# Understanding time-odd mean fields in covariant density functional theories

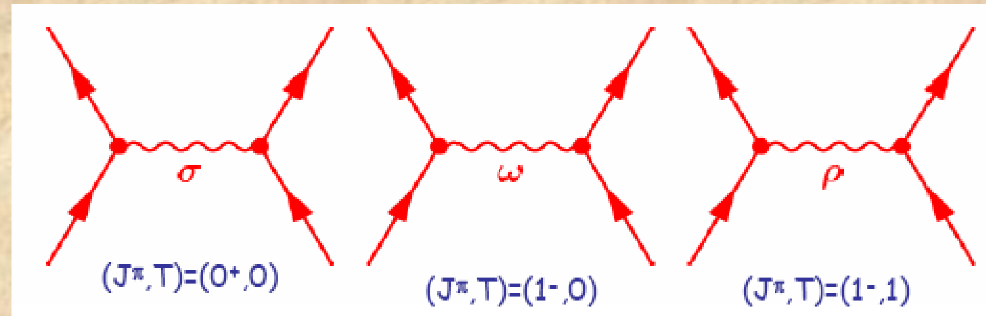
Anatoli Afanasjev
Mississippi State University

1. Brief overview of formalism
2. The effect of time-odd mean fields on
   - binding energies
   - odd-even mass scatterings
   - proton emitters
   - odd-odd nuclei
   - rotating systems
3. Conclusions.

In collaboration with Hazem Abusara

# Covariant density functional (CDF) theory

The nucleons interact via the exchange of effective mesons →
→ **effective Lagrangian**




Long-range attractive scalar field

Short-range repulsive vector field

Isovector field

$$E_{\mathrm{RMF}}[\hat{\rho}, \phi_m] = \mathrm{Tr}[(\boldsymbol{\alpha p} + \beta m)\hat{\rho}] \pm \int \left[\frac{1}{2}(\nabla\phi_m)^2 + U(\phi_m)\right] \mathrm{d}^3 r + \mathrm{Tr}[(\Gamma_m \phi_m)\hat{\rho}]$$

density matrix $\hat{\rho}$

$\phi_m \equiv \{\sigma, \omega^\mu, \vec{\rho}^\mu, A^\mu\}$ - meson fields

$$\hat{h} = \frac{\delta E}{\delta \hat{\rho}}$$

**Mean field**

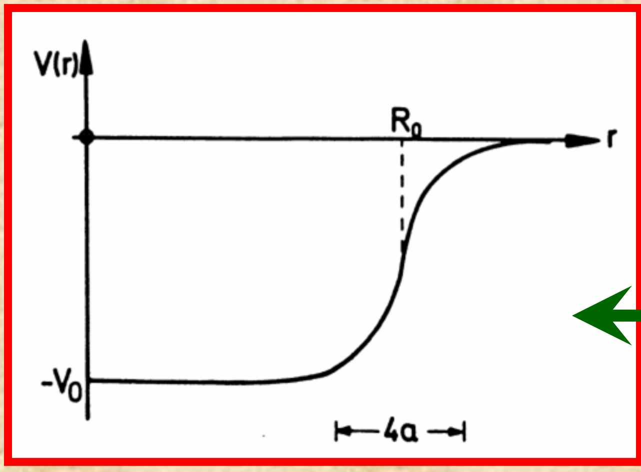
$$\hat{h}|\varphi_i\rangle = \varepsilon_i |\varphi_i\rangle$$

**Eigenfunctions**

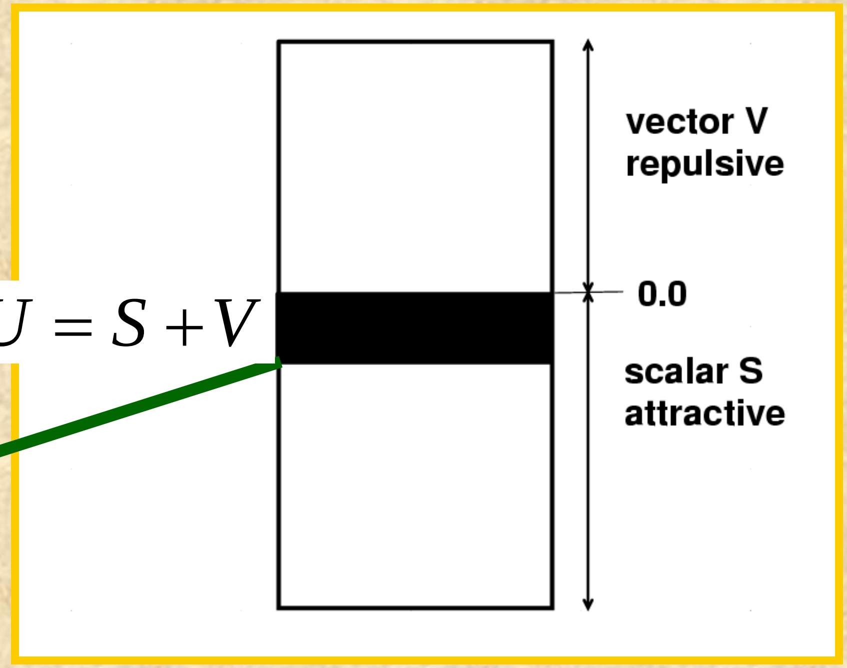
$$V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau}\vec{\rho}(\mathbf{r}) + eA(\mathbf{r})$$

$$S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r})$$

## U – nucleonic potential




$$U = S + V$$




V ~ 350 MeV/nucleon
S ~ - 400 MeV/nucleon
U ~ - 50 MeV/nucleon

Mean fields

Action of time-reversal operator T

Time-even mean fields

Time-odd mean fields

**Little attention has been paid to time-odd mean fields in CDF theory**

80th - **magnetic moments**

U. Hoffman and P. Ring, PLB 214, 307 (1988)

R.J. Furnstahl, PRC 38, 370 (1988)

and some other authors

90th - **moments of inertia**

J.Konig and P.Ring, PRL 71,3079 (1993)

A.A. and P.Ring, PRC C62, 031302(R) (2000)

00th - **terminating states**

A.A., PRC 78, 054303 (2008)

# Tool: cranked relativistic mean field theory

1. The Dirac equations for the fermions in the rotating frame (one-dimensional cranking approximation)

$$\hat{h}_D = \boldsymbol{\alpha}(-i\boldsymbol{\nabla} - \boldsymbol{V}(\boldsymbol{r})) + V_0(\boldsymbol{r}) + \beta(m + S(\boldsymbol{r})) - \Omega_X \hat{J}_X$$

Magnetic potential

$$\boldsymbol{V}(\boldsymbol{r}) = g_\omega \boldsymbol{\omega}(\boldsymbol{r}) + g_\rho \tau_3 \boldsymbol{\rho}(\boldsymbol{r}) + e\frac{1-\tau_3}{2}\boldsymbol{A}(\mathbf{r})$$

⇨ Nuclear magnetism

-space-like components of vector mesons

-behaves in Dirac equation like a magnetic field

2. Klein-Gordon equations for mesons:

$$\begin{aligned}
\{-\Delta + m_\sigma^2\}\ \sigma(\boldsymbol{r}) &= -g_\sigma[\rho_s^n(\boldsymbol{r}) + \rho_s^p(\boldsymbol{r})] \\
&\quad - g_2\sigma^2(\boldsymbol{r}) - g_3\sigma^3(\boldsymbol{r}) \\
\{-\Delta + m_\omega^2\}\,\omega_0(\boldsymbol{r}) &= g_\omega[\rho_v^n(\boldsymbol{r}) + \rho_v^p(\boldsymbol{r})], \\
\{-\Delta + m_\omega^2\}\ \boldsymbol{\omega}(\boldsymbol{r}) &= g_\omega[\boldsymbol{j}^n(\boldsymbol{r}) + \boldsymbol{j}^p(\boldsymbol{r})]
\end{aligned}$$

Two sources of time-reversal symmetry breaking:

- Coriolis term
- magnetic potential

⇦ "time-odd" mean fields in non-relativistic theory

# Nuclear magnetism (NM)=Time-odd (TO) mean fields

**Microscopic nature of nuclear magnetism**

Dirac spinors

$$\psi_i(\mathbf{r}) = \begin{pmatrix} f_i(\mathbf{r}) \\ i g_i(\mathbf{r}) \end{pmatrix}$$

Baryonic current: product of small and large components of Dirac spinor

$$\begin{aligned} \mathbf{j}_i^B(\mathbf{r}) &= \psi_i^+(\mathbf{r})\hat{\alpha}\,\psi_i(\mathbf{r}) \\ &= i f_i^+(\mathbf{r})\hat{\sigma} g_i(\mathbf{r}) - i g_i^+(\mathbf{r})\hat{\sigma} f_i(\mathbf{r}) \end{aligned}$$

Klein-Gordon equations

$$\mathbf{j}_i^B(\mathbf{r}) \to \boldsymbol{\omega}(\mathbf{r}), \boldsymbol{\rho}(\mathbf{r}) \to \mathbf{V}(\mathbf{r}) \to \text{Dirac eq.}$$

Pairing is neglected in the calculations

Single-particle states are characterized by signature

$$r = \pm i$$

**Abbreviations:**

NM – nuclear magnetism is included
WNM – nuclear magnetism is neglected

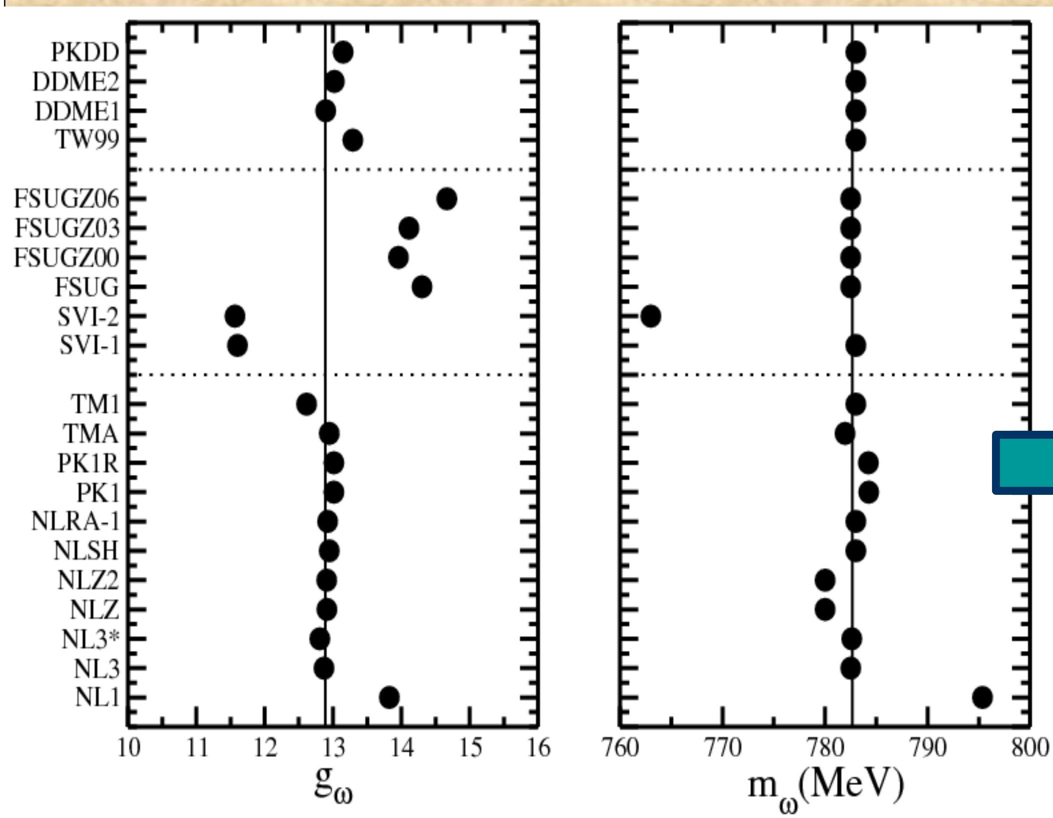
# 1. Dominance of the ω-meson in time-odd mean fields

$$\{-\Delta + m_\omega^2\}\,\omega_0(\boldsymbol{r}) = g_\omega[\rho_v^n(\boldsymbol{r}) + \rho_v^p(\boldsymbol{r})]$$

$$\{-\Delta + m_\omega^2\}\,\boldsymbol{\omega}(\boldsymbol{r}) = g_\omega[\boldsymbol{j}^n(\boldsymbol{r}) + \boldsymbol{j}^p(\boldsymbol{r})]$$ ← isoscalar

$$\{-\Delta + m_\rho^2\}\,\rho_0(\boldsymbol{r}) = g_\rho[\rho_v^n(\boldsymbol{r}) - \rho_v^p(\boldsymbol{r})].$$

$$\{-\Delta + m_\rho^2\}\,\boldsymbol{\rho}(\boldsymbol{r}) = g_\rho[\boldsymbol{j}^n(\boldsymbol{r}) - \boldsymbol{j}^p(\boldsymbol{r})]$$ ← isovector

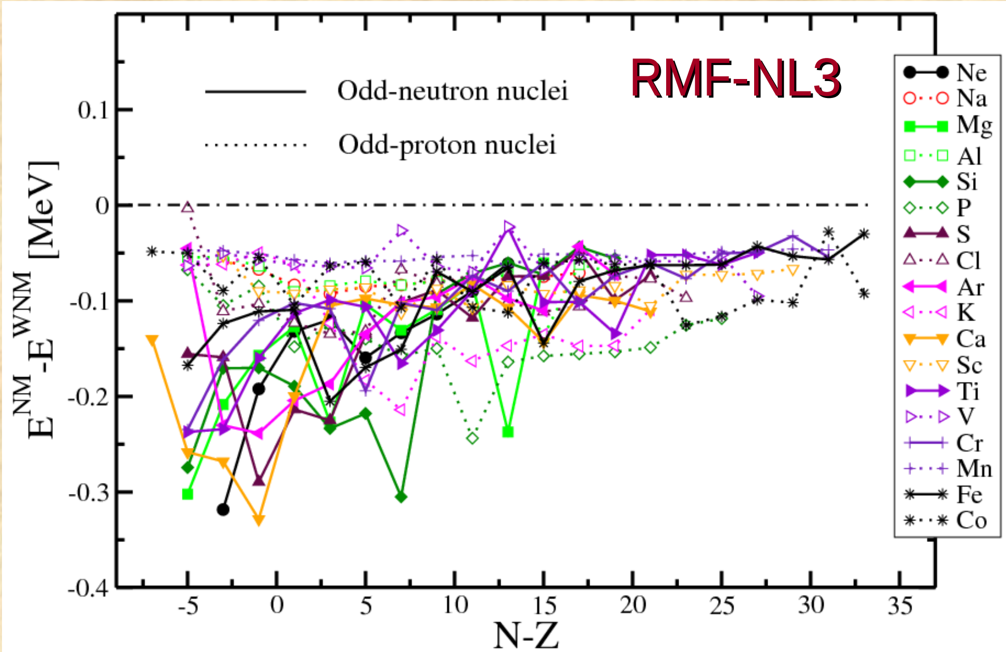


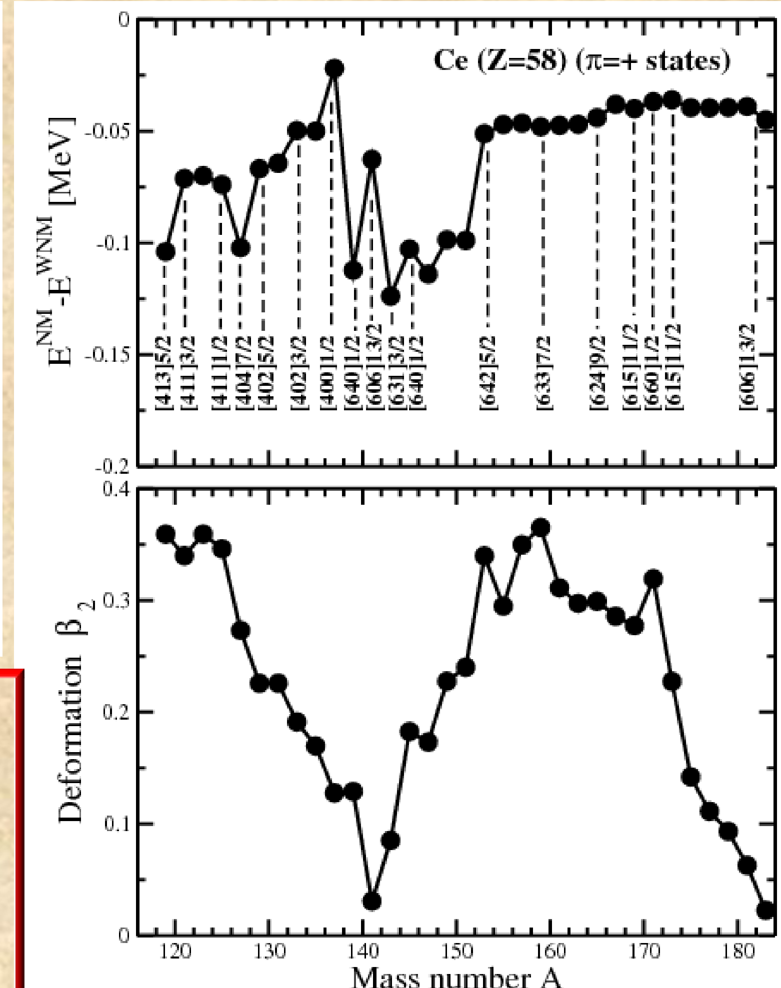

The contribution of the ρ-meson is very small in the majority of the cases

Weak dependence on the parametrization

BT states: time-odd mean fields are defined with ~15% accuracy
A.A., PRC 78, 054303 (08)

# 2. Impact of time-odd mean fields on binding energies of odd-mass nuclei: general features



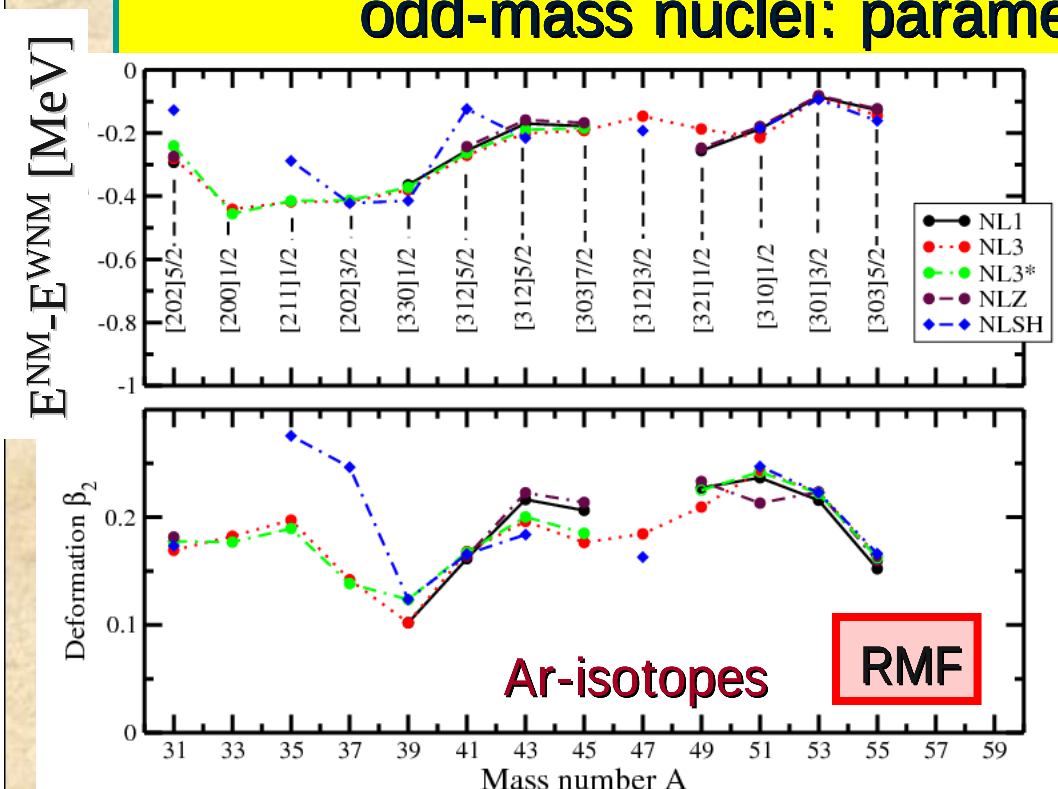




1. NM always leads to additional binding

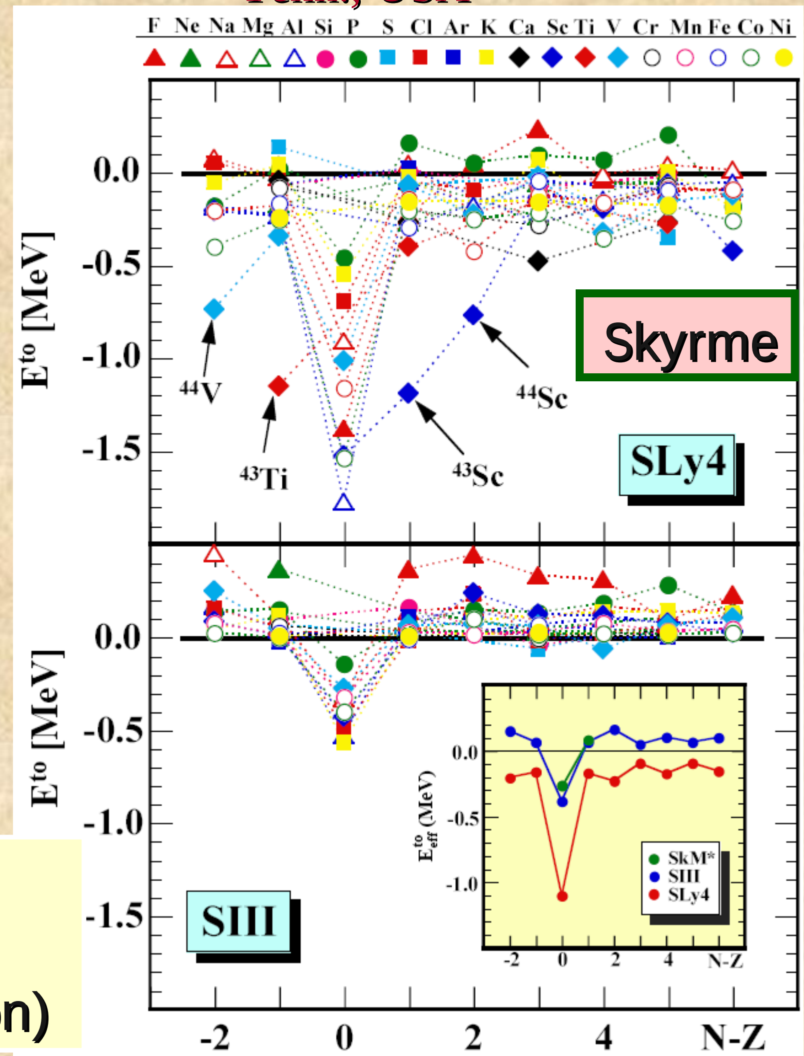
2. Additional binding due to NM is not clearly correlated with the structure of blocked state or deformation

Such state-dependence is seen in the effects of TO mean fields in Skyrme HF calculations, T. Duguet et al, PRC 65, 014310 (2001)

# 2. Impact of time-odd mean fields on binding energies of odd-mass nuclei: parametrization dependence



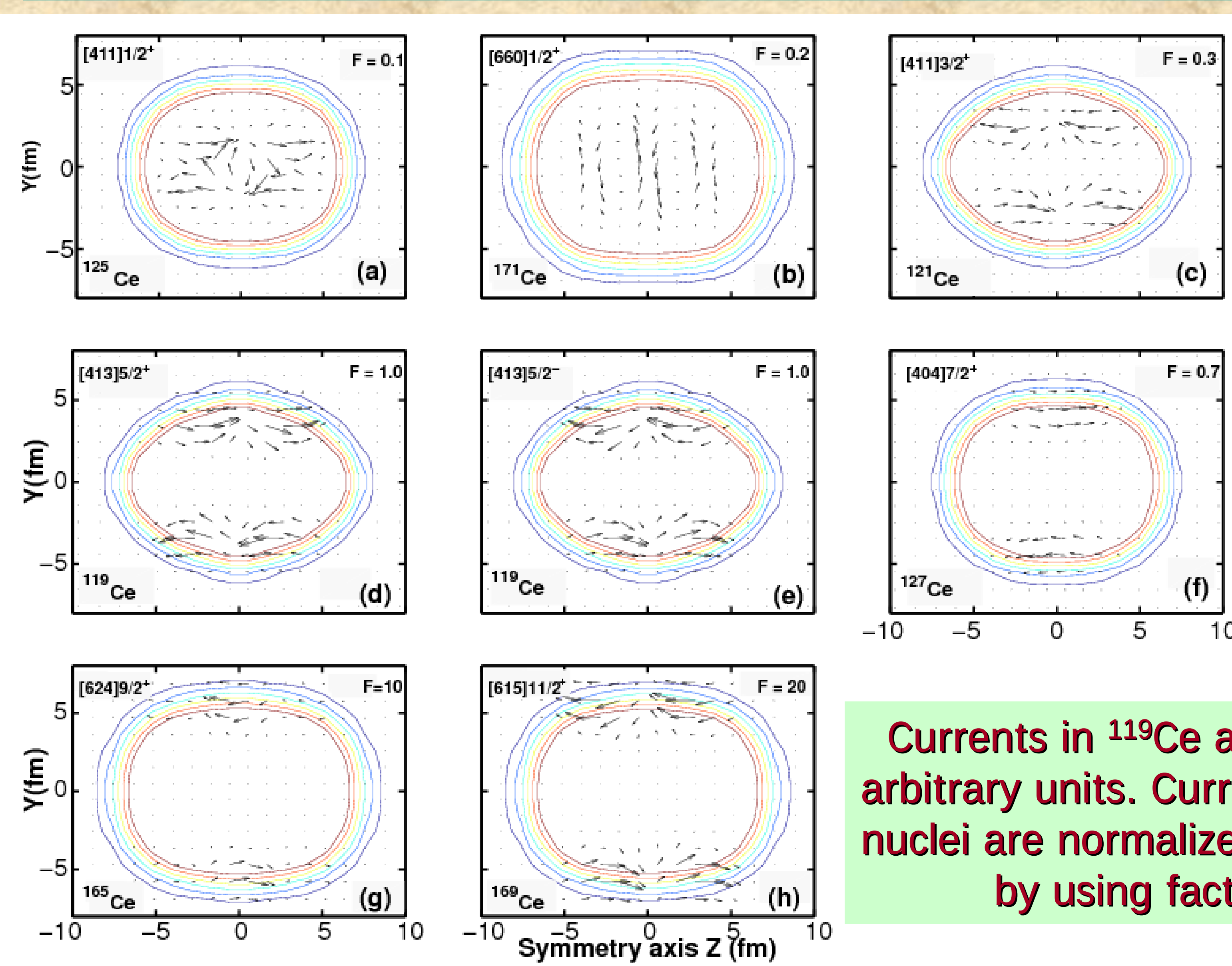

W. Satula, Proceedings of Nuclear Structure'98 conference, Gatlinburg, Tenn., USA




Additional binding due to NM (always attractive) only weakly depends on the RMF parametrization.

In Skyrme DFT calculations the time-odd mean fields can be both attractive and repulsive (sign even depends on mass region)

# 3. Neutron current distributions $j^n(r)$ in Ce isotopes: dependence on $\Omega$ of blocked state

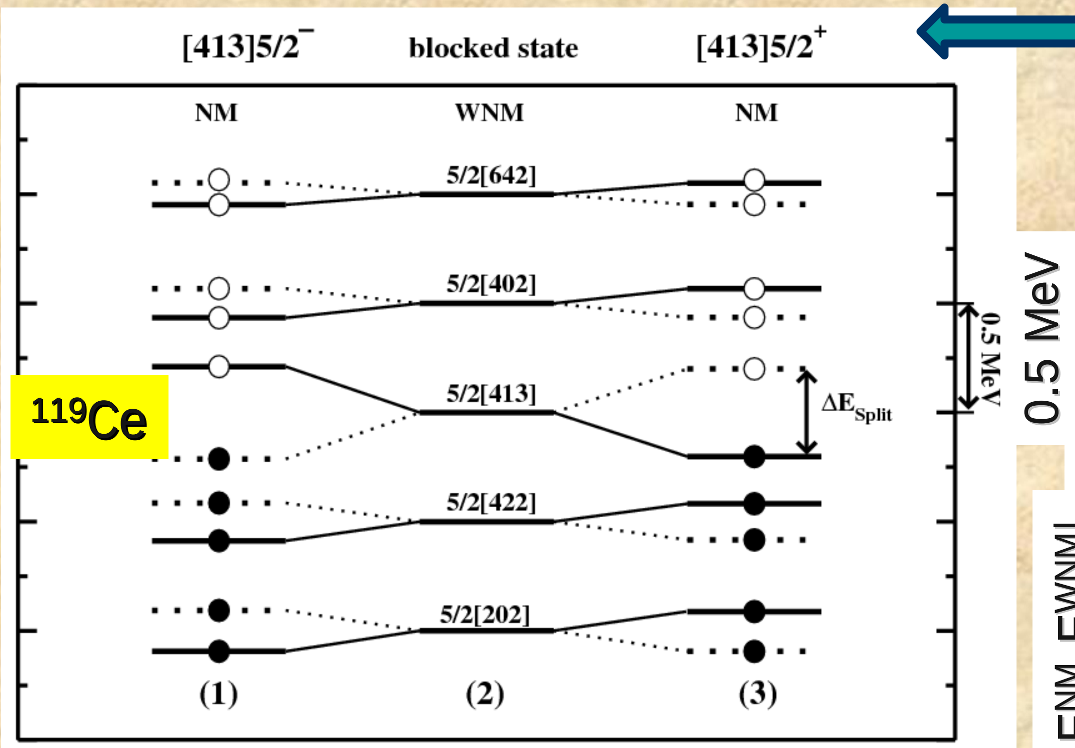


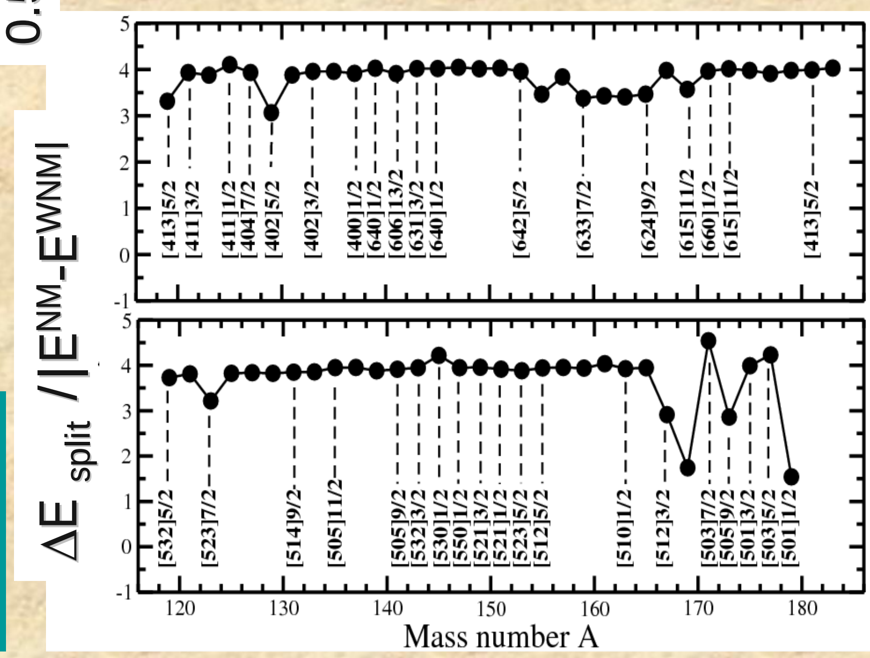

Currents in $^{119}$Ce are given at arbitrary units. Currents for other nuclei are normalized to $^{119}$Ce by using factor F.

# 4. Breaking of Kramer's degeneracy of single-particle states in the presence of time-odd mean fields

**The energy splitting $\Delta E_{split}$ between different signatures of the s-p states**

superscript = sign of *r* signature




**Signature splitting in the presence of TO mean field correlated with additional binding due to NM**

# 5. Microscopic mechanism of impact of TO fields on binding energies

$$E_{tot} = E_{part} + E_{cm} - E_\sigma - E_{\sigma NL} - E_\omega^{TL} - E_\rho^{TL} - E_\omega^{SL} - E_\rho^{SL} - E_{Coul},$$

$$E_{part} = \sum_i^A \varepsilon_i,$$

$$E_\sigma = \frac{1}{2}\, g_\sigma \int d^3r\; \sigma(\boldsymbol{r}) \left[\rho_s^p(\boldsymbol{r}) + \rho_s^n(\boldsymbol{r})\right],$$

$$E_{\sigma NL} = \frac{1}{2} \int d^3r \left[\frac{1}{3}\, g_2\, \sigma^3(\boldsymbol{r}) + \frac{1}{2}\, g_3\, \sigma^4(\boldsymbol{r})\right]$$

$$E_\omega^{TL} = \frac{1}{2}\, g_\omega \int d^3r\, \omega_0(\boldsymbol{r}) \left[\rho_v^p(\boldsymbol{r}) + \rho_v^n(\boldsymbol{r})\right]$$

$$E_\omega^{SL} = -\frac{1}{2}\, g_\omega \int d^3r\, \boldsymbol{\omega}(\boldsymbol{r}) \left[\boldsymbol{j}^p(\boldsymbol{r}) + \boldsymbol{j}^n(\boldsymbol{r})\right]$$

$$E_\rho^{TL} = \frac{1}{2}\, g_\rho \int d^3r \rho_0(\boldsymbol{r}) \left[\rho_v^n(\boldsymbol{r}) - \rho_v^p(\boldsymbol{r})\right]$$

$$E_\rho^{SL} = -\frac{1}{2}\, g_\rho \int d^3r\, \boldsymbol{\rho}(\boldsymbol{r}) \left[\boldsymbol{j}^n(\boldsymbol{r}) - \boldsymbol{j}^p(\boldsymbol{r})\right]$$

| Quantity | $E_i^{WNM}$ | $E_i^{NM} - E_i^{WNM}$ | |
|---|---|---|---|
| 1 | 2 | 3 | |
| $E_{part}$ | −2849.889 | −0.41 | ½ TO |
| $E_\sigma$ | −17079.532 | −2.231 | Pol |
| $E_{\sigma NL}$ | 343.341 | −0.017 | Pol |
| $E_\omega^{TL}$ | 14356.156 | 2.054 | Pol |
| $E_\omega^{SL}$ | 0.0 | −0.124 | TO |
| $E_\rho^{TL}$ | 2.044 | 0.003 | Pol |
| $E_\rho^{SL}$ | 0.0 | −0.010 | TO |
| $E_{Coul}$ | 481.196 | 0.017 | Pol |
| $E_{cm}$ | −6.252 | 0.0 | |
| $E_{tot}$ | −959.349 | −0.104 | |

## 6. Physical consequences: impact of time-odd mean field on odd-even mass staggering

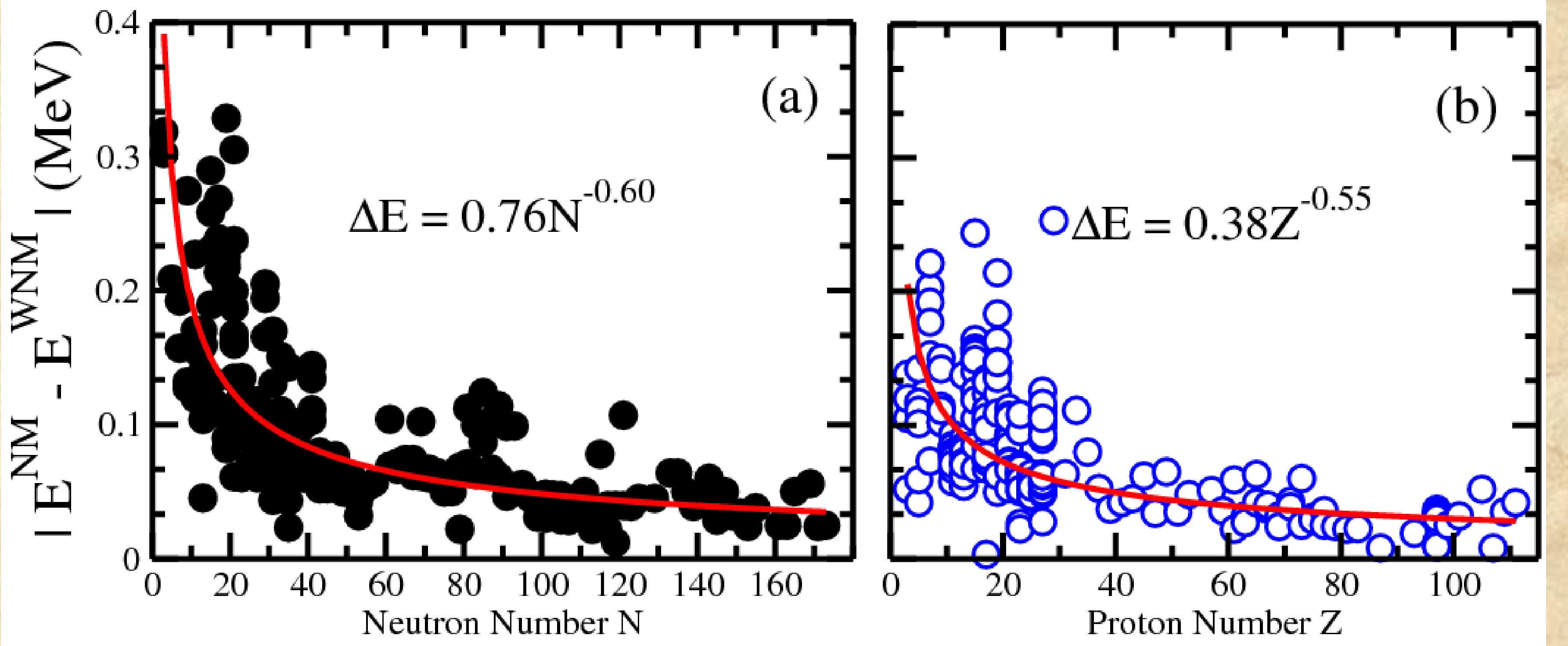
The three-point indicator [56]

$$\Delta^{(3)}(N) = \frac{\pi_N}{2}\left[B(N-1) + B(N+1) - 2B(N)\right]$$

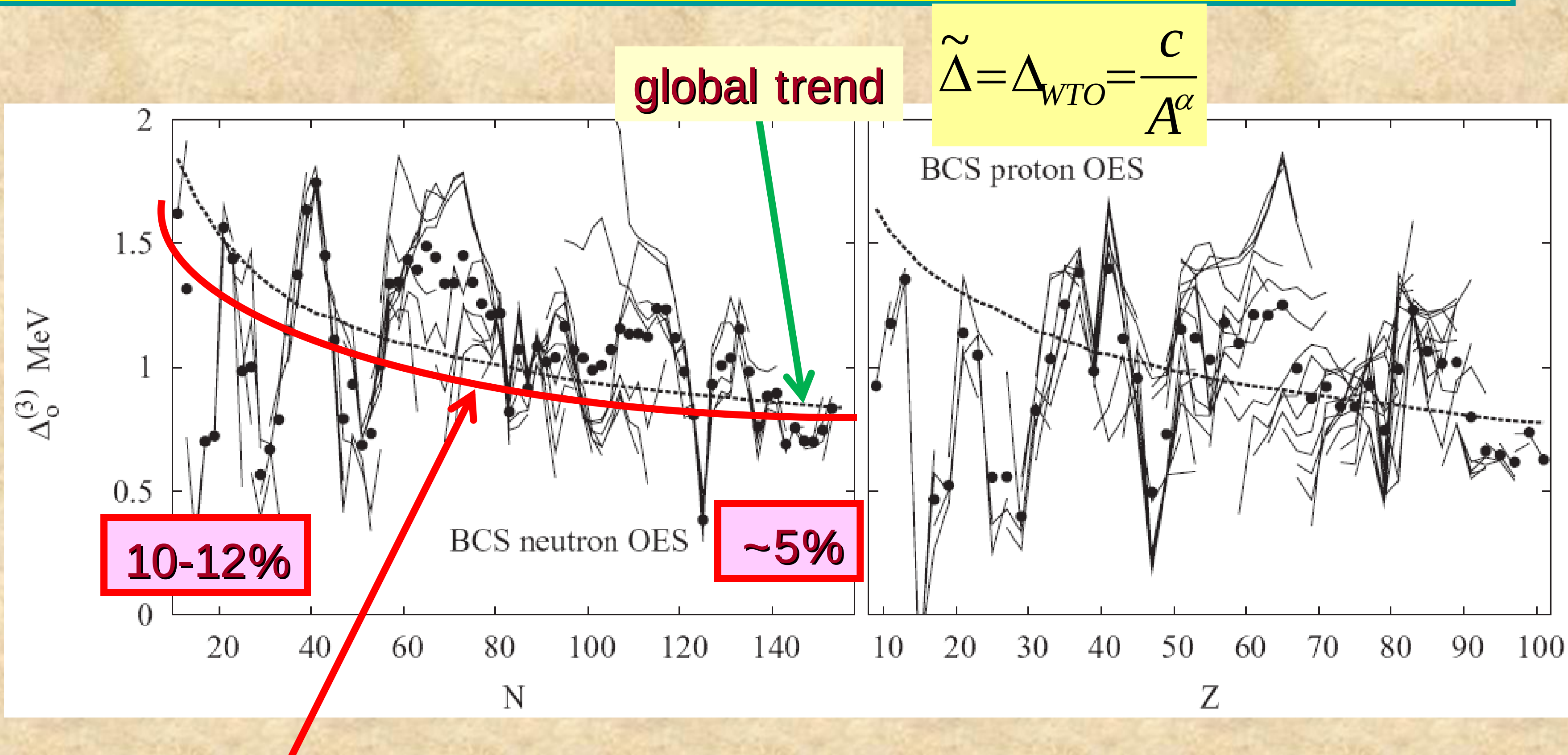
$$\Delta^{(3)}_{TO}(N) = \Delta^{(3)}_{WTO}(N) + \delta E_{TO}$$

$$\delta E_{TO} = E_{NM} - E_{WNM}$$

with without time-odd mean fields



# 6. Physical consequences: impact of time-odd mean field on odd-even mass staggering (OES)



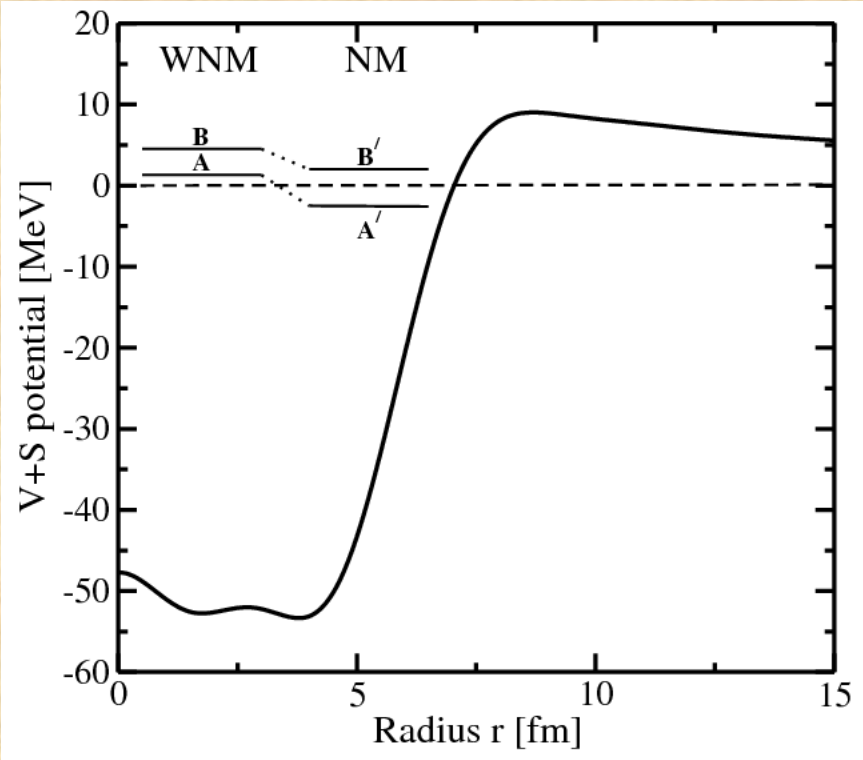

Global trend when TO mean fields are included ($\Delta_{TO}$)

Effect on proton OES is smaller by a factor of 2

Figure from G.F. Bertsch et al, PRC 79, 034306 (2009)

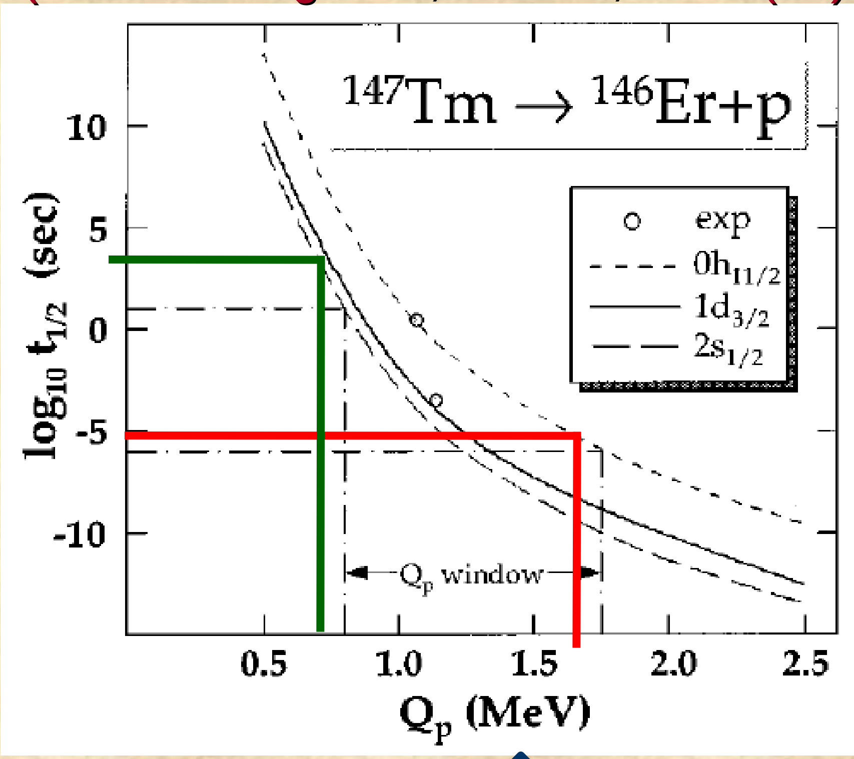
# 7. Physical consequences: impact of time-odd mean field on properties of proton emitters

Schematic figure




## Proton partial half-lives

(from S.Aberg et al, PRC 56, 1762 (97))




A → A' : proton-unbound nucleus → proton bound

B → B' : still proton-unbound, but proton partial-half live changes

## 7. Physical consequences: impact of time-odd mean field on properties of proton emitters

The impact of NM can be dramatic on the half-lives of proton emitters in lighter nuclei, since

(1)The general increase of additional binding due to NM and the magnitude of $\Delta E_{split}$ with decreasing mass

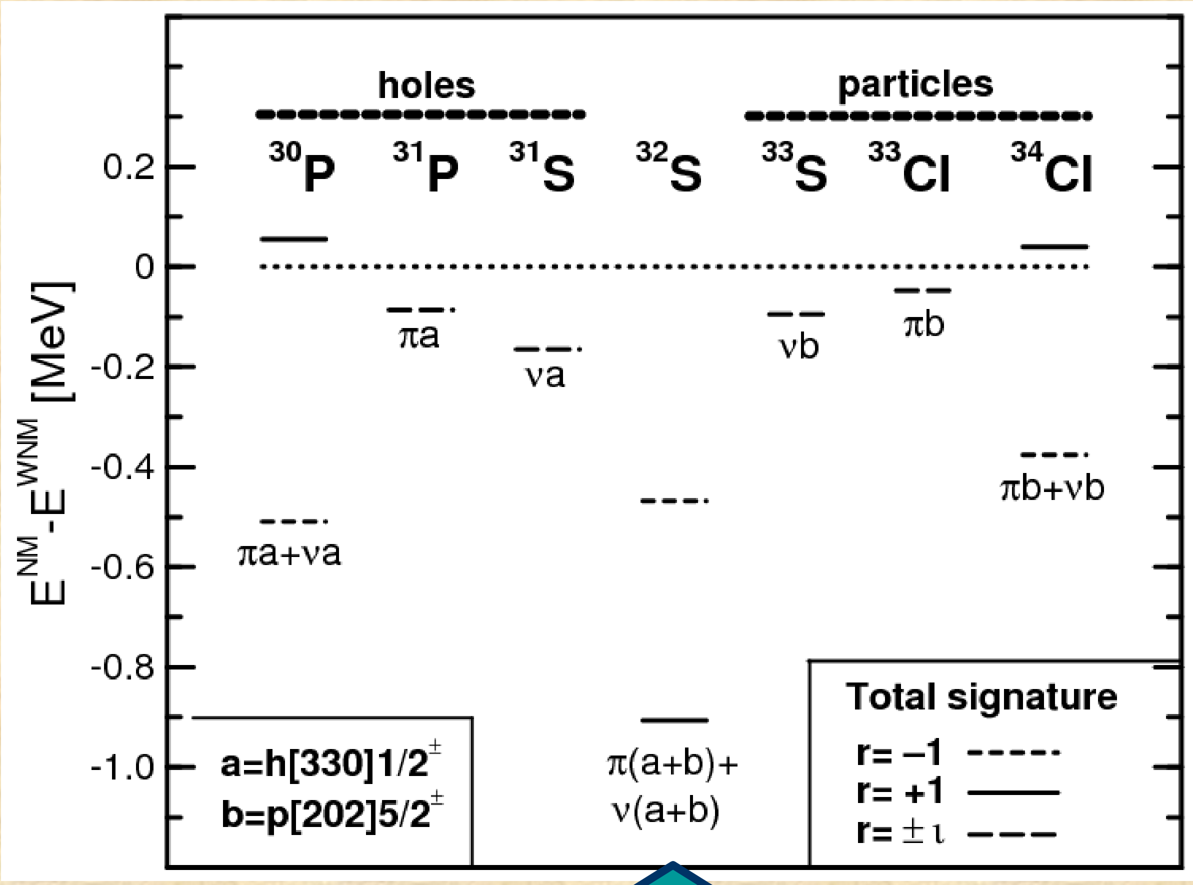
(2)The narrowing of the $Q_p$ window with the decrease of mass due to lowering of the Coulomb barrier

**Examples:** $^{69}$Br – the change in proton energy of around 300 keV causes a change in the proton decay lifetime of 11 orders of magnitude
[P.J.Woods et al, Ann.Rev.Nucl.Part.Sc. 47, 541 (97)]

Z=20 – half-life window of 10 to $10^{-4}$ s corresponds to proton energies of 100-150 keV in nuclei around Z=20
[V.I.Goldansky, NP 19, 482 (1960)]

$^{7}$B - variation of $Q_p$ value between 3 to 50 keV changes half-lives by 30 orders of magnitude
[S.Aberg et al, PRC 56, 1762 (97)]

# 8. Impact of time-odd mean field on single-particle states in odd-odd nuclei



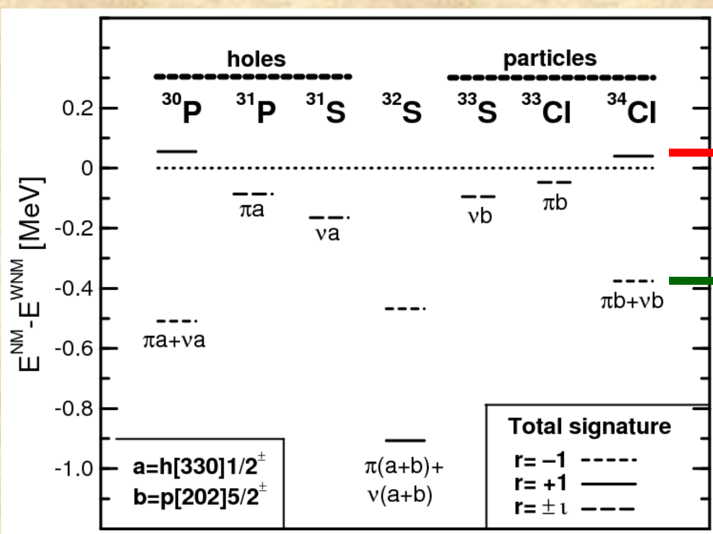

Neutron states are stronger affected by NM than proton states

Skyrme EDF: 1. Energy of r=-1 state is not affected by TO mean fields

2. |E(r=+1)-E(r=-1)| ~ 2 MeV

From Molique et al, PRC 61, 044304 (2000)

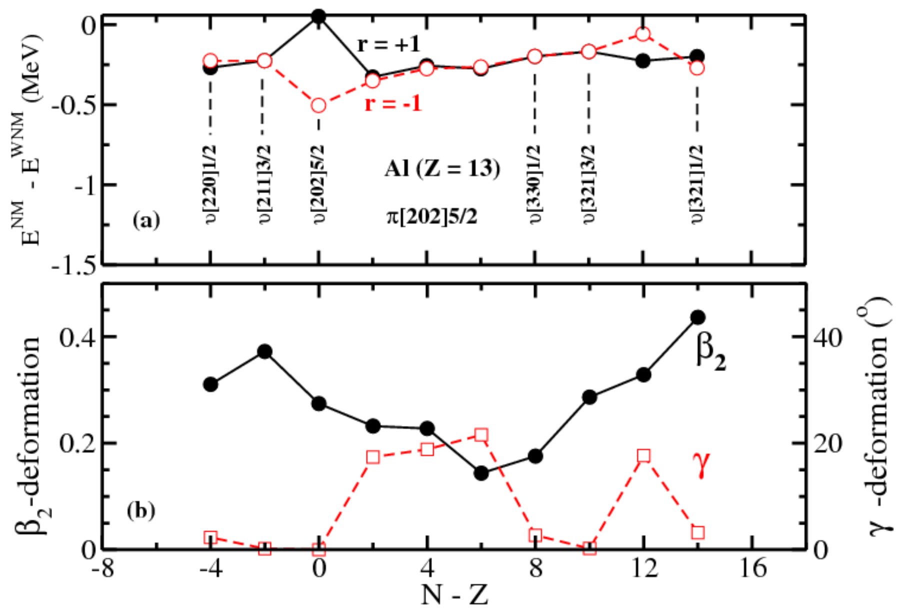
| Quantity | $E_i^{WNM}(A,B)$ | $E_i^{NM}(A) - E_i^{WNM}(A)$ | | $E_i^{NM}(B) - E_i^{WNM}(B)$ |
|---|---|---|---|---|
| 1 | 2 | 3 | | 4 |
| $E_{part}$ | $-836.044$ | $-1.47$ | ½ TO | $-0.004$ |
| $E_{\sigma}$ | -4416.351 | $-7.824$ | Pol | $+0.004$ |
| $E_{\sigma NL}$ | 84.864 | $-0.036$ | Pol | $-0.001$ |
| $E_{\omega}^{TL}$ | 3698.757 | 7.126 | Pol | $-0.003$ |
| $E_{\omega}^{SL}$ | 0.0 | $-0.413$ | TO | 0.0 |
| $E_{\rho}^{TL}$ | 0.061 | 0.0 | Pol | 0.0 |
| $E_{\rho}^{SL}$ | 0.0 | 0.0 | TO | $-0.043$ |
| $E_{Coul}$ | 59.839 | 0.051 | Pol | $-0.002$ |
| $E_{cm}$ | $-9.492$ | 0.0 | | 0.0 |
| $E_{tot}$ | $-272.705$ | $-0.374$ | | 0.041 |

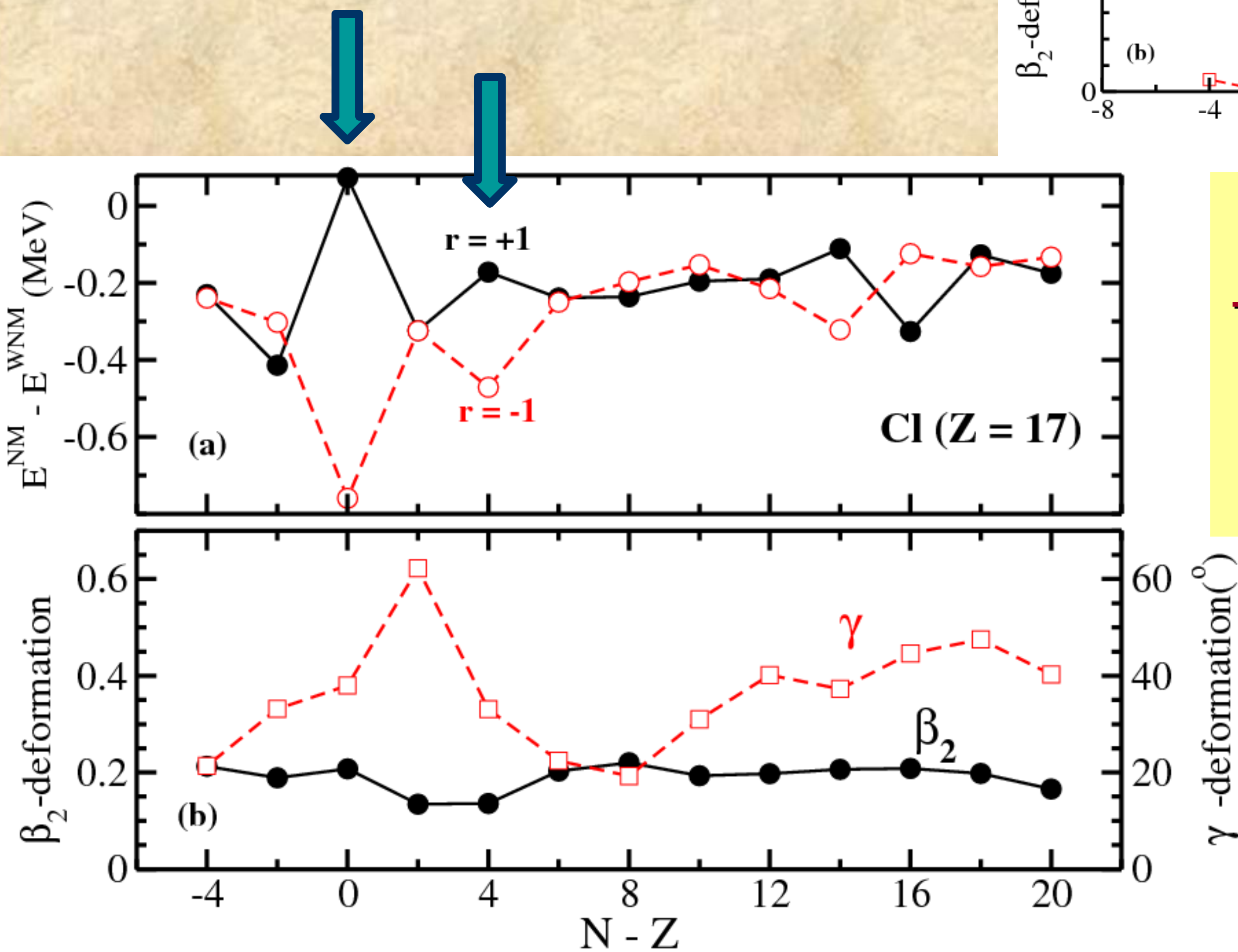



π and ν currents
in opposite directions

π and ν currents
in the same direction

# 8. Impact of time-odd mean fields on single-particle states in odd-odd nuclei



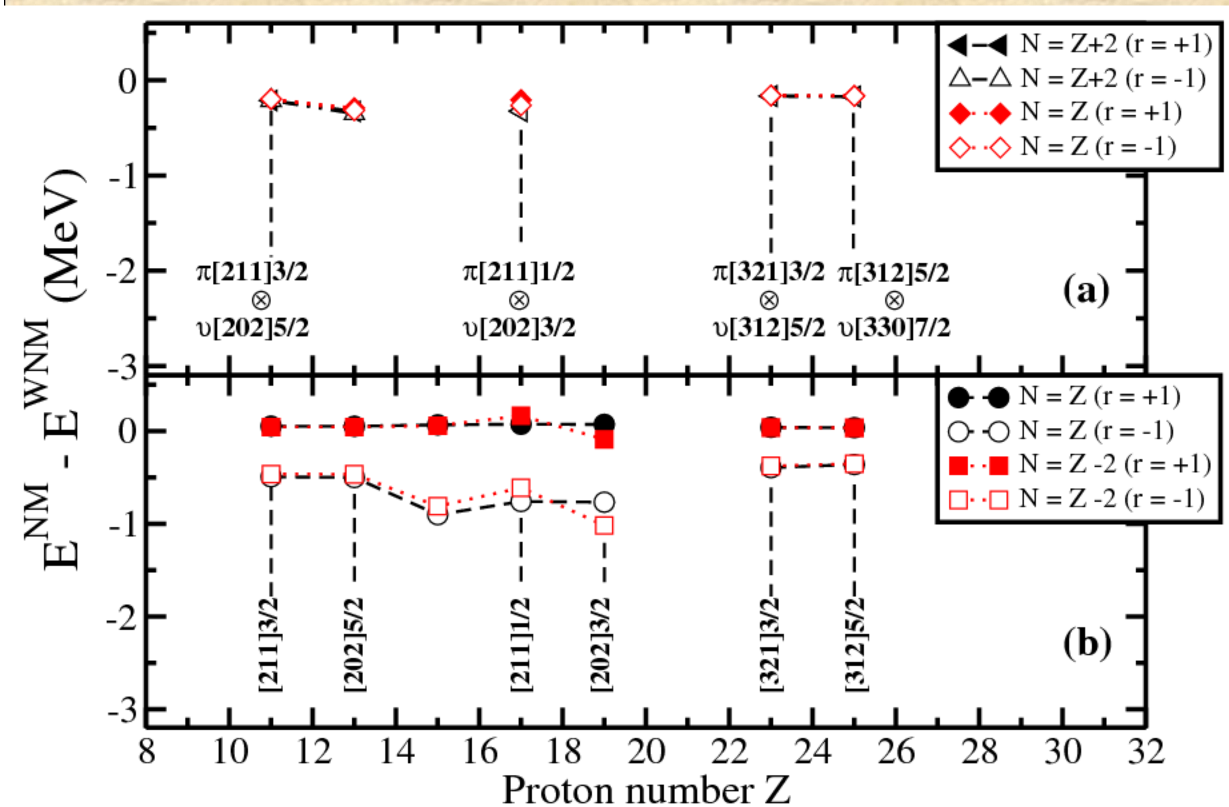




Signature separation is

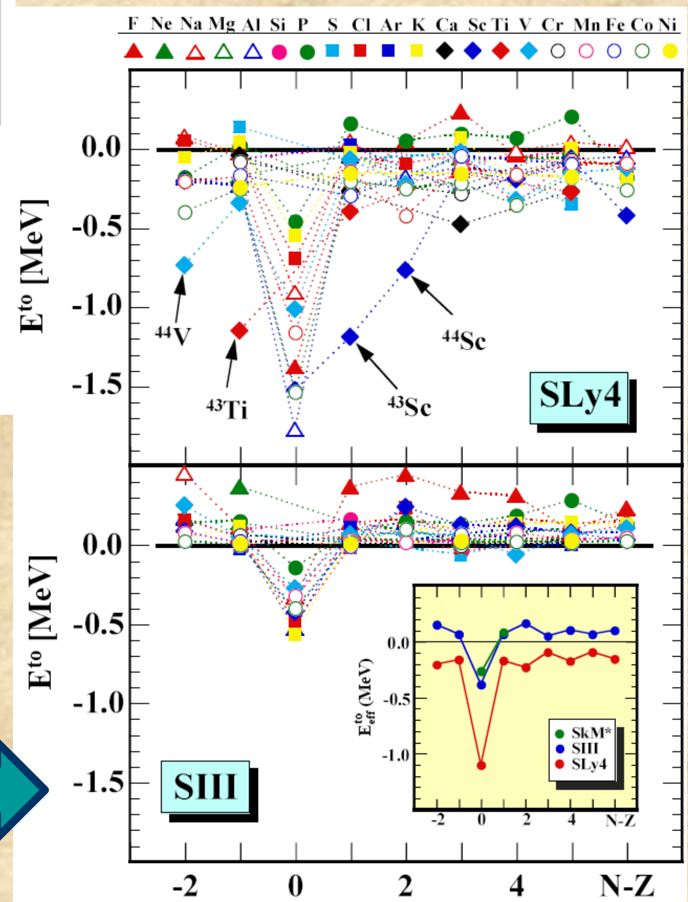
- **large** when the proton and neutron currents are of the same order of magnitude

- **small** when the currents are much stronger in one subsystem than in another

# 9. Are time-odd mean fields enhanced at N=Z ?

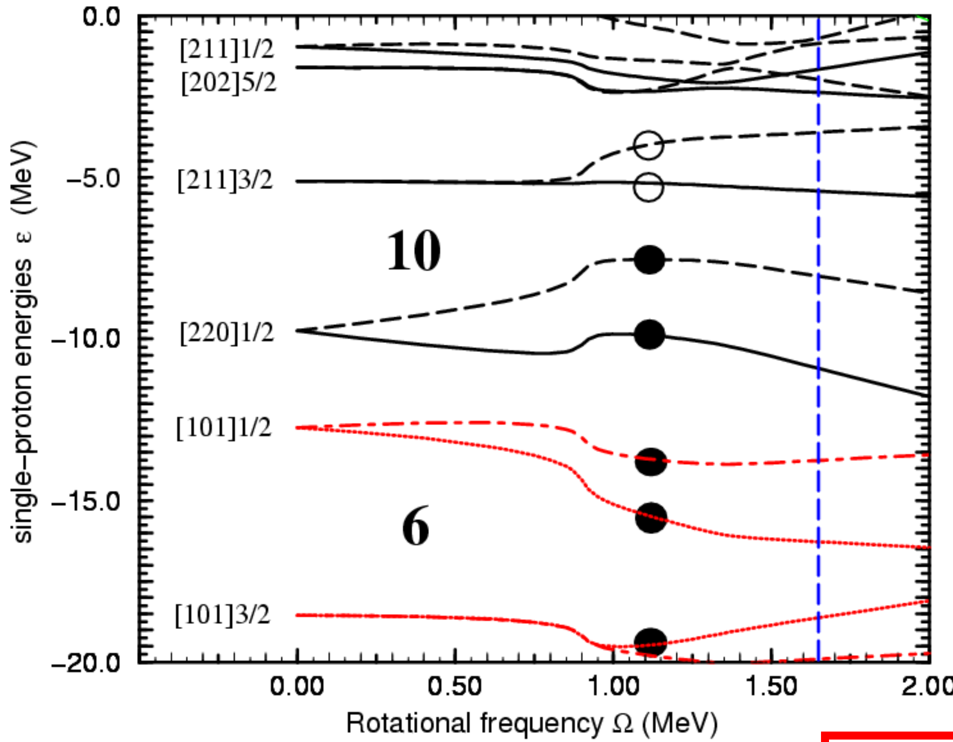



Skyrme




W. Satula, Proceedings of Nuclear Structure'98 conference, Gatlinburg, Tenn., USA

# 10. Time-odd mean fields in terminating bands

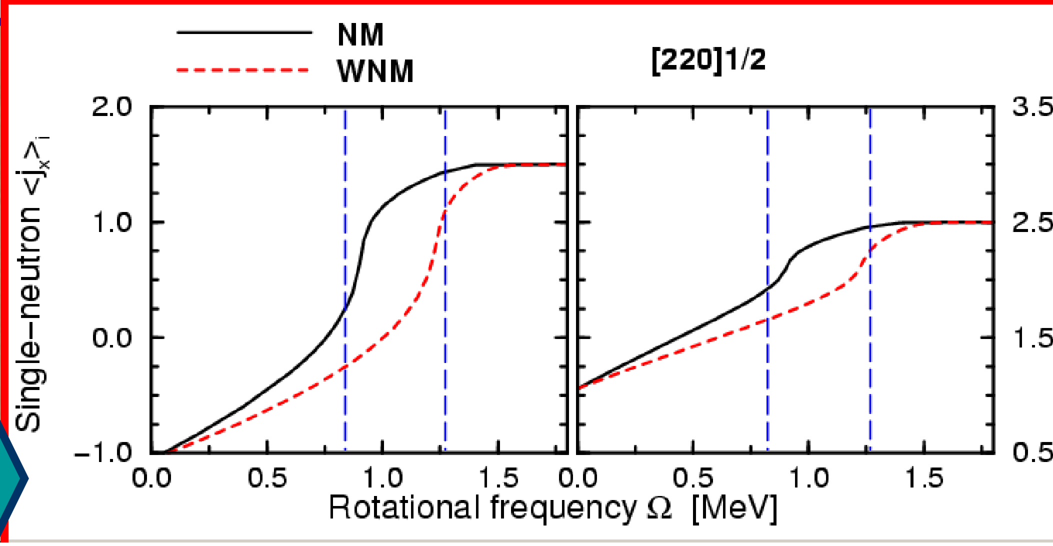



## $^{20}$Ne: the band terminating at I=8$^{+}$

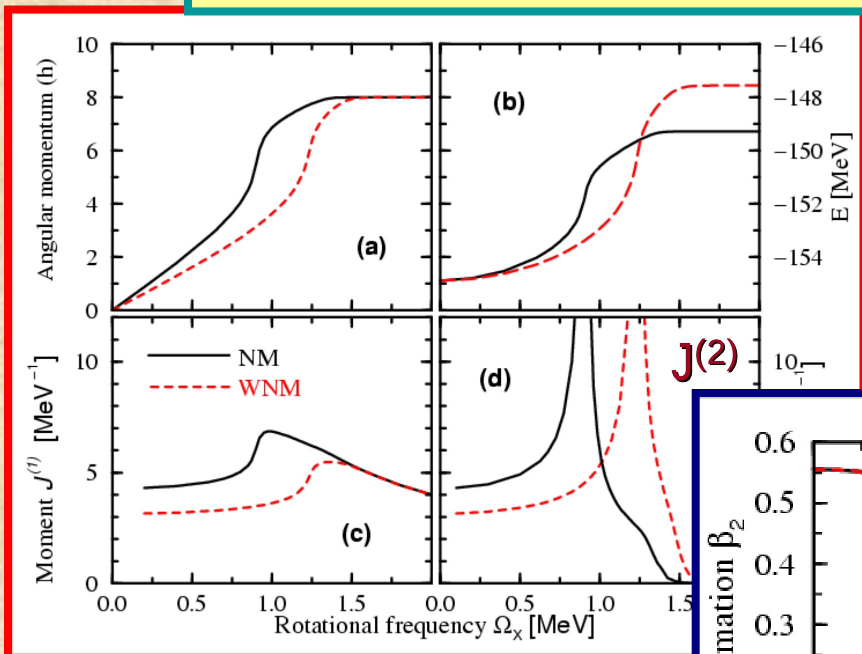
Structure of the band with respect of $^{16}$O core:

$$\pi(d_{5/2})^2\nu(d_{5/2})^2$$

The aligned single-particle angular momentum $<j_x>$ at band termination does not depend on presence or absence of nuclear magnetism

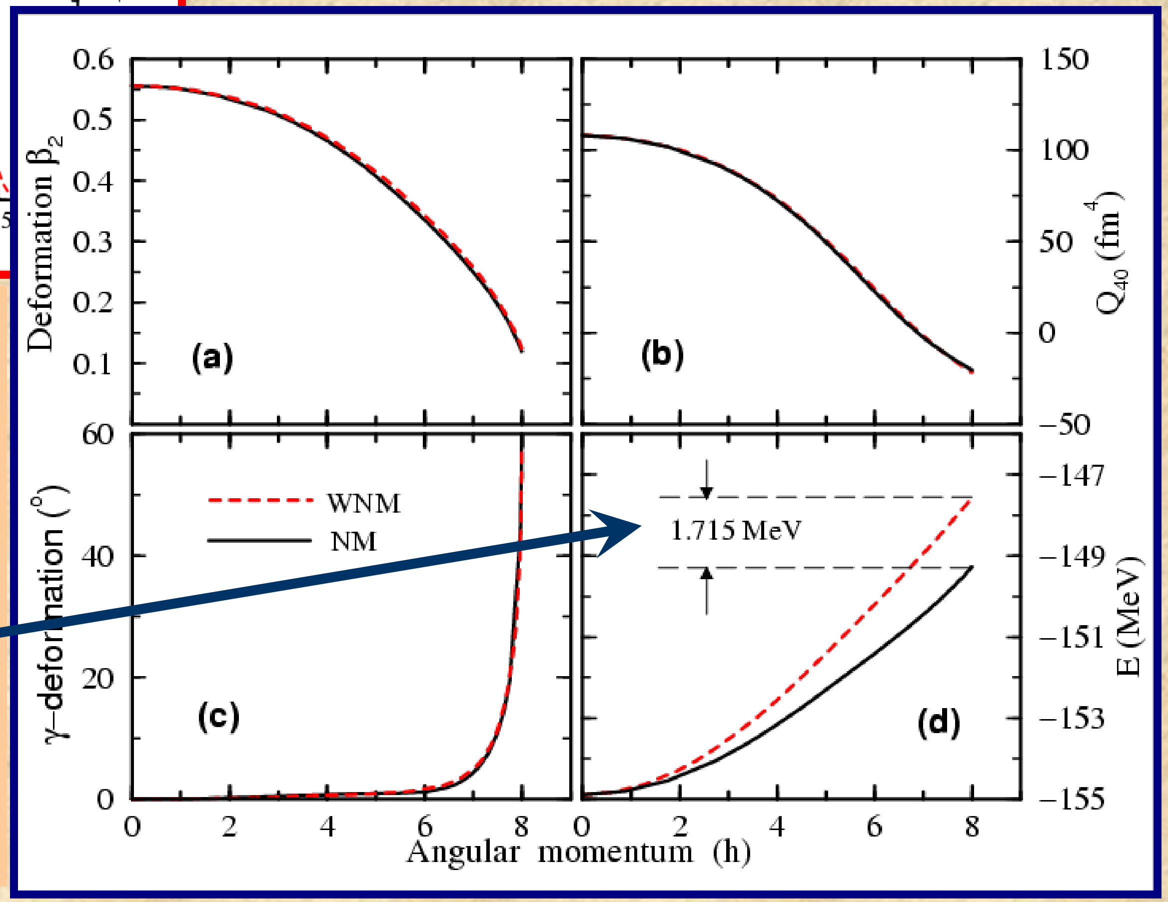


# 10. Time-odd mean fields in terminating bands



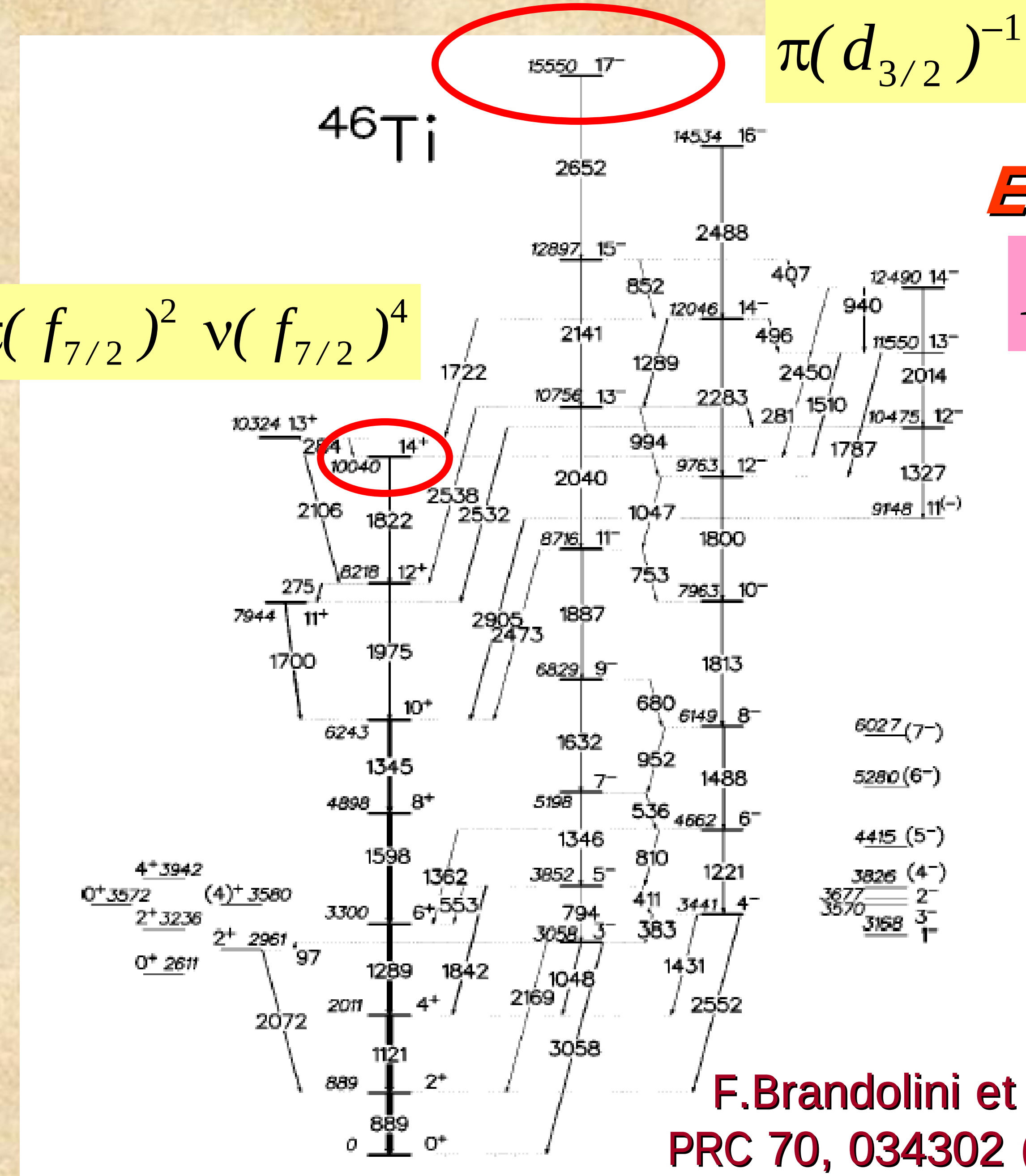

The maximum spin which can build within the configuration is the same in the calculations with and without nuclear magnetism

Additional binding energy at band termination due to time-odd mean fields depends on single-particle configuration:
It is maximum at the terminating state for a given configuration



$\pi(d_{3/2})^{-1}(f_{7/2})^{3}\ \nu(f_{7/2})^{4}$

$\pi(f_{7/2})^{2}\ \nu(f_{7/2})^{4}$




**_Energy differences_**

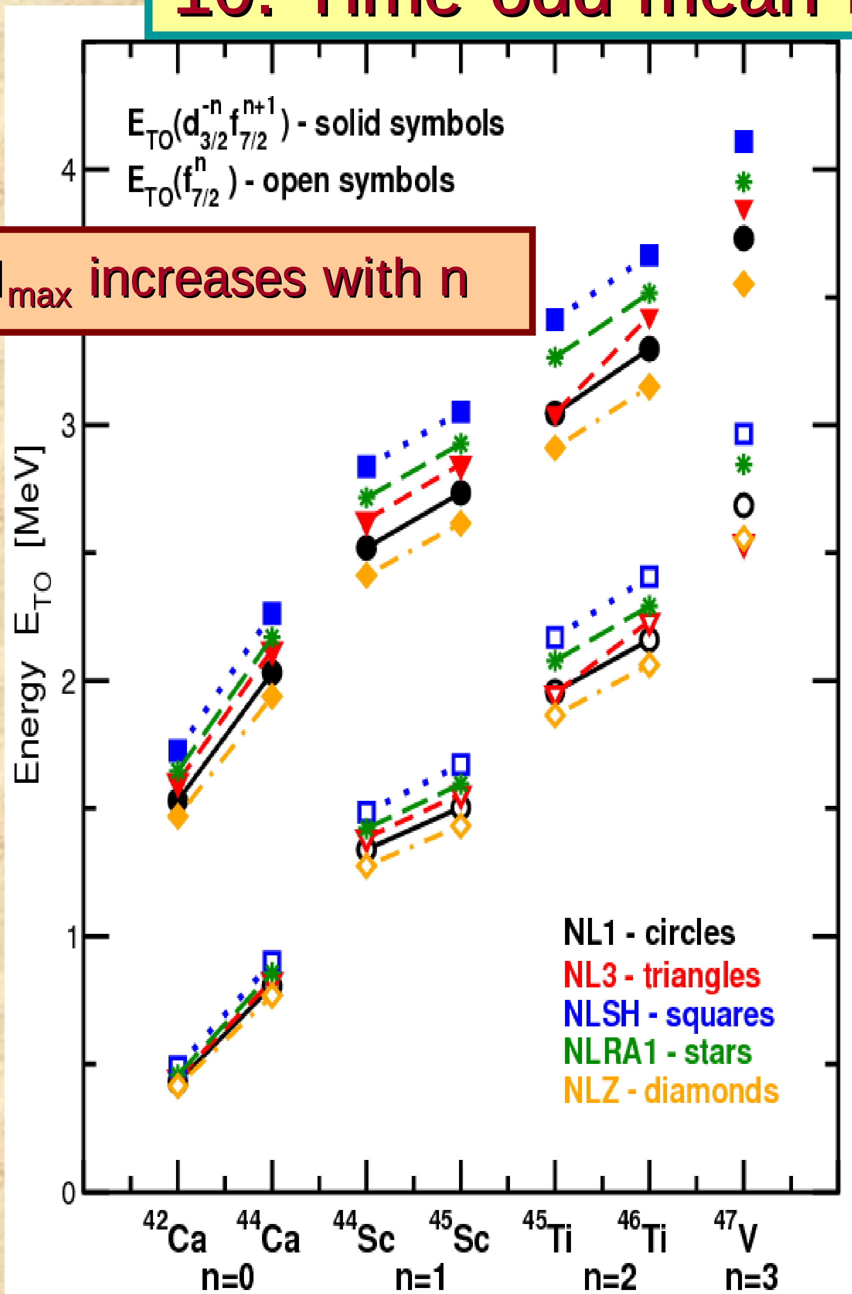
$E\left[\left(d_{3/2}^{-1} f_{7/2}^{n+1}\right)-\left(f_{7/2}^{n}\right)\right]$

F.Brandolini et al,
PRC 70, 034302 (2004)

# 10. Time-odd mean fields in terminating bands

$E_{TO}$ - additional binding due to time-odd mean fields

$$E_{TO} = \left| E^{NM} - E^{WNM} \right|$$

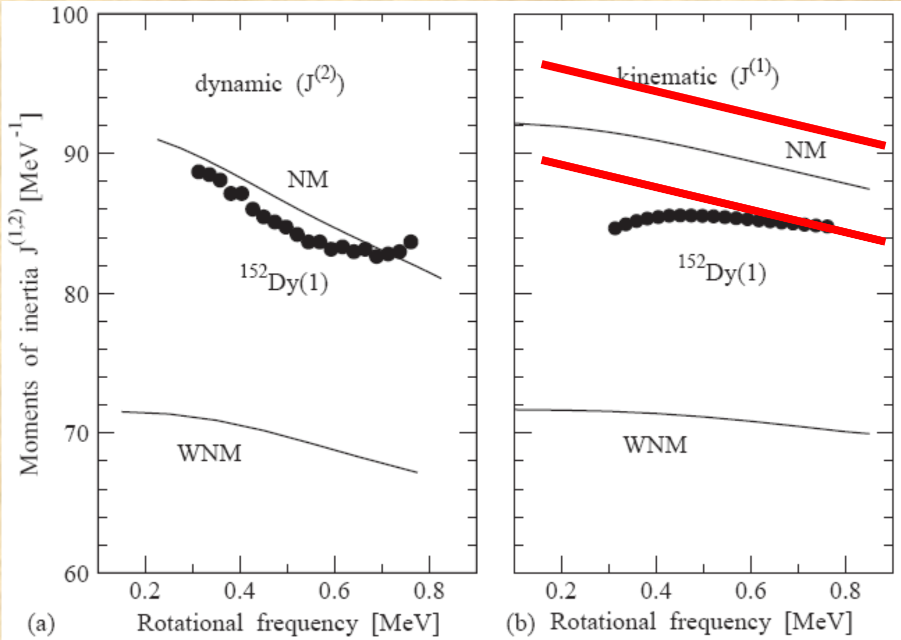



$I_{max}$ increases with n

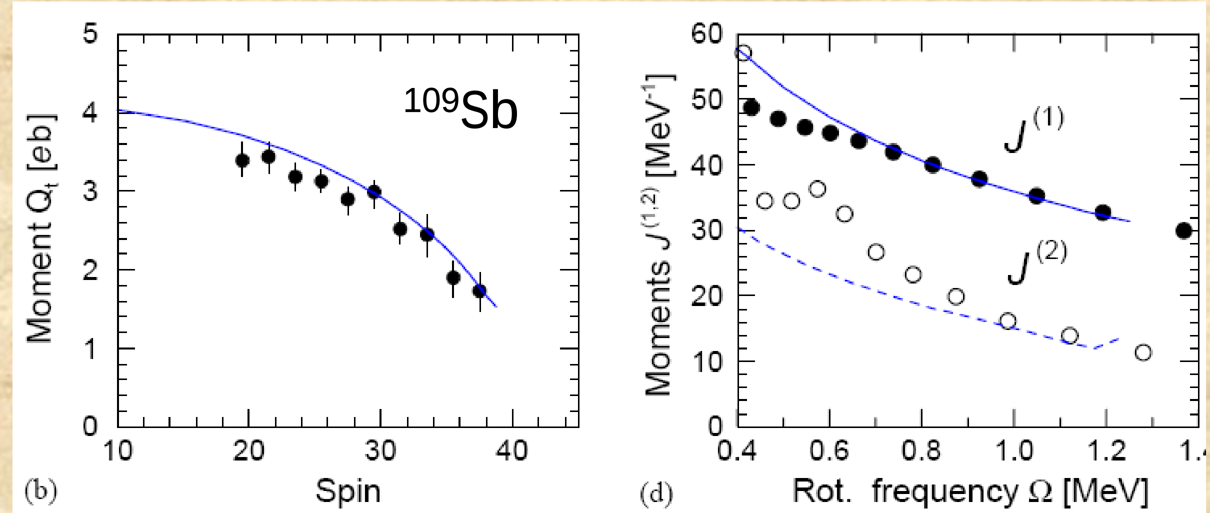
time-odd mean fields are defined with ~15% accuracy in the non-linear RMF parametrizations A.A., PRC 78, 054303 (08)

# 11. Are there experimental data which will allow to fix time-odd mean fields precisely

1. **magnetic moments**
   **NO**, complex quantity (meson-exchange corrections, coupling to magnetic resonances ...)
2. **moments of inertia**
   **PARTIALLY**,
   requires systematic calculations
3. - **terminating states**
   related to moments of inertia
   **NOT LIKELY**




Superdeformed band




Smooth terminating band

# Conclusions

Time-odd mean fields (nuclear magnetism)

- are dominated by $\omega$-meson
- are always attractive in odd-mass nuclei (additional binding due to NM)
- show weak dependence on the parametrization for the non-linear parametrizations
- should be taken into account when the strength of pairing is defined using odd-even mass differences
- affect the properties of proton emitters
- odd-odd nuclei – more complicated and can be more attractive than in odd-mass nuclei
  - enhanced when proton and neutron occupy the same single-particle states
- should be taken into account for mass tables

***Outlook:*** 1. manuscript submitted to PRC

2. systematic study of rotating nuclei, density-dependent meson couplings and scalar-vector couplings – manuscript in preparation