Workshop on New Approaches in Nuclear Many-Body Theory

EFT inspired approaches to solving few- and many-body systems

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Outline

- motivation
- NCSM and EFT: separately and together
- first applications: few-body systems
- the nuclear many-body problem and trapped fermion systems
- renormalization of the interaction (LS, new approaches)
- results: nuclear few bodies & cold atoms
- three cold atoms: untrapped case as a limit of trapped systems
- conclusions and outlook

Motivation

- connection to QCD
- all the current *ab initio* few- and many-body methods have limitations
- need for reliable methods to extrapolate outside the valley of stability

For the NCSM:

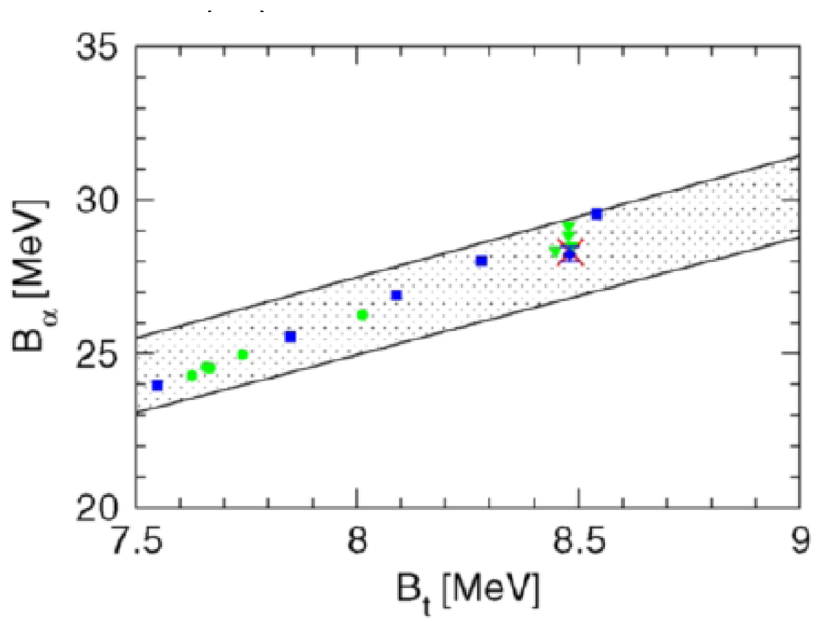
- different types of interaction (motivates the cluster approximation)
- mitigate long- and short-range degrees of freedom (better description of low-momentum observables)

EFT

Original motivation: to understand the gross features of nuclear systems from a QCD perspective.

• separation of scales (integrates out high momenta)
• long-distance physics included explicitly
• short-distance physics added as corrections in powers of relevant scales present in the problem (e.g., for NN interaction at low mom
• general application to other systems (nucleon-cor
• results are *improvable* order by order and *model* .

EFT approach:
- identify relevant degrees of freedom
- identify symmetries
- write the most general Lagrangian (infinite number o
- organize the interaction (power counting)
- adjust parameters to observables
- predictions...



L. Platter et. al., PLB **607**, 254 (2005)

NCSM

- all particles are allowed to interact
- truncation in energy in a HO basis
- usual separation P & Q spaces
- effective interaction constructed via a unitary transformation
- ``cluster approximation''
- short-range effects accounted by the effective interaction
- long-range and many-body effects accounted by increasing the model space

NCSM and (pionless) EFT

Shell model: truncation using a finite number of HO states:

$$P = \sum_{\substack{n,l \\ 2n+l \leq N_{max}}} |nl\rangle\langle nl|$$

(projector into the model space)

EFT without explicit pions:

$$\langle m, l=0|\delta|n, l=0\rangle \sim \left(\frac{n!m!}{\Gamma(n+3/2)\Gamma(m+3/2)}\right)^{1/2} L_n^{1/2}(0) L_m^{1/2}(0)$$

$$\begin{aligned} V_{NN}(\vec{p}, \vec{p}\,') \quad = \quad & C_0^{(S)} + C_0^{(T)} \vec{\sigma_1} \cdot \vec{\sigma_2} \\ & + C_2^{(1)} q^2 + C_2^{(2)} k^2 + \left(C_2^{(3)} q^2 + C_2^{(4)} k^2 \right) \vec{\sigma_1} \cdot \vec{\sigma_2} \\ & + i C_2^{(5)} \frac{\vec{\sigma_1} + \vec{\sigma_2}}{2} (\vec{q} \times \vec{k}) + C_2^{(6)} \vec{q} \cdot \vec{\sigma_1} \vec{k} \cdot \vec{\sigma_2} + \ldots \end{aligned}$$

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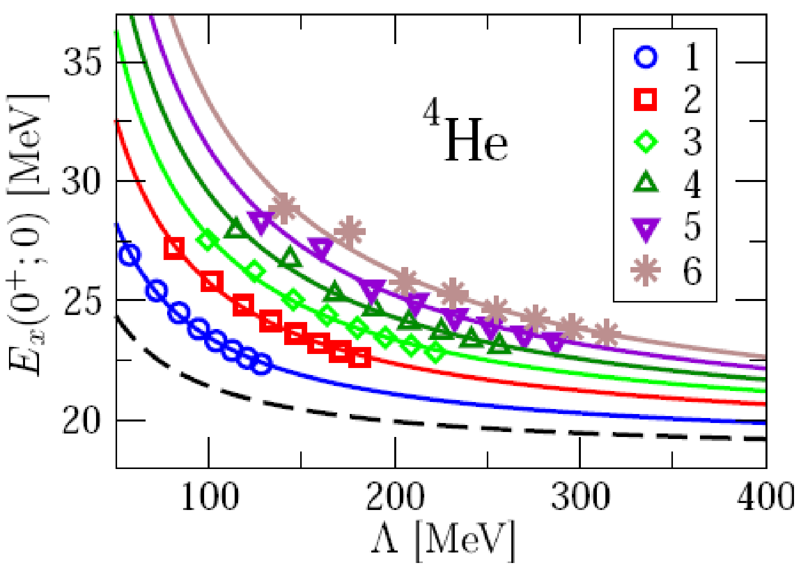
Direct application of EFT to NCSM

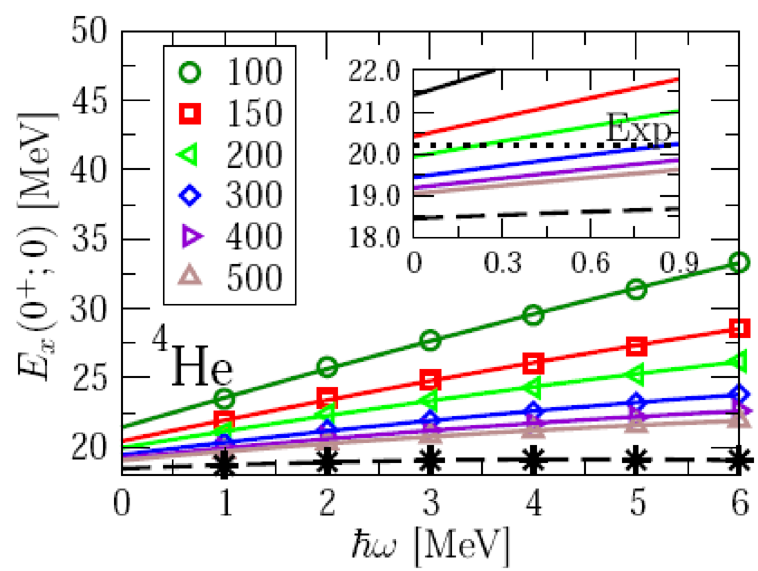
fix the deuteron BE

fix ^{3}H and ^{4}He BE

$$H = \frac{1}{2m_N A}\sum_{[i<j]}(\vec{p}_i - \vec{p}_j)^2$$

$$+C_0^1 \sum_{[i<j]^1} \delta(\vec{r}_i - \vec{r}_j) + C_0^0 \sum_{[i<j]^0} \delta(\vec{r}_i - \vec{r}_j) + D_0 \sum_{[i<j<k]} \delta(\vec{r}_i - \vec{r}_j)\delta(\vec{r}_j - \vec{r}_k)$$





$$\Lambda = \sqrt{2\mu(N_{max} + 3/2)\omega}$$

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Stetcu, Barrett, van Kolck, PLB **653** (2007) 358

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About the new method

- The good:
  - underlying QCD
  - the approximations motivated by general principles
- The bad:
  - LO requires large model spaces
- The ugly:
  - using few-body observables to fix the two-body force quickly becomes cumbersome

Motivation for searching alternatives to fix the two-body interaction

The nuclear many-body problem and trapped atoms

$$H_{int} = \frac{1}{A}\sum_{i>j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m} + \sum_{i>j=1}^{A} V_{ij} + \sum_{i>j>k=1}^{A} V_{ijk} + \ldots$$

$$\begin{aligned} H &= H_{int} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2}mA\omega^2\vec{R}_{CM}^2 \\ &= \sum_{i=1}^{A}\left(\frac{\vec{p}_i^2}{2m} + \frac{1}{2}m\omega^2 r^2\right) + \sum_{i<j=1}^{A}\left(V_{ij} - \frac{m\omega^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right) + \sum_{i>j>k=1}^{A} V_{ijk} + \ldots \end{aligned}$$

Lipkin 1958

$$H_A = \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + C_0\sum_{i<j}\delta^{(3)}(\vec{r}_i-\vec{r}_j)$$

trapped fermions

Interaction renormalization

from addition of the CM term

Unitary transformation (Lee-Suzuki)
[Navratil et. al.]

Effective Field Theory in HO basis
[Stetcu et. al.]

$$h = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + C_0 V_{12}$$

$$h^{eff} = U h U^\dagger$$

(preserves lowest *D* eigenvalues of *h*)

truncation introduces many-body forces

- form of interaction preserved
- the coupling constants adjusted in each model space to reproduce some observables

truncation introduces higher-order terms

PRESENT METHOD COMBINES BOTH APPROACHES

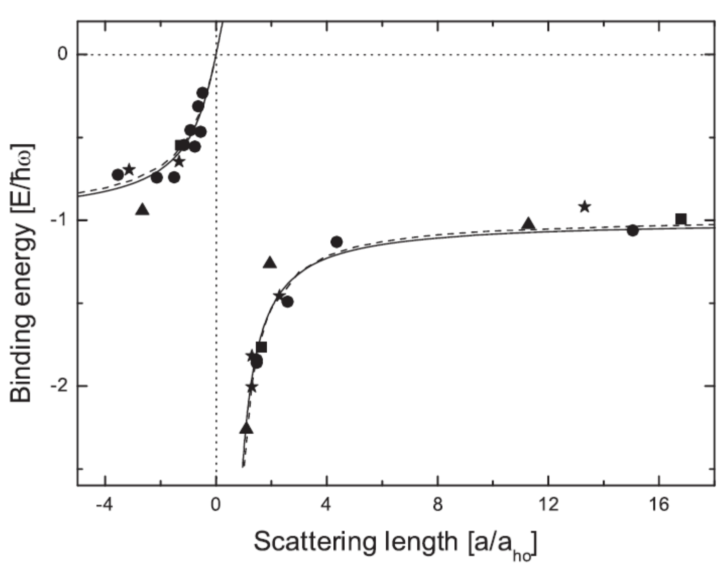
adjust C_0's to reproduce as many eigenvalues (but *no* unitary transformation)

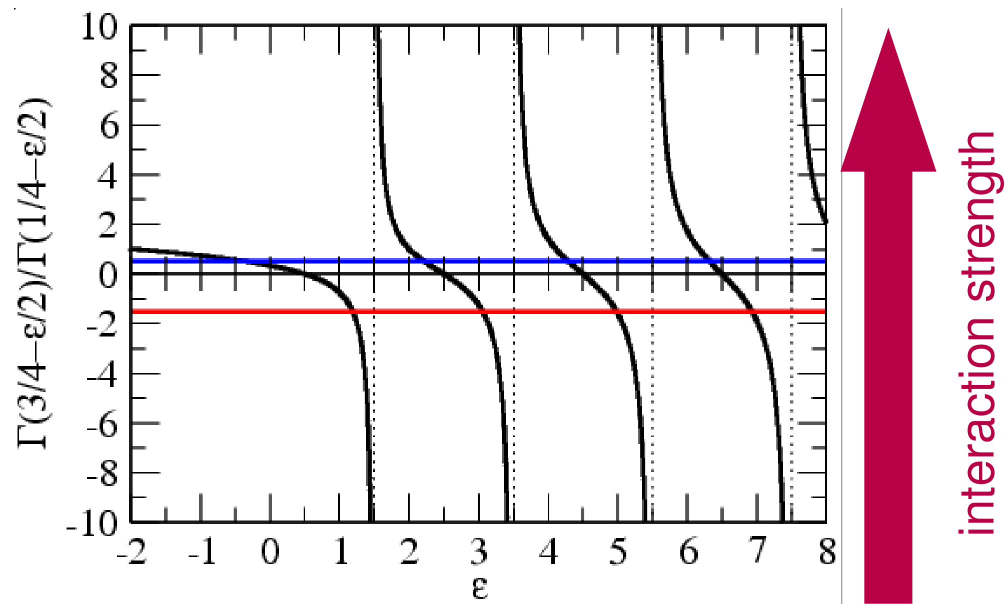
preserve the form of the interaction (power counting)

Trapped molecules: experiment and theory

Eingenvalues: $$\frac{\Gamma(3/4 - E/2\omega)}{\Gamma(1/4 - E/2\omega)} = \frac{b}{2a_2}$$

scattering observable





T. Busch et. al., Found. Phys. **28** (1998) 549

T. Stöferle et. al., Phys. Rev. Lett. **96** (2006) 030401

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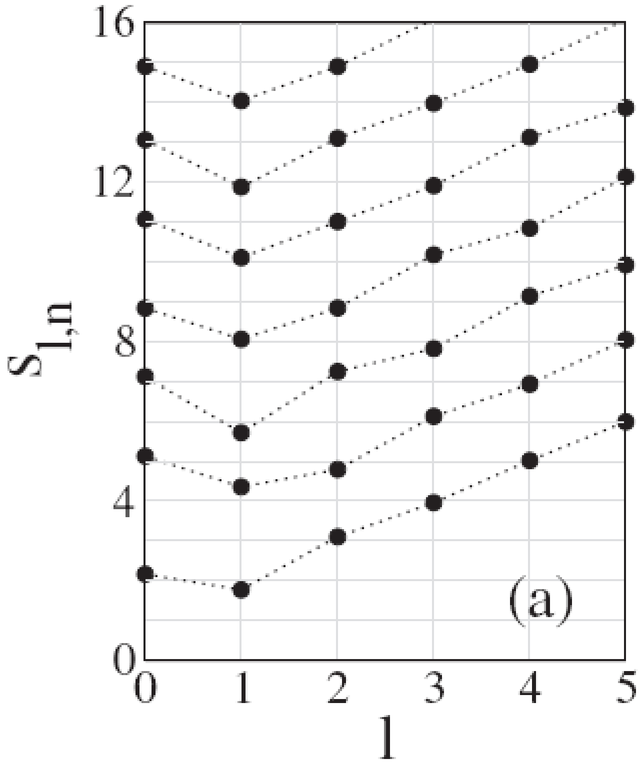
Three-body solution in unitary limit (analytical results)

Solve the free Schrodinger Eq. w/ boundary condition:

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) A(\mathbf{R}_{ij}, \mathbf{r}_k) + O(r_{ij})$$

$$E = E_{\text{c.m.}} + (s_{l,n} + 1 + 2q)\hbar\omega.$$



[F. Werner and Y. Castin, Phys. Rev. Lett. **97**, 150401 (2006)]

Scales involved

- trapping potential of frequency ω: $b=1/(\mu\omega)^{1/2}$
- free-space two-body scattering length: a_2
- range of the interaction: r_0

Assumptions:

- $b>>r_0$
- $a_2>>r_0$
- arbitrary b/a_2

- short-range physics: approximated as a series of contact interactions + derivatives
- observables: expressed as expansions in powers of Qr_0
- three-body and many-body forces

Two-body renormalization

Start with the two-body Schrodinger Equation:

$$\left[b^2p^2 + \frac{r^2}{b^2} + 2\mu C_0 b^2 \delta^{(3)}(\vec{r})\right]\psi(\vec{r}) = 2\frac{E}{\omega}\psi(\vec{r})$$

Finite-space eigenvalues:

$$\frac{1}{C_0(N_{max},\omega)} = -\sum_{n=0}^{N_{max}/2} \frac{|\phi_n(0)|^2}{(2n+3/2)\omega - E}$$

E_0

$N_{max} \to \infty$

Pseudo-potential eigenvalues:

$$\frac{\Gamma(3/4 - E/2\omega)}{\Gamma(1/4 - E/2\omega)} = \frac{b}{2a_2}$$

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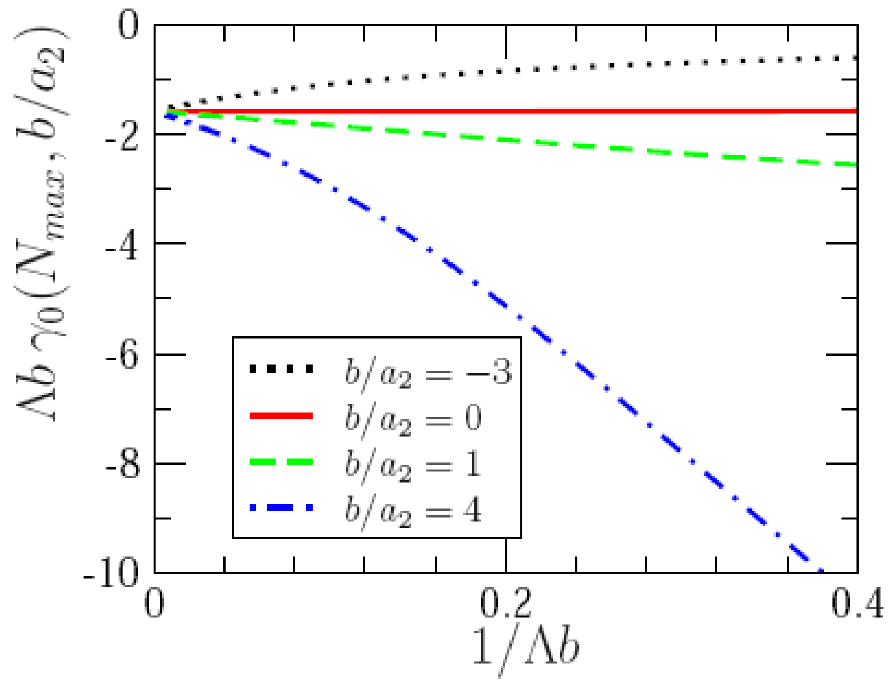
Running of the coupling constant



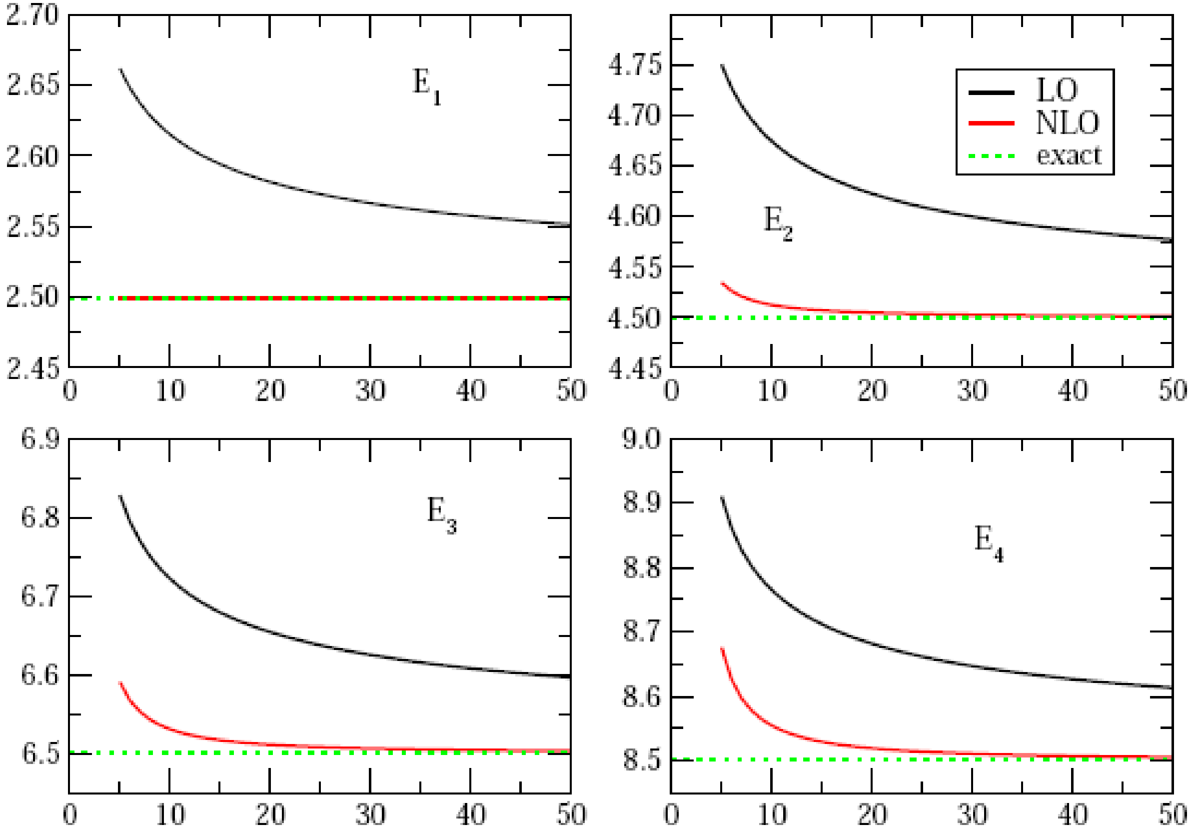
$$\gamma_0(N_{max}, b/a_2) = \frac{\mu}{2\pi b} C_0(N_{max}, \omega)$$

$$\Lambda = \sqrt{2\mu(N_{max} + 3/2)\omega}$$

NLO

Running of two-body energies of trapped two particles in unitary limit

E_0 (g.s.) fixed in LO, E_1 fixed in NLO



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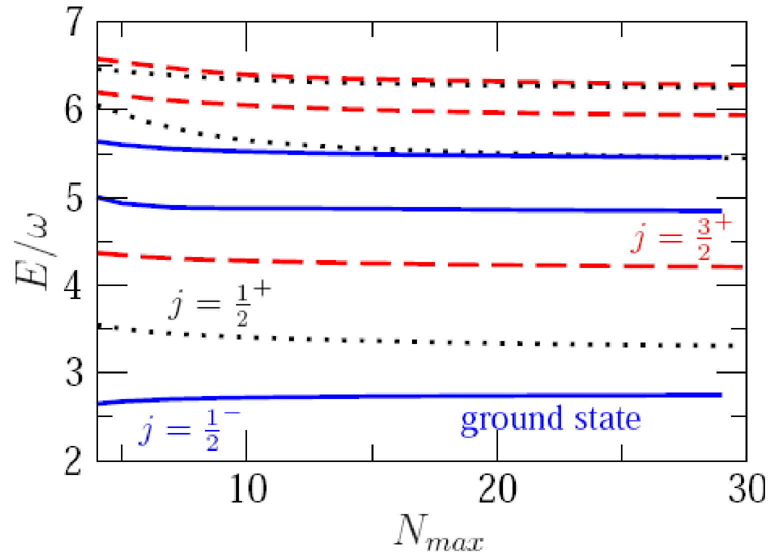
Three-body system in unitary limit
(benchmark against analytical results)

$$E = E_{\mathrm{c.m.}} + (s_{l,n} + 1 + 2q)\hbar\omega$$



(unperturbed levels not shown)

TABLE I: Comparison between the results of the present approach (E_∞/ω) and of the semi-analytical formula from Ref. [13] (Eq. (10)), for $j^\pi = \frac{1}{2}^-$. Extrapolation errors on the last figure are shown in parantheses; where absent, the errors are too small.

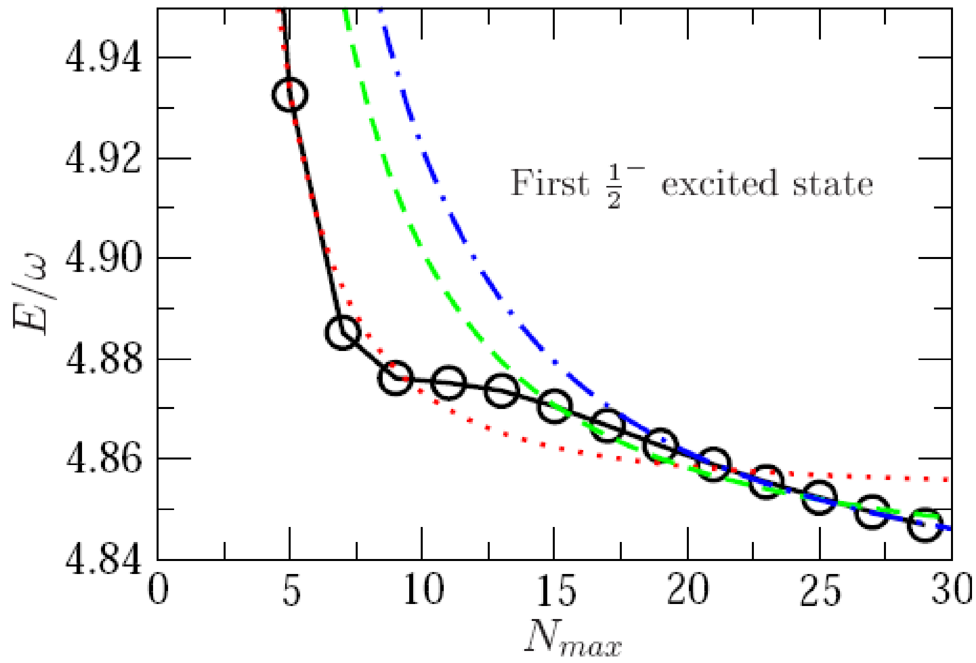
n	l	q	s	Eq. (10)	E_∞/ω
0	1	0	1.77	2.77	2.76
0	1	1	1.77	4.77	4.71(2)
1	1	0	4.36	5.36	5.39

Variation with N_{max}: about 10%

(error because of missing higher order terms in the expansion)

Ultraviolet behavior

Missing higher order terms in the truncated space: ultraviolet dependence of observables which are not fixed.



$$E = E_\infty + \frac{E_c}{(N_{max} + 3/2)^\alpha}$$

$$\Lambda = \sqrt{2\mu(N_{max} + 3/2)\omega}$$

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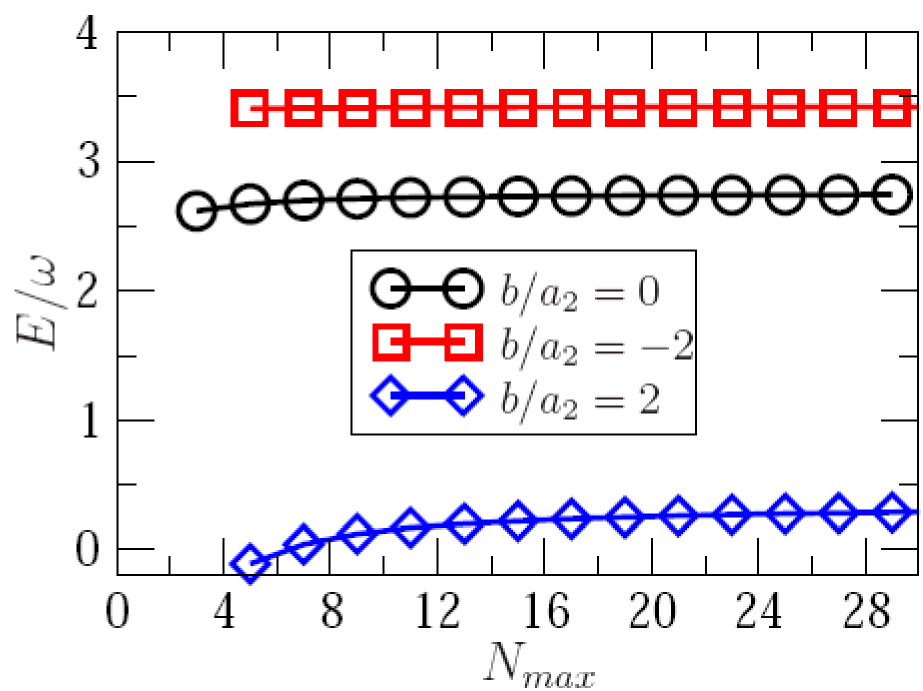
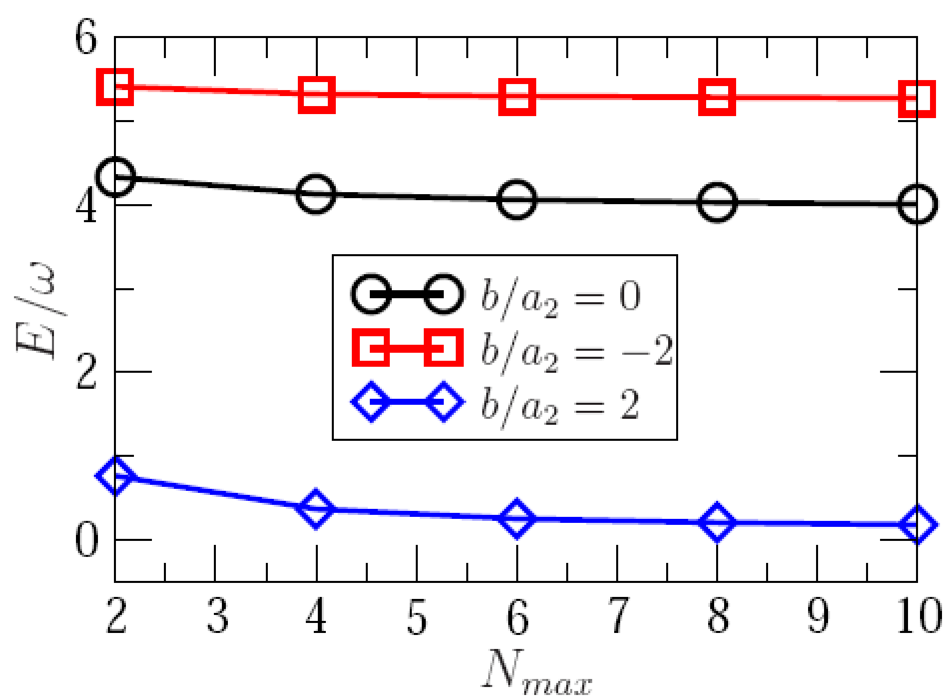
Away from unitarity



Running of the lowest positive-parity state for the four-body system.



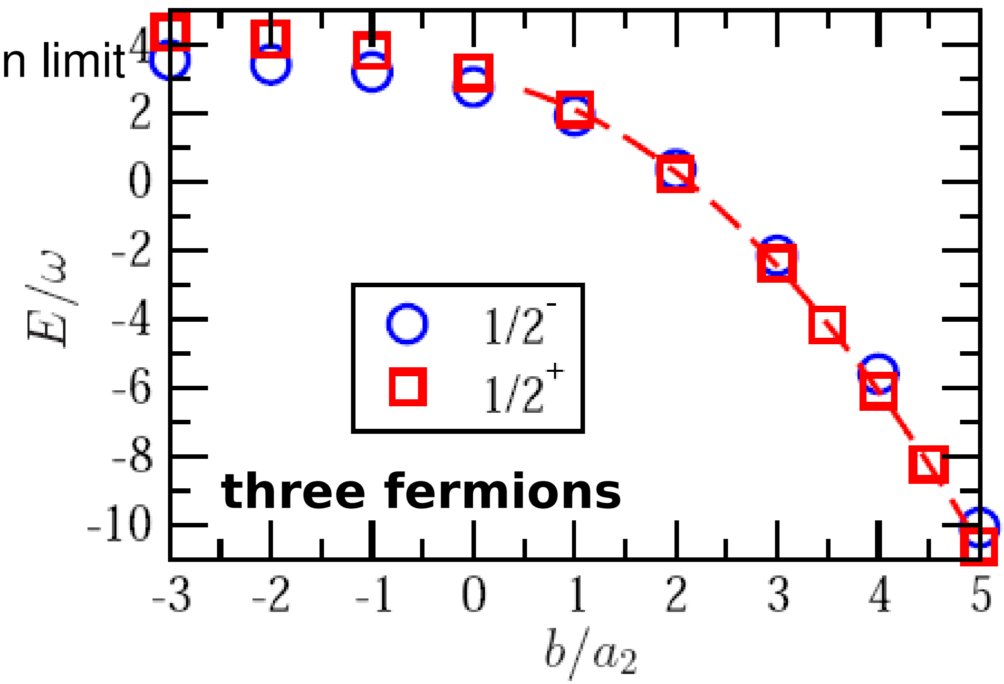
Running of the lowest negative-parity state for the three-body system.

Stetcu et. al., arXiv:0705.4335

Large b/a_2 range

Stetcu et. al., arXiv:0705.4335

no interaction limit



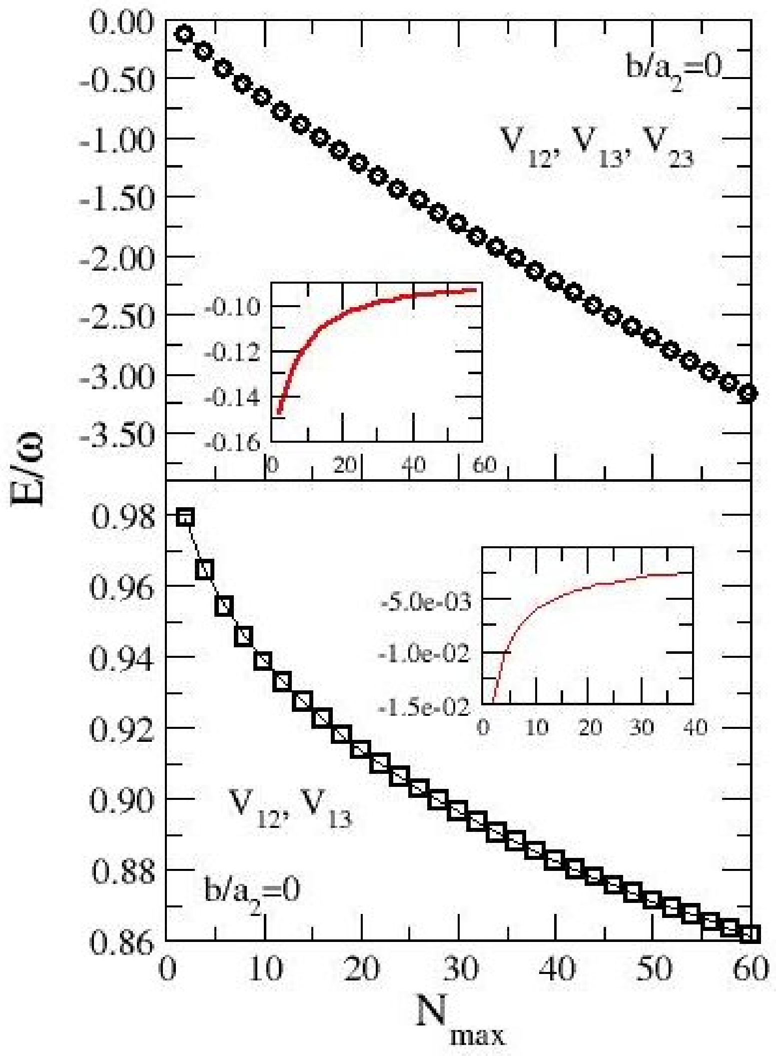
untrapped limit

Untrapped particle limit: $b/a_2 \to \infty$

three-particle energy: $$E \simeq -\frac{1}{2\mu a_2^2}$$

four-particle energy: $$E \sim 0.6\left(-\frac{2}{2\mu a_2^2}\right)$$ (large errors)

Three distinguishable particles



Summary and outlook

First application of the NCSM and EFT to few particles:

- ✓ formulation of NCSM truncation as a EFT
- ✓ renormalization of the two-body interaction (based on power counting)
- ✓ results at the unitary limit and away from it
- ✓ continuum limit (from trapped systems)

To do:

- obtain more precise solutions for four particles
- include next-to-leading corrections
- extend to bosons and arbitrary spin fermions (power counting modifies)
- **further application to few-body problems in nuclear physics**

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