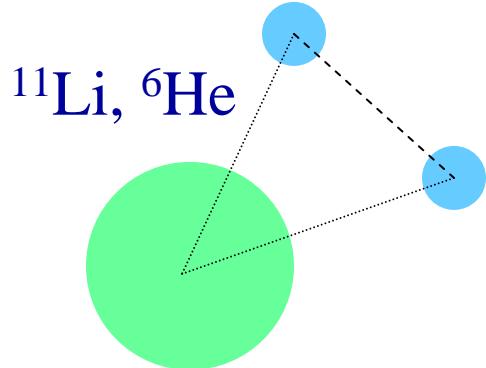


Di-neutron correlation in light neutron-rich nuclei



K. Hagino (Tohoku University)

in collaboration with
H. Sagawa (University of Aizu)

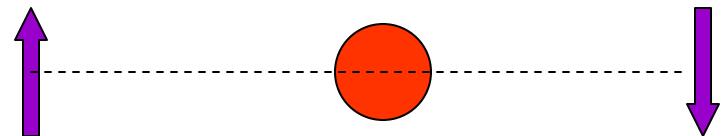
1. *Introduction: Pairing correlations in nuclei*
2. *Three-body model with density-dep. contact int.*
3. *E1 excitations and Geometry of Borromean nuclei*
4. *Coexistence of BCS- and BEC- like pair structures*
5. *Summary*

Introduction: Pairing correlations in nuclei

Spatial structure of a Cooper pair?

Coherence length of a Cooper pair:

$$\xi = \frac{\hbar^2 k_F}{m\Delta}$$



→ much larger than the nuclear size

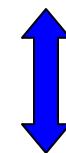
(note)

$$k_F = 1.36 \text{ fm}^{-1}$$

$$\Delta = 12/\sqrt{140} = 1.01 \text{ MeV}$$

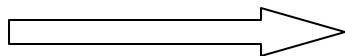
(for $A=140$)

$$\rightarrow \xi = 55.6 \text{ fm}$$

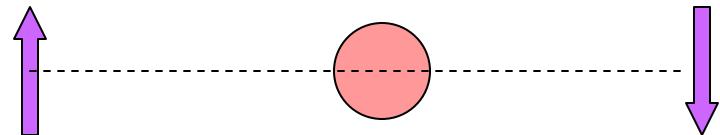
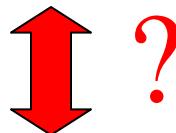


$$R = 1.2 \times 140^{1/3} = 6.23 \text{ fm}$$

Coherence length of a Cooper pair: $\xi = \frac{\hbar^2 k_F}{m\Delta}$



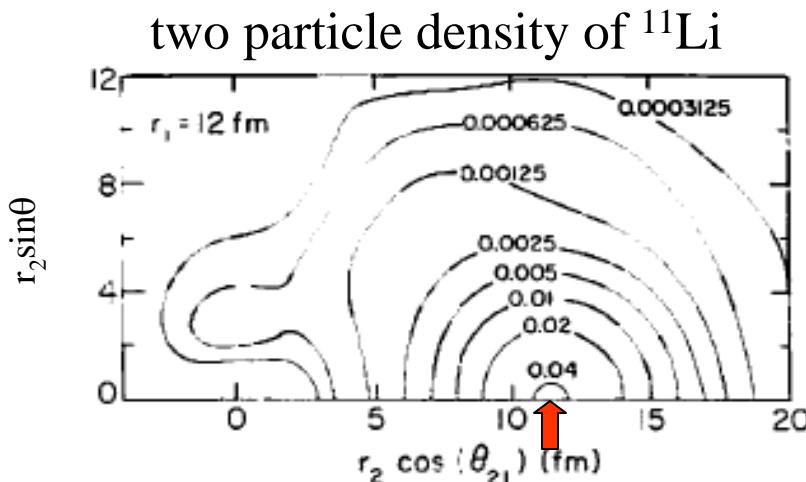
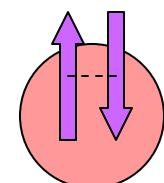
much larger than the nuclear size



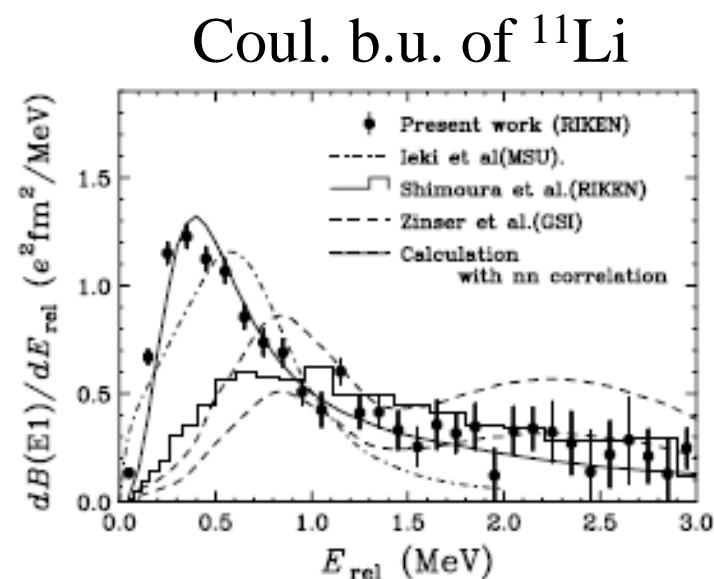
Di-neutron correlations in neutron-rich nuclei

cf. HFB calculations for $^{18-24}\text{O}$, $^{50-58}\text{Ca}$, $^{80-86}\text{Ni}$:

M. Matsuo, K. Mizuyama, Y. Serizawa, PRC71('05)064326



G.F. Bertsch, H. Esbensen,
Ann. of Phys., 209('91)327



T. Nakamura et al., PRL96('06)252502

Another motivation: Spatial correlation of valence neutrons?

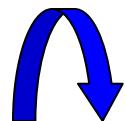
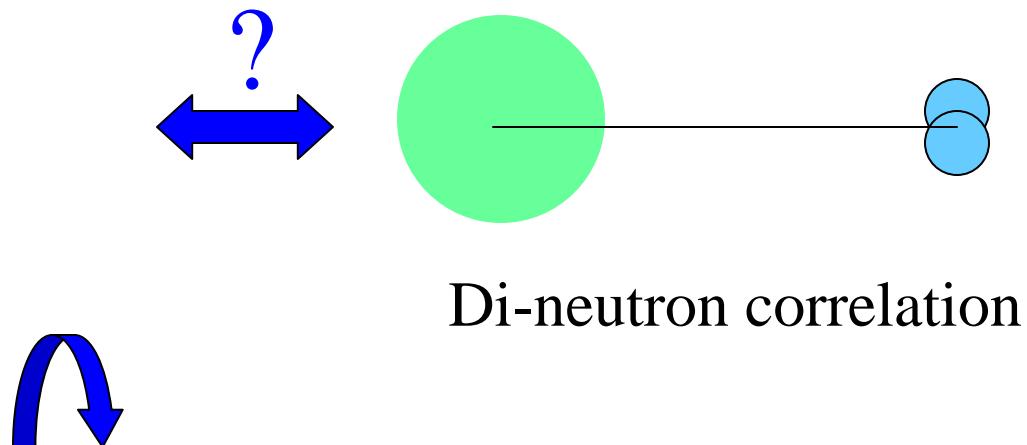
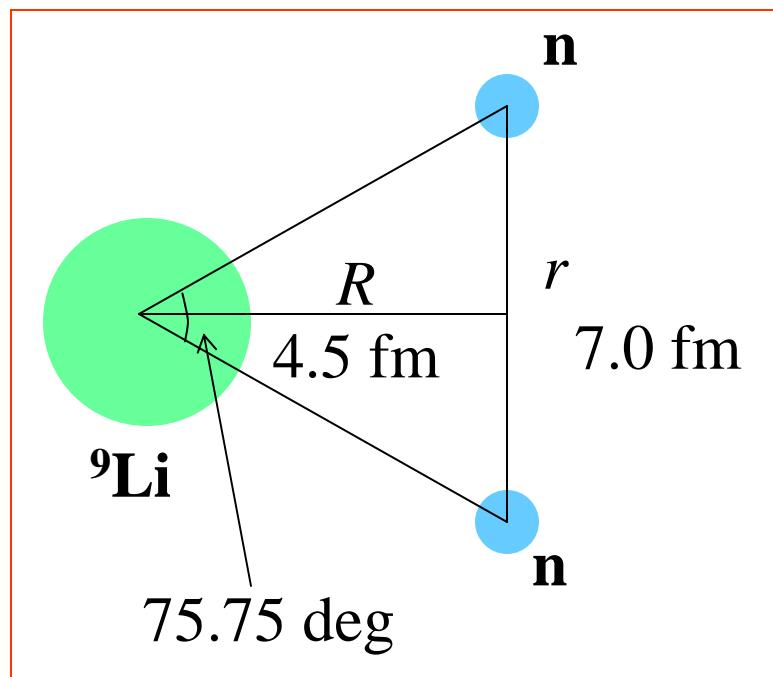
Analysis of Coul. Dissociation for ^{11}Li

S. Shimoura et al., PLB348('95) 29

$$\Psi(\mathbf{R}, \mathbf{r}) \sim \int dk a(k) \psi_k(\mathbf{R}) \phi_k(\mathbf{r})$$

$$\psi_k(\mathbf{R}) \sim \exp(-\eta(k)R)/R$$

$$\phi_k(\mathbf{r}) \sim \exp(ik \cdot \mathbf{r})/r$$



Detailed theoretical re-analysis of
➤ Density distribution
➤ Dipole excitations

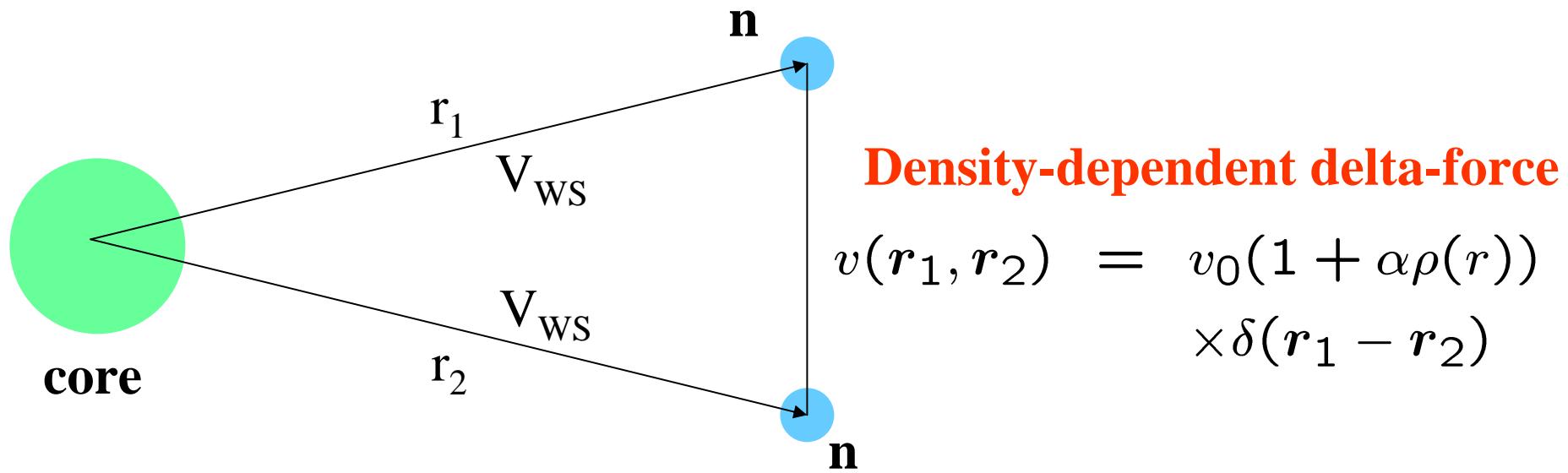
Three-body model

G.F. Bertsch and H. Esbensen,

Ann. of Phys. 209 ('91) 327

H. Esbensen, G.F. Bertsch, K. Hencken,

Phys. Rev. C 56 ('99) 3054



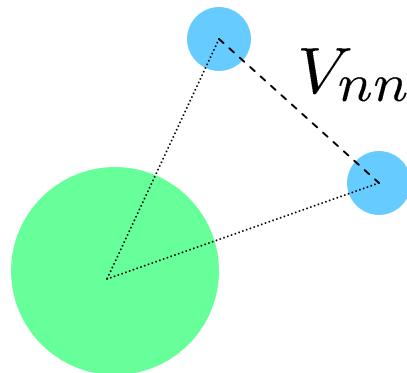
$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(p_1 + p_2)^2}{2A_c m}$$

(note) recoil kinetic energy of the core nucleus

Density-dependent delta interaction

H. Esbensen, G.F. Bertsch, K. Hencken, *Phys. Rev. C56*('99)3054

$$V_{nn}(\mathbf{r}_1, \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2) \left(v_0 + \frac{v_\rho}{1 + \exp[(r_1 - R_\rho)/a_\rho]} \right)$$



➤ Two neutron system in the vacuum:

$$\text{V}_{nn}^{(0)}(\mathbf{r}_1, \mathbf{r}_2) = v_0 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$
$$a_{nn} = \frac{\pi}{2} \cdot \frac{\alpha}{1 + \alpha k_c}$$

$$\alpha = \frac{v_0}{2\pi^2} \frac{m}{\hbar^2}, \quad E_{\text{cut}} = \frac{\hbar^2 k_c^2}{m}$$

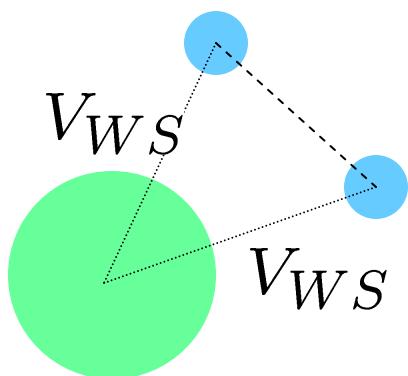


$$v_0 = \frac{2\pi^2 \hbar^2}{m} \cdot \frac{2a_{nn}}{\pi - 2k_c a_{nn}}$$

➤ Two neutron system in the medium:

v_ρ, R_ρ, a_ρ : adjust so that S_{2n} can be reproduced

Two-particle wave functions (J=0 pairs)



$$\hat{h} \psi_{nljm}(\mathbf{r}) = \epsilon_{nlj} \psi_{nljm}(\mathbf{r})$$

$\Psi_{nn'lj}^{(2)}(\mathbf{r}, \mathbf{r}') = \sum_m \langle jmj - m | 00 \rangle \psi_{nljm}(\mathbf{r}) \psi_{n'lj-m}(\mathbf{r}')$



Hamiltonian diagonalization

$$\Psi_{gs}(\mathbf{r}, \mathbf{r}') = \mathcal{A} \sum_{nn'lj} \alpha_{nn'lj} \Psi_{nn'lj}^{(2)}(\mathbf{r}, \mathbf{r}')$$

- Continuum: box discretization

- Energy cut-off:

$$\epsilon_{nlj} + \epsilon_{n'lj} \leq \frac{A_c + 1}{A_c} E_{\text{cut}}$$

Application to ^{11}Li , ^6He

^{11}Li , ^6He : Typical Borromean nuclei
Esbensen et al.

^{11}Li : $a_{nn} = -15$ fm, $E_{\text{cut}} = 30$ MeV, $R_{\text{box}} = 40$ fm

WS: adjusted to $p_{3/2}$ energy in ^8Li & n - ^9Li elastic scattering

Parity-dependence ← to increase the s-wave component

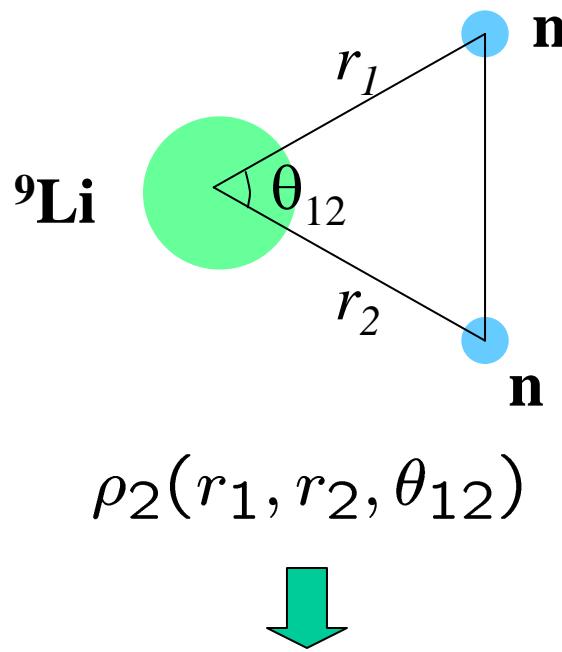
^6He : $a_{nn} = -15$ fm, $E_{\text{cut}} = 40$ MeV, $R_{\text{box}} = 30$ fm

WS: adjusted to n - α elastic scattering

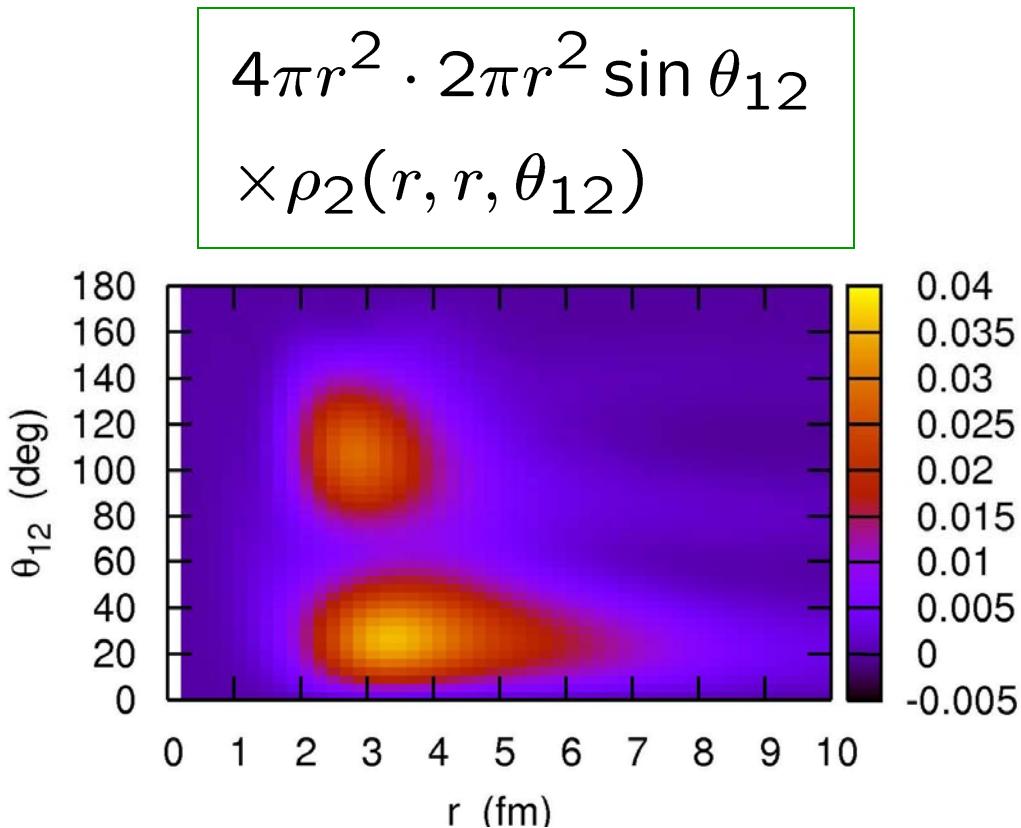
Results for ground state properties

Nucleus	S_{2n} (MeV)	$\langle r_{nn}^2 \rangle$ (fm 2)	$\langle r_{c-2n}^2 \rangle$ (fm 2)	dominant config.	fraction (%)	$S=0$ (%)
^6He	0.975	21.3	13.2	$(p_{3/2})^2$	83.0	87.0
^{11}Li	0.295	41.4	26.3	$(p_{1/2})^2$	59.1	60.6

Two-particle density for ^{11}Li



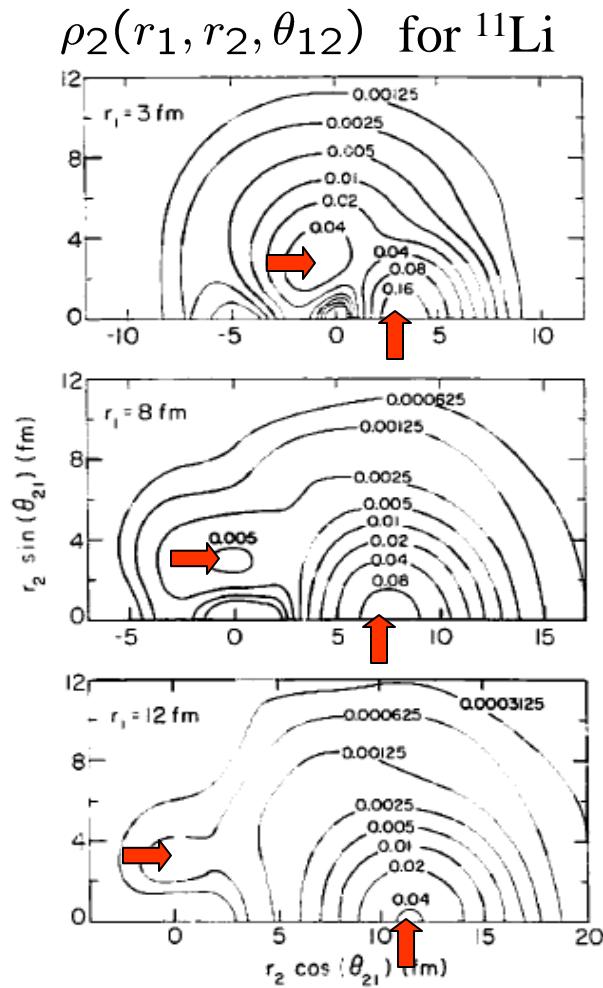
Set $r_1=r_2=r$, and plot ρ_2 as a function of r and θ_{12}



- two-peaked structure
- Long tail for “di-neutron”

(note)

$$\int_{-\infty}^0 q^{1/2} J_0(qr) \int_{-\infty}^0 q^{1/2} J_0(qr) \int_{-\pi}^0 J_0 \sin \theta J_0(q\theta) J_0(b\theta) (\alpha, \beta, \theta) = 1$$



G.F. Bertsch, H. Esbensen,
Ann. of Phys., 209('91)327

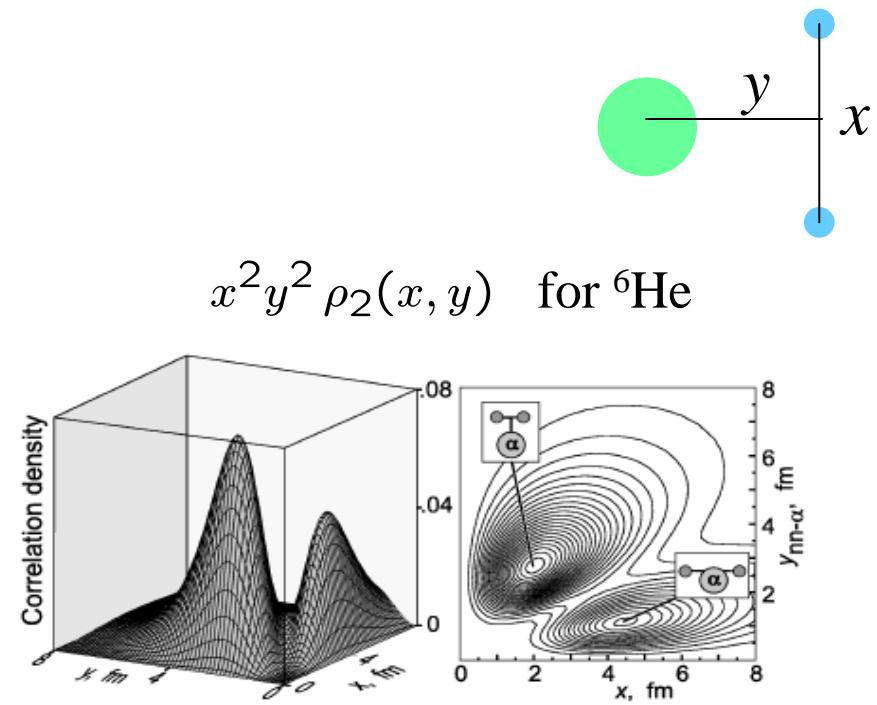
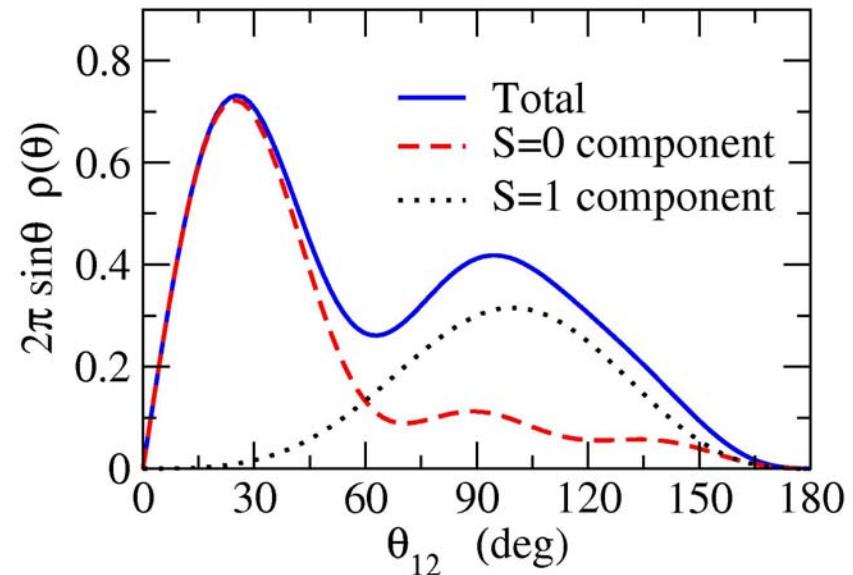
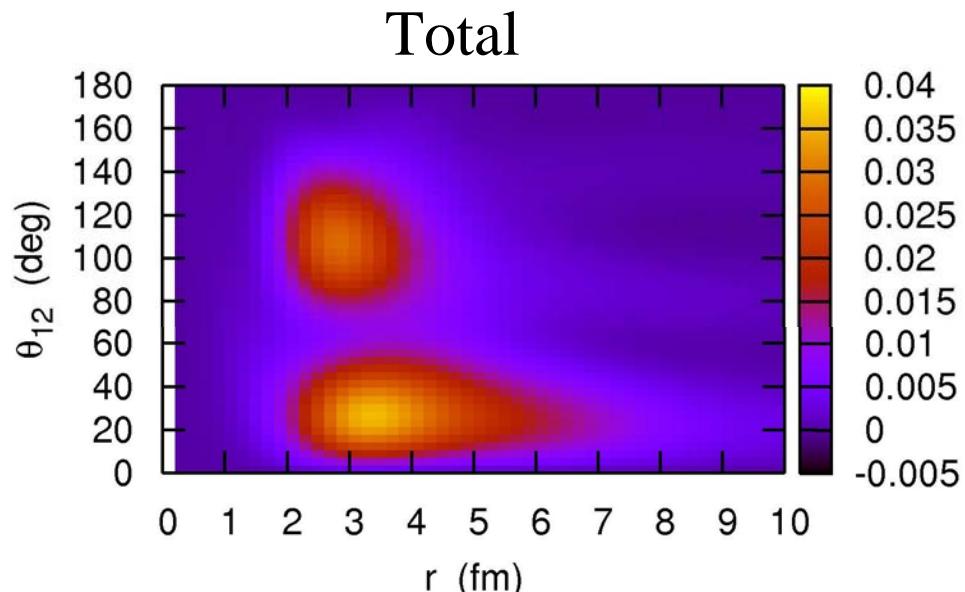


FIG. 1. Spatial correlation density plot for the 0^+ ground state of ^6He . Two components—di-neutron and cigarlike—are shown schematically.

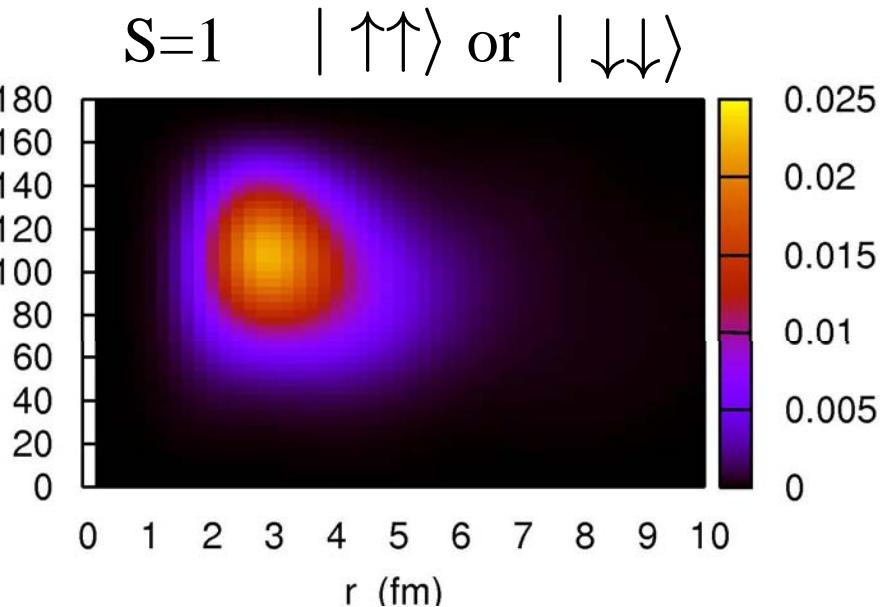
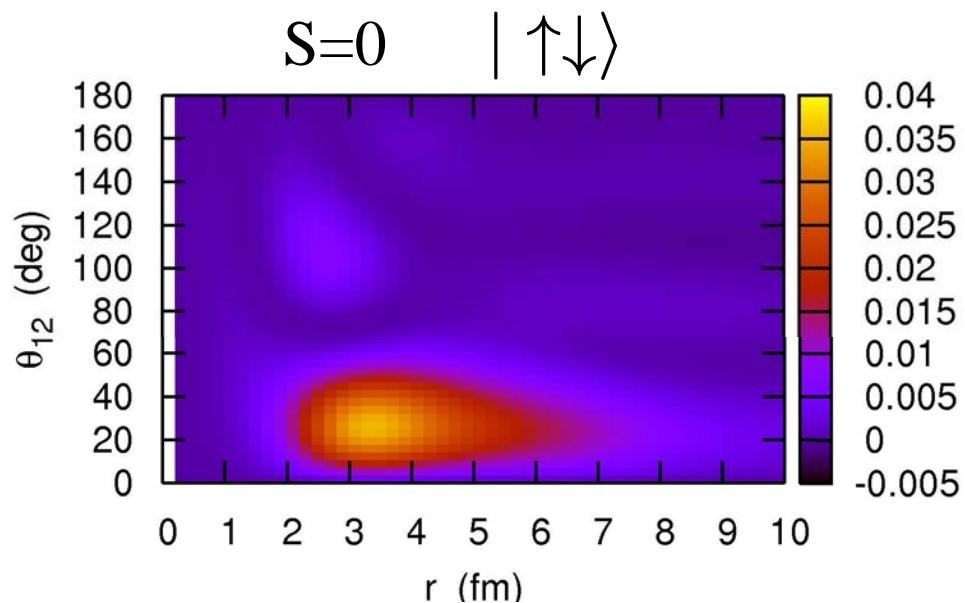
Yu.Ts. Oganessian, V.I. Zagrebaev,
and J.S. Vaagen, *PRL82*('99)4996
M.V. Zhukov et al., *Phys. Rep.* 231('93)151

“di-neutron” and “cigar-like”
configurations

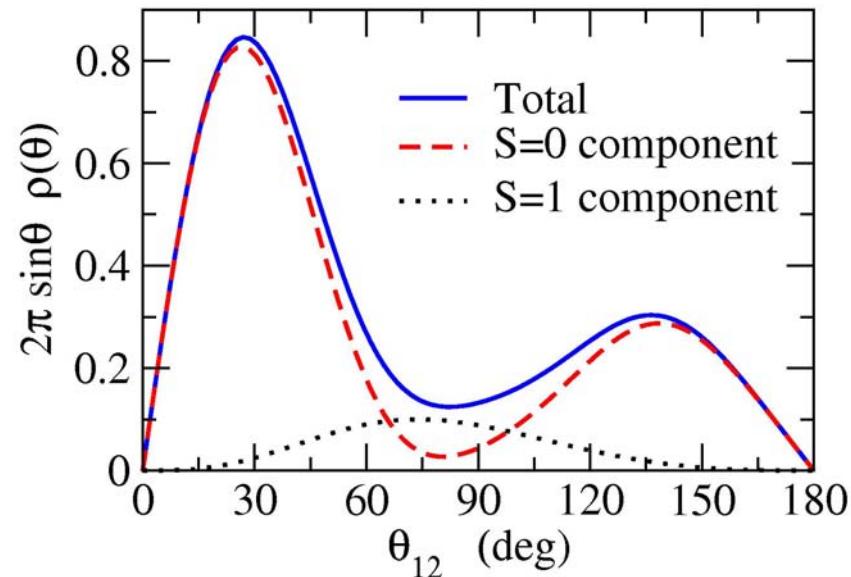
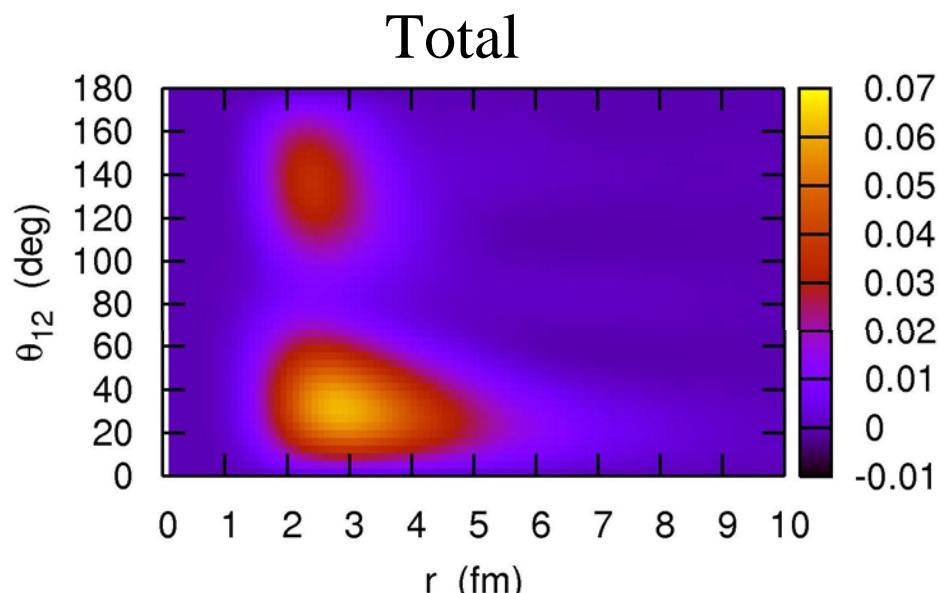
Two-particle density for ^{11}Li



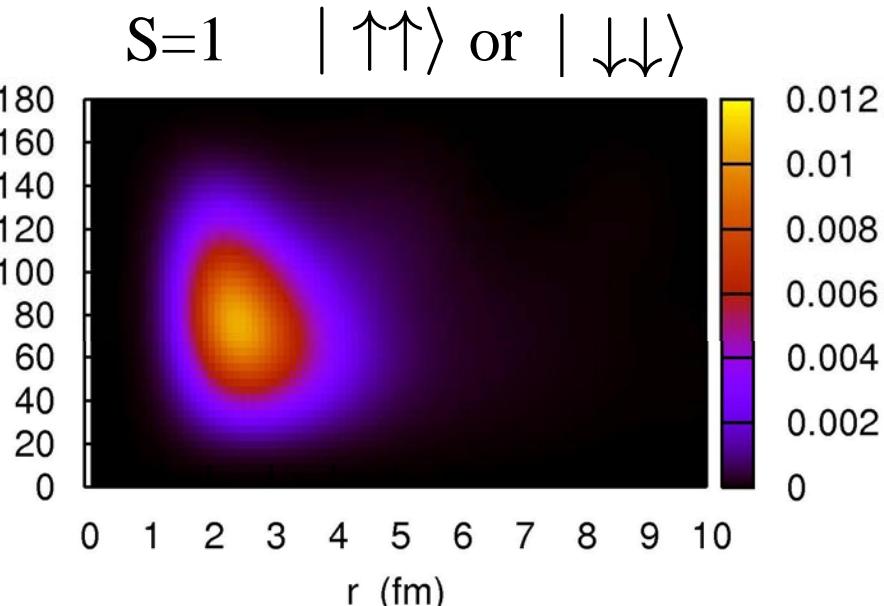
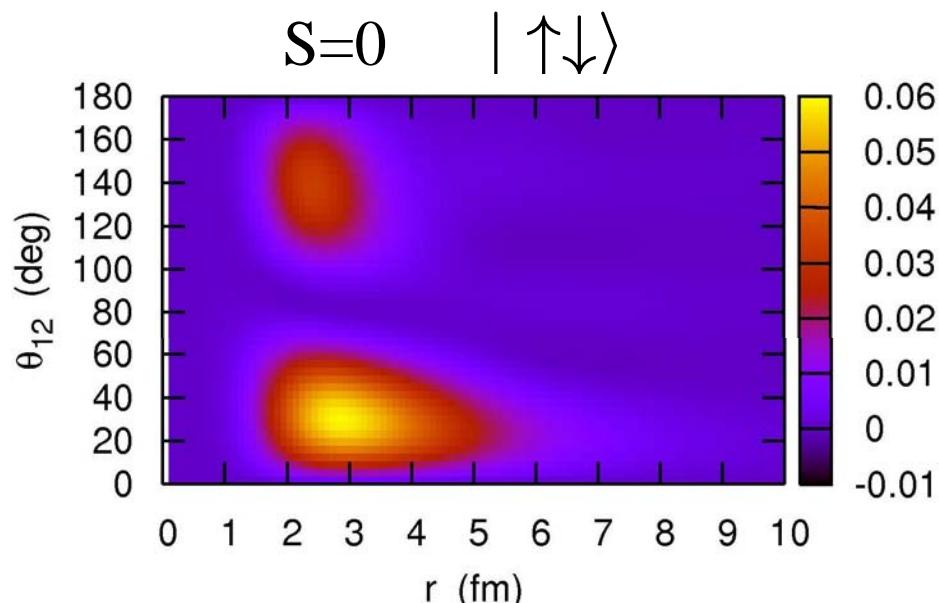
$$\rightarrow \langle \theta_{12} \rangle = 65.29 \text{ deg.}$$



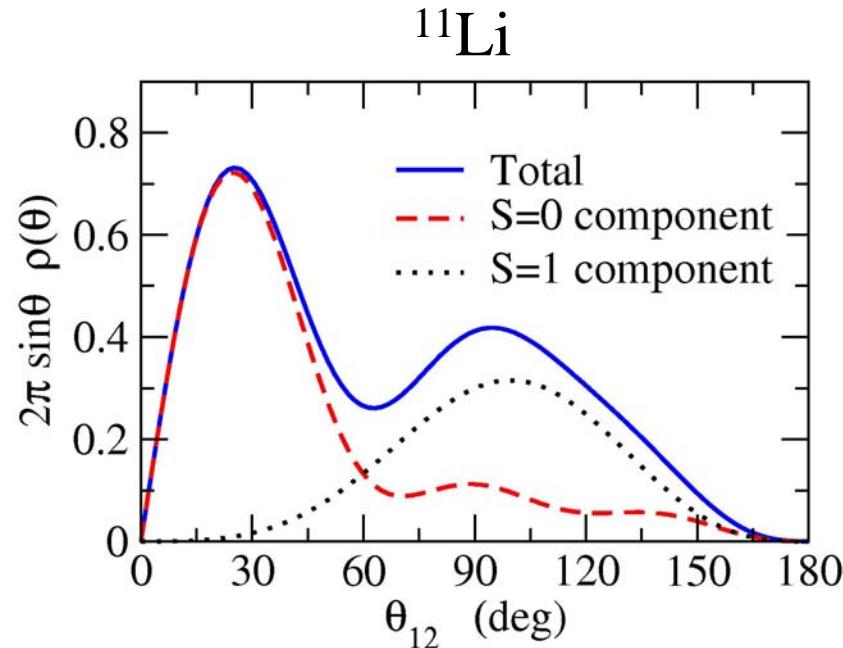
Two-particle density for ${}^6\text{He}$



$$\rightarrow \langle \theta_{12} \rangle = 66.33 \text{ deg.}$$

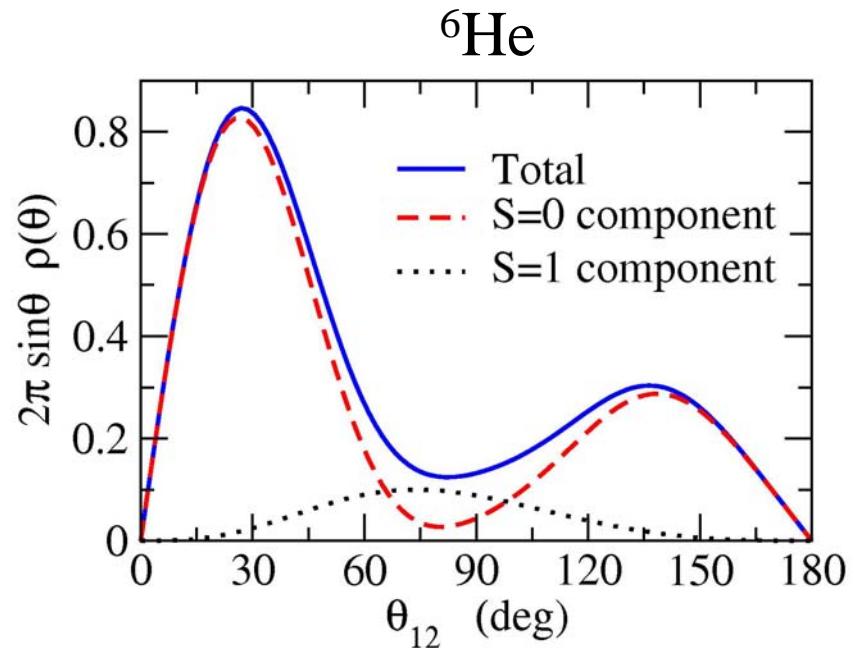


Comparison between the two nuclei



$(p_{1/2})^2 : 59.1\%$

$(s_{1/2})^2 : 22.7\% \quad (d_{5/2})^2 : 11.5\%$



$(p_{3/2})^2 : 83.0\%$

$(d_{5/2})^2 : 6.11\%,$

$(p_{1/2})^2 : 4.85\%$

$(s_{1/2})^2 : 3.04\%, \quad (d_{3/2})^2 : 1.47\%$

$$\begin{aligned} \rho^{S=0}(\theta) &\propto \cos^2 \theta \\ \rho^{S=1}(\theta) &\propto \sin^2 \theta \end{aligned}$$

for $(p_{1/2})^2$ or $(p_{3/2})^2$

E1 excitations and geometry of Borromean nuclei

Dipole excitations

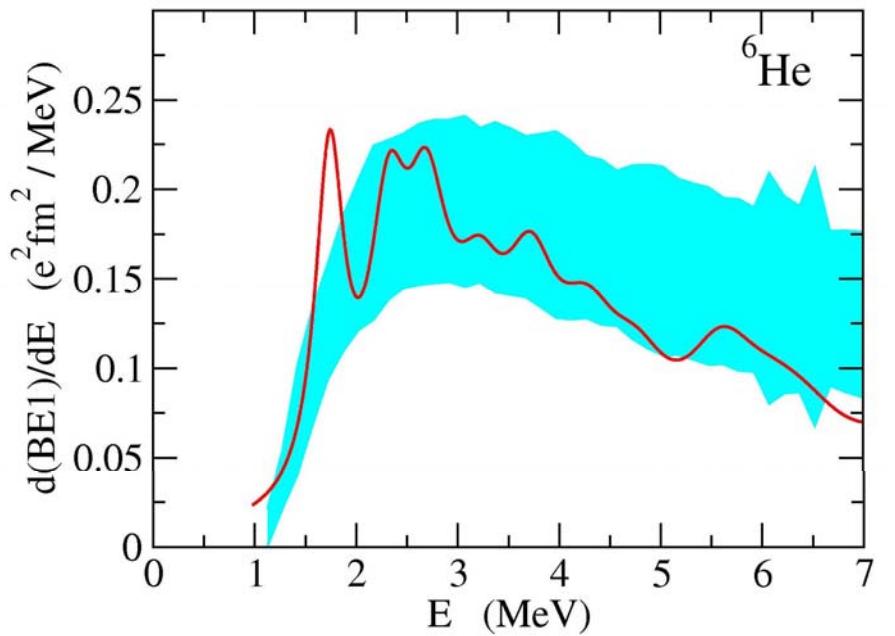
Response to the dipole field:

$$B_k(E1) = 3 |\langle \Psi_{1-}^k | \hat{D}_0 | \Psi_{gs} \rangle|^2$$

$$\hat{D}_M = -\frac{Ze}{A} \sum_{i=1,2} r_i Y_{1M}(\hat{\mathbf{r}}_i)$$

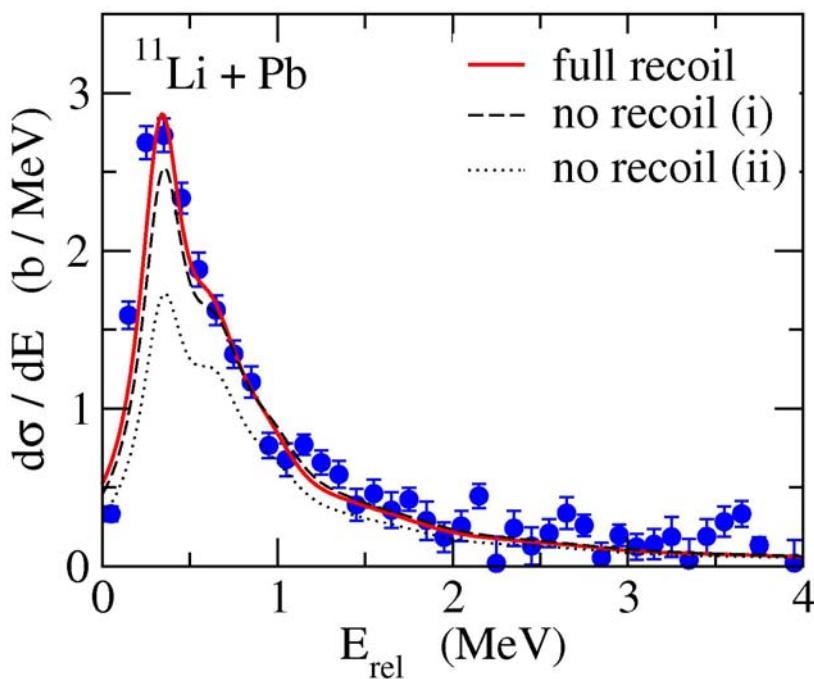
Smearing:

$$B(E1) = \sum_k \frac{\Gamma}{\pi} \frac{B_k(E1)}{(E - E_k)^2 + \Gamma^2}$$

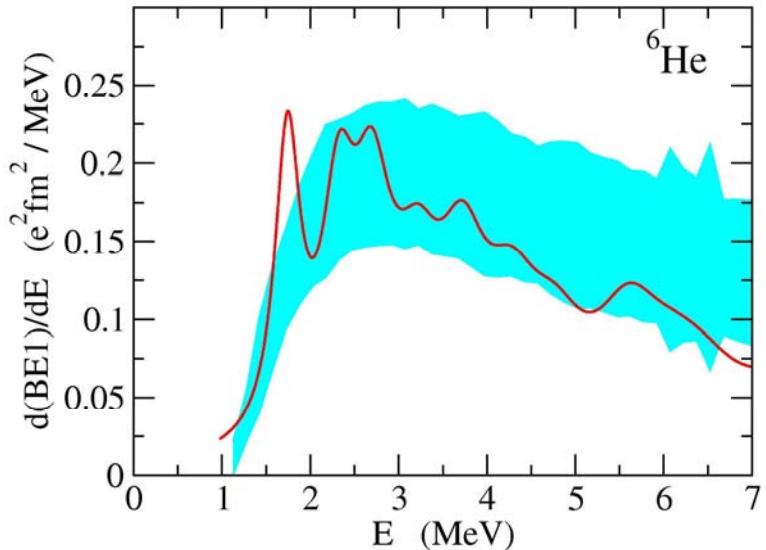
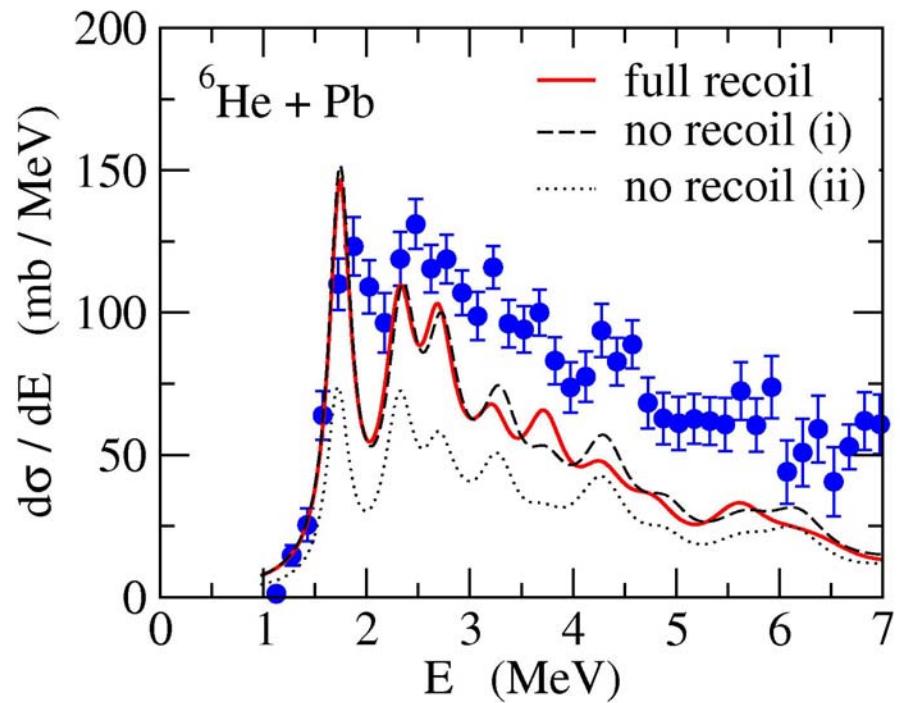


$$\Gamma = 0.2 \cdot \sqrt{E - E_{th}} \quad (\text{MeV})$$

Dipole excitations



$$\Gamma = 0.25 \cdot \sqrt{E - E_{\text{th}}} \quad (\text{MeV})$$

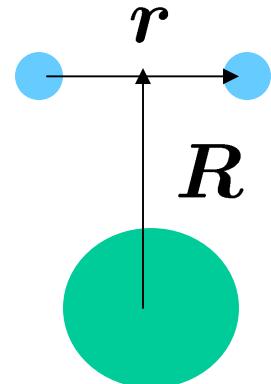


Geometry of Borromean nuclei

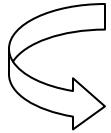
“experimental” mean opening angle

$$\sqrt{\langle R^2 \rangle}$$

$$\begin{aligned} B(E1) &= \sum_i B(E1; gs \rightarrow i) \\ &= \frac{3}{\pi} \left(\frac{Ze}{A} \right)^2 \langle R^2 \rangle \end{aligned}$$



(note) forbidden transition



$$\langle R^2 \rangle_{\text{exp}} = \frac{B(E1; E \leq E_{\text{max}})_{\text{exp}}}{B(E1; E \leq E_{\text{max}})_{\text{cal}}} \cdot \langle R^2 \rangle_{\text{cal}}$$

H. Esbensen, K.H., P. Mueller, and H. Sagawa,
PRC76('07)924302

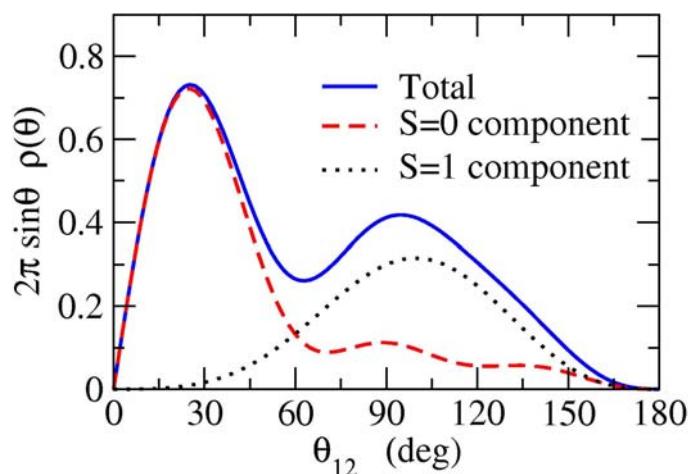
$$\sqrt{\langle r^2 \rangle}$$

$$\langle r_m^2 \rangle = \frac{A_c}{A} \langle r_m^2 \rangle_{A_c} + \frac{2A_c}{A^2} \langle R^2 \rangle + \frac{1}{2A} \langle r^2 \rangle$$

or also from HBT-type 2n correlation study

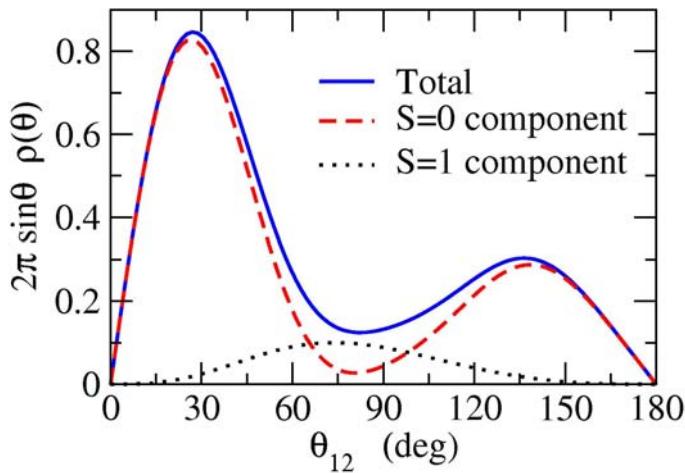
Geometry of Borromean nuclei

^{11}Li

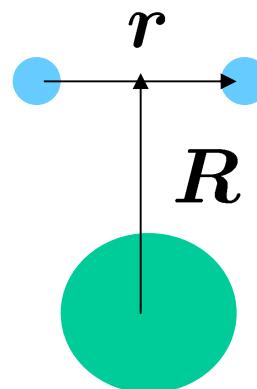


$$\rightarrow \langle \theta_{12} \rangle = 65.29 \text{ deg.}$$

^6He



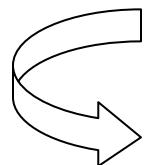
$$\rightarrow \langle \theta_{12} \rangle = 66.33 \text{ deg.}$$



“experimental” mean opening angle

$$\sqrt{\langle R^2 \rangle} \longleftrightarrow \text{B(E1)}$$

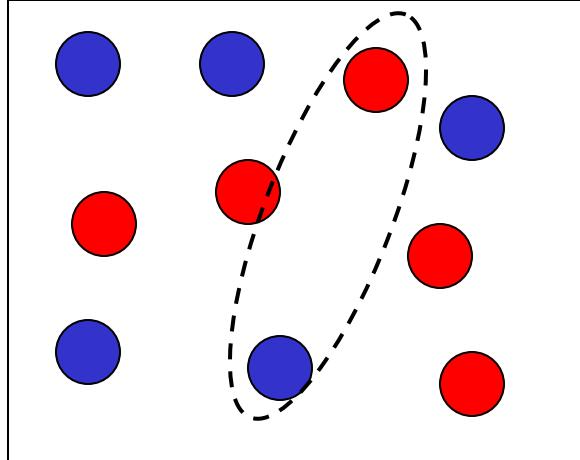
$$\sqrt{\langle r^2 \rangle} \longleftrightarrow \text{matter radius or HBT}$$



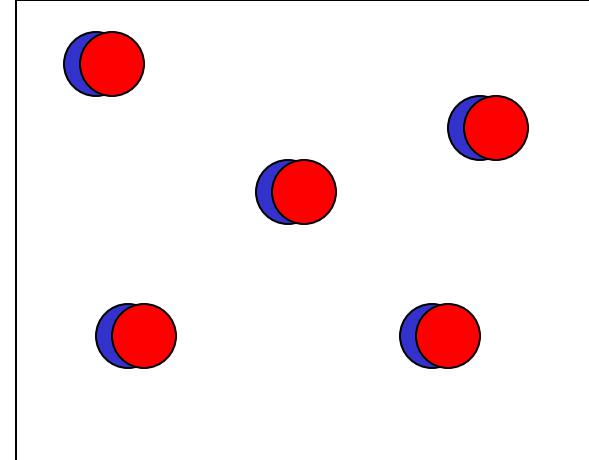
$$\begin{aligned} \langle \theta_{12} \rangle &= 65.2^{+11.4}_{-13.0} \quad (^{11}\text{Li}) \\ &= 74.5^{+11.2}_{-13.1} \quad (^6\text{He}) \end{aligned}$$

Nucleus	Method	$\sqrt{\langle r_{nn}^2 \rangle}$ (fm)	$\sqrt{\langle r_{c-2n}^2 \rangle}$ (fm)	$\langle \theta_{nn} \rangle$ (deg.)
${}^6\text{He}$	Matter radii	3.75+/-0.93	3.88+/-0.32	$51.6^{+11.2}_{-12.4}$
	HBT	5.9+/-1.2		$74.5^{+11.2}_{-13.1}$
	3body calc.	4.65	3.63	66.33
${}^{11}\text{Li}$	Matter radii	5.50+/-2.24	5.15+/-0.33	$56.2^{+17.8}_{-21.3}$
	HBT	6.6+/-1.5		$65.2^{+11.4}_{-13.0}$
	3body calc.	6.43	5.13	65.29

BCS-BEC crossover phenomenon

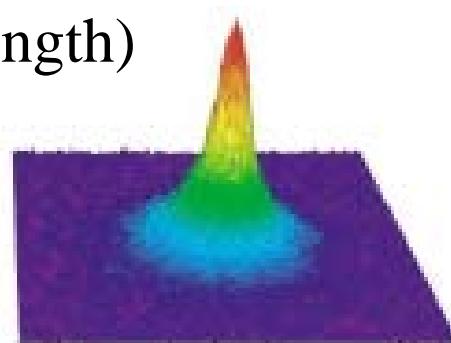
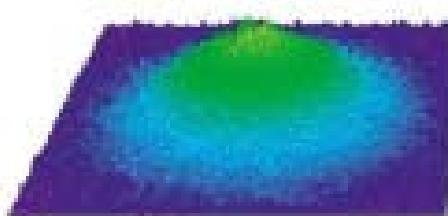


$|v_{\text{pair}}| \rightarrow \infty$
crossover



BCS (weak coupling)

- Weakly interacting fermions
- Correlation in **p** space (large coherence length)



BEC (strong coupling)

- Weakly interacting “diatomic molecules”
- Correlation in **r** space (small coherence length)

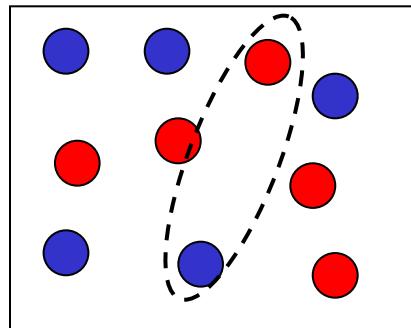
cf. BEC of ^{40}K molecules

M. Greiner et al., Nature 426('04)537

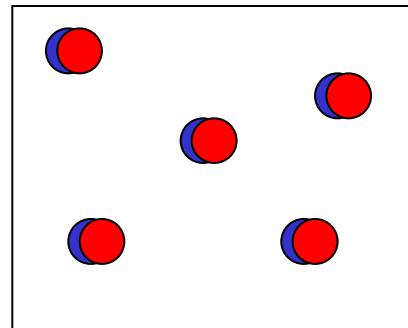
cf. BCS-BEC crossover in color superconductivity: Y. Nishida and H. Abuki,
PRD72('05)096004

BCS-BEC crossover

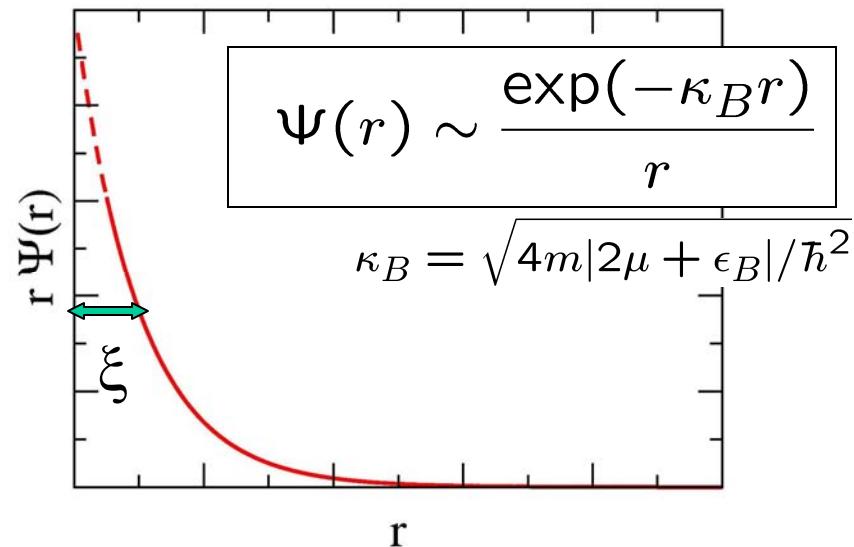
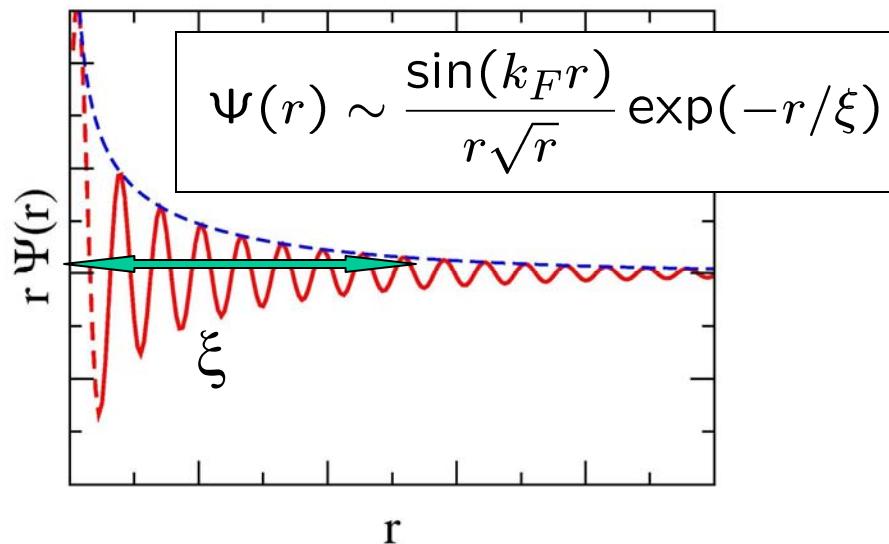
Cooper pair wave function: $\Psi(r, r') \sim \langle \Phi_0 | c^\dagger(r, \uparrow) c^\dagger(r', \downarrow) | \Phi_0 \rangle$



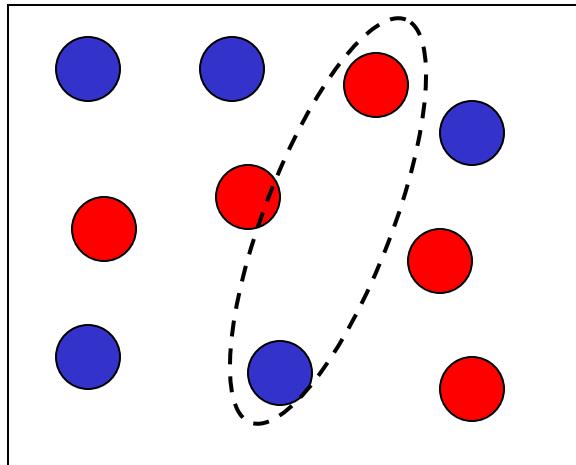
$|v_{\text{pair}}| \rightarrow \infty$
crossover



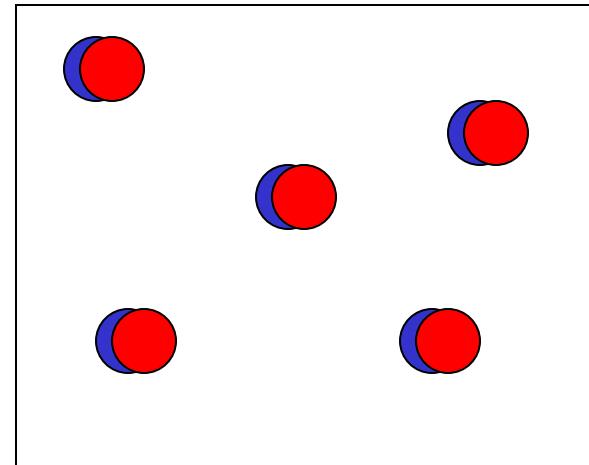
BCS (weak coupling)
Correlation in p space
(large coherence length)



BCS-BEC crossover

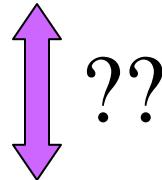


$|v_{\text{pair}}| \rightarrow \infty$
crossover



BCS (weak coupling)

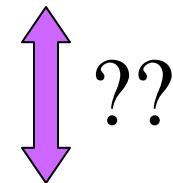
- Weakly interacting fermions
- Correlation in **p** space (large coherence length)



Pairing in stable nuclei

BEC (strong coupling)

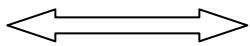
- Weakly interacting “diatomic molecules”
- Correlation in **r** space (small coherence length)



Di-neutron correlations in neutron-rich nuclei

BCS-BEC crossover behavior in infinite nuclear matter

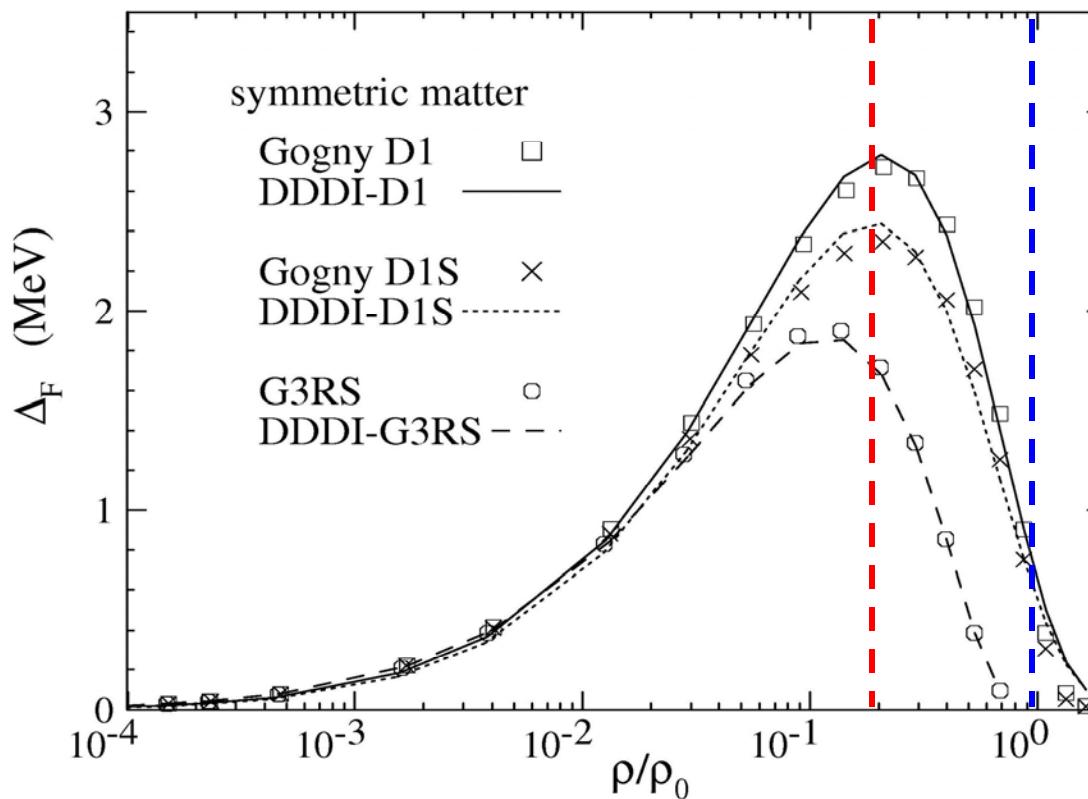
Neutron-rich nuclei



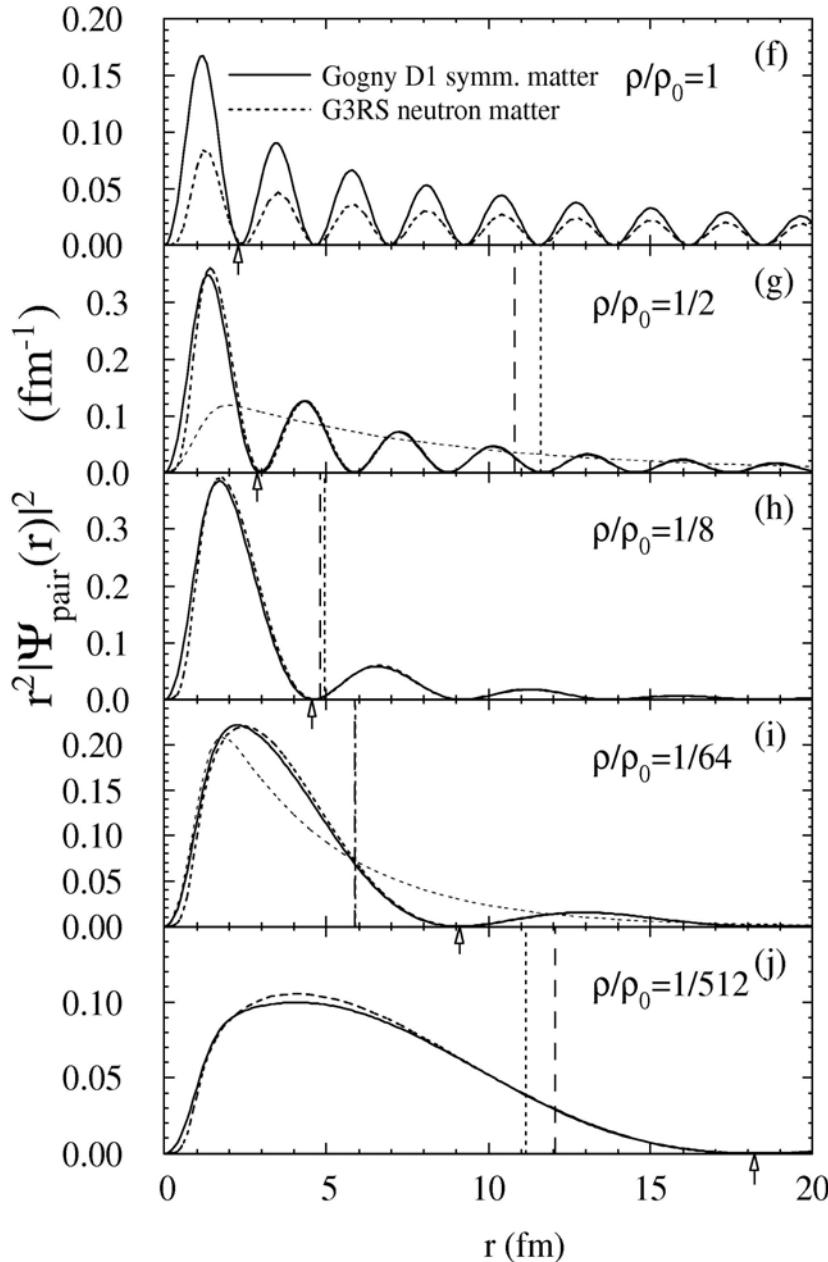
- Weakly bound levels

- Unsaturated density around surface
(halo/skin)

pairing gap in infinite nuclear matter



Spatial structure of neutron Cooper pair in infinite matter

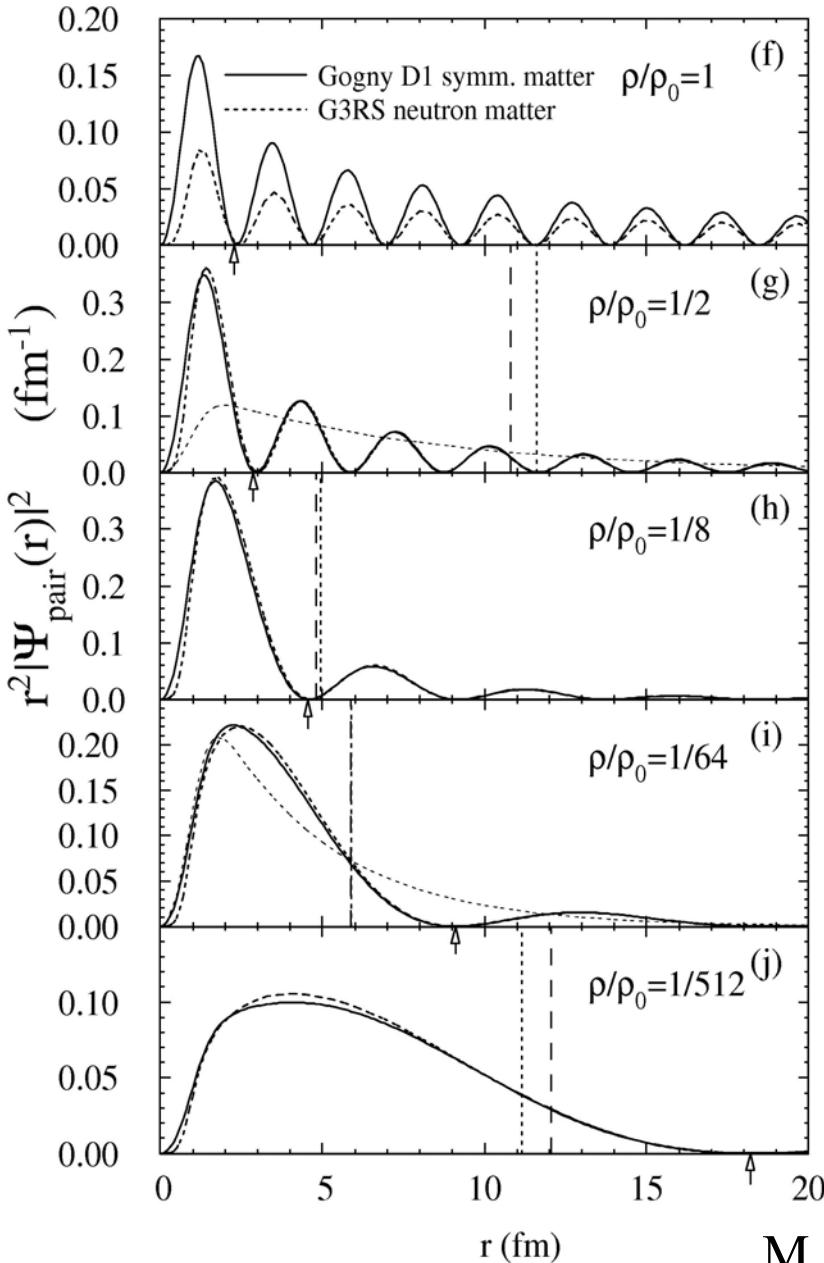


BCS



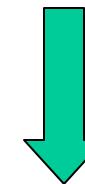
Crossover region

Spatial structure of neutron Cooper pair in infinite matter



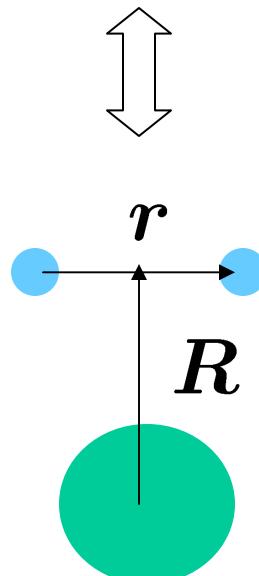
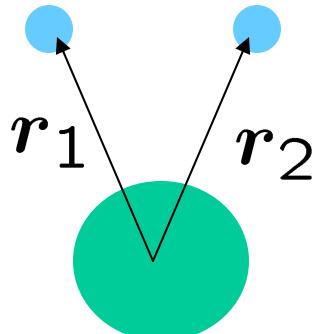
Our Motivations

- How can this behavior be seen in *finite* nuclei?
- Relation to di-neutron correlation?



Di-neutron wave function in Borromean nuclei

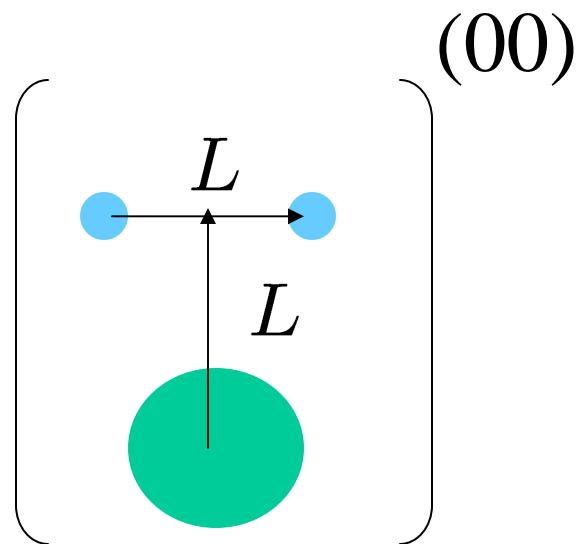
$$\Psi^{(S=0)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_L f_L(r, R) [Y_L(\hat{\mathbf{r}}) Y_L(\hat{\mathbf{R}})]^{(00)}$$



$$\begin{aligned}
 f_L(r, R) &= \sum_{n' \leq n} \sum_{l,j} \alpha_{nn'lj} (-)^{l+L} \frac{\sqrt{2\pi(2j+1)}}{\sqrt{2(1+\delta_{n,n'})}} \\
 &\quad \times \int_0^\pi \sin \theta d\theta Y_{L0}(\theta) \sum_m (-)^m \frac{(l-m)!}{(l+m)!} \\
 &\quad \times P_l^m(\cos \theta_1) P_l^m(\cos \theta_2) \\
 &\quad \times \phi_{nlj}(r_1) \phi_{n'lj}(r_2) \\
 r_1 &= \sqrt{R^2 + r^2/4 + Rr \cos \theta} \\
 r_2 &= \sqrt{R^2 + r^2/4 - Rr \cos \theta} \\
 \cos \theta_1 &= (R + r \cos \theta/2)/r_1 \\
 \cos \theta_2 &= (R - r \cos \theta/2)/r_2
 \end{aligned}$$

Di-neutron wave function in Borromean nuclei

$$\Psi^{(S=0)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_L f_L(r, R) [Y_L(\hat{\mathbf{r}}) Y_L(\hat{\mathbf{R}})]^{(00)}$$



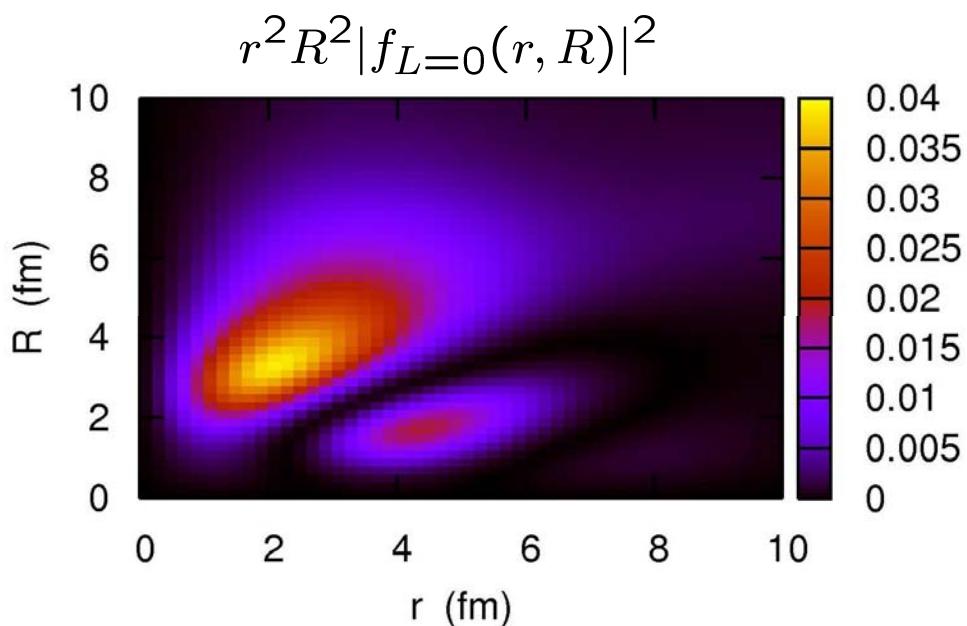
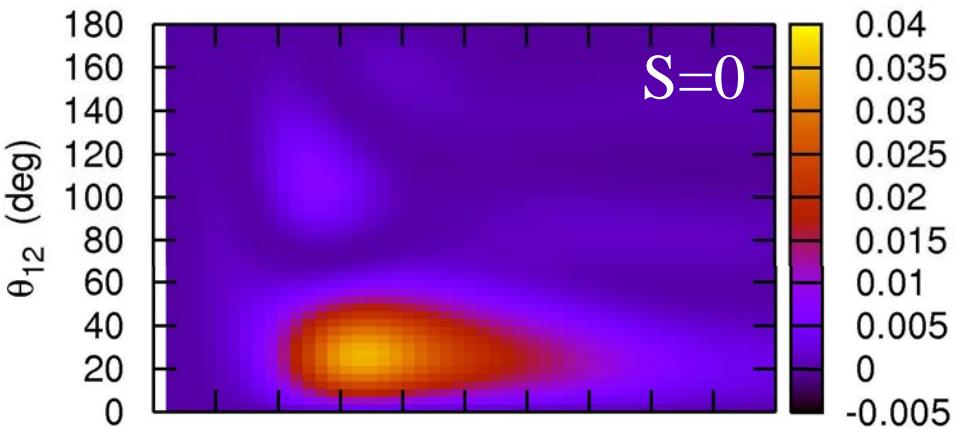
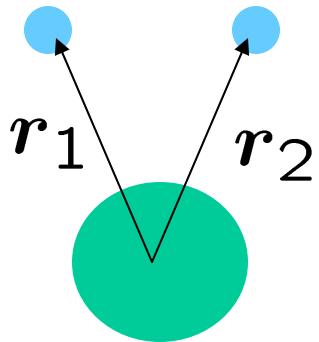
$$P_L = \int r^2 dr \int R^2 dR |f_L(r, R)|^2$$

$P_{L=0} = 0.578$	
$P_{L=2} = 0.020$	
$P_{L=4} = 0.00452$	

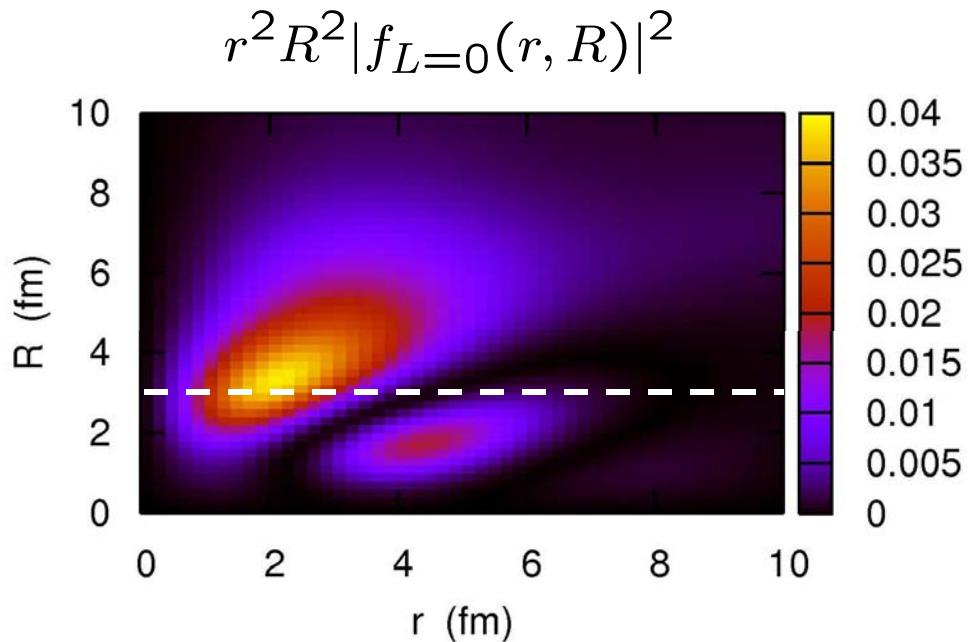
$$\text{Sum} = 0.603$$

$$\iff P_{S=0} = 0.606$$

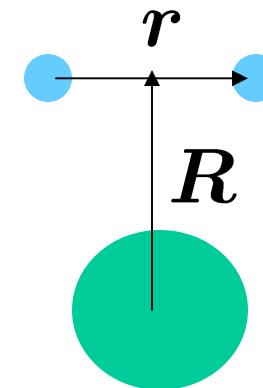
$$\Psi^{(S=0)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_L f_L(r, R) [Y_L(\hat{\mathbf{r}}) Y_L(\hat{\mathbf{R}})]^{(00)}$$



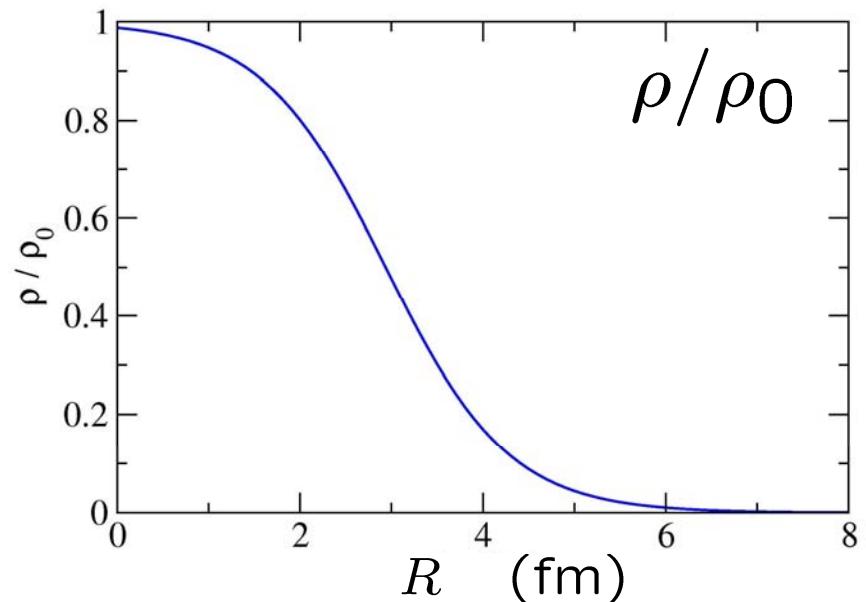
Plot the wf at several values of R



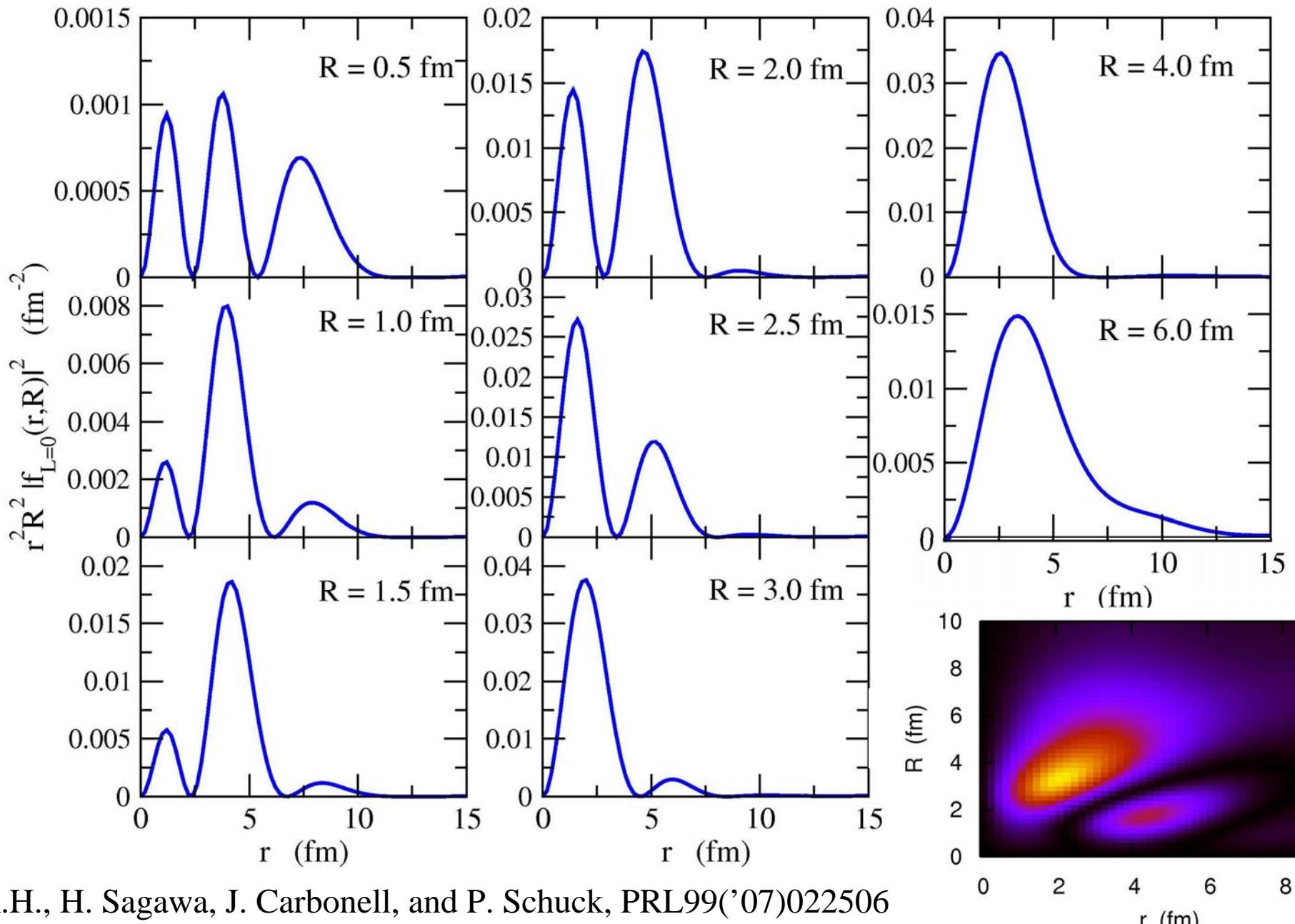
Probing the behavior
at several densities



$$v_{nn} = F[\rho(R)] \cdot \delta(r)$$



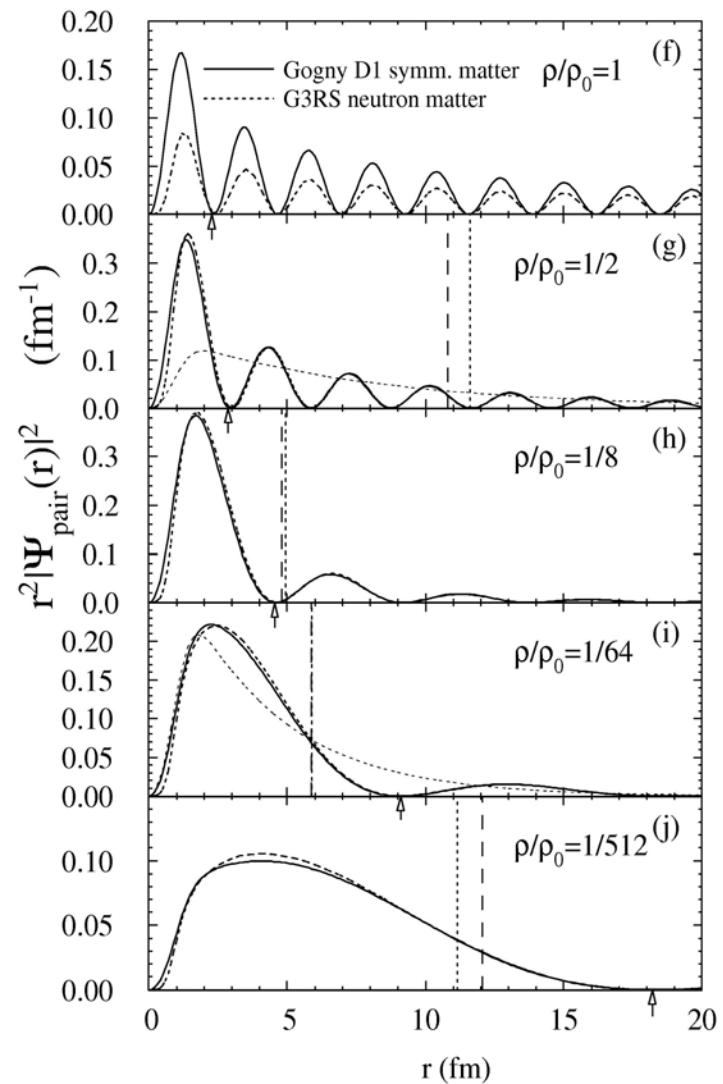
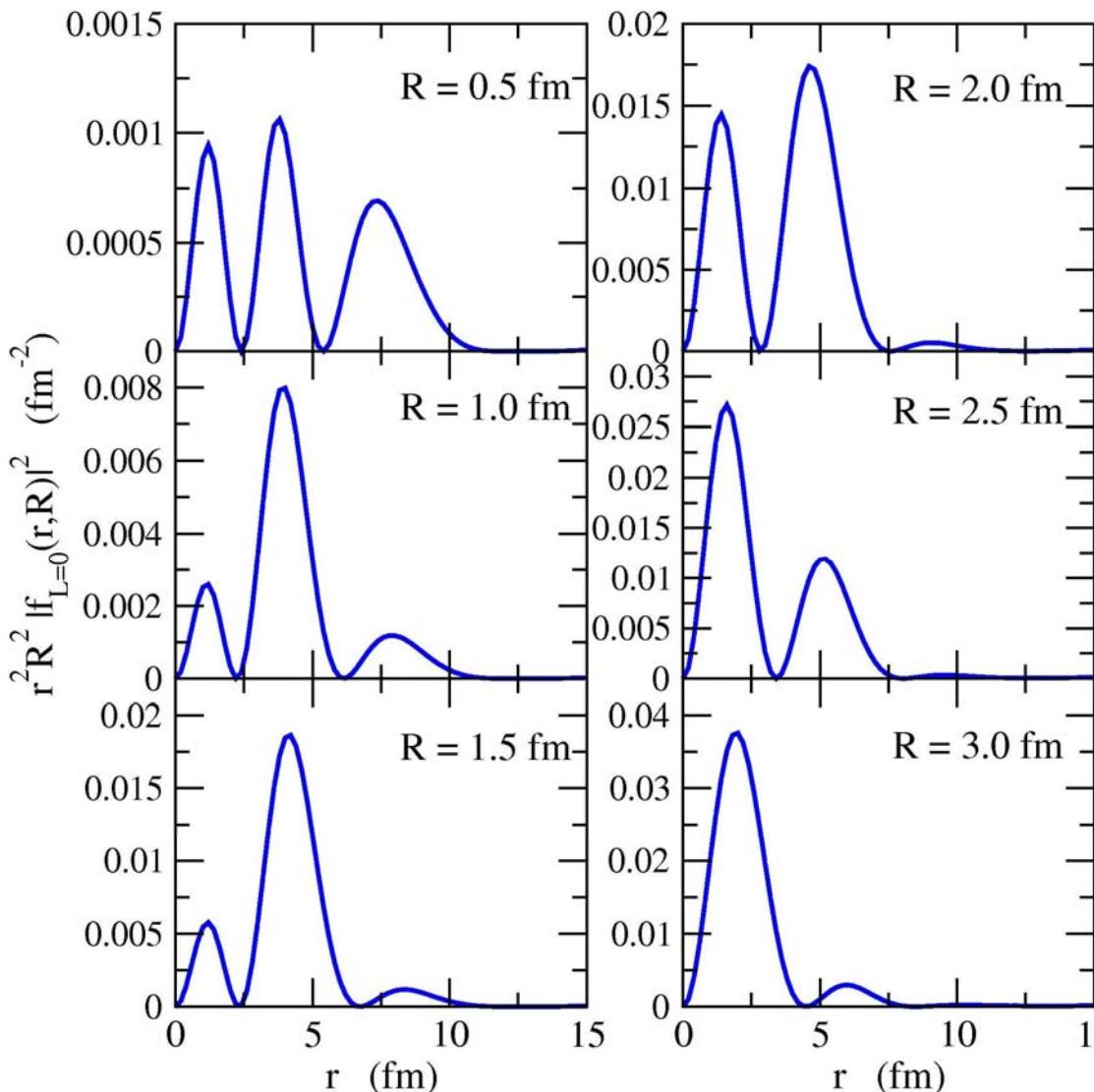
$$r^2 R^2 |f_{L=0}(r, R)|^2$$



^{11}Li

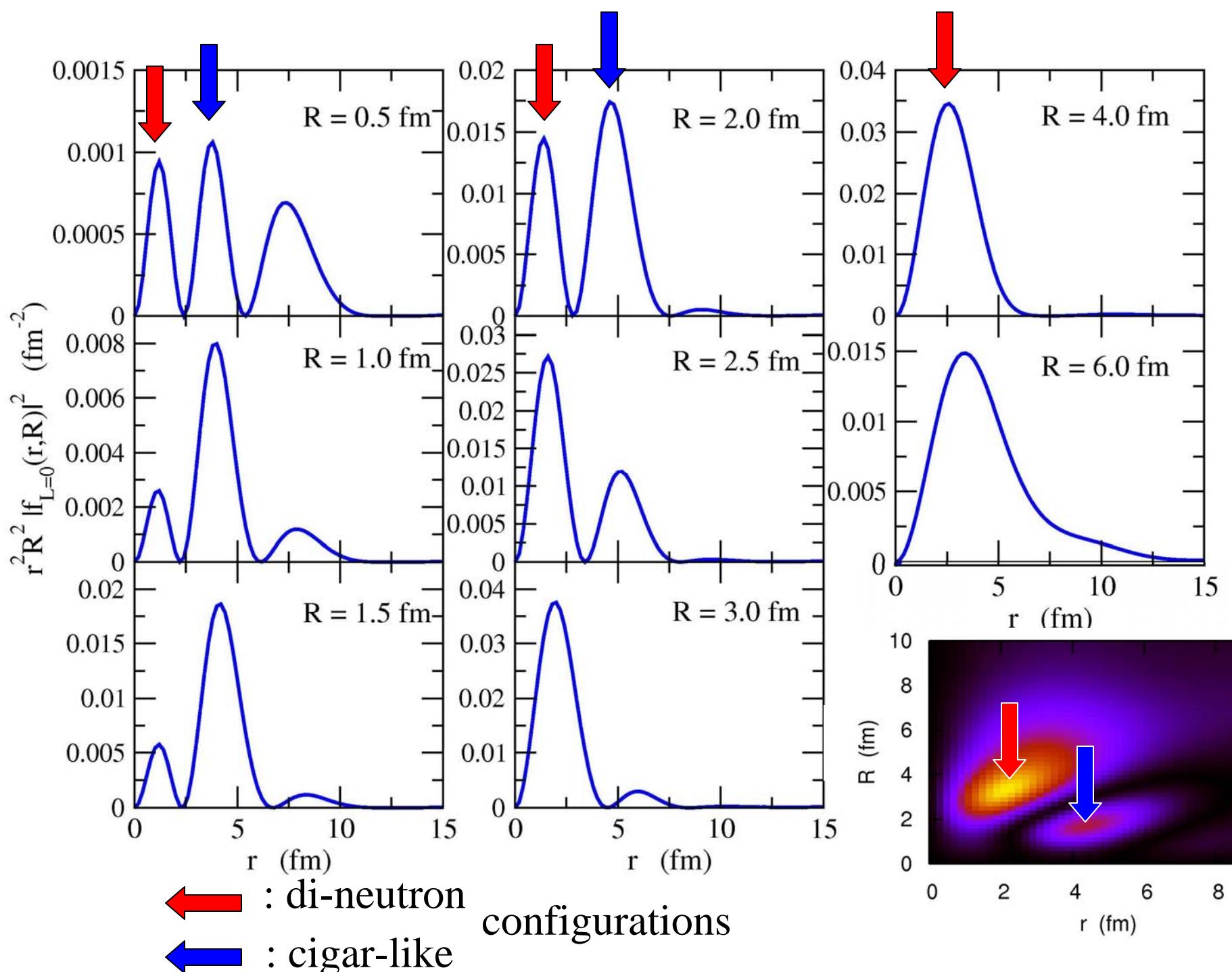
good correspondence

Nuclear Matter Calc.



K.H., H. Sagawa, J. Carbonell, and P. Schuck,
PRL99('07)022506

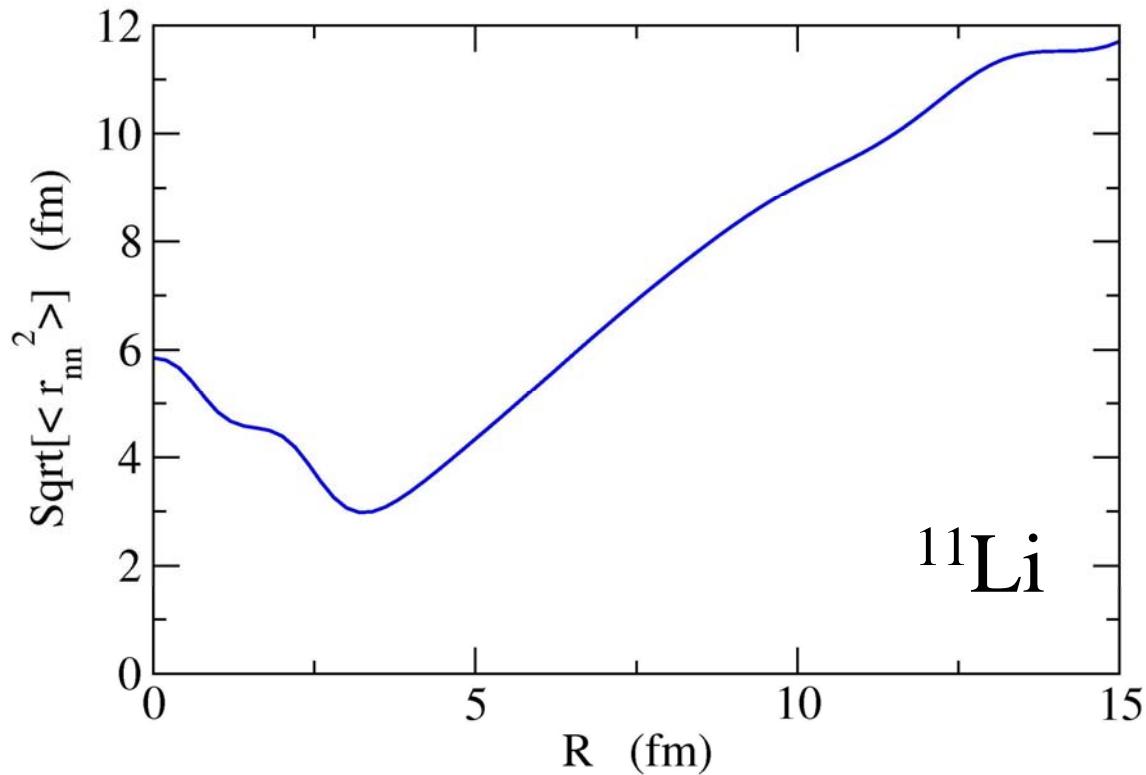
M. Matsuo, PRC73('06)044309



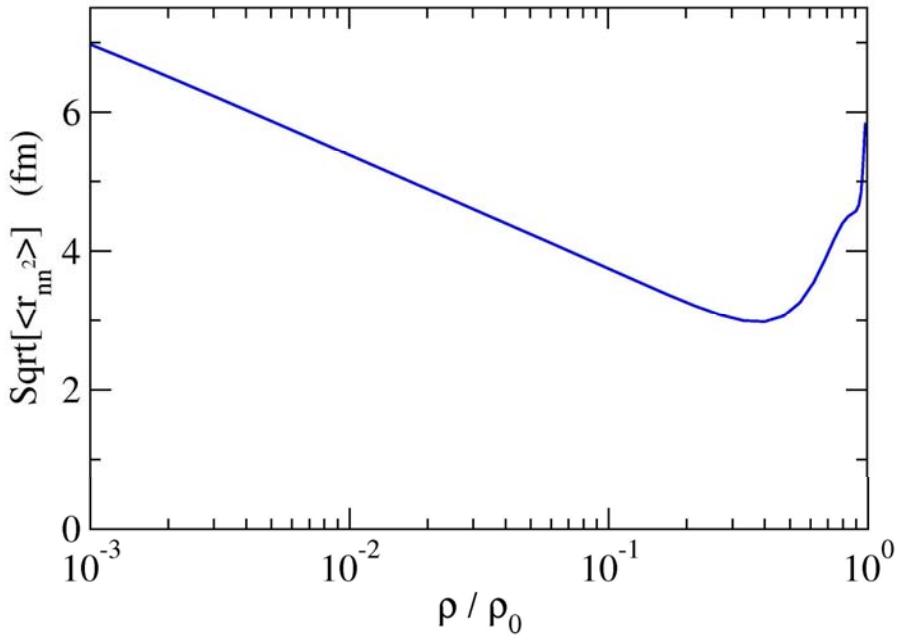
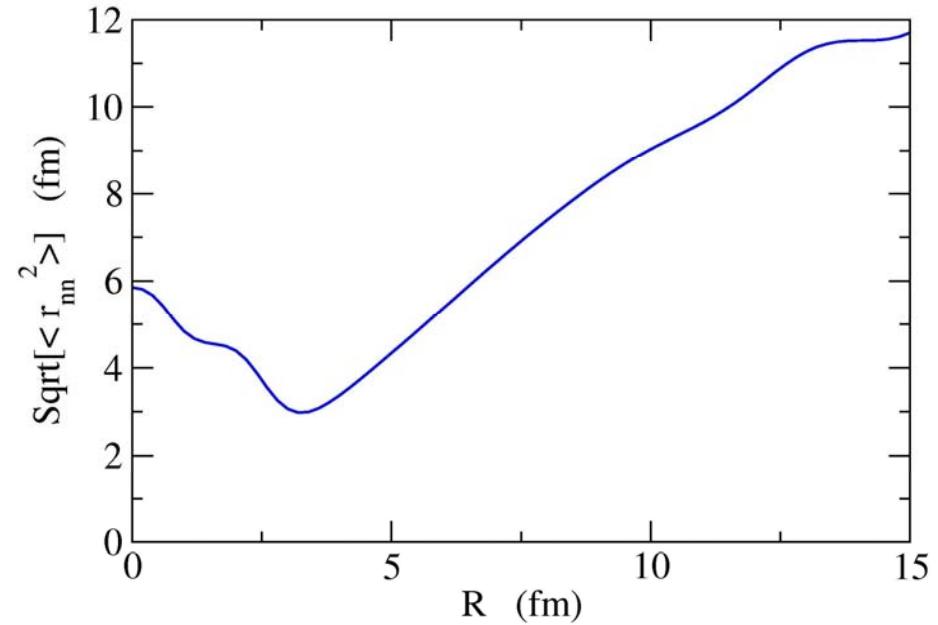
2-neutron rms distance

$$\sqrt{\langle r_{nn}^2 \rangle}(R) = \sqrt{\frac{\int r^4 dr |f_{L=0}(r, R)|^2}{\int r^2 dr |f_{L=0}(r, R)|^2}}$$

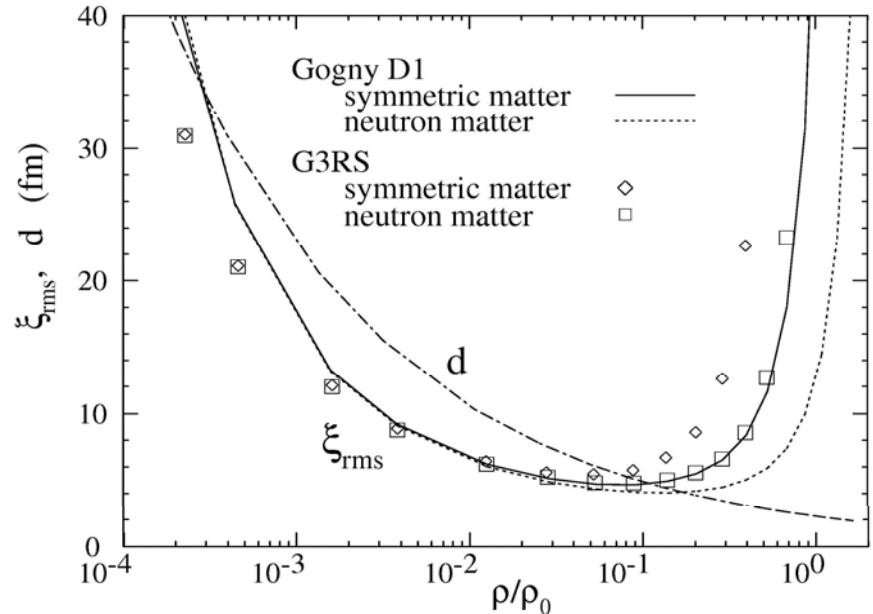
$$\Psi^{(S=0)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_L f_L(r, R) [Y_L(\hat{\mathbf{r}}) Y_L(\hat{\mathbf{R}})]^{(00)}$$



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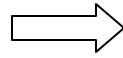


Matter Calc.



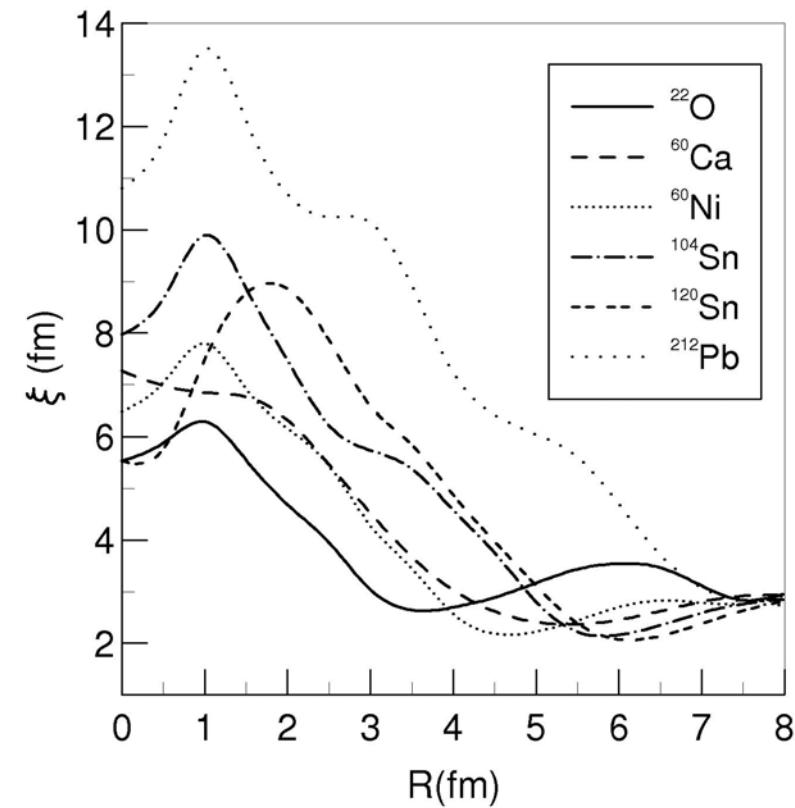
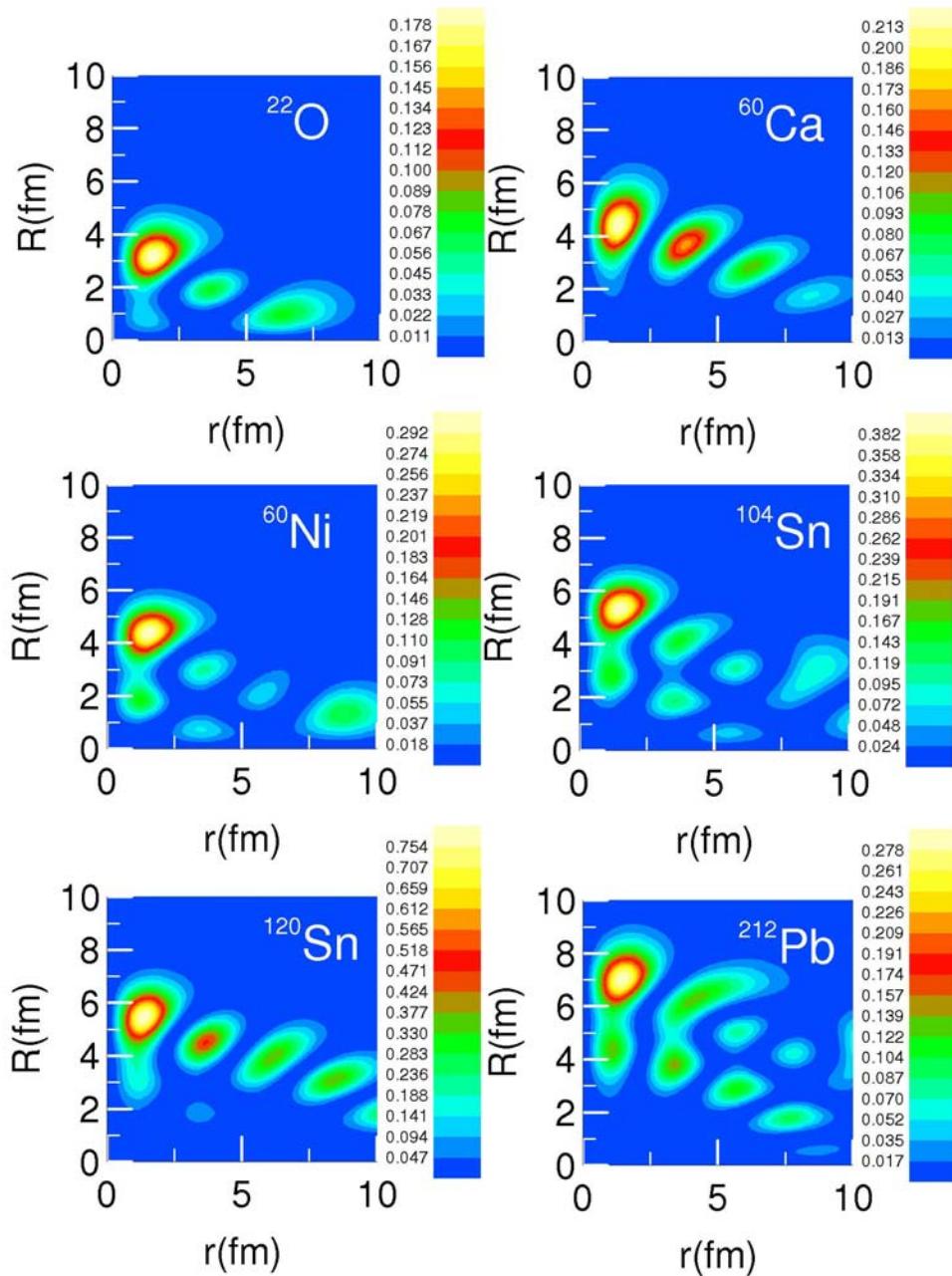
M. Matsuo, PRC73('06)044309

cf. Free n-n system
virtual state around zero energy



$\langle r \rangle \sim 12$ fm
(Nijmegen potential)

Gogny HFB calculations



N. Pillet, N. Sandulescu,
and P. Schuck,
PRC76('07)024310

Di-neutron correlation in ${}^8\text{He}$

${}^8\text{He}(0^+)$

M.V. Zhukov et al., PRC50('94)R1

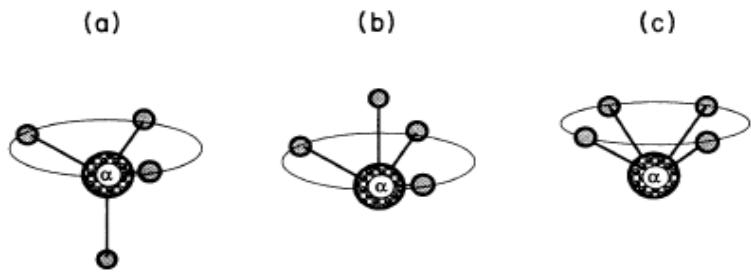


FIG. 2. Three configurations with maximum probability for the angular part of the spatial correlation function [(6)] are shown.

(1p_{3/2})⁴ configuration in H.O. pot.



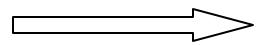
lack of continuum couplings
(mixing of several ang. mom.)

$\alpha + 4n$ model (see also Varga, Suzuki, Ohbayasi, PRC50('94)189)

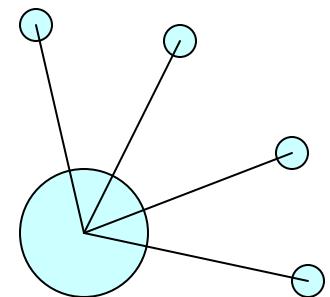
$$H = \sum_{i=1}^4 \left(\frac{\mathbf{p}_i^2}{2m} + V_{n\alpha}(\mathbf{r}_i) \right) + \sum_{i < j} v_{nn}(\mathbf{r}_i - \mathbf{r}_j)$$

$V_{n\alpha}$: Woods-Saxon

v_{nn} : density-dependent contact force



Hartree-Fock-Bogoliubov (HFB) + PNP (VBP)



Hartree-Fock-Bogoliubov approximation

quasi-particle operators: $\beta_k^\dagger = \sum_l (U_{lk} c_l^\dagger + V_{lk} c_l)$

$$\longrightarrow |\text{HFB}\rangle = \prod_k \beta_k |0\rangle \quad \boxed{\begin{pmatrix} \hat{h} - \lambda & \Delta(r) \\ \Delta(r) & -\hat{h} + \lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}}$$

For $H = \sum_{i=1}^4 \left(\frac{\mathbf{p}_i^2}{2m} + V_{n\alpha}(\mathbf{r}_i) \right) + \sum_{i < j} v_0 \left(1 - \frac{\rho_t(\mathbf{r})}{\rho_0} \right) \delta(\mathbf{r}_i - \mathbf{r}_j); \quad \rho_t(\mathbf{r}) = \rho_\alpha(\mathbf{r}) + \rho_{4n}(\mathbf{r})$

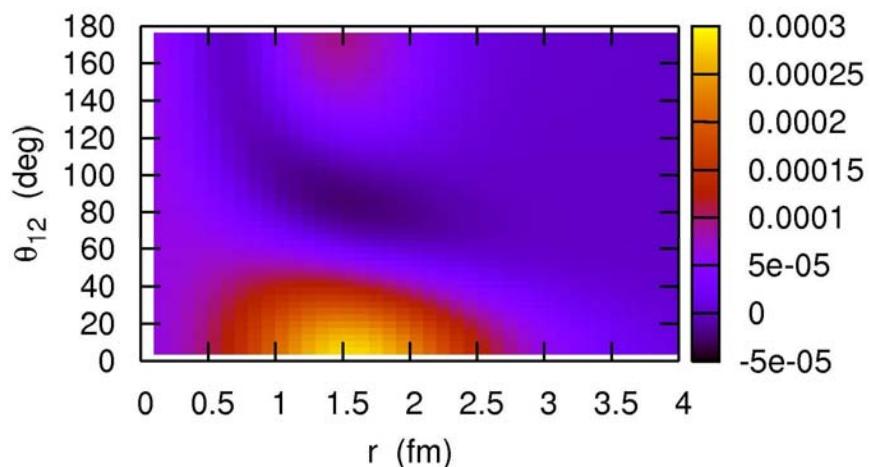


$$\hat{h} = \frac{\delta E}{\delta \rho} = -\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} v_0 \left(1 - \frac{\rho_\alpha(r)}{\rho_0} \right) \cdot \rho_{4n}(r) + V_{n\alpha}(r) \\ - \frac{3}{4} v_0 \frac{\rho_{4n}(r)^2}{\rho_0} - \frac{1}{4} v_0 \frac{\tilde{\rho}_{4n}(r)^2}{\rho_0}$$

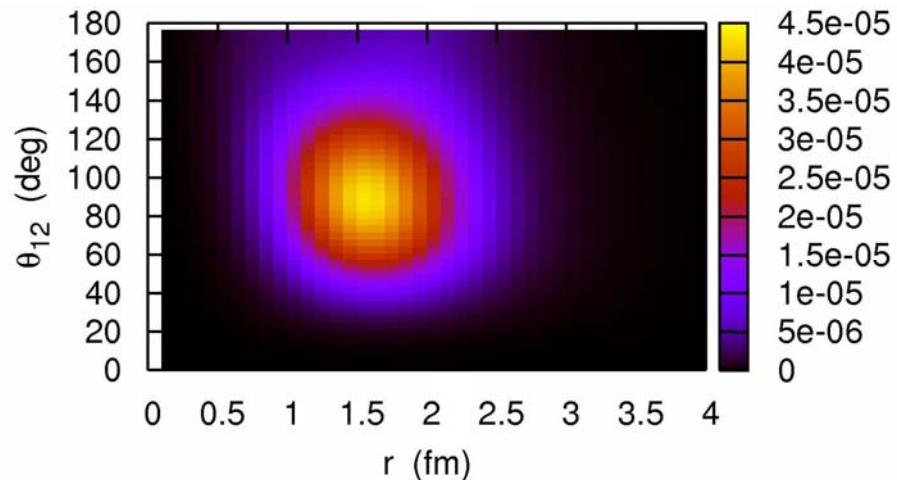
$$\Delta(r) = \frac{\delta E}{\delta \tilde{\rho}} = \frac{1}{2} v_0 \left(1 - \frac{\rho_t(r)}{\rho_0} \right) \cdot \tilde{\rho}_{4n}(r)$$

$$\rho_{4n}(\mathbf{r}) = \sum_k V_k(\mathbf{r}) V_k^*(\mathbf{r}); \quad \tilde{\rho}_{4n}(\mathbf{r}) = - \sum_k V_k(\mathbf{r}) U_k^*(\mathbf{r})$$

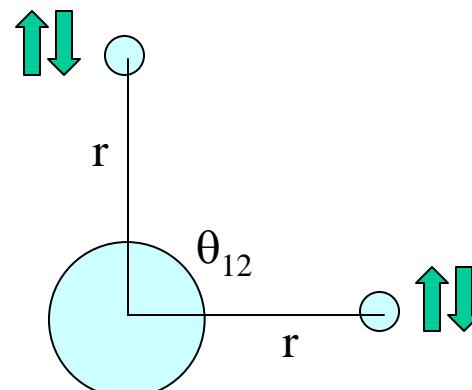
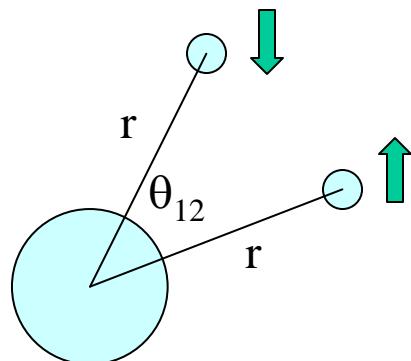
two-body correlation density



4 particle density (dineutron-dineutron configuration)

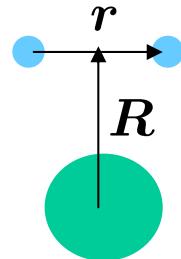
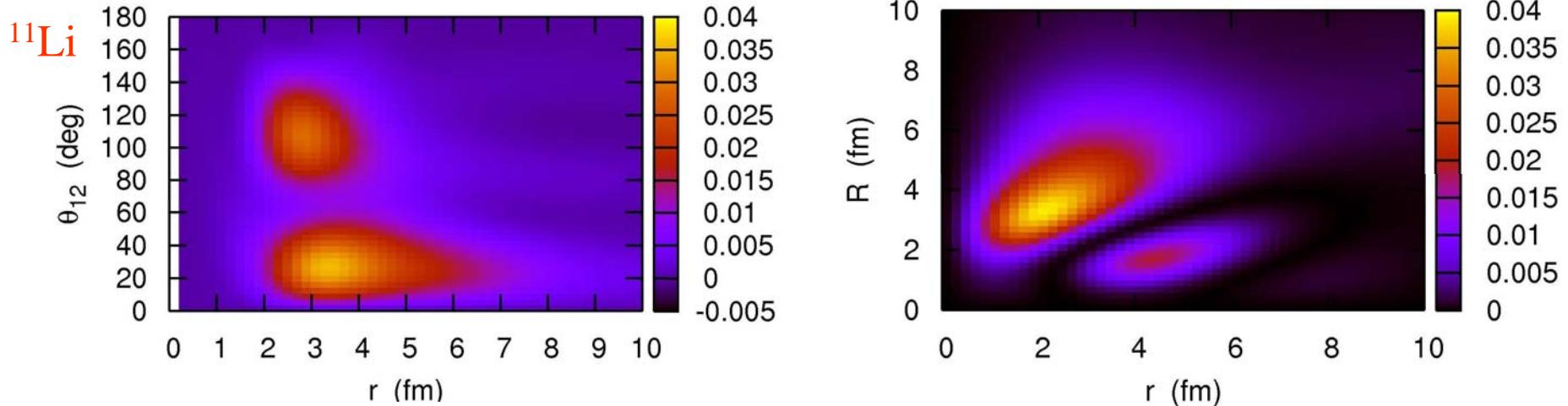


$$\rho_2(r, r') \sim \langle \text{HFB} | c_{\mathbf{r}\uparrow}^\dagger c_{\mathbf{r}'\downarrow}^\dagger c_{\mathbf{r}'\downarrow} c_{\mathbf{r}\uparrow} | \text{HFB} \rangle, \quad \rho_4(r, r') = \langle \text{HFB} | c_{\mathbf{r}\uparrow}^\dagger c_{\mathbf{r}\downarrow}^\dagger c_{\mathbf{r}'\uparrow}^\dagger c_{\mathbf{r}'\downarrow}^\dagger \\ \times c_{\mathbf{r}'\downarrow} c_{\mathbf{r}'\uparrow} c_{\mathbf{r}\downarrow} c_{\mathbf{r}\uparrow} | \text{HFB} \rangle$$



Summary

➤ Application of three-body model to Borromean nuclei



➤ E1 response and geometry of Borromean nuclei

➤ Di-neutron wave function for each R

- Close correspondence to the matter calculations
- BCS/BEC crossover phenomenon
- Concentration of a Cooper pair on the nuclear surface
- Also in other superfluid nuclei (universality)

➤ Strong di-neutron correlation in ^8He