Recent developments in NN and 3N interactions for nuclear structure

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TRIUMF Theory Group

New Approaches in Nuclear Many-Body Theory, INT, Oct. 15, 2007

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Outline

1. NN interactions: Chiral EFT and RG

2. Two-body operators

3. Three-nucleon interactions: a frontier - status and challenges

4. Nuclear forces from lattice QCD

5. Convergence in nuclear structure calculations

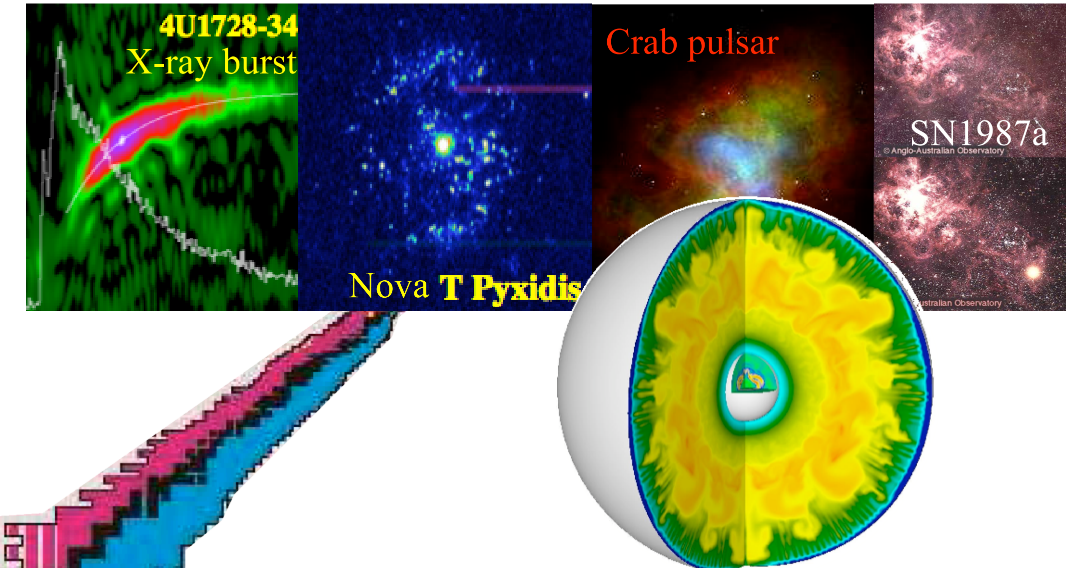
Strong interaction physics in the lab and cosmos

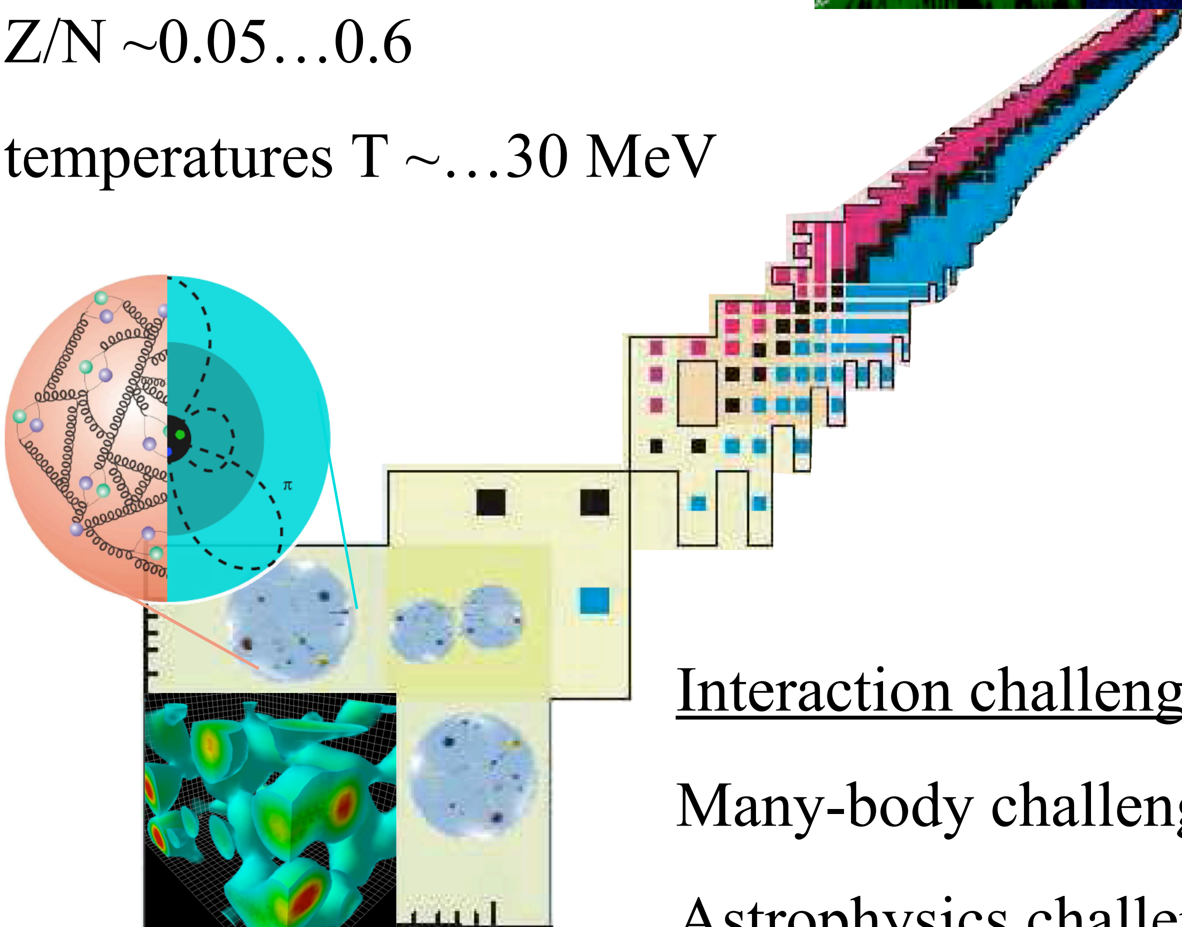
Matter at the extremes:

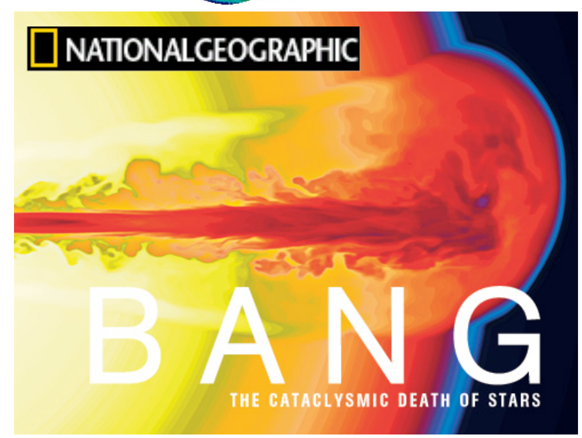
density $\rho \sim 10^{11} \ldots 10^{15}$ g/cm^3

neutron-rich to proton-rich
Z/N ~0.05…0.6

temperatures T ~…30 MeV







<u>Interaction challenges</u>

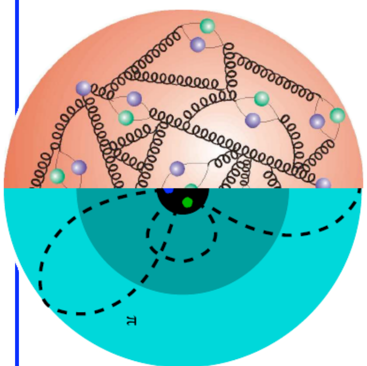
Many-body challenges

Astrophysics challenges

Resolution   Scale dependence of nuclear interactions

with high-energy probes:
deconfined quarks+gluons

cf. scale/scheme dependence
of parton distribution functions



Lattice QCD

Effective theory for NN, many-N interactions, operators depend on resolution scale Λ

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory

nucleons interacting via pion exchanges and contact interactions

typical Fermi momenta in nuclei $\sim m_\pi$

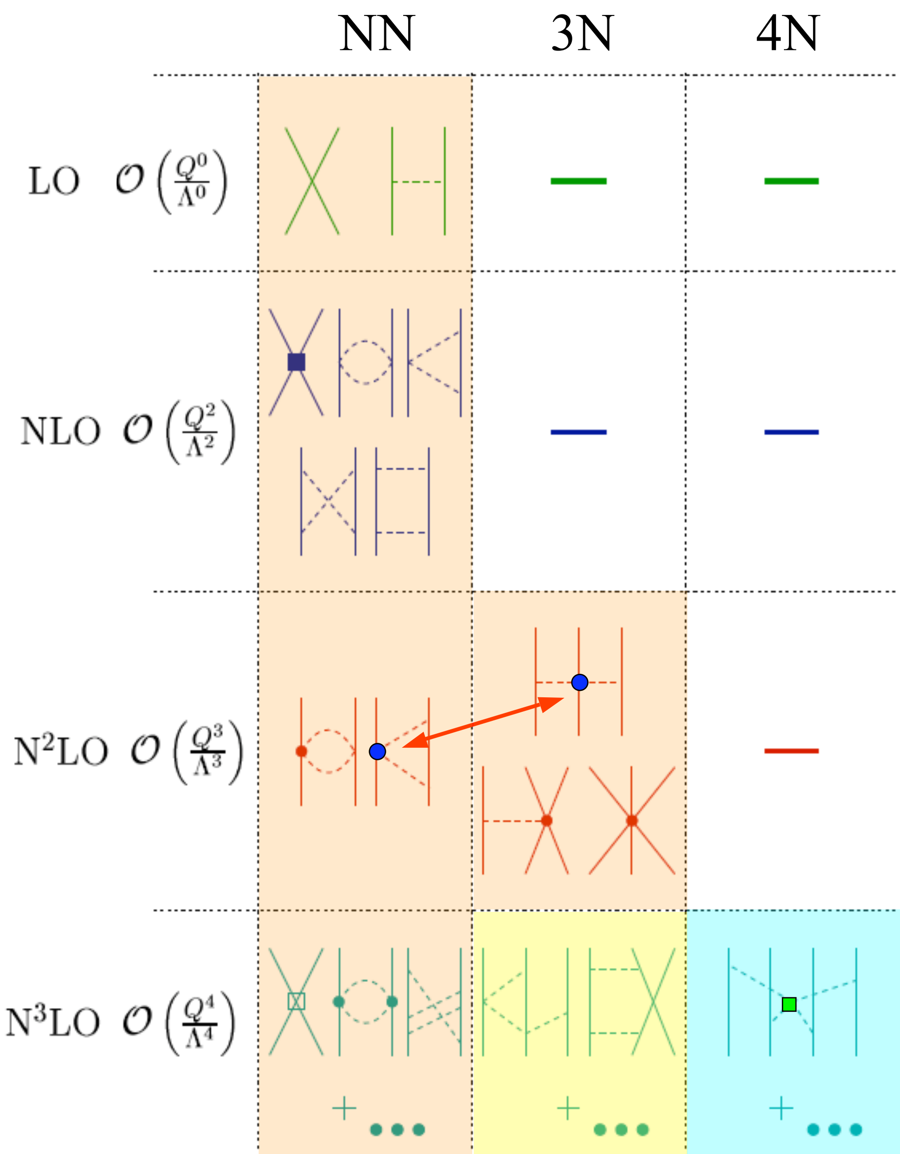
$Q \ll m_\pi$=140 MeV - pion not resolved:
pionless effective field theory

nucleons and contact interactions, large scattering lengths + corrections

applicable to loosely-bound, dilute systems, reactions at astro energies

Chiral effective field theory (EFT) for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale Λ_b



limited resolution at low energies, can expand in powers Q/Λ_b

details of short-distance physics not resolved

capture in few short-range couplings, fit to experiment once

include long-range physics explicitly (pions for chiral EFT)

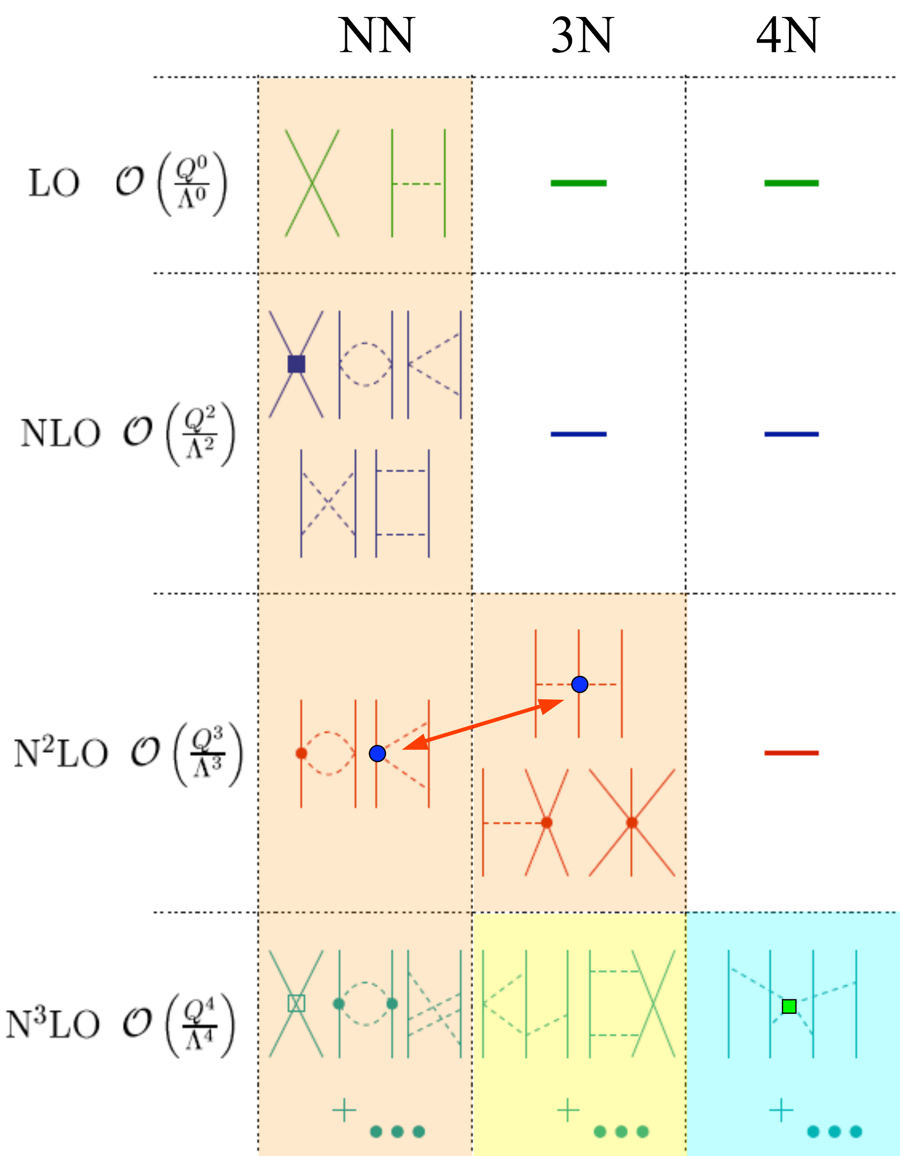
systematic: can work to desired accuracy and obtain error estimates

can connect to lattice QCD

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Chiral effective field theory (EFT) for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale Λ_b

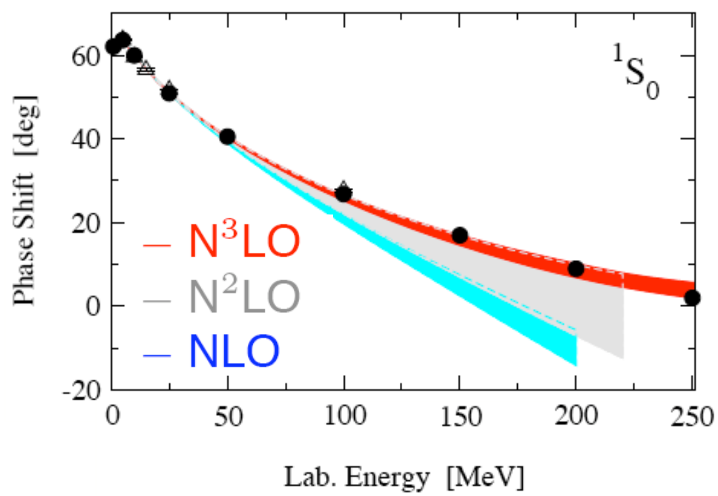


explains pheno hierarchy:
NN > 3N > 4N > …

NN-3N, πN, ππ, electro-weak,…
consistency

3N,4N: 2 new couplings to N^{3}LO!

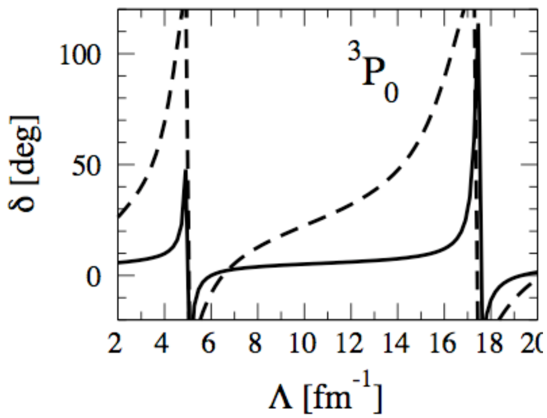
theoretical error estimates



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Open problems

Power counting with singular pion exchanges, tensor parts ~ $1/r^n$?
promotion of contact interactions to lower order (needed beyond …LO?)
Nogga, Timmermans, van Kolck (2005)



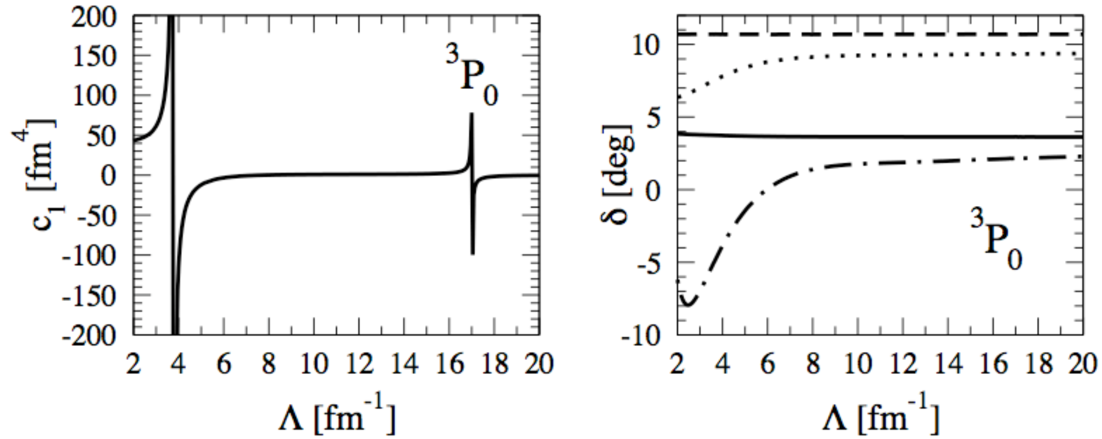


FIG. 10: Fit result for the counterterm c_1 as a function of the cutoff, and the resulting cutoff dependence of the $^3P_0$ phase shift at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

Delta-full vs. Delta-less EFT, $m_\Delta - m_N \sim m_\pi$?

Counting of 1/m corrections, $m \sim \Lambda^2$ or Λ? could be important for A_y

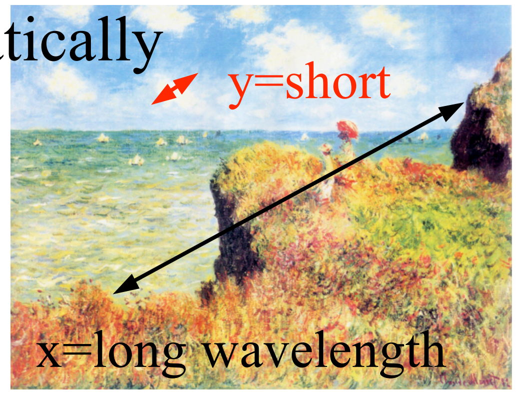
Explore breakdown scale value with Lepage plots

……

Renormalization group (RG) for nuclear forces

integrate out high-momentum modes that are not resolved
and incorporate their effects in couplings of effective theory

schematically



$$Z = \int dx \int dy \, e^{-S(x,y)} = \int dx \int dy \, e^{-a(x^2+y^2)-b(x^2+y^2)^2}$$

$$= \int dx \, e^{-S_{\text{eff}}(x)} = \int dx \, e^{-a'x^2-b'x^4-c'x^6+\ldots}$$

separate into slow and fast modes $\phi(\omega,k) = \begin{cases} \phi_<(\omega,k) & \text{for } \omega, k < \Lambda \\ \phi_>(\omega,k) & \text{else} \end{cases}$
and integrate out fast modes

$$Z = \int \prod d\phi_<(\omega,k) \, e^{-S_{\text{free}}[\phi_<]} \int \prod d\phi_>(\omega,k) \, e^{-S_{\text{free}}[\phi_>]-S_{\text{int}}[\phi_<,\phi_>]}$$

$$= \int \prod d\phi_<(\omega,k) \, e^{-S_{\text{eff}}[\phi_<]}$$

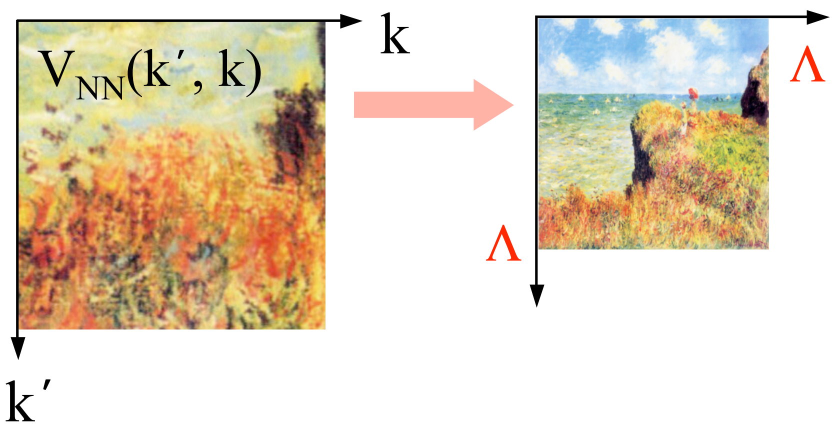
leads to resolution/Λ-dependent couplings, Λ is not breakdown scale

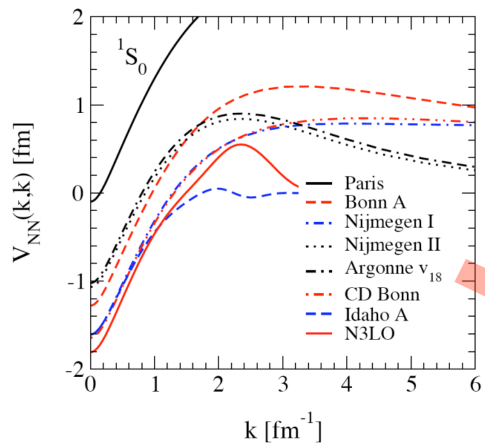
applied to pionless EFT $C_0(\Lambda) = \dfrac{4\pi}{m} \dfrac{1}{\frac{1}{a_s} - \frac{2}{\pi}\Lambda}$

low-energy physics determined by scattering length a_s

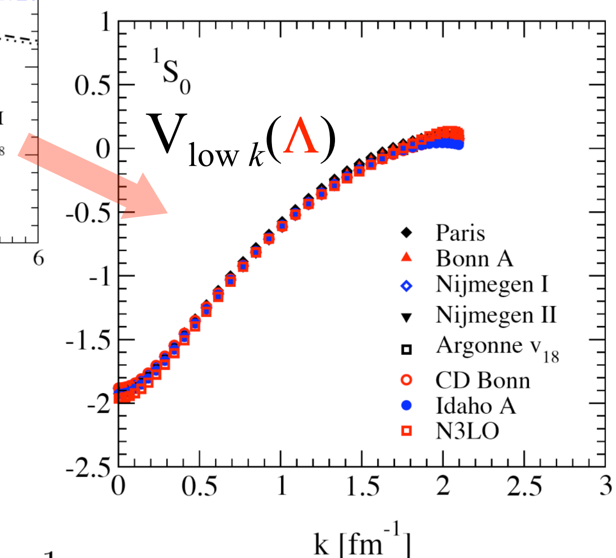
Low-momentum interactions from the Renormalization Group

evolve to lower resolution/cutoffs by integrating out high-momenta, can be carried out exactly for NN interactions





reproduces low-energy NN observables

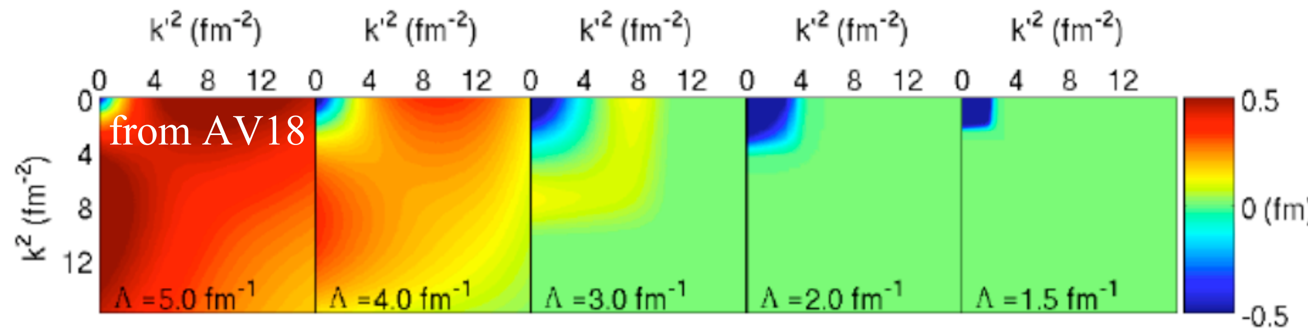


implemented by discretized RG equation or equivalent Lee-Suzuki transformation

universal low-momentum $V_{\text{low }k}(\Lambda)$ for $\Lambda \lesssim 2.1\,\text{fm}^{-1}$

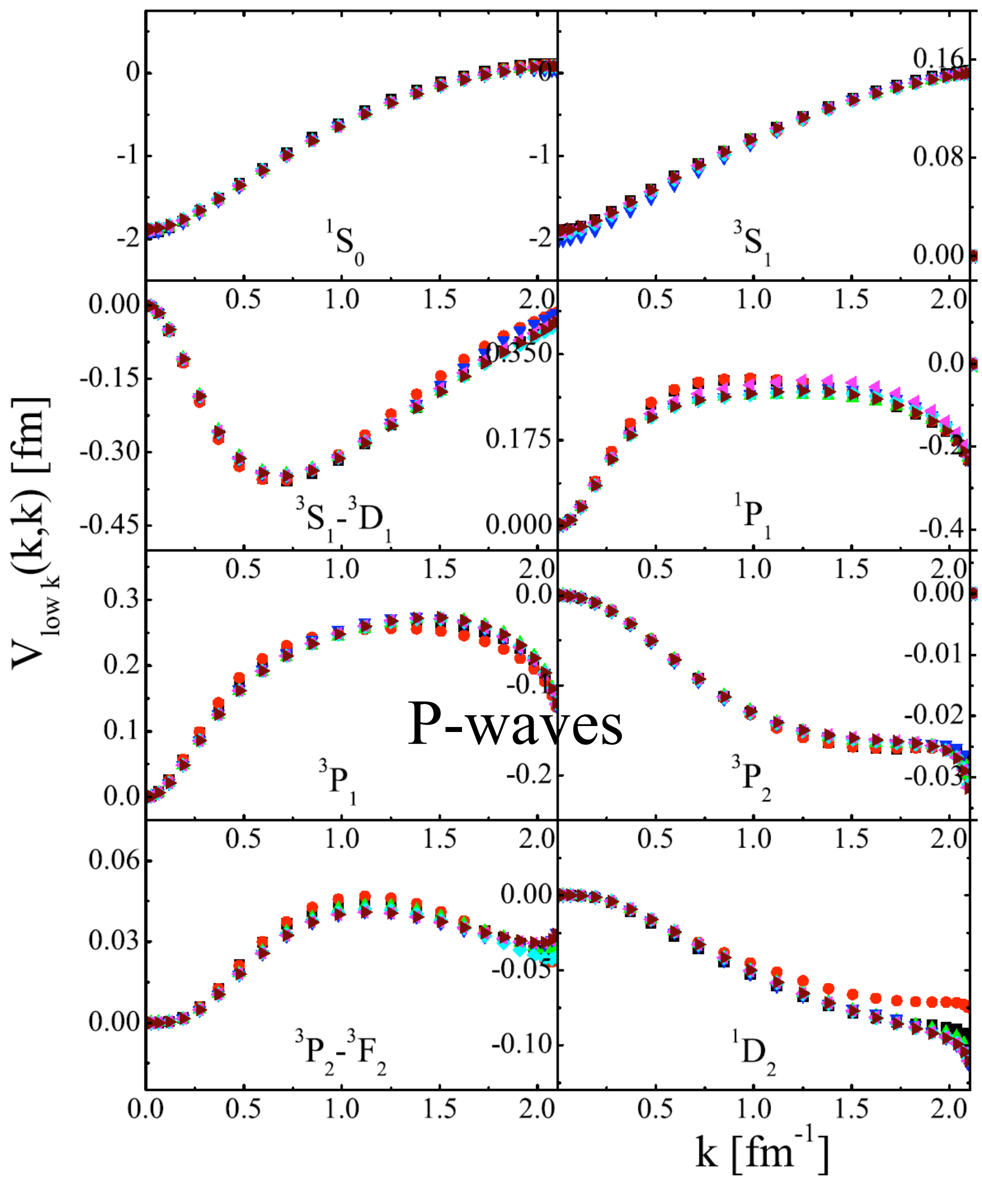
Bogner, Kuo, AS (2003); Coraggio, Covello et al.

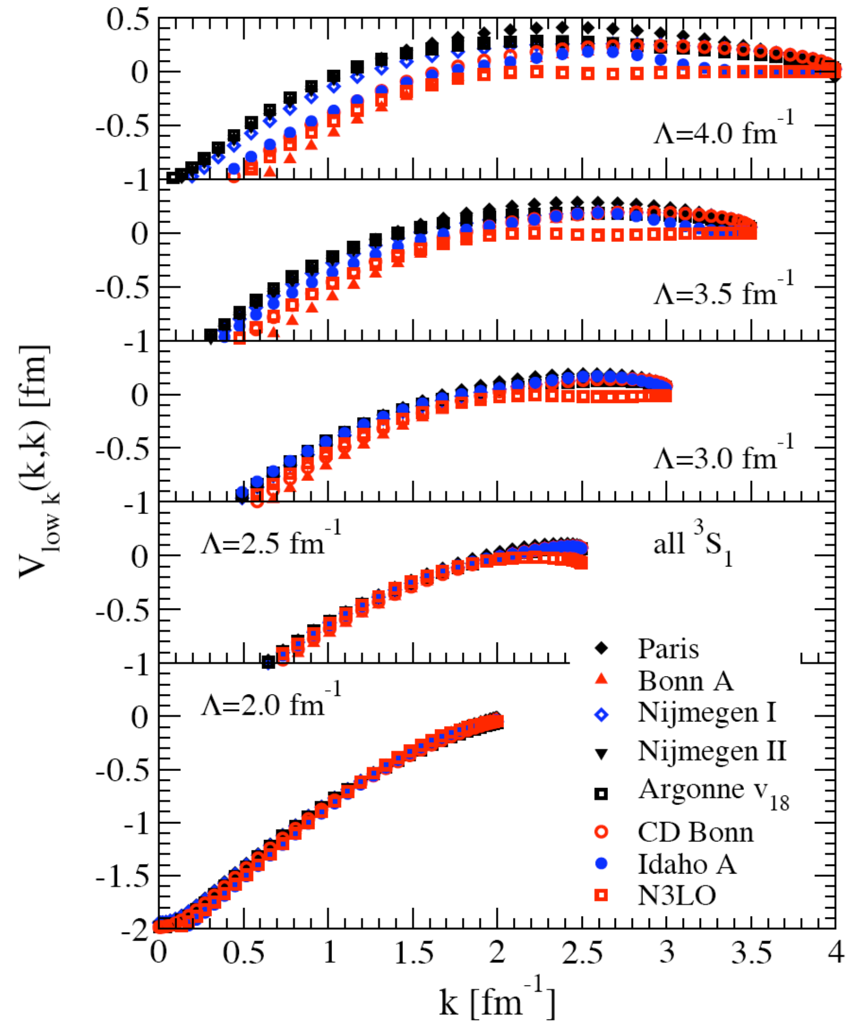
evolution to $V_{\text{low }k}(\Lambda)$ weakens off-diagonal coupling



Collapse in all partial waves

due to same long-distance pion exchange and phase shift equivalence





small differences correlate with fits to data

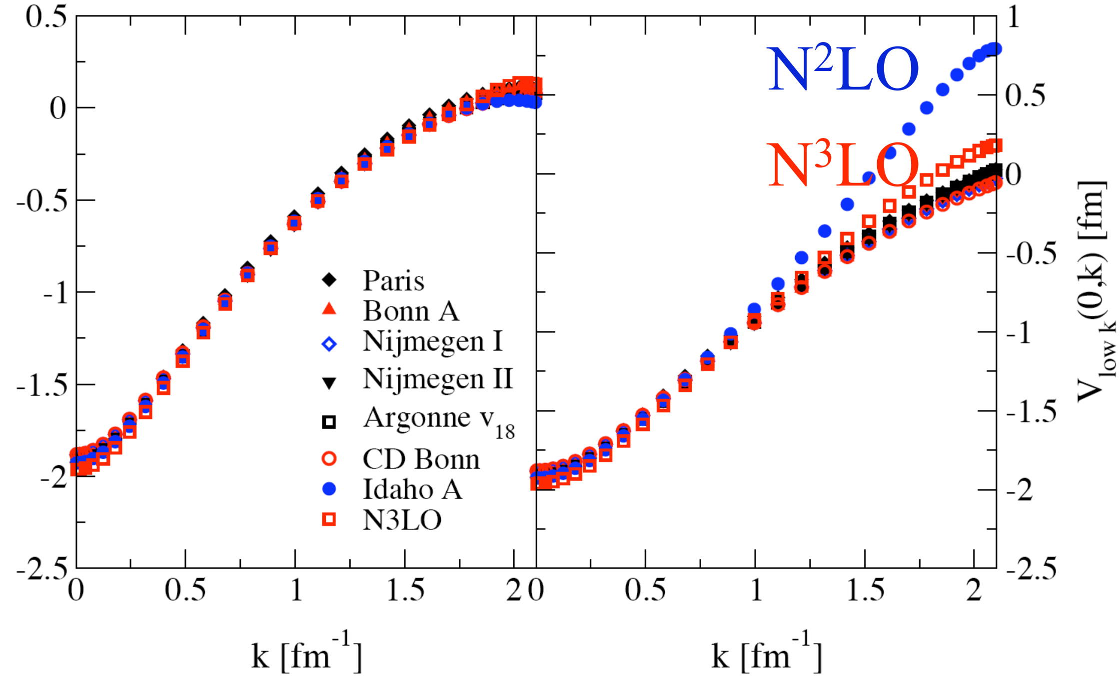
$V_{low\,k}(\Lambda)$ defines class of NN interactions

Connections to EFT

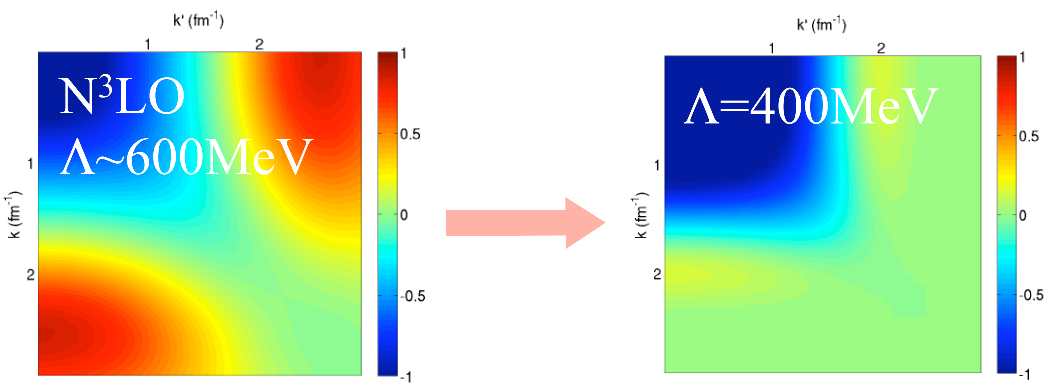
Collapse to $V_{\text{low } k}$ band for higher orders in chiral EFT

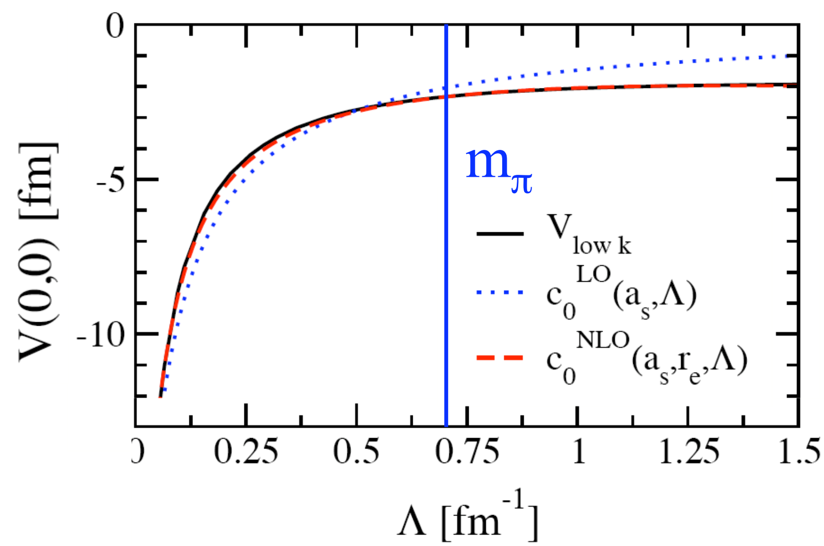
Renormalization generates higher-order contacts

Evolution of $V_{\text{low } k}(0,0;\Lambda)$ follows contact interaction $c_0(\Lambda)$ at NLO



Evolution of chiral EFT interactions to low-momentum beneficial





Weinberg eigenvalue diagnostic

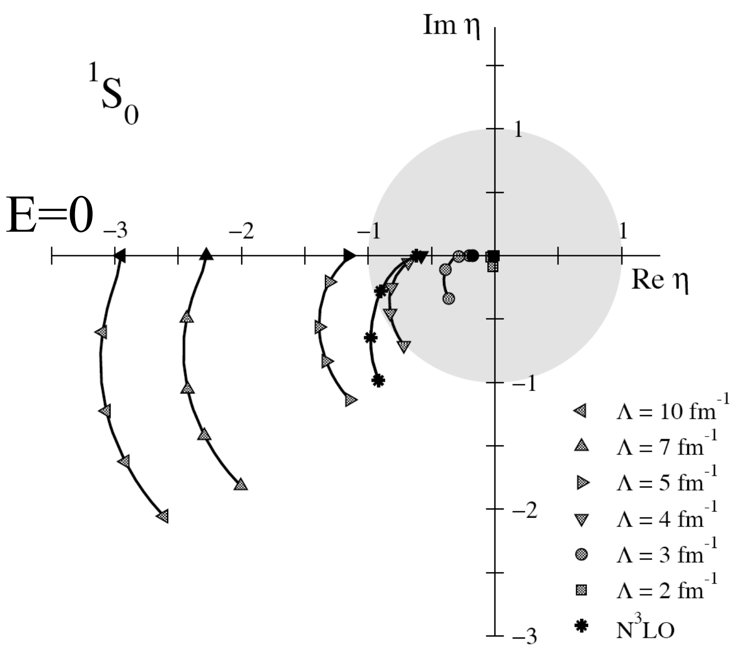
study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

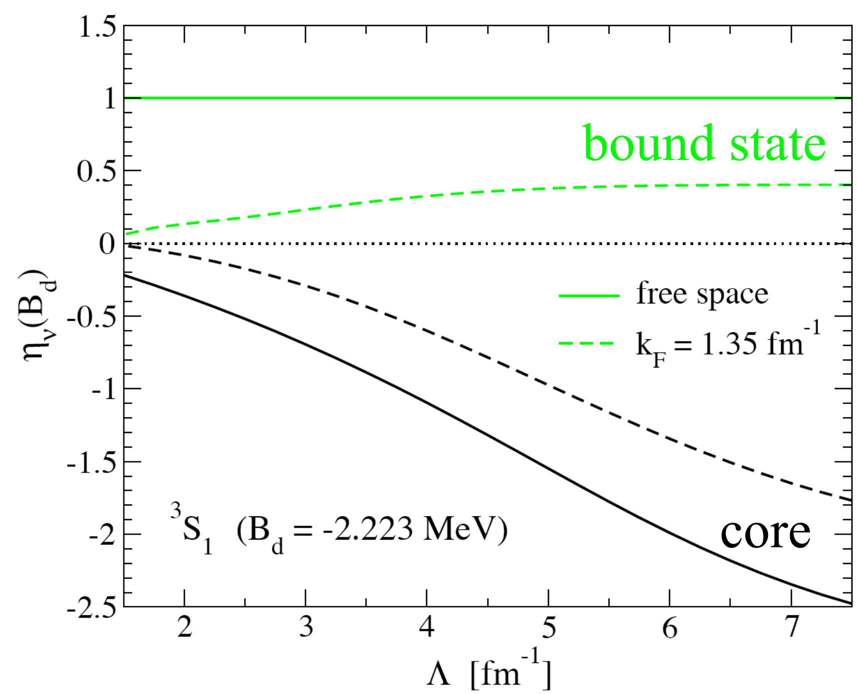
governs convergence $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

can write as Schrödinger eqn $\left(H_0 + \frac{1}{\eta_\nu(z)}V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

high momenta/large cutoffs lead to flipped-potential bound states of -λV for small λ/large η ⇒ Born series always nonperturbative with cores

Repulsive core eigenvalues small for lower cutoffs

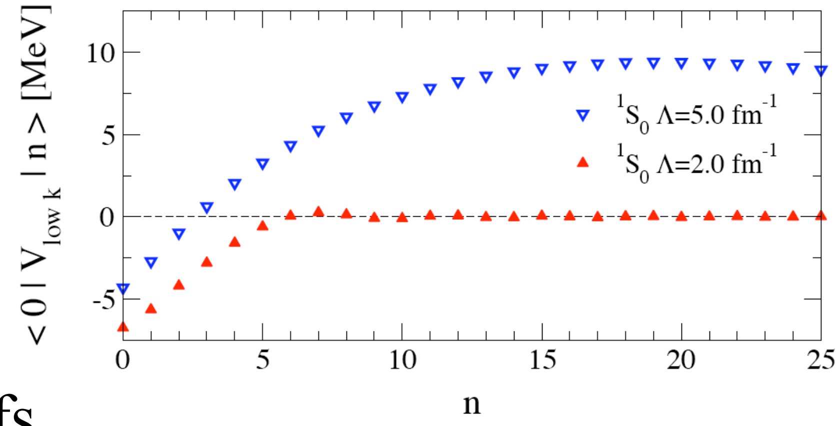




Bogner, AS, Furnstahl, Nogga (2005)

Advantages of lower cutoffs for nuclear structure

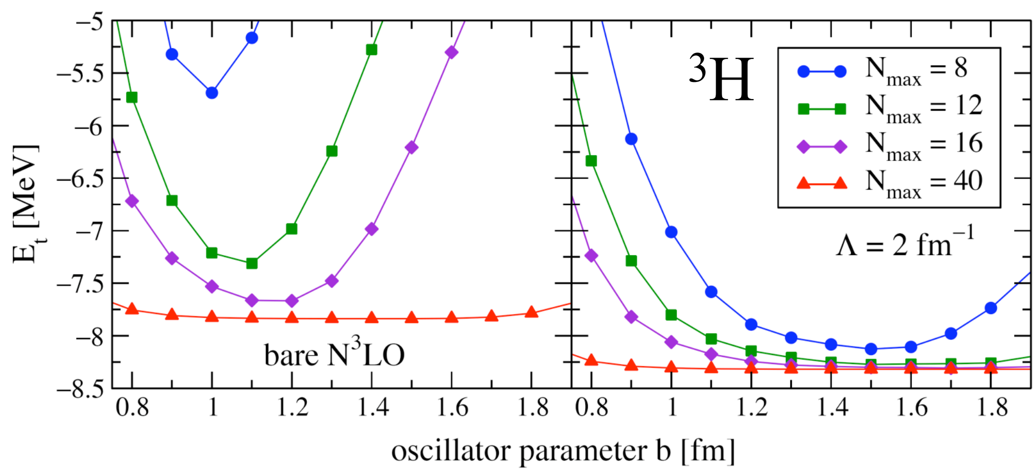
tractable in an oscillator basis

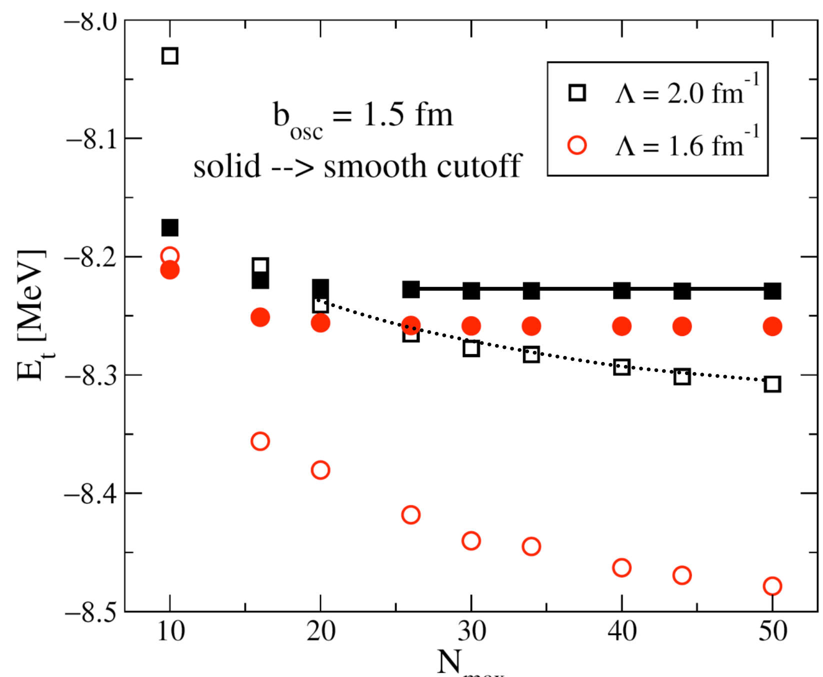


direct convergence for structure calculations

slow at 10-100keV level for sharp cutoffs

smooth cutoffs lead to great improvements Bogner, Furnstahl, Ramanan, AS (2007)

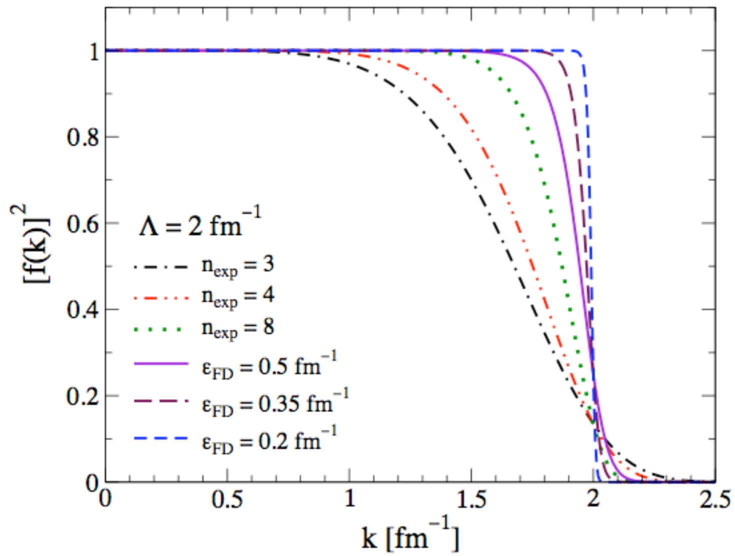




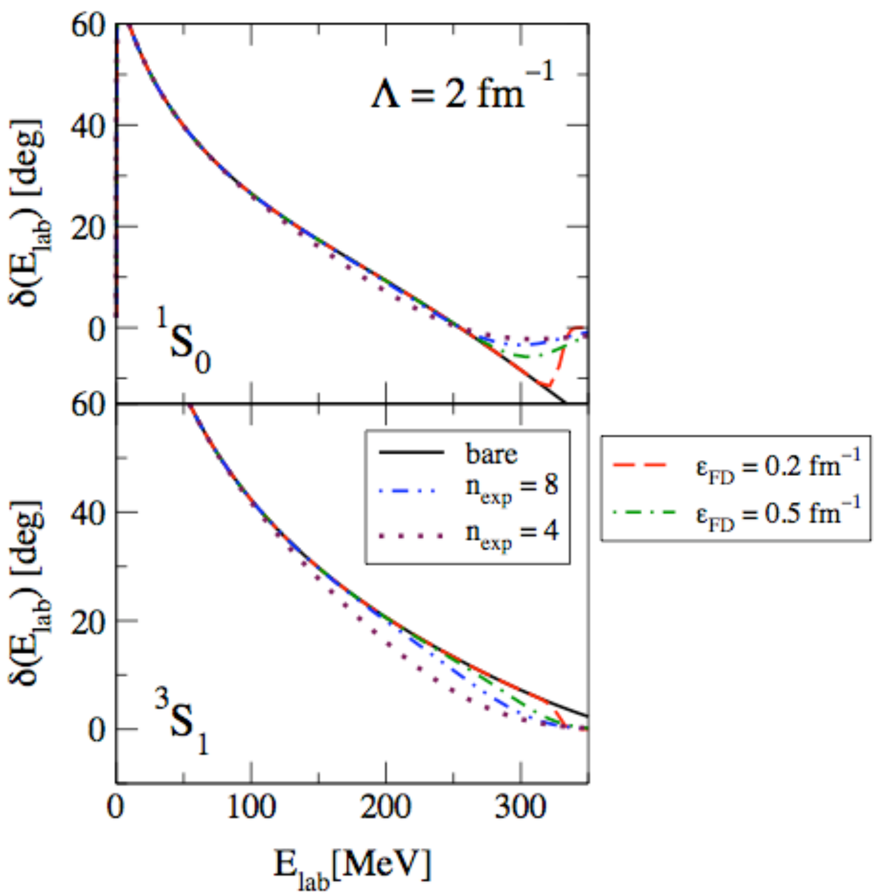
Low-momentum interactions with smooth cutoffs

P $V_{\text{low }k}(\Lambda)$ P replaced by smooth regulators Bogner, Furnstahl, Ramanan, AS (2007)

$$V_{\text{low }k}(k', k) = f(k')\, v(k', k)\, f(k)$$



constructed by effective interaction methods
or equivalent smooth-cutoff RG equation



regulator choice = scheme dependence
exp, Fermi-Dirac,… regulator

smooth cutoff $V_{\text{low }k}(\Lambda)$ reproduces
NN observables up to $f^2(k)$ factor

Similarity RG interactions (SRG)

Unitary transformations to band-diagonal $V_{srg}(\lambda)$ with cutoff $\lambda = s^{-1/4}$

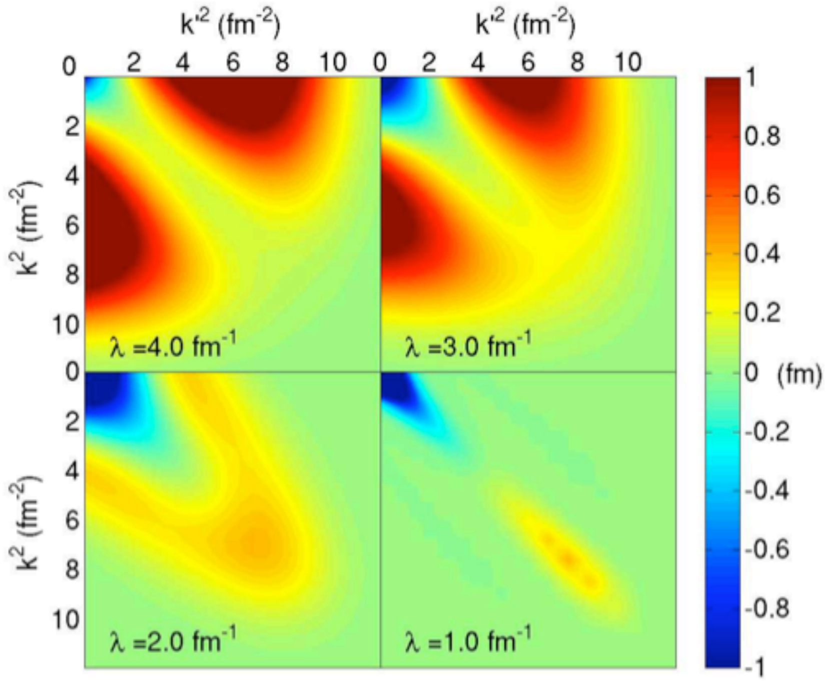
Bogner, Furnstahl, Perry (2007) evolution of 3N seems feasible

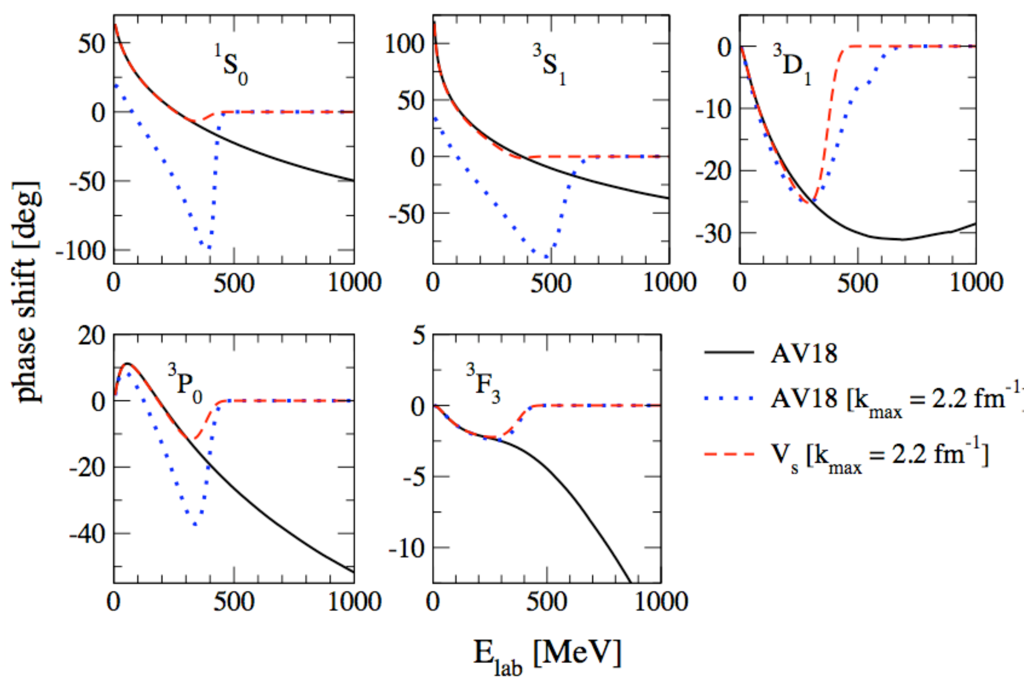
$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)^2\, V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2 + k'^2 - 2q^2)\, V_s(k,q)\, V_s(q,k')$$

reproduces all NN observables

low momentum transfer around the diagonal

intermediate momenta $k > k_{max} \sim \lambda$ can be neglected for low energies



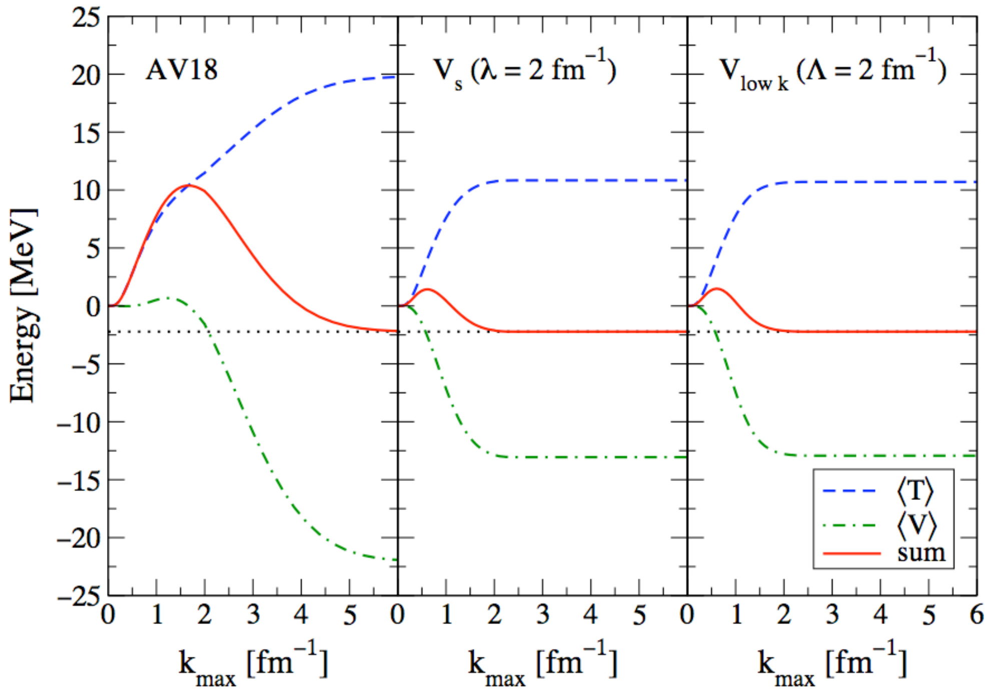


Are low-energy nuclear observables sensitive to high-energy phase shifts?

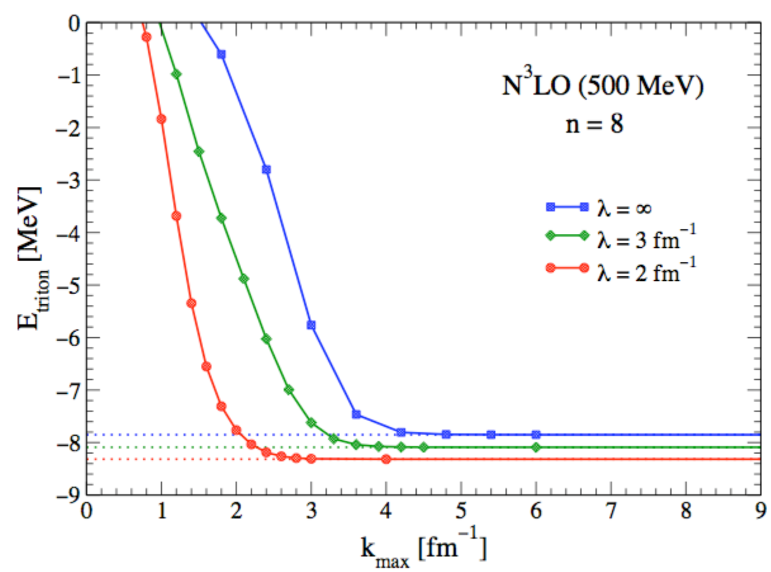
No, but one could be mislead by large cutoffs Bogner, Furnstahl, Perry, AS (2007)

SRG decouples high- from low-energy physics, while reproducing all NN

high-momentum components (and high-energy phase shifts) can be set to zero when using low-momentum interactions, without loss of information



deuteron and triton binding



Decoupling: $V_{srg}(\lambda)$ with $k_{max} \sim \lambda \Rightarrow$ smooth $V_{\text{low }k}(\Lambda)$ with $\Lambda \sim \lambda$

keep diagonal high momenta if you worry about high-energy phase shifts

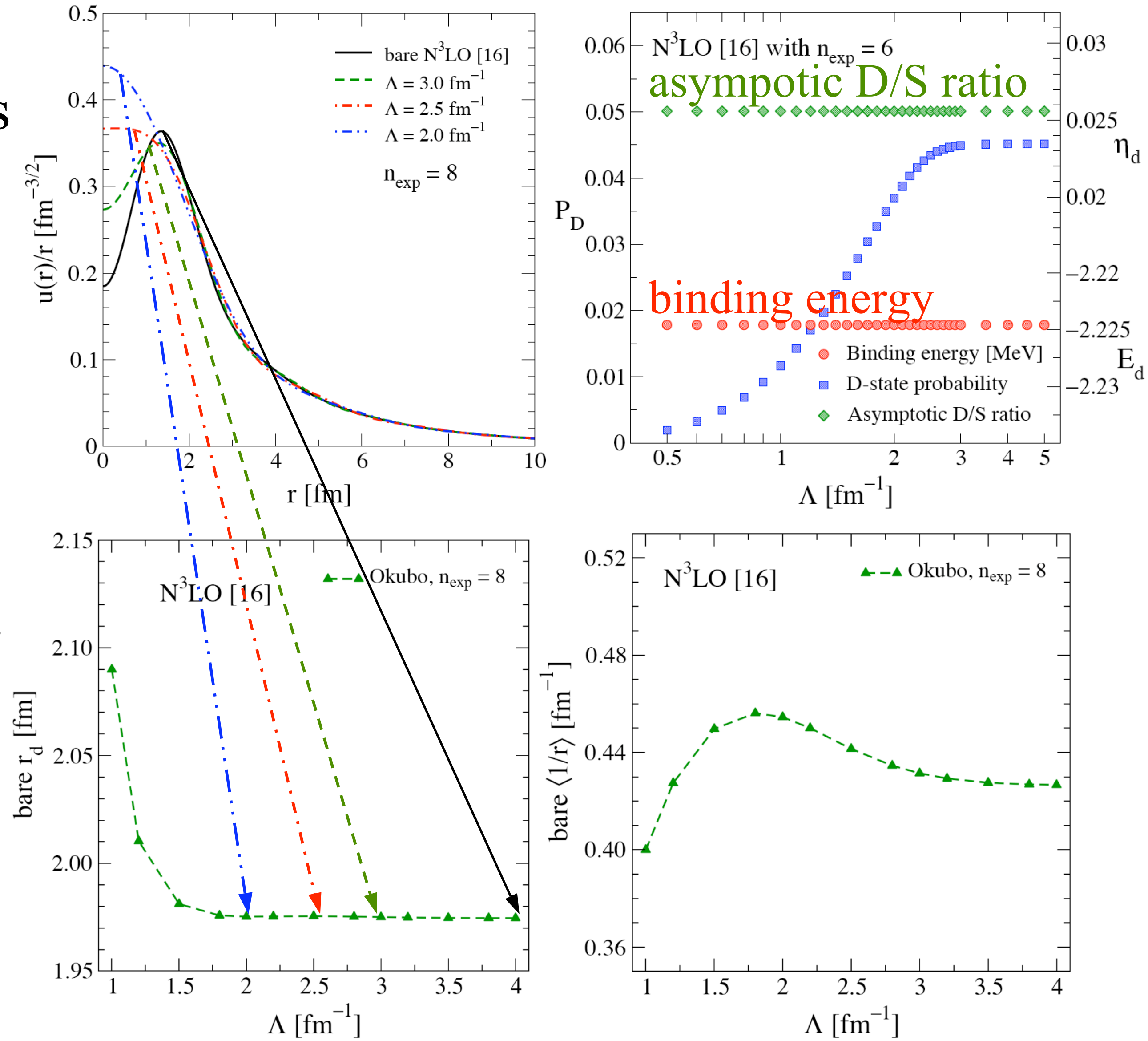
Short-range correlations ?? ?

RG preserves long-range parts of interactions, deuteron observables, with dramatically different wave functions/correlations

short-range correlations depend on resolution scale, cf. parton df

weak renormalization of long-range operators r, Q, 1/r,…

short-range operators are never “bare” (exchange currents,…)

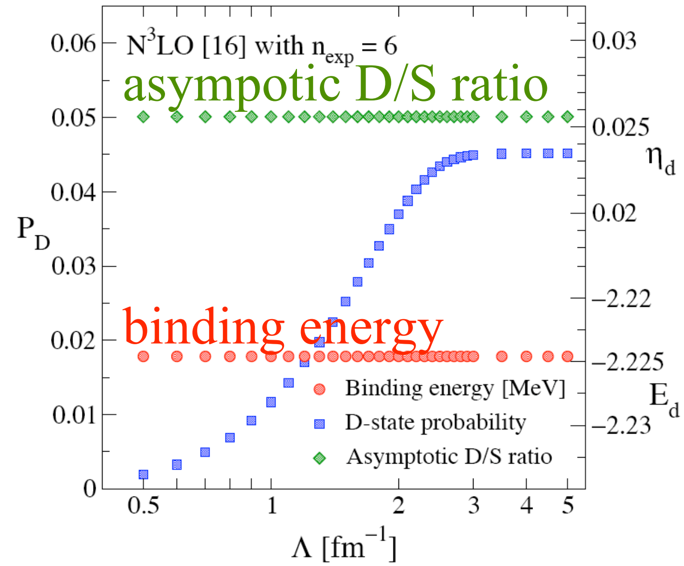


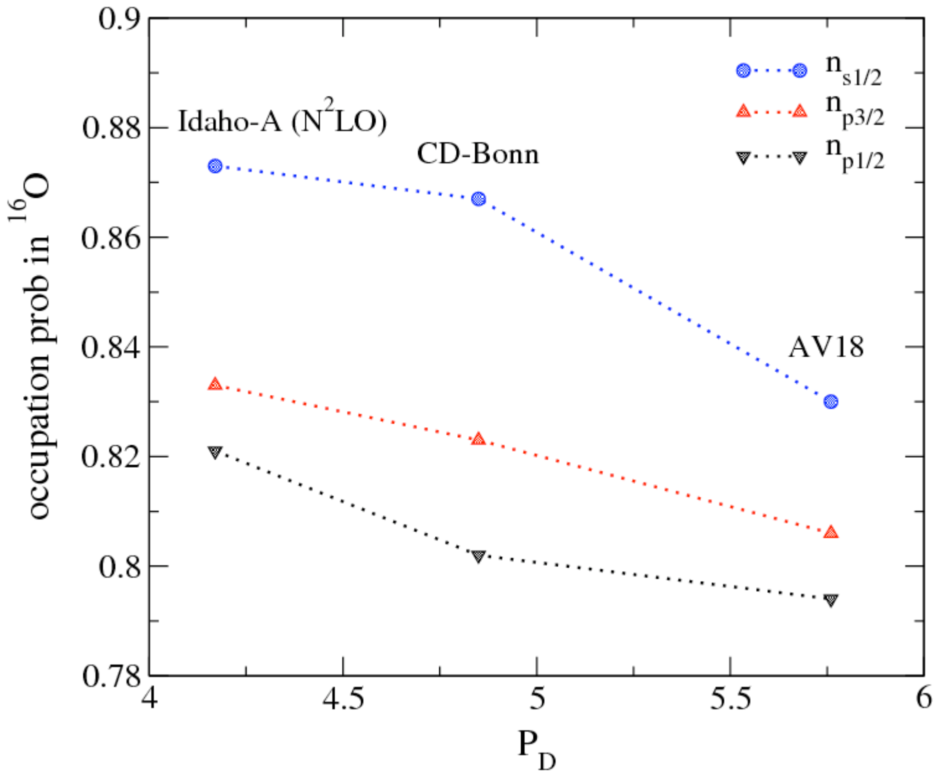
Occupation numbers ??

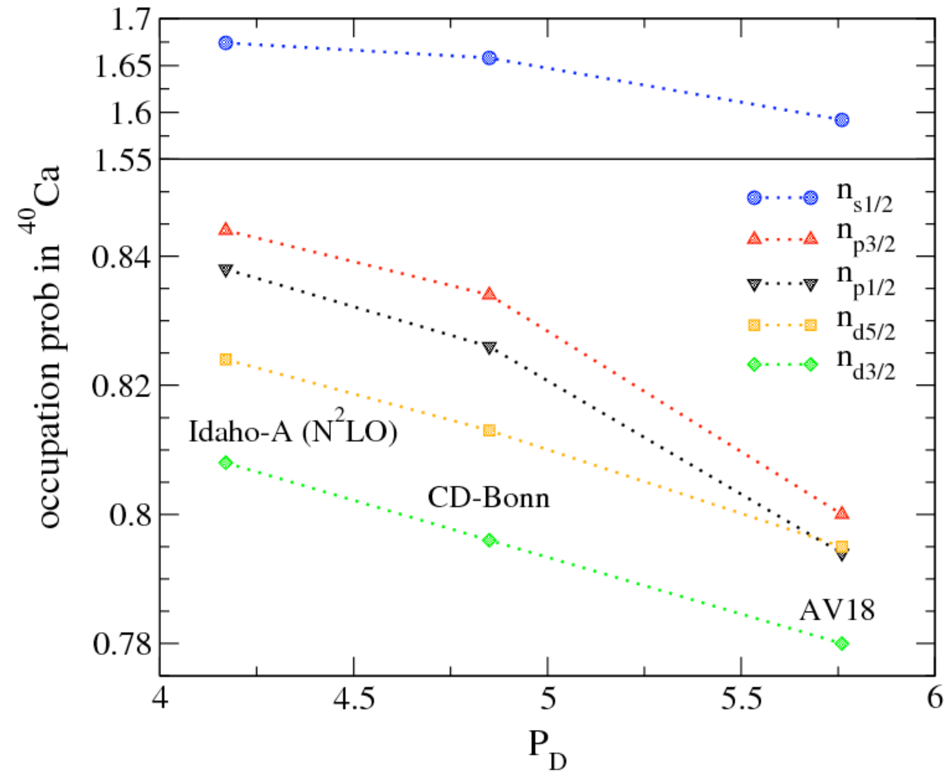
depend on resolution scale

simplest occupation number: deuteron
D-state probability P_D is not an observable

occupation numbers in ^{16}O and ^{40}Ca
correlate with D-state probability
[occupation numbers from Gad, Muether (2002)]







Tjon line

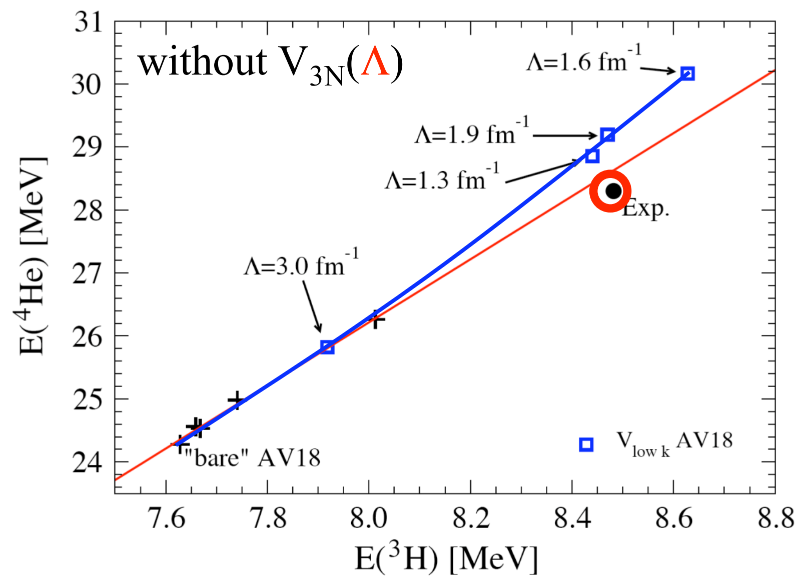
$V_{\text{low } k}(\Lambda)$ defines class of NN interactions with cutoff-independent low-energy NN observables

cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

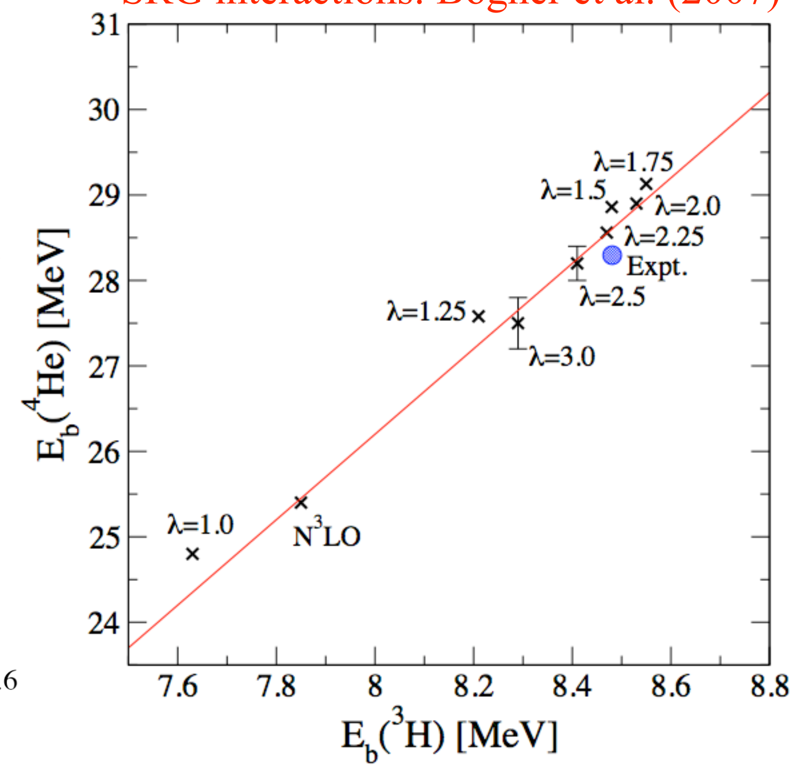
Cutoff dependence explains Tjon line, 3N required by renormalization

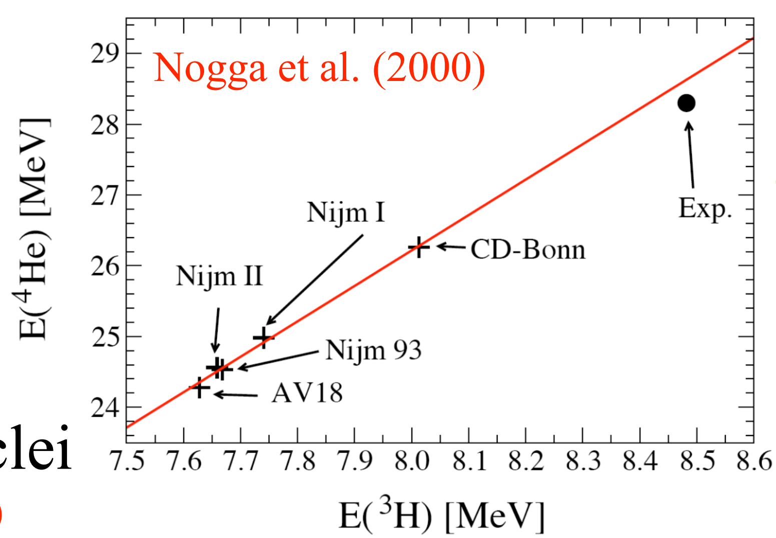
Experiment breaks from line ⇒ 3N

Nogga, Bogner, AS (2004)



SRG interactions: Bogner et al. (2007)

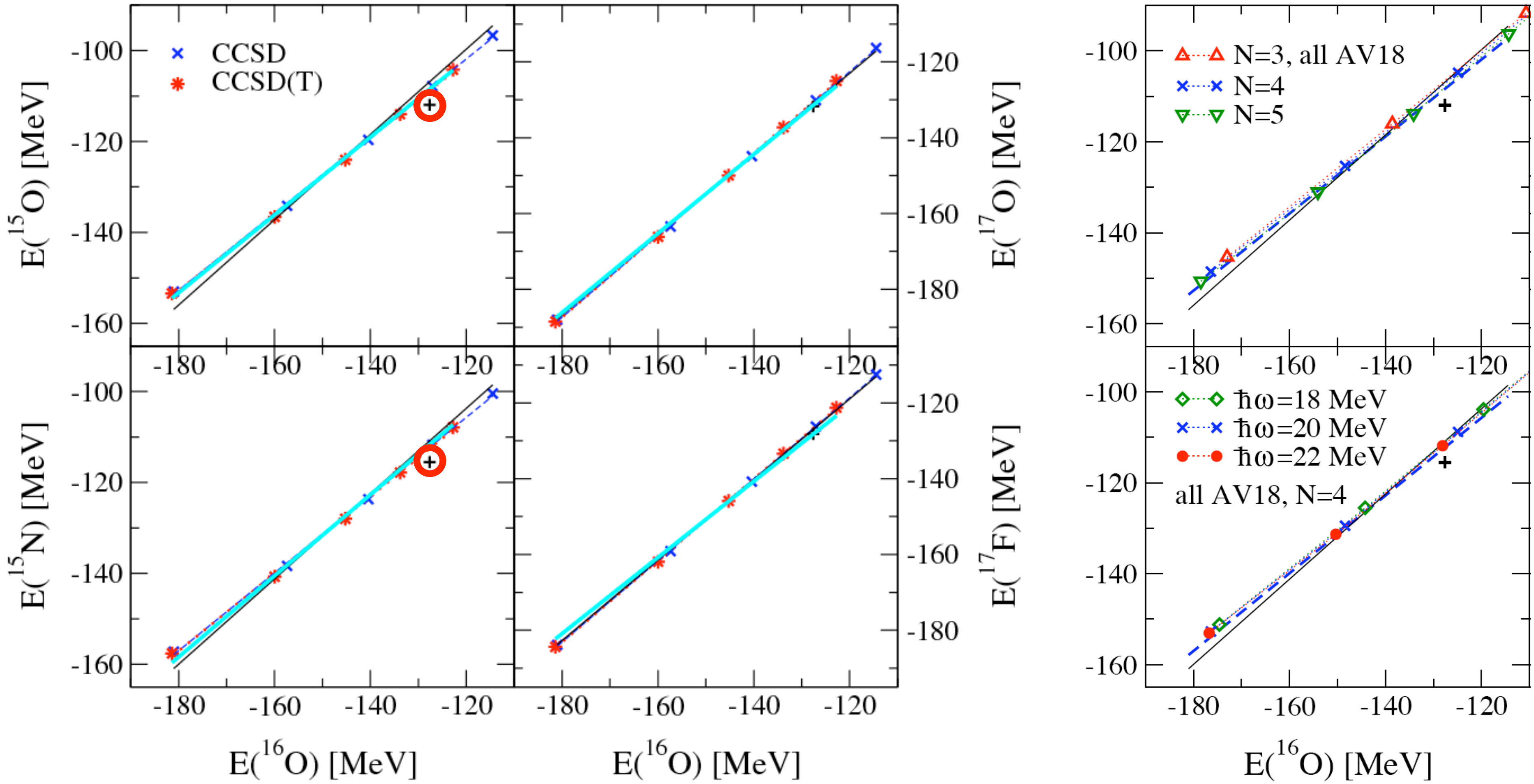




Tjon lines in p-shell nuclei

Bogner et al. (2007)

Tjon lines in medium-mass nuclei



NN-only results lead to Tjon lines in ^{16}O±1 Hagen, Dean, AS, in prep. ⇒ 3N

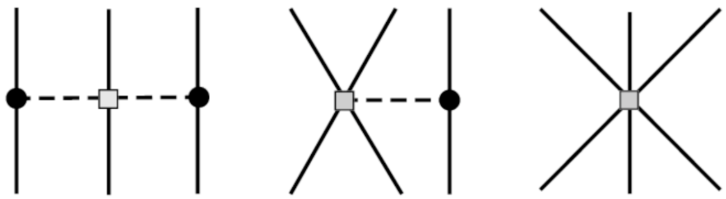
truncations in oscillator shells N, different hω approx on same lines

slopes agree with nuclear matter limit A±1/A + surface/Coulomb corr.

Chiral EFT 3N interactions

leading $N^2LO \sim (Q/\Lambda)^3$ van Kolck (1994), Epelbaum et al. (2002)

long (2π) intermed. (π) short-range



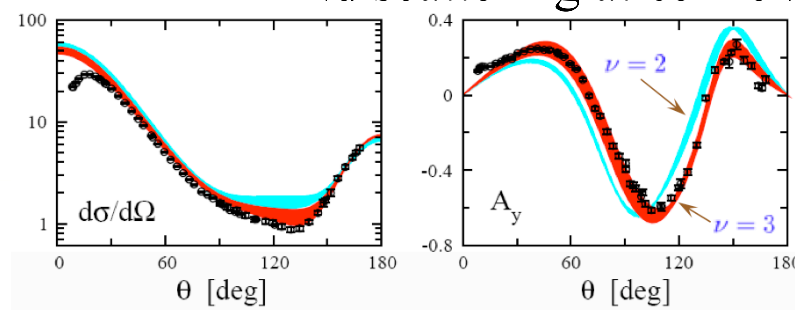
c_1,c_3,c_4 terms $D(\Lambda)$ term $E(\Lambda)$ term

$$= \frac{g_A^2}{8F_\pi^4}\frac{(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_3\cdot\vec{q}_3)}{[\vec{q}_1^{\,2}+M_\pi^2][\vec{q}_3^{\,2}+M_\pi^2]}\left\{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3(-4c_1M_\pi^2+2c_3\,\vec{q}_1\cdot\vec{q}_3)+c_4[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3]\cdot\boldsymbol{\tau}_2[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2\right\} + \text{perm.}$$

$$= -\frac{g_A D}{8F_\pi}\frac{(\vec{\sigma}_3\cdot\vec{q}_3)}{\vec{q}_3^{\,2}+M_\pi^2}(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)(\vec{\sigma}_2\cdot\vec{q}_3) + \text{perm.}$$

$$= \frac{1}{2}E(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3) + \text{perm.}$$

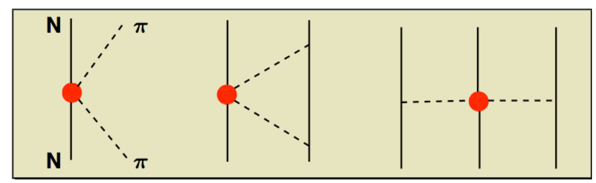
Nd scattering at 65MeV



c_i relate πN, NN, 3N: determination from πN

Meissner (2007)

$$c_1 = -0.9^{+0.2}_{-0.5}\,,\; c_3 = -4.7^{+1.2}_{-1.0}\,,\; c_4 = 3.5^{+0.5}_{-0.2}$$



consistent with peripheral NN: c_1=-0.76(7), c_3=-4.78(10), c_4=3.96(22)

Rentmeester et al. (2003)

c_3,c_4 important for structure, but large uncertainties at present

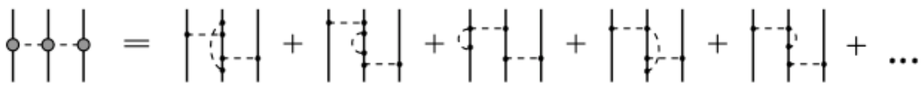
D term can be fixed by tritium beta decay, kinematics matches 3N

Subleading chiral EFT 3N interactions

parameter-free $N^3LO \sim (Q/\Lambda)^4$ Status from Epelbaum @ TRIUMF 3N workshop (2007)

- 1/m-corrections to 1 insertion from $\mathcal{L}^{(2)}_{1/m} =$  $+ \mathcal{O}(\pi^3)$
  - — rich operator structure (includes spin-orbit interactions)
- 1-loop diagrams with all vertices from $\mathcal{L}^{(0)}_{\text{eff}}$

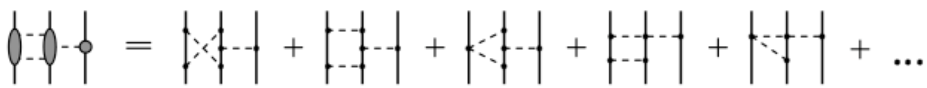
2π – exchange



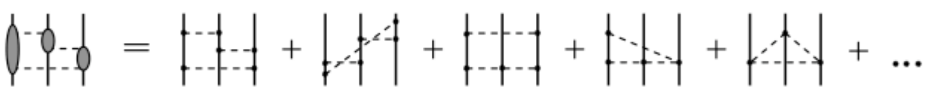
The calculated corrections simply shift the LECs c_i as follows:

$$\delta c_1 = \frac{g_A^2 M_\pi}{64\pi F_\pi^2} \sim 0.13 \text{ GeV}^{-1} \qquad \delta c_3 = \frac{3 g_A^4 M_\pi}{16\pi F_\pi^2} \sim 2.5 \text{ GeV}^{-1} \qquad \delta c_4 = -\frac{g_A^4 M_\pi}{16\pi F_\pi^2} \sim -0.85 \text{ GeV}^{-1}$$

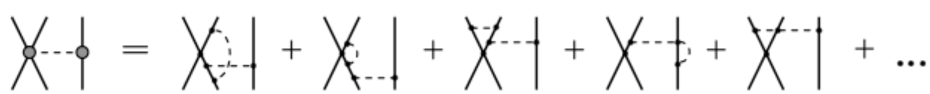
2π-1π – exchange



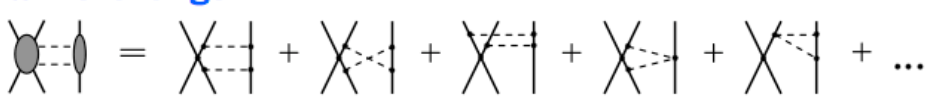
ring diagrams



contact-1π – exchange



contact-2π – exchange



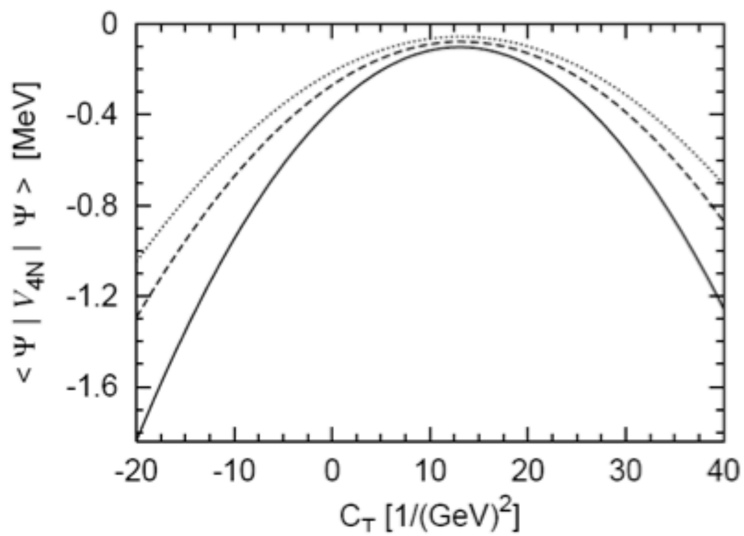
Chiral EFT 4N interactions

from Epelbaum @ TRIUMF 3N workshop (2007)

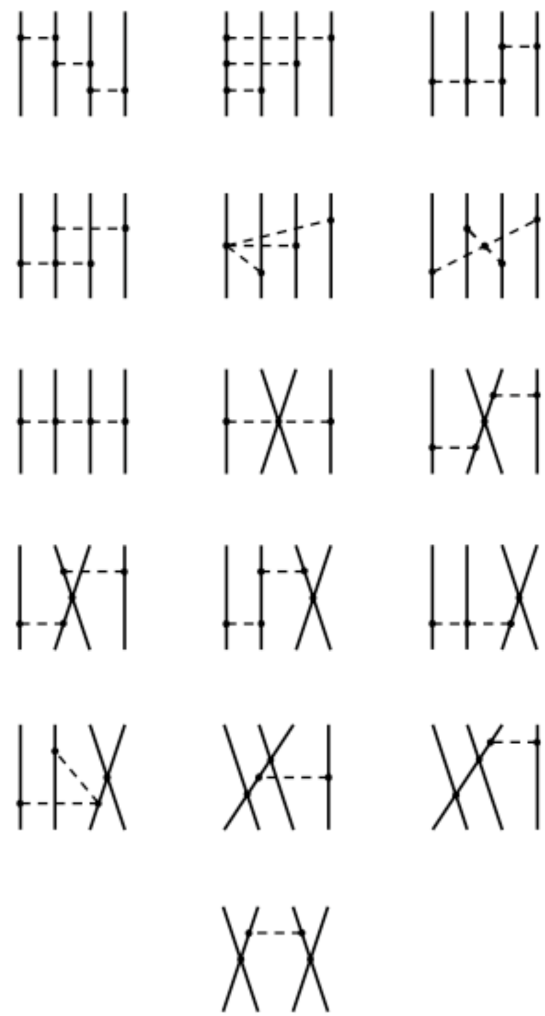
Four-nucleon force *(E.E. '05)*

- first shows up at order $\nu = 4$
- chiral symmetry plays a crucial role
- parameter-free

Contribution of the 4NF to the ^{4}He BE is attractive and of the order of few 100 keV
(Rozpedzik et al. ´06)



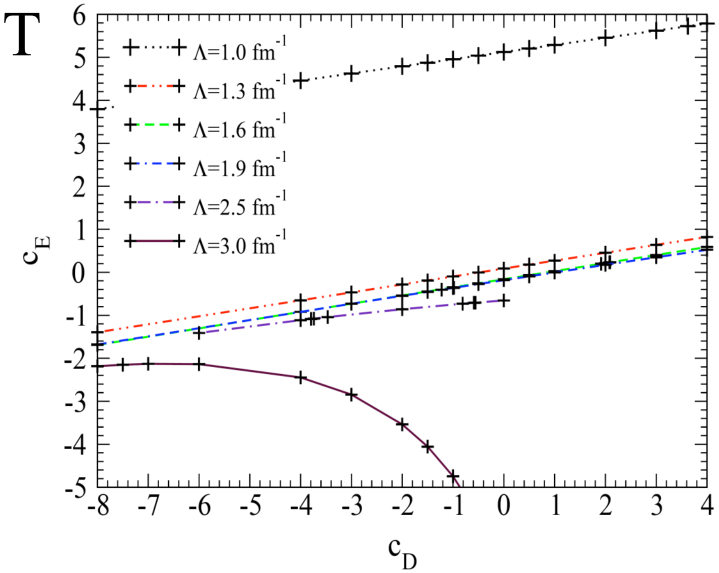
Results from: Rozpedzik et al., nucl-th/0606017



4N contributions ~ 1 MeV at saturation density not unreasonable

Low-momentum 3N interactions $V_{3N}(\Lambda)$

corresponding $V_{3N}(\Lambda)$ from leading chiral EFT



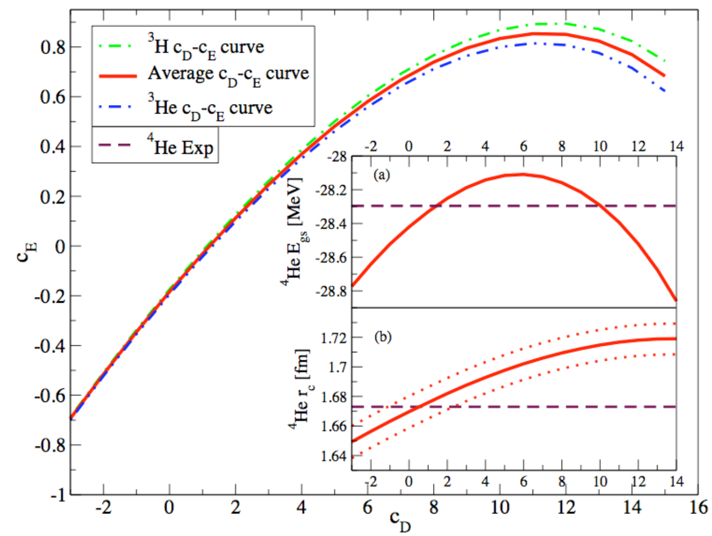
fit D,E couplings to A=3,4 binding energies for range of cutoffs

linear dependences in fits to triton binding

3N interactions perturbative for $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$

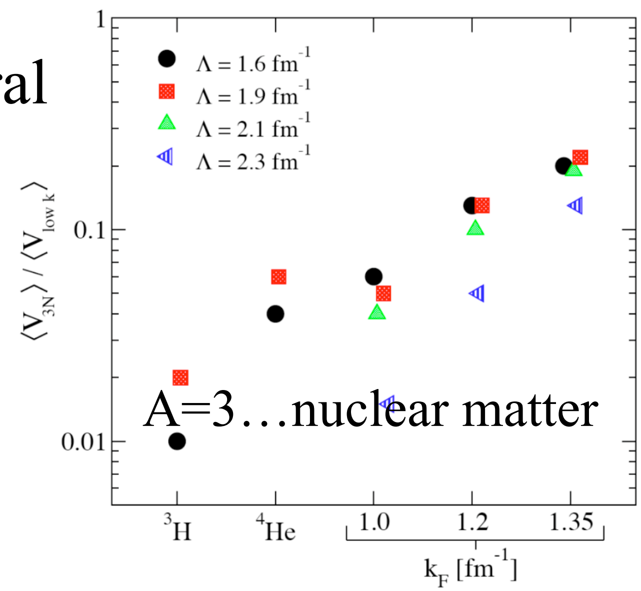
Nogga, Bogner, AS (2004)

nonperturbative at larger cutoffs
cf. chiral EFT $\Lambda \approx 3\ \mathrm{fm}^{-1}$

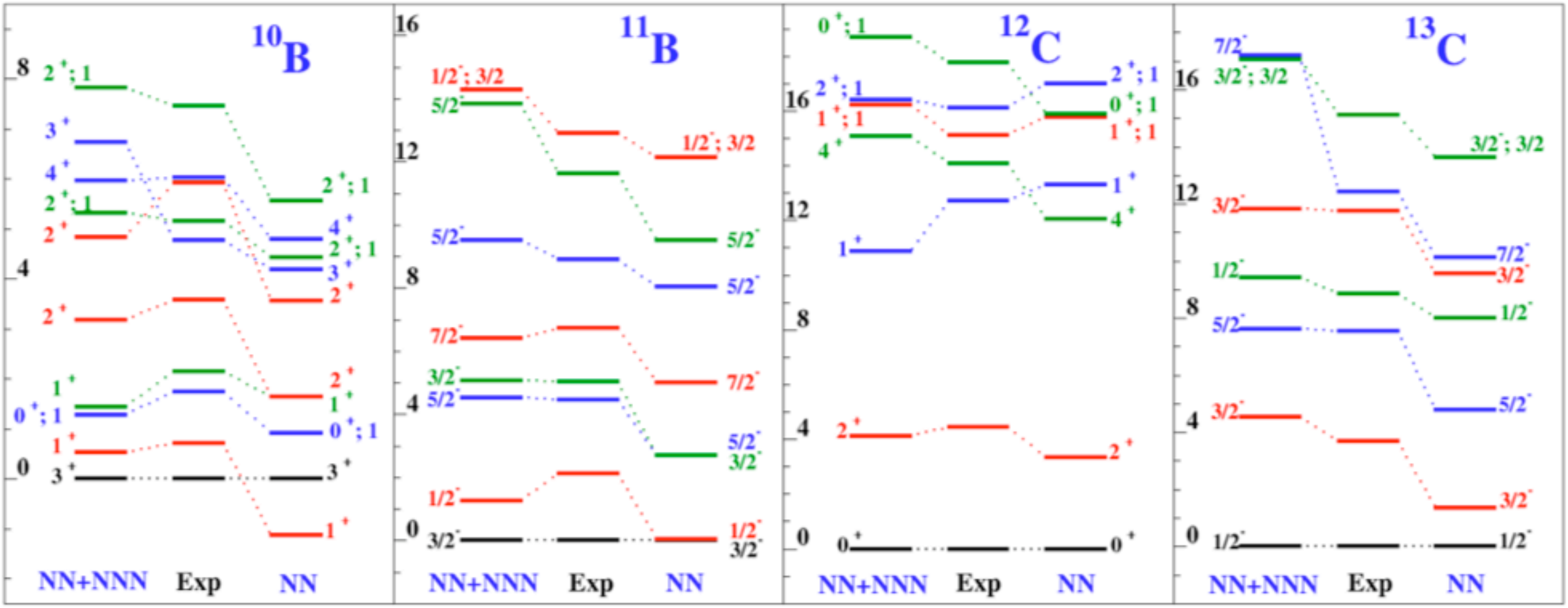


Navratil et al. (2007)

3N exp. values natural size $\sim (Q/\Lambda)^3\ V_{NN}$



NCSM highlights with chiral EFT interactions



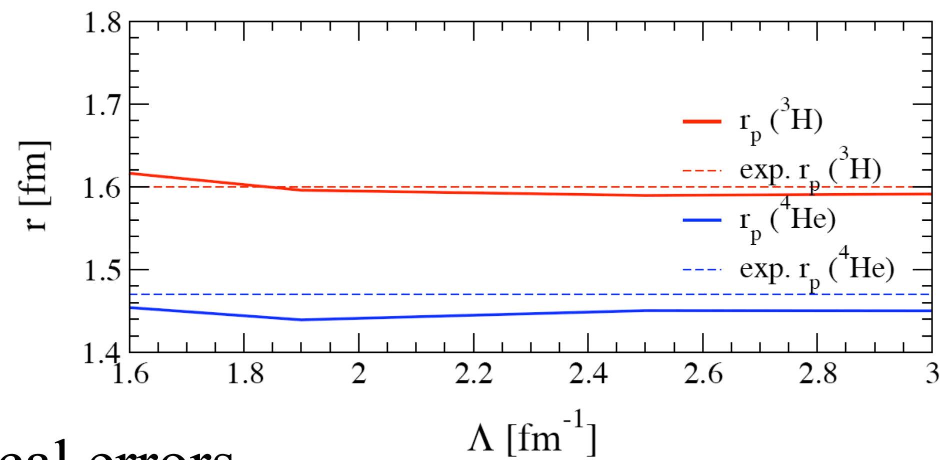
Navratil et al. (2007)

impressive agreement, highlights the importance of 3N interactions

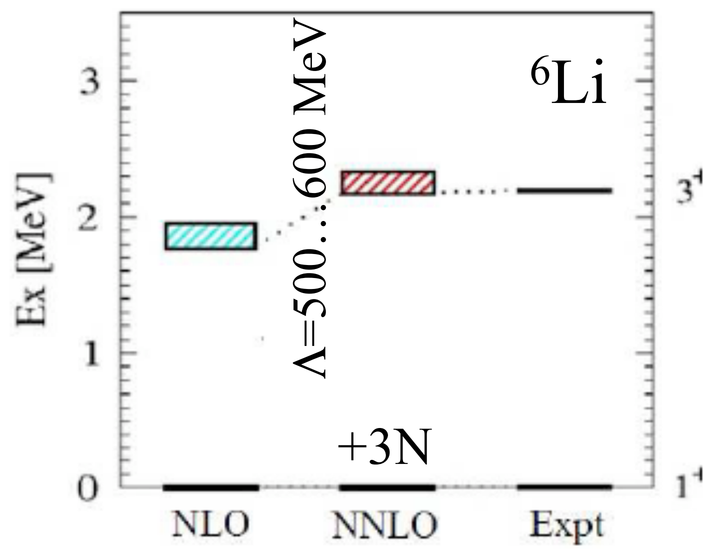
Theoretical uncertainties

$V_{\text{low }k}(\Lambda)$ + leading chiral $V_{3N}(\Lambda) \Rightarrow$ cutoff dependence of observables probes neglected many-body int.

Radii of light nuclei approximately cutoff-independent, agree with exp.



Can provide lower limits on theoretical errors



from A. Nogga

long-term: uncertainties of matrix elements needed in fundamental symmetry tests

neutrinoless double-beta decay

atomic EDMs

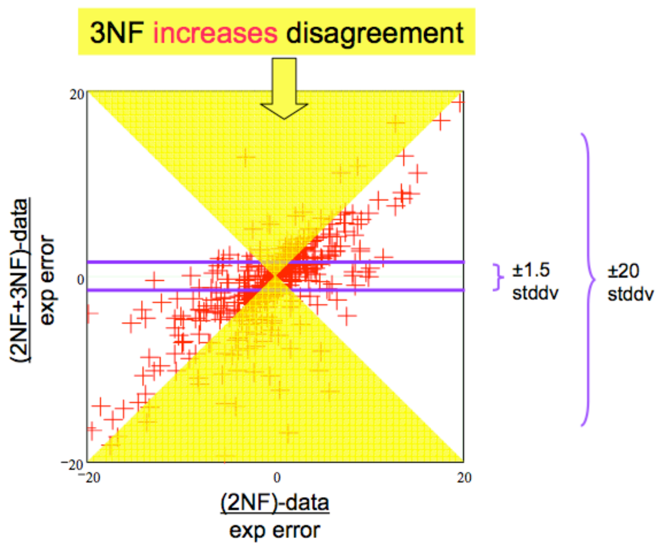
isospin-symmetry breaking corrections for V_{ud} ……

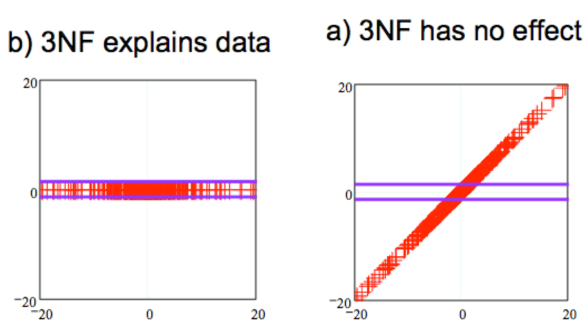
3N interactions: a frontier

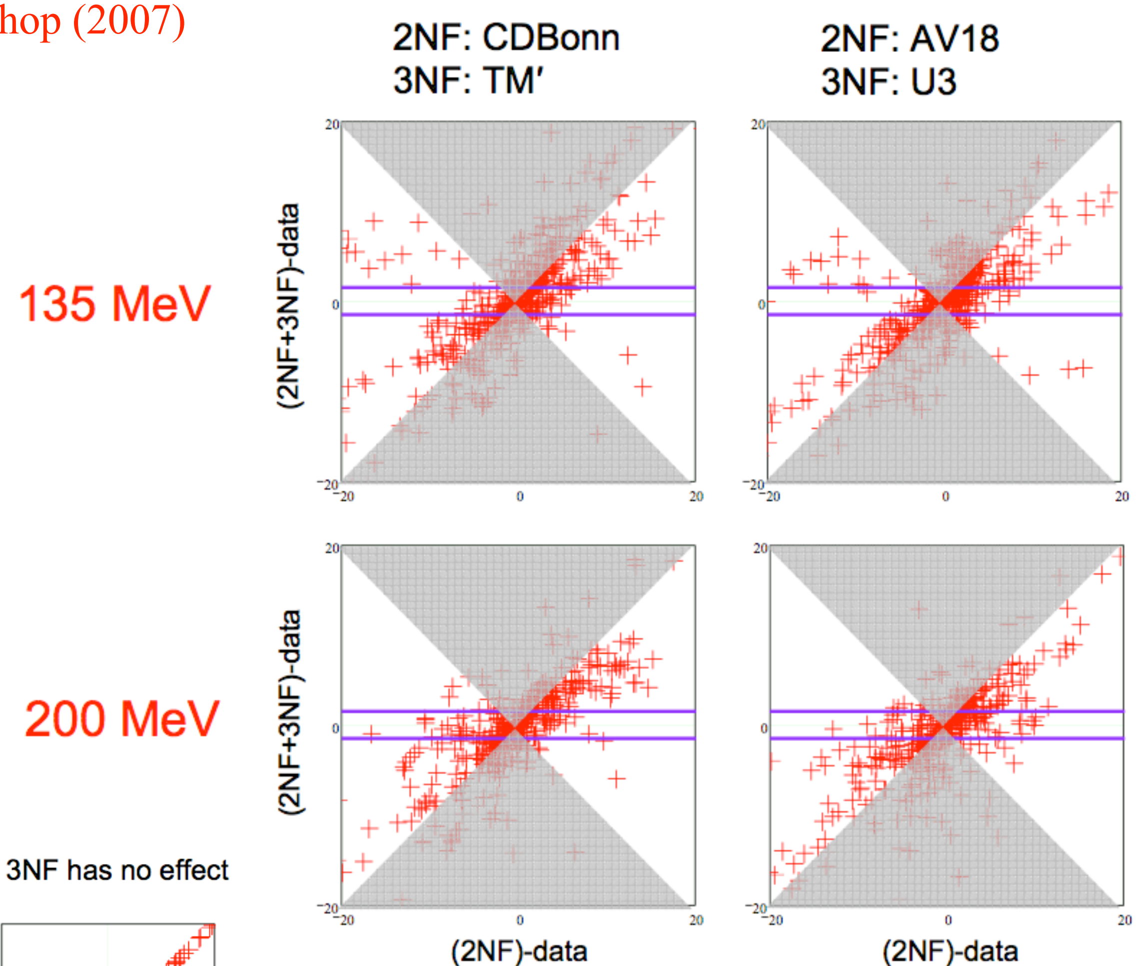
from H.-O. Meyer @ TRIUMF 3N workshop (2007)

pd scattering

a way to look at
880 data points…







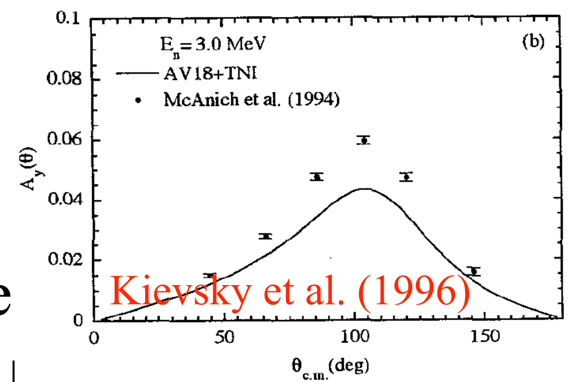
coherent 3N effort needed with theoretical uncertainties

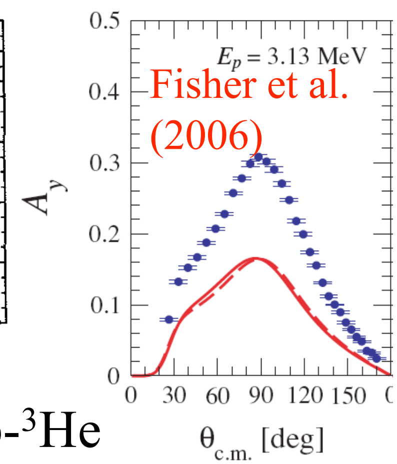
3N interactions crucial for many-nucleon systems

Effect of 3N interactions are amplified in nuclei,
constrain 3N with few- and many-body data ⇒ controlled predictions

3N interaction crucial:

to break off Tjon, Coester,… lines

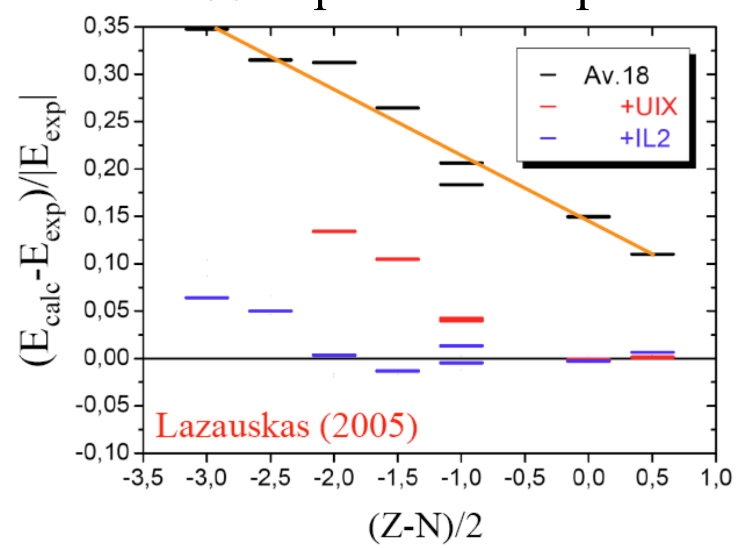




spin-orbit dependence and A_y puzzle

$$A_y = \frac{\frac{d\sigma}{d\Omega}\big|_\uparrow - \frac{d\sigma}{d\Omega}\big|_\downarrow}{\frac{d\sigma}{d\Omega}\big|_\uparrow + \frac{d\sigma}{d\Omega}\big|_\downarrow}$$

30% in nd, 100% in p-^{3}He
~0% in p-^{3}H ⇒ isospin



isospin dependence, n/p-rich systems

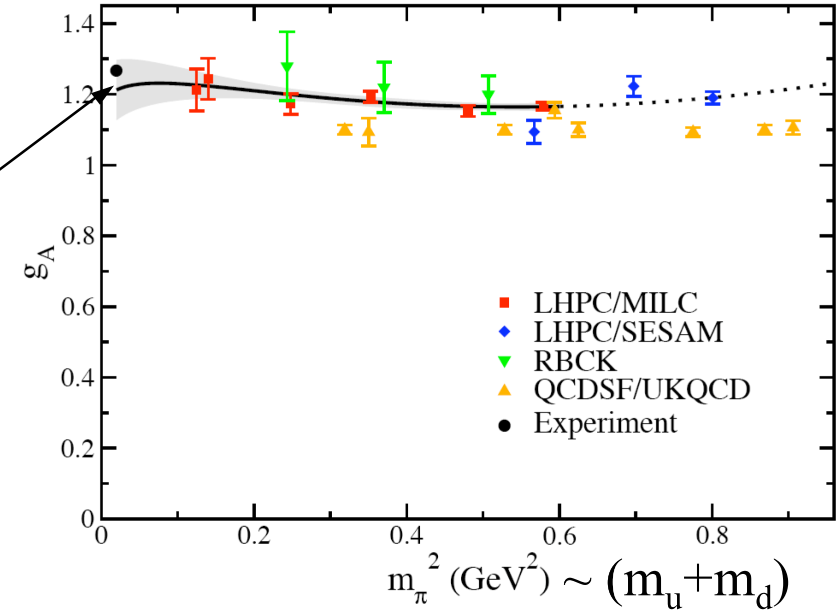
impact on nuclear matter, saturation

pivotal for extrapolations
to extremes of astrophysics

Lattice QCD and nuclear forces

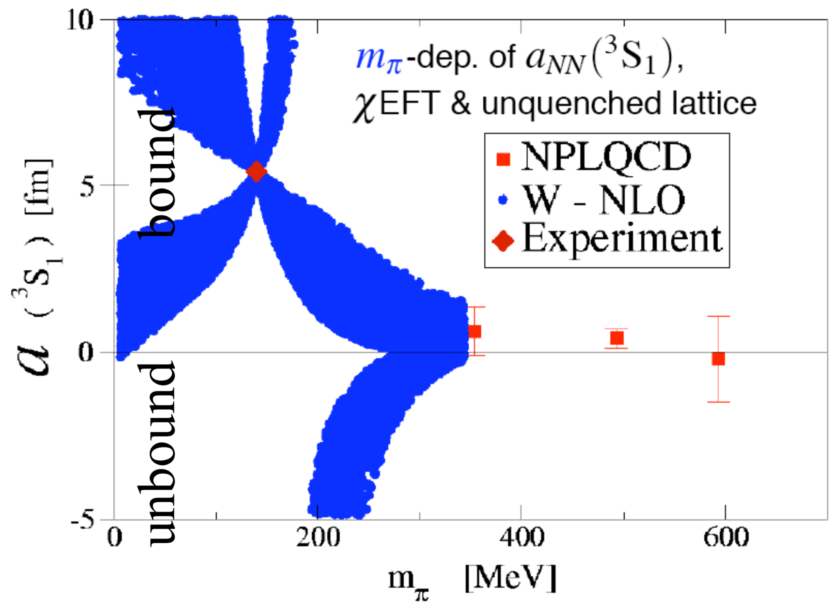
pion-NN coupling g_a from full QCD
Edwards et al. (2006)

chiral EFT extrapolation to physical pion mass agrees with experiment



How does the deuteron binding depend on quark masses? Beane et al. (2006)

First coherent effort to connect nuclear forces to underlying QCD



Constrain some low-energy couplings

Constrain experimentally difficult observables: 3-neutron properties
⇒ isospin T=3/2 part of 3N interactions ……

NCSM with low-momentum interactions

Bogner, Furnstahl, Maris, Perry, AS, Vary, arXiv:0708.3754.

very promising convergence from N_{max}~6-10 with smooth-cutoff and SRG low-mom. interactions

future: include 3N interactions and push limits towards heavier systems

	^{3}H		^{4}He		^{6}He		^{6}Li		^{7}Li	
λ	$\hbar\Omega$	E_{gs}	$\hbar\Omega$	E_{gs}	$\hbar\Omega$	E_{gs}	$\hbar\Omega$	E_{gs}	$\hbar\Omega$	E_{gs}
∞	–	−7.85	42	−26.1(8)						
3.0	28	−8.29	34	−27.5(3)	28	−28(1)	28	−31.5(8)	24	−38.7(30)
2.5	24	−8.41	28	−28.2(2)	24	−28.9(3)	24	−32.1(3)	24	−38.7(20)
2.25	22	−8.47	24	−28.6(1)	22	−29.4(2)	22	−32.5(2)	22	−40.3(10)
2.0	18	−8.53	24	−28.90	20	−30.0(1)	20	−33.1(1)	20	−41.2(5)
1.75	16	−8.55	20	−29.13	16	−30.6	18	−33.6	18	−41.7(4)
1.5	12	−8.48	18	−28.86	14	−30.7	16	−33.7	16	−42.0(3)
1.25	10	−8.21	14	−27.58	12	−29.9	12	−32.9	12	−41.1(2)
1.0	8	−7.63	14	−24.80	10	−27.4	10	−30.4	12	−37.8(2)

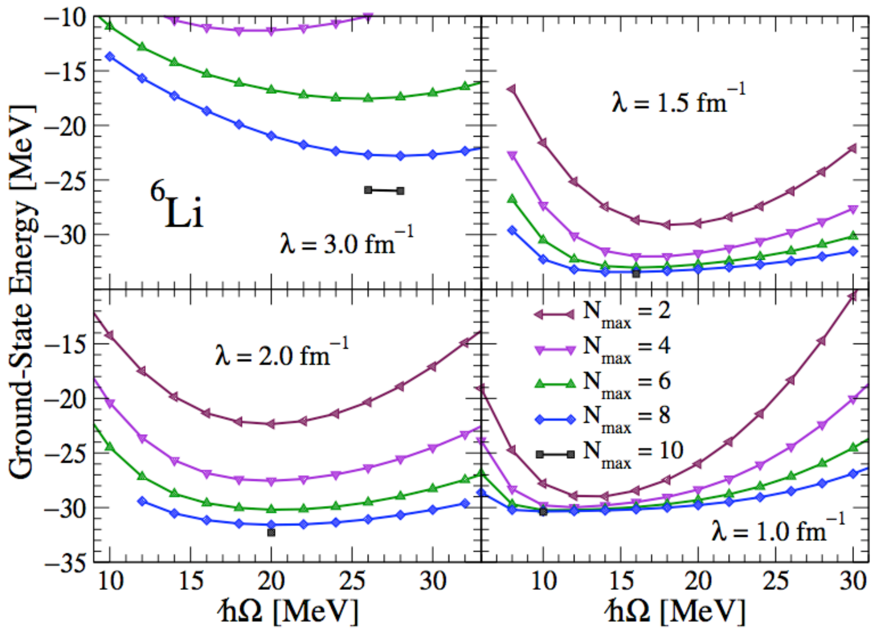


Fig. 5. Ground-state energy of ^{6}Li as a function of $\hbar\Omega$ at four different values of λ (3, 2, 1.5, 1 fm^{-1}). The initial potential is the 500 MeV N^{3}LO NN-only potential from Ref. [11].

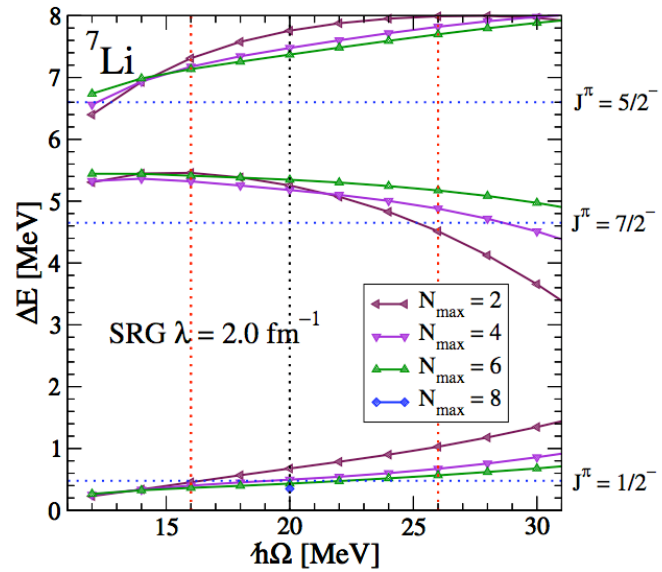


Fig. 21. Excitation energies of the lowest natural-partiy states of ^{7}Li as a function of $\hbar\Omega$ for $\lambda = 2.0\,\text{fm}^{-1}$. The initial potential is the 500 MeV N^{3}LO NN-only potential from Ref. [11]. The horizontal dotted lines are the experimental values while the vertical dotted lines mark the optimal $\hbar\Omega$ value for the ground-state energy (middle) and the range for which estimates are close to this.

Coupled-cluster theory: pushing the limits to A=40

based on $V_{\mathrm{low}\,k}(\Lambda)$, full $V_{3N}(\Lambda)$ in progress

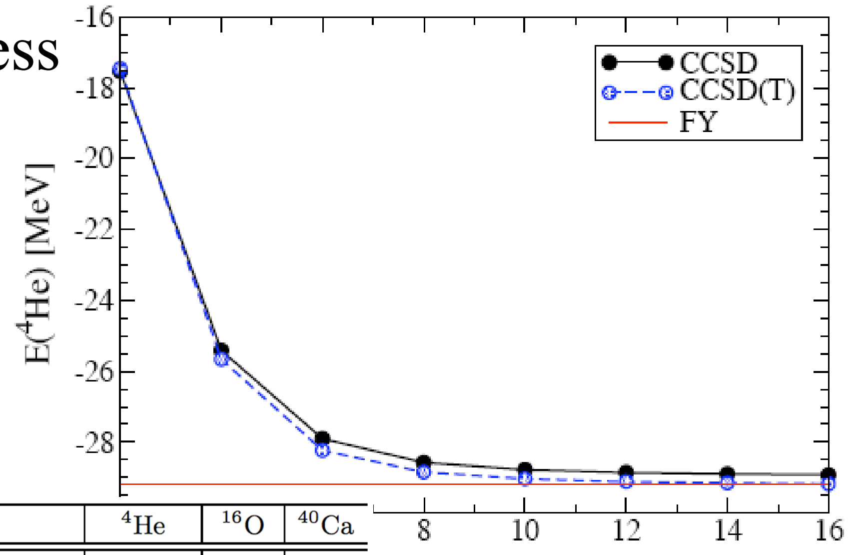
Hagen et al. (2007)

CC theory meets and sets benchmarks:

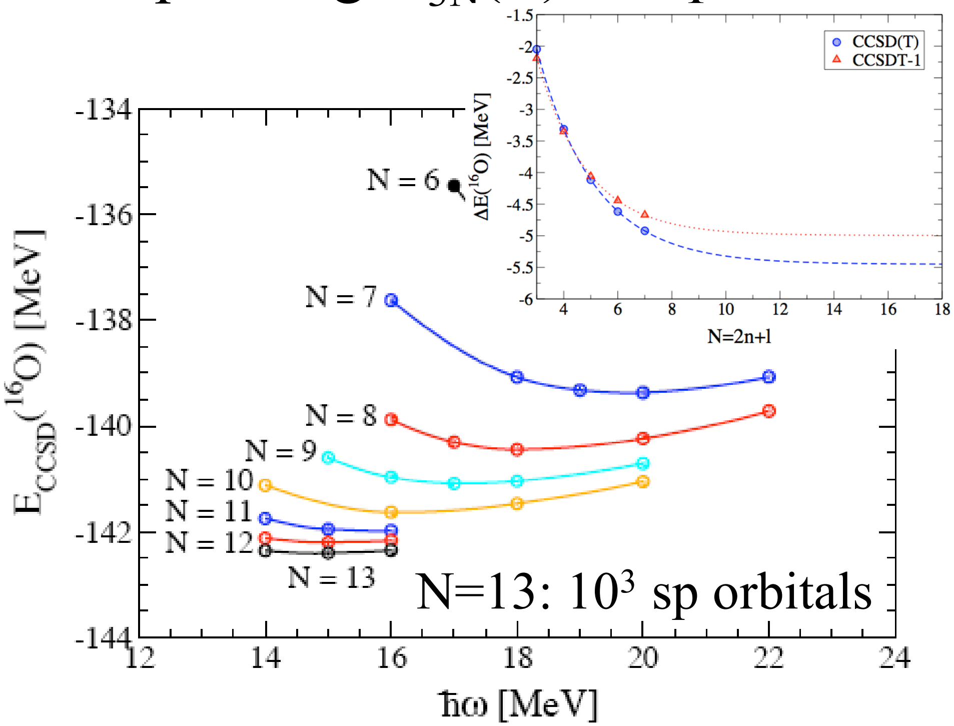
within 10 keV of Faddeev-Yakubovsky

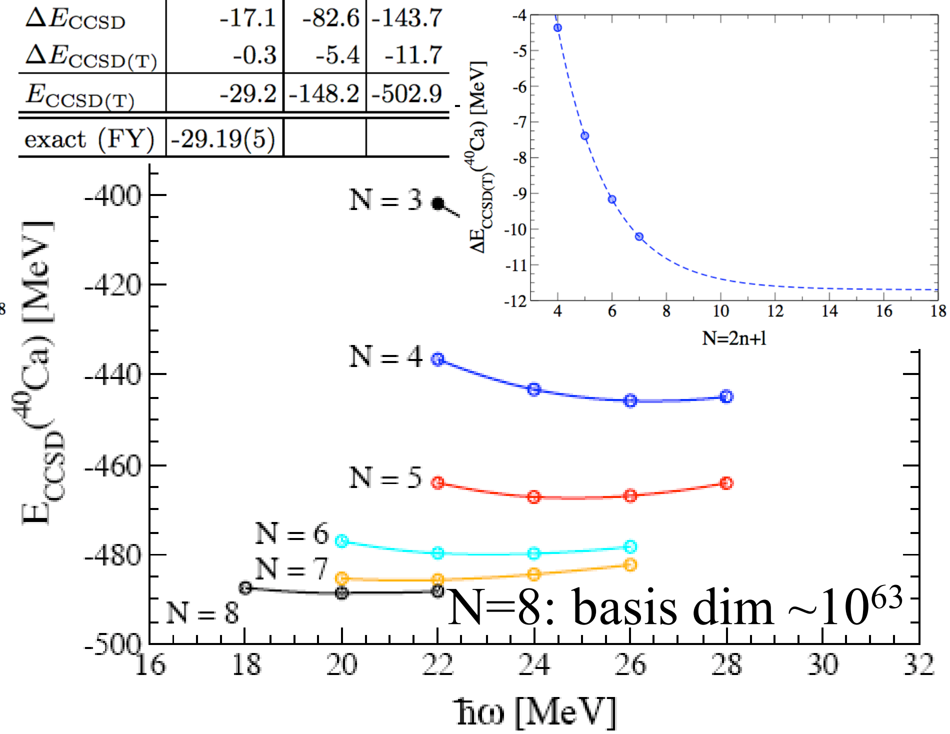
accurate for ^{16}O and ^{40}Ca

corresponding $V_{3N}(\Lambda)$ is repulsive



	^4He	^{16}O	^{40}Ca
E_0	-11.8	-60.2	-347.5
ΔE_{CCSD}	-17.1	-82.6	-143.7
$\Delta E_{CCSD(T)}$	-0.3	-5.4	-11.7
$E_{CCSD(T)}$	-29.2	-148.2	-502.9
exact (FY)	-29.19(5)		



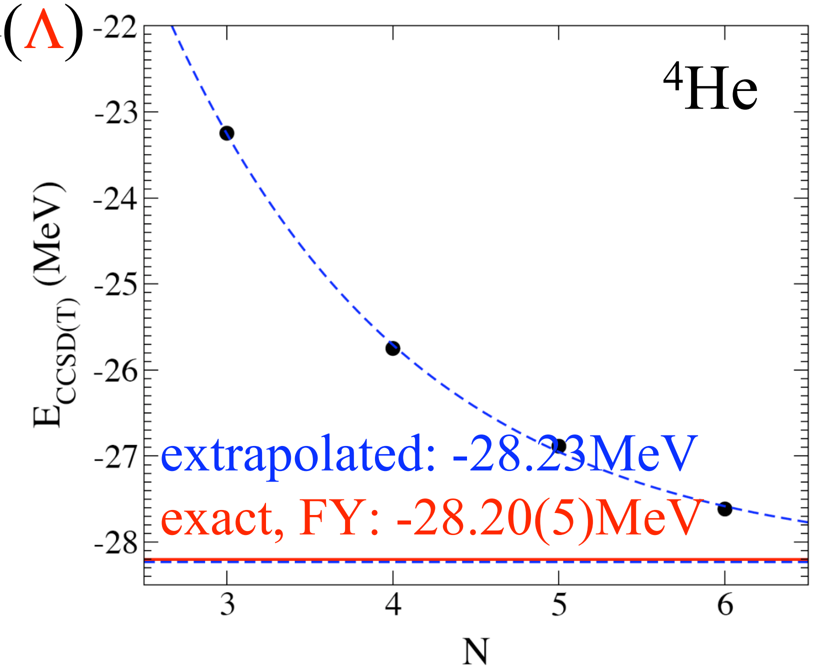


Towards 3N interactions in medium-mass nuclei

based on low-momentum $V_{\text{low k}}(\Lambda) + V_{3N}(\Lambda)$

Hagen et al. (2007)

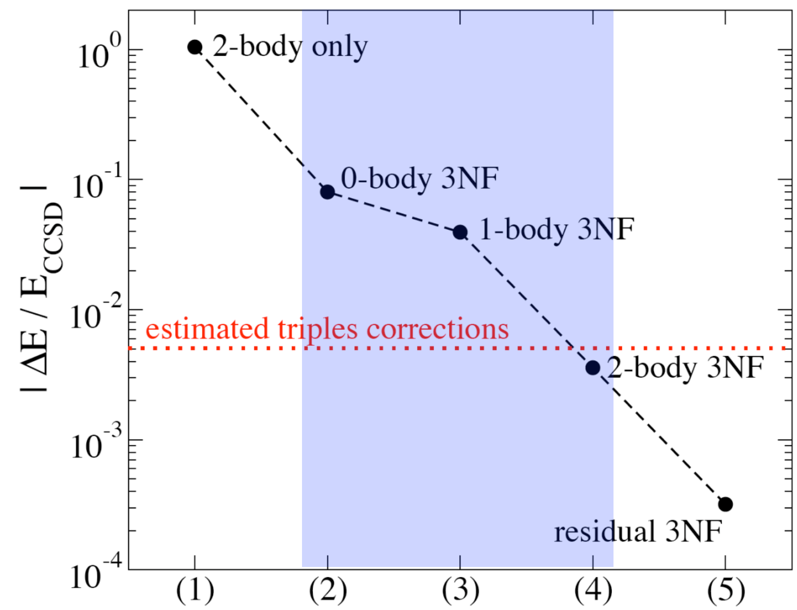
developed coupled-cluster theory with 3N interactions, first benchmark for ^{4}He



Results show that 0-, 1- and 2-body parts of 3N interaction dominate

residual 3N interaction can be neglected!
very promising

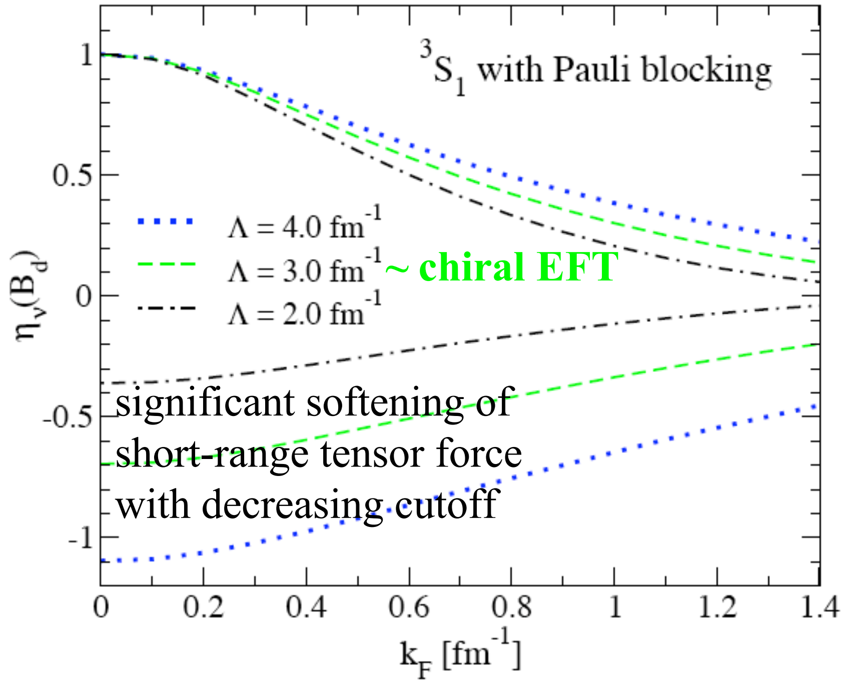
supports that monopole corrections for valence shell interactions due to 3N

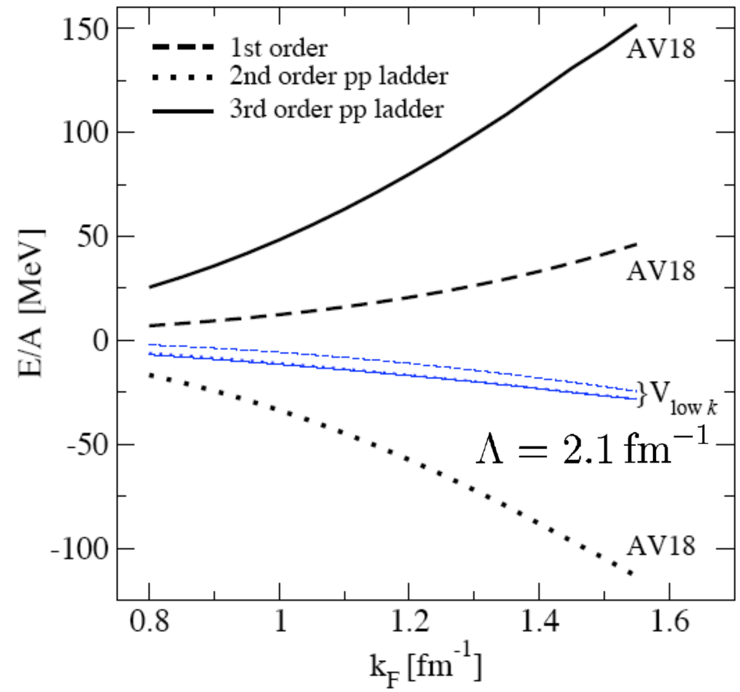


Perturbative two-body ladders

Weinberg analysis anticipates rapidly converging particle-particle contributions to nuclear matter energy for low-momentum interactions

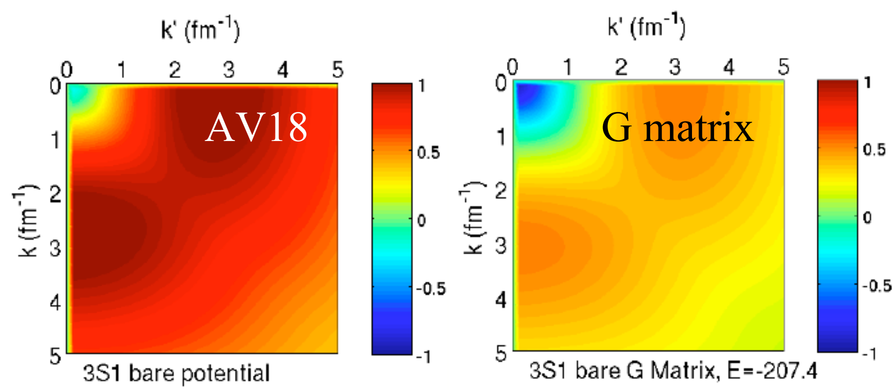
Bogner, AS, Furnstahl, Nogga (2005)





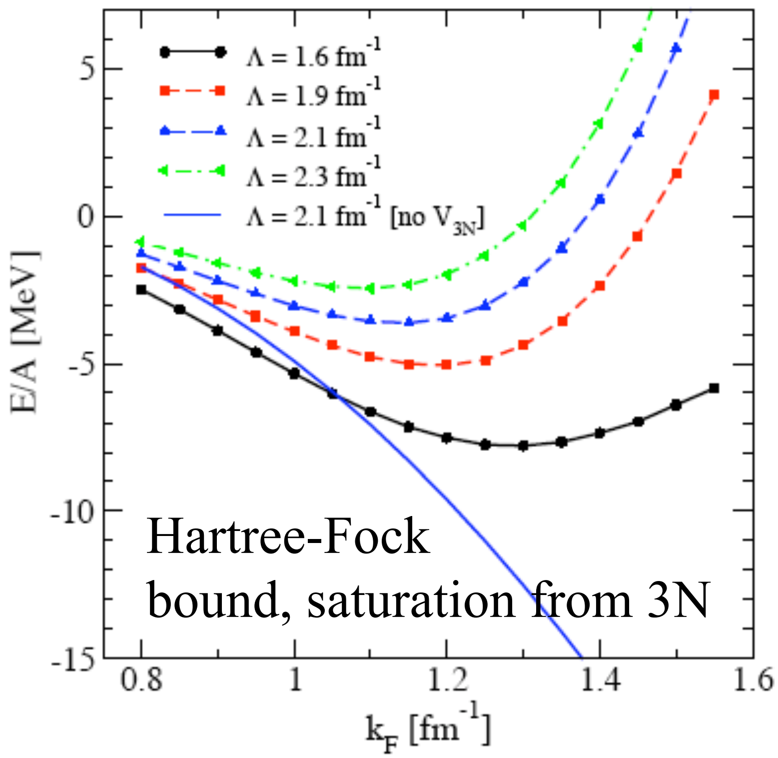
Perturbation theory for bulk properties in place of ladder resummations

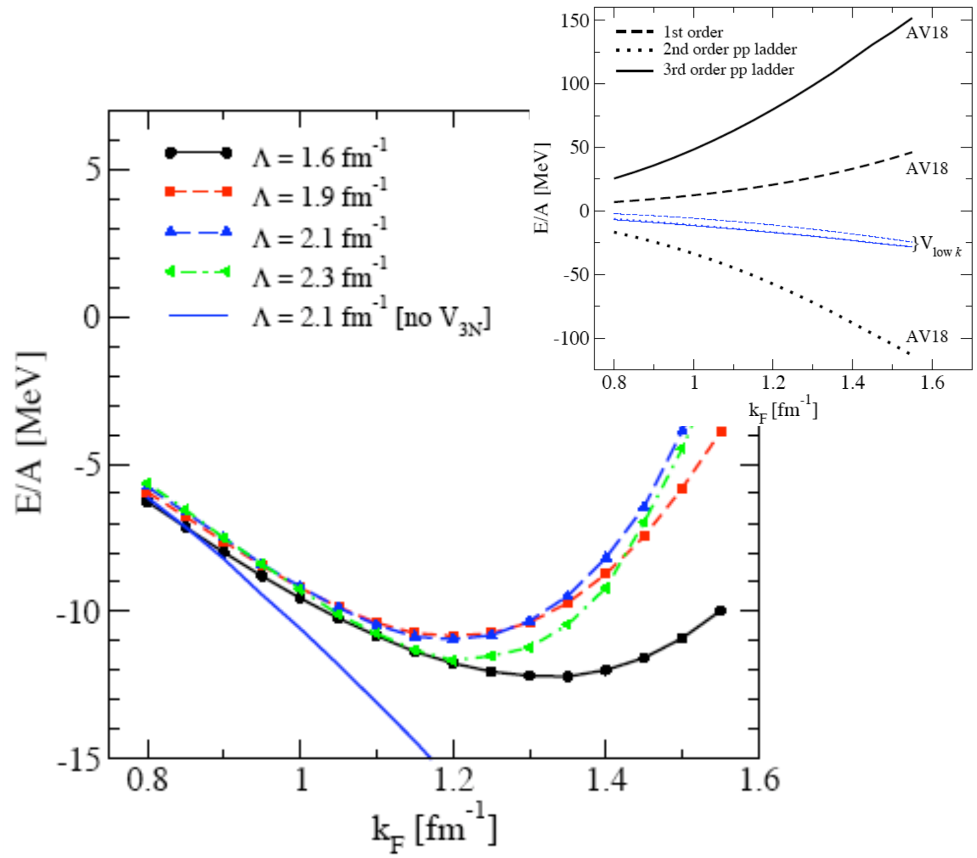
G matrix does not tame off-diagonal coupling



Possibility of perturbative nuclear matter with NN and 3N

motivated by Weinberg analysis





Hartree-Fock + ≈2nd-order
cutoff dep. strongly reduced

Bogner, AS, Furnstahl, Nogga (2005)

3N drives saturation but expectation values natural, consistent with EFT

will provide key guidance to microscopic DFT for SciDAC UNEDF

Many on-going developments

CC, NCSM calculations in progress

3N contributions to shell model Jason Holt

new power counting for nuclear matter

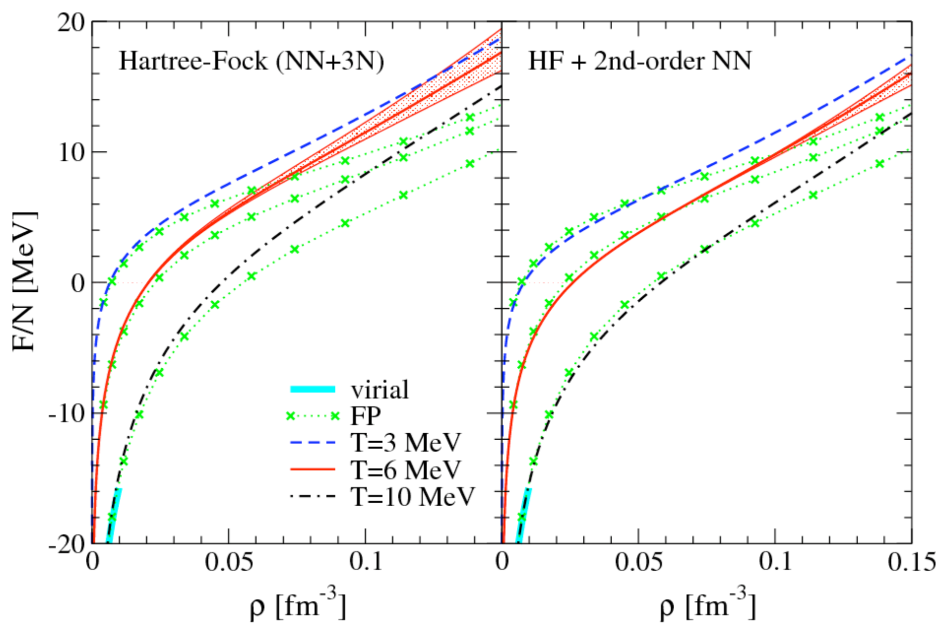
microscopic guidance for DFT Bogner, Furnstahl, Platter

constructing microscopic interactions for HF+GCM+… Rotival, Duguet et al.

superfluidity in neutron stars
Hebeler et al.

finite temperature equation of state for astrophysics
Tolos, Friman, AS (2006)

……



One-line summary and Thanks to my collaborators

Exciting era with advances on many fronts!

J. Braun, J.D. Holt

THE OHIO STATE UNIVERSITY R.J. Furnstahl, R.J. Perry, S. Ramanan

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STONY BROOK UNIVERSITY G.E. Brown, T.T.S. Kuo

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