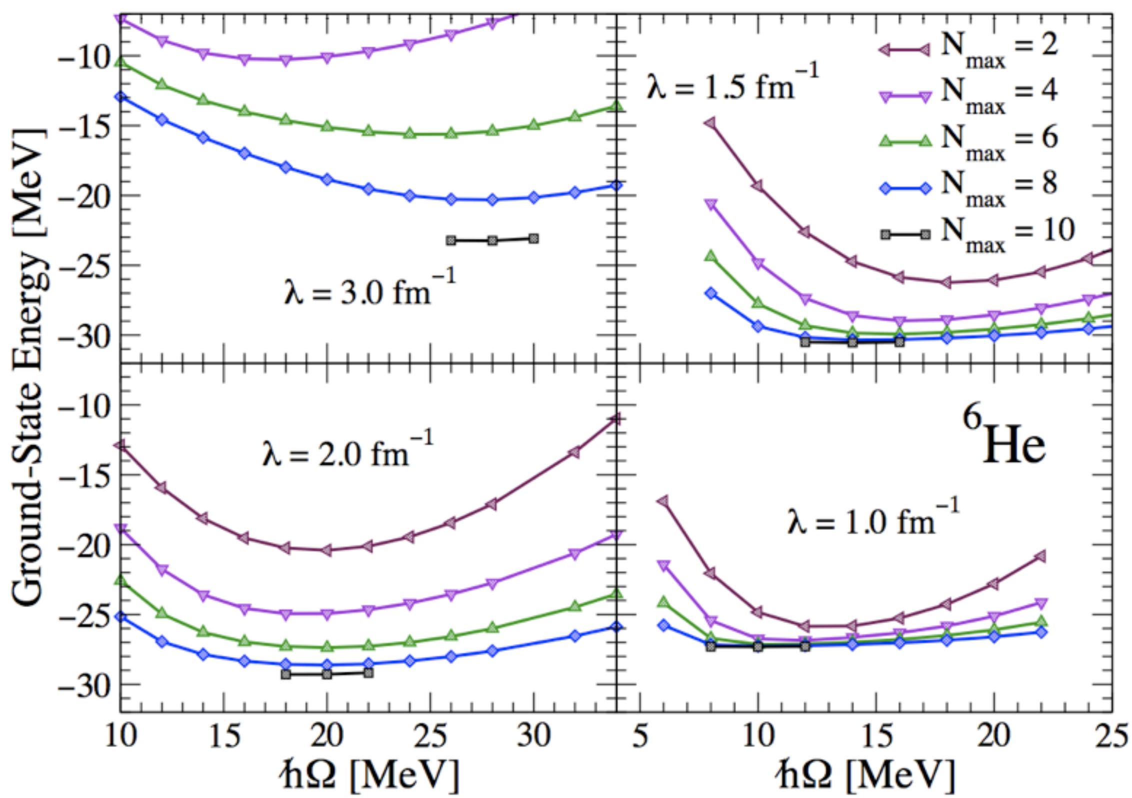

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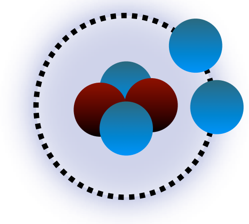
What we want to do more for the helium halo nuclei?

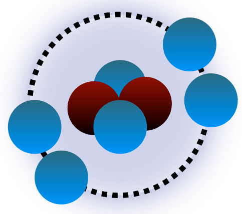
- Improve ^6He and ^8He description
- Study the cutoff dependence to assess the error due to neglected 3NF

Our Ab-initio Methods

- Hyper-spherical Harmonics Expansion for ^{6}He



- Cluster Cluster Theory for ^{8}He



Hyper-spherical Harmonics

- Few-body method - uses relative coordinates $|\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle$

$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM}$ $\quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

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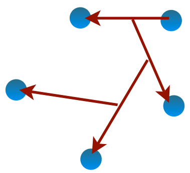

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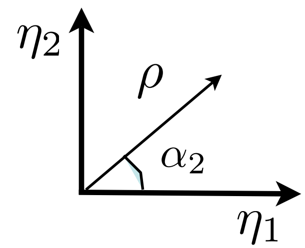
A=3

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \end{cases}$$



TRIUMF

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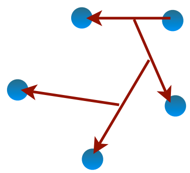

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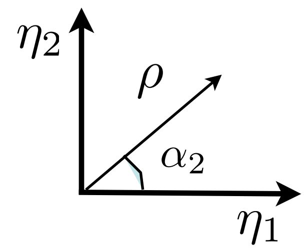
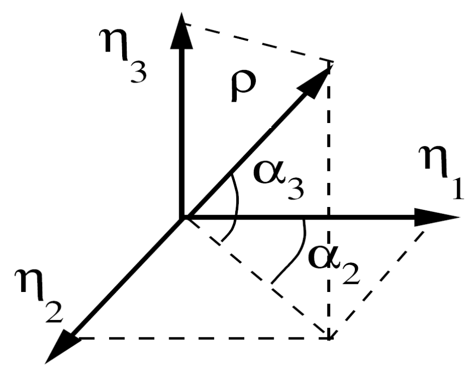
A=3

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \end{cases}$$



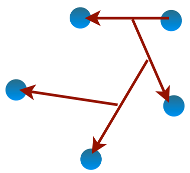
A=4

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \\ \vec{\eta}_3 = \{\eta_3, \theta_3, \phi_3\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \\ \sin\alpha_3 = \frac{\eta_3}{\rho} \end{cases}$$



Hyper-spherical Harmonics

- Few-body method - uses relative coordinates $|\psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1,\vec{\eta}_2,\ldots,\vec{\eta}_{A-1})\rangle$



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

$$H_0(\rho,\Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right)[\mathcal{Y}_{[K]}^{\mu}(\Omega)\chi_{ST}^{\bar{\mu}}]_{JT}^{a}$$

Asymptotic $e^{-a\rho}$ $\rho \to \infty$

Model space truncation $K \leq K_{max}$, Matrix Diagonalization $\langle\psi|H_{(2)}|\psi\rangle = \frac{A(A-1)}{2}\langle\psi|H_{(A,A-1)}|\psi\rangle$

Can use non-local interactions

Most applications in few-body; challenge in A>4 Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

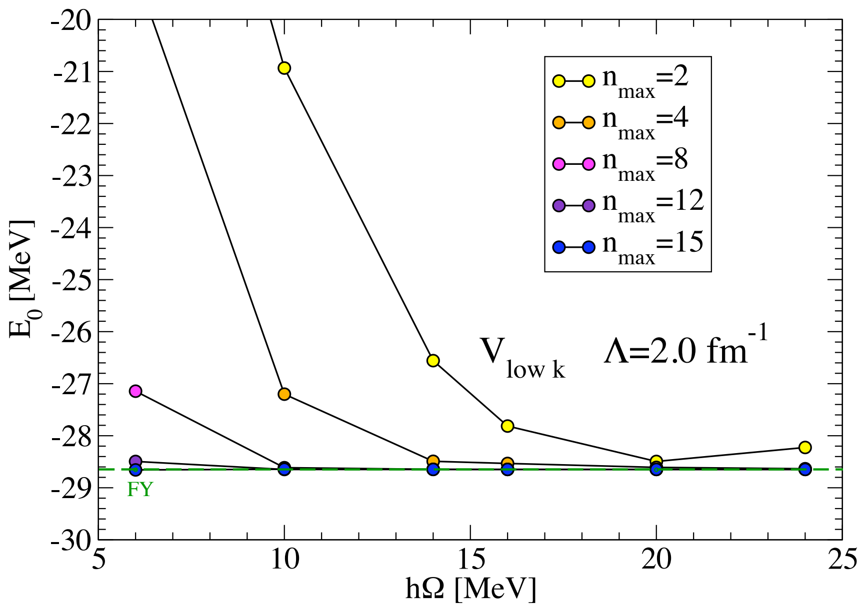
Hyper-spherical Harmonics

How to use non-local interactions with HH ?

$$\hat{V} = \sum_{nn',\ell\ell'}^{n_{max},\ell_{max}} |n(\ell s)j\rangle v_{nn'\ell\ell's}^{j,\hbar\Omega} \langle n'(\ell' s')j| \text{ with } v_{nn'\ell\ell's}^{j,\hbar\Omega} = \langle n(\ell s)j|\hat{V}|n'(\ell' s')j\rangle$$

- Expansion of Hilbert space size $\longrightarrow$ $K_{\max}$
- Expansion of the potential $\longrightarrow$ $n_{max}, \ell_{max} \quad \rightarrow \quad \hbar\Omega$

^{4}He



A=6 with a "soft interaction"

HH for ^{6}He with JISP16 NN interaction

D. Gazit et al., arxiV:0903.1048

K_{max}	B.E.
4	18.367
6	24.103
8	26.392
10	27.560
12	28.112
14	28.424
∞	28.70(13)

Comparison with No-Core-Full-Configuration

(1) A.M. Shirokov et al., Phys.Lett. B644, 33 (2007) B.E.=28.32(28) MeV $N_{max} = 12, \hbar\Omega = 17.5\text{MeV}$

(2) P. Maris et al., PRC 79, 104308 (2009) B.E.=28.68(12) MeV extrapolation from $N_{max} = 14$

HH consistent with NCFC, but have the advantage of not having HO parameter dependence

Coupled Cluster Theory

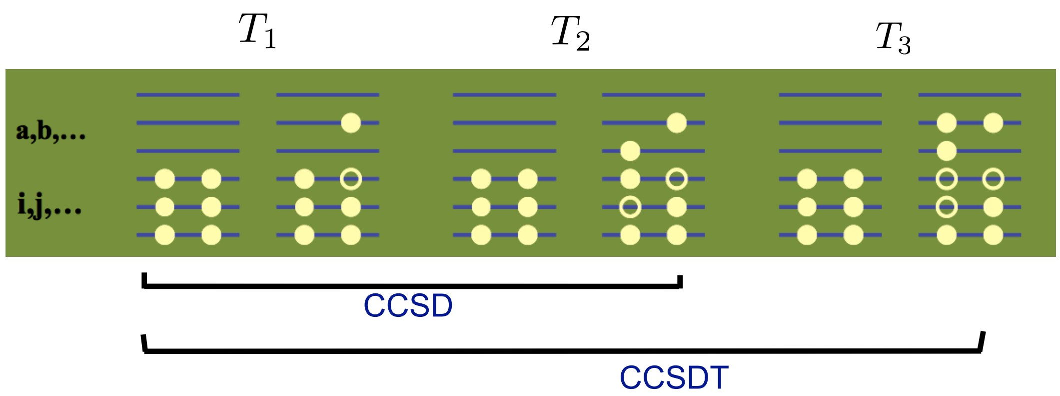
- Many-body method- uses particle coordinates

$$|\psi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle = e^T |\phi(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\rangle$$

→ reference SD

$$T = \sum T_{(A)} \qquad T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$

$$T_2 = \sum_{ia} t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i$$



CCSD Equations

$$E_0 = \langle \phi | e^{-T} H e^T | \phi \rangle$$

$$0 = \langle \phi_i^a | e^{-T} H e^T | \phi \rangle$$

$$0 = \langle \phi_{ij}^{ab} | e^{-T} H e^T | \phi \rangle$$

CC is flexible in single particle w.f. adopted

If use HF

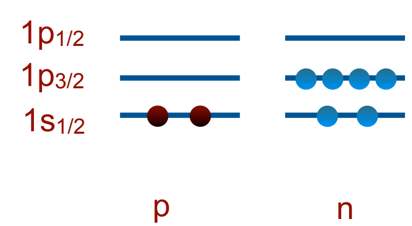
Asymptotic $\phi_i \sim e^{-k_i r_i} \quad r \to \infty$

Model space truncation $N \leq N_{max}$

Can use non-local interactions

Applicable to medium-mass nuclei

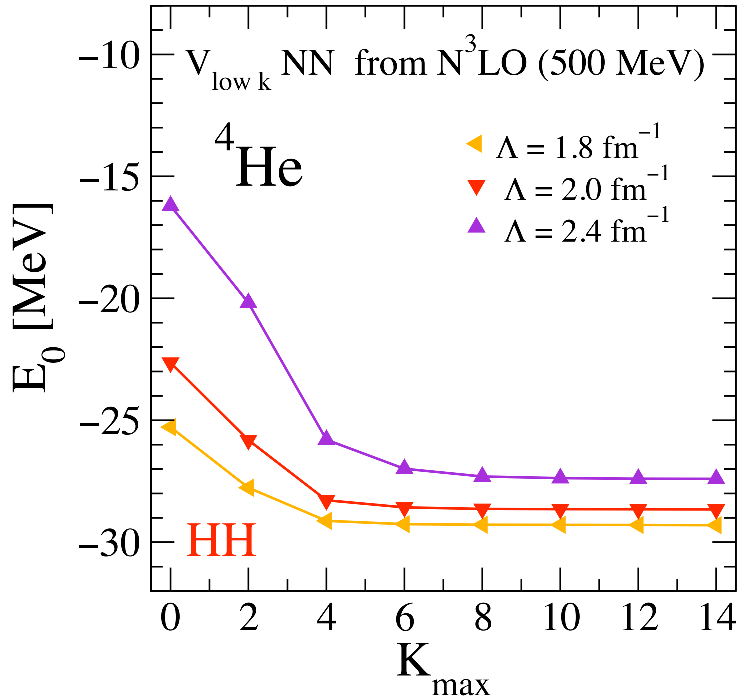
Use it for ^{8}He, closed shell nucleus



Results for binding energies

Benchmark on ^{4}He

$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$



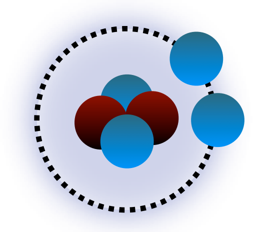
- Benchmark HH-CC-FY on ^{4}He -

Method	$\Lambda = 2.0\ fm^{-1}$	$E_0(^4He)$ [MeV]
Faddeev-Yakubovsky (FY)		-28.65(5)
Hyperspherical harmonics (HH)		-28.65(2)
CCSD level coupled-cluster theory (CC)		-28.44
Lambda-CCSD(T) (CC with triples corrections)		-28.63

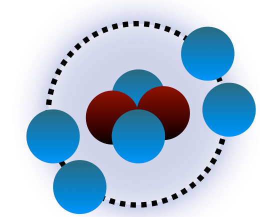
E^{exp}=-28.296 MeV

Helium Halo Nuclei

- Binding Energy -



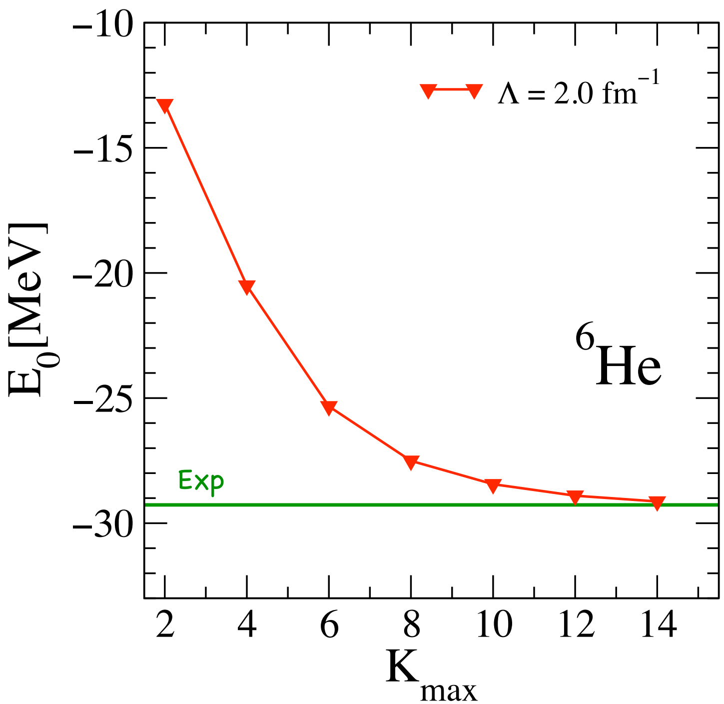
$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$

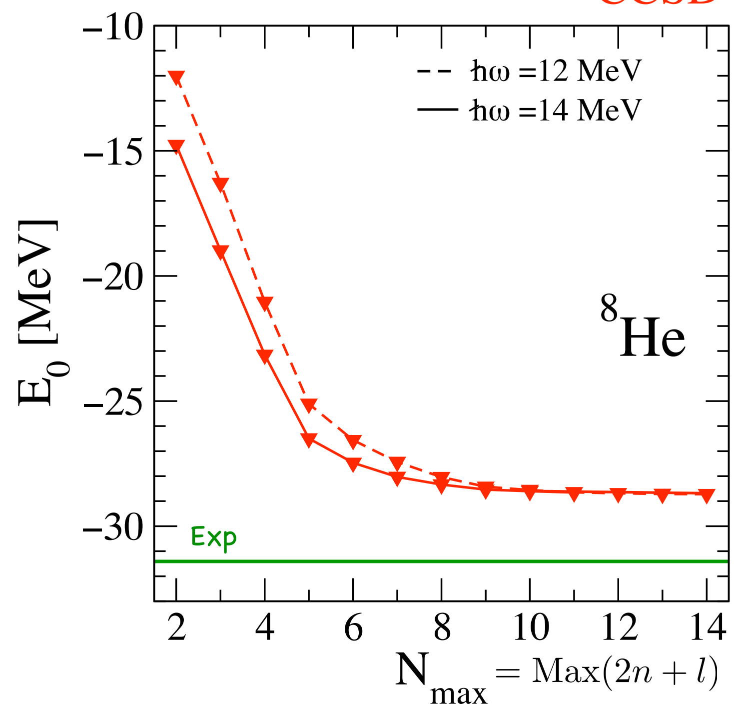


HH

$V_{\text{low k}}$ NN from N^3LO

CCSD



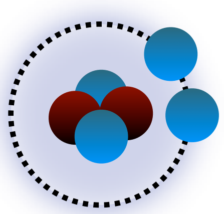


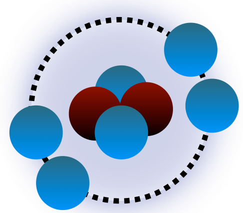
S.Bacca et al., arXiv:0902.1696

Helium Halo Nuclei

- Binding Energy -

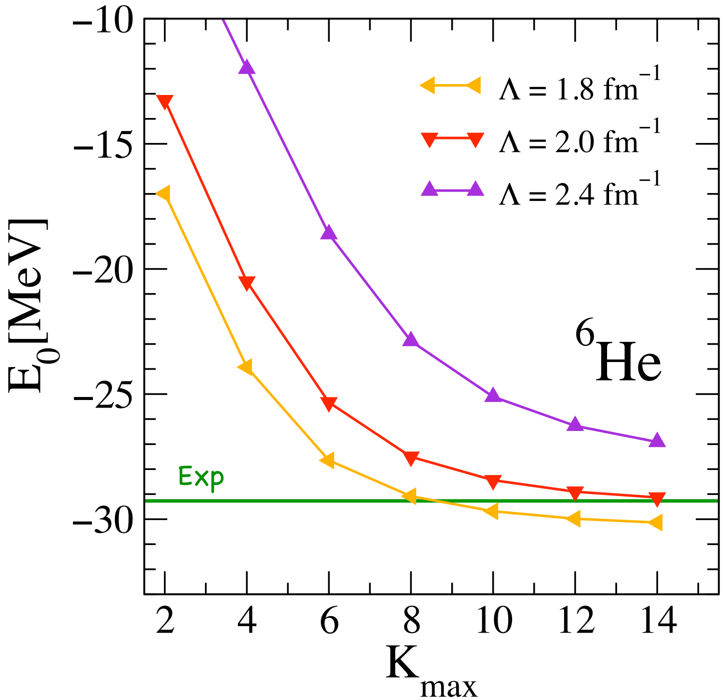
$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$



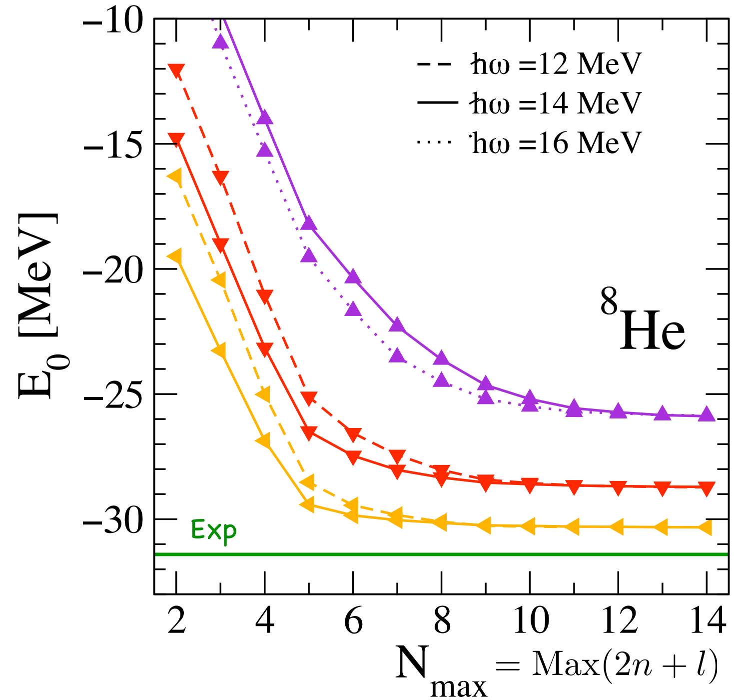


$V_{\text{low k}}$ NN from N^3LO

HH



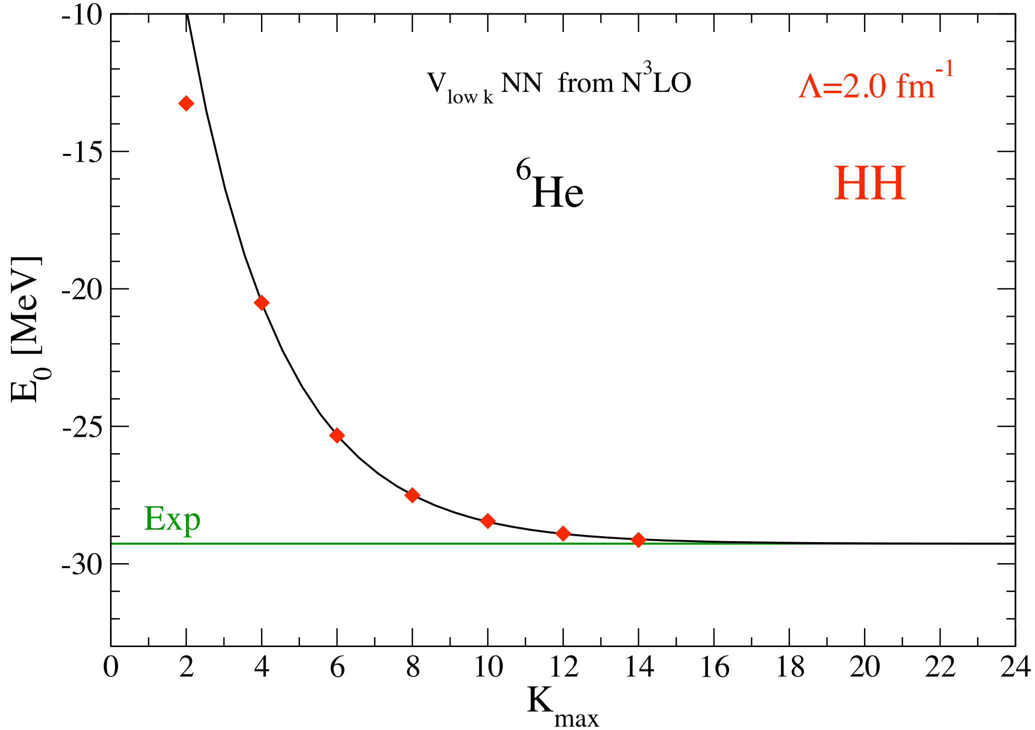
CCSD



S.Bacca et al., arXiv:0902.1696

Binding Energy ^{6}He

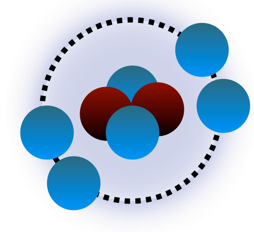
- Extrapolation -



$$E(K_{max}) = E^{\infty} + Ae^{-BK_{max}}$$

Λ	$E(K_{max} = 14)$	E^{∞}
1.8	-30.13	-30.28(3)
2.0	-29.13	-29.35(13)
2.4	-26.91	-27.62(19)

Binding Energy ^{8}He



- CC Theory: Add Triples Correction -

Hilbert space: 15 major shell

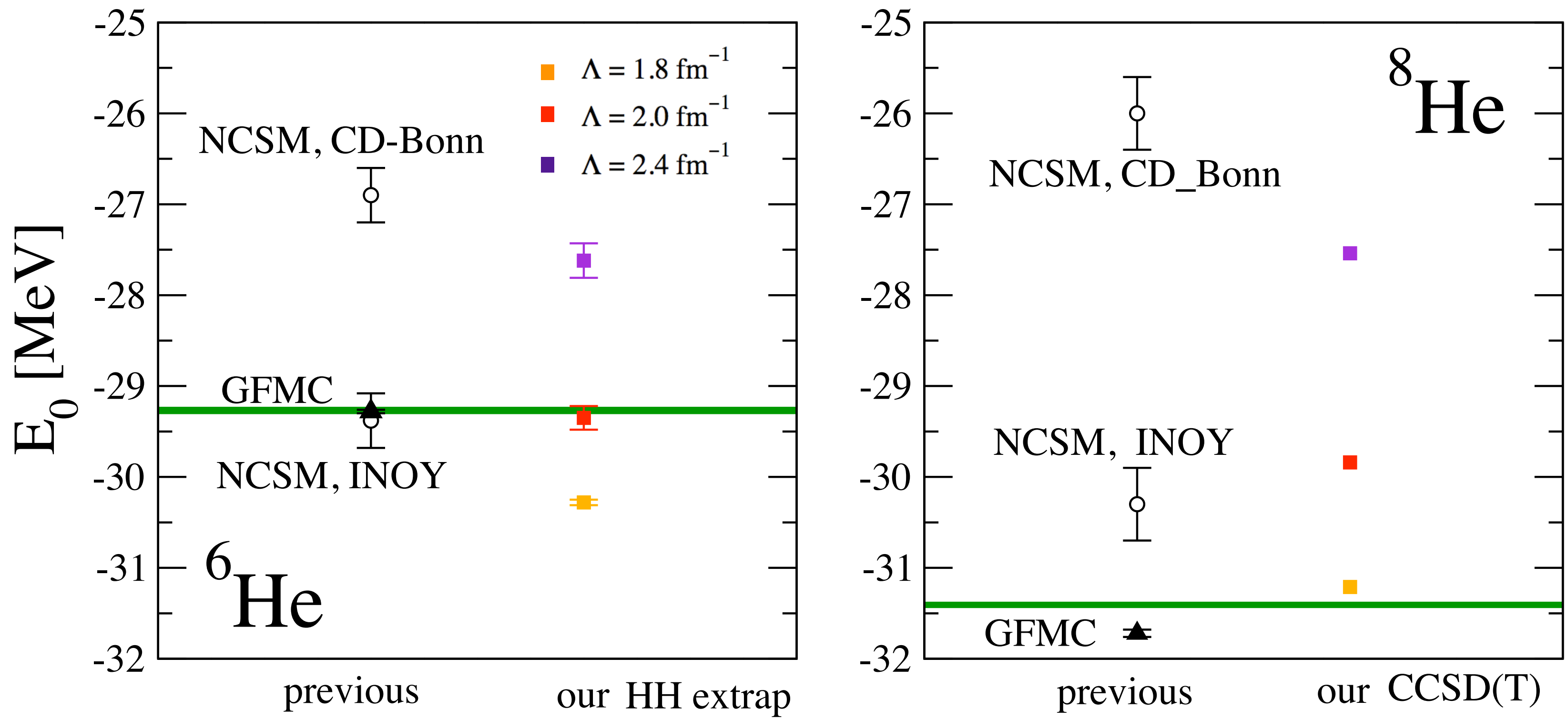
Values in MeV

Λ	E[CCSD]	E[Lambda-CCSD(T)]	Δ
1.8	-30.33	-31.21	0.88
2.0	-28.72	-29.84	1.12
2.4	-25.88	-27.54	1.66

- Triples corrections are larger for larger cutoff
- Their relative effect goes from 3 to 6%

TRIUMF

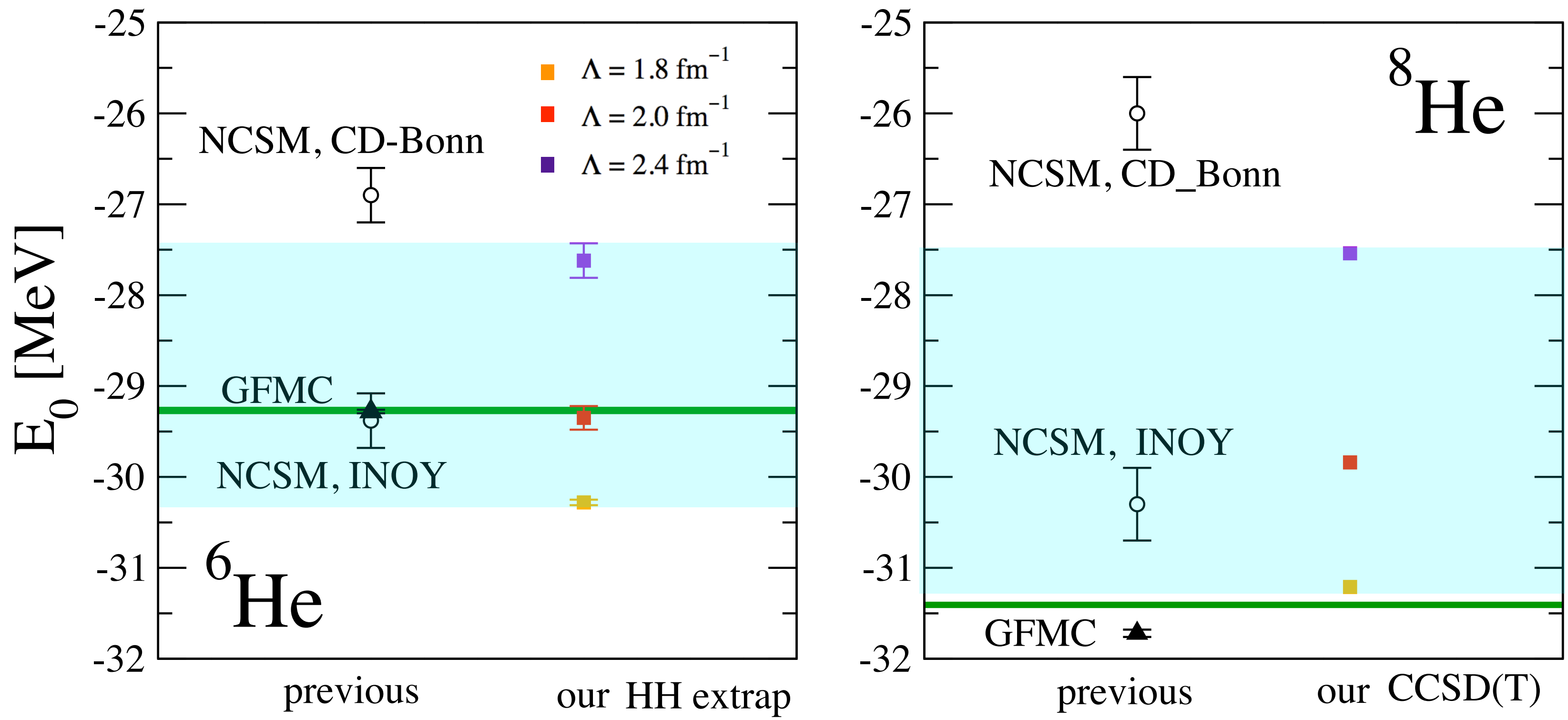
Binding Energy Summary



Experimental data

— TITAN ISAC-TRIUMF

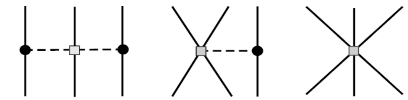
Binding Energy Summary



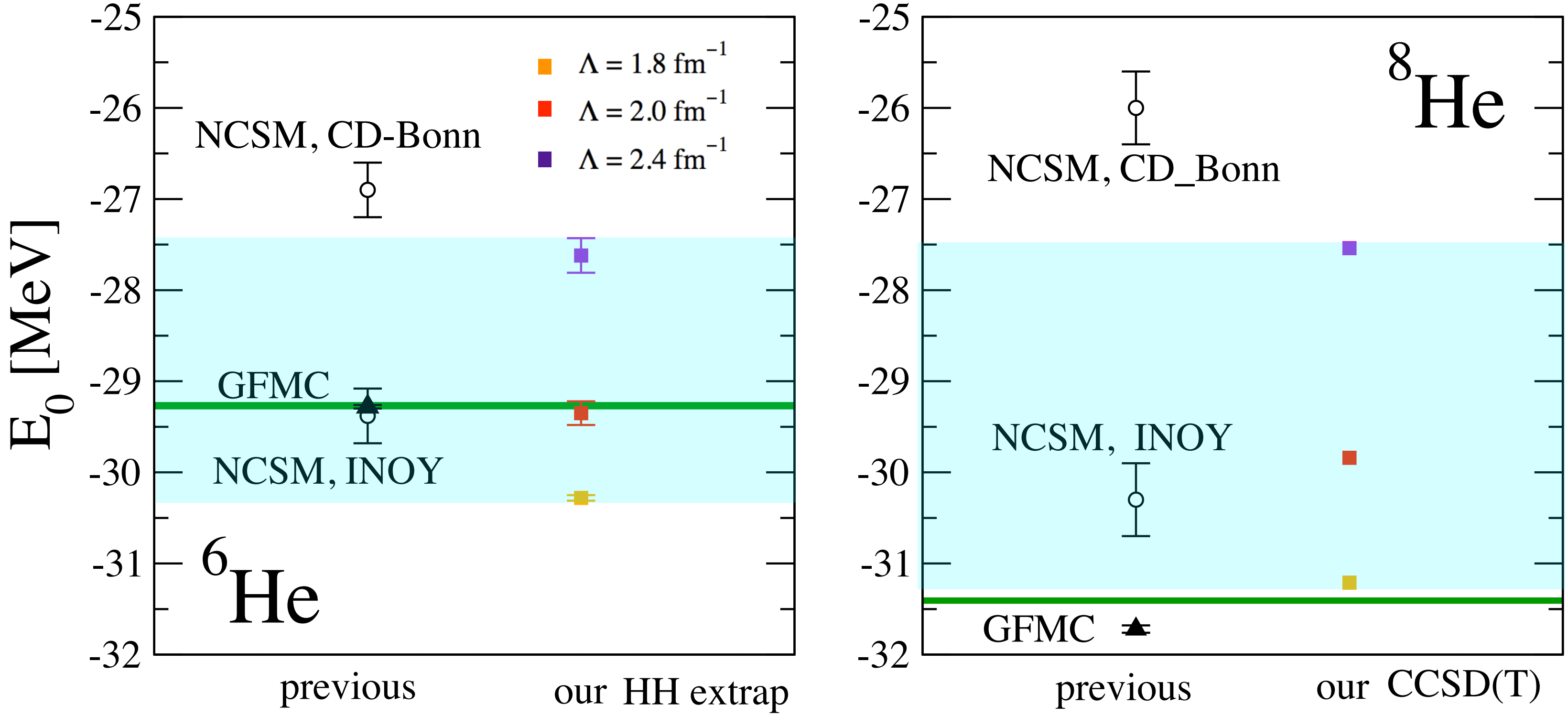
Experimental data

TITAN
ISAC-TRIUMF

Our estimated error in neglected



Binding Energy Summary



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TITAN ISAC-TRIUMF

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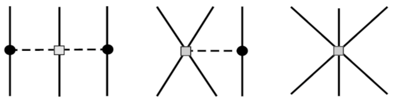
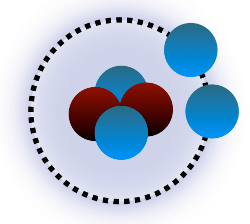


- For cutoff 2.0 fm^{-1} ^{4}He and ^{6}He are close to experiment, but ^{8}He is under-bound
- Low momentum 3NF are overall repulsive in s-shell nuclei and nuclear matter, but two-pion exchange c_i are attractive in ^{4}He and could provide further attractive spin-orbit (LS) contributions for the halo neutrons

TRIUMF

Results for radii

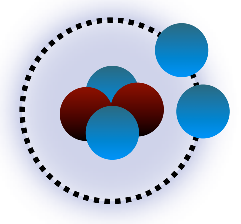
Radii for ^{6}He



- Matter radius -

$$r^2 = \frac{1}{A^2}\sum_{i<j}(\vec{r}_i - \vec{r}_j)^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2 \rangle}$$

Radii for ^{6}He



- Matter radius -

$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i' - r_j')^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2 \rangle}$$

$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i - \cancel{R_{cm}} - (r_j - \cancel{R_{cm}}))^2$$

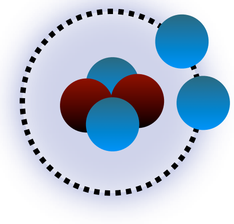
Radii for ^{6}He

- Matter radius -

$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2 \rangle}$$

TRIUMF

Radii for ^{6}He



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Is matter radius HO parameter independent?

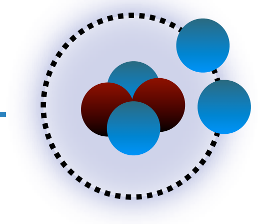
V_{lowk} with $\Lambda = 2.0 \text{fm}^{-1}$

K_{max}	$\hbar\omega$ 10 MeV	$\hbar\omega$ 30 MeV
2	1.909838	1.909850
4	1.880131	1.880159
6	1.944250	1.944330
8	2.019431	2.019712
10	2.087983	2.088125

values in fm

TRIUMF

Radii for ^{6}He



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Is matter radius HO parameter independent? Yes!

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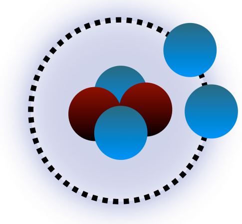
Radii for ^{6}He

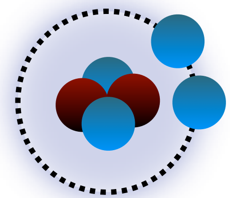


- Matter and proton radius -

$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2\rangle}$$

$$r_p^2 = \frac{1}{ZA}\sum_{i<j}(r_i - r_j)^2(q_i + q_j) - \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 \longrightarrow r_p = \sqrt{r_p^2}$$

Radii for ^{6}He



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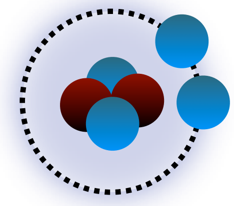
V_{lowk} with $\Lambda = 2.0 \mathrm{fm}^{-1}$

K_{max}	r [fm]	r_p [fm]
6	1.94	1.76
8	2.02	1.83
10	2.09	1.90
12	2.15	1.96
14	2.20	2.00

TRIUMF

Radii for ^{6}He



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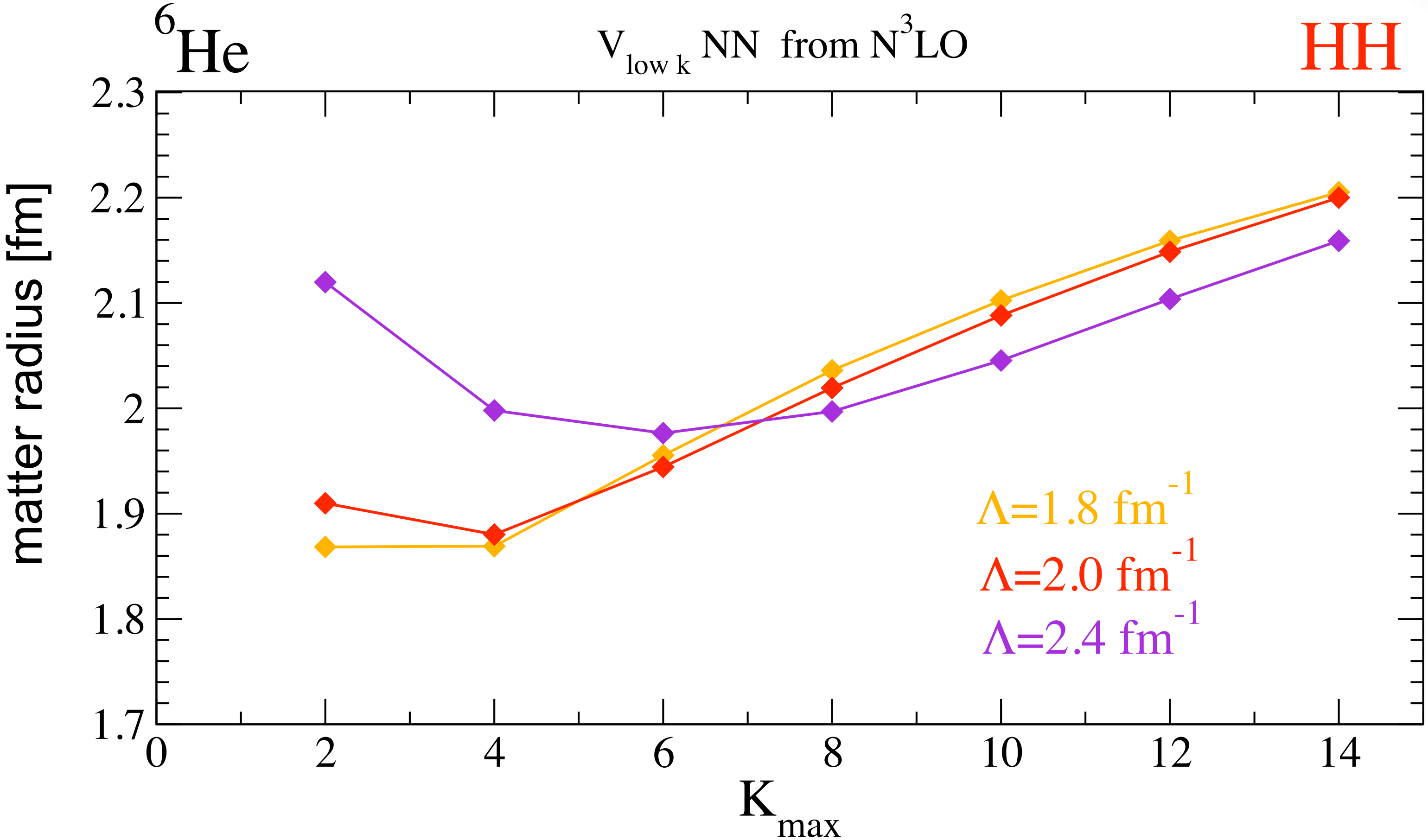
V_{lowk} with $\Lambda = 2.0\text{fm}^{-1}$

K_{max}	r [fm]	r_p [fm]
6	1.94	1.76
8	2.02	1.83
10	2.09	1.90
12	2.15	1.96
14	2.20	2.00

Convergence is slow...

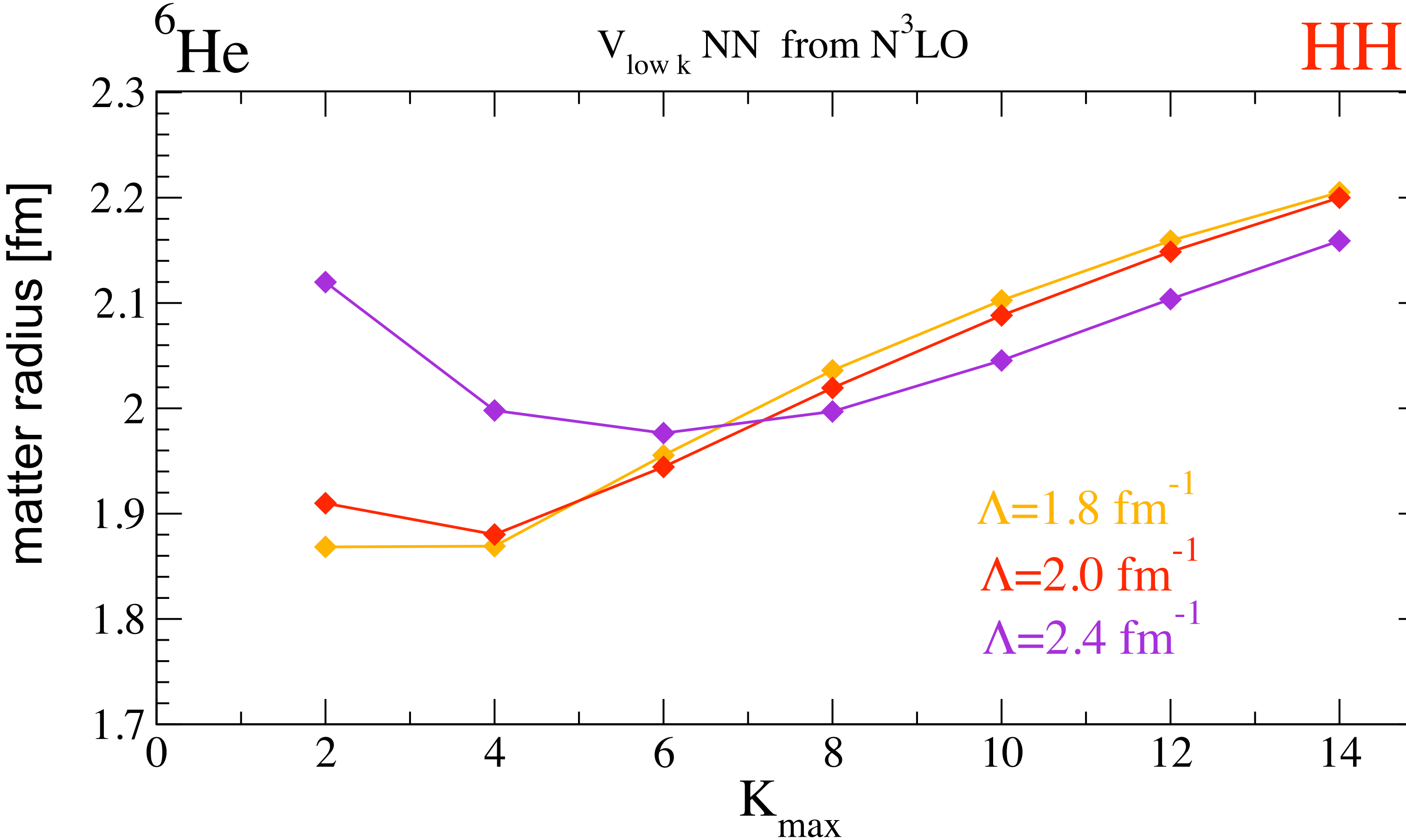
Radius for ^{6}He

- Matter Radius -



Radius for ^{6}He

- Matter Radius -



- What is going on?

- Binding energies before extrapolation -

$\Lambda = 2.4 \text{ fm}^{-1}$

^{6}He is not bound!

K_{max}	^{4}He [MeV]	^{6}He [MeV]	S_n[MeV]
10	-27.37	-25.10	
12	-27.39	-26.27	
14	-27.40	-26.91	
∞	-27.40	-27.62	0.22

- Binding energies before extrapolation -

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should start to bind ^{6}He in larger spaces

K_{max}	^{4}He [MeV]	^{6}He [MeV]	S_n[MeV]
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TRIUMF

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$\Lambda = 2.0 \text{ fm}^{-1}$

^{6}He bound

K_{max}	^{4}He [MeV]	^{6}He [MeV]	S_n[MeV]
10	-28.64	-28.44	
12	-28.65	-28.92	0.27
14	-28.65	-29.13	0.48
∞	-28.65	-29.35	0.70

- Binding energies before extrapolation -

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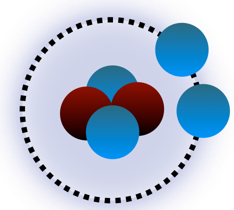
∞	-28.65	-29.35	0.70

$\Lambda = 1.8\ \mathrm{fm}^{-1}$

^{6}He bound

K_{max}	^{4}He [MeV]	^{6}He [MeV]	S_n[MeV]
10	-29.29	-29.69	
12	-29.29	-29.98	0.69
14	-29.29	-30.13	0.84
∞	-29.29	-30.28	0.98

- Binding energies before extrapolation -



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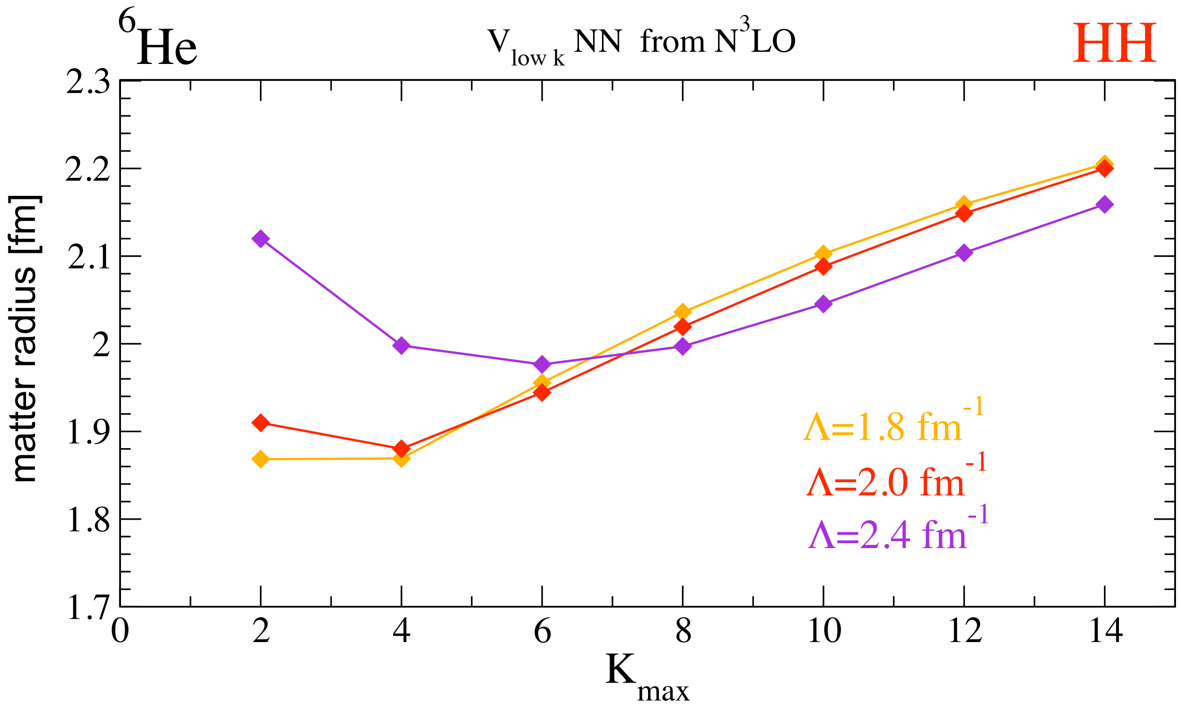
10	-29.29	-29.69	
12	-29.29	-29.98	0.69
14	-29.29	-30.13	0.84
∞	-29.29	-30.28	0.98

Exp=0.972

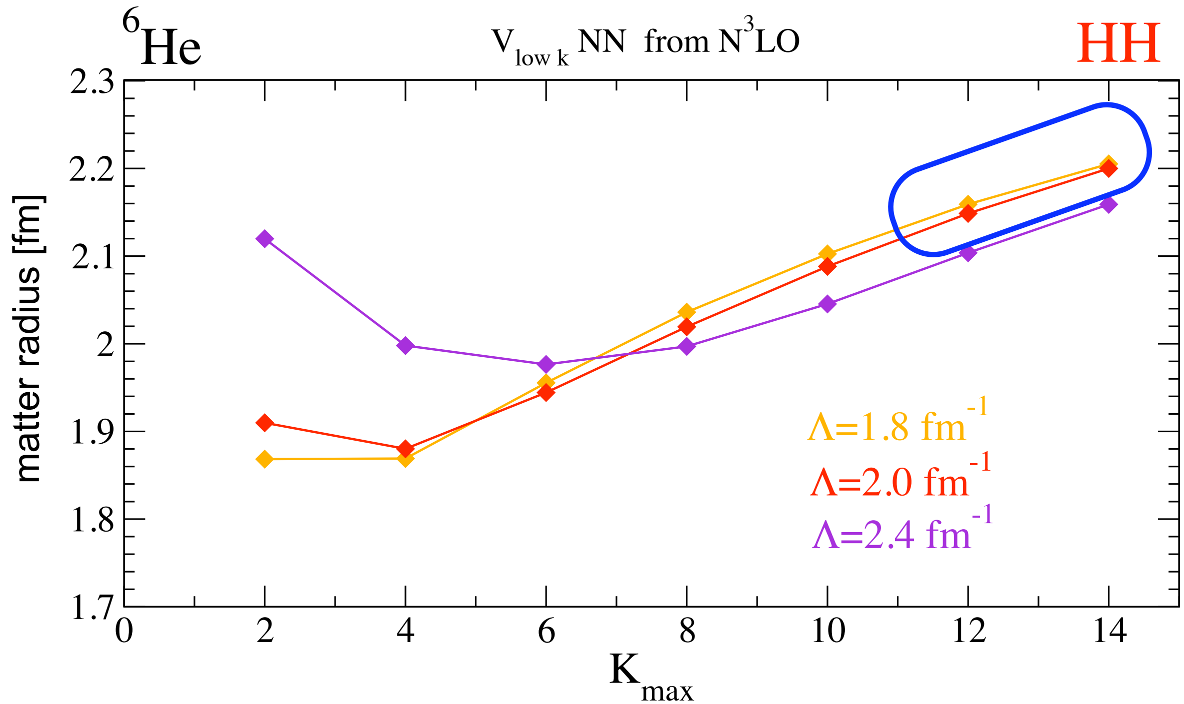
Radius for ^{6}He

- Matter Radius -



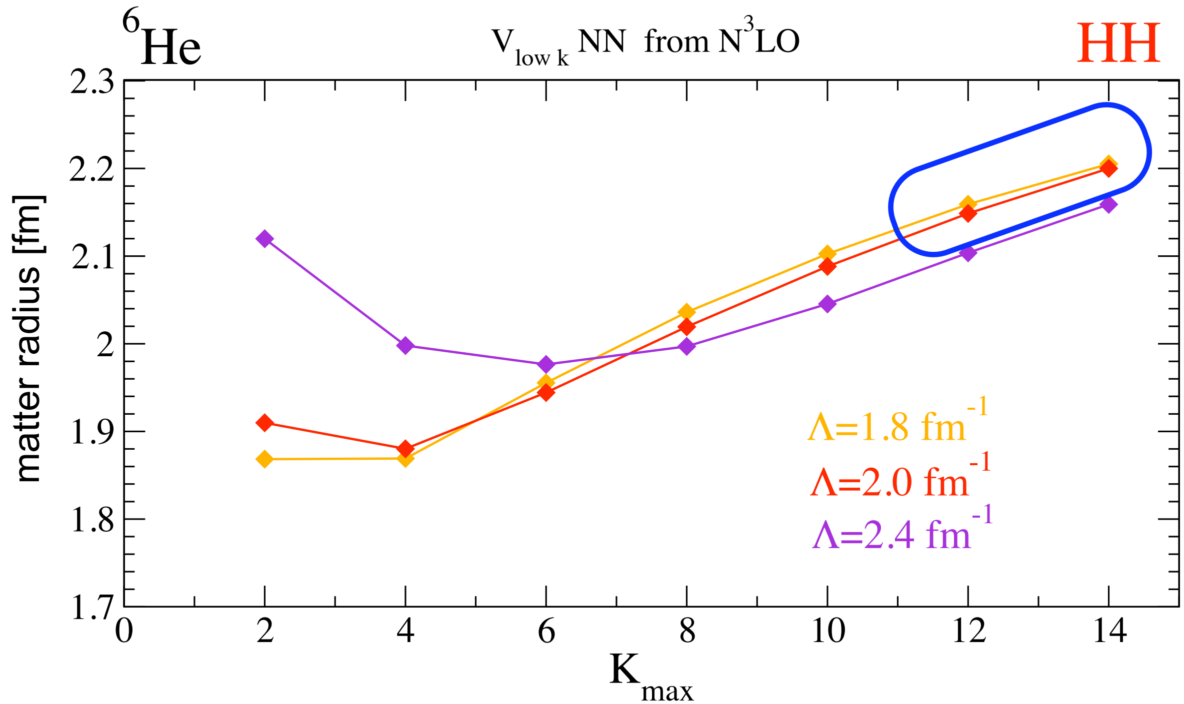
Radius for ^6He

- Matter Radius -



Radius for ^{6}He

- Matter Radius -

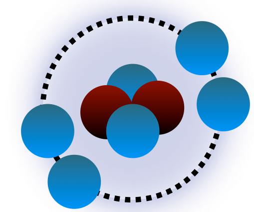


- To converge we need K_{max}=16

- It looks like for large cutoffs, were we recover N^3LO, ^{6}He will not be bound (as for AV18)

- The radius is very correlated to the S_n.

Same conclusions could be drawn by looking at proton radii



Radii for ^{8}He

- Matter and Point-proton Radius -

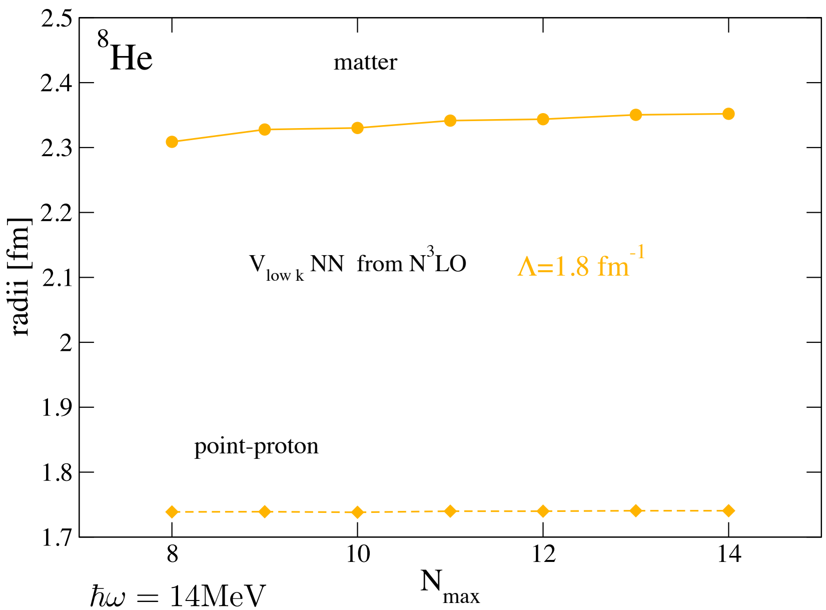
$V_{\mathrm{low\ k}}$ from N^{3}LO $\quad H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$

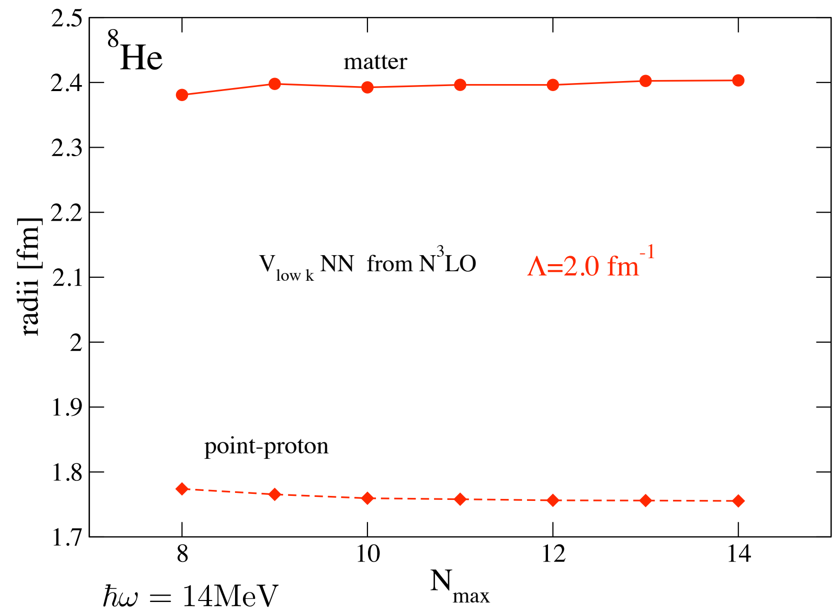
$$r^2 = \frac{1}{A}\sum_i^A r_i^2$$

$\sqrt{\langle r^2 \rangle}$ matter radius

$$r_p^2 = \frac{1}{Z}\sum_i^A r_i^2 \left(\frac{1+\tau_i^3}{2}\right)$$

$\sqrt{\langle r_p^2 \rangle}$ point-proton radius





- Point-proton radius is smaller than matter radius

Radii for ^{8}He

- Matter Radius: Comments... -

$$r^2 = \frac{1}{A}\sum_i^A r_i^2$$

What about c.m.?

Need to consider relative coordinates:

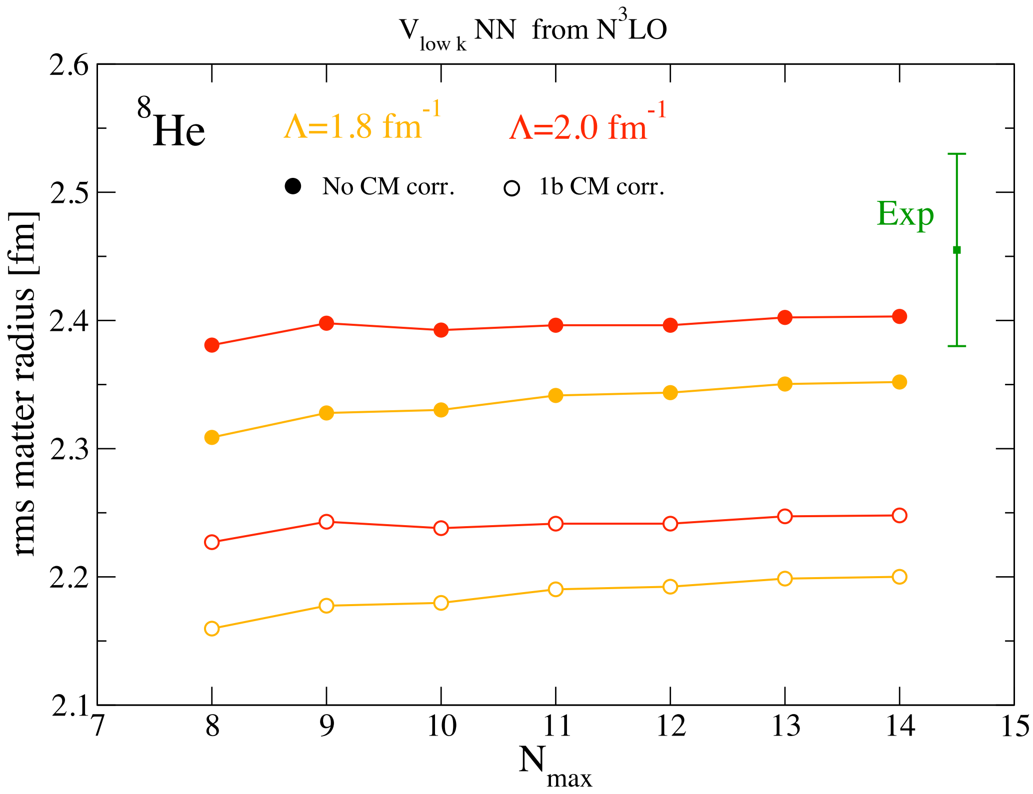
$$r^2 = \frac{1}{A}\sum_i^A (r_i - R_{\mathrm{c.m.}})^2 = \frac{1}{A}\sum_i^A r_i^2 - R_{\mathrm{c.m.}}^2$$

c.m. correction

$$r^2 = \frac{1}{A}\sum_i^A r_i^2\left(1-\frac{1}{A}\right) - \frac{2}{A^2}\sum_{i<j}^A r_i r_j$$

1b c.m. correction

2b c.m. correction



Radii for ^{8}He

- Matter Radius: Comments... -

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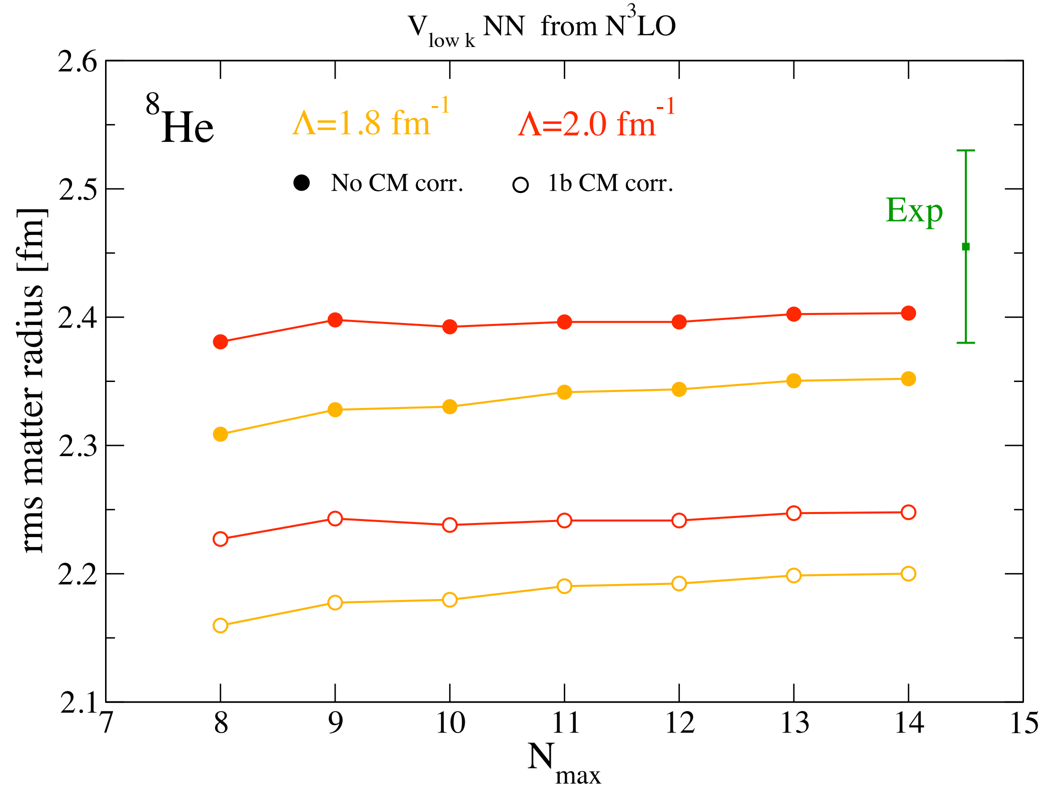

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c.m. correction

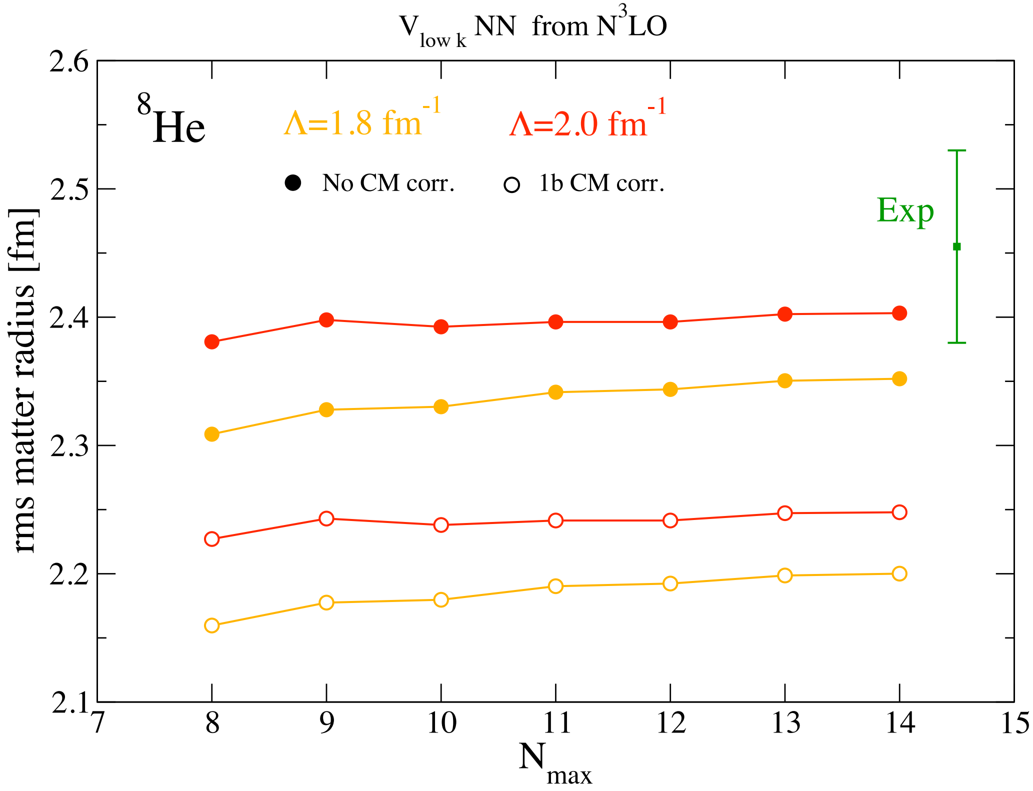
arXiv:0905.3167: Coupled cluster wave function factorizes into a Gaussian for the CM for ^{4}He and ^{16}O

Can estimate it analytically:

$$\langle r^2\rangle = \frac{1}{A}\langle\sum_i r_i^2\rangle - \langle R^2_{c.m.}\rangle$$

$$\langle T\rangle = \langle V\rangle = \frac{3}{4}\hbar\tilde{\omega}$$

$$\frac{1}{2}mA\tilde{\omega}^2\langle R^2_{c.m.}\rangle = \frac{3}{4}\hbar\tilde{\omega}$$



Radii for ^{8}He

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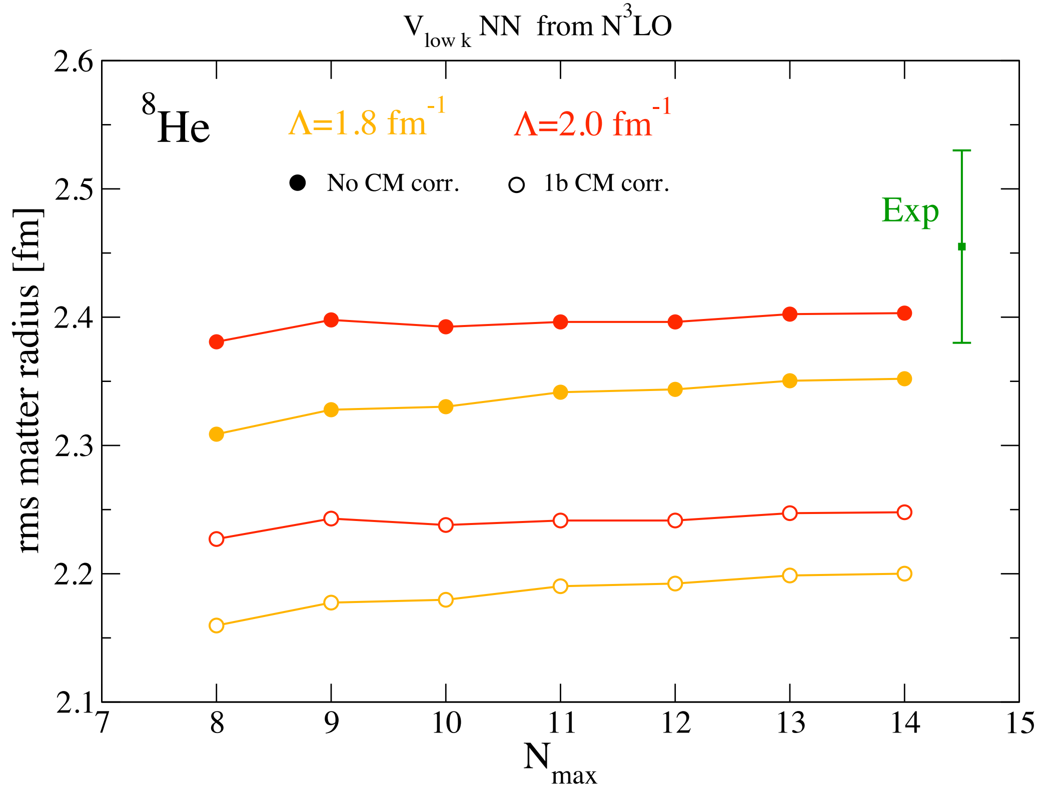
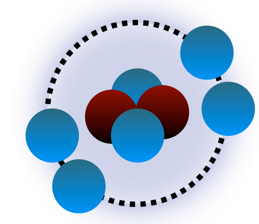
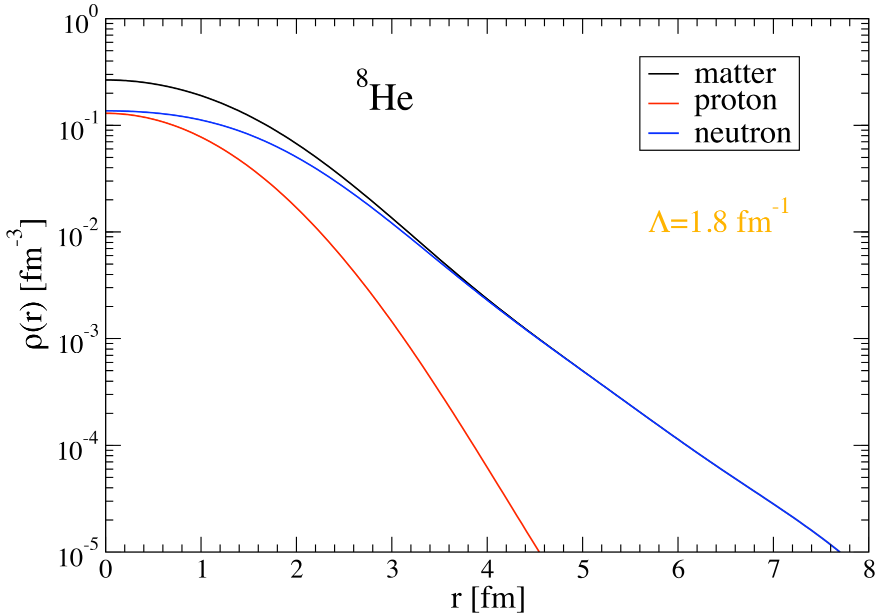


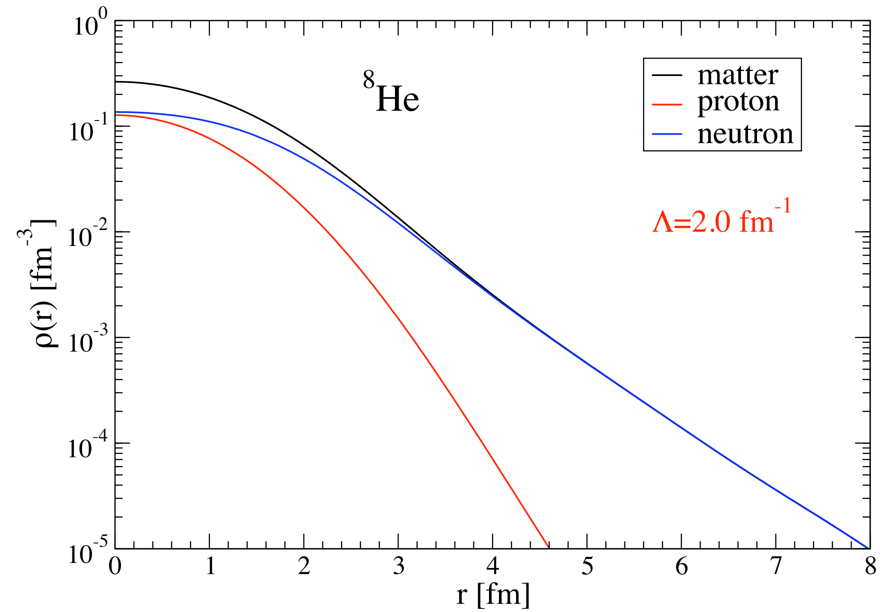
- Check CM factorization for ^{8}He
- Look for eigenfrequency and HO parameter (in)dependence
- Calculate CM corrected radii and benchmark on ^{4}He

Densities for ^{8}He



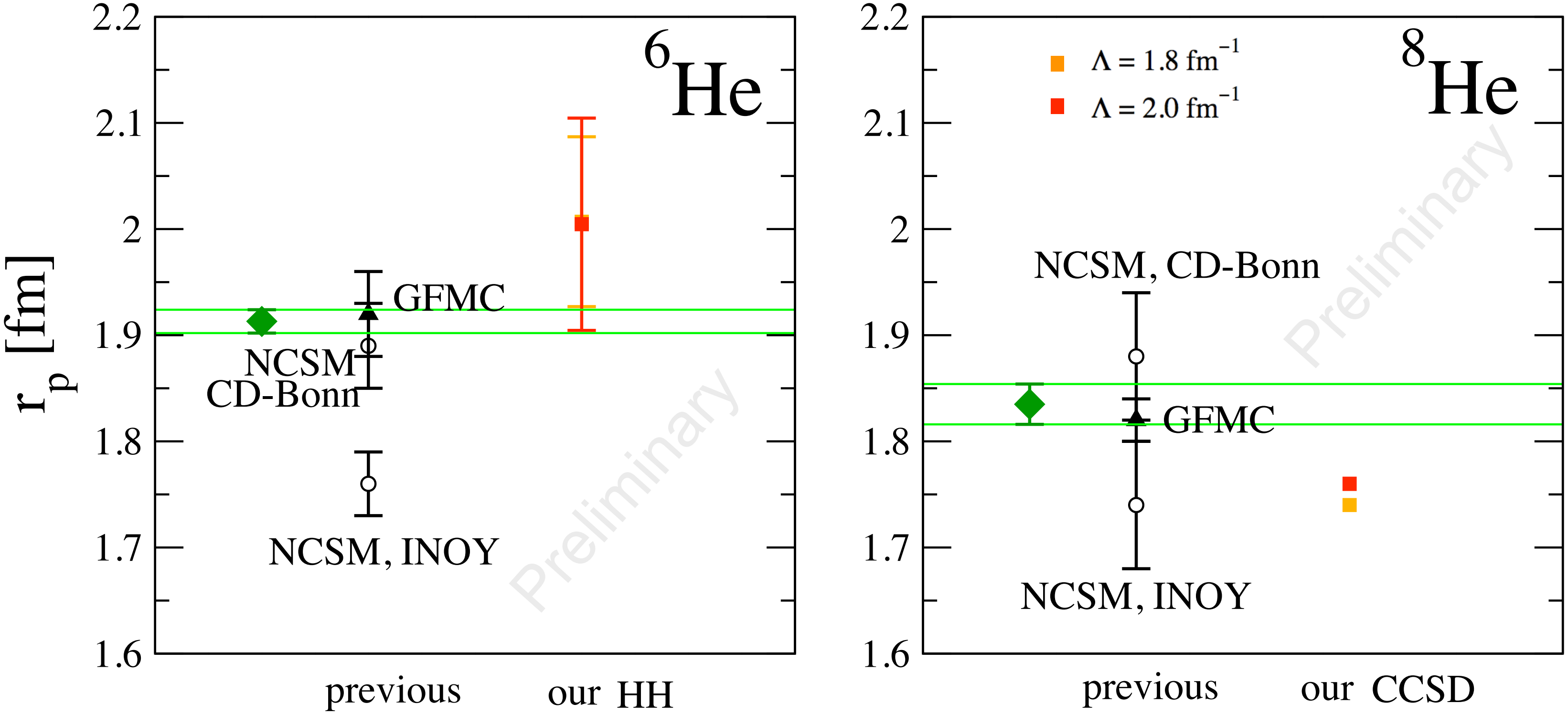
$V_{\text{low k}}$ from N^{3}LO $\quad H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$





- Theory predicts the neutron structure: very much extended neutron tail
- Small cutoff dependence of the neutron tail: consistent with radii

Proton radii Summary

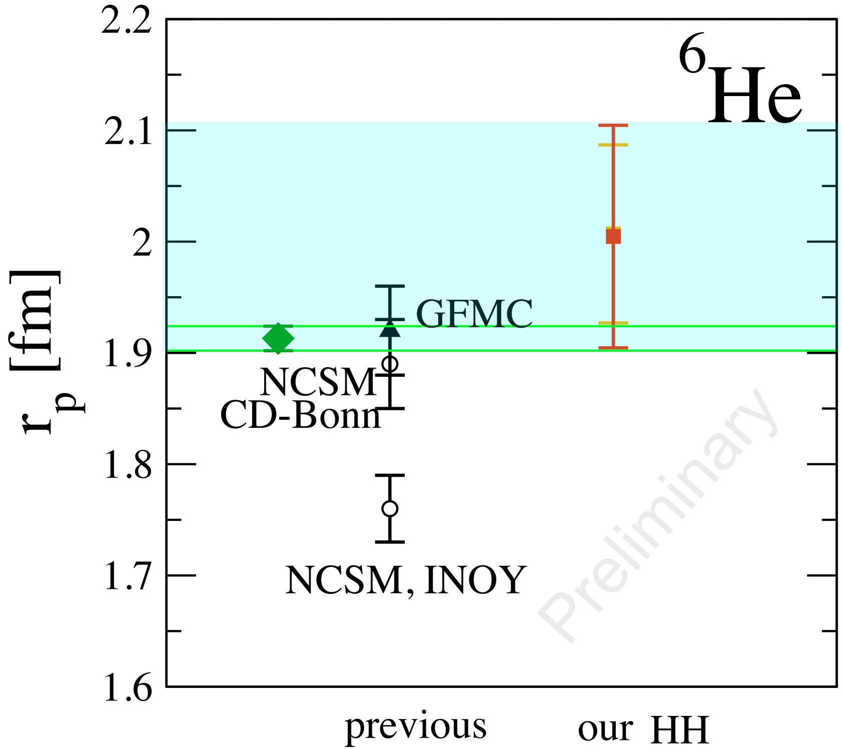


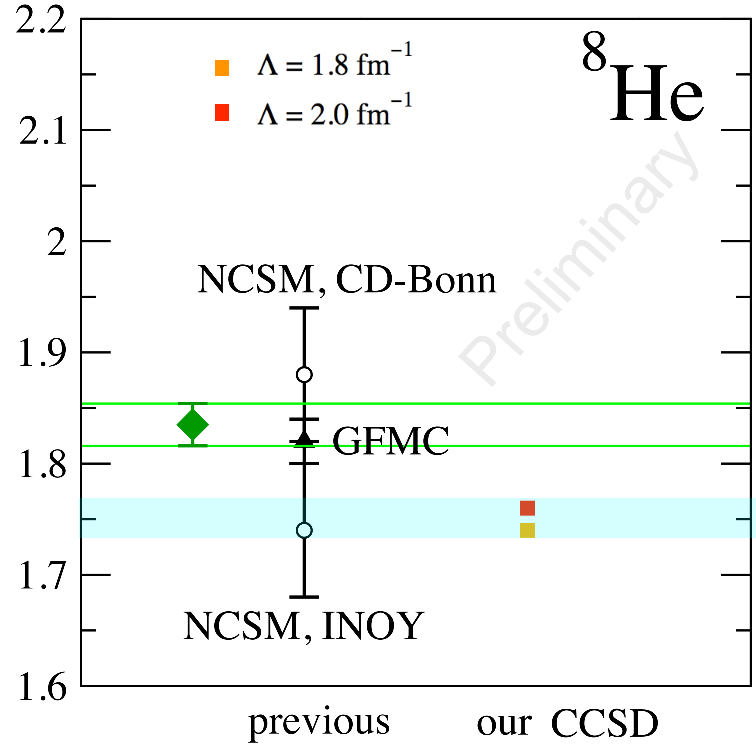
◆ from laser spectroscopy

using TITAN ISAC-TRIUMF

binding energy as input

Proton radii Summary



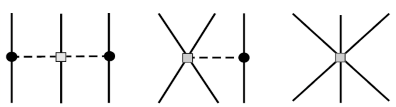


from laser spectroscopy

using TITAN ISAC-TRIUMF

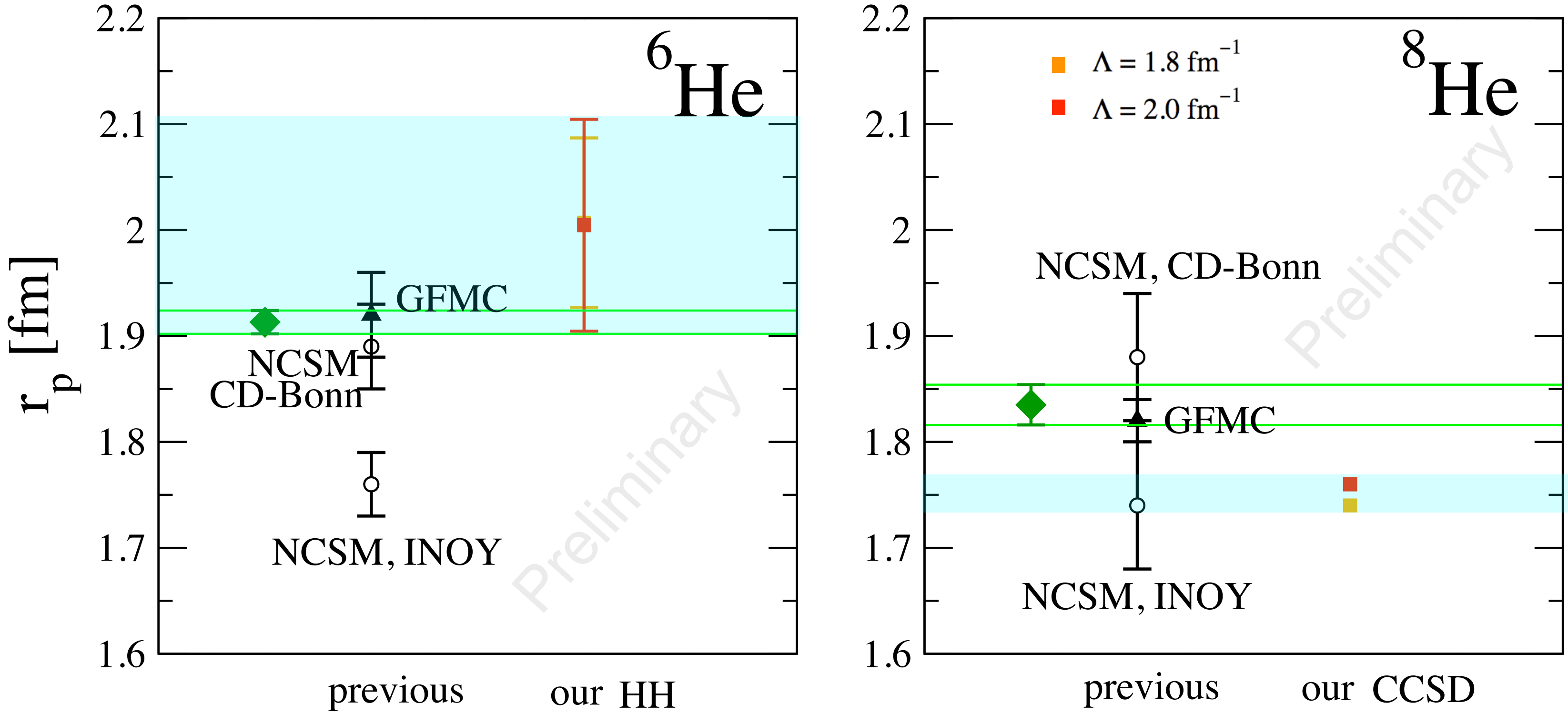
binding energy as input

Our estimated error in neglected



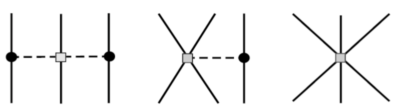
is in percentage consistent with the results for binding energy

Proton radii Summary



◆ from laser spectroscopy using TITAN ISAC-TRIUMF binding energy as input

Our estimated error in neglected

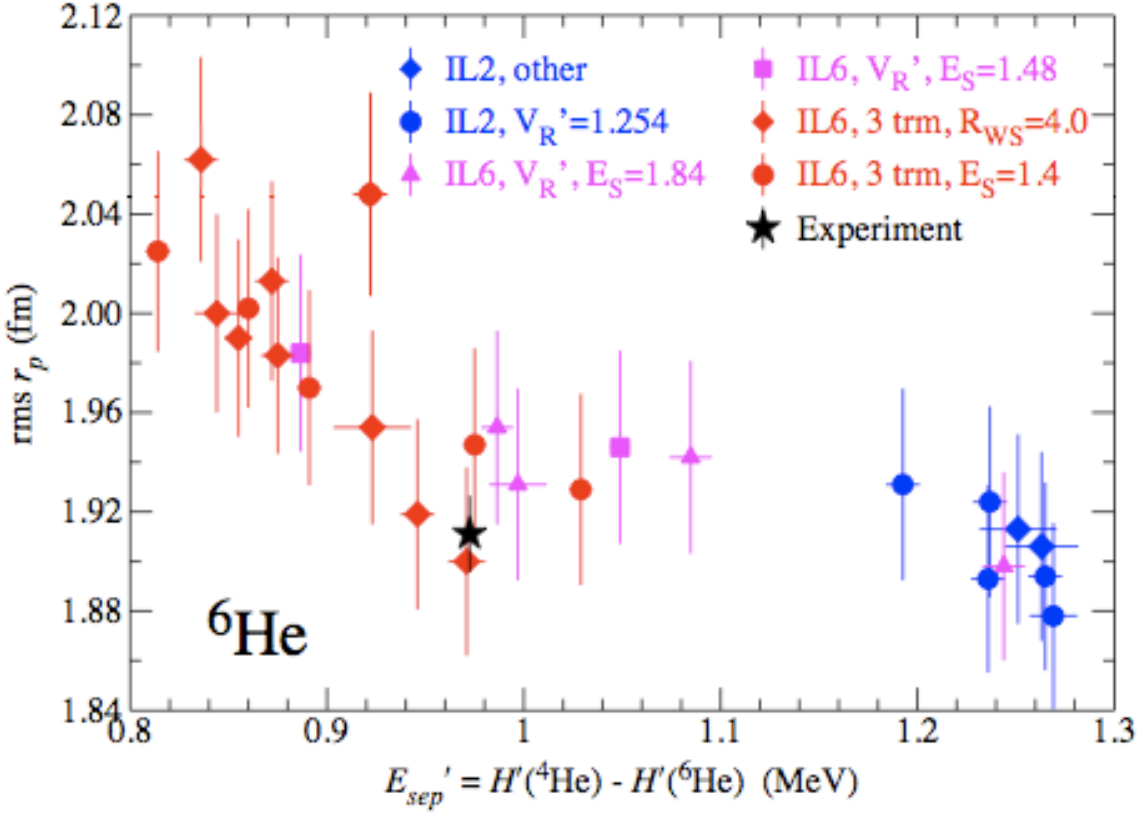


is in percentage consistent with the results for binding energy

- The fact that for some "choice" of the NN force one gets correct radii and wrong energies (or vice-versa) shows that halo nuclei provide important tests of the different aspects of nuclear forces, which includes 3NF

Nota Bene

- GFMC estimation of the proton radius -



S.C. Pieper, arXiv:0711.1500, proceedings of Enrico Fermi School

Summary

- We provide improved description of helium halo nuclei from evolved EFT interactions with the correct asymptotic in the wave function

- We estimate the effect of three-nucleon forces on binding energies by varying the cutoff of the evolved interaction

- We made the first steps towards providing accurate estimates for radii in helium halo nuclei

Future:

- Include three-nucleon forces

- Extend coupled cluster theory calculations to heavier neutron rich nuclei, e.g. lithium → $^{11}\mathrm{Li}$ or oxygen isotope chain