A description of few-particle correlations and clustering

Y. Suzuki & W. Horiuchi (Niigata)

Outline

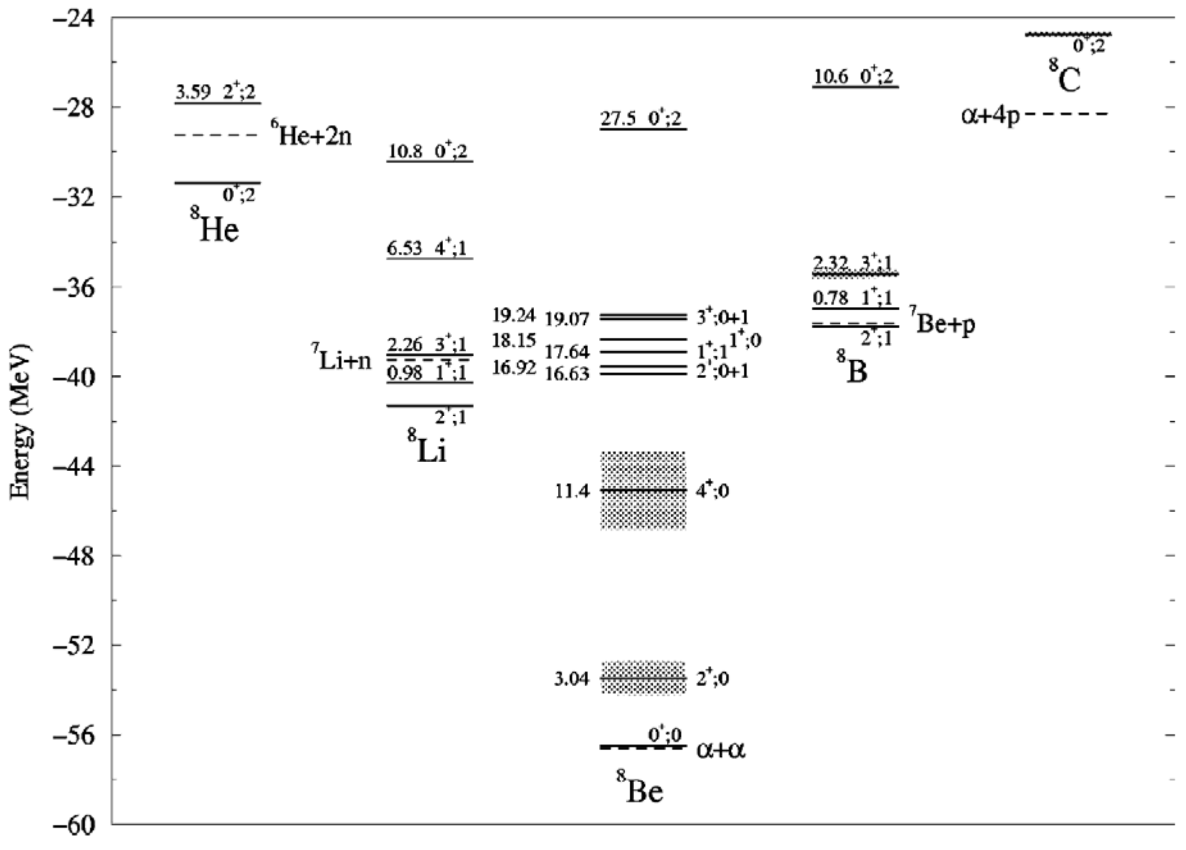
1. Motivation
   -- Mean-field motion vs clustering
2. Explicitly correlated Gaussian (ECG) basis
3. Test of the basis
   -- Spectrum of ^{4}He
4. Examples of two-nucleon correlation
   -- Momentum distribution of A=6 nuclei
5. Summary and Outlook

Alpha-clustering

– ^{8}Be, ^{12}C, ^{16}C, ^{20}Ne etc.

– Tight binding of alpha particle

– Tensor force, Distortion of alpha particle



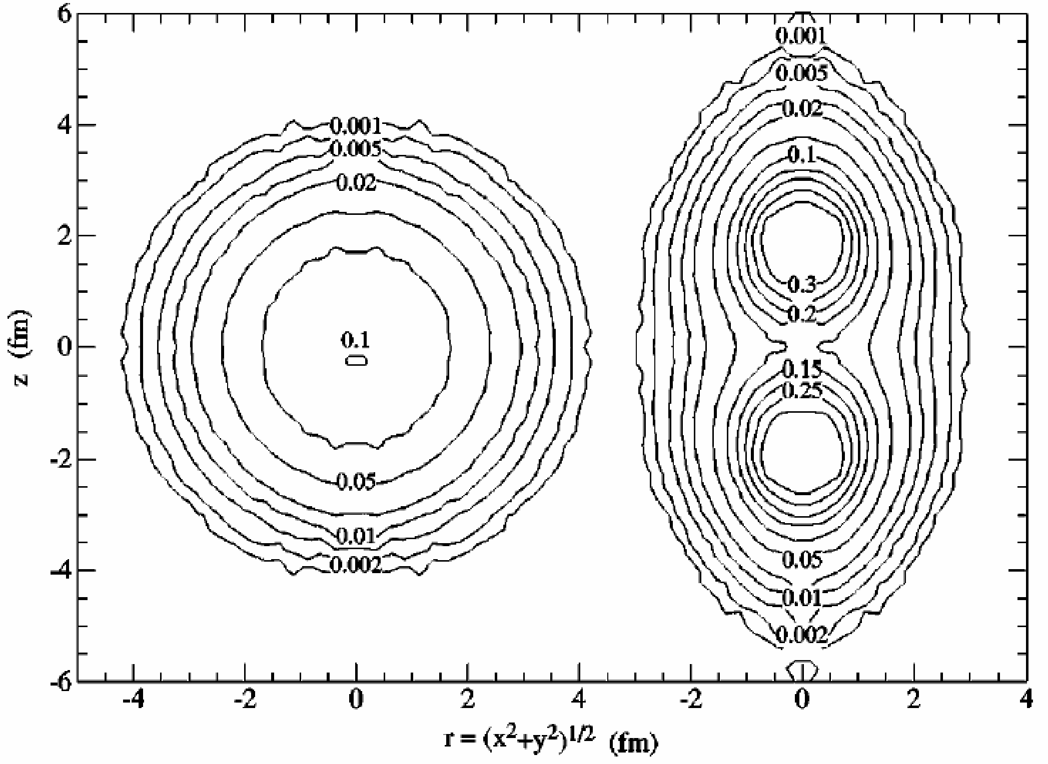
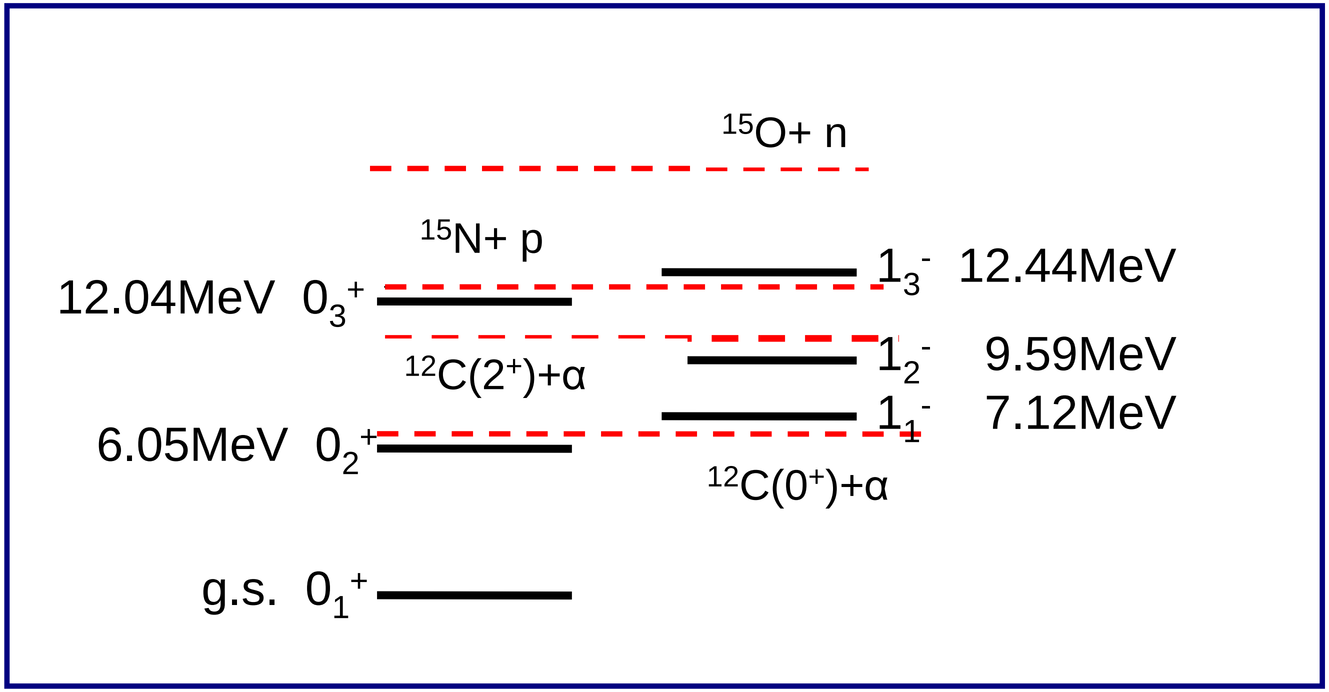


FIG. 15. Contours of constant density, plotted in cylindrical coordinates, for $^8Be(0^+)$. The left side is in the "laboratory" frame while the right side is in the intrinsic frame.

R.B. Wiringa et al., PRC62, 014001 (2000)

^{16}O spectrum

– Coexistence of shell and cluster states



Motivation

^{16}O: Testing ground to study dynamics of nucleon motion
from multi ph excitations (deformed) and clustering

- Still challenging despites theoretical progress
  - Green's function Monte Carlo (A ~ 12)
  - No core shell-model
  - Coupled cluster theory
- $^{12}C(\alpha, \gamma)^{16}O$ hard !
  - Measurement at energies of astrophysical interest
  - Reliable calculation with isospin mixture due to CSB forces and p-n mass difference

Ab-Initio Coupled-Cluster Study of ^{16}O

M. Włoch PRL **94,** 212501 (2005)

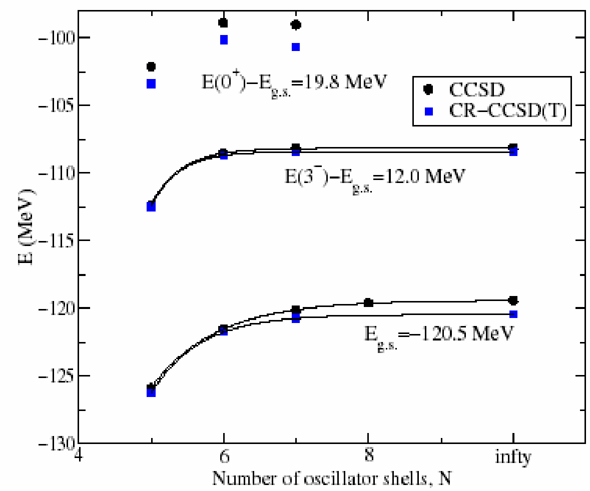
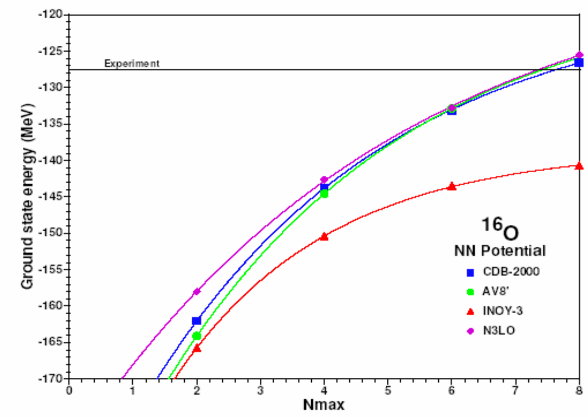


FIG. 1 (color online). The coupled-cluster energies of the ground-state (g.s.) and first-excited 3^- and 0^+ states as functions of the number of oscillator shells N obtained with the Idaho-A interaction.

Ab initio No-Core Shell Model

J.P. Vary[1,a], Eur. Phys. J. A 25, s01, 475–480 (2005)



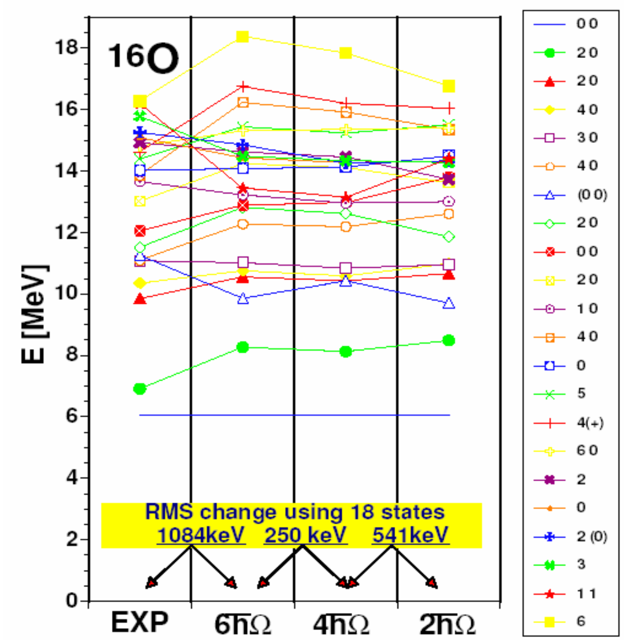


Fig. 4. Low-lying positive-parity ^{16}O states from the CD-Bonn interaction at the $a = 2$ cluster approximation in the NCSM with $\hbar\Omega = 15$ MeV. The spectra are aligned with the experimental first-excited 0^+ state.

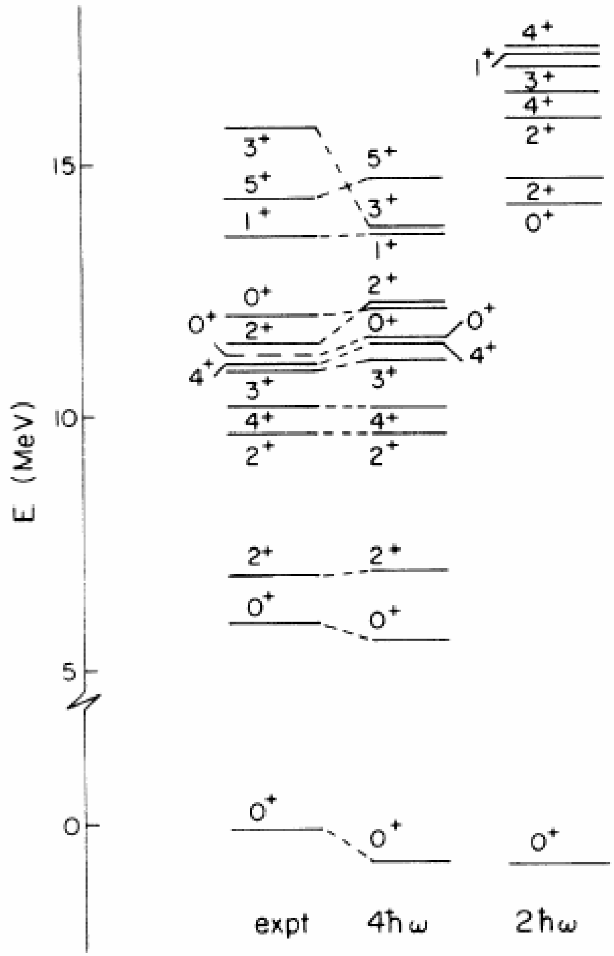
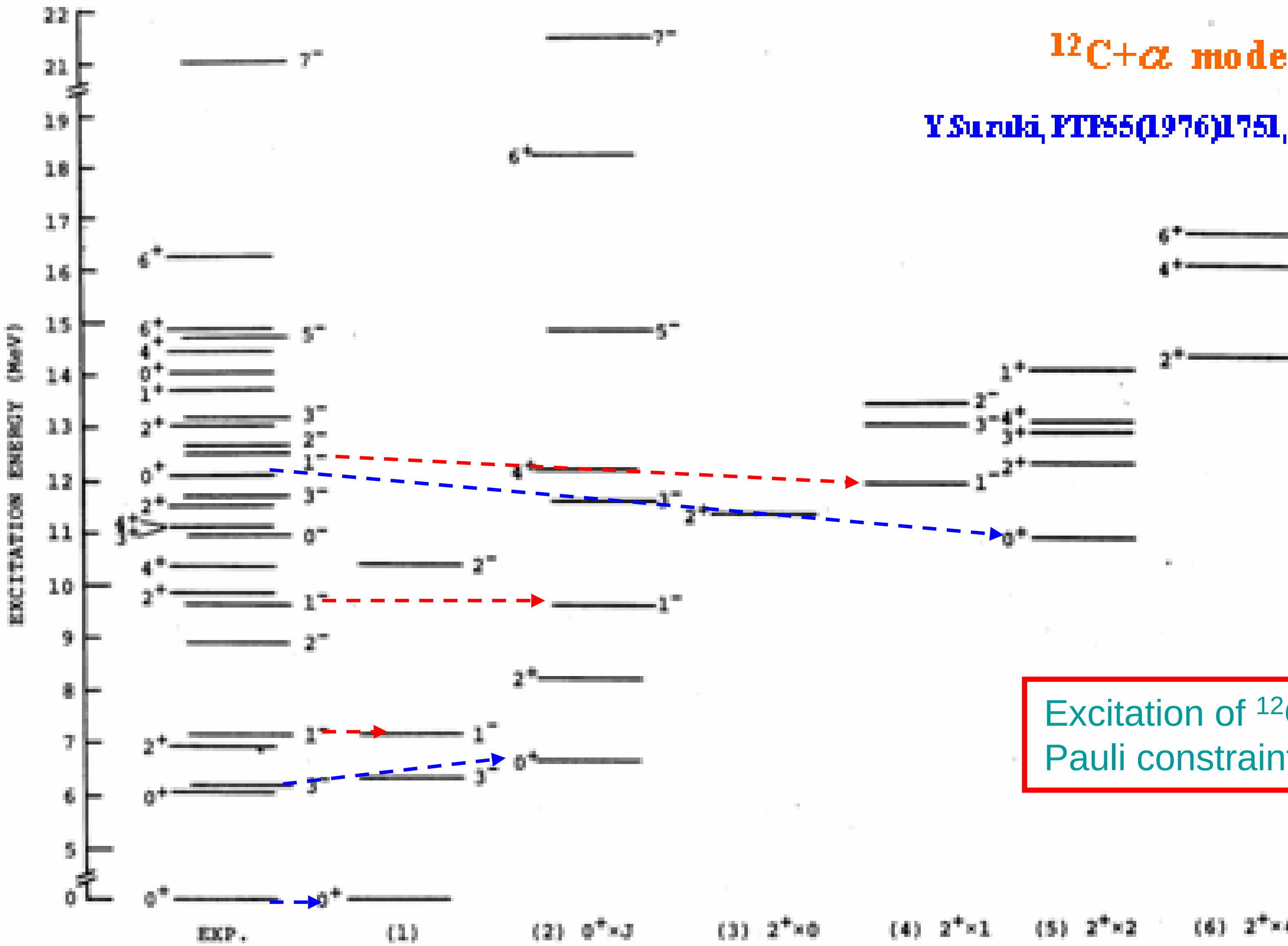


FIG. 1. A comparison of experiment and the $4\hbar\omega$ ^{16}O shell-model spectrum of $T=0$ states. The spectrum resulting from diagonalizing the same Hamiltonian in a $2\hbar\omega$ space is also shown.

$^{12}C+\alpha$ model

Y. Suzuki, PTP55(1976)1751, 56(1976)111



Energy levels of ^{16}O. (T=0)

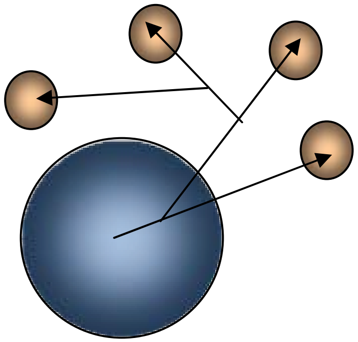
Excitation of ^{12}C
Pauli constraint

Can we found the $^{12}C+\alpha$ model on the dynamics of four nucleons (without assuming αcluster) interacting with a ^{12}C core ?

core + few-nucleons approach

Features of the model

- Few nucleons interacting via realistic potential
- Macroscopic core-nucleon interaction
- Few nucleons receive Pauli constraint from core, otherwise move unconditionally
- Core excitation



Basis function for orbital motion

Explicitly correlated Gaussian (ECG) with angular functions specified by global vectors (GV)

- L, parity=$(-1)^L$ $\quad \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)\mathcal{Y}_{LM}(\widetilde{u_1}\boldsymbol{x})$

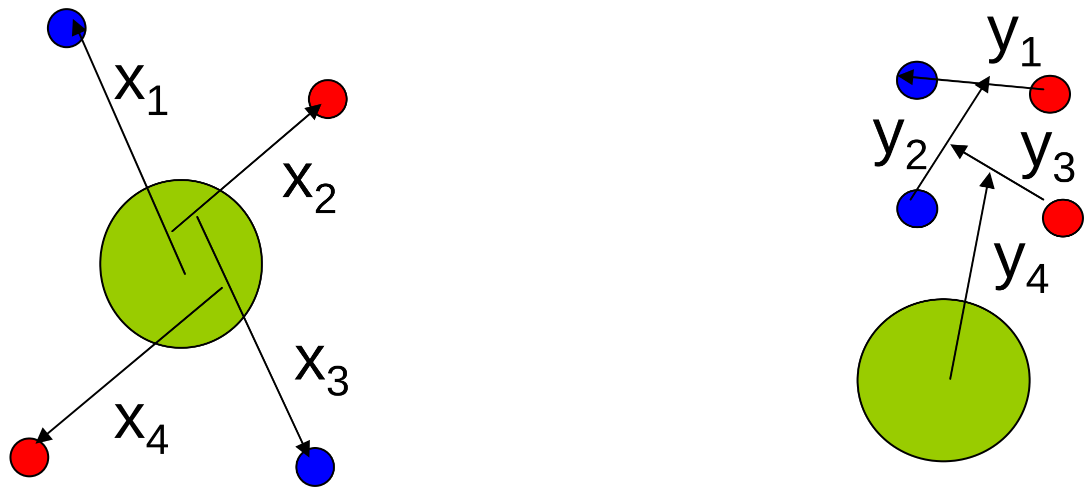
$$\tilde{\boldsymbol{x}}A\boldsymbol{x} = \sum_{i,j=1}^{N-1} A_{ij}\boldsymbol{x}_i \cdot \boldsymbol{x}_j \qquad A_{ij} \neq 0$$

$$\mathcal{Y}_{LM}(\widetilde{u_1}\boldsymbol{x}) = |\widetilde{u_1}\boldsymbol{x}|^L Y_{LM}(\widehat{\widetilde{u_1}\boldsymbol{x}}) \qquad \widetilde{u_1}\boldsymbol{x} = \sum_{i=1}^{N-1} u_{1_i}\boldsymbol{x}_i$$

- L, parity=$(-1)^{L+1}$ $\quad \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_L(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_1(\widetilde{u_2}\boldsymbol{x})]_{LM}$

Varying A and u enables to include all correlations.
NB: L=0, parity= -1

Unifying shell and cluster correlations



$$y = Tx \quad \Longrightarrow \quad \tilde{y}By = \tilde{x}\tilde{T}BTx \qquad \tilde{v}y = \widetilde{\tilde{T}v}x$$

Both types of correlations can be described in a single coordinate set.
Permutations of identical particles induce linear transformation of coordinates.
No need of coord. trans. Only suitable choice of A and u needed.

Advantage of the basis functions

Variational Solution

$$\Phi(A,u,\alpha) = \mathcal{A}\left\{\left[\psi_L^{(\text{orbital})}(A,u)\psi_S^{(\text{spin})}(S_{12},S_{123},\ldots)\right]_{JM} \times\psi_{TM_T}^{(\text{isospin})}(T_{12},T_{123},\ldots)\right\}$$

$$\alpha = (L,S,S_{12},\ldots,T_{12},\ldots)$$

$$\Psi_{JMTM_T} = \sum_{i=1}^{K} C_i\Phi(A_i,u_i,\alpha_i)$$

$$H_{ij} = \langle\Phi(A_i,u_i,\alpha_i)|H|\Phi(A_j,u_j,\alpha_j)\rangle$$

$$B_{ij} = \langle\Phi(A_i,u_i,\alpha_i)|\Phi(A_j,u_j,\alpha_j)\rangle$$

$$\sum_{j=1}^{K}(H_{ij}-EB_{ij})C_j = 0$$

Symmetry, Center of mass motion
Variation after projection

$$\Psi_{JM} = \sum_{LS} C_{LS}[\psi_L \psi_S]_{JM}$$

L, S coupling scheme
useful to know tensor correlation

Example: A=4, J=0
(L,S)=(0,0), (1,1), (2,2)

Relationship between Partial-Wave Expansion and GVR

Successive coupling $[\cdots[[y_{l_1}(\mathbf{x}_1)y_{l_2}(\mathbf{x}_2)]_{l_{12}}\,y_{l_3}(\mathbf{x}_3)]_{l_{123}}\cdots]_{LM}$

Small ℓ values are used. Calculation of matrix elements is involved.

Global vectors $y_{LM}(u_1\mathbf{x}_1+u_2\mathbf{x}_2+\cdots+u_{N-1}\mathbf{x}_{N-1})$

$$y_{LM}(a\mathbf{x}_1+b\mathbf{x}_2)=\sum_{l=0}^{L}\sqrt{\frac{4\pi(2L+1)!}{(2l+1)!(2L-2l+1)!}}a^l b^{L-l}[y_l(\mathbf{x}_1)y_{L-l}(\mathbf{x}_2)]_{LM}$$

Cross terms of Correlated Gaussians add additional ℓ values.

$$\exp(A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j)\to\sum_n(\boldsymbol{x}_i\cdot\boldsymbol{x}_j)^n\sim\sum_{\ell=n,n-2,\ldots}[y_\ell(\boldsymbol{x}_i)y_\ell(\boldsymbol{x}_j)]_{00}$$

Y.S, J.Usukura, K.Varga, JPB31(1998)

G3RS vs AV8'

Tamagaki, PTP39 (1968)
Pudliner et al., PRC56(1997)

$$\Psi_{JM} = \sum_{LS} C_{LS}[\psi_L \psi_S]_{JM}$$

$$E = \sum_{LS} \sum_{L'S'} C_{LS} C_{L'S'} \langle LS; JM | H | L'S'; JM \rangle$$

Deuteron

$d(1^+)$	(0,1)	(2,1)	$P(L,S)$
(0,1)	4.198	−12.73	0.952
(2,1)		6.257	0.048

-2.27

(0,1)	(2,1)	$P(L,S)$
7.355	−18.94	0.942
	9.338	0.058

-2.24 MeV

Triton

$t(\frac{1}{2}^+)$	(0,1/2)	(2,3/2)	(1,1/2)	(1,3/2)	$P(L,S)$
(0,1/2)	4.089	−22.50	−0.000	−0.001	0.930
(2,3/2)		10.83	−0.155	−0.070	0.070
(1,1/2)			0.069	0.012	0.000
(1,3/2)				0.029	0.000

-7.70

(Arai)

(0,1/2)	(2,3/2)	(1,1/2)	(1,3/2)	$P(L,S)$
9.717	−33.58	−0.018	−0.013	0.914
	16.33	−0.276	−0.137	0.086
		0.128	0.031	0.000
			0.058	0.000

-7.76

(Hiyama et al.)

AV8' gives larger ME than G3RS for T, V_t, V_b but smaller for Vc.
The net result is similar between the two.

Algorithm of the SVM

Possibility of the stochastic optimization
1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

1. **Generate** $(A_k^1, A_k^2, \cdots\cdots, A_k^m)$ **randomly**
2. **Get the eigenvalues** $(E_k^1, E_k^2, \cdots\cdots, E_k^m)$
3. **Select** A_k^n corresponding to the lowest E_k^n and **Include** it in a basis set
4. **k → k+1**

Y. S. and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).
K. Varga and Y. S., Phys. Rev. C52, 2885 (1995).

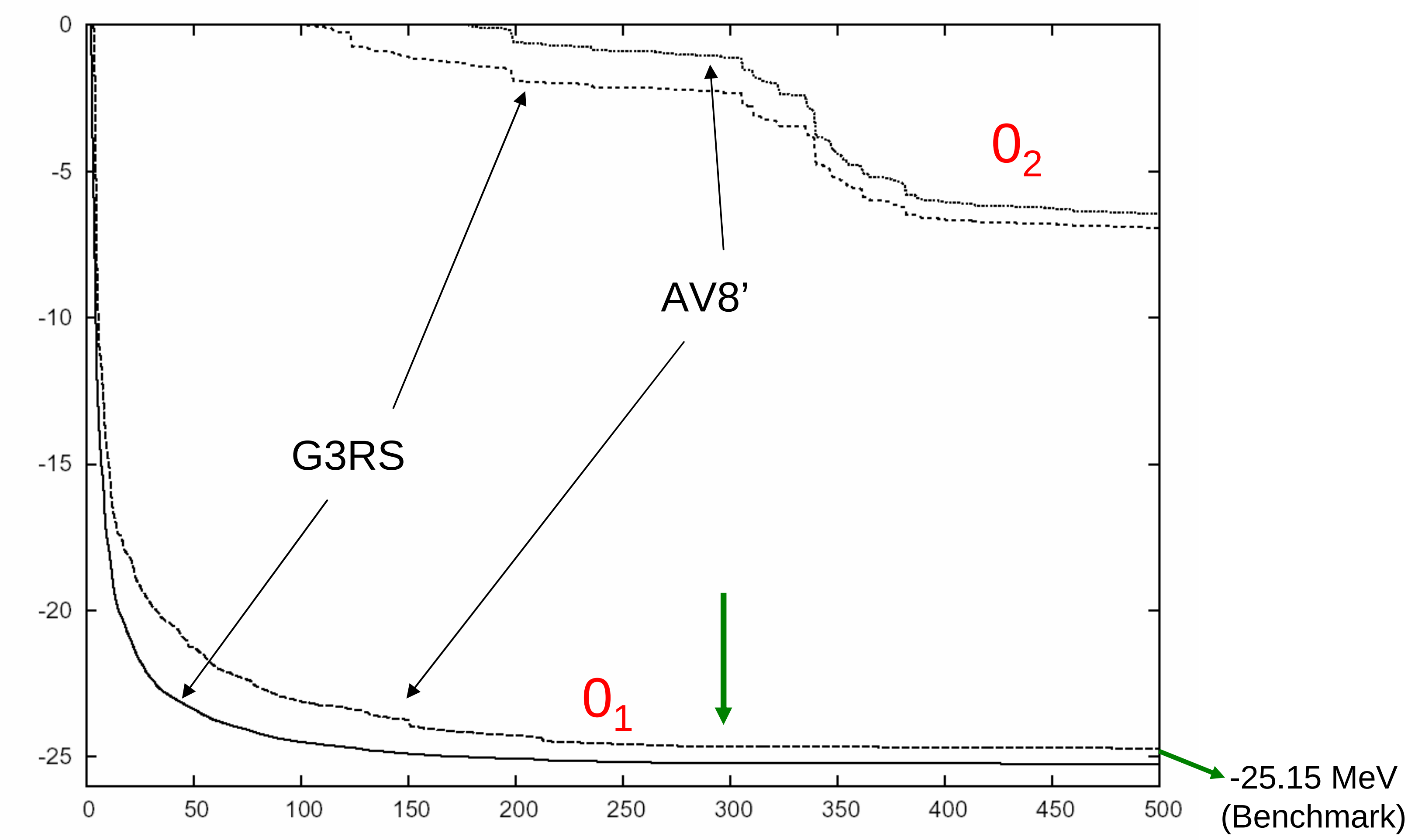
As preparation for core+4N calculations,

we focus on

1. Test of GV representation
2. Spectrum of 4N system (^{4}He)
   - -- 3N+N clustering
   - -- evidence for tensor correlation
3. core+2N problem (^{6}He, ^{6}Li)
   - -- characteristics of N-N correlation

Test of ECG: ^{4}He 0^+ states

(W. Coulomb)

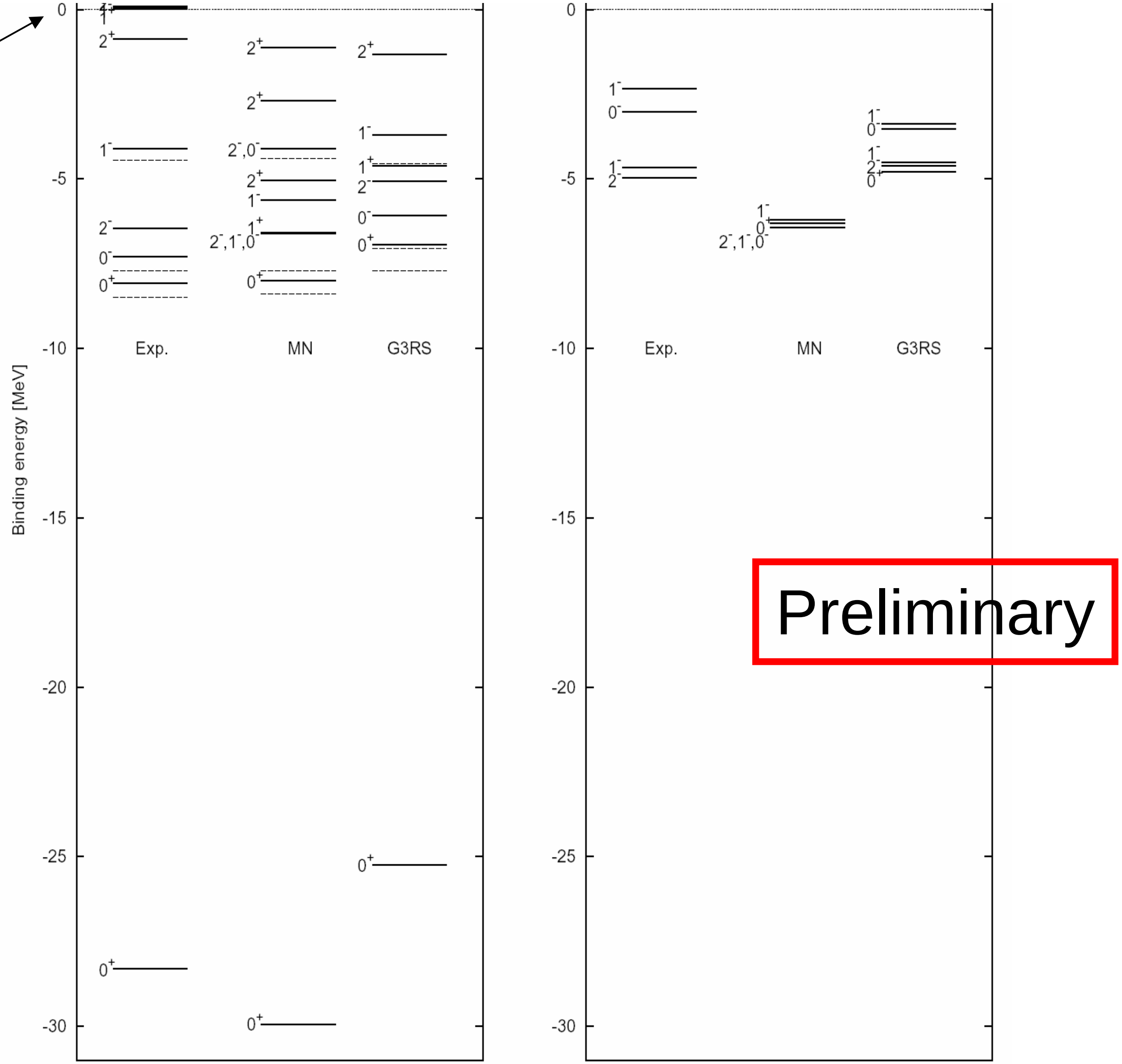


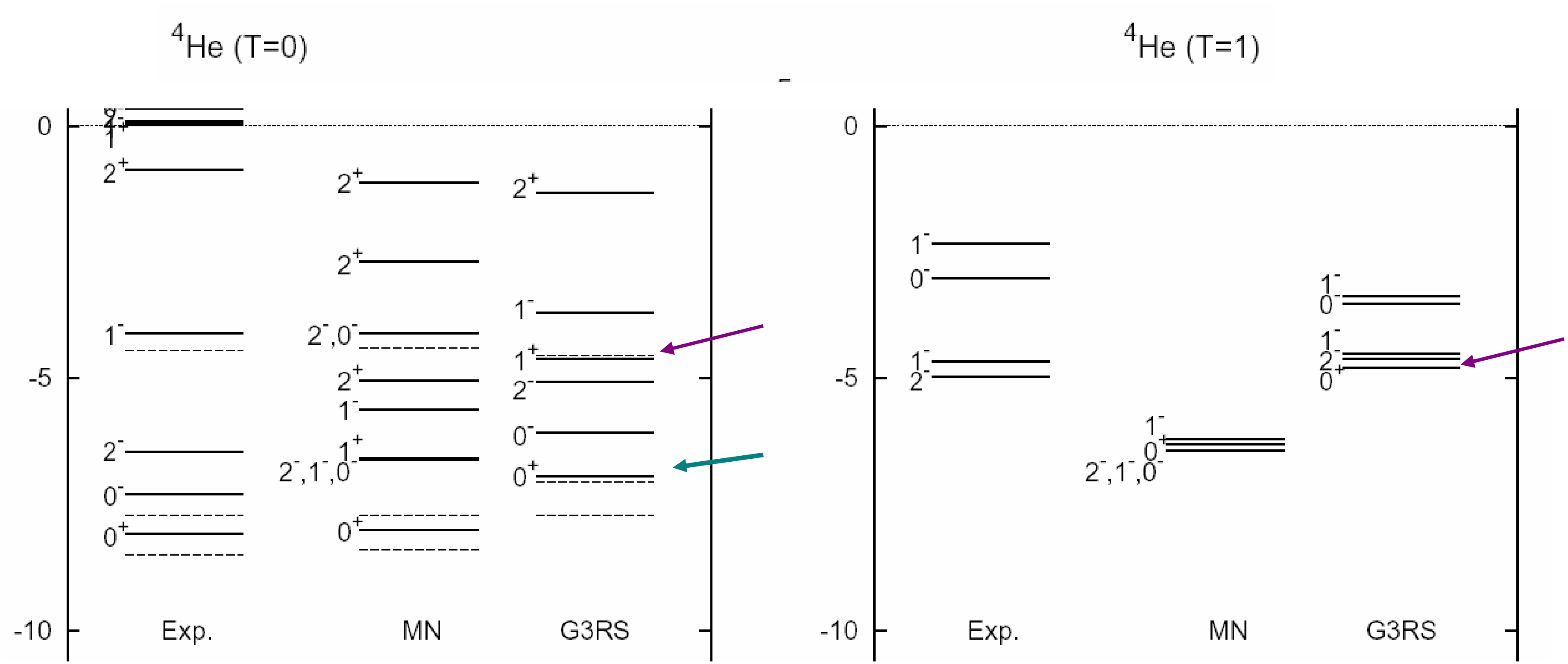
Basis dimension

-25.28 MeV
(Arai)

Spectrum of ^{4}He: Purely central vs realisitc forces

p+p+n+n





Realistic potential reproduces correctly splitting of levels, which are degenerate in pure central force model.
Theory predicts 0^+ and 1^+ states with 3N+N cluster structure.

(S, T)=(1/2, 1/2); L=0

Contribution of unnatural parity components

^{4}He: G3RS

Ground state

0_1^+	(0,0)	(2,2)	(1,1)	$P(L,S)$
(0,0)	0.919	−46.51	−0.006	0.885
(2,2)		21.17	−1.497	0.112
(1,1)			0.692	0.002

ΔE~ -0.8 MeV

Excited state

0_2^+	(0,0)	(2,2)	(1,1)	$P(L,S)$
(0,0)	4.821	−22.54	−0.002	0.931
(2,2)		10.94	−0.282	0.068
(1,1)			0.139	0.001

3N+N cluster state

Cf. triton

$t(\frac{1}{2}^+)$	(0,1/2)	(2,3/2)	(1,1/2)	(1,3/2)	$P(L,S)$
(0,1/2)	4.089	−22.50	−0.000	−0.001	0.930
(2,3/2)		10.83	−0.155	−0.070	0.070
(1,1/2)			0.069	0.012	0.000
(1,3/2)				0.029	0.000

Most distinct role of unnatural parity: 0^-

Experiment:
Ex=-7.29 MeV
(1.20 MeV above t+p)
Γ=0.84 MeV

Total

0^-_1	(1,1)	(2,2)	$P(L,S)$
(1,1)	1.137	−13.85	0.954
(2,2)		6.644	0.046

Calculation:
-6.07 MeV
(1.63 MeV above t+p)

Kinetic

$\langle T \rangle$	(1,1)	(2,2)
(1,1)	41.87 (43.89)	
(2,2)		7.385 (160.2)

Central

$\langle V_c \rangle$	(1,1)	(2,2)
(1,1)	−27.98 (−29.33)	
(2,2)		−1.129 (−24.49)

Tensor

$\langle V_t \rangle$	(1,1)	(2,2)
(1,1)	−13.55 (−14.21)	−13.86 (−66.09)
(2,2)		0.367 (7.955)

Spin-orbit

$\langle V_b \rangle$	(1,1)	(2,2)
(1,1)	0.327 (0.343)	0.010 (0.050)
(2,2)		0.000 (0.006)

Coulomb

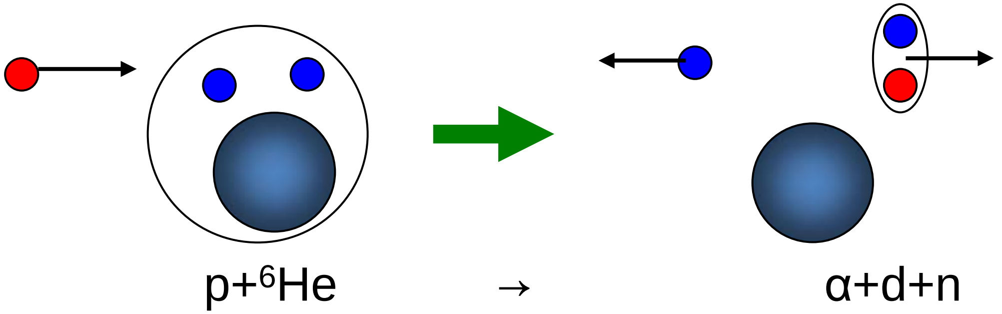
$\langle V_{\text{Coul.}} \rangle$	(1,1)	(2,2)
(1,1)	0.469 (0.492)	
(2,2)		0.021 (0.454)

Vital to reproduce 0- is coupling between natural and unnatural states, which arises from tensor force.

Momentum distribution measures correlations

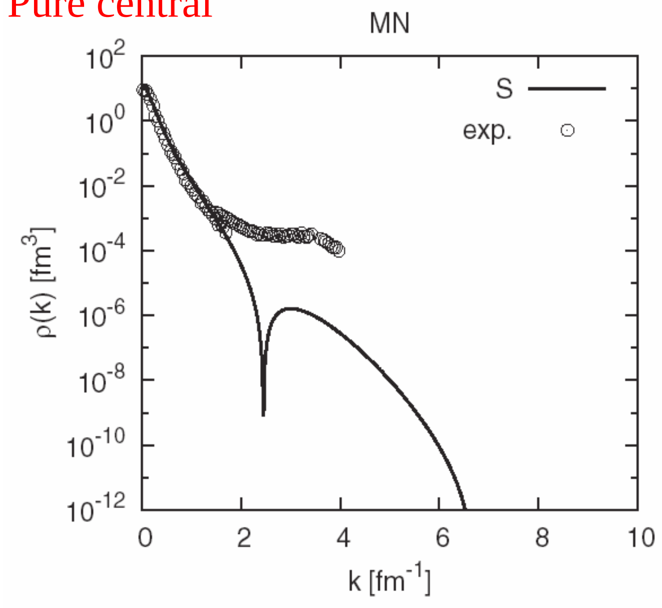
- Quantity reflecting two-nucleon correlation
- Experiments on nucleon correlation
  - Intensity interferometry F. M. Marques et al., PLB476 (2000).
  - $^{12}C(e,e'np)$, $^{12}C(e,e'pp)$ E. Piasetzky et al.
    - Theoretical analysis R. Schiavilla et al., PRL98 (2007).
  - Recent experiment at RIKEN T. Suda et al.

One nucleon exchange reaction

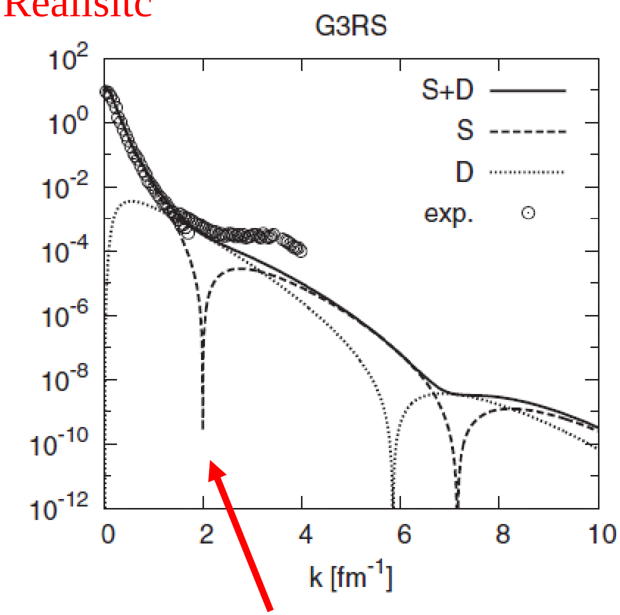


Deuteron momentum distribution

Pure central



Realisitc



D-wave fills the dip of S-wave

Magnitude of high mpmentum components

Effect of a short-ranged repulsion

Non-nucleonic effect in k>2.5 fm^{-1}

Data: K. I. Blomqvist et al., Phys. Lett. B424 (1998).

Core + 2N model ^{6}He: α+n+n
^{6}Li: α+n+p

MN or G3RS for NN
Phenomenological pot. for αN

W.Horiuchi, Y.S.PRC76(2007)

Theory

	^{6}He		^{6}Li		d	
	Effective	Realistic	Effective	Realistic	Effective	Realistic
Energy(MeV)	-0.421	-0.460	-3.91	-3.31	-2.20	-2.27
Tensor(MeV)	-	0.107	-	-12.3	-	-11.5
N-N distance(fm)	5.05	4.86	3.48	3.58	3.90	3.96

Experiment

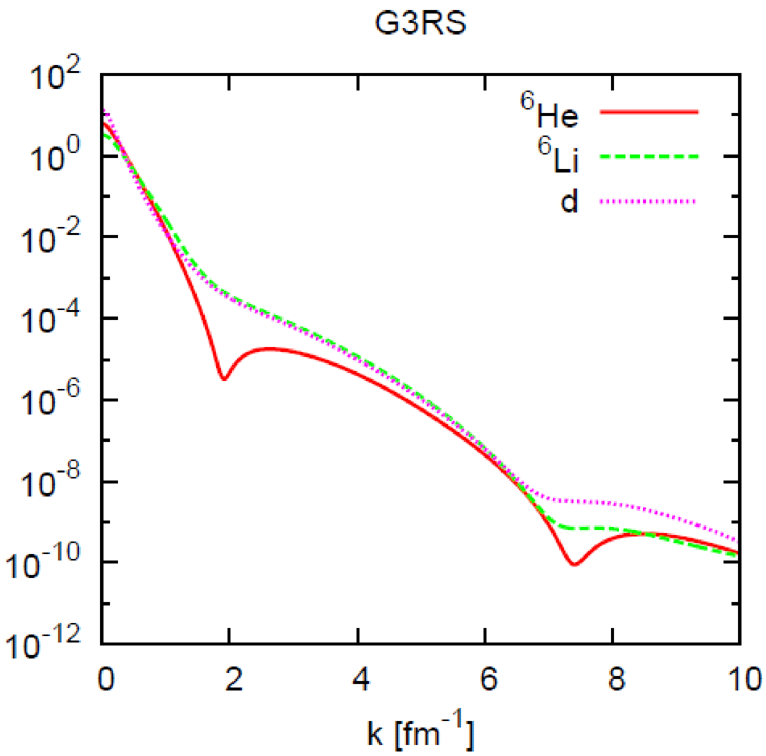
	^{6}He	^{6}Li	d
Energy(MeV)	-0.975	-3.90	-2.22
N-N distance(fm)	5.9±1.2	not measured	3.91

Intensity interferometry experiment (Marques *et al*. PLB476 (2000).)

Relative momentum distribution

-- information on N-N correlation --

W.Horiuchi, Y.S.PRC76(2007)



Momentum distribution of ^{6}He differs from that of ^{6}Li.
Momentum distribution of ^{6}Li is similar to that of deuteron.

$P(LS)$	^{6}He (0^+)	^{6}Li (1^+)
(00)	87.5	
(11)	12.5	0.8
(10)		3.9
(01)		90.3
(21)		5.0

The dip of ^{6}He reflects S-wave dominance.
D-wave in ^{6}Li fills the dip of S-wave.

Shrinkage due to the interaction with core

	^{6}Li (1^+)		d (1^+)	
	MN	G3RS	MN	G3RS
E	−3.91	−3.31	−2.20	−2.28
$\langle T_r \rangle$	17.56	23.28	10.48	16.48
$\langle v_{12}^{C} \rangle$	−13.41	−7.71	−12.69	−7.29
$\langle v_{12}^{T} \rangle$	—	−12.25	—	−11.46
$\langle v_{12}^{LS} \rangle$	—	—	—	—
$\langle T_R \rangle$	13.29	11.49	—	—
$\langle U_1^C + U_2^C \rangle$	−19.00	−16.44	—	—
$\langle U_1^{LS} + U_2^{LS} \rangle$	−2.34	−1.69	—	—
$\sqrt{\langle r^2 \rangle}$	3.48	3.58	3.90	3.96

E(pn)~ +3.3 MeV

PRL 98, 132501 (2007) PHYSICAL REVIEW LETTERS week ending 30 MARCH 2007

Tensor Forces and the Ground-State Structure of Nuclei

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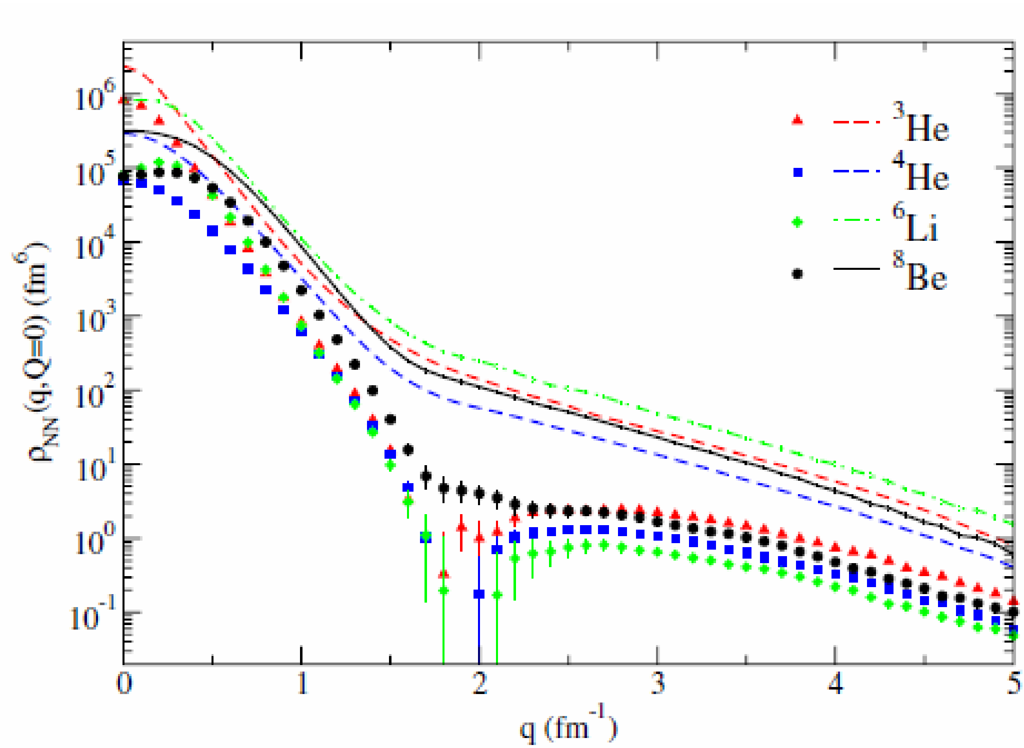
(Received 10 November 2006; published 27 March 2007)



Lines: n-p pair (no dip)
Dots: p-p pair

^{12}C(e,e'np), ^{12}C(e,e'pp)

Summary

- Explicitly correlated Gaussian applied to ^{4}He and α+N+N model for ^{6}He and ^{6}Li
  -- Spectrum of ^{4}He well reproduced with realistic potential
  -- (3N)+N cluster states with 0^+, 1^+ (T=0, 1). Need to be examined
- Dominance of tensor correlation in 0^-, E_x=20.01 MeV.
  -- Unnatural parity component described with DGV
- Relative momentum distributions in ^{6}He, ^{6}Li and d
  - Effect of a short-ranged repulsion at large k
  - Distribution of ^{6}He differs from ^{6}Li, which is similar to d
  - Effect of tensor force evident at k ~ 2 fm^{-1}.

Outlook

Application to ^{12}C+(few-nucleons) system; ^{16}O, ^{15}C, ^{16}C etc.