Alpha-cluster states and 4α particle condensation in ^{16}O

Yasuro Funaki (RIKEN)

Taiichi Yamada (Kanto Gakuin Univ.)

Peter Schuck (IPN, Orsay, Paris-Sud Univ.)

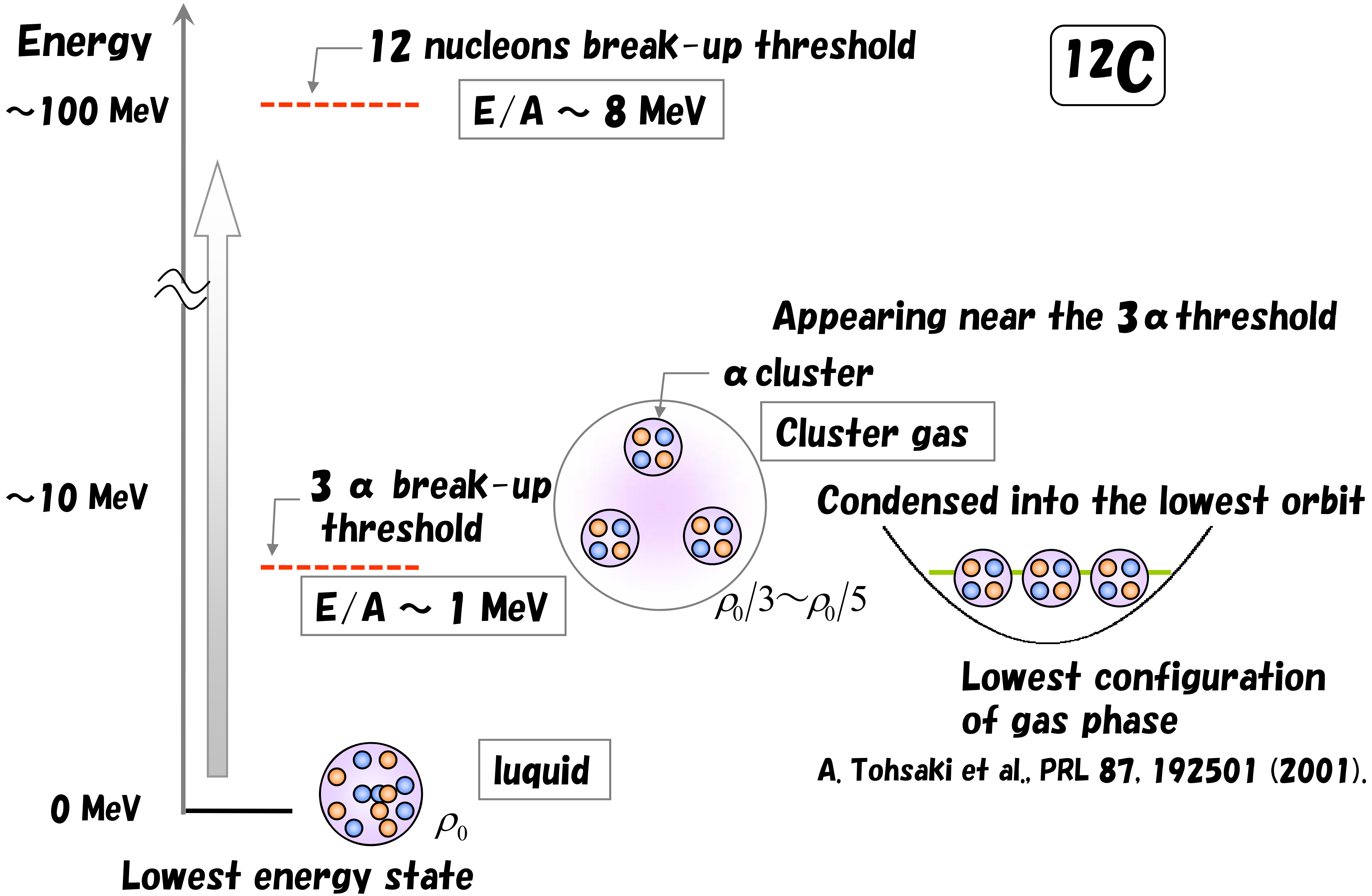
Hisashi Horiuchi (RCNP)

Akihiro Tohsaki (RCNP)

Gerd Röpke (Rostock Univ.)

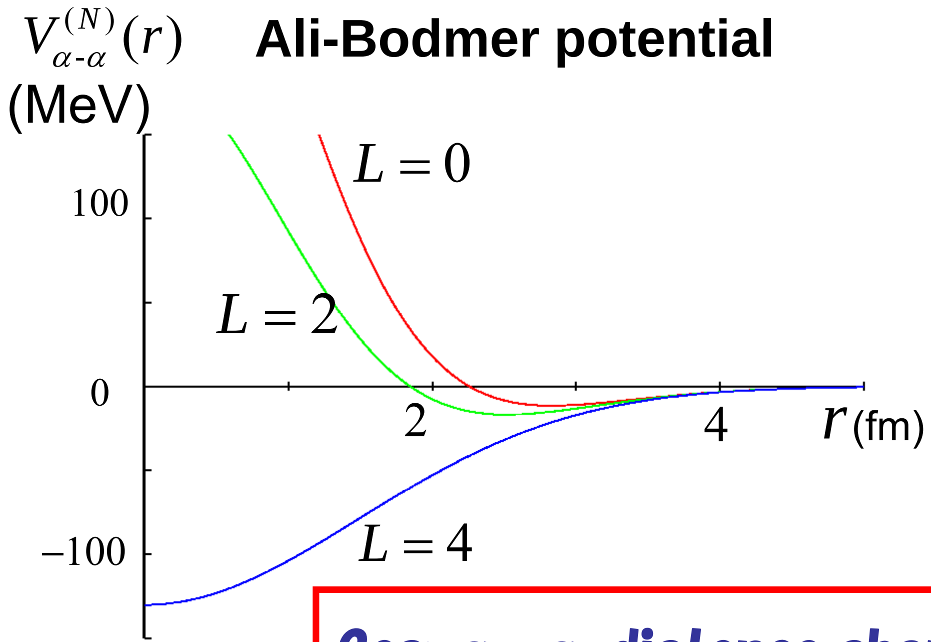
INT Workshop on ``Correlations in Nuclei: From Di-Nucleons to Clusters”, University of Washington, Seattle, USA, 26 - 29 November, 2007

Appearing of cluster gas state and "BEC" state in finite nuclei



α－α interaction range and potentials α particles feel in the gas states

Ali-Bodmer potential

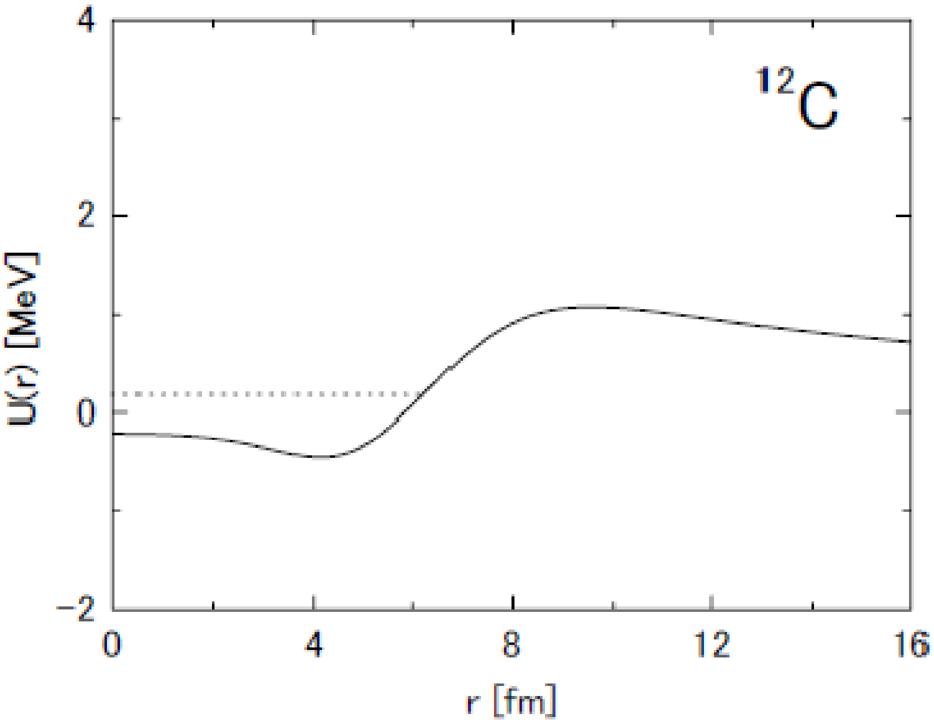


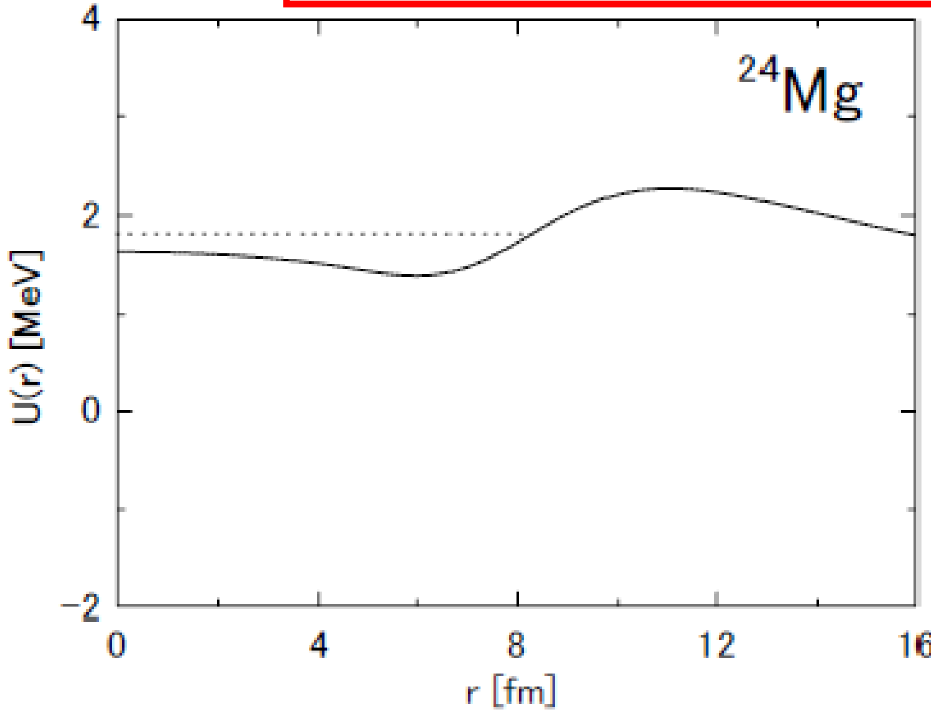
Coulomb barrier

Position is outside the α－α interaction range(～4 fm)

Stabilization of α condensate state

Gas: α－α distance should be more than 4 fm





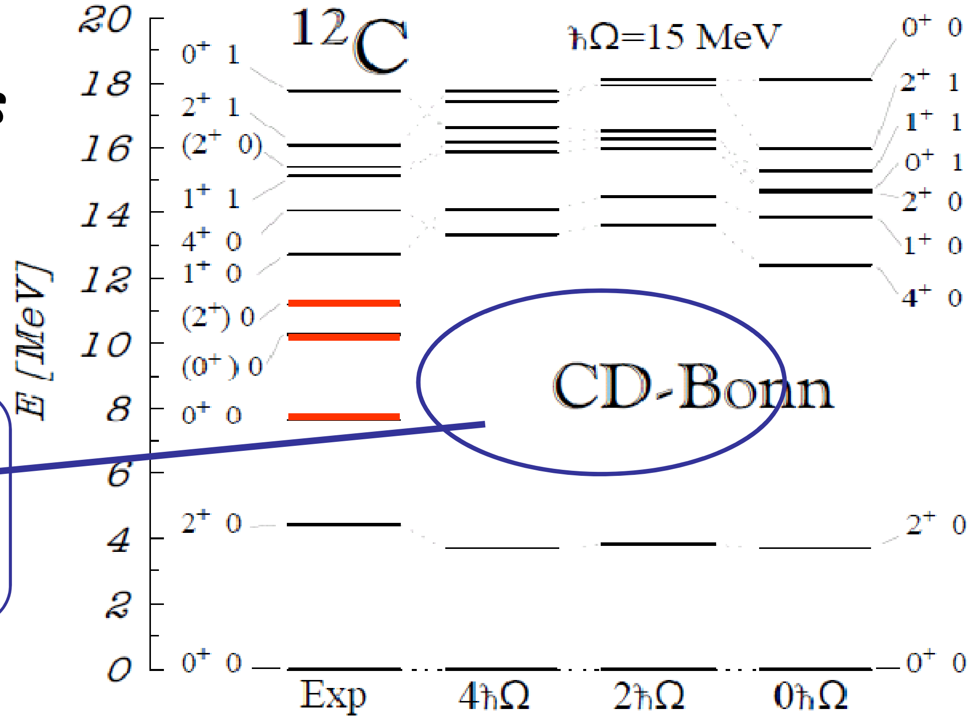
Gross-Pitaevsky eq.

T. Yamada and P. Schuck, PRC 69, 024309 (2004)

0_2^+ state of ^{12}C (Hoyle state)

P. Navratil et al., Phys. Rev. Lett. 84 (2000), 5728.

Ab initio non-core shell model calculation



- Importance for ^{12}C synthesis in stars
- Typical mysterious 0^+ state

(One of the typical excited states which resist a shell model description)

0_2^+ state : missing
(excitation energy is not lower than 20 MeV)

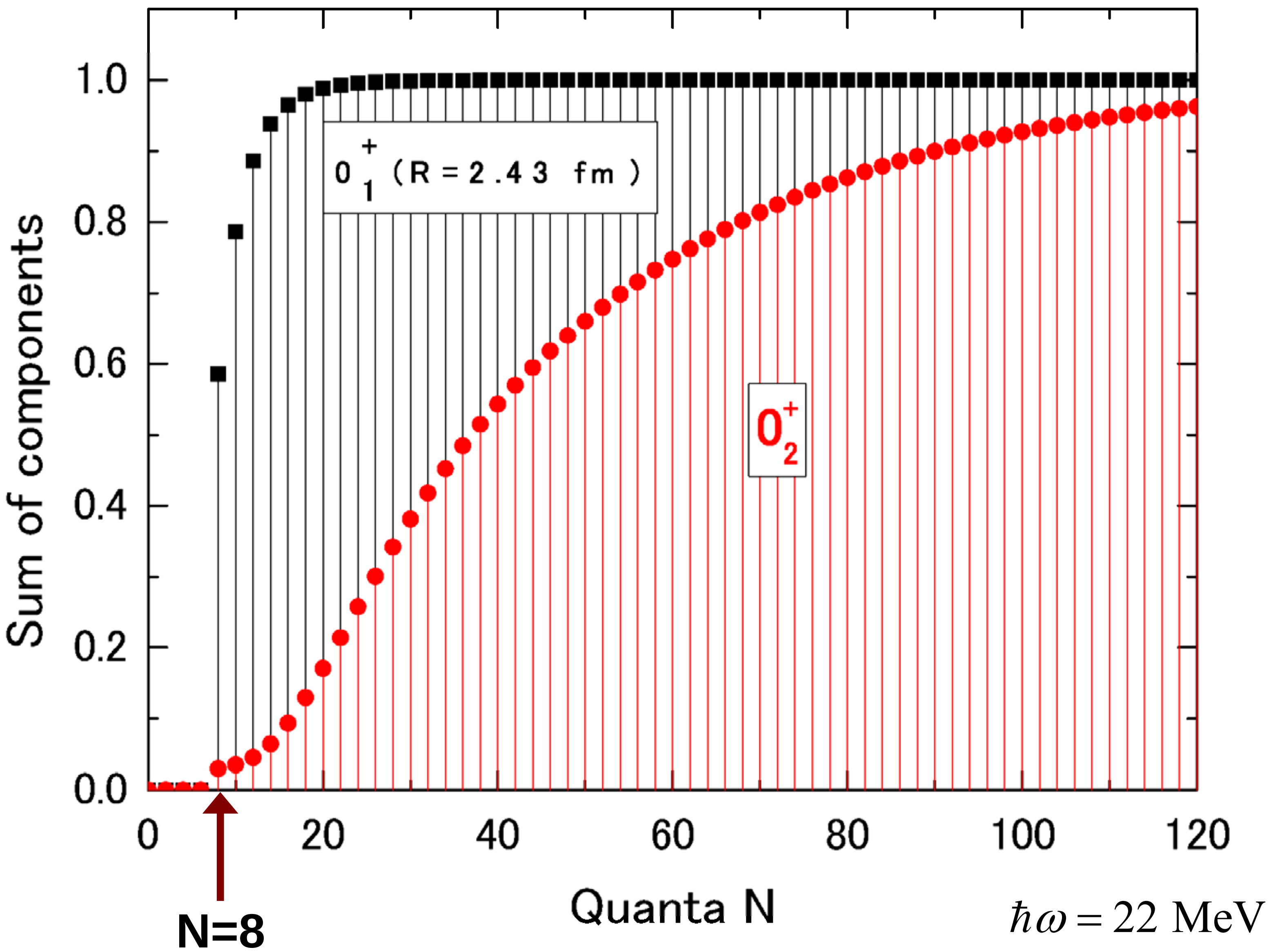
- 3α problem is fully solved about 30 years ago.
- Most of experimental data are nicely reproduced. (Kamimura et al. (RGM), Uegaki et al. (GCM))

	Exp.	Theor.
Energy (MeV)	7.65	7.74
α decay width (eV)	8.7±2.7	7.7
$M(0_2^+ \rightarrow 0_1^+)$ (fm^2)	5.4±0.2	6.7
$B(E2: 0_2^+ \rightarrow 2_1^+)$ (e^2 fm^4)	13±4	5.6

3α Resonating Group Method (RGM)

$$\langle \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3) | (H-E) | \mathcal{A}\{\chi(\vec{s},\vec{r})\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}\rangle = 0$$

Expansion of 0^+_1 and 0^+_2 wfs with H.O. basis



The expansion was done wrt relative motions of α's.

Calculated by T. Yamada

$n\alpha$ condensate wave function (THSR-w.f.)

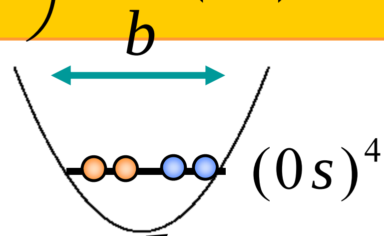
$$\Phi_{n\alpha}(\beta,b)=\mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}\vec{X}_i^2\right)\phi(\alpha_i)\right)\right\} \quad (B^2=b^2+2R_0^2)$$

$$\propto \left\langle \vec{r}_1 i_1,\cdots,\vec{r}_{4n} i_{4n} \middle| \left(C_\alpha^\dagger\right)^n \middle| \mathrm{VAC} \right\rangle$$

$$C_\alpha^\dagger=\int d^3\vec{R}\exp\left(-\frac{R^2}{R_0^2}\right)B_\alpha^\dagger\left(\vec{R}\right)$$

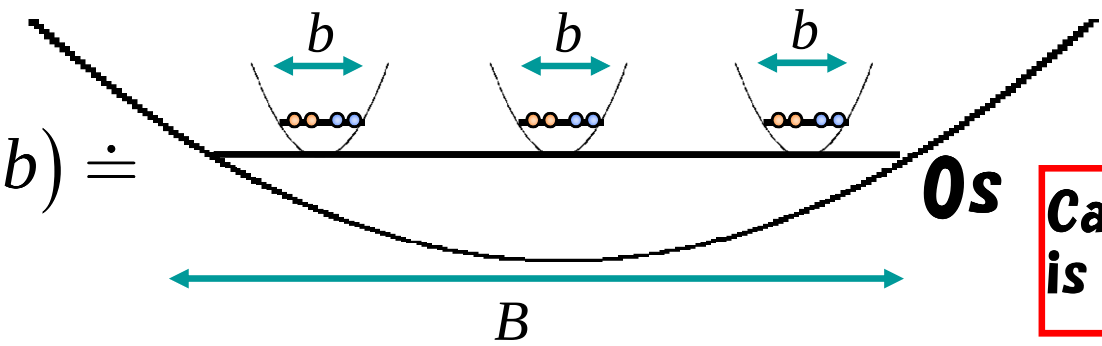
Brink's wave function

$$\phi(\alpha)\propto\left\langle \vec{r}_1 i_1,\cdots,\vec{r}_4 i_4 \middle| B_\alpha^\dagger(\vec{R}) \middle| \mathrm{VAC} \right\rangle \doteq$$



$(0s)^4$ configuration around $\vec{R}$

$$\Phi_{n\alpha}(R_0,b)\doteq$$



Calculation of matrix elements is owing to Tohsaki's technique

Hill-Wheeler equation (R_0: generator coordinate, b: fixed)

$$\sum_{R_0'}\left\langle \Phi_{n\alpha}(R_0,b)\middle|H-E^\lambda\middle|\Phi_{n\alpha}\left(R_0',b\right)\right\rangle f_{R_0'}^\lambda=0 \qquad \Psi_{n\alpha}^\lambda=\sum_{R_0} f_{R_0}^\lambda \Phi_{n\alpha}(R_0,b)$$

The comparison of 3α condensate model (H.W.) with microscopic 3α model (3α RGM)

Volkov No.2 force: M=0.59 $E_{3\alpha}^{\text{ths}} = -82.04\ \text{MeV}$

Calculated values of r.m.s radius (fm) and monopole matrix element (fm²)

Calculated values of binding energy (MeV)

	H. W.	3α RGM
0_1^+	-89.52	-89.4
0_2^+	-81.79	-81.7
2_1^+	-86.71	-86.7

	H. W.	3α RGM
r.m.s radius 0_1^+	2.40	2.40
r.m.s radius 0_2^+	3.83	3.47
r.m.s radius 2_1^+	2.38	2.38
$M\,(0_2^+ \to 0_1^+)$	6.45	6.7

First example of α condensate state in finite nuclei

3α break-up threshold : 7.27 MeV

Hoyle state (0_2^+ state in ^{12}C (excitation energy : 7.65 MeV))

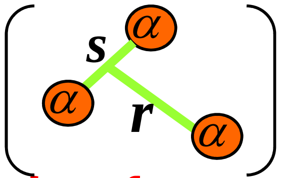
Indicating 3α condensate character

Microscopic approach (3α cond. model w.f.)

The Solution of 3α RGM eq. of motion, RGM

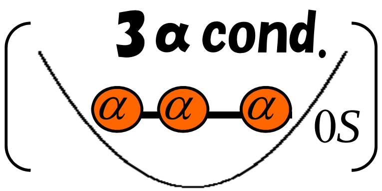
$$\left\langle \phi^3(\alpha) \middle| H - E \middle| \mathcal{A}\left[\chi(\mathbf{s},\mathbf{r})\phi^3(\alpha)\right] \right\rangle = 0$$

M. Kamimura, NPA **351**, 456 (1981). $\mathcal{A}$



is almost equivalent to the 3α cond. w.f.

$$\chi(\mathbf{s},\mathbf{r}) = \exp\left(-\frac{2}{B^2}\sum_{i=1}^{3}(X_i - X_G)^2\right)\mathcal{A}$$



X_i : c.o.m of α-particle

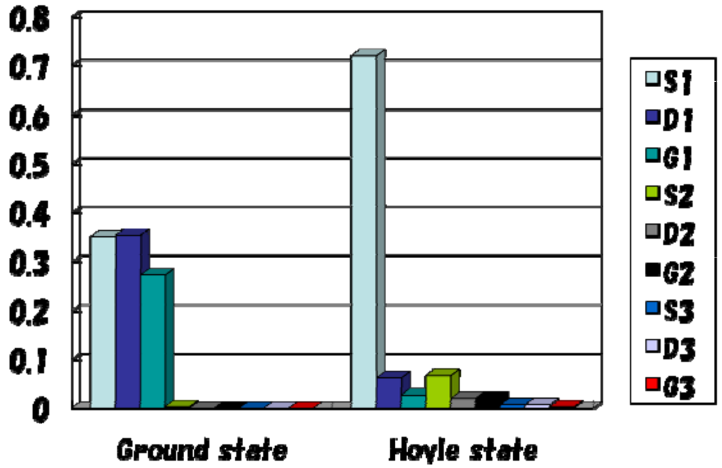
Y. F et al., PRC **67**, 051306(R) (2003).

- Occupation probability of α-particle orbit
  Huge 0S occupancy (>70 %)
- Momentum distribution
  Delta-function-like behavior

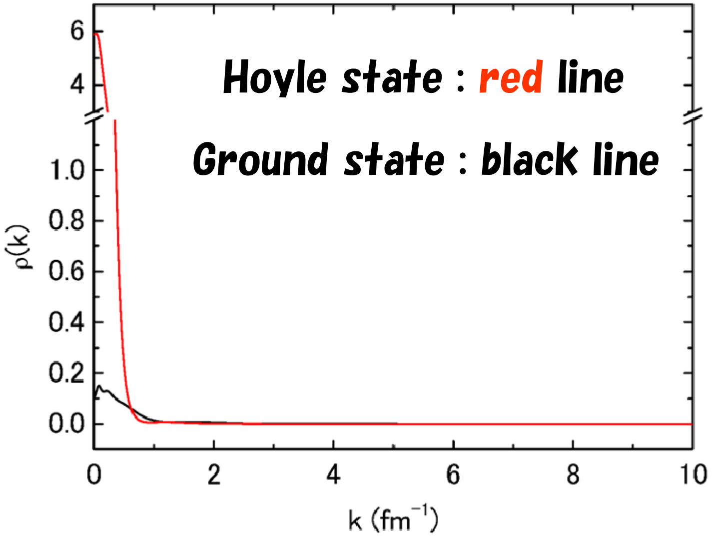
H. Matsumura and Y. Suzuki, NPA **739**, 238 (2004).

T. Yamada and P. Schuck EPJA **26**, 185 (2005).

Occupation probability of α-particle orbit



Momentum distribution of α-particle



OCM in ^{12}C

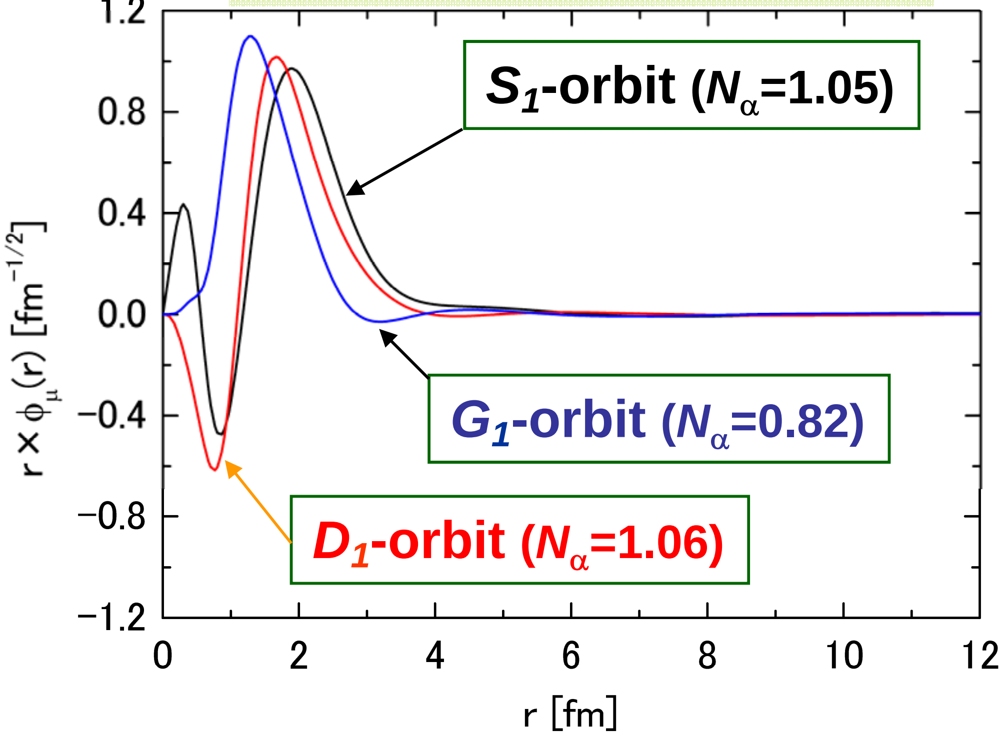
$$\int \rho(\vec{r},\vec{r}') f^{\lambda}(\vec{r}')dr = \mu^{\lambda} f^{\lambda}(\vec{r})$$

Single α-orbits in ^{12}C(0^+)

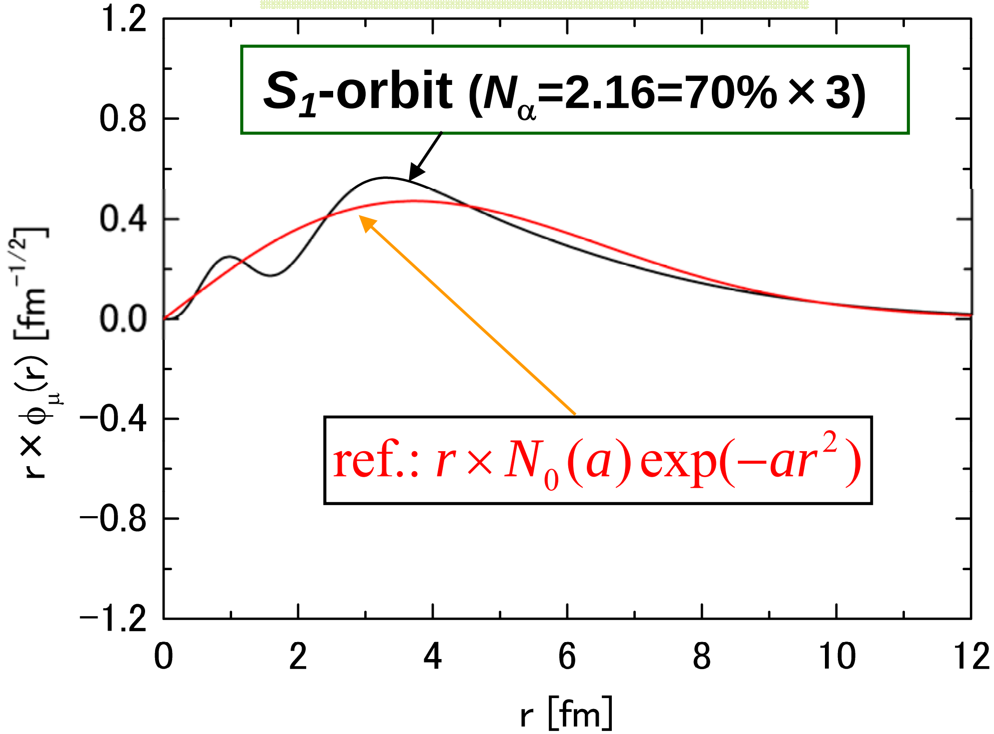
$$\mu^{\lambda}/3 = N_{\alpha}$$

N_α: Occupation probability

0_1^+ (g.s.): R_{rns}=2.44 fm



0_2^+ : R_{rms}=4.31 fm



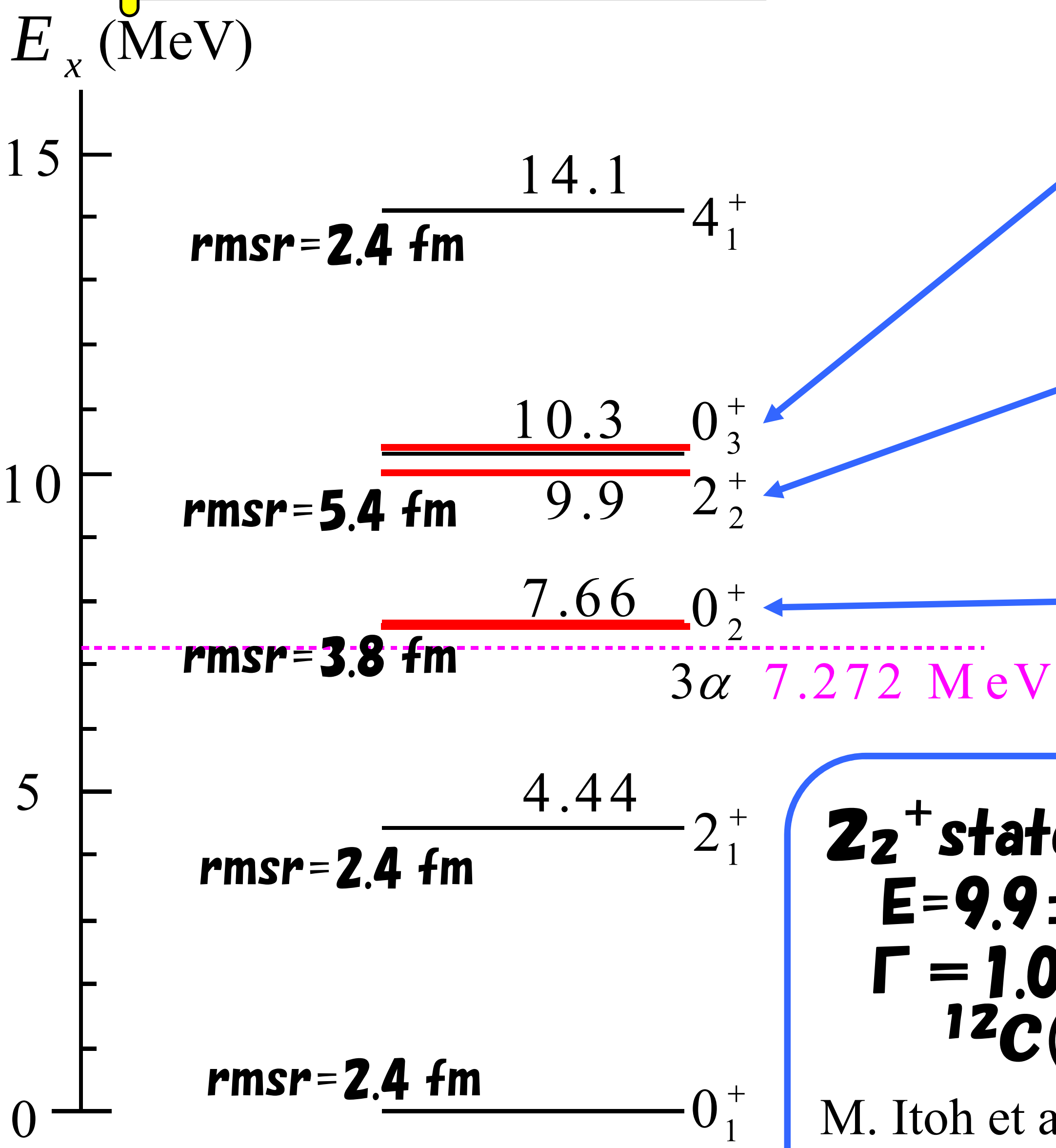
Large oscillation : **strong** Pauli blocking effect
Compact structure
SU(3) model: [f](λμ)=[444]**(04)-like structure**

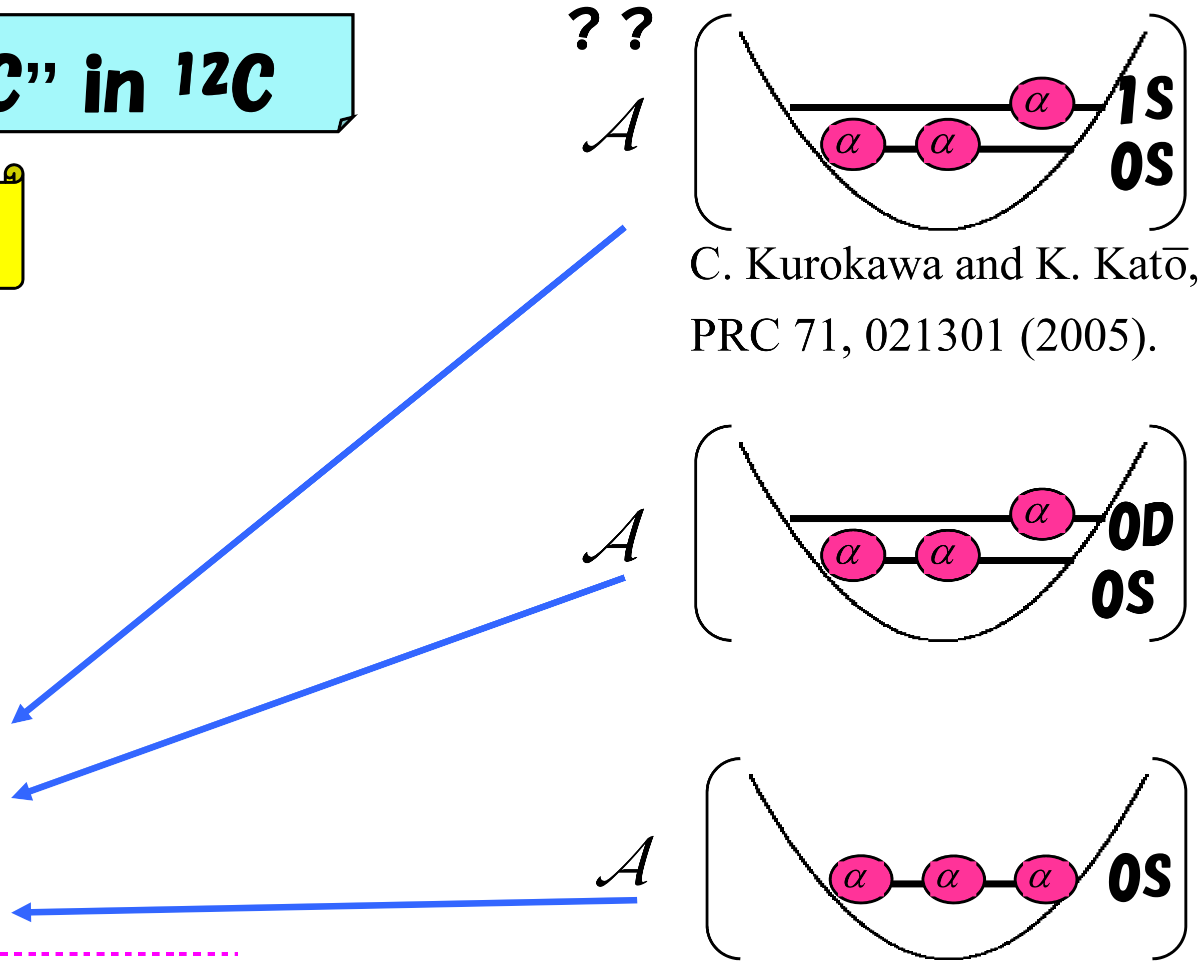
Small oscillation: **weak** Pauli blocking effect
Long tail: **dilute** structure
Radial behavior: **Gaussian** form with a= 0.04 fm^{-2}

T. Yamada and P. Schuck, Euro. Phys. Jour. A, 26 (2005), 185.

"BEC" in ^{12}C

Observed levels of ^{12}C





C. Kurokawa and K. Katō, PRC 71, 021301 (2005).

2_2^+ state :
E=9.9±0.3 MeV
Γ=1.0±0.3 MeV
$^{12}C(\alpha, \alpha')$

M. Itoh et al., Nucl. Phys. A 738 (2004) 268-272

α cond. +ACCC
E=9.38 MeV
Γ=0.64 MeV
Volkov No. 1 force is adopted

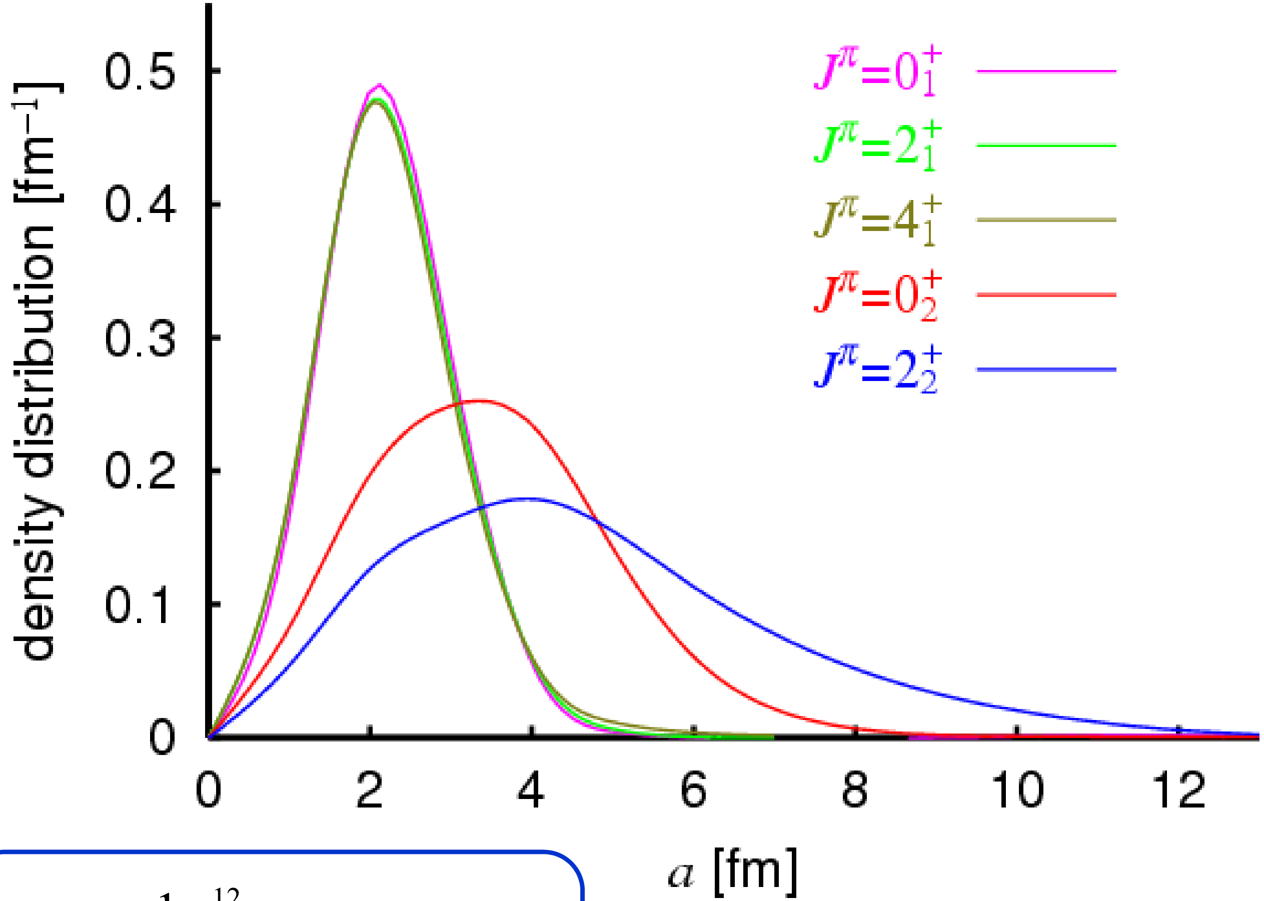
Y. F. et al., EPJA 24, 321 (2005).

Density distribution of 0_1^+, 2_1^+, 4_1^+ states (shell model structure) and 0_2^+, 2_2^+ states (gas-like structure)

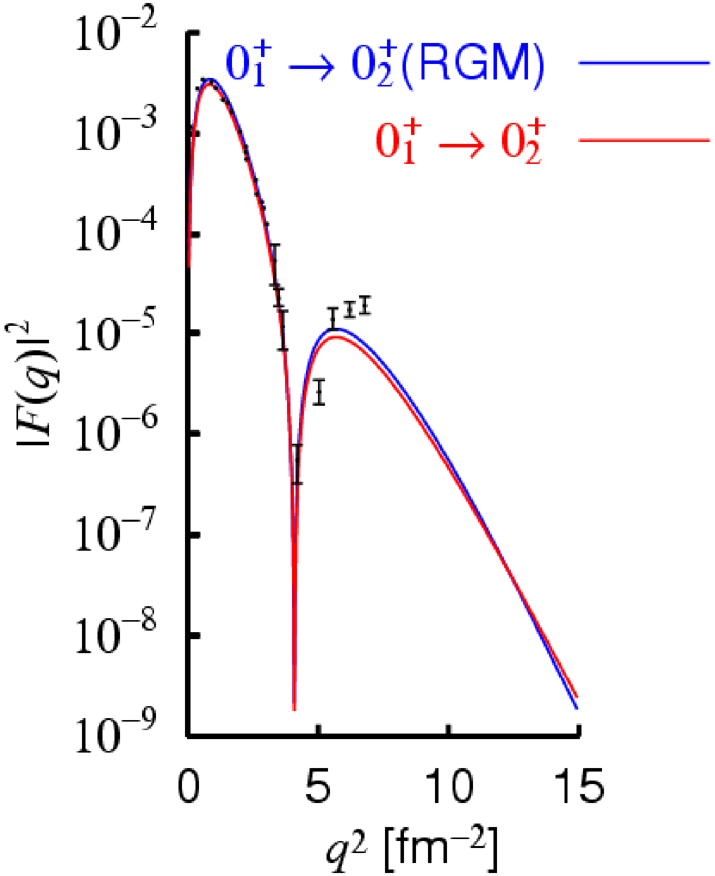
$0_1^+, 2_1^+, 4_1^+$: $R_{r.m.s.} \sim 2.4$ fm

0_2^+: $R_{r.m.s.} = 3.83$ fm ($\sim \rho_0/4$)

2_2^+: $R_{r.m.s.} = 5.4$ fm ($\sim \rho_0/10$) "α-halo state"



Form factor



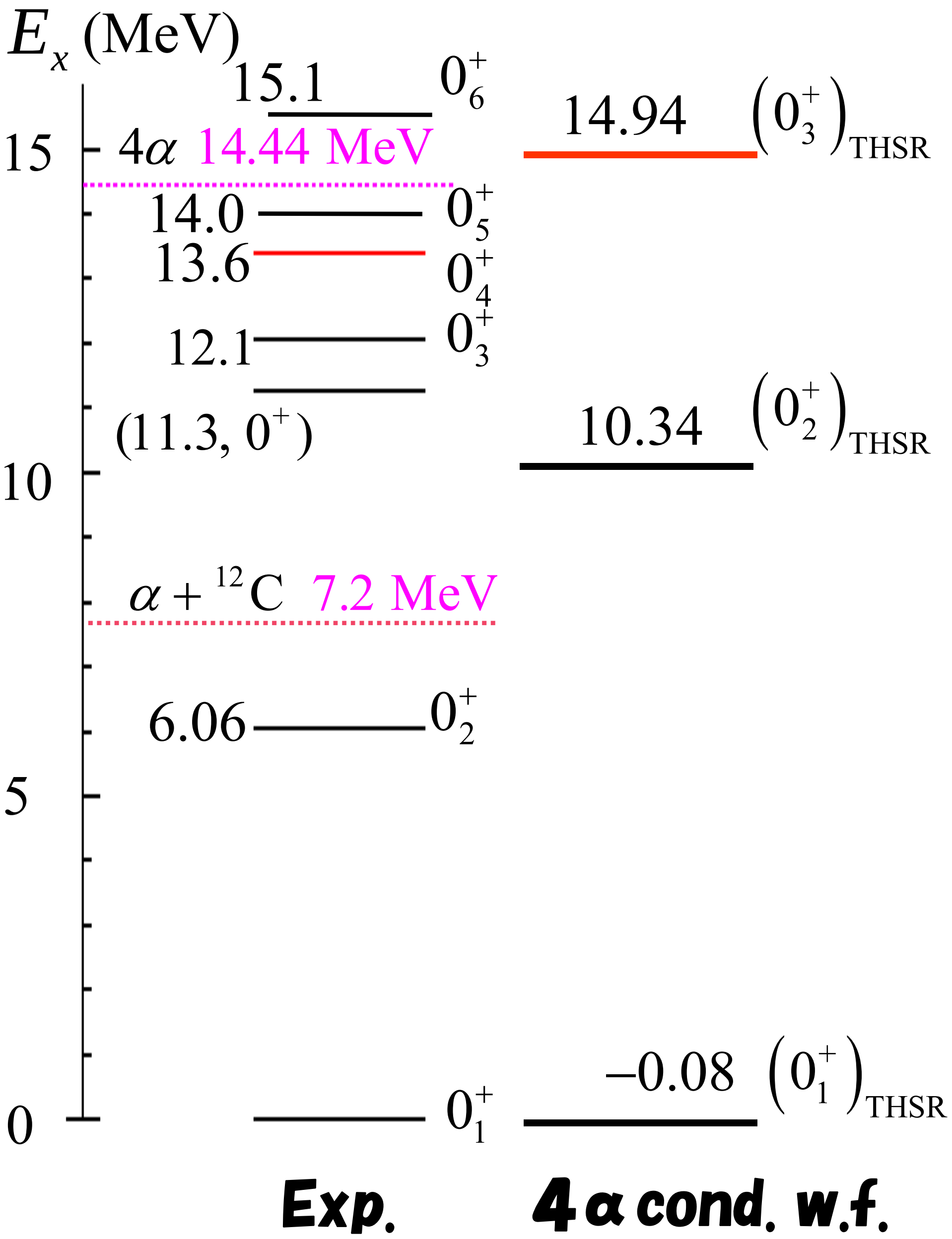
$$\rho(a) = \frac{1}{12}\sum_{i=1}^{12}\delta\left(\left|\vec{r}_i - \vec{X}_G\right| - a\right)$$

Density operator

Volkov No. 2 force is adopted.

First attempt to explore 4α condensate state in ^{16}O

Low lying 0^+ levels of ^{16}O

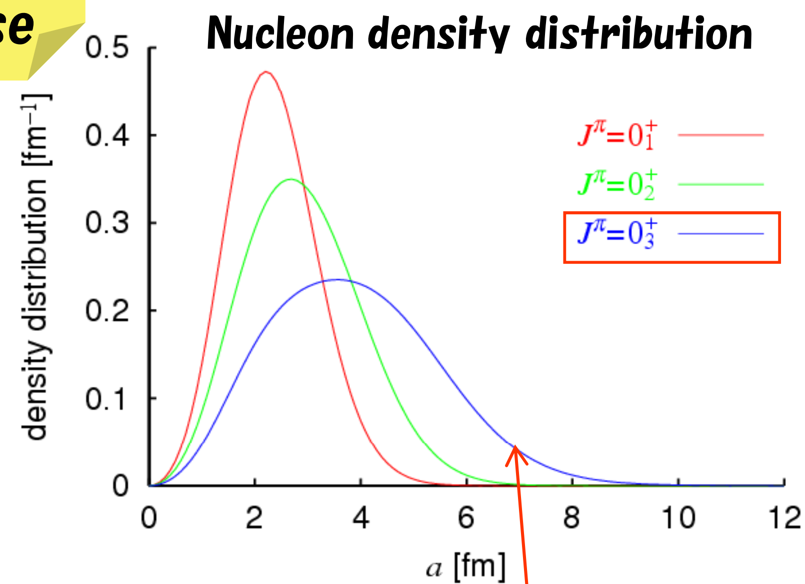


nα condensate model w.f. (THSR-w.f.)

$$\Phi_{n\alpha} = \mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}X_i^2\right)\phi(\alpha_i)\right)\right\}$$

A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke, PRL 87, 192501 (2001).

n=4 case



$(0_3^+)_{\text{THSR}}$: **Very dilute density.**
~~4 α condensate state~~

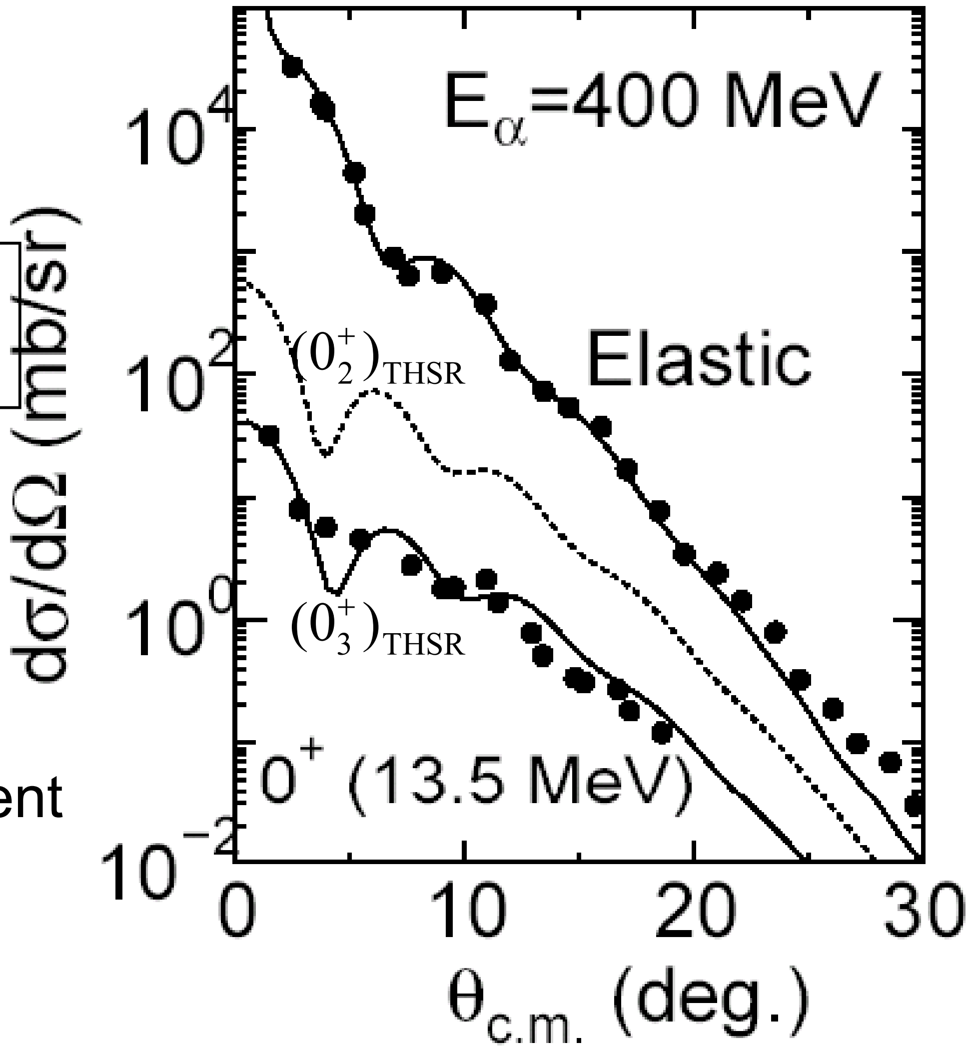
α+^{16}O inelastic scattering

0_5^+ state:
A candidate of 4α condensate
E = 13.6 MeV
Γ = 0.8 MeV
^{16}O(α, α') Wakasa et al.

The result of the calculation is consistent with the experimental data.

The 0^+ state at E_x =13.5 MeV can be assigned to the four-α condensed state.

The 0^+ state wave function obtained at E_x=10.3 MeV leads to a largely different absolute value.

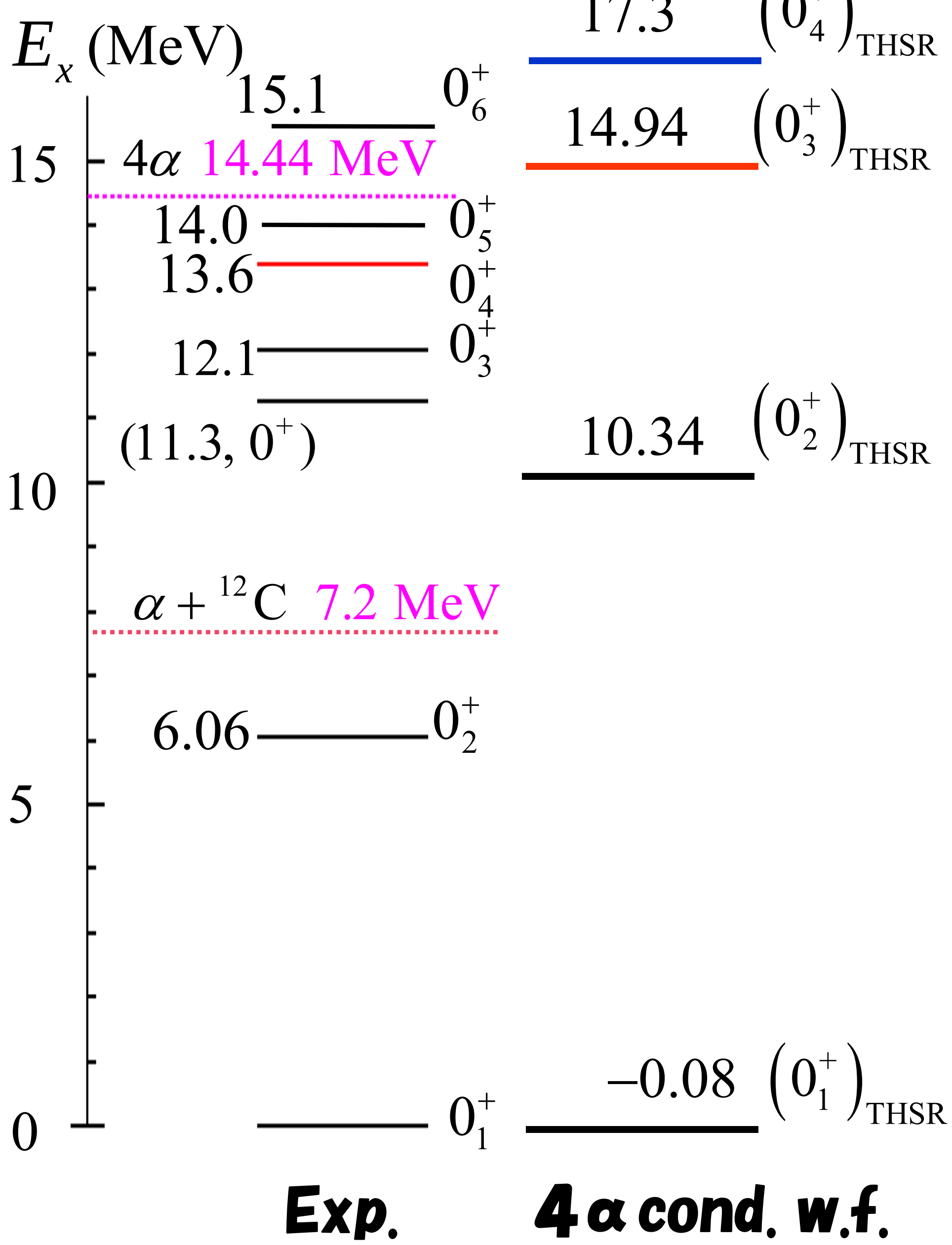


$(0_3^+)_{THSR}$:
E = 14.9 MeV
Γ = 1.5 MeV
(based on R-matrix theory)

T. Wakasa, E. Ihara, M. Takashina and Y. F. et al, PLB 653, 173 (2007).

First attempt to explore 4α condensate state in ^{16}O

Low lying 0^+ levels of ^{16}O



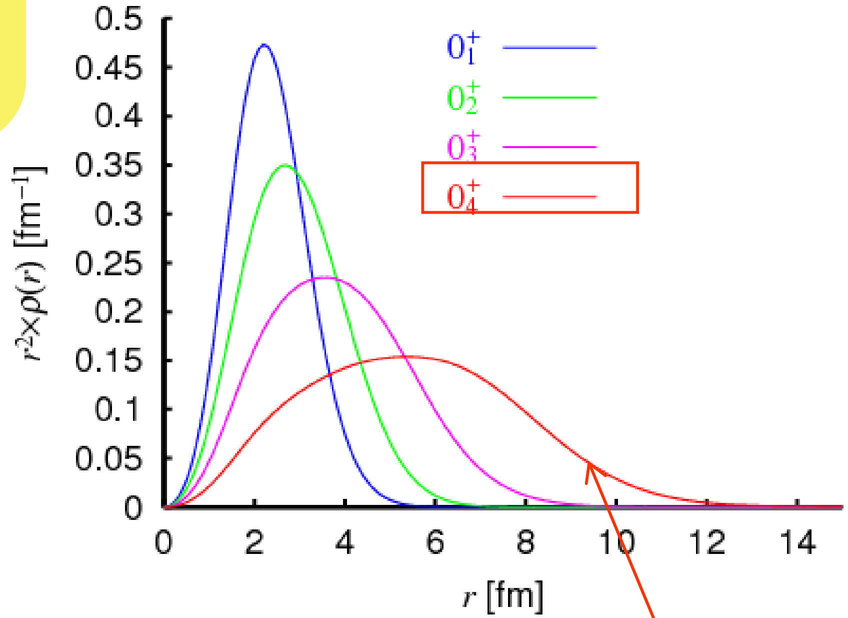
nα condensate model w.f. (THSR-w.f.)

$$\Phi_{n\alpha} = \mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}X_i^2\right)\phi(\alpha_i)\right)\right\}$$

A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke, PRL 87, 192501 (2001).

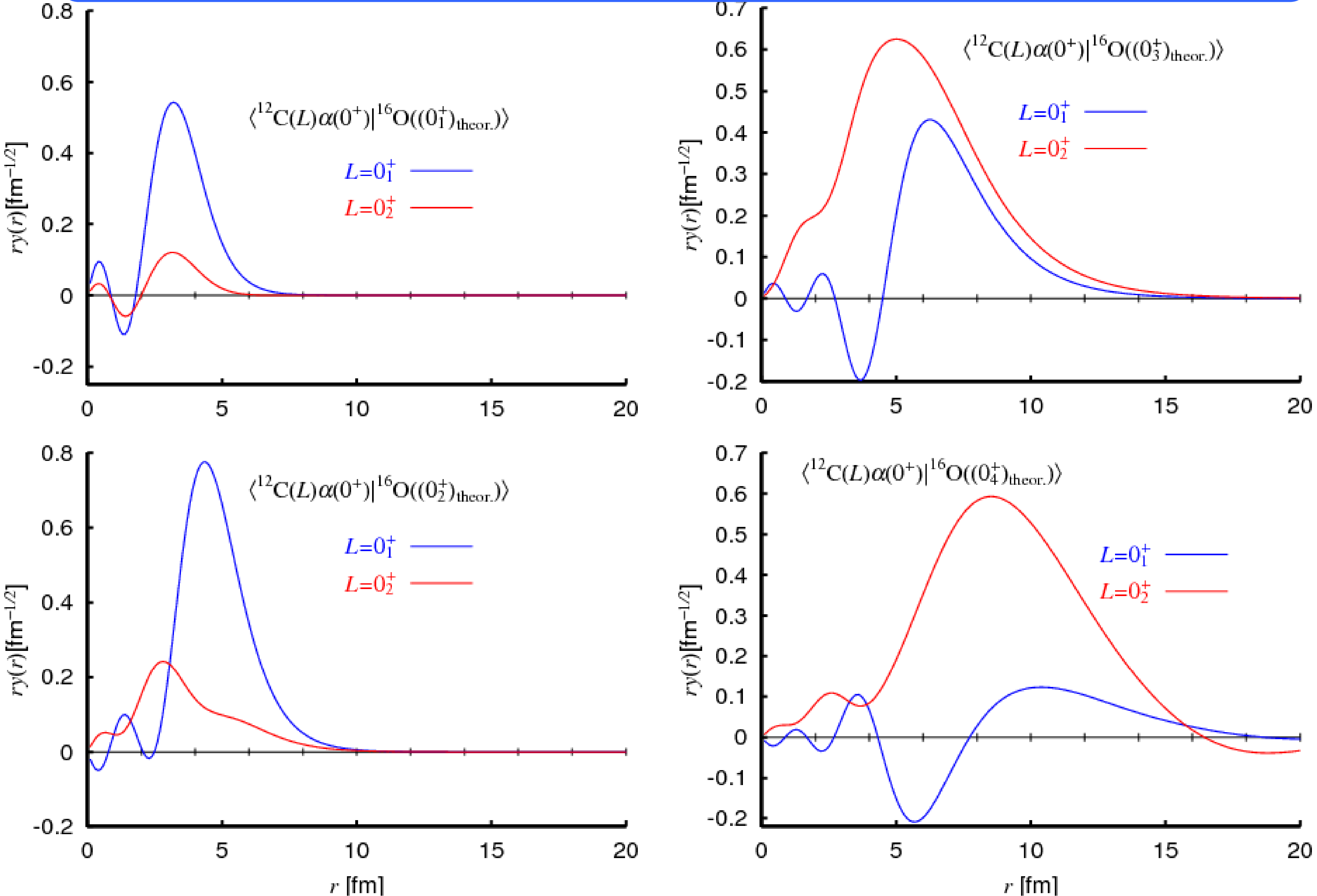
n=4 case

Nucleon density distribution



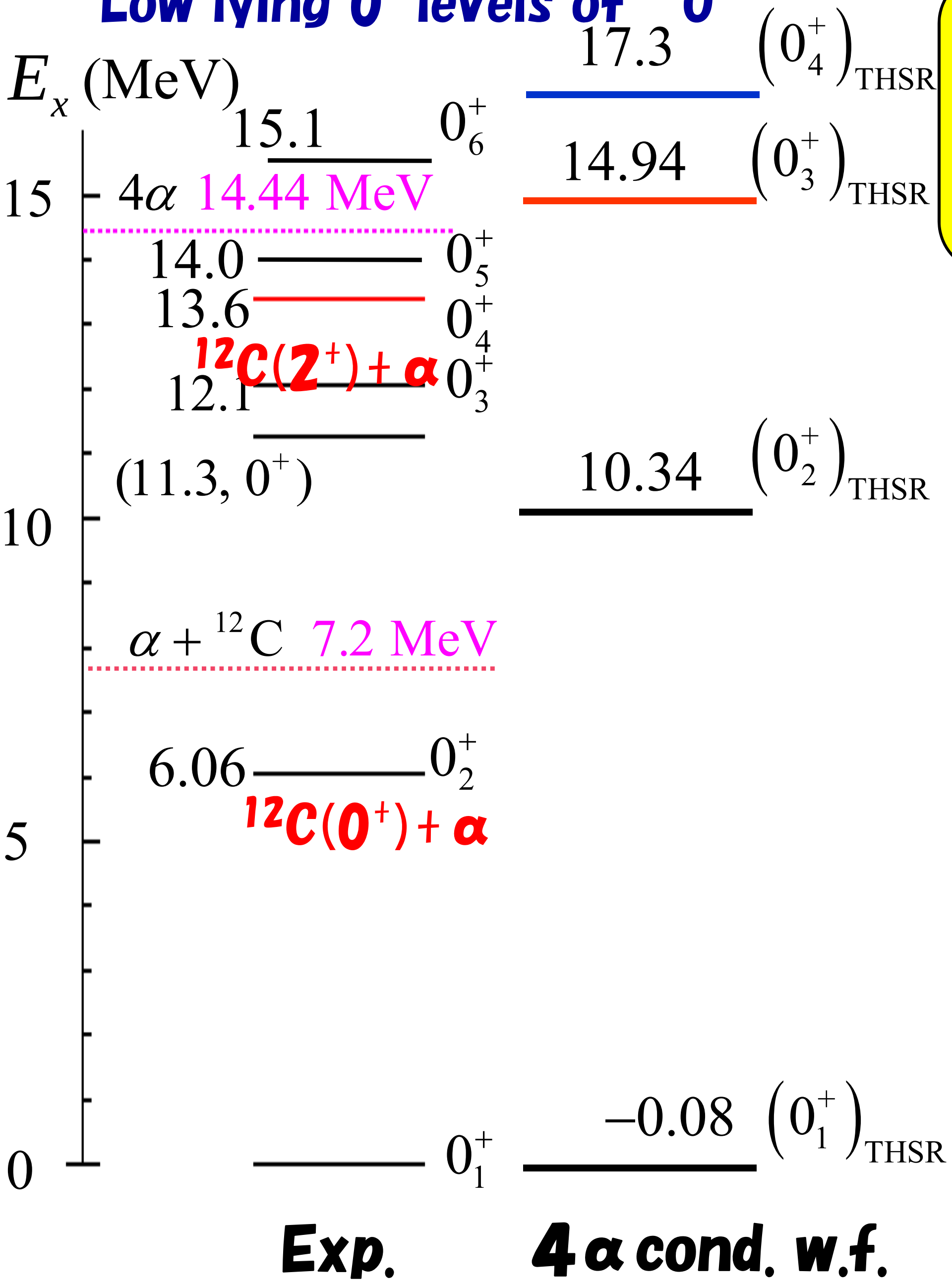
$(0_4^+)_{THSR}$: **Very much dilute density.**
4 α condensate state

Reduced width amplitudes of 0_1^+ - 0_4^+ states obtained with THSR w.f.(overlap amplitude between α plus $^{12}C(0_1^+$ or $0_2^+)$ and ^{16}O wfs)



Motivation for 4α OCM (Orthogonality Condition Model)

Low lying 0^+ levels of ^{16}O



n α cond. w. f. (microscopic)

$$\Phi_{n\alpha}(R_0, b) = \mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}\vec{X}_i^2\right)\phi(\alpha_i)\right)\right\}$$

- Need to check the existence of 4 α cond. state in larger (4α)model space.
- 4α cond. w. f. may have a difficulty to represent the ^{12}C+α structure.

4 α cond. w.f. hardly describes these states

- $0_2^+(\alpha + {}^{12}\mathrm{C}(0^+))$ △
- (0_3^+) ?
- $0_4^+(\alpha + {}^{12}\mathrm{C}(2^+))$ ✕

⇩

Need to solve full 4 α problem, 4 α OCM (semi microscopic)

Model space of 4α OCM (Orthogonality Condition Model)

Present: Larger model space

$$\varphi_{\ell m}(\mathbf{r},\nu) = N_\ell(\nu) r^\ell \exp\left(-\nu r^2\right) Y_{\ell m}(\mathbf{r})$$

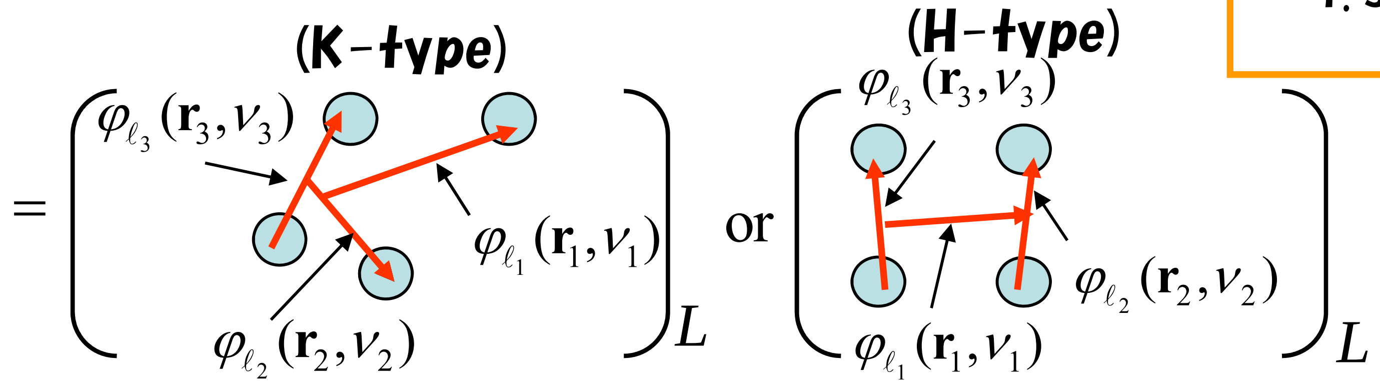
Gaussian basis ⟷

$^{12}C+\alpha$: succeeded
dilute 4α: not reproduced

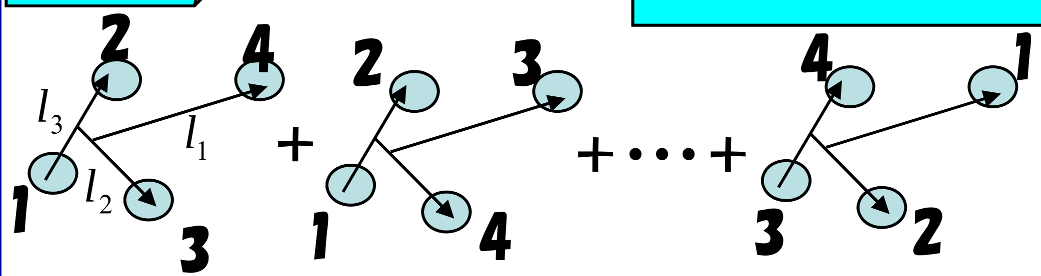
4α OCM (H. O. basis)
K. Fukatsu and K. Kato, PTP **87**, 151 (1992).

$^{12}C+\alpha$ coupled channel OCM (H. O. basis)
Y. Suzuki, PTP **55**, 1751 (1976); **56**, 111 (1976).

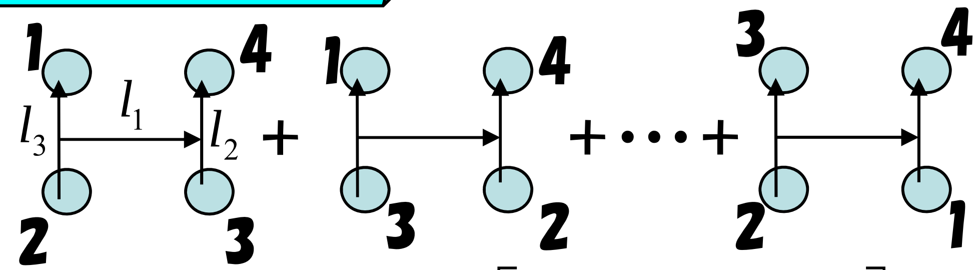
$$\Phi^{(ijkl)}_{c_K \text{ or } c_H}(\nu_1,\nu_2,\nu_3) = \left[\varphi_{\ell_1}(\mathbf{r}_1,\nu_1)\varphi_{\ell_2}(\mathbf{r}_2,\nu_2)\varphi_{\ell_3}(\mathbf{r}_3,\nu_3)\right]_L$$



K-type | **Symmetrization on α particles** | **H-type**



$$\Phi^{4\alpha}_{c_K}(\nu_1,\nu_2,\nu_3) = \hat{S}\left[\Phi^{(ijkl)}_{c_K}(\nu_1,\nu_2,\nu_3)\right]$$



$$\Phi^{4\alpha}_{c_H}(\nu_1,\nu_2,\nu_3) = \hat{S}\left[\Phi^{(ijkl)}_{c_H}(\nu_1,\nu_2,\nu_3)\right]$$

Adopted angular momentum channels (10 channels)

$$c_K \text{ and } c_H = [[l_3, l_2]_{l_{32}}, l_1]_L$$

K-type channel

$$c_K : \quad [[0,0]_0,0]_0 \quad [[2,0]_2,2]_0 \quad [[0,2]_2,2]_0 \quad [[2,2]_0,0]_0 \quad [[0,1]_1,1]_0 \quad [[2,1]_1,1]_0$$

H-type channel

$$c_H : \quad [[0,0]_0,0]_0 \quad [[2,0]_2,2]_0 \quad [[0,2]_2,2]_0 \quad [[2,2]_0,0]_0$$

Total wave function

$$\Phi_L(^{16}\mathrm{O}) = \sum_{c_K, \nu_1, \nu_2, \nu_3} A_{c_K}(\nu_1, \nu_2, \nu_3) \Phi^{4\alpha}_{c_K}(\nu_1, \nu_2, \nu_3) + \sum_{c_H, \nu_1, \nu_2, \nu_3} A_{c_H}(\nu_1, \nu_2, \nu_3) \Phi^{4\alpha}_{c_H}(\nu_1, \nu_2, \nu_3)$$

Hamiltonian of 4α OCM

$$H = T + \sum_{i<j}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij})\right] + V_{3\alpha} + V_{4\alpha} + V_{Pauli}$$

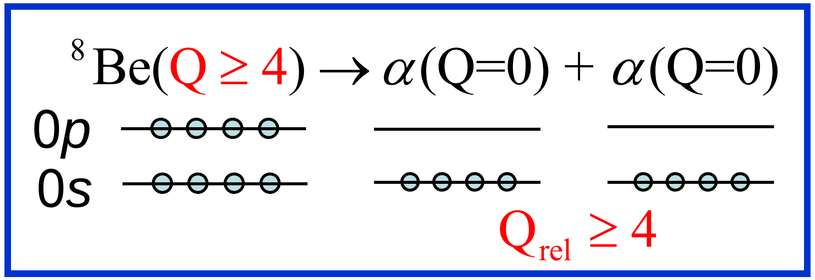
Equation of motion

$$\delta\left[\left\langle \Phi_L(^{16}\mathrm{O}) \middle| H - E \middle| \Phi_L(^{16}\mathrm{O}) \right\rangle\right] = 0$$

Pauli blocking operator on α - α motions

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda \sum_{2n+\ell<4} \sum_{ij} \left| u_{n\ell}(r_{ij}) \right\rangle \left\langle u_{n\ell}(r_{ij}) \right|$$

Pauli forbidden state: h.o.w.f.



α - α motion with $Q_{rel} < 4$ should be eliminated

Q_{rel} : H.O. quantum number regarding α - α motion

2-body force (folding MHN force)

$$V_{2\alpha}(r) = \sum_n V_n^{(2)} \exp\left(-\beta_n^{(2)} r^2\right)$$

Coulomb force

$$V_{2\alpha}^{Coul}(r) = \frac{4e^2}{r}\mathrm{erf}(ar)$$

Phenomenological 3-body force (repulsive)

$$V_{3\alpha} = V^{(3)} \sum_{i<j<k} \exp\left[-\beta(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)\right]$$

$$V^{(3)} = 87.5\ \mathrm{MeV}, \quad \beta = 0.15\ \mathrm{fm}^{-2}$$

Phenomenological 4-body force (repulsive)

$$V_{4\alpha} = V^{(4)} \exp\left[-\beta(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)\right]$$

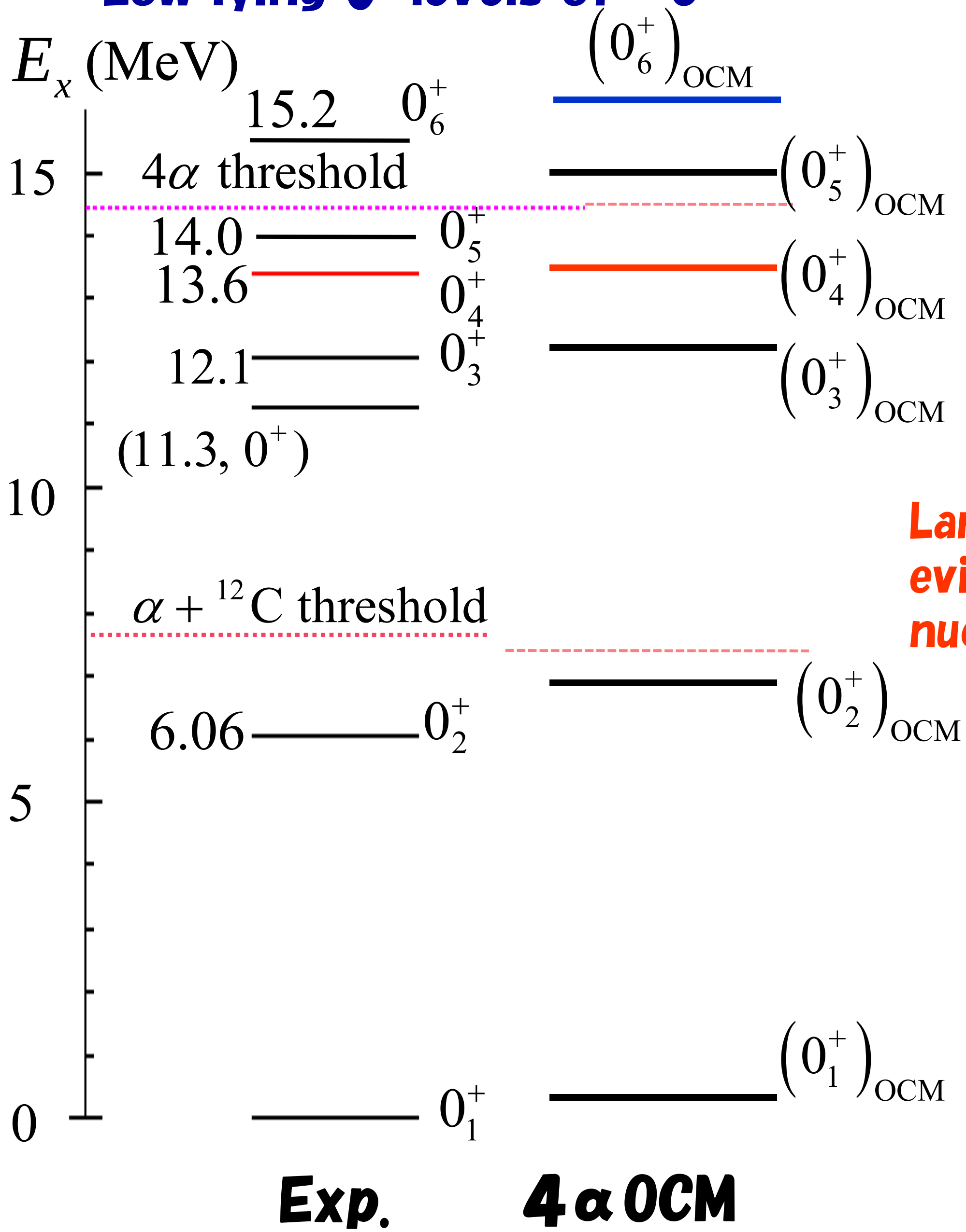
$$V^{(4)} = 12000\ \mathrm{MeV}, \quad \beta = 0.15\ \mathrm{fm}^{-2}$$

Energies from 4α threshold

	Cal. (MeV)	Exp. (MeV)
^{12}C(g.s.)	−7.32	−7.28
^{12}C(2_1^+)	−4.88	−2.84
^{12}C(4_1^+)	2.06	6.43
^{12}C(0_2^+)	0.70	0.38
^{16}O(g.s.)	−14.2	−14.44

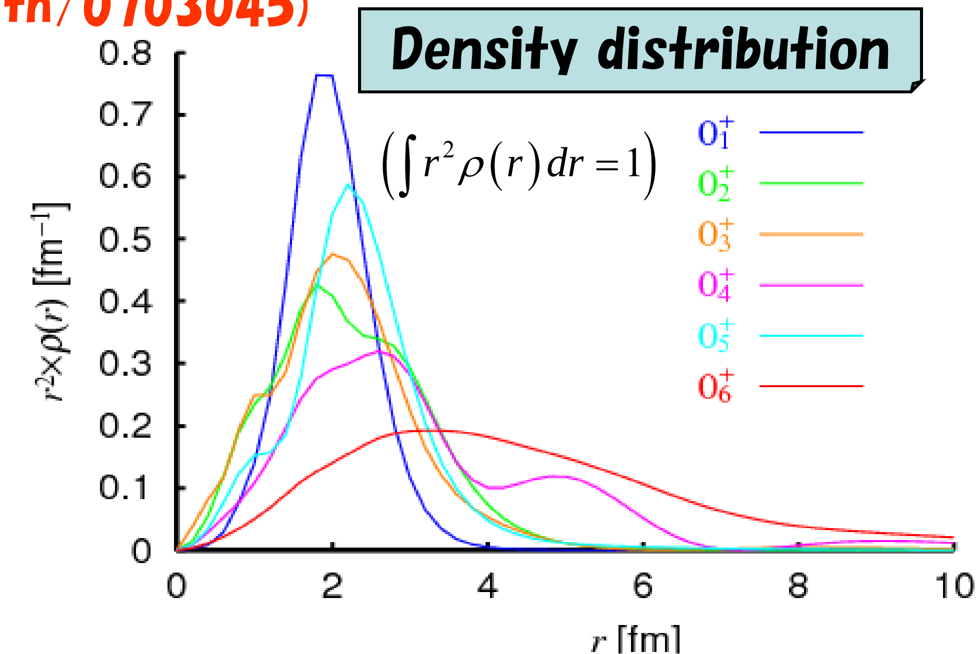
Energy levels, rms radii, monopole matrix elements and density distribution.

Low lying 0^+ levels of ^{16}O



	R_{rms} (fm)	M(E0)(fm^2)	M(E0)(fm^2) Exp.
$(0_1^+)_{OCM}$	2.7		
$(0_2^+)_{OCM}$	3.1	4.2	0_2^+: 3.55
$(0_3^+)_{OCM}$	2.9	4.1	0_3^+: 4.03
$(0_4^+)_{OCM}$	3.9	2.4	0_4^+: no data
$(0_5^+)_{OCM}$	3.1	2.0	0_5^+: 3.3
$(0_6^+)_{OCM}$	5.4	1.4	

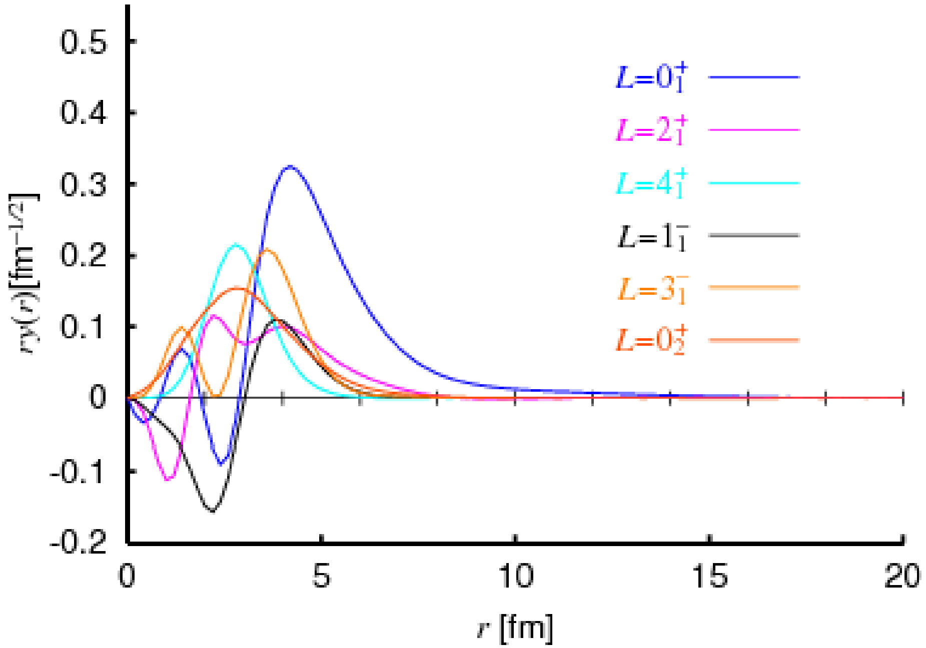
Large monopole matrix element can be the evidence of cluster states (Yamada, Y.F. et el., nucl-th/0703045)



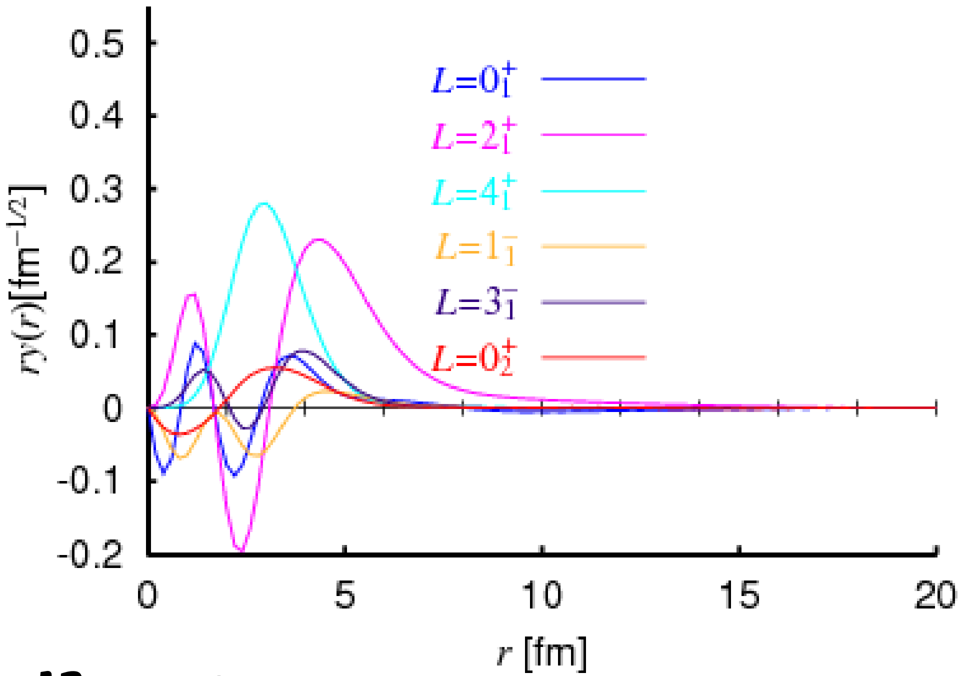
Reduced width amplitude for the lower states

$$r \times \mathcal{Y}_{L,J_k=0_k}(r_1) = r \times \left\langle \left[Y_L(\mathbf{r}) \Phi_L(^{12}\mathrm{C}) \right]_0 \middle| \Phi_{J_k=0_k}(^{16}\mathrm{O}) \right\rangle$$

$(0_2^+)_{\mathrm{OCM}}$



$(0_3^+)_{\mathrm{OCM}}$



$^{12}C(0_1^+) + \alpha$ component is dominant in surface region (~5 fm).

$\Downarrow$

$\alpha + {}^{12}C(0_1^+)$ structure

$^{12}C(2_1^+) + \alpha$ component is dominant in surface region (~5 fm).

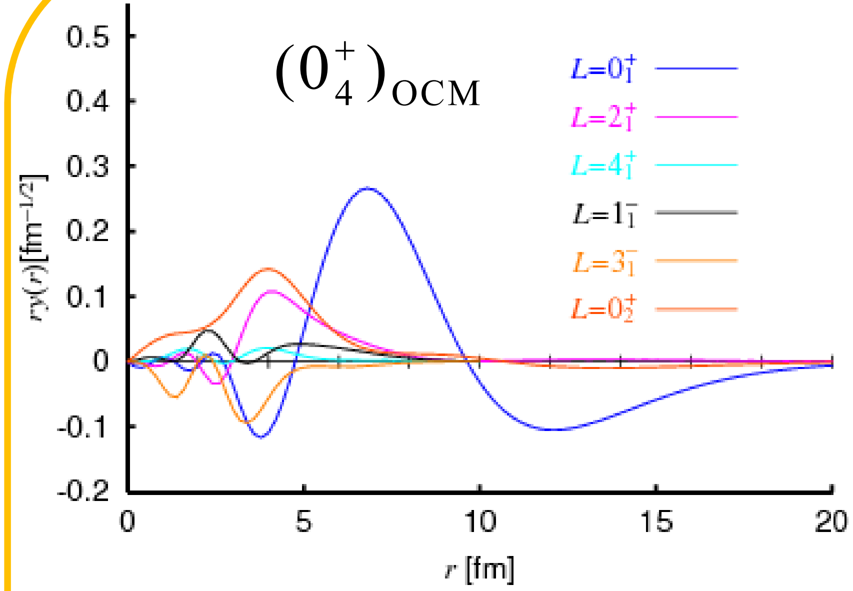
$\Downarrow$

$\alpha + {}^{12}C(2_1^+)$ structure

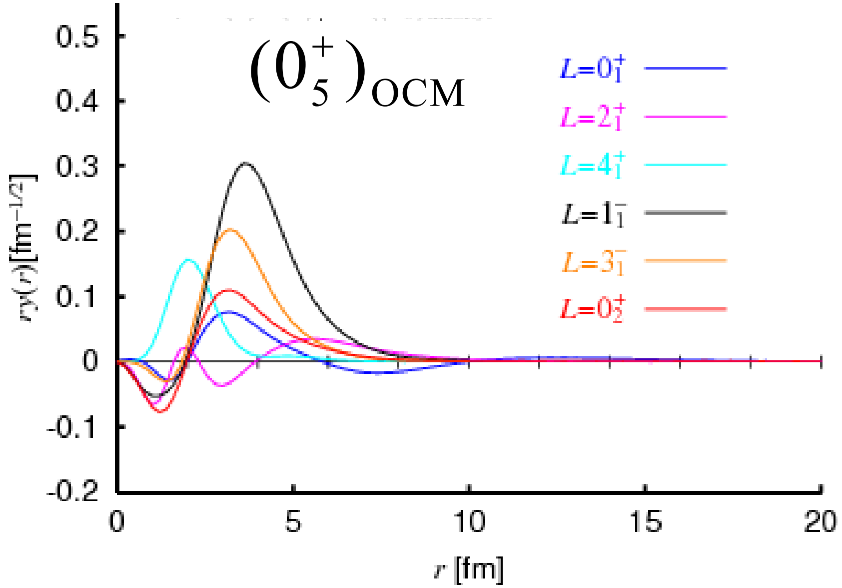
Consistent with the previous Suzuki and Kato results !

Reduced width amplitudes of 0_4^+ and 0_5^+ states obtained with 4α OCM

Defined as $r \times \mathcal{Y}_{L,J=0_k}(r) = r \times \left\langle \left[Y_L(\mathbf{r}) \Phi_L(^{12}\mathrm{C}) \right]_0 \middle| \Phi_{J_k=0_k}(^{16}\mathrm{O}) \right\rangle$



- Very well developed α cluster structure
- ^{12}C(g.s.) + α component is dominant. higher nodal structure

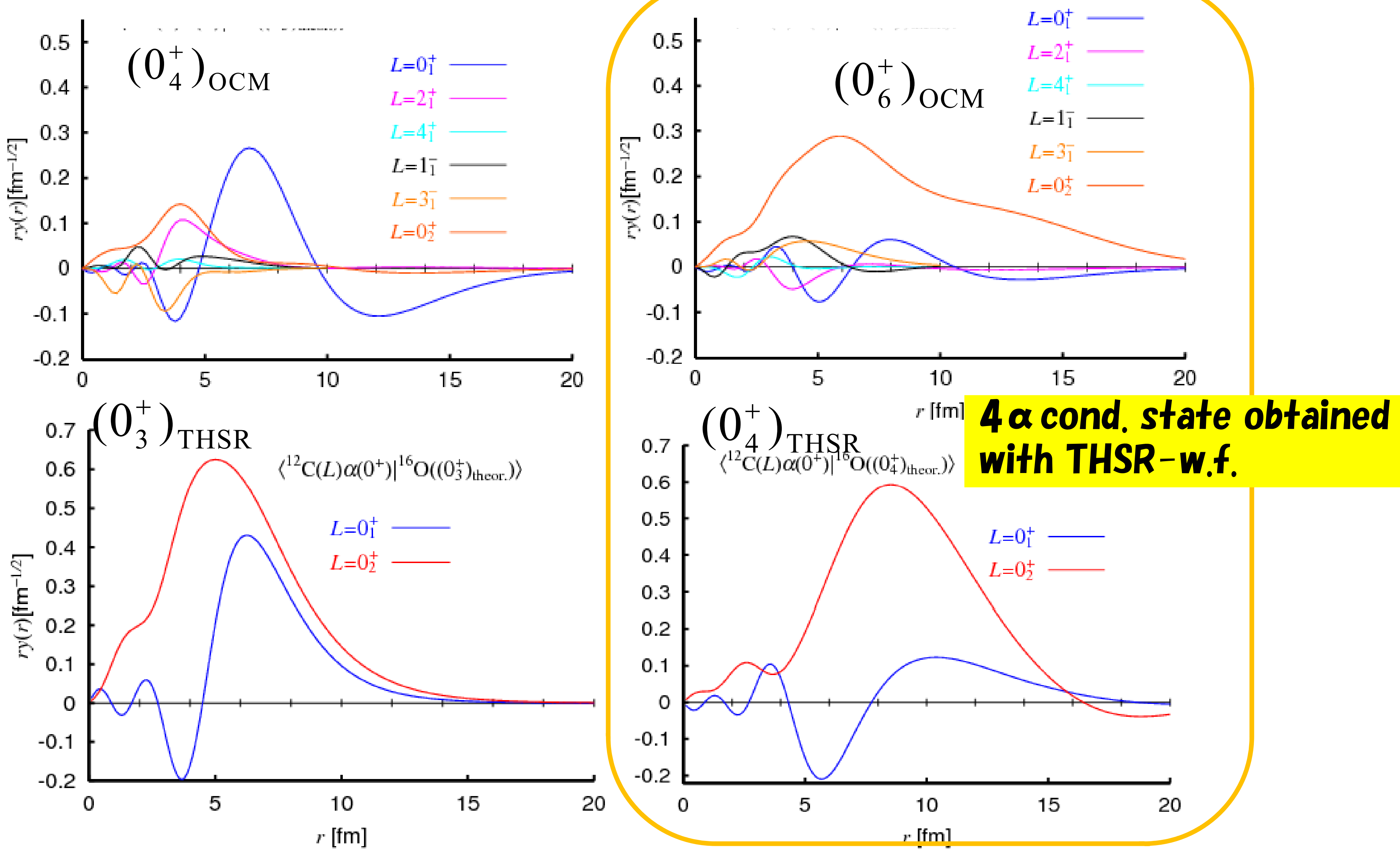


- Large amplitude in surface region(∼5 fm)
- developed α cluster structure
- mixing of ^{12}C(1^-) + α , ^{12}C(3^-) + α and ^{12}C(0_2^+) + α structures

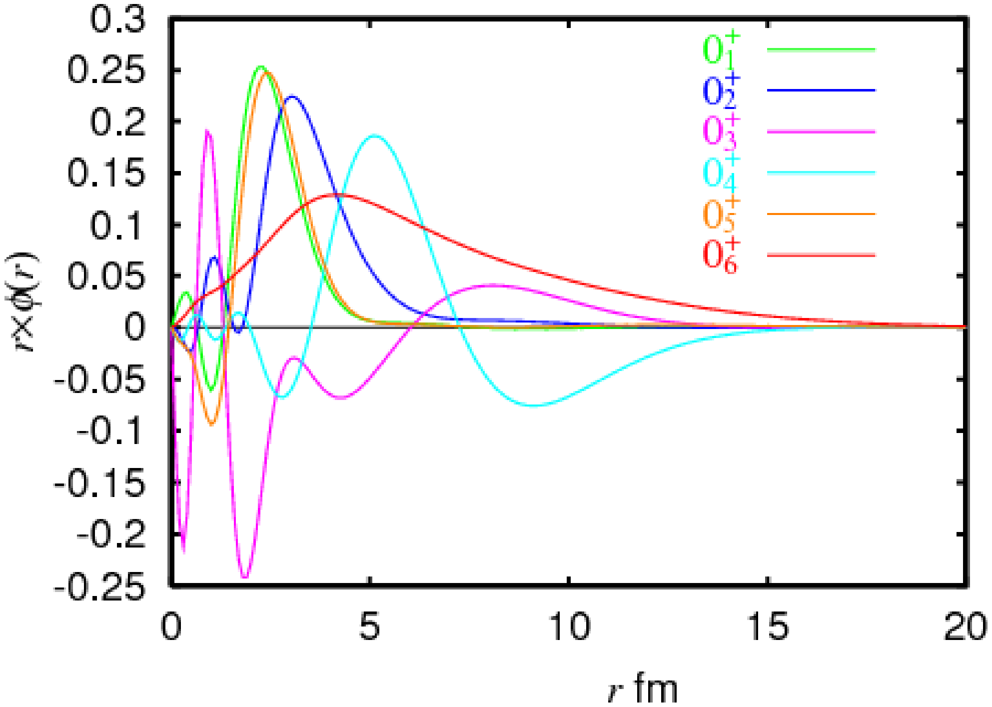
- New (not discussed so far) α + ^{12}C cluster states.
- α + ^{12}C dynamics survives up to around the 4α threshold.

Reduced width amplitudes of 0_4^+ and 0_6^+ states obtained with 4α OCM

Defined as $r \times \mathcal{Y}_{L,J=0_k}(r) = r \times \left\langle \left[Y_L(\mathbf{r}) \Phi_L(^{12}\mathrm{C}) \right]_0 \middle| \Phi_{J_k=0_k}(^{16}\mathrm{O}) \right\rangle$



Single particle occupancy and single particle orbit for the 0_1^+ - 0_6^+ states obtained with 4α OCM (Only the S orbit (L=0))



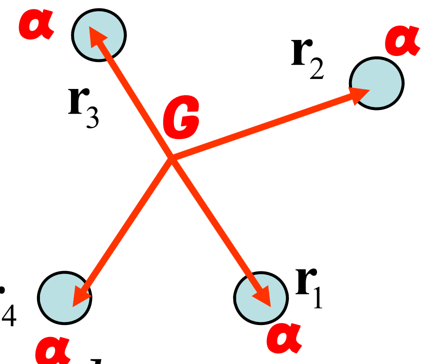
The largest (for $L = 0$) $\mu^{\lambda} = 2.33$

0_6^+ state : 2.33/4 =58 %

Large 0S occupancy !

Strongly evidence that the 0_6^+ state is the 4α condensate

The largest occupancies for the other states are less than 20 %.



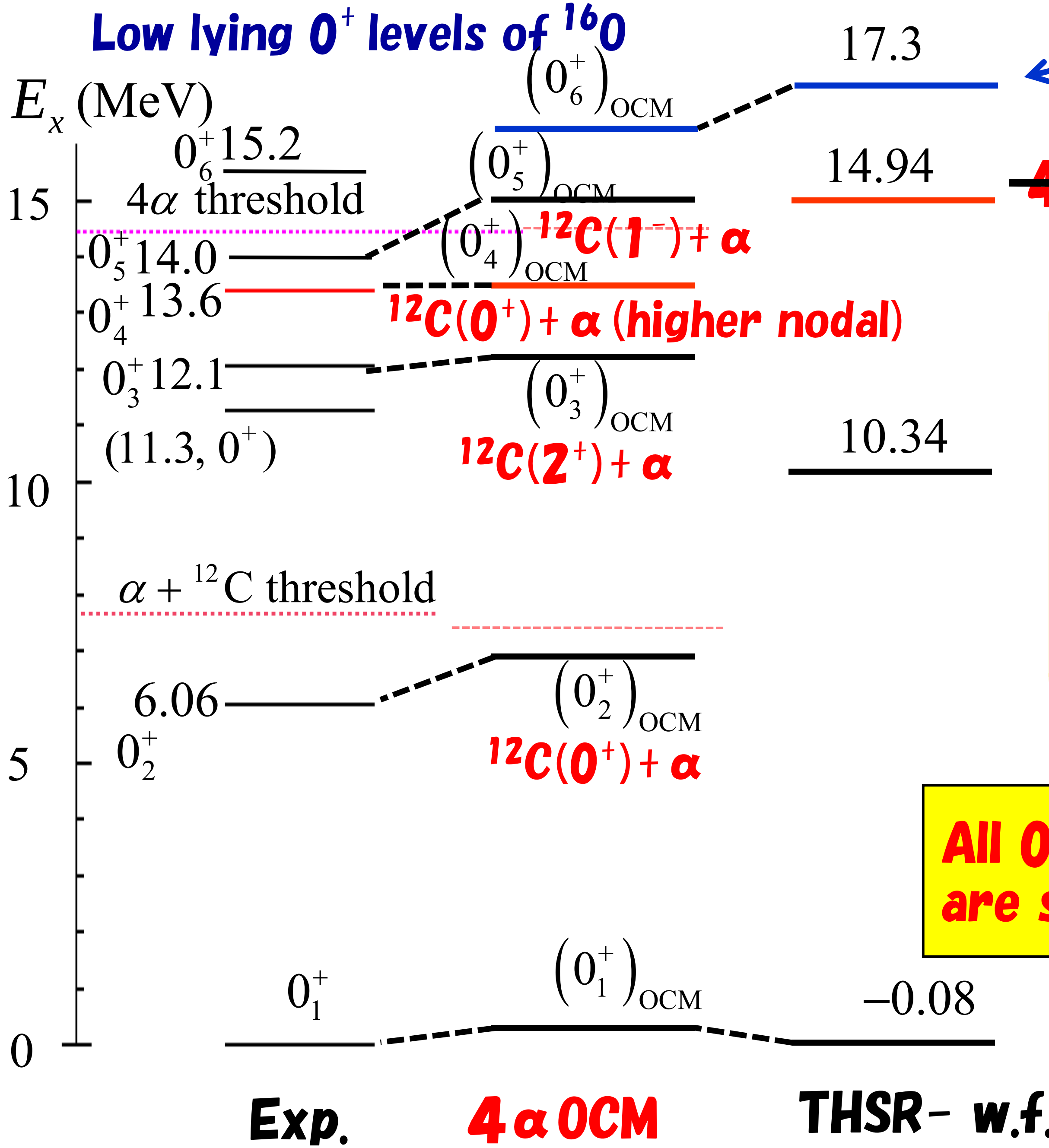
$$\int dr' \, \rho(r,r') f^{\lambda}(r') = \mu^{\lambda} f^{\lambda}(r)$$

$$\rho(r,r') = \sum_{i=1}^{4} \int \Phi^*(r_1,\cdots r,\cdots r_4)\Phi(r_1,\cdots r',\cdots r_4) dr_1 \cdots dr_{i-1} dr_{i+1} \cdots dr_4$$

$\lambda = (n, L)$, μ^{λ} : single particle occupancy, $f^{\lambda}(r)$: single particle orbit

$$\sum_{\lambda} \mu^{\lambda} = 4$$

Possible assignment of the two calculations and observations



$\Gamma(0_4^+)_{theor} \sim 0.2$ MeV
$\Gamma(0_5^+)_{theor} < 0.05$ MeV
(calculated based on R-matrix theory)

$\Gamma(0_4^+$ at 13.6 MeV$) = 0.6$ MeV
$\Gamma(0_5^+$ at 14.0 MeV$) = 0.17$ MeV

All 0^+ states up to 4 α threshold are simultaneously reproduced !

Summary

- Beyond doubt the Hoyle state is the 3α condensate state. (THSR and OCM)
- 4α condensate w. f. (4α THSR-w.f.) predicts the existence of 4α condensate state. (not as the third 0^+ state but as the fourth 0^+ state)

Analysis by using 4α OCM(orthogonality condition model) in order to describe both $^{12}C+\alpha$, 4α gas states and others, if any, in larger model space.

4α condensate state and other cluster states are simultaneously obtained.

Large 0S-occupancey, 60 %

- Two new resonance states are obtained near the 4α threshold.
  One has a developed α cluster structure (R_{rms}~3.0 fm) in which $^{12}C(1^-)+\alpha$, $^{12}C(3^-)+\alpha$ and $^{12}C(0_2^+)+\alpha$ components are mixed.
  The other has a very well developed α cluster structure (R_{rms}~4.0 fm).
  $^{12}C(0^+)+\alpha$ (higher nodal)
  ⇒ corresponding to the observed 0_4^+ and 0_5^+ states, respectively
- Successfully reproducing the well known $^{12}C(g.s.)+\alpha$ (6.05 MeV) and $^{12}C(2^+)+\alpha$ (12.05 MeV) structures,

For future work,

- Analyses of condensate fraction and 4α CSM are necessary for more reliable conclusion.
- 4α linear chain structure (the band head is estimated at 16.7 MeV)
  We are able to discuss it simultaneously with the lower structures !