Multi-meson systems in QCD

William Detmold

arXiv:0710.1827

Multi-boson energies

- *n*-boson systems in a box (L^3)
  [Bogoliubov '47][Huang, Yang '57][Wu '59][Lüscher '85]
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[Beane, WD, Savage '07]

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Many mesons in LQCD

- Consider *n* π^+ correlator (m_u=m_d)

$$C_n(t) = \left\langle 0 \middle| \left[\sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(0,0) \right]^n \middle| 0 \right\rangle$$

$$\rightarrow A e^{-E_n t}$$

- *n!*² Wick contractions

$$C_3(t) = \mathrm{tr}\left[\Pi\right]^3 \ - \ 3\ \mathrm{tr}\left[\Pi\right]\mathrm{tr}\left[\Pi^2\right] + \ 2\ \mathrm{tr}\left[\Pi^3\right]$$

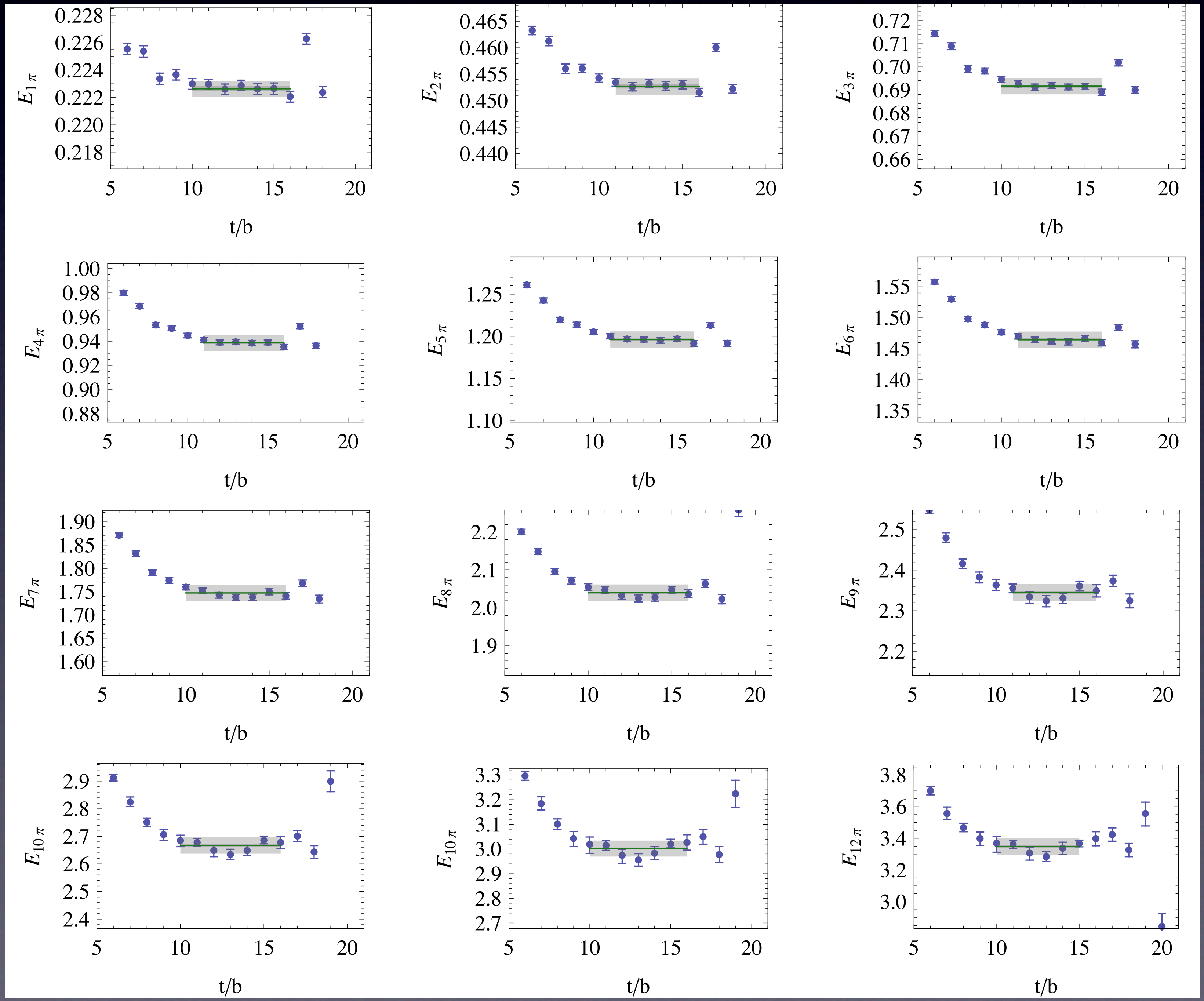
$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$

- π^+ contractions: only a single quark propagator

PRELIMINARY

n-meson energies

- Effective energy plots: $\log[C_n(t)/C_n(t+1)]$



DW propagators on coarse MILC gauge configs

m_π = 352 MeV

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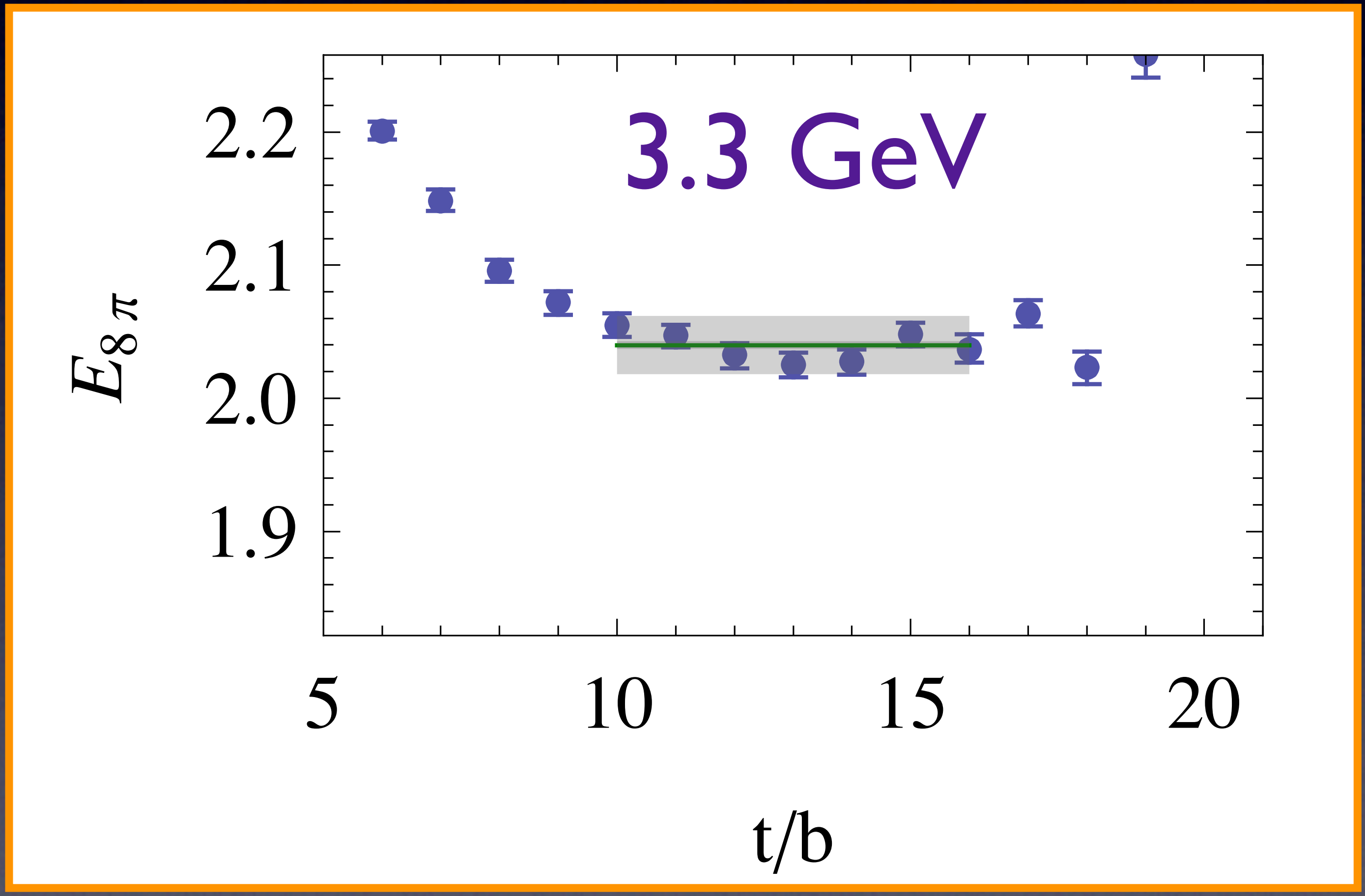

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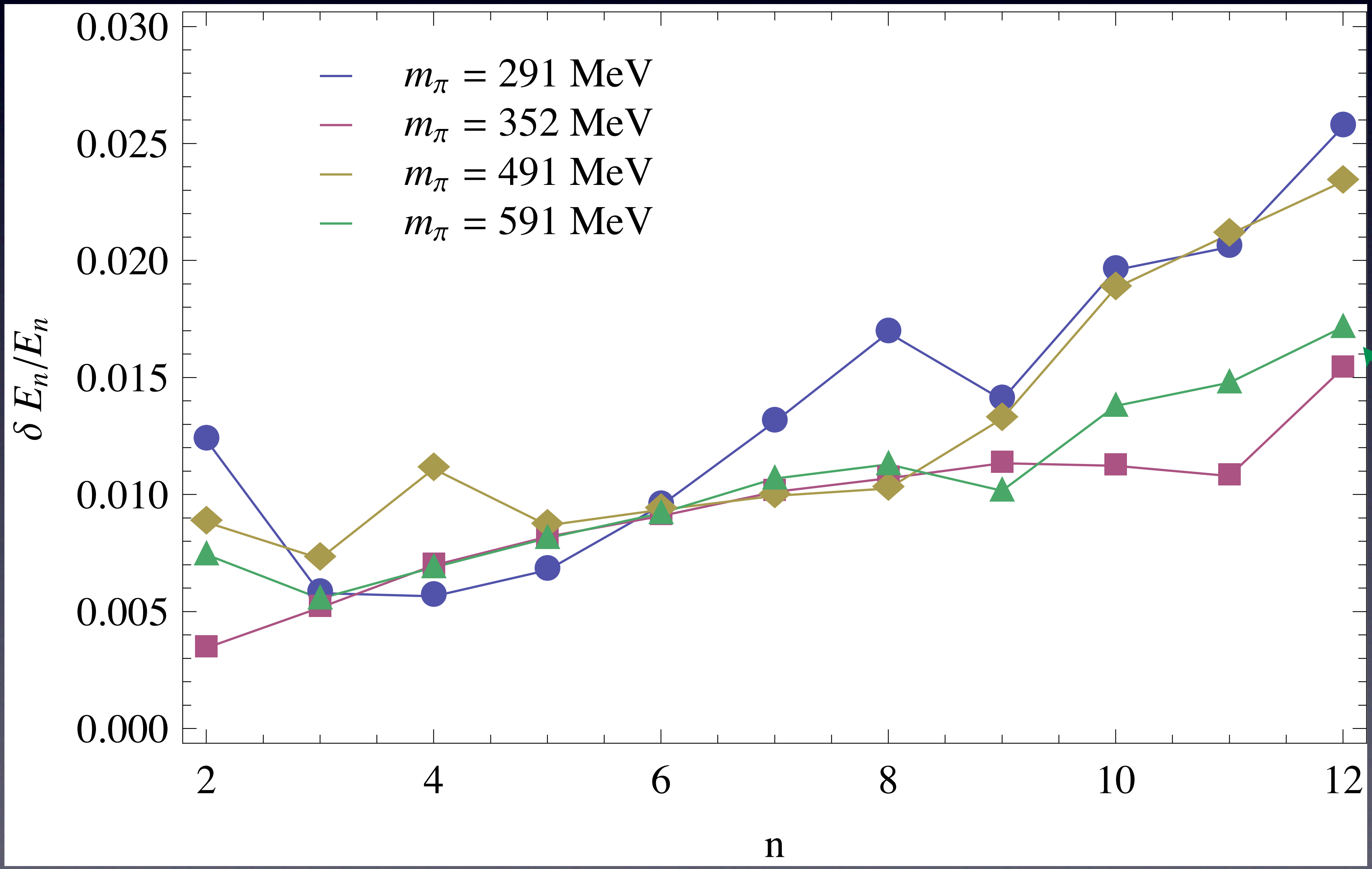
PRELIMINARY

n-meson energies

- Clear signals for n=1,...,12



~ 8 GeV !

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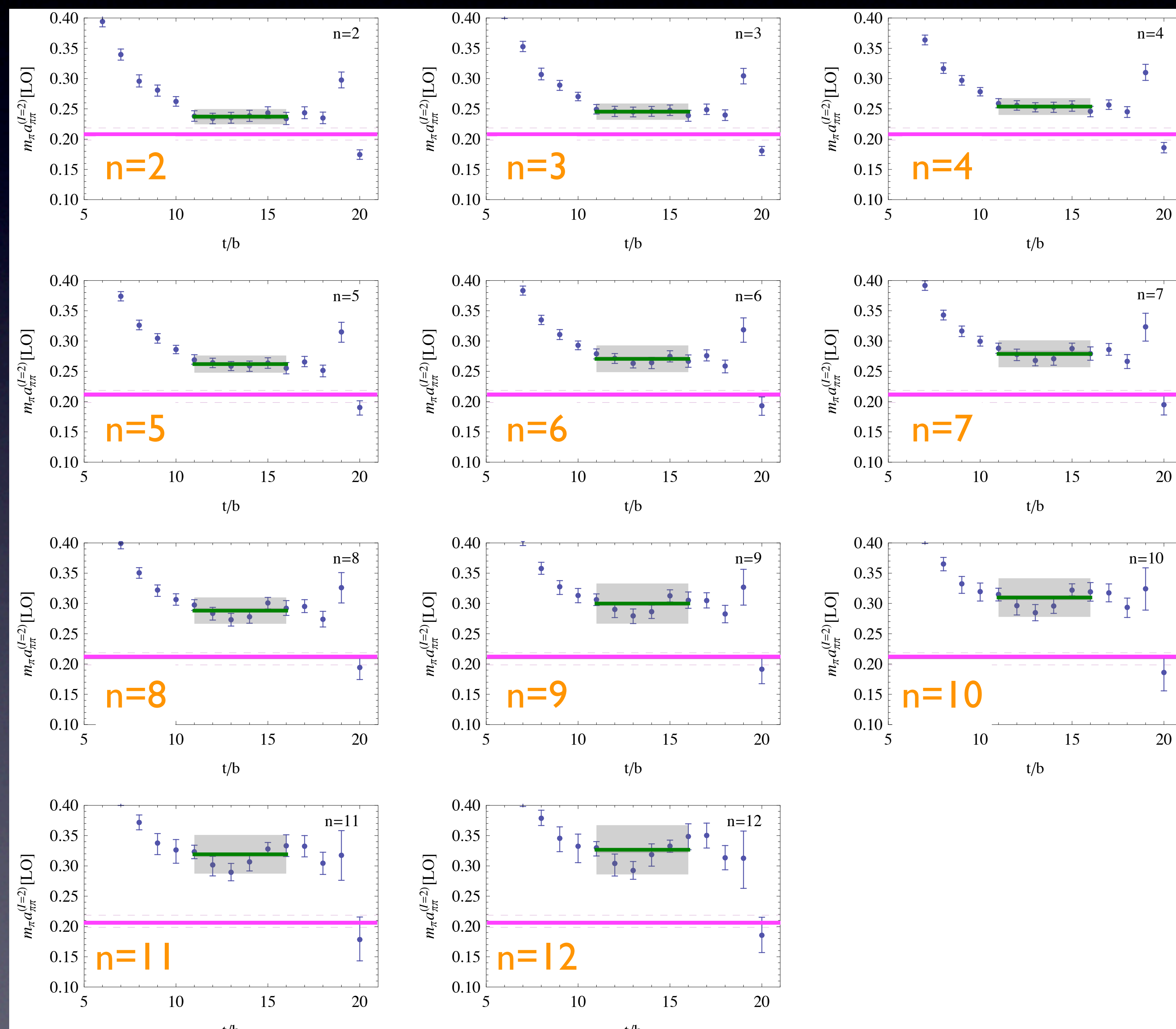
PRELIMINARY

Pion scattering

- Extractions of $m_\pi a$ from four orders in L

LO



Lüscher exact two-body

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Pion scattering

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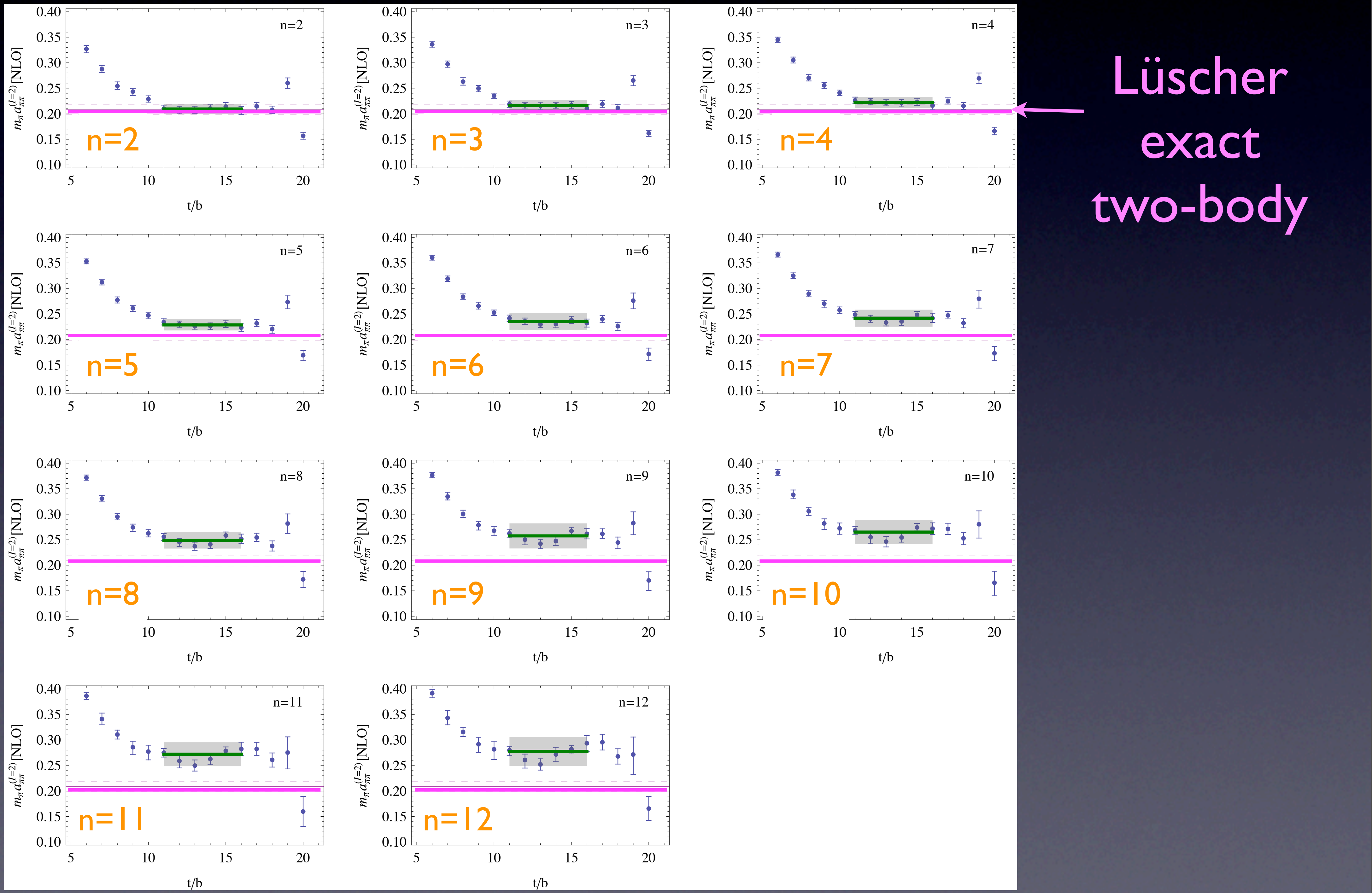
NLO

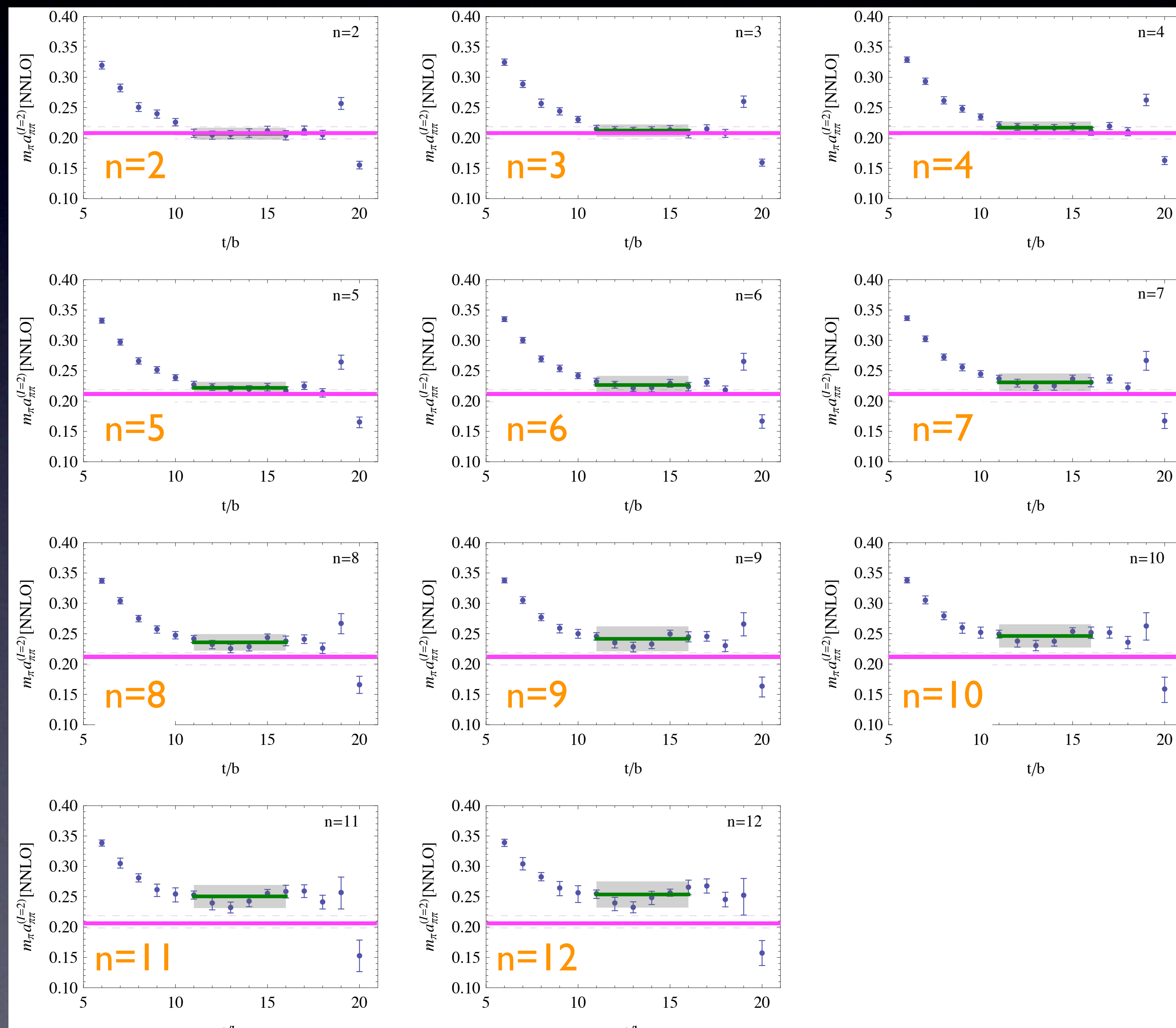


PRELIMINARY

Pion scattering

- Extractions of $m_\pi a$ from four orders in L

NNLO



Lüscher exact two-body

N^{3}LO

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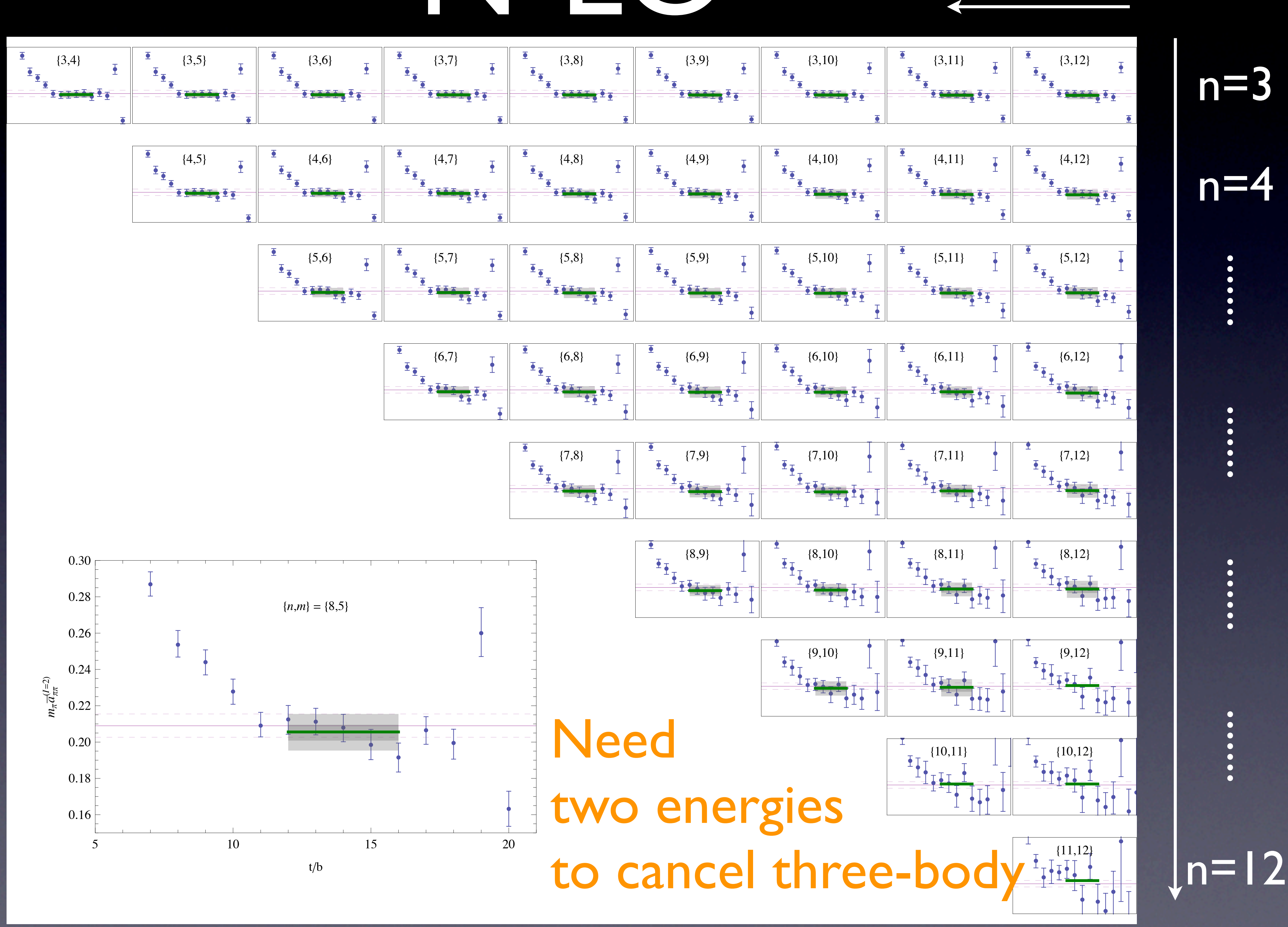
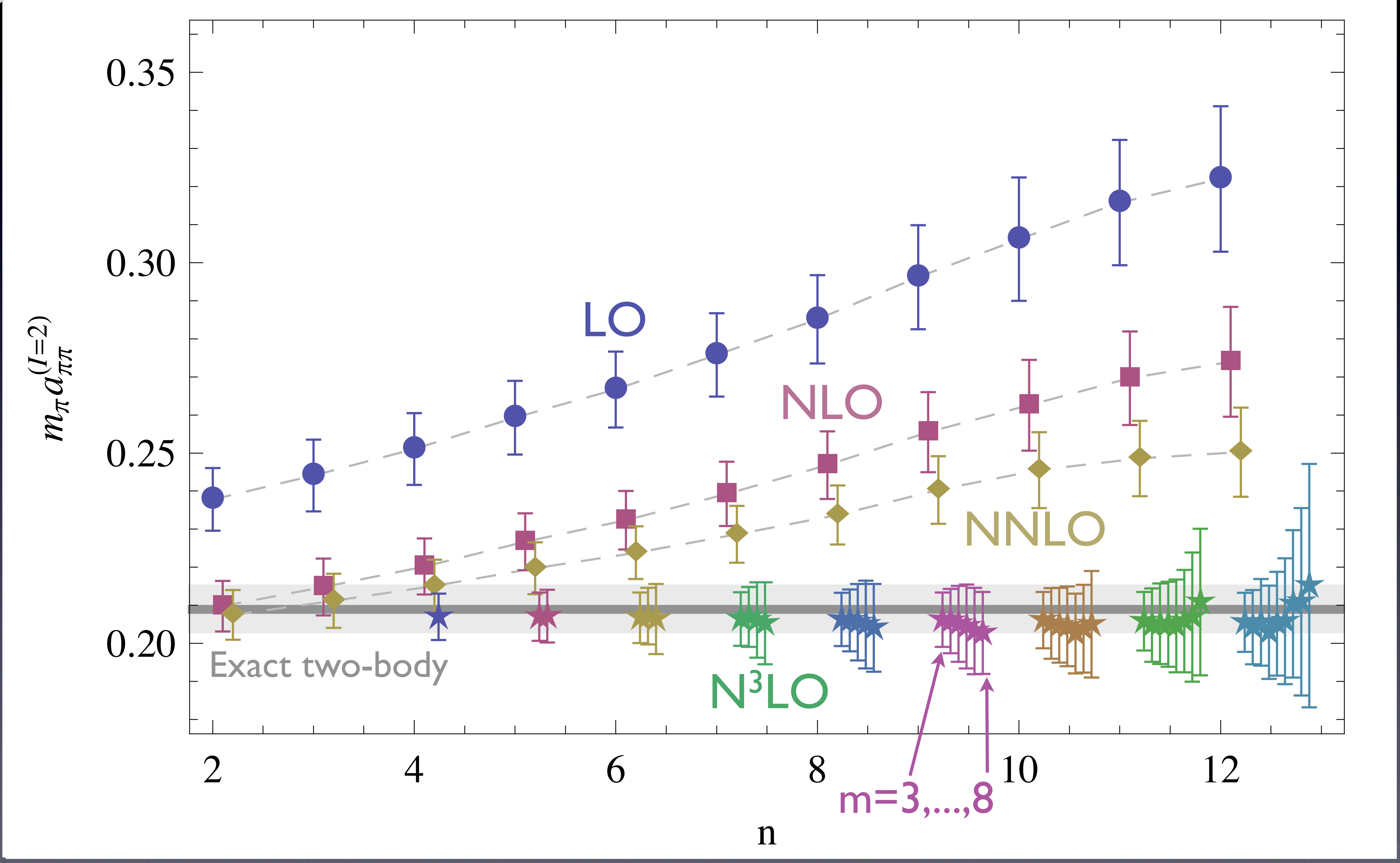


Need
two energies
to cancel three-body

Pion scattering

PRELIMINARY



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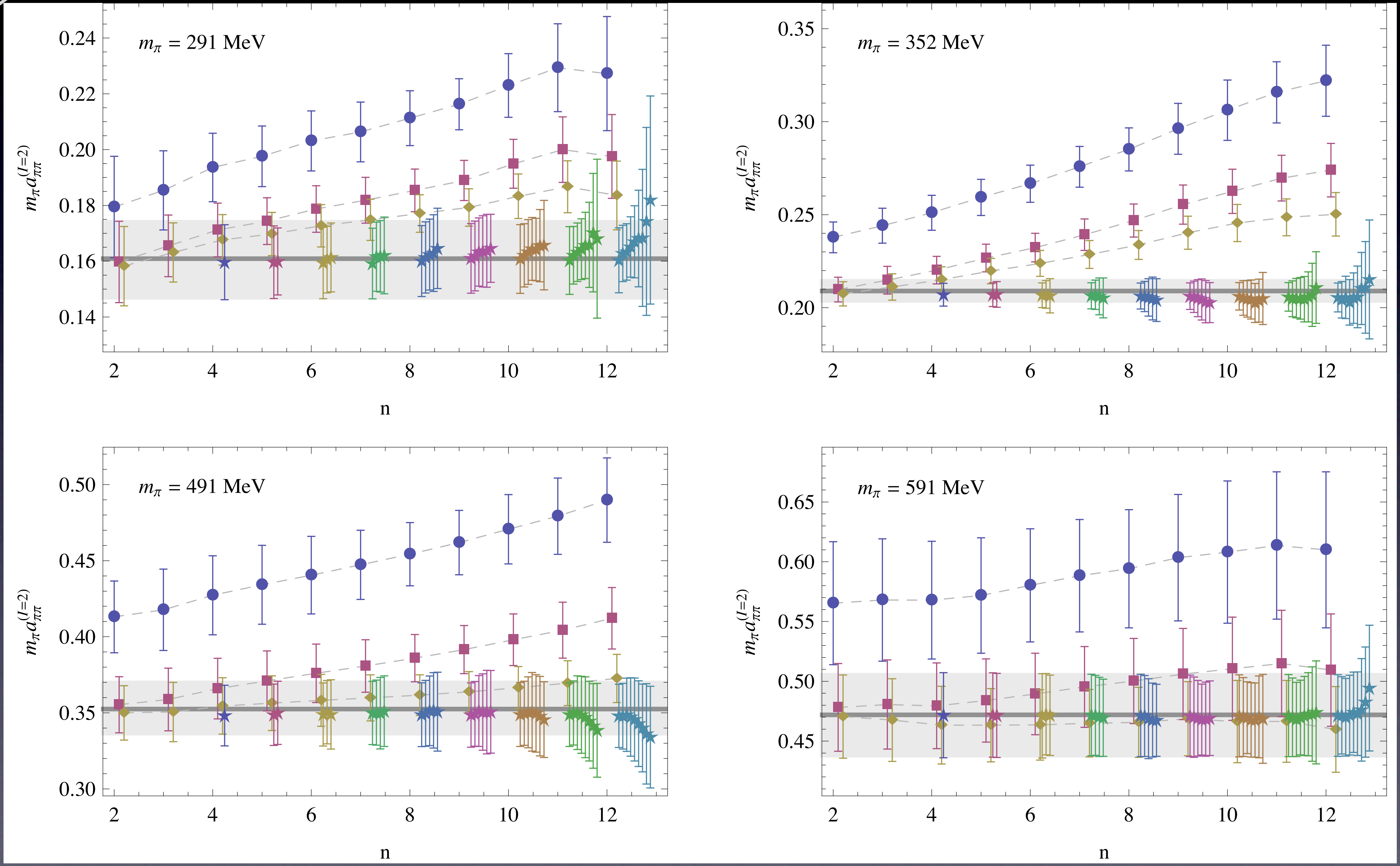
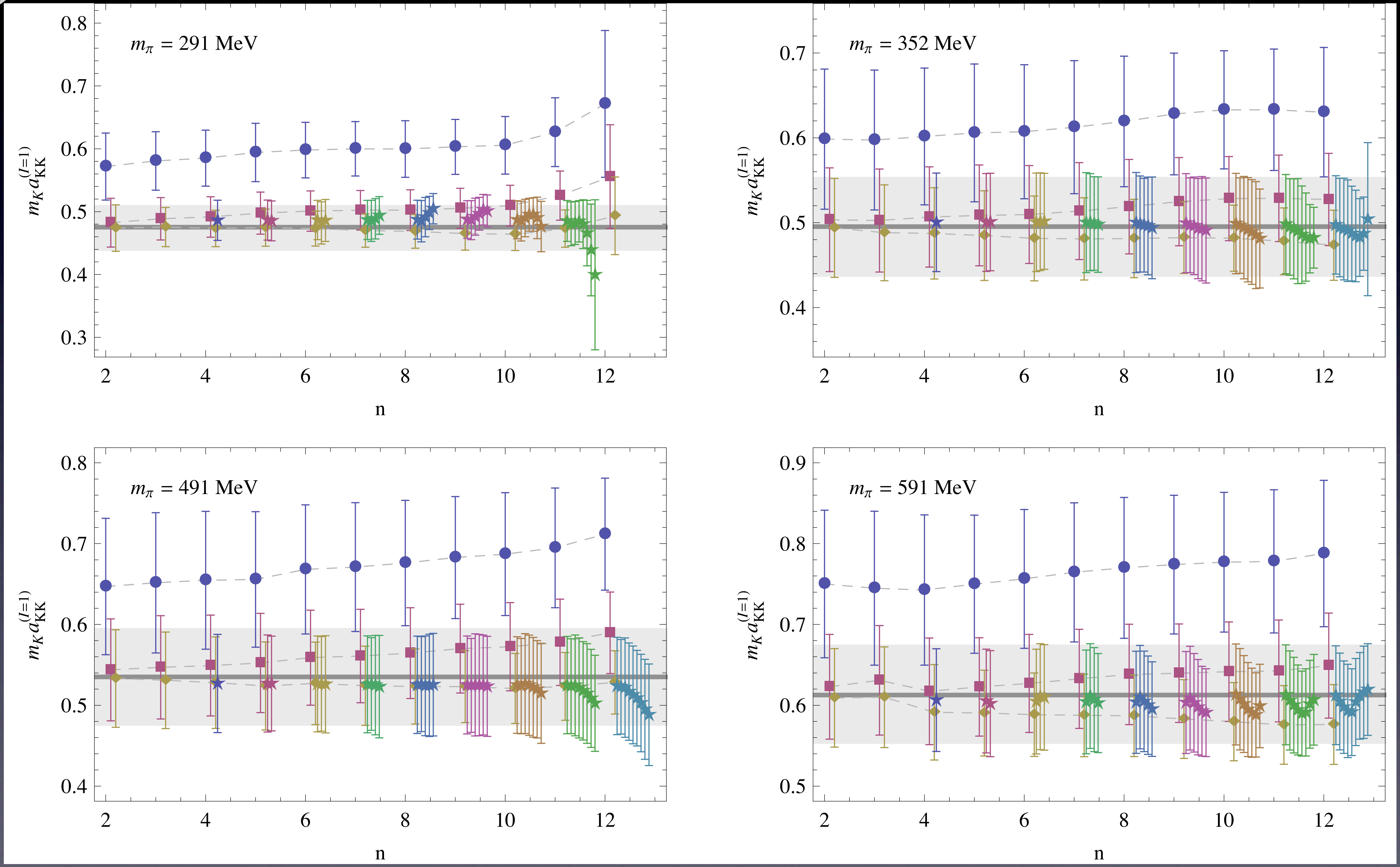


PRELIMINARY

Kaon scattering



Scattering lengths

- Scattering lengths equally well extracted for two mesons or ten mesons
- Described by analytic prediction
- Shows presence of contribution that scales as $\binom{n}{3}$
  - varies by two-orders of magnitude

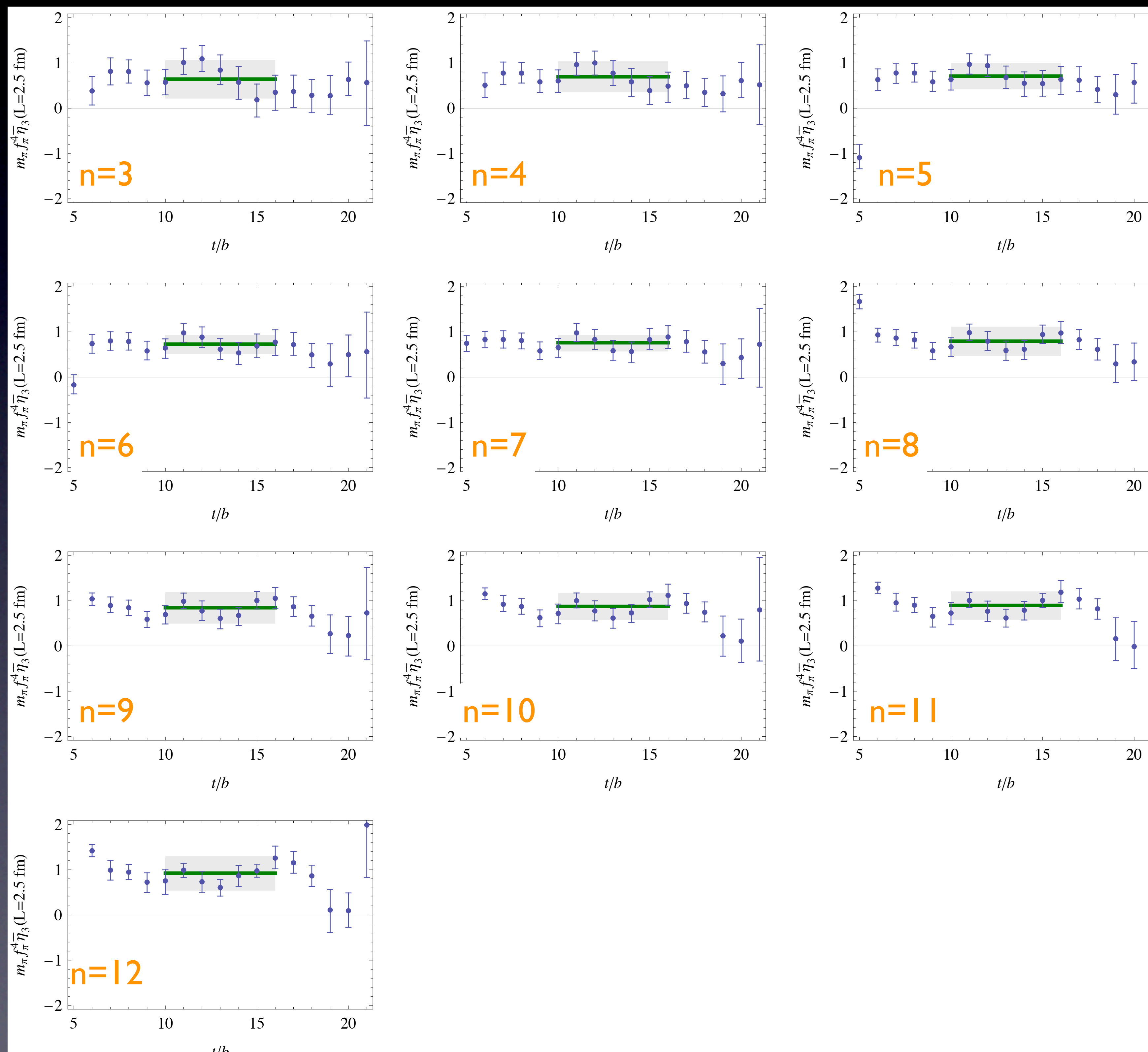
Three meson interactions

- At $1/L^6$, point-like three-boson interaction must occur [Braaten, Nieto '95]
  - RGI 3BI: $\overline{\eta}_3^{(L)}$ physically meaningful
  - Depends logarithmically on L

- Naive dimensional-analysis $m_\pi f_\pi^4 \overline{\eta}_3^{(L)} \sim 1$
- Combinations of energy shifts isolates the RGI interaction

PRELIMINARY

$\pi^+\pi^+\pi^+$ interaction



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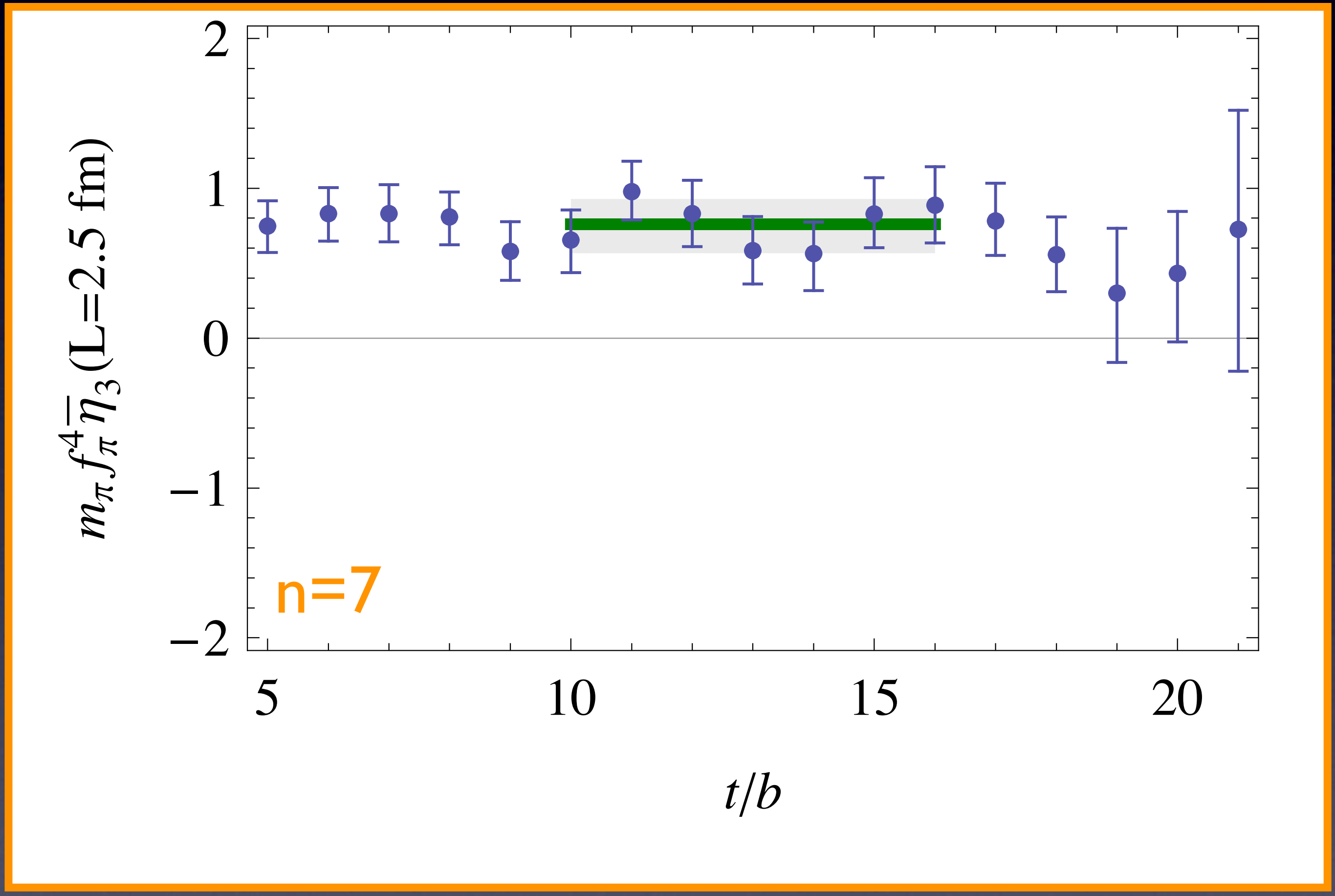

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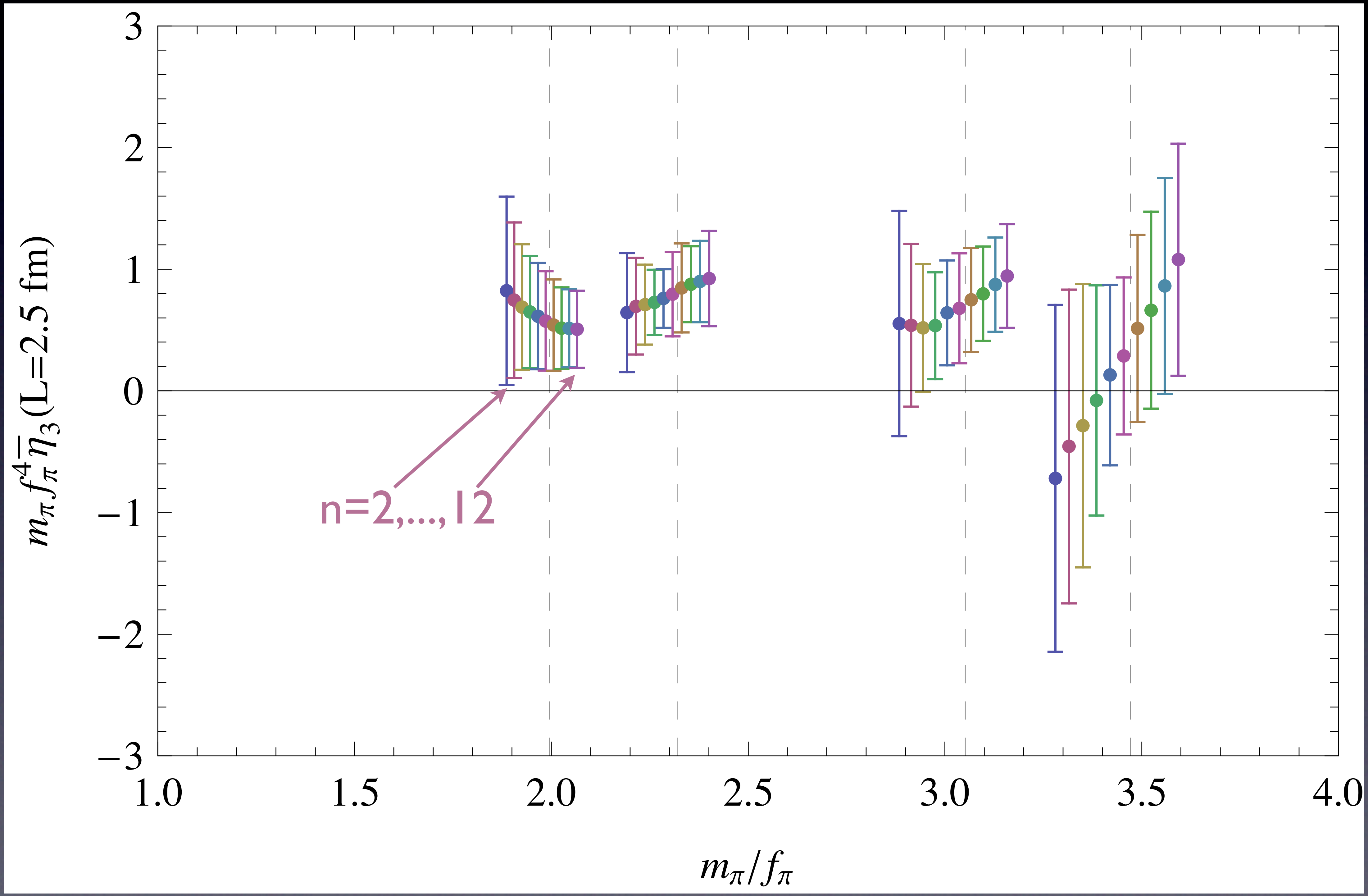
Mass dependence



$^{314}\pi^+$?

- Limits on larger systems
  - More and more propagators: Pauli EP
  - Ground state - condensation?
- In progress
  - Different volumes, lattice spacings/actions
  - Analyse at $1/L^7$
    - Reduce errors
    - Disentangle *a*, *r*, p-wave interaction, relativity
  - Mixed systems: *n* pions and *m* kaons
  - Fermions: issues