Decay out of Superdeformed Bands in a Two-Level Model

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Correlations in Nuclei: From Di-Nucleons to Clusters
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SIMON FRASER UNIVERSITY

Acknowledgments

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- Charles A. Stafford
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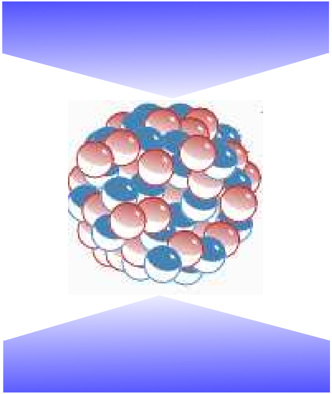
Why collectivity?

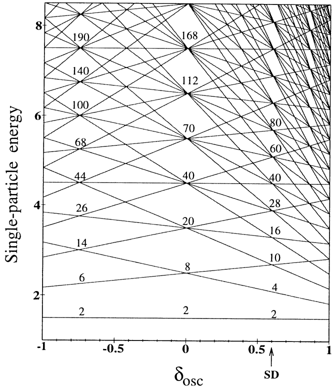
“Top Down”: Collective Motion



“Bottom Up”: Microscopic Approaches

Outline

Superdeformation

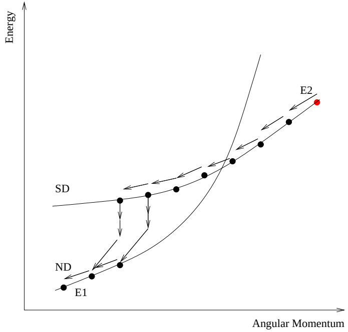


from Wong (1998).

- General prediction of shell models.
- Ellipsoidal and highly deformed: $\frac{\text{major}}{\text{minor}} \approx 2$.
- Clear experimental signature
  - Large electric quadrupole: $Q \approx .007ZA^{2/3}\text{eb}$.
  - Little centrifugal stretching: rigid rotor spectrum.
- For very high angular momenta, SD states can be yrast.

Outline

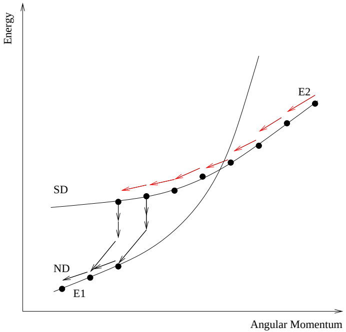
Life and Death of an SD Nucleus



Typical Decay Experiment

1. Nucleus is created in a high angular momentum SD yrast state.
2. Decay via E2 transitions along SD rotational band.
3. Transistion to a lower-lying ND band.
4. Decay down ND band via E1-dominated transitions.

Life and Death of an SD Nucleus



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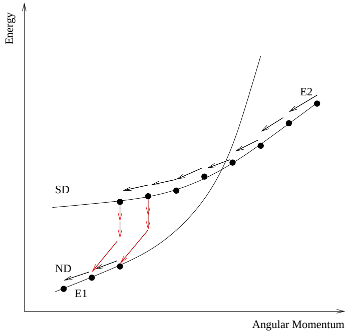

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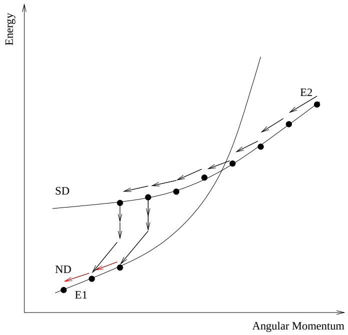

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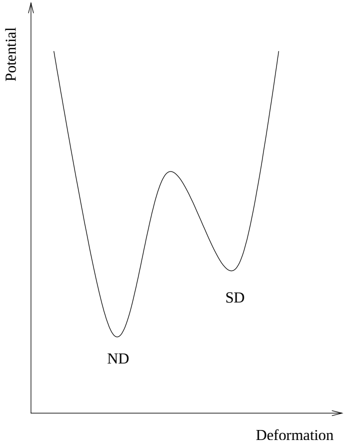
Modeling the Decay



Schematic Potential

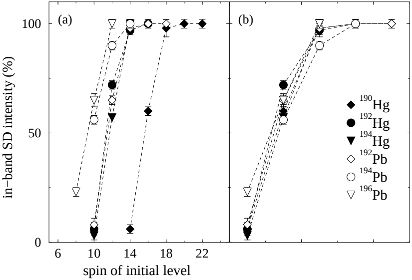
- Double well.
- Function of angular momentum.

In principle, each SD state can decay to all ND states.

Interesting Questions

A shopping list

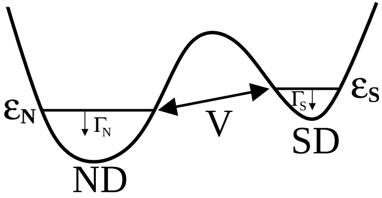
- How many states do we need to keep in the ND well?
- How important is electromagnetic broadening?
- Can we extract information about the potential barrier from a decay experiment?
- Why are the decay profiles for $A \approx 190$ so similar?



from Wilson *et al.* *PRC*, **71**, 34319 (2005).

Outline

Overview



Basic Assumption

Only one ND state mixes significantly with the decaying SD state.

C. A. Stafford & B. R. Barrett, PRC 60, 51305 (1999).

Benefits

- Elegant, intuitive model.
- Treats all interactions (nuclear and EM) on the same footing.
- Exactly solvable via Dyson's Equation.
- Just four parameters: V, $\Delta = \varepsilon_N - \varepsilon_S$, Γ_S, Γ_N.
- F_N is an experimental input.

Electromagnetic Decay Rates

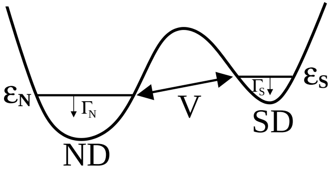
- Γ_S: lifetimes, quadrupole moments.
- Γ_N:
  - Cranking model Fermi-gas level density (Åberg 1988): $\rho(U) = \frac{\sqrt{\pi}}{48a^{1/4}} U^{-5/4} e^{2\sqrt{aU}}$
  - Giant Dipole Resonanace (Døssing & Vigezzi 1995): $\Gamma_N \approx \Gamma_{E1}(U) \approx 4! \frac{4}{3\pi} \frac{e^2}{\hbar c} \frac{1}{mc^2} \frac{\Gamma_{GDR}}{E_0^4} \frac{NZ}{A} \left(\frac{U}{a}\right)^{5/2}$
  - a, E_0, Γ_{GDR}, U(backshift) fit to nuclear data.

Non-Unitary Time Evolution

1. $t = 0$: The nucleus has just decayed via E2 and is localized in SD well.
2. Coherent Rabi oscillations + decoherent virtual interactions with EM field.
3. Nucleus escapes double-well by a real E1 or E2 decay.

Total wavefunction:

- $|\psi(t)\rangle = a_S(t)|S\rangle + a_N(t)|N\rangle$
- $|a_S(t)|^2 + |a_N(t)|^2 \leq 1$
- $|\psi(0)\rangle = |S\rangle$



Analytic Solution

Treat tunneling between wells as perturbation

$$G_0^{-1} = \begin{pmatrix} E + i\Gamma_S/2 & 0 \\ 0 & E - \Delta + i\Gamma_N/2 \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}$$

Dyson's Equation - exact to all orders in $\hat{V}$

$$G = G_0 + G_0 \hat{V} G$$

$$G^{-1} = G_0^{-1} - \hat{V} = \begin{pmatrix} E + i\Gamma_S/2 & -V \\ -V & E - \Delta + i\Gamma_N/2 \end{pmatrix}$$

Complex Rabi Frequency

Stafford & Barrett *PRC* **60**, 51305 (1999)

$$P_N(t) = |G_{NS}(t)|^2 = \frac{2V^2}{|\omega|^2} e^{-(\Gamma_N+\Gamma_S)t/2} \left(\cosh \omega_i t - \cos \omega_r t\right)$$

$$\omega \equiv \omega_r + i\omega_i = \sqrt{4V^2 + \left[\Delta - \frac{i}{2}\left(\Gamma_N - \Gamma_S\right)\right]^2}$$

- $\Gamma_N, \Gamma_S \sim .1\mathrm{meV}$
- $V \gtrsim 1\mathrm{eV}$
- $\Delta \sim D_N \equiv 1/\rho(U) \gtrsim 1\mathrm{eV}$

$\Rightarrow$ The nucleus coherently oscillates $\gtrsim 10^4$ times before decaying!

Results

DMC, C. A. Stafford, & B. R. Barrett, PRL 91, 102502 (2003)

Branching ratios

$$F_S = \frac{\Gamma_S}{\Gamma_S + \Gamma_N \Gamma^{\downarrow}/(\Gamma_N + \Gamma^{\downarrow})} = \frac{\Gamma_S}{\Gamma_S + \Gamma_{out}}$$

$$\Gamma^{\downarrow} = \frac{2\bar{\Gamma} V^2}{\Delta^2 + \bar{\Gamma}^2}, \quad \bar{\Gamma} \equiv \frac{\Gamma_S + \Gamma_N}{2}$$

Tunneling width is a measurable quantity

$$\Gamma^{\downarrow} = \frac{F_N \Gamma_N \Gamma_S}{\Gamma_N - F_N(\Gamma_S + \Gamma_N)}$$

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Limiting Cases of $\Gamma^{\downarrow}$

Return, for a moment, to the full ND spectrum. The net tunneling rate throught the barrier is approximated by Fermi's Golden Rule:

$$\Gamma^{\downarrow} = 2\pi \int_{-\infty}^{\infty} V^2 \rho_S(E)\rho_N(E)dE.$$

Two-level limit

$$V \ll D_N \rightarrow \Gamma^{\downarrow} = \frac{2\overline{\Gamma}V^2}{\Delta^2 + \overline{\Gamma}^2}$$

Many-level limit

$$V \gg D_N \rightarrow \Gamma^{\downarrow} = 2\pi \frac{\langle V^2 \rangle}{D_N}$$

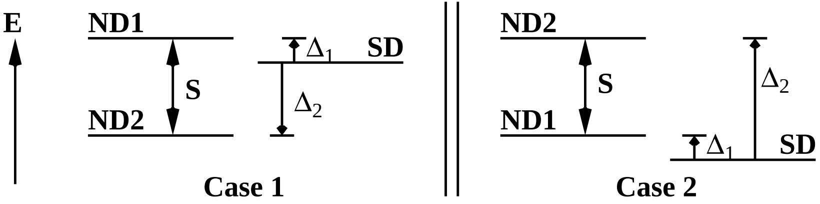
Outline

Gaussian Orthogonal Ensemble

A tool for calculating typical detunings

"Structureless" statistical model for ND states

- Assumes only time-reversal and rotational symmetry for nuclear Hamiltonian.
- Wigner surmise: $P(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4}s^2}, \quad s \equiv \frac{\mathbb{S}}{D_N}$



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$$\mathcal{P}(\Delta_1|s) = \frac{1}{sD_N}\Theta\left(\frac{s}{2} - \frac{|\Delta_1|}{D_N}\right)$$

$$\mathcal{P}(\Delta_2|s) = \frac{1}{sD_N}\Theta\left(\frac{|\Delta_2|}{D_N} - \frac{s}{2}\right)\Theta\left(s - \frac{|\Delta_2|}{D_N}\right)$$

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$$\mathcal{P}(\Delta_{1,2}) = \int_0^\infty ds P(s) \mathcal{P}(\Delta_{1,2}|s)$$

$$\mathcal{P}(\Delta_1) = \frac{\pi}{2D_N} \mathrm{erfc}(\sqrt{\pi}\frac{|\Delta_1|}{D_N})$$

$$\mathcal{P}(\Delta_2) = \frac{\pi}{2D_N}\left[\mathrm{erf}\left(\sqrt{\pi}\frac{|\Delta_2|}{D_N}\right) - \mathrm{erf}\left(\frac{\sqrt{\pi}}{2}\frac{|\Delta_2|}{D_N}\right)\right]$$

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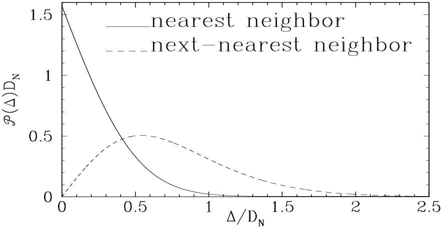

$$\langle|\Delta_1|\rangle = \frac{D_N}{4}$$

$$\langle|\Delta_2|\rangle = \frac{3D_N}{4}$$

Statistical Theory of V

$$\Gamma^{\downarrow} = \frac{2\bar{\Gamma}V^2}{\Delta^2 + \bar{\Gamma}^2} \rightarrow |\Delta| = \sqrt{\frac{2\bar{\Gamma}}{\Gamma^{\downarrow}}\left(V^2 - \frac{\Gamma^{\downarrow}\bar{\Gamma}}{2}\right)} \rightarrow V_{min} = \sqrt{\frac{\Gamma^{\downarrow}\bar{\Gamma}}{2}}$$

$$\mathcal{P}(V) = 2\mathcal{P}(\Delta)\left|\frac{d\Delta}{dV}\right|$$

The most one can say about V with current experiments

$$\mathcal{P}(V \geq V_{min}) = \frac{2\pi\bar{\Gamma}V}{\Gamma^{\downarrow}|\Delta|D_N}\operatorname{erfc}\left(\sqrt{\pi}\frac{|\Delta|}{D_N}\right)$$

$$\langle V\rangle = \sqrt{\frac{\Gamma^{\downarrow}}{2\bar{\Gamma}}\left[\frac{D_N}{4} + \mathcal{O}\left(\frac{\bar{\Gamma}^2}{D_N}\right)\right]}$$

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Extraction of $\langle V \rangle$

	$F_N = P_{out}$	Γ_S (meV)	Γ_N (meV)	D_N (eV)	$\Gamma^{\downarrow}$ (meV)	$\langle V \rangle$ (eV)
$^{192}\mathrm{Pb}(14)$	0.02	0.266	0.201	1,258	0.0056	34
$^{192}\mathrm{Pb}(12)$	0.34	0.132	0.200	1,272	0.10	170
$^{192}\mathrm{Pb}(10)$	0.88	0.048	0.188	1,410	1.9	1000
$^{194}\mathrm{Hg}(12)$	0.40	0.108	21	344	0.072	5.0
$^{194}\mathrm{Hg}(10)$	0.97	0.046	20	493	1.6	35

F_N, Γ_S, Γ_N, and D_N:

- $^{192}\mathrm{Pb}$: Wilson *et al.*, *PRL* **90**, 142501 (2003).
- $^{192}\mathrm{Pb}$: Wilson & Davidson, *PRC* **69**, 41303 (2004).
- $^{194}\mathrm{Hg}$: Lauritsen *et al.*, *PRL* **88**, 042501 (2002).

$\Gamma^{\downarrow}$ for $^{192}\mathrm{Pb}(10)$ is the median value given $\Gamma^{\downarrow} \geq 0$ and $\sigma_{\Gamma_N} = \Gamma_N$.

Outline

Adding a Third Level

Three-level Green function

$$G^{-1} = \begin{pmatrix} E + i\Gamma_S/2 & -V_1 & -V_2 \\ -V_1 & E - \Delta_1 + i\Gamma_N/2 & 0 \\ -V_2 & 0 & E - \Delta_2 + i\Gamma_N/2 \end{pmatrix}$$

Total ND branching ratio

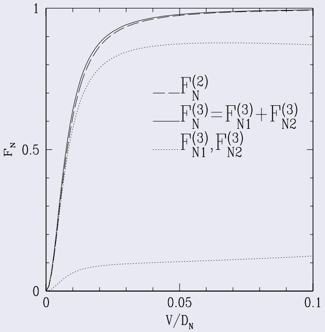
$$F_N = F_{N1} + F_{N2}$$

- Second ND level will take some strength from each of the other levels.
- New possibilty: quantum interference effects.

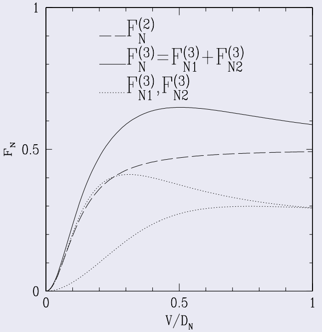
Three-Level Results

Levels taken at their mean detunings

$\Gamma_S/\Gamma_N = 10^{-3}$

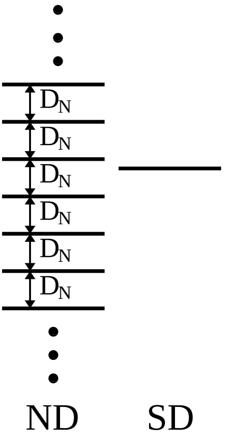


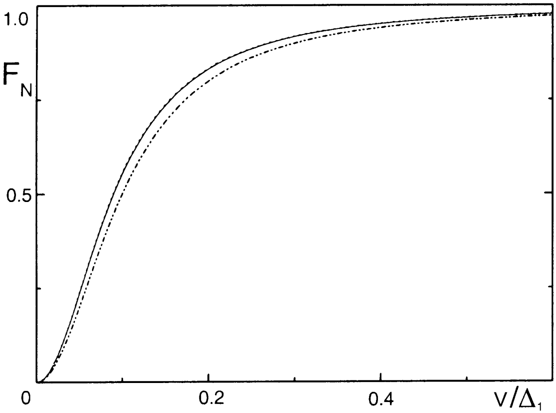
$\Gamma_S = \Gamma_N$



Infinite ND Band Approximation

Dzyublik & Utyuzh, PRC 68, 024311 (2003)





From Dzyublik & Utyuzh. $A \approx 190$. Δ_1 is taken at its mean value in the GOE.

The Shopping List Revisited

- How many states do we need to keep in the ND well?
- How important is electromagnetic broadening?
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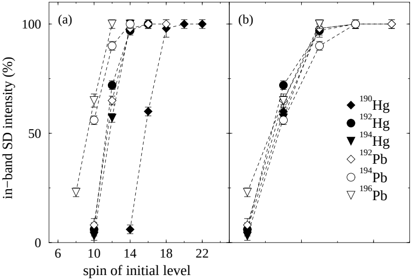
- Why are the decay profiles for $A \approx 190$ so similar?



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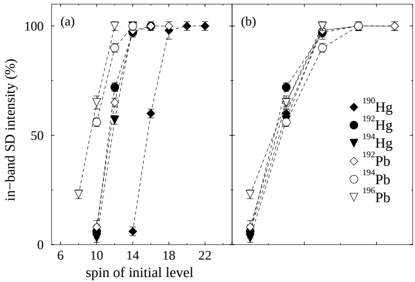

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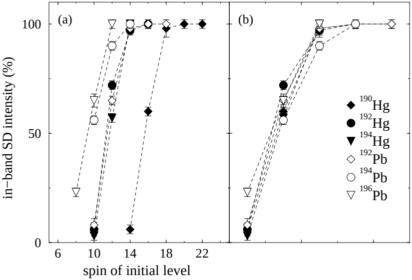

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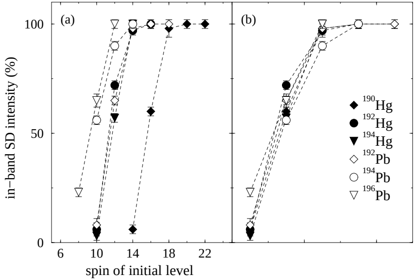
abruptness $\longrightarrow$



$\longleftarrow$ universality!

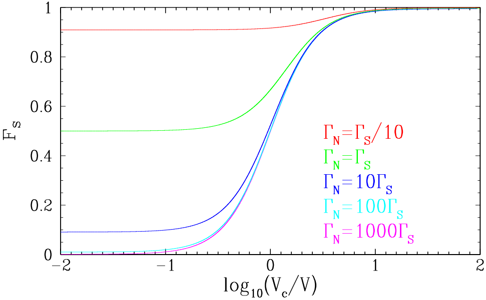
from Wilson *et al.* *PRC*, **71**, 34319 (2005).

Under what Conditions can Decay Occur?

DMC, B. R. Barrett, & C. A. Stafford, nucl-th/0702072.

Rewrite F_S:

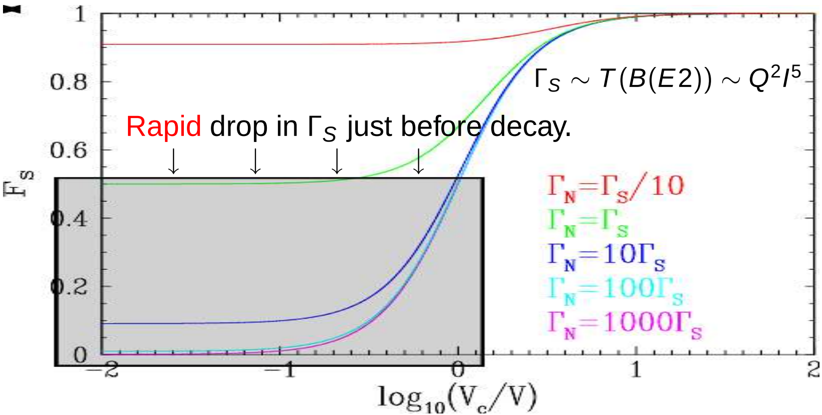
$$F_S = 1 - \frac{1}{1 + (V_c/V)^2 + \Gamma_S/\Gamma_N}, \qquad V_c^2 = \left(\Delta^2 + \overline{\Gamma}^2\right) \frac{\Gamma_S/\Gamma_N}{1 + \Gamma_S/\Gamma_N}$$



- Problem separates naturally into two energy scales.
- Only when conditions are favorable in both can decay occur.
- Γ_N is in competition with Γ_S, V with V_c (renormalized or effective Δ).

$\rightarrow$ Decay occurs only when $V \gtrsim V_c$ and $\Gamma_N \gtrsim \Gamma_S$.

Universality in the 190 Mass Region



$\Gamma_S \sim T(B(E2)) \sim Q^2 I^5$

Rapid drop in Γ_S just before decay.

Summary

- The two-level approximation yields an elegant, exactly solvable model.
- The decay is governed by competition between direct decay down the SD band and a two-step series decay, through the barrier and into the ND band.
- Three and infinite-level models indicate the two-level approximation is extremely accurate, especially for the $A \approx 190$ decay-out.
- Making use of the GOE allows a statistical extraction of V.
- Universality in the two-level model is a natural result of falling Γ_S.