

Role of shell evolution in the structure of exotic nuclei in the sd-pf shell

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Outline

- Introduction—deducing the shell evolution around $N=20$ from a phenomenology
- A source of the shell evolution: tensor force
- Construction of a new shell model interaction in the sd-pf shell towards high predictive power
- Yrast-state properties: single-particle state and its effect on the collectivity
- Single-particle structure from the spectroscopic factor

Aim of the present study

- Observables

- In general, reflecting a complicated wave function (correlated)
- In many cases, the **single-particle property (or shell structure)** plays an important role in the wave function, even though it does not appear directly.

- Strategy

- To extract how the shell structure develops over the chart
- Models that can describe correlation well → using shell model

- Step

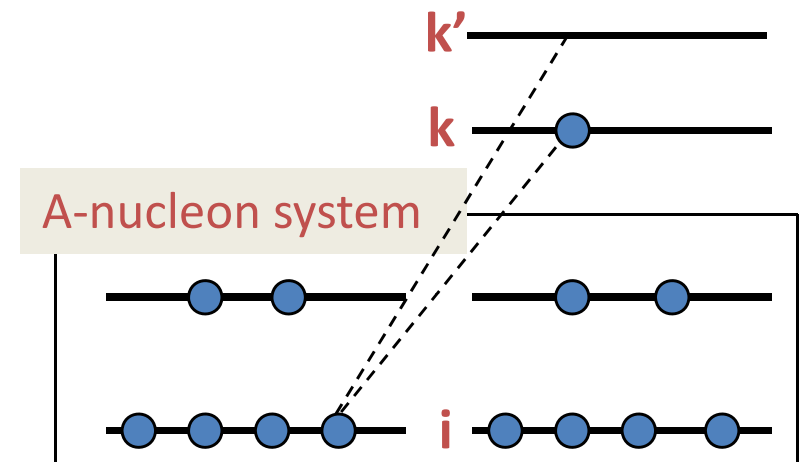
1. Phenomenology
2. Deducing the mechanism
3. Testing and predicting

Monopole interaction and effective single particle energy

- Effective single-particle energy (ESPE):

Shell-model viewpoint of generalized SPE

- Filling configuration is assumed for the A-nucleon system.
- Additional binding energy in the (A+1) nucleon system defines ESPE.
- Total energy for a fully occupied system can be evaluated simply by counting the number of “bonds” and their **monopole** centroids.



sum of the monopole centroids associated with the orbit k

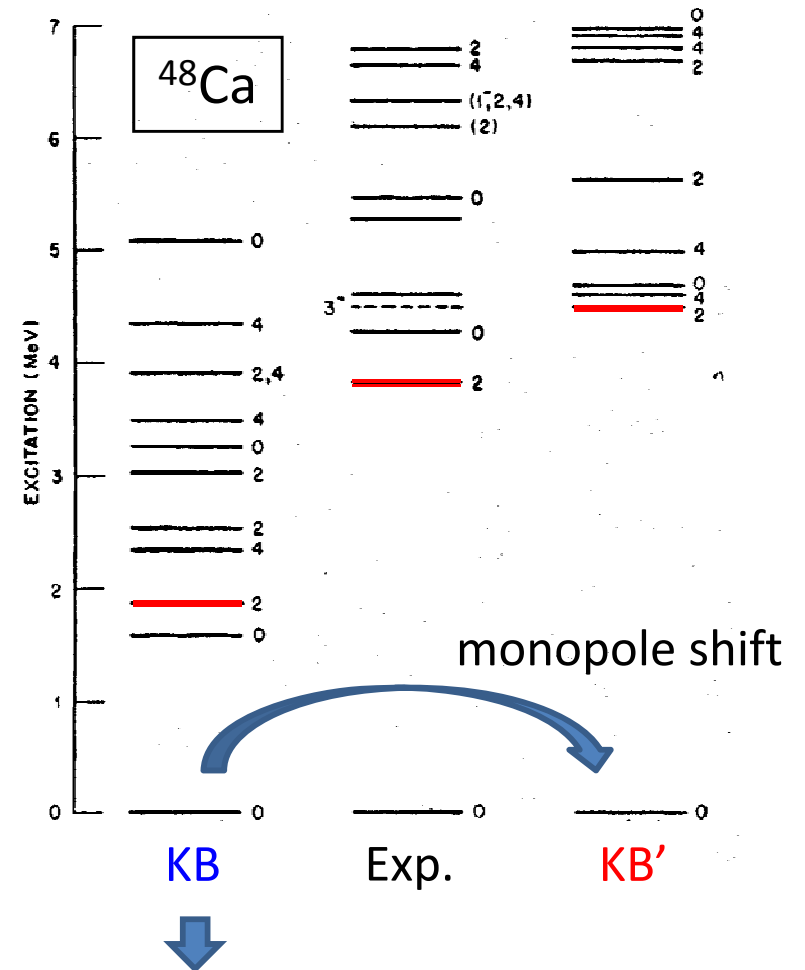
$$V_{i,j}^T = \frac{\sum_J (2J + 1) \langle i, j | V | i, j \rangle_{J,T}}{\sum_J (2J + 1)}$$

- Orbital dependence of the monopole centroid causes the **shell evolution**. ($V_{i,k} \neq V_{i,k'}$ gives rise to the variation of the shell gap.)

Problem coming from inaccurate monopole

- $f_{7/2}$ closure cannot be achieved by an interaction without good monopole property.
- KB3 interaction, further modified one to KB', succeeded in reproducing more pf shell nuclei.
(A. Poves and A. Zuker, Phys. Rep. 70, 235 (1981) and succeeding papers)
 - Monopole shift is a phenomenological but powerful way to reproduce the structure.

Obtaining a good monopole interaction is a key issue in the shell model.

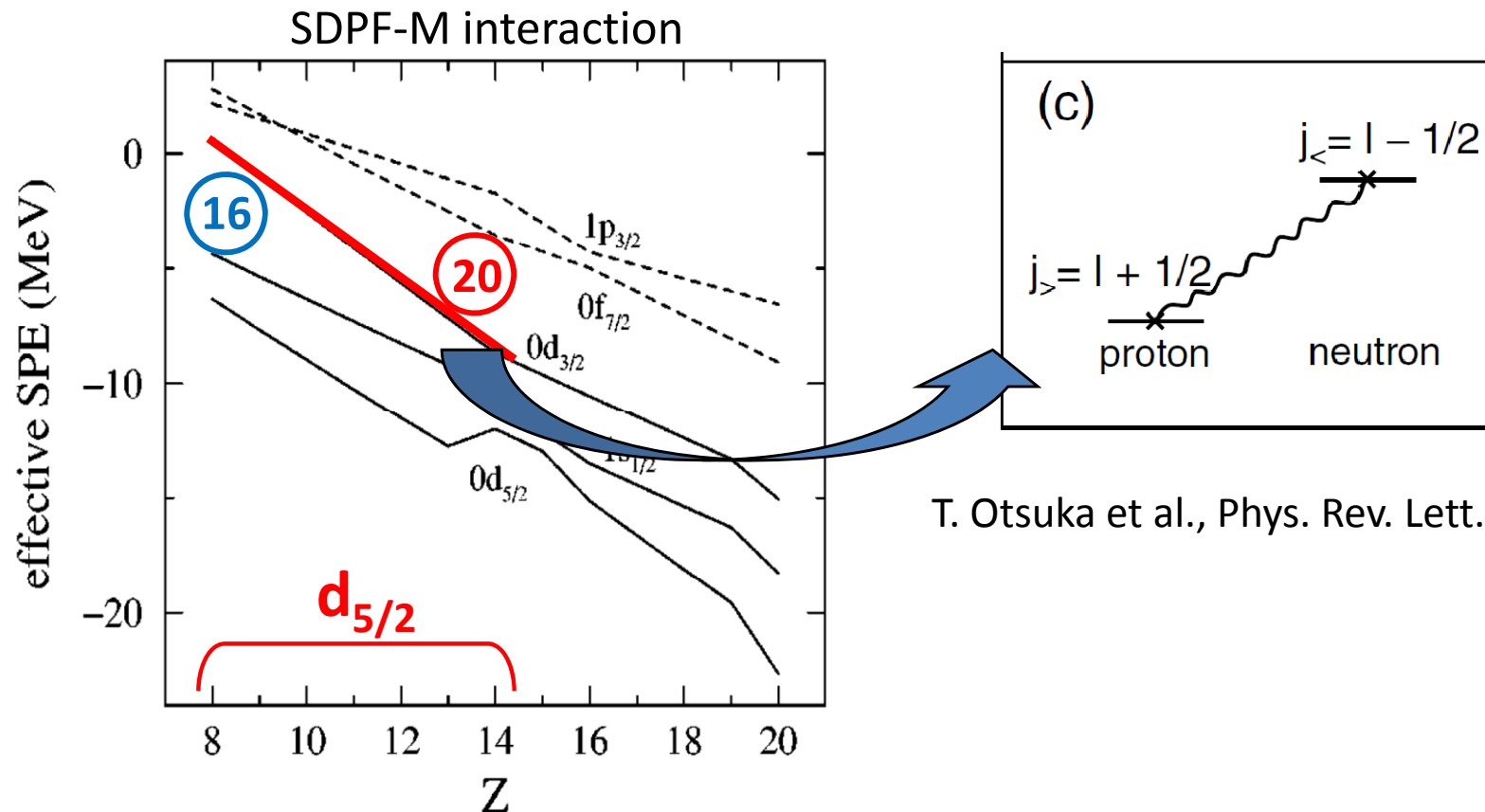


Collapsed N=28 shell closure

J. B. McGrory et al., Phys. Rev. C 2, 186 (1970) .

Phenomenology of N=20 region

- “**SDPF-M**” interaction accounts for the **disappearance of N=20 magic** and the **appearance of N=16 magic** as a result of its strongly attractive T=0 $d_{3/2}$ - $d_{5/2}$ monopole interaction.



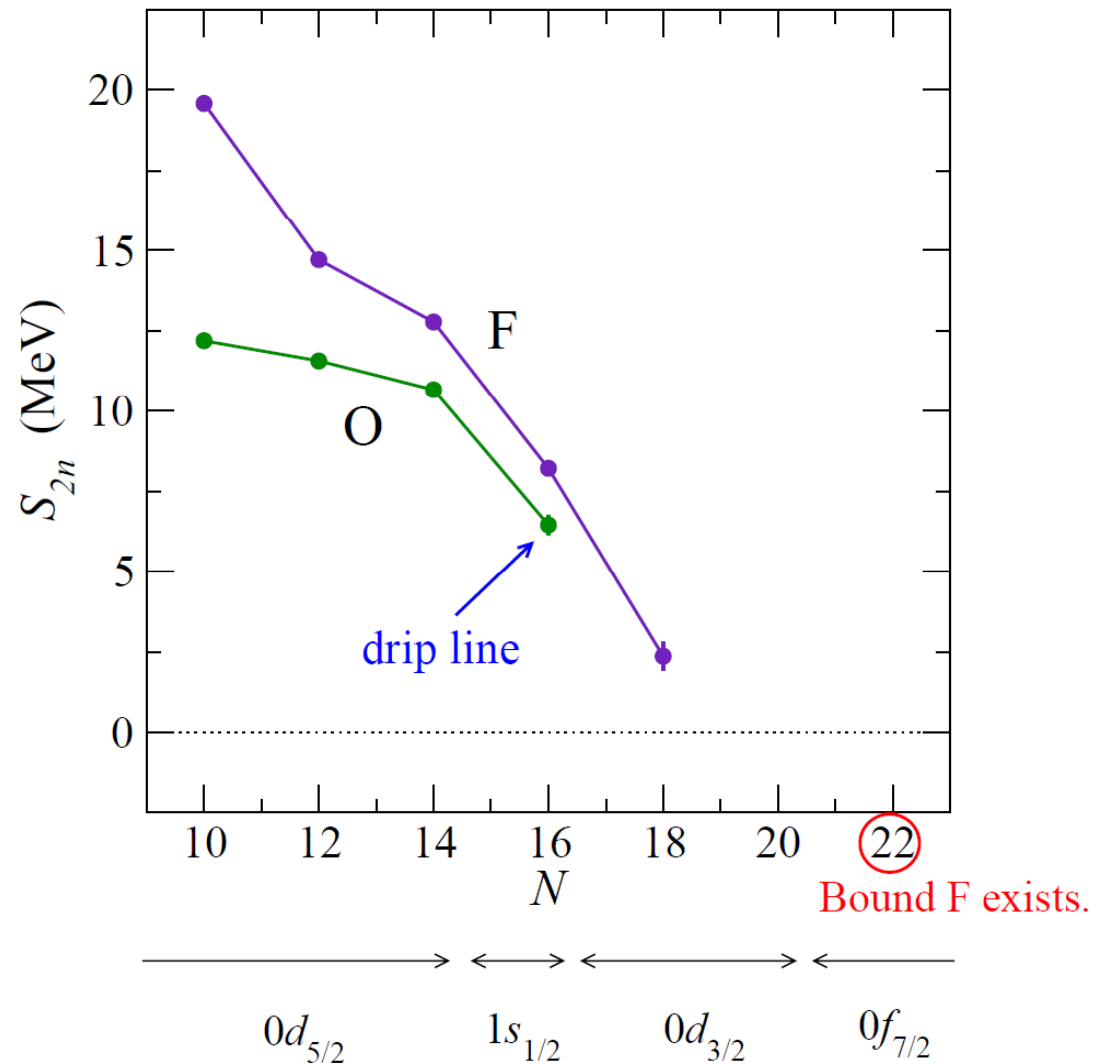
T. Otsuka et al., Phys. Rev. Lett. 87, 082502 (2001).

Y. Utsuno et al., Phys. Rev. C 60, 054315 (1999) and following MCSM papers.

Unexpected extension of the drip line

- No bound oxygen isotopes heavier than $N=16$.
(D. Guillemaud-Mueller et al., M. Fauerbach et al., O. Tarasov et al., H. Sakurai et al.)
- Fluorine isotopes are bound at least up to $N=22$.
(H. Sakurai et al.)
- The standard USD shell model predicts bound oxygen up to $N=18$.
- If there is a big $N=20$ shell gap, fluorine with $N=22$ must not be a bound nucleus.

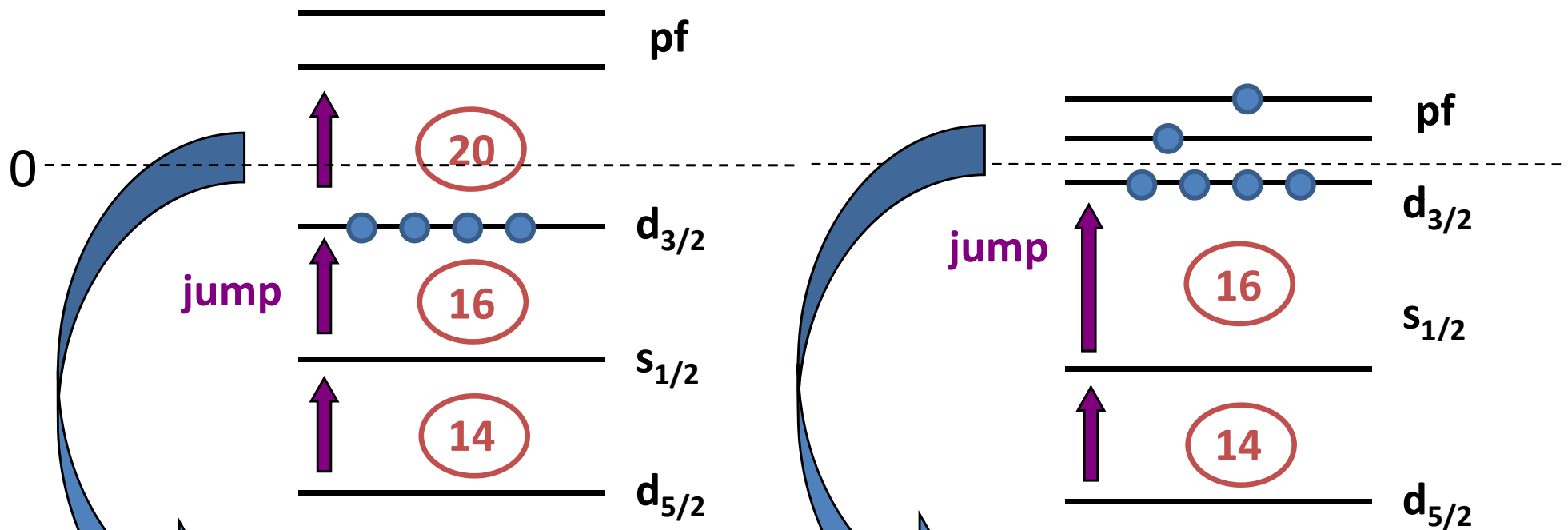
Experimental S_{2n} of O and F



Schematic explanation about the drip line

i) Normal shell structure

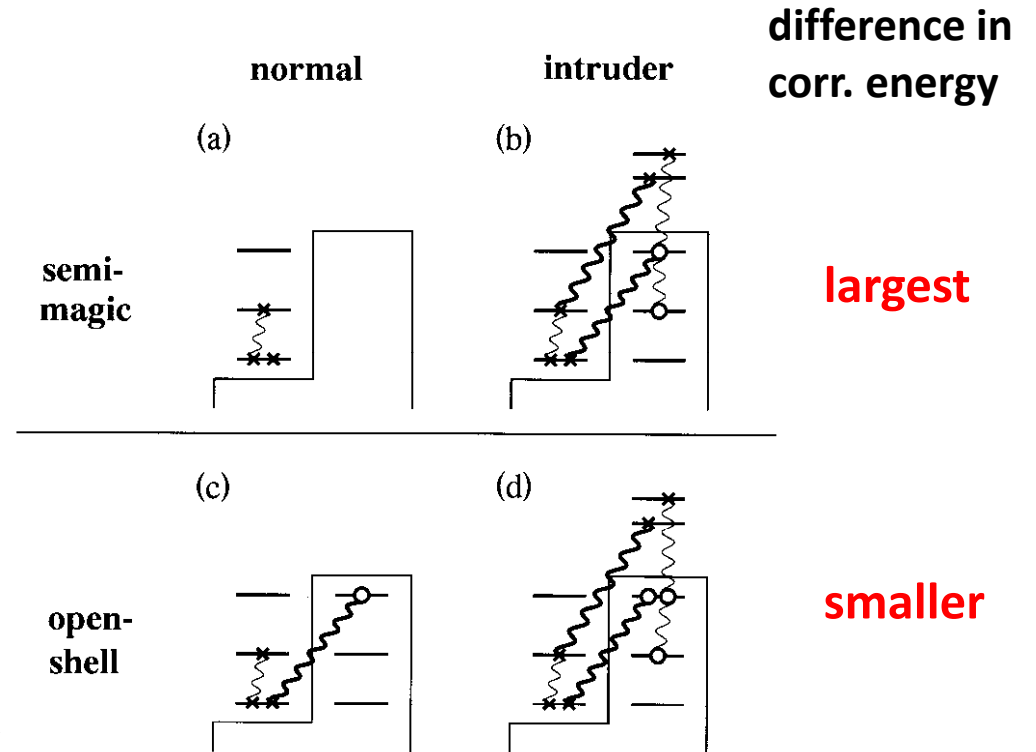
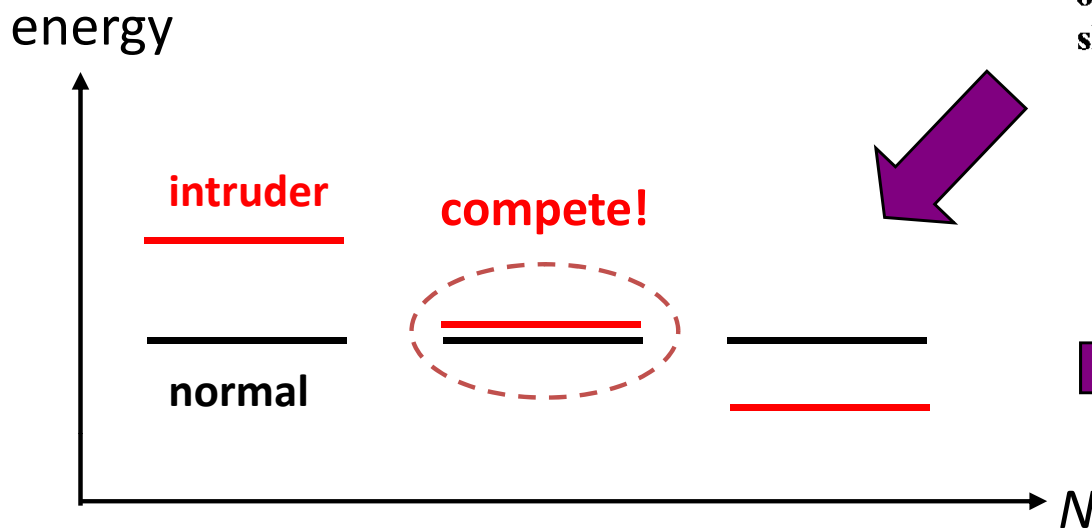
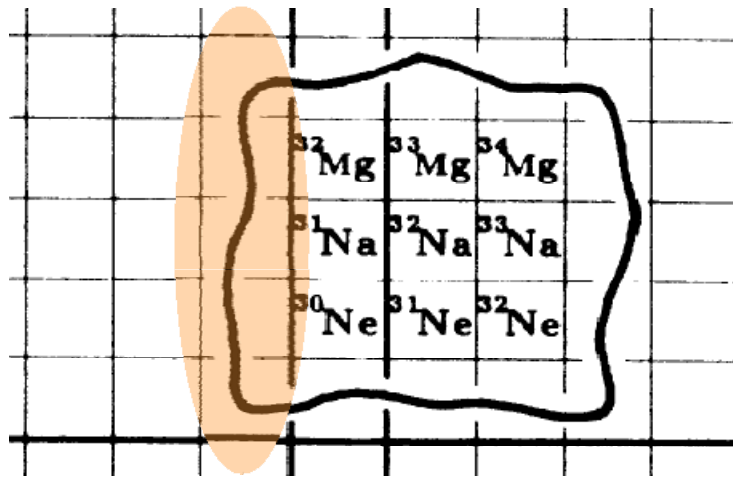
ii) Quenched N=20 shell gap



If there is a certain N=20 shell gap in F, the drip line would not persist so far away (at most 4 in $d_{3/2}$).

Large correlation energy due to the degeneracy can extend the drip line further.

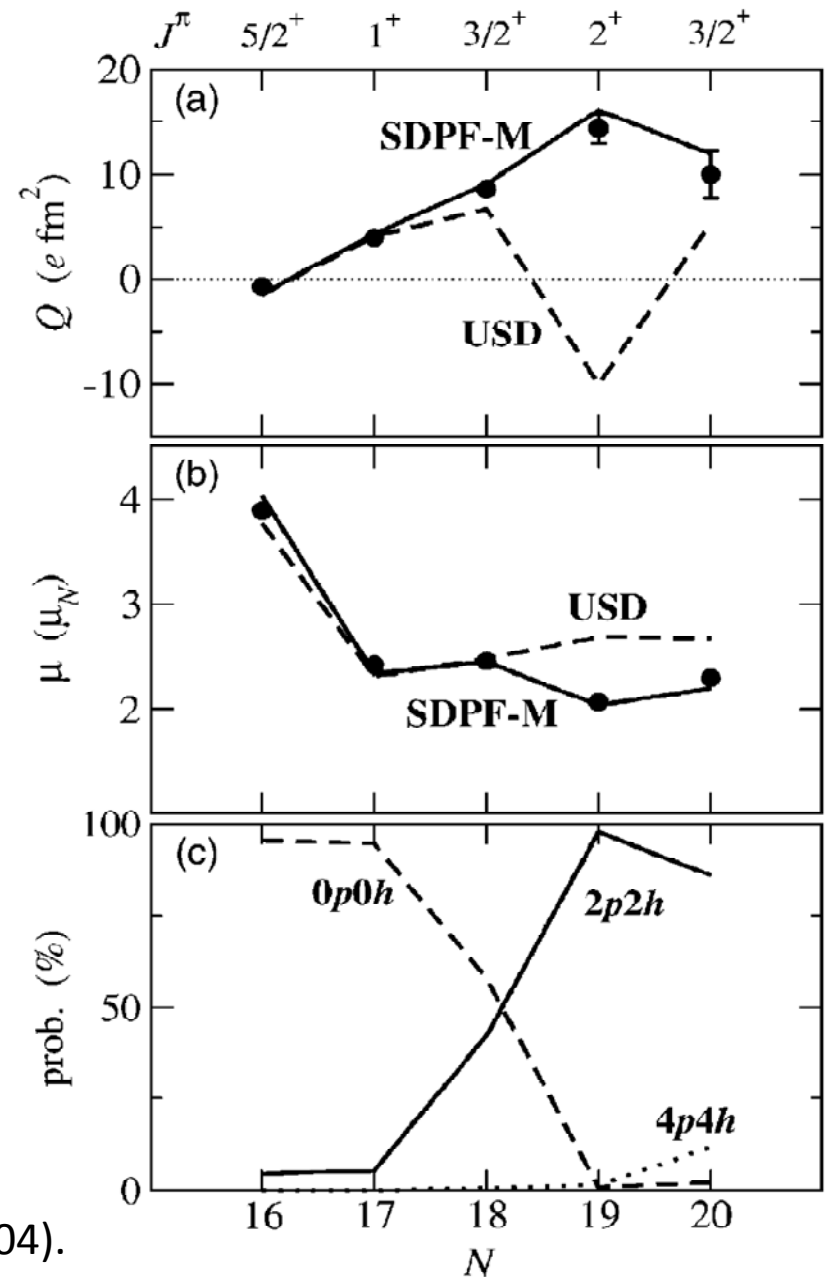
Importance of determining “western” boundary



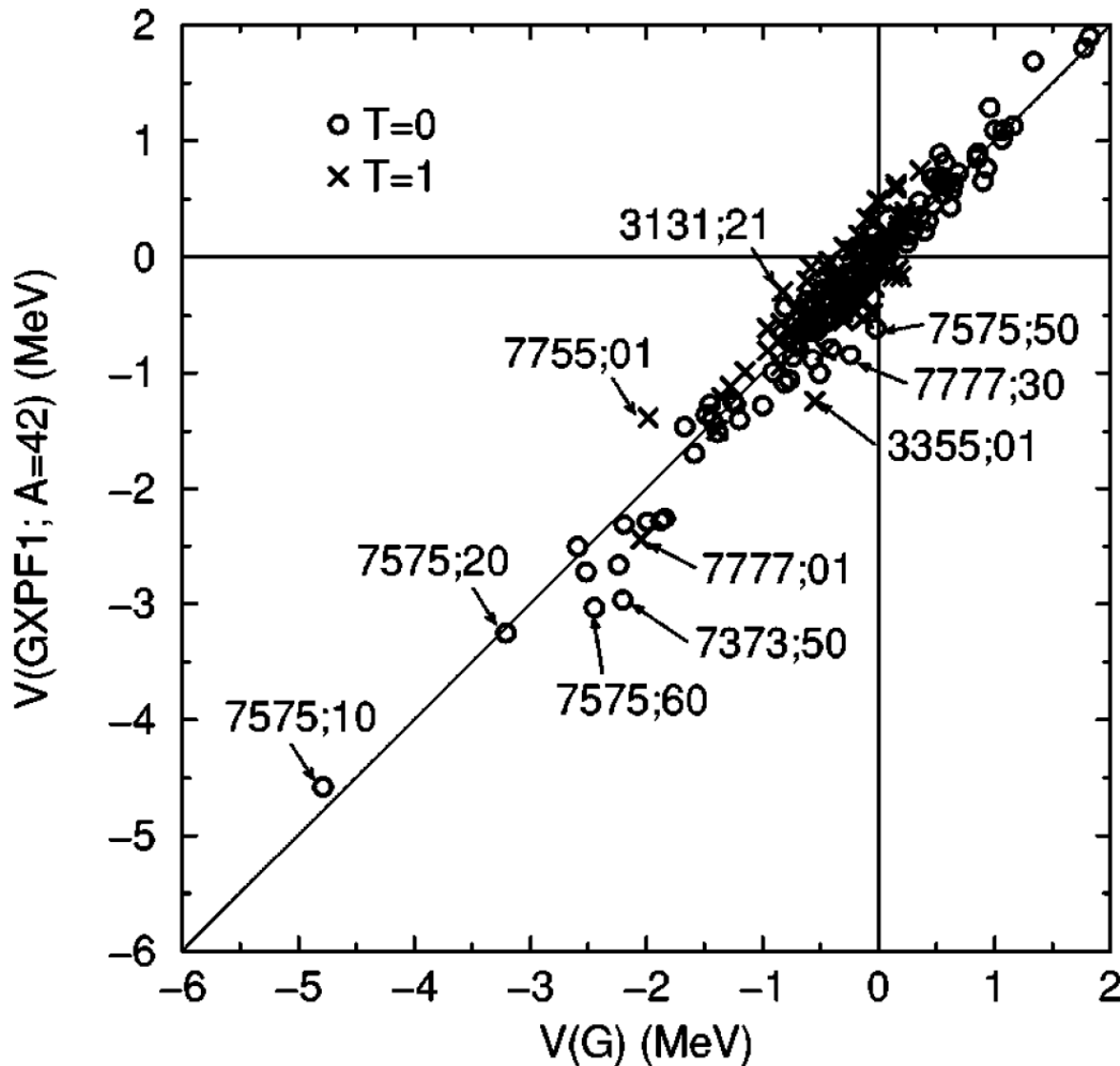
The location of the western boundary is quite sensitive to the shell gap.

The case for Na isotopes

- Moment: direct information on the ground-state wave function
 - Quadrupole moment and magnetic moment of Na isotopes compared between theory and experiment
 - Clear evidence for the dominance of the intruder state at N=19 which occurs earlier than the prediction by “island of inversion”.



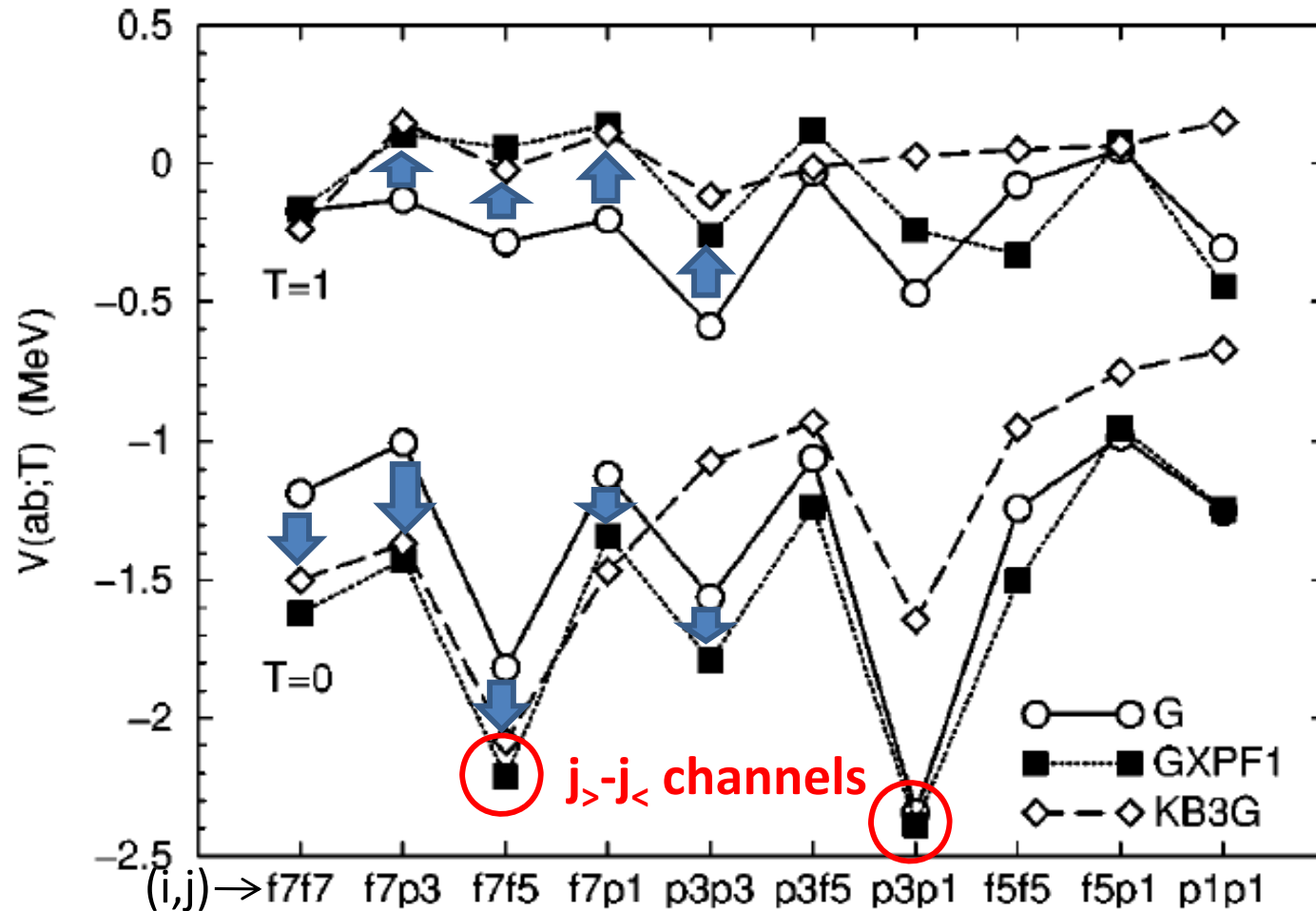
Microscopic vs. semi-empirical



Notation: $abcd;JT = \langle ab | V | cd \rangle_{JT}$ $7=f_{7/2}$, $5=f_{5/2}$, $3=p_{3/2}$, and $1=p_{1/2}$

- **GXPF1**
M. Honma et al., Phys. Rev. C 65, 061301(R) (2002); 69, 034335 (2004).
 - Semi-empirical interaction starting from G (Bonn C)
 - Good description over the **pf shell** region: reflecting good isospin behavior
- Strong correlation between G and GXPF1

Microscopic vs. semi-empirical (monopole)



Taken from M. Honma et al.,
Phys. Rev. C 69, 034335 (2004).

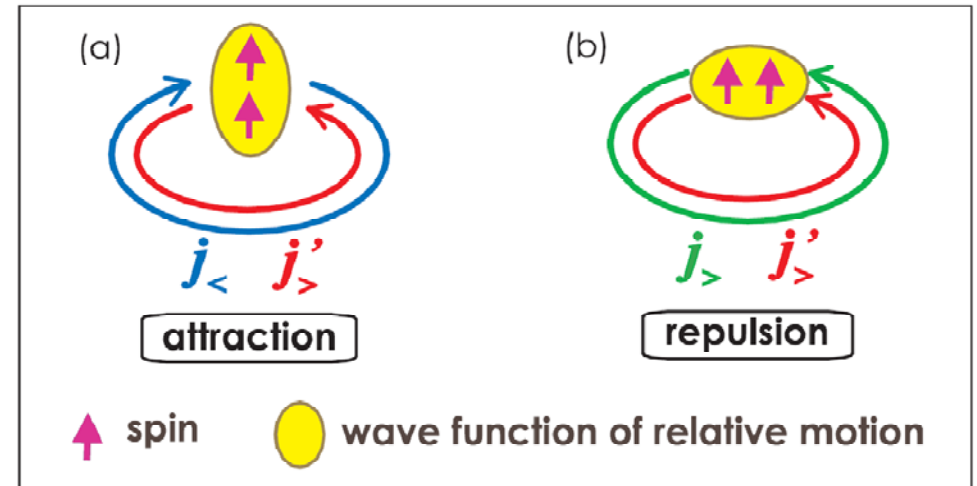
$f7 = f_{7/2}$ etc.

- Systematic deviation from the G matrix (T=0 and 1)
 - Related to monopole correction by the Strasbourg group
- **Strong $j_>-j_<$ attraction is also seen!**

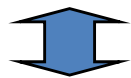
Shell evolution due to the tensor force

- **Tensor force**

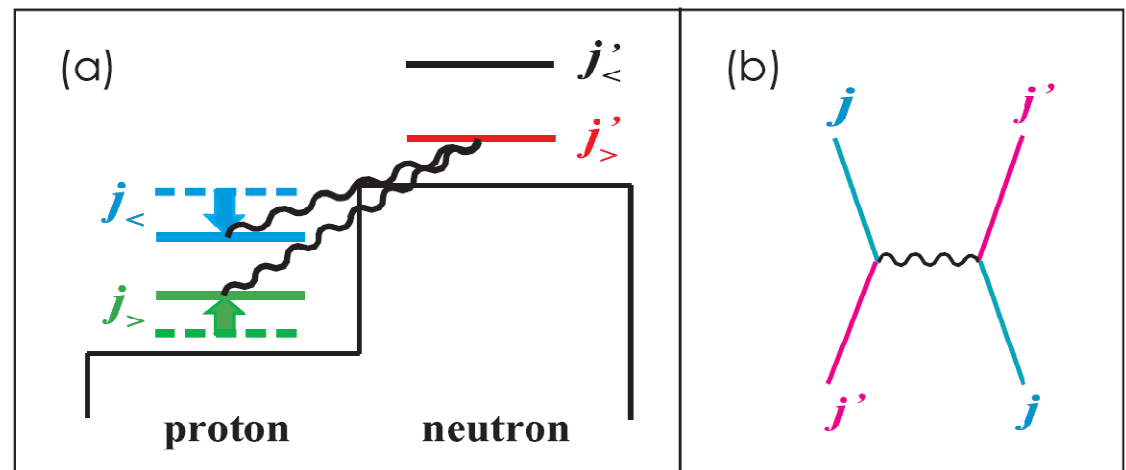
- Working not only between the same l but also different l 's strongly dependent on the spin direction.
- Most promising quantitatively



$$(2j_{>} + 1)V_{j_{>},j'}^T + (2j_{<} + 1)V_{j_{<},j'}^T = 0;$$



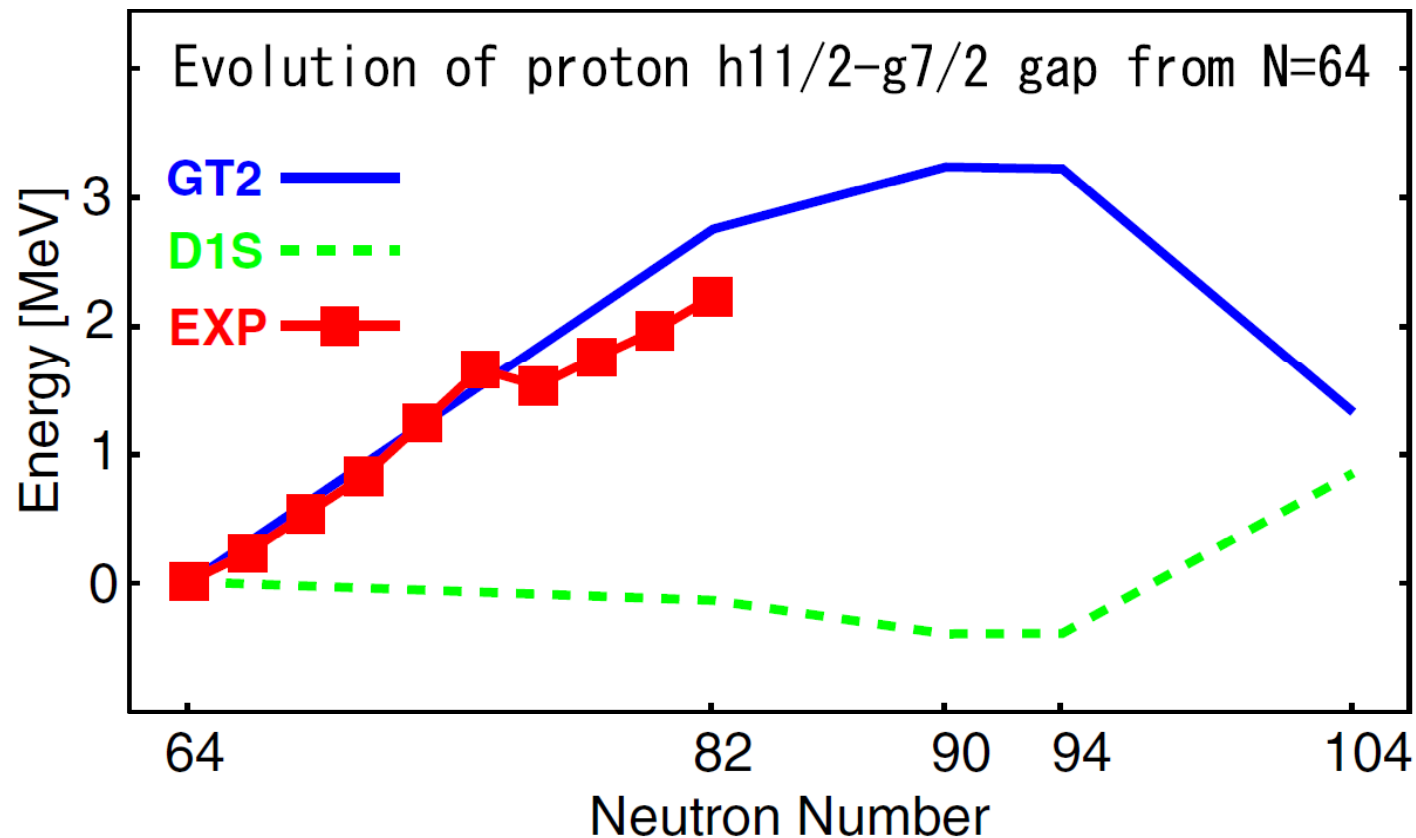
Effect of the tensor vanishes for an LS-closed configuration.



T. Otsuka et al., Phys. Rev. Lett. 92, 232502 (2005).

Mean-field calculation with the tensor force

- GT2 interaction (T. Otsuka et al., Phys. Rev. Lett. 97, 162501 (2006).) successfully accounts for the spherical single-particle structure on top of $Z=50$ owing to the tensor shell evolution.

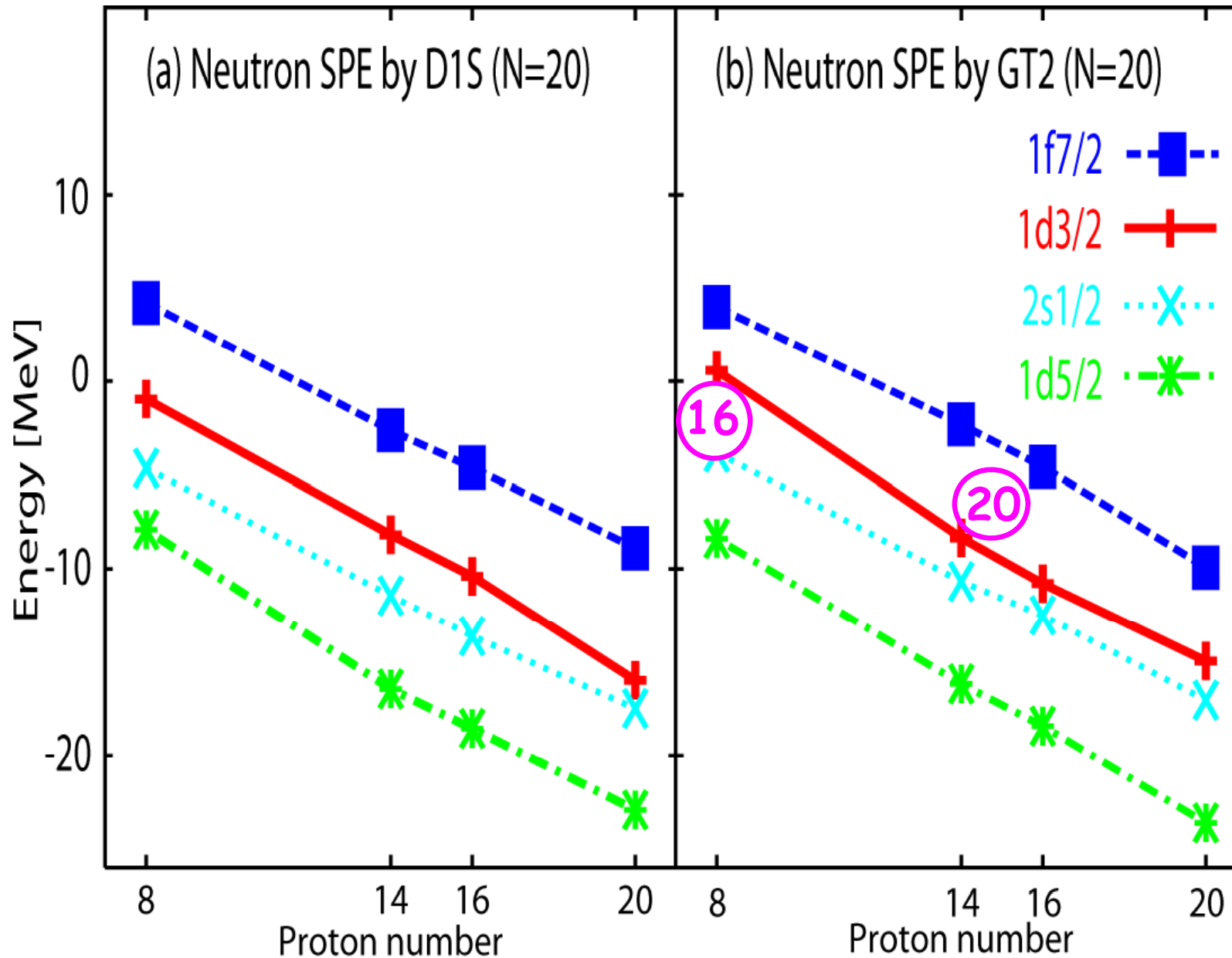


Two Gogny(-type) interactions : D1S and GT2

(Thanks to T. Otsuka,
T. Matsuo, D. Abe)

w/o tensor

with tensor



The same
change as
in the shell
model with
SDPF-M int.

16

20

How to extract the tensor force from a shell model interaction

- **Spin-tensor decomposition** (M.W. Kirson, Phys. Lett. 47B, 110 (1973))
 - Any interaction can be decomposed into central, LS and tensor parts even though it is given in numbers.

$$V = \sum_{k=0}^2 V_k = \sum_{k=0}^2 U^{(k)} \cdot X^{(k)}$$



$$\langle ABLSJ'T | V_k | CDL'S'J'T \rangle$$

$$= (-1)^{J'} (2k+1) \begin{Bmatrix} L & S & J' \\ S' & L' & k \end{Bmatrix} \sum_J (-1)^J (2J+1) \begin{Bmatrix} L & S & J \\ S' & L' & k \end{Bmatrix}$$

$$\times \langle ABLSJT | V | CDL'S'JT \rangle$$

(U and X stands for the two-body operator with rank k in the coordinate and spin spaces, respectively: central for k=0, LS for k=1, and tensor for k=2)

(A to D are the label of the orbit having quantum numbers n and l)

Application to the GXPF1 interaction

- **GXPF1**

M. Honma, T. Otsuka, B.A. Brown and T. Mizusaki, Phys. Rev. C 65, 061301(R) (2002); 69, 034335 (2004).

- Giving a quite quantitative description for pf-shell nuclei

- **Millener-Kurath (MK)**

D.J. Millener and D. Kurath, Nucl. Phys. A 255, 315 (1975).

- Often adopted as the cross-shell interaction in the p-sd and sd-pf shell regions.
- Yukawa-type potential

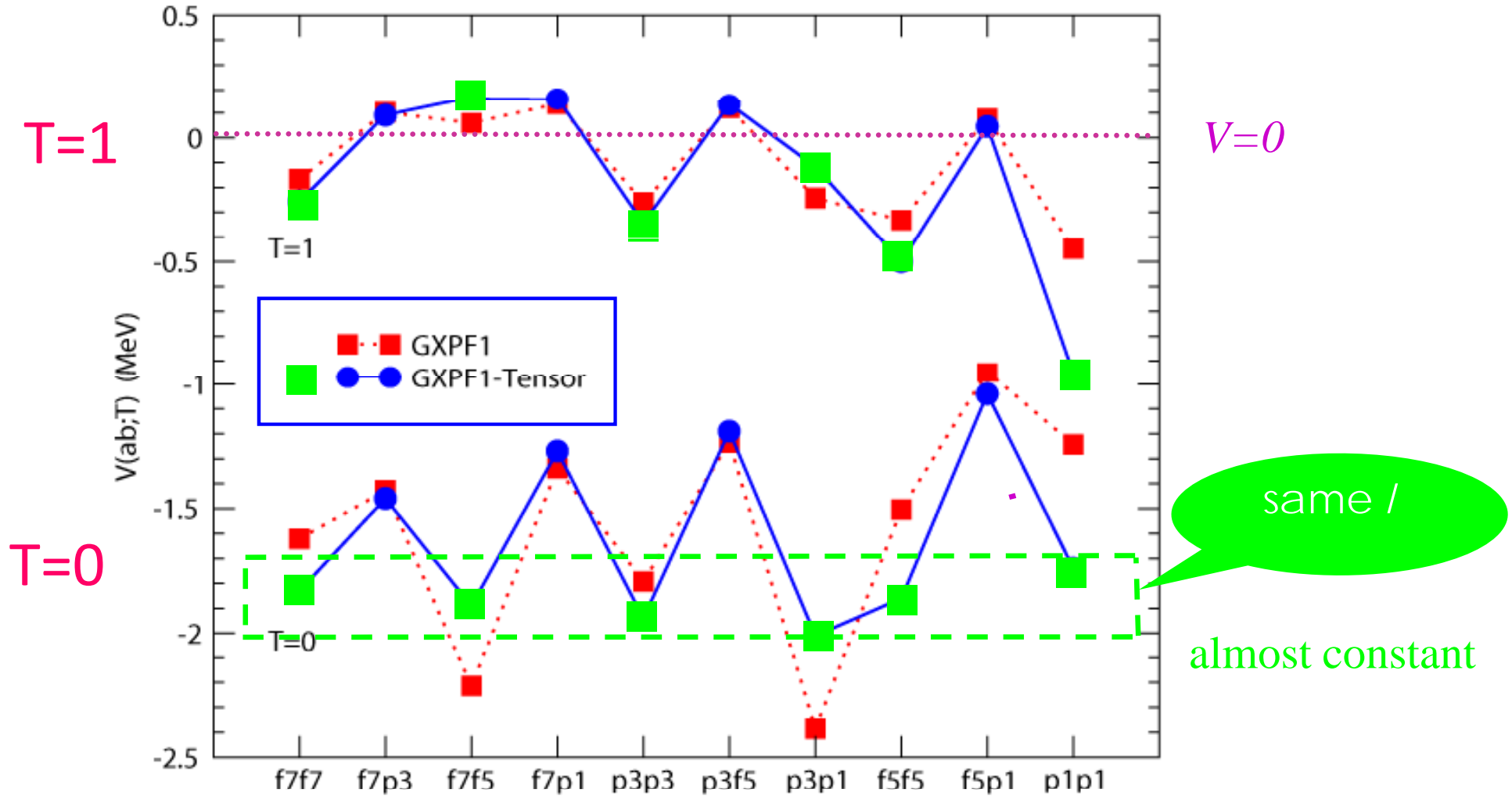
T=0 monopole interaction (MeV)

i	j	GXPF1	$\pi+\rho$	GT2	MK
f7	f7	0.223	0.210	0.244	0.080
f7	p3	0.036	0.035	0.044	0.013
f7	f5	-0.335	-0.315	-0.365	-0.120
f7	p1	-0.073	-0.070	-0.088	-0.026
p3	p3	0.092	0.150	0.154	0.064
p3	f5	-0.048	-0.046	-0.058	-0.017
p3	p1	-0.229	-0.376	-0.386	-0.160
f5	f5	0.382	0.360	0.418	0.137
f5	p1	0.097	0.093	0.117	0.034
p1	p1	0.306	0.501	0.515	0.213

Tensor force in the shell model interaction is rather similar to that of the one-boson exchange potential and that for a mean-field calculation. Universality?

Monopole interaction of GXPF1 :
 full (original) and
 part after subtraction of tensor part ($\pi + \rho$)

■ GXPF1 - Tensor between same l



Message from the GXPF1 study

- **Tensor force**

- Rather strong compared with conventional potential interactions such as MK etc.
- Quite similar to $\pi+\rho$ (with cutoff at 0.7 fm) and that of GT2 which successfully accounts for the spherical shell evolution

- **Central force**

- The dependence of spin on its monopole part almost disappears.
- Very simple and characteristic feature

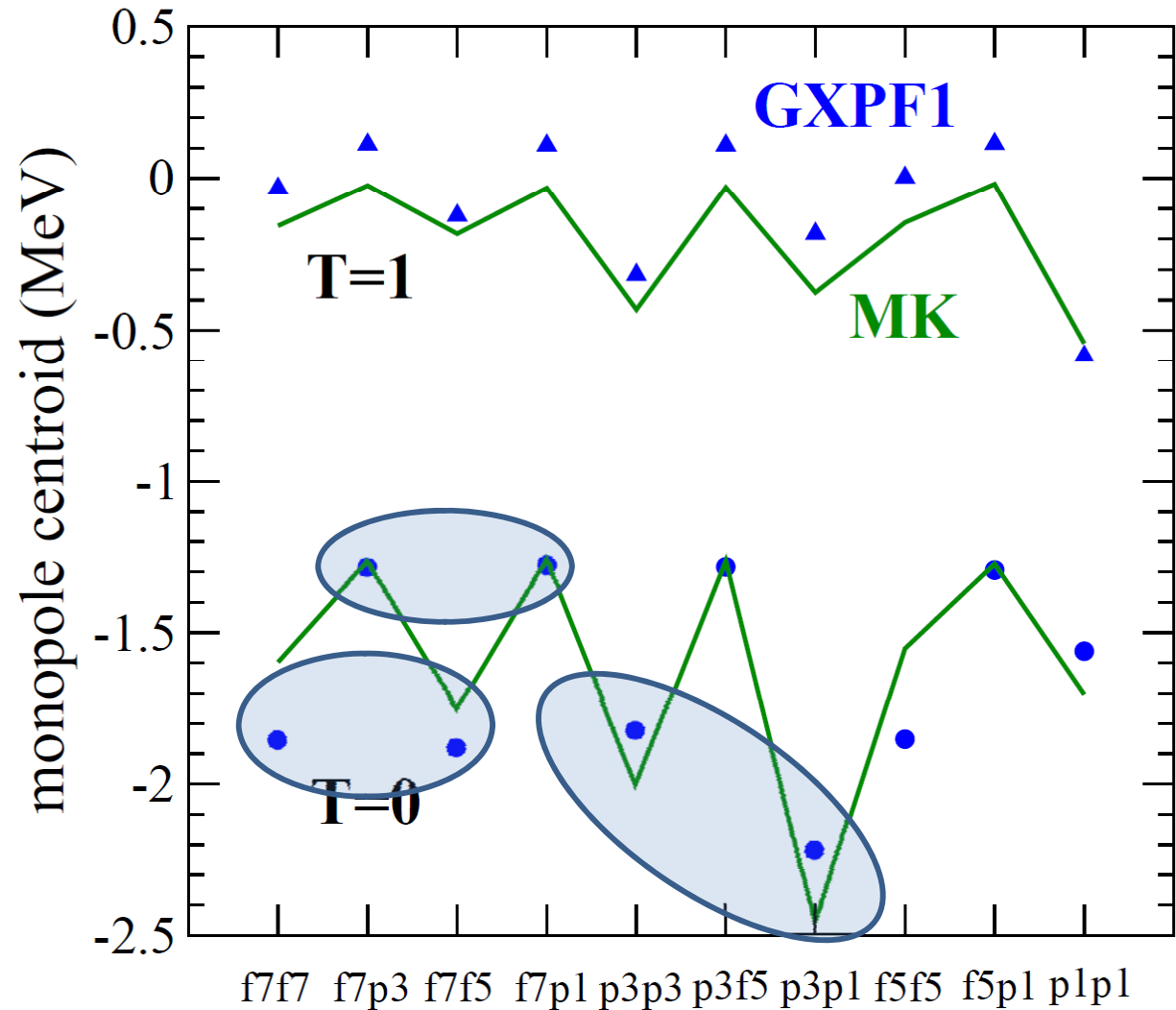


Suggesting a way how to make a new interaction which does not need an empirical monopole correction after experiment

Comparison with the MK central

- MK
 - Yukawa (pion range)
 - Four parameters
- Agreement
 - Basic pattern
- Disagreement
 - Overall T=1
 - f-f vs. f-p
 - f-f vs. p-p

How to fix those?



Procedure to obtain a better central

1. f-f vs. f-p

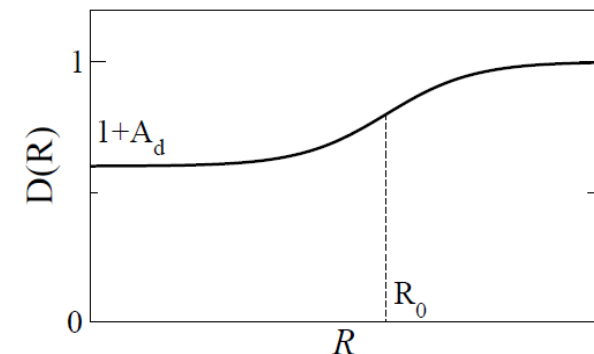
- Sensitive to the range of two-body interaction
- Medium range contributes much to the binding, so that there is less reason to adopt OPEP as the effective interaction.
- At the present study, the Gaussian with 1.1 fm range is adopted.
- The Gaussian does not improve p-p.

2. f-f vs. p-p

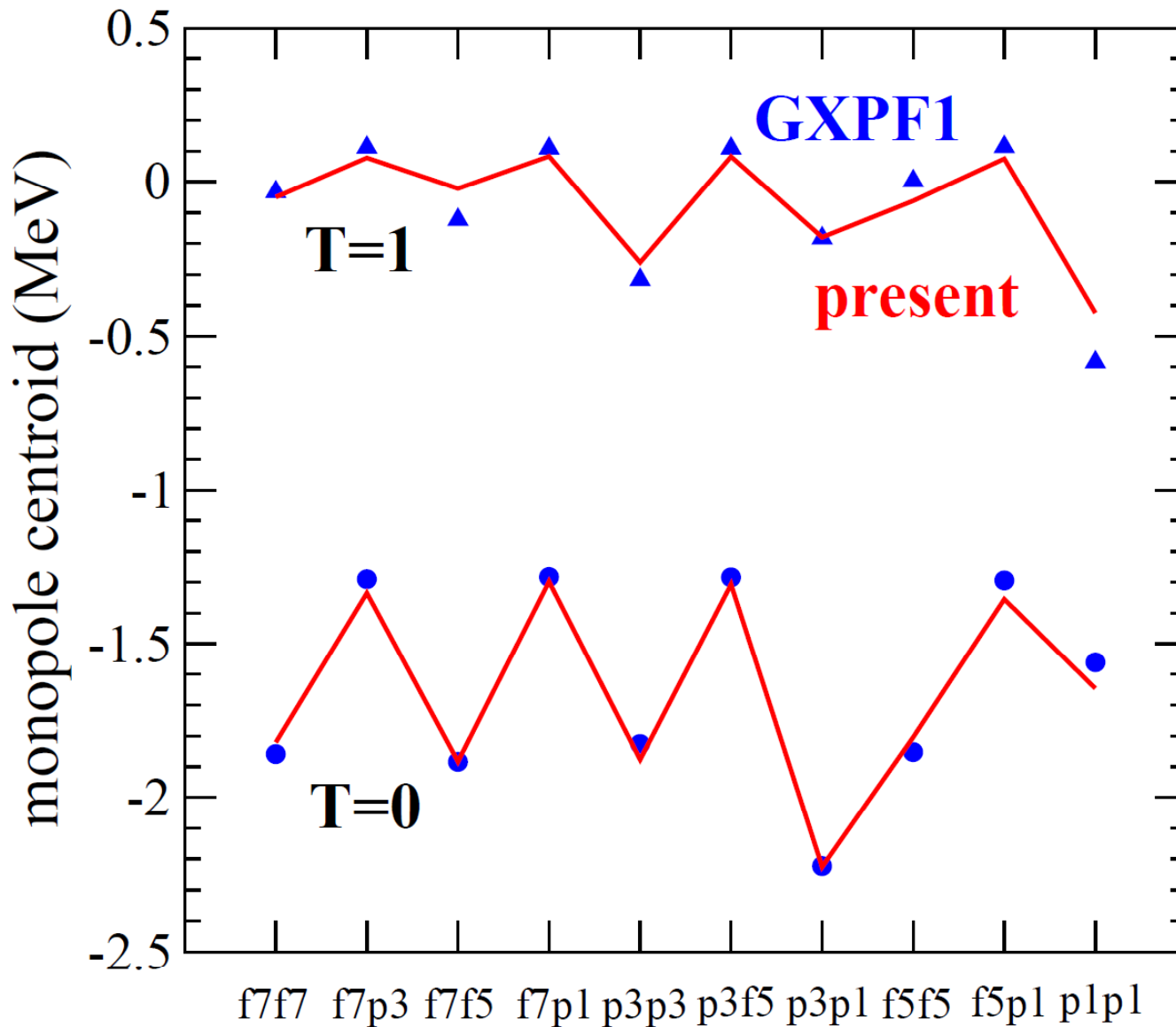
- Generally, potential type interactions make the interaction between larger node (in this case, $n=2$) too attractive.
- This can be remedied to introduce a density dependent interaction (or $R=(r_1+r_2)/2$ dependence) as demonstrated by Brown et al. in the sd shell.

$$D(R) = 1 + A_d F(R)^{B_d}$$

$$F(R) = \{1 + \exp[(R - R_0) / a]\}^{-1}$$



How close to GXPF1 (central)?



Only 6 parameters!
(i.e., strength of each (S,T) channel, range of Gaussian, and density dependence parameter)

New sd-pf shell interaction

- Intra-shell interaction
 - Standard interactions are used: USD for sd and GXPF1A for pf
 - Monopole correction like SDPF-M adopted in sd to have a good tensor force
 - Small modification in pairing and quadrupole pairing in $f_{7/2}$ to reproduce better 2+ energy for lower pf shell nuclei
 - No any other changes (applicable model space: 0hw and 1hw space)
- Cross-shell interaction
 - Not determined very well
 - Construct a new one with proper tensor force
- Single-particle energies
 - sd shell: the same as USD's
 - pf shell gaps: to be the same as those of GXPF1A in ^{48}Ca
 - Overall position of the pf shell: to reproduce unnatural parity states in K isotope chain

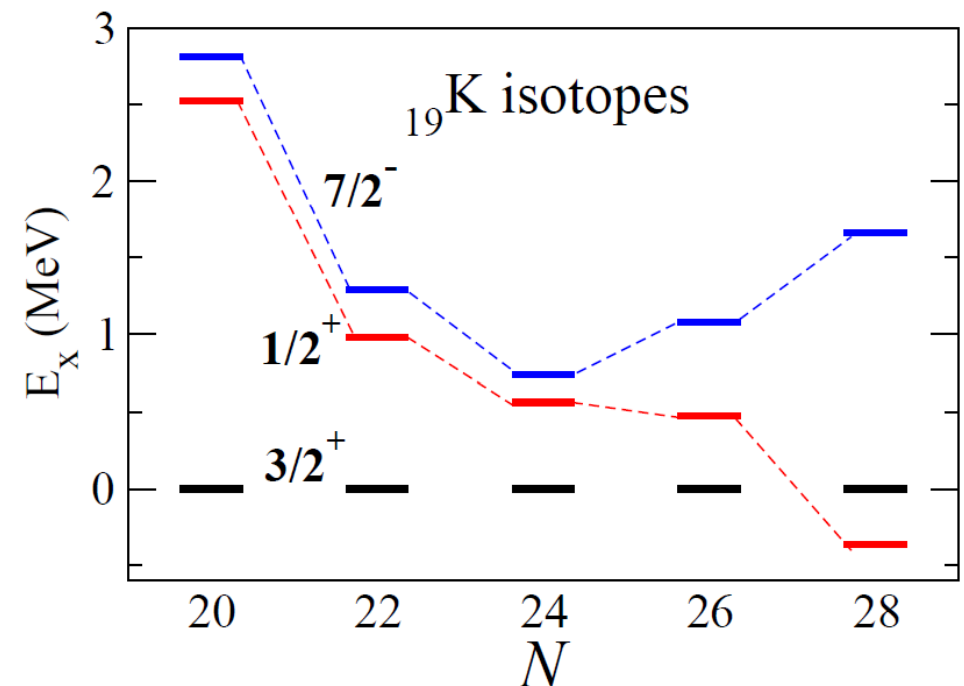
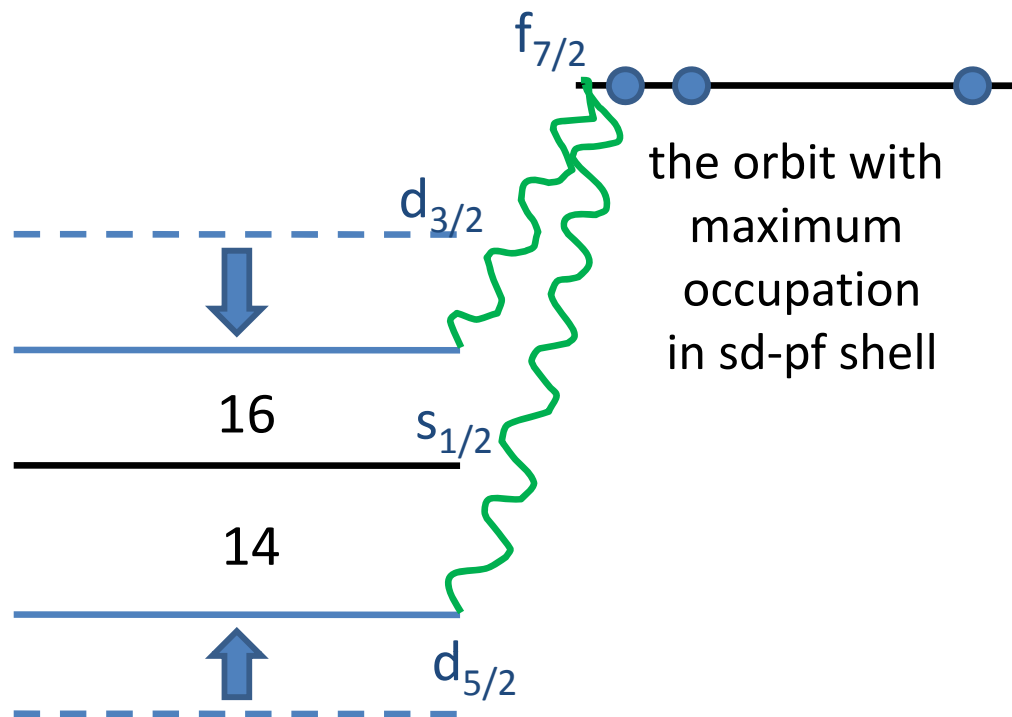
Summary of the cross-shell interaction

- Tensor part
 - The above-mentioned analysis suggests the tensor force close to $\pi+\rho$.
 - Fixed to be $\pi+\rho$.
- (two-body) LS part
 - Less effect except between s and p orbits.
 - Fixed to be that of M3Y.
- Central part
 - To be as close to GXPF1 as possible and as simple as possible
 - Potential type interaction
 - Gaussian potential (range 1.1 fm) is better than OPEP especially for the strength of f-p relative to f-f.
 - Density dependence (here, R dependence) improves the strength of p-p relative to f-f as adopted in the FPD6 interaction.

What nuclei are to be studied to see the tensor effect?

- Monopole interaction

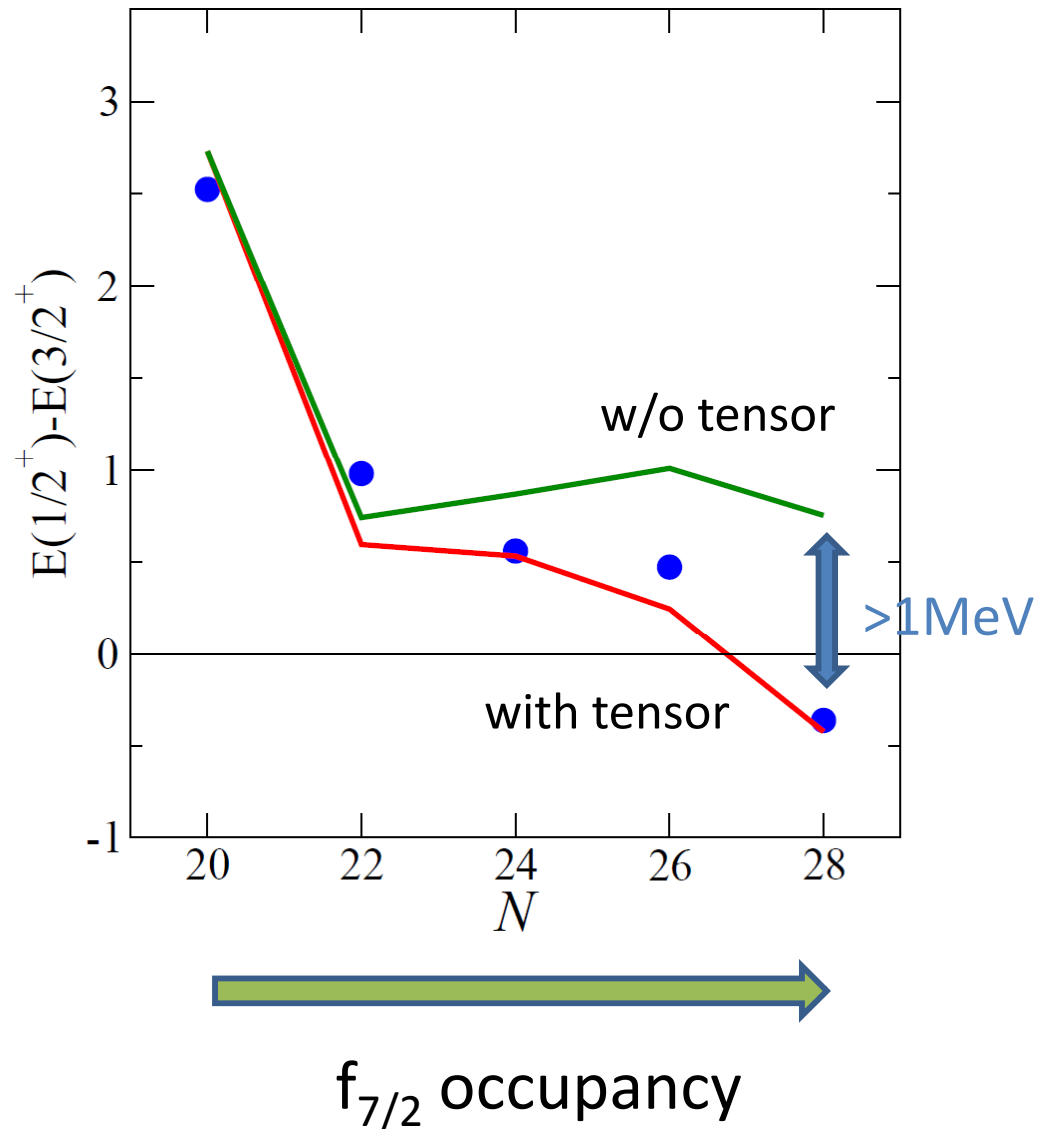
- Piled-up effect with the nucleon number
- No occupation vs. maximum occupation in $f_{7/2}$ is most clear.
- Proton particle or hole state such as K isotope chain



K isotopes: proton hole state

- Large discrepancy at N=28 without the tensor force
 - For N=22, 24 and 26, $1/2^+$ has large $\pi(d_{3/2})^{-1}\nu(2^+)$ component being not sensitive to $s_{1/2}$.
- Rough estimate of the shift from N=20 to 28
 - Exp.: $2.52 - (-0.36) = 2.88$ MeV
 - T=0 monopole (for A=42):
central: $0.375 * 8 * 0.5 = 1.50$ MeV
tensor: $0.313 * 8 * 0.5 = 1.25$ MeV
 - About half of the shift is accounted for by the tensor force.

Evolution of $1/2^+ - 3/2^+$ spacing



Comparison with mean-field description

SLy4

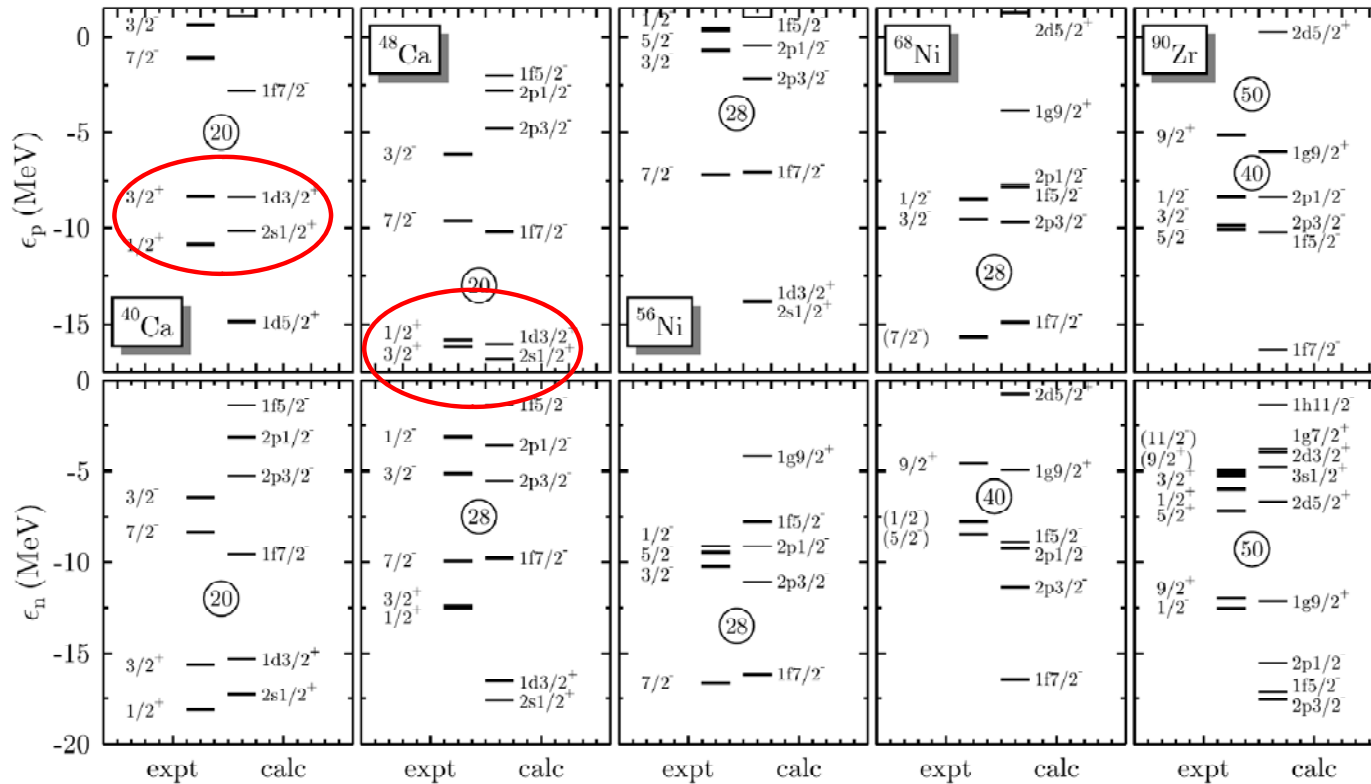


TABLE IV: Neutron particle states in ^{49}Ca .

Shell	$\epsilon_j - \epsilon_{3/2}$	Experimental
$p_{3/2}$	0.0	g.s.
$p_{1/2}$	1.9	2.0
$f_{5/2}$	4.2	~ 4.0

TABLE V: Proton hole states in ^{47}K .

Shell	$\epsilon_j - \epsilon_{1/2}$	Experimental
$s_{1/2}$	1.0	g.s.
$d_{3/2}$	0.0	0.4
$d_{5/2}$	6.0	~ 5.0

Taken from a lecture by G. Bertsch (@CNS, Japan, 2005)

Where is the $d_{5/2}$ hole state?

- Natural consequence from tensor force
 - Not only $d_{3/2}$ is pulled down but also $d_{5/2}$ is shifted up at the same time.
 - Explaining experimental reduction (~ 2 MeV) of the gap.

$d_{5/2}$ - $d_{3/2}$ gap compared with experiment

	^{39}K	^{47}K
Exp.	6.74	4.84
with tensor	7.42	5.25
w/o tensor	7.42	7.93

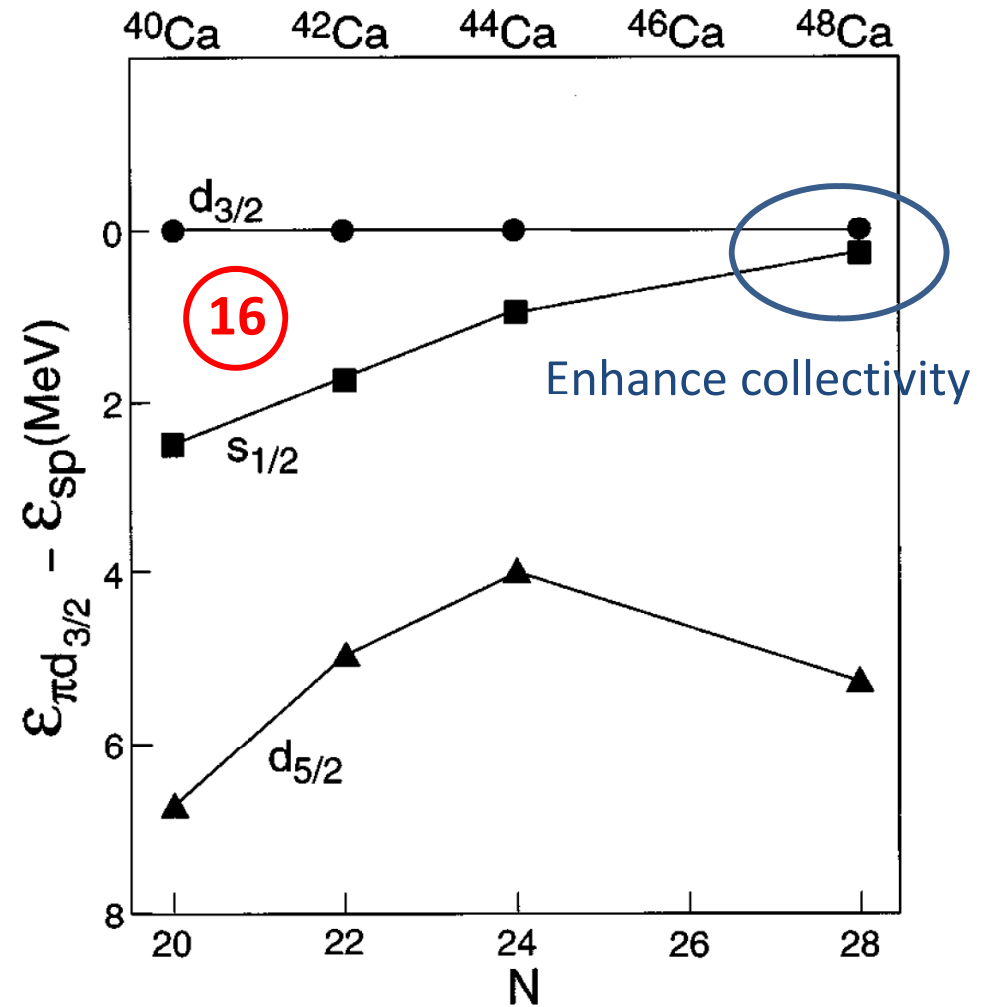
Exp.) ^{39}K : P. Doll et al., Nucl. Phys. A 263, 210 (1976)

^{47}K : G.J. Kramer et al., Nucl. Phys. A 679, 267 (2001).

Calc.) Deduced from effective single-particle energy on top of ^{40}Ca and ^{48}Ca

Influence on collectivity

- It was argued that the collectivity in neutron-rich sulfur ($Z=16$) isotopes would be caused predominantly by the degeneracy of proton orbits $d_{3/2}$ and $s_{1/2}$.
- We can confirm the picture comparing two shell model calculations: with and w/o tensor force.



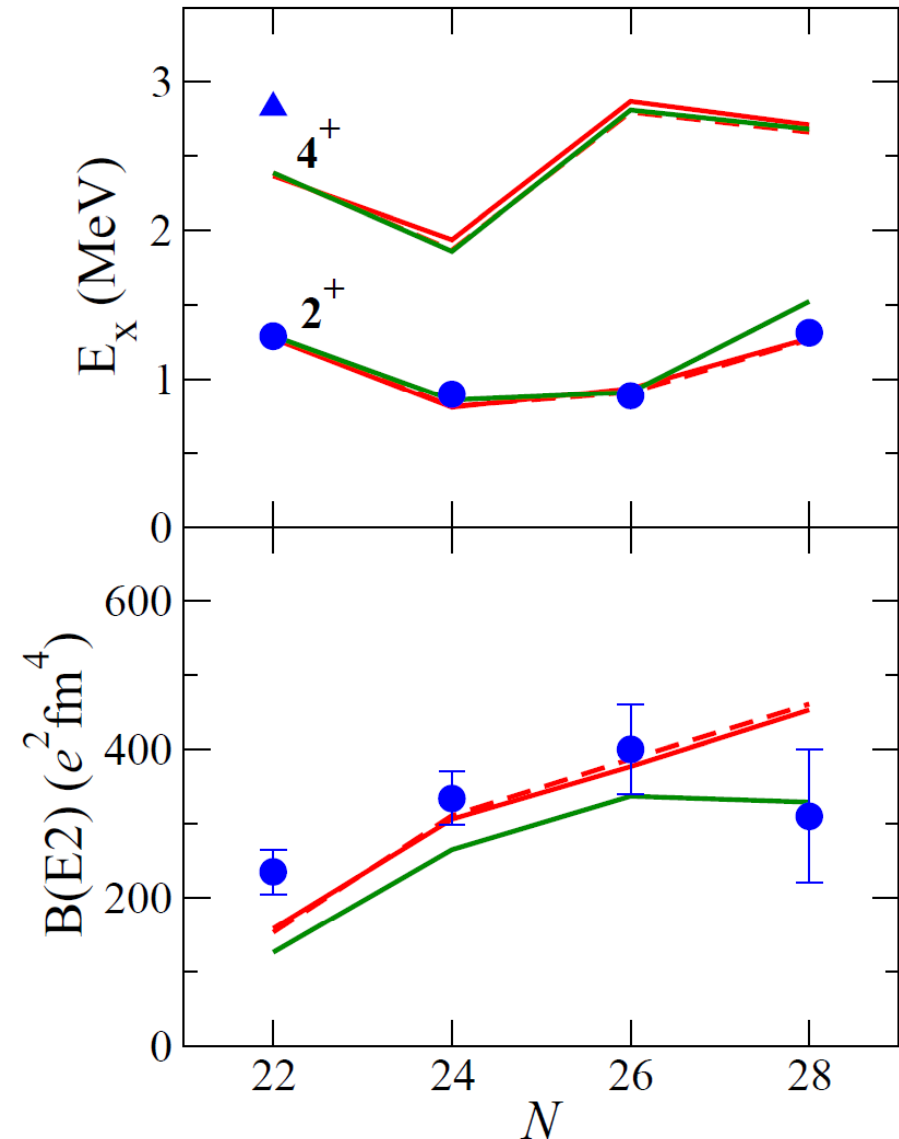
Sulfur (Z=16) isotopes

- Sensitivity to tensor force

- Not so clear except some difference at N=28
- Dominance in neutron 2^+ state for lower N
- Difference at N=28 is not very large, probably because of the near degeneracy without the tensor force.
- Tensor force predominantly works as the monopole shift.

effective charge

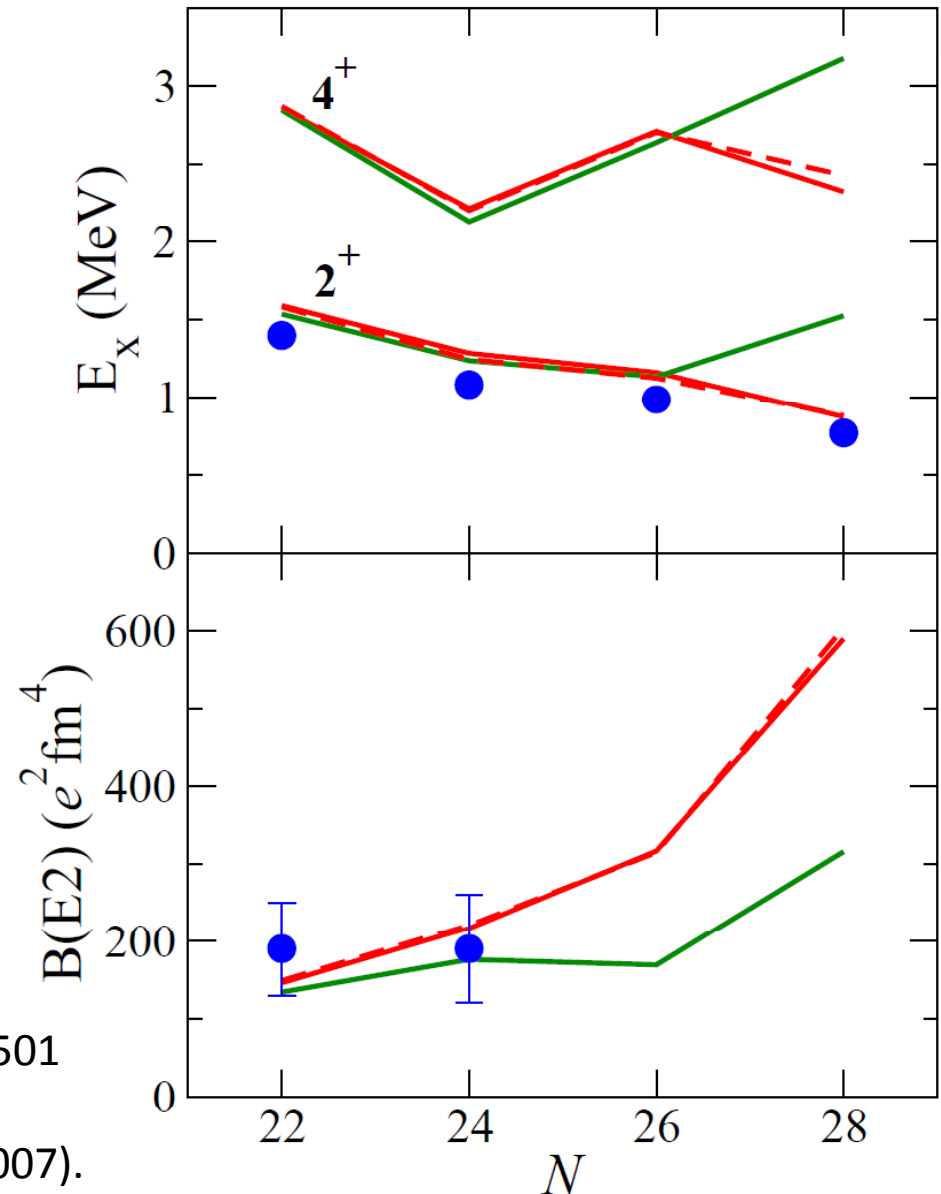
$(e_p, e_n) = (1.25e, 0.45e)$ to reproduce $B(E2)$ of lower sulfur isotopes.



Note: dashed line: tensor force as monopole int.

Silicon (Z=14) isotopes

- ^{42}Si (N=28) is very sensitive to the tensor force.
- 2^+ level of ^{40}Si (N=26) is not sensitive to the N=28 magicity, but its B(E2) might be a good measure.
- The disappearance of the magicity strongly affects the spectra of neighboring nuclei such as ^{41}Si and ^{43}P .

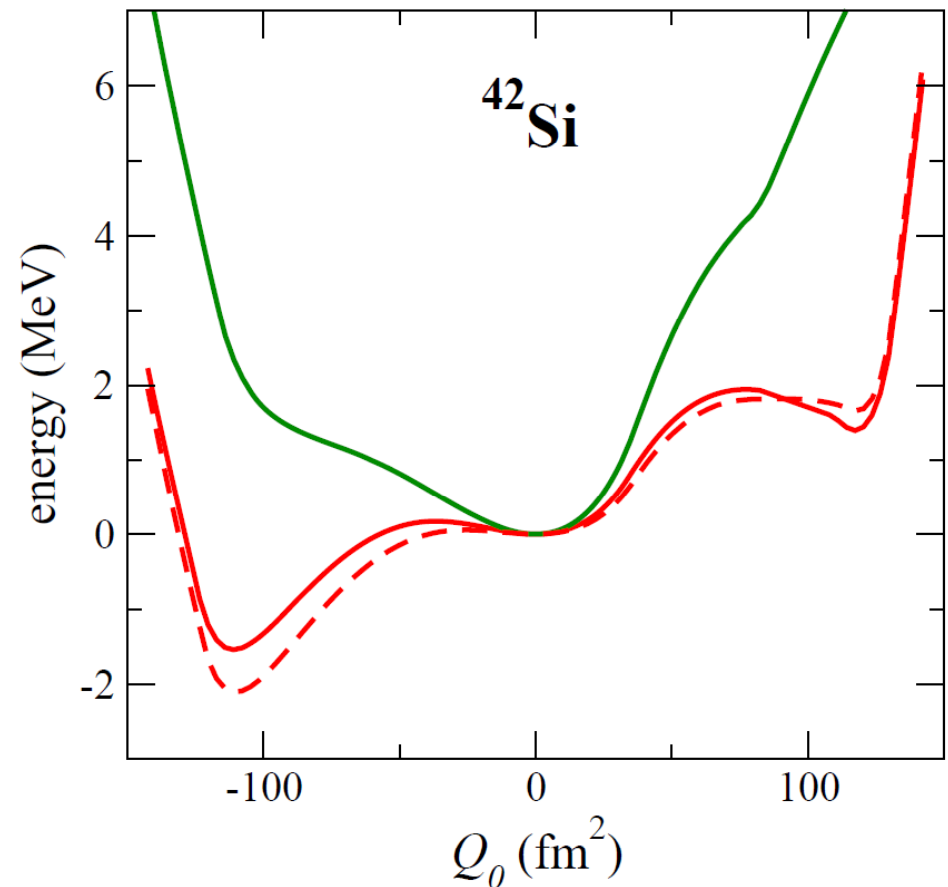


Exp.) ^{40}Si : C.M. Campbell et al., Phys. Rev. Lett. 97, 112501 (2006).

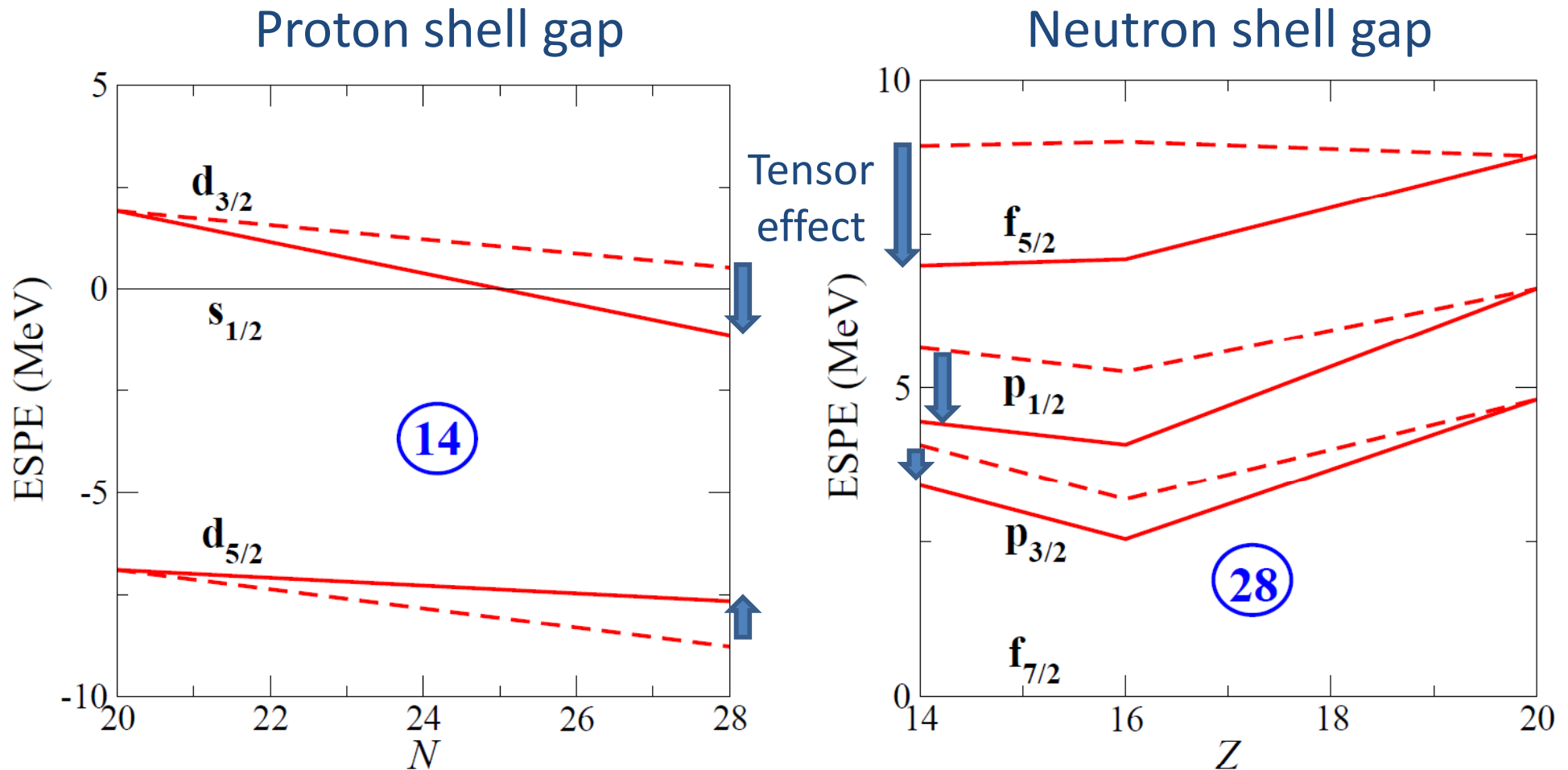
^{42}Si : B. Bastin et al. Phys. Rev. Lett. 99, 022503 (2007).

Deformation of ^{42}Si

- PES analysis
 - About 3 MeV (relative) energy gain for the **oblate deformed** state due to the tensor force
 - **No magic!**
- Role of tensor force
 - Working mainly as the monopole shift (from comparison with dashed line, where the tensor force is included as the monopole interaction only)



Effective shell gap



- There still remain certain shell gaps both for proton and neutron at ^{42}Si , although the tensor force reduces them.

Spectroscopic factor

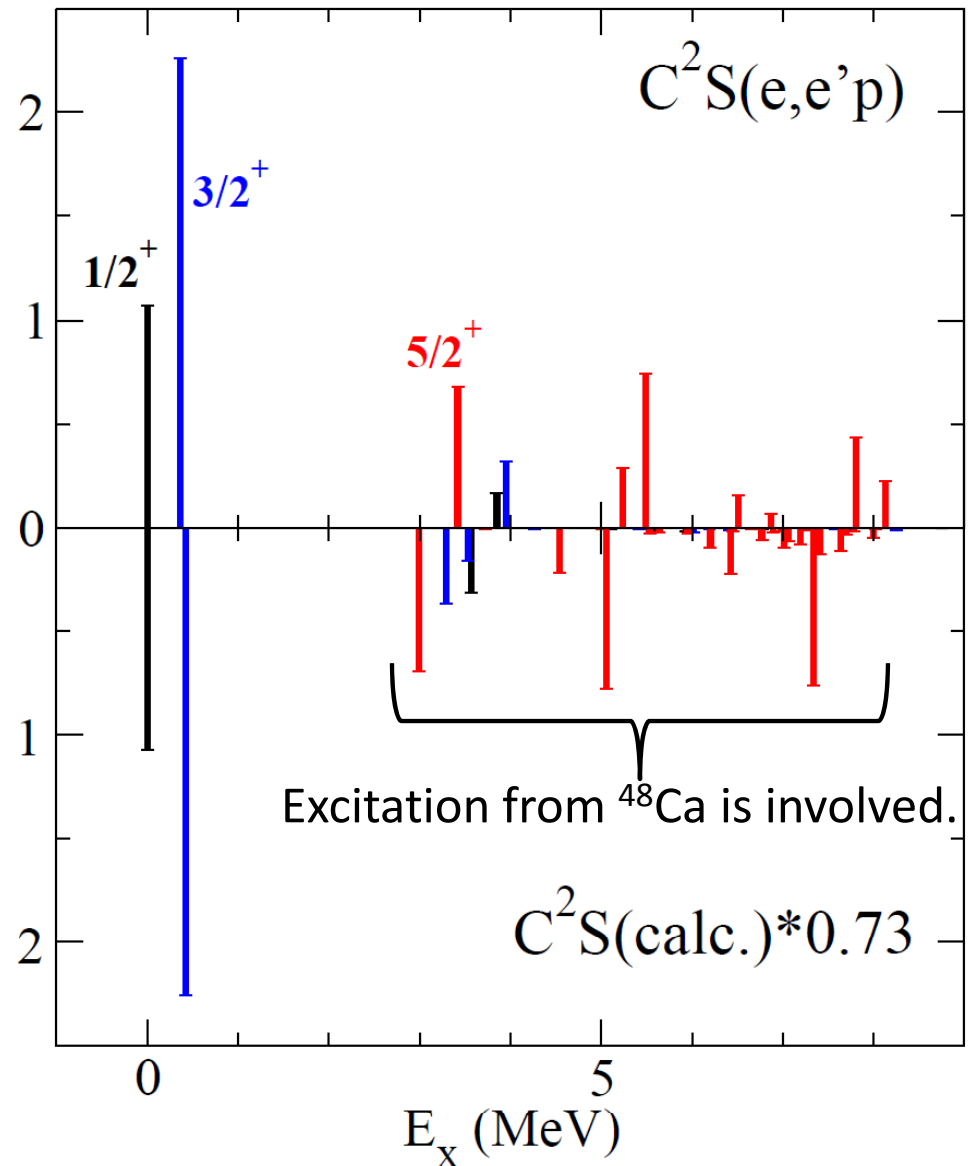
- Good information to obtain a single-particle structure
- For exotic nuclei, MSU is actively working to extract the spectroscopic factor (or cross section) from the knockout reaction.
- Two-proton knockout can be also regarded as the direct reaction, which enables to relate to the spectroscopic factor.

NuShell at MSU (Brown and Rae)

- Based on NuShell - a new set of core programs by Bill Rae written in 2007
- Techniques the same as Oxbash – JT-scheme basis expanded in terms of the M-scheme basis
- Fortran 95, allocated variables and OpenMP multiprocessing
- Wavefunctions, one- and two-nucleon spectroscopic factors, one- and two-body transition densities, cluster overlaps, LS basis for atomic cluster physics
- Input and output in user-friendly Oxbash style
- MPI version is being developed by Hang Liu for the MSU HPCC
- Open-source code with executables ready to run under Windows and Unix, 32 and 64 bit

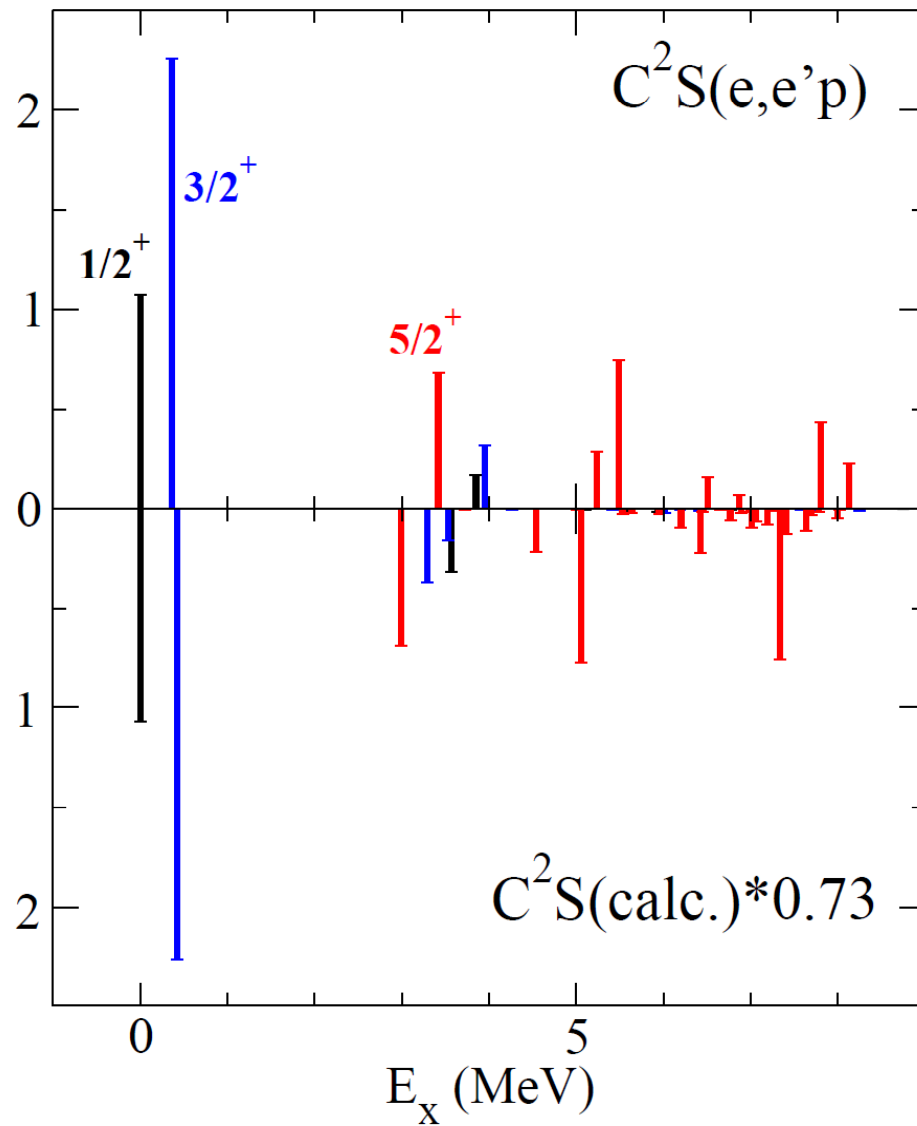
One-hole state of ^{48}Ca

- Proton hole state of ^{48}Ca explored by the spectroscopic factor
explored by the spectroscopic factor
 - Distribution: good agreement, especially for the existence of three large peaks of $5/2^+$
 - Need for overall quenching by about 0.73 compared with electron scattering data
 - Systematic lower positioning by about 500 keV

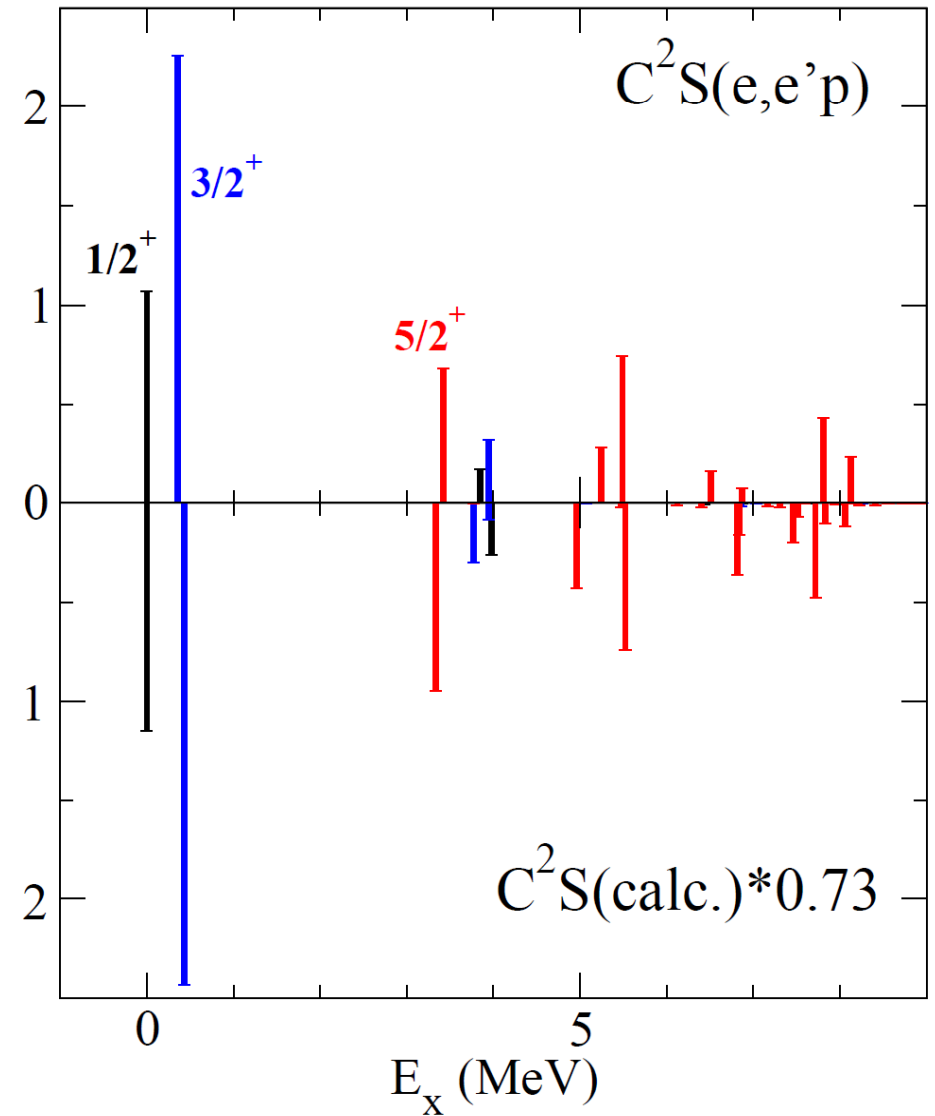


Effect of the excitation from N=28

Original interaction



A modified N=28 gap



^{47}Ar : reduction of LS splitting?

- Measurement of $E_x(1/2^-)$ of ^{47}Ar and its spectroscopic factor (L. Gaudefroy et al., Phys. Rev. Lett. 97, 092501 (2006); see right).
 - N=29; ground state $3/2^-$
 - $P_{1/2}$ dominance from the large spectroscopic factor
 - Reduction of $p_{3/2}$ - $p_{1/2}$ shell gap from ^{49}Ca to ^{47}Ar is deduced to be 890(75) keV.
 - Claiming that such a large reduction of the LS splitting is hard to understand

What causes the reduction?
 LS splitting by the tensor force or something?

TABLE I. Experimental energies in keV (E^*), angular momenta (ℓ), vacancies $(2J+1)C^2S$ of the levels identified in ^{47}Ar are compared to shell model calculations.

Experiment			Shell model		
E^*	ℓ	$(2J+1)C^2S$	E^*	J^π	$(2J+1)C^2S$
0	1	2.44(20)	0	$3/2^-$	2.56
1130(75)	1	1.62(12)	1251	$1/2^-$	1.62
1740(95)	3	1.36(16)	1365	$7/2^-$	0.8
2655(80)	3,(4)	1.32(18)	2684	$5/2^-$	0.78
3335(80)	3,(4)	2.58(18)	3266	$5/2^-$	2.76
3985(85)	4,(3)	3.40(40)
4790(95)
5500(85)	4	2.10(10)
6200(100)

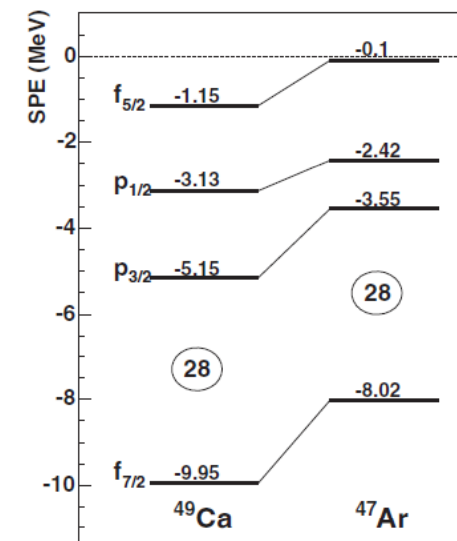


FIG. 3. Neutron single-particle energies (SPE) of the fp orbitals for the $^{47}\text{Ar}_{29}$ and $^{49}\text{Ca}_{29}$ nuclei (see text for details).

From shell model calculation

- Reduction of $3/2^-$ - $1/2^-$ splitting:
1.057 MeV (calc.) vs. **0.893(75)** MeV (exp.)
- Good agreement with the experimental spectroscopic factor

^{47}Ar	$(2J+1)C^2S$	
	Exp.	Calc.
$3/2^-$	2.44(20)	2.47
$1/2^-$	1.62(12)	1.73

^{49}Ca	$(2J+1)C^2S$	
	Exp.	Calc.
$3/2^-$	3.36(48)	3.79
$1/2^-$	1.82(30)	1.93



- Good wave function
- Possible to relate to the single-particle energy

Reduction of the gap by monopole int.

- Simple estimate by the filling configuration of $(d_{3/2})^{-2}$

$$\begin{aligned}
 & \frac{1}{2} \times 0.96 \times 2 \times \left\{ \underbrace{(-0.077 - 0.001 + 0.205)}_{T=0} + \underbrace{(-0.010 - 0.032 + 0.074)}_{T=1} \right\} \\
 & = 0.15 \text{ MeV}
 \end{aligned}$$

- Taking into account the fraction of protons

$$\begin{aligned}
 d_{3/2} & \quad \frac{1}{2} \times 0.96 \times 1.12 \times \left\{ (-0.077 - 0.001 + 0.205) + (-0.010 - 0.032 + 0.074) \right\} \\
 s_{1/2} & \quad + \frac{1}{2} \times 0.96 \times 0.73 \times \left\{ (0 - 0.030 + 0) + (0 + 0.208 + 0) \right\} \\
 d_{5/2} & \quad + \frac{1}{2} \times 0.96 \times 0.15 \times \left\{ (0.052 + 0.001 - 0.137) + (0.007 + 0.032 - 0.096) \right\} \\
 & = 0.14 \text{ MeV}
 \end{aligned}$$

Tensor (wrt $d_{3/2}$) and LS (wrt $s_{1/2}$) reduces the gap to a certain extent.
But it is not enough to explain the observed reduction.

The correlation plays an important role!

(see also A. Signoracci and B. A. Brown, Phys. Rev. Lett. 99, 099201 (2007).)

Summary

- To account for the disappearance of the N=20 magic number and appearance of a new N=16 magic and some other phenomena, the shell structure in exotic nuclei must be varying rather sharply, which we call **shell evolution**.
- The tensor force is the most promising source of the shell evolution.
- We constructed a new sd-pf cross-shell interaction including a proper tensor force and “good” central force to simulate GXPF1.
- Without any arbitrary monopole shift, the present interaction reproduces the proton hole states in K isotopes chain and 2^+ energy level of ^{42}Si , both of which seem to be a fingerprint of the shell evolution by tensor force.
- We started studying the spectroscopic factor in unstable nuclei to obtain more about the single-particle structure.

List of collaboration

- **Constructing effective interaction and shell evolution**
 - Takaharu Otsuka (University of Tokyo/RIKEN)
 - Takahiro Mizusaki (Senshu University)
 - Michio Honma (University of Aizu)
- **Application to spectroscopic factor**
 - Alex Brown (Michigan State University)
 - Experimentalists at MSU