Embedding collective models in the shell model

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Initial objective:

To give the collective models a microscopic foundation in shell-model terms.

We find that the many nuclear models are associated with shell-model coupling schemes.

New objective (in the light of the Draayer, Lourney, Dytrych, Vary, program):

What does this understanding imply for the construction of optimal choices of effective shell-model spaces.

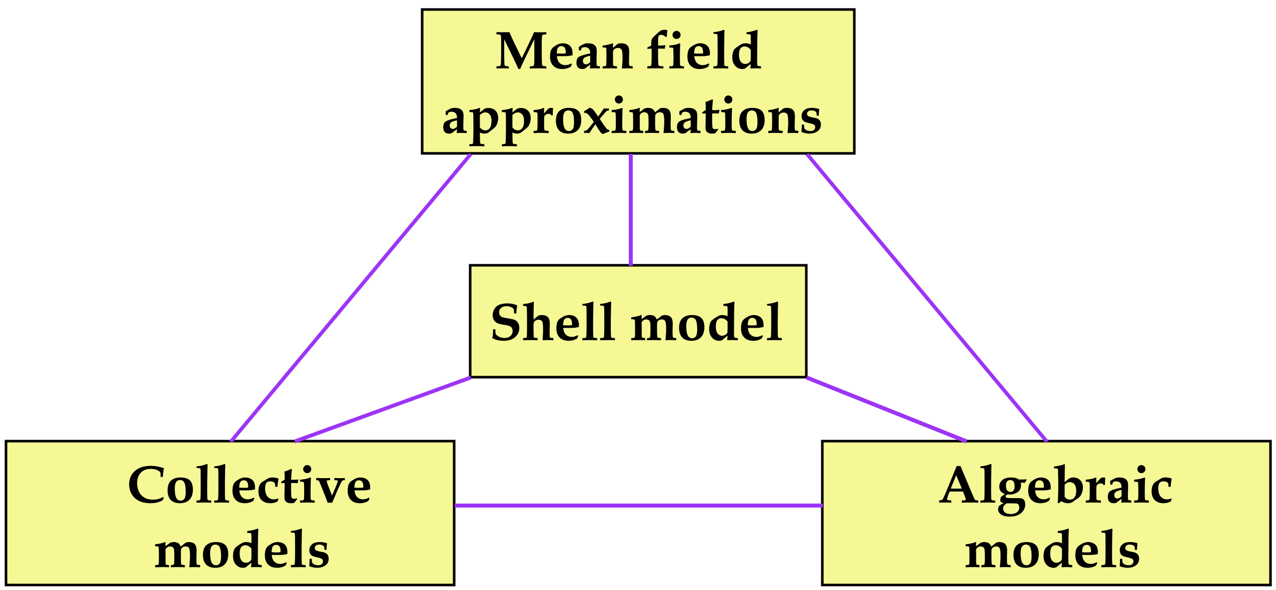
Major colleagues:

G,T. Rosensteel, C. Bahri, J.P. Draayer, and numerous students and post-docs (many debts to K.T. Hecht)

Basic ansatz

The harmonic oscillator shell model provides the formal framework for the description of nuclear structure.

It is an exceedingly rich algebraic model with a wealth of dynamicalsubgroups chains and solvable submodels.



The algebraic strategy

Express the model in terms of a unitary irreducible representation of a Lie algebra of observables (i.e., a spectrum generating algebra).

The model can then be embedded as a submodel of the shell model if its SGA is expressible in terms of many-nucleon coordinates (position, momentum, and spin).

E.g. Elliott's SU(3) model is an example of a complete algebraic model and a submodel of the shell model.

What about the Bohr model?

Originally expressed in terms of liquid drop shape coordinates

$$R(\vartheta,\varphi) = R_0\left[1 + \sum_{\lambda\nu} \alpha_{\lambda\nu} Y^*_{\lambda\nu}(\vartheta,\varphi)\right]$$

It is an algebraic model with many dynamical symmetries (cf. Caprio's talk)

Heisenberg-Weyl algebra:

$$\pi^{\lambda\nu} = -i\hbar \frac{\partial}{\partial \alpha_{\lambda\nu}} \qquad \left[\alpha_{\lambda\mu}, \pi^{\lambda\nu}\right] = i\hbar\delta^{\nu}_{\mu}$$

How do you make α microscopic?
Change to quadrupole-moment coordinates

The microscopic collective model

Change to quadrupole-moment coordinates

$$\alpha_\nu \rightarrow Q_{ij} = \sum_{n=1}^{A} x_{ni} x_{nj} \qquad \pi^{\lambda\nu} = -i\hbar \frac{\partial}{\partial \alpha_\nu} \rightarrow T_{ij} = -i\hbar \sum_{n=1}^{A} x_{ni} \frac{\partial}{\partial \alpha_{nj}}$$

The microscopic version has a cm(3) SGA

$$Q_{ij} = \sum_{n=1}^{A} x_{ni} x_{nj}, \qquad T_{ij} = \sum_{n=1}^{A} x_{ni} p_{nj}$$

$$\left[Q_{ij}, T_{lk} \right] = i\hbar \left(\delta_{jk} Q_{il} + \delta_{ik} Q_{ijl} \right)$$

Weaver, Biedenharn, Cusson, Ann. Phys. 77, 250 (1973)

The CM(3) model

To complete the construction of the CM(3) model one must determine its irreducible unitary representations.

Its algebraic structure is of the same type as the Euclidean and Poincare groups which admit unitary representations with intrinsic spin.

— GT Rosensteel and DJ Rowe, Ann. Phys.96, 1 (1976)

CM(3) irreps are characterized by a vorticity quantum number.
A zero vorticity irrep is an irrotational flow model (cf. Original Bohr model).

However, the microscopic CM(3) model adds the possibility of vorticity to the vibrational and rotational degrees of freedom of the Bohr model.

Construction of a model Hamiltonian: What to do about the kinetic energy?

A breakthrough came when we realised that the model is easily extended to include the many-nucleon kinetic energy to its Lie algebra.

The extension of CM(3) to Sp3,R)

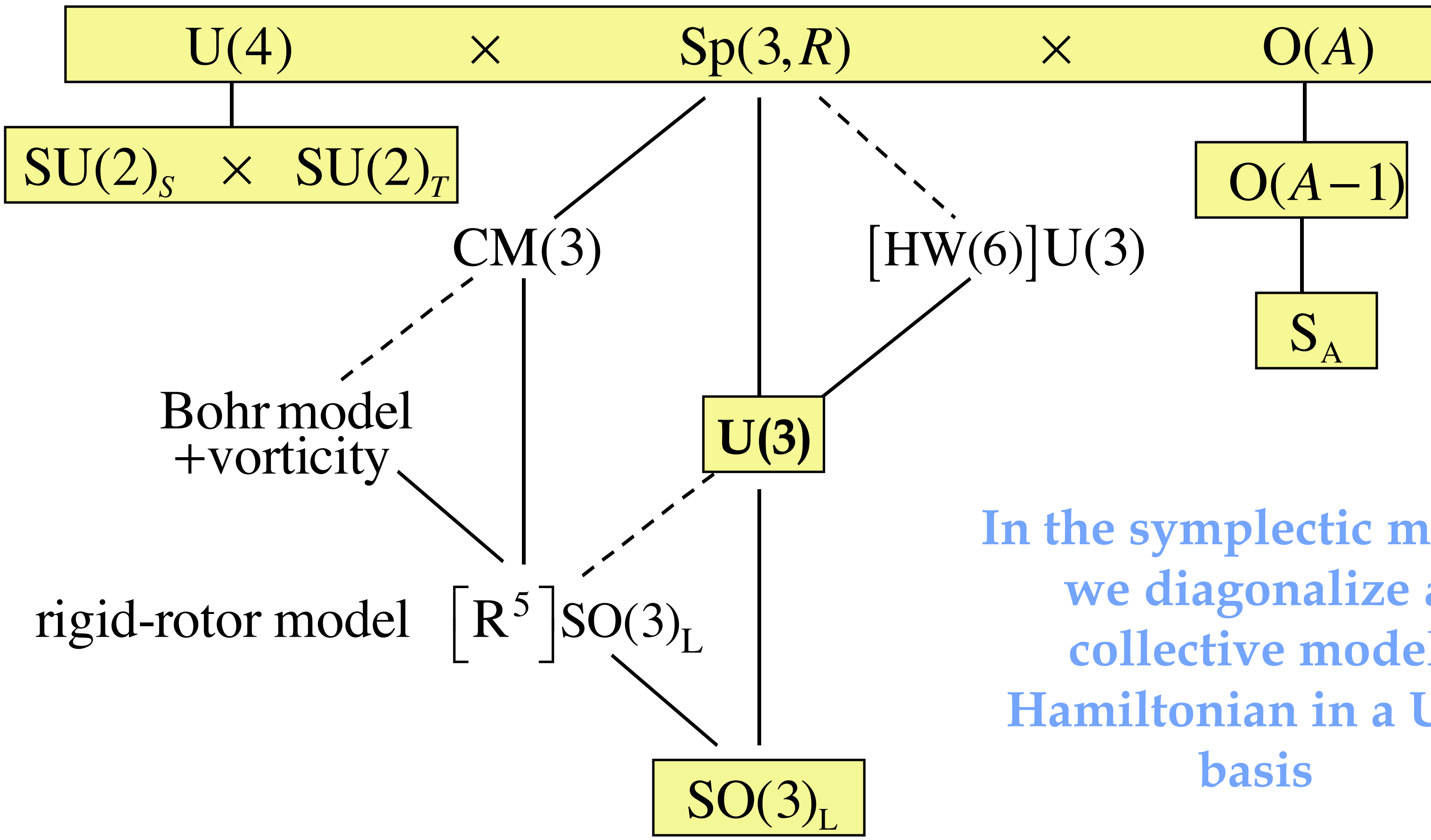
$$\mathrm{cm}(3) \rightarrow \mathrm{sp}(3,R)$$

$$\sum_{n=1}^{A} x_{ni} x_{nj}, \quad \sum_{n=1}^{A} (x_{ni} p_{nj} + p_{nj} x_{ni}), \quad \sum_{n=1}^{A} p_{ni} p_{nj}$$

GT Rosensteel and DJ Rowe, Phys. Rev. Lett. 38, 10 (1977)

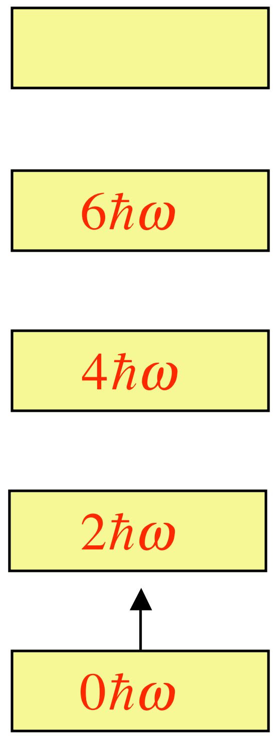
Sp(3,R) is semisimple. Its representations sit nicely within the space of the spherical harmonic oscillator shell model. Its matrix elements are easily computed.

Dynamical groups of an LST coupling scheme



In the symplectic model we diagonalize a collective model Hamiltonian in a U(3) basis

Structure of an Sp(3,R) irrep in a U(3) basis



$N+6 \quad (\lambda\mu)\otimes(20)\otimes(20)\otimes(20)$

$N+4 \quad (\lambda\mu)\otimes(20)\otimes(20)$

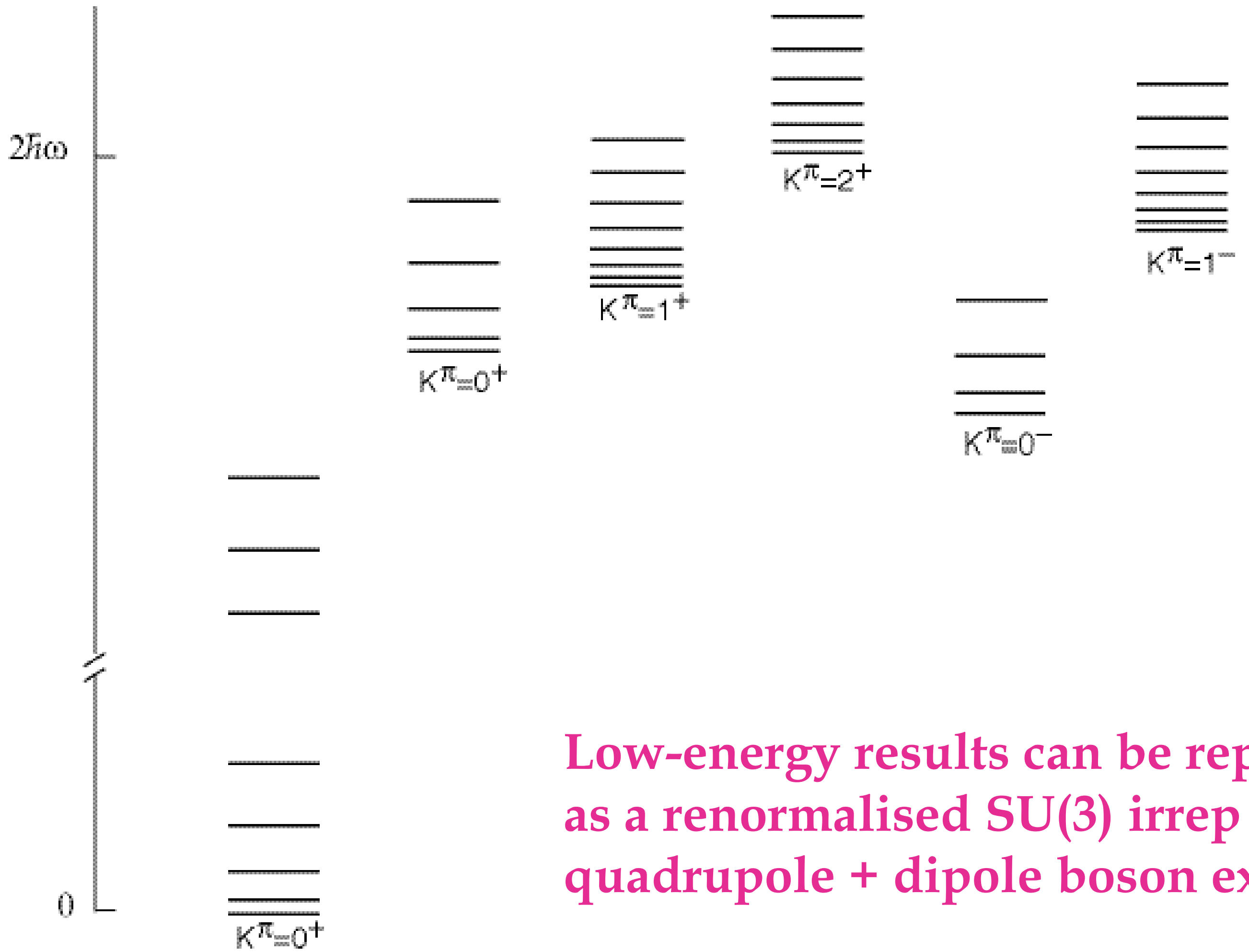
$N+2 \quad (\lambda\mu)\otimes(20)$

Giant monopole/quadrupole raising operators

$N \quad (\lambda\mu)$

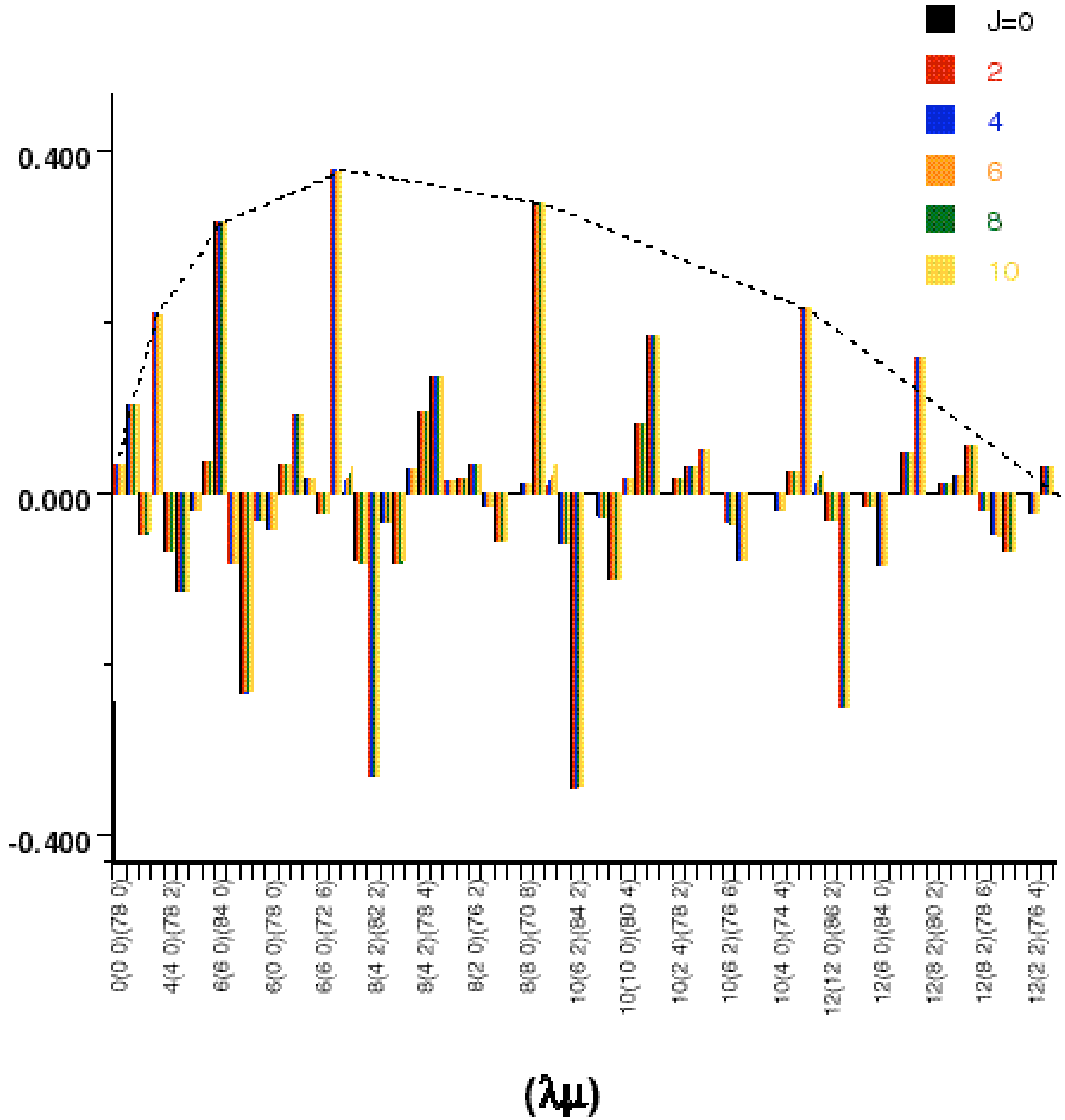
For large N, the monople/quadrupole raising operators contract to s and d boson creation operators.

A single Sp(3,R) irrep plus Giant dipole



Low-energy results can be represented as a renormalised SU(3) irrep + monopole quadrupole + dipole boson excitations

Sp(3,R) model wave functions for ^{166}Er expanded on an SU(3) basis



Bahri & Rowe
Nucl. Phys. A662, 125 (2000).

Ground-state rotational band of ^{166}Er

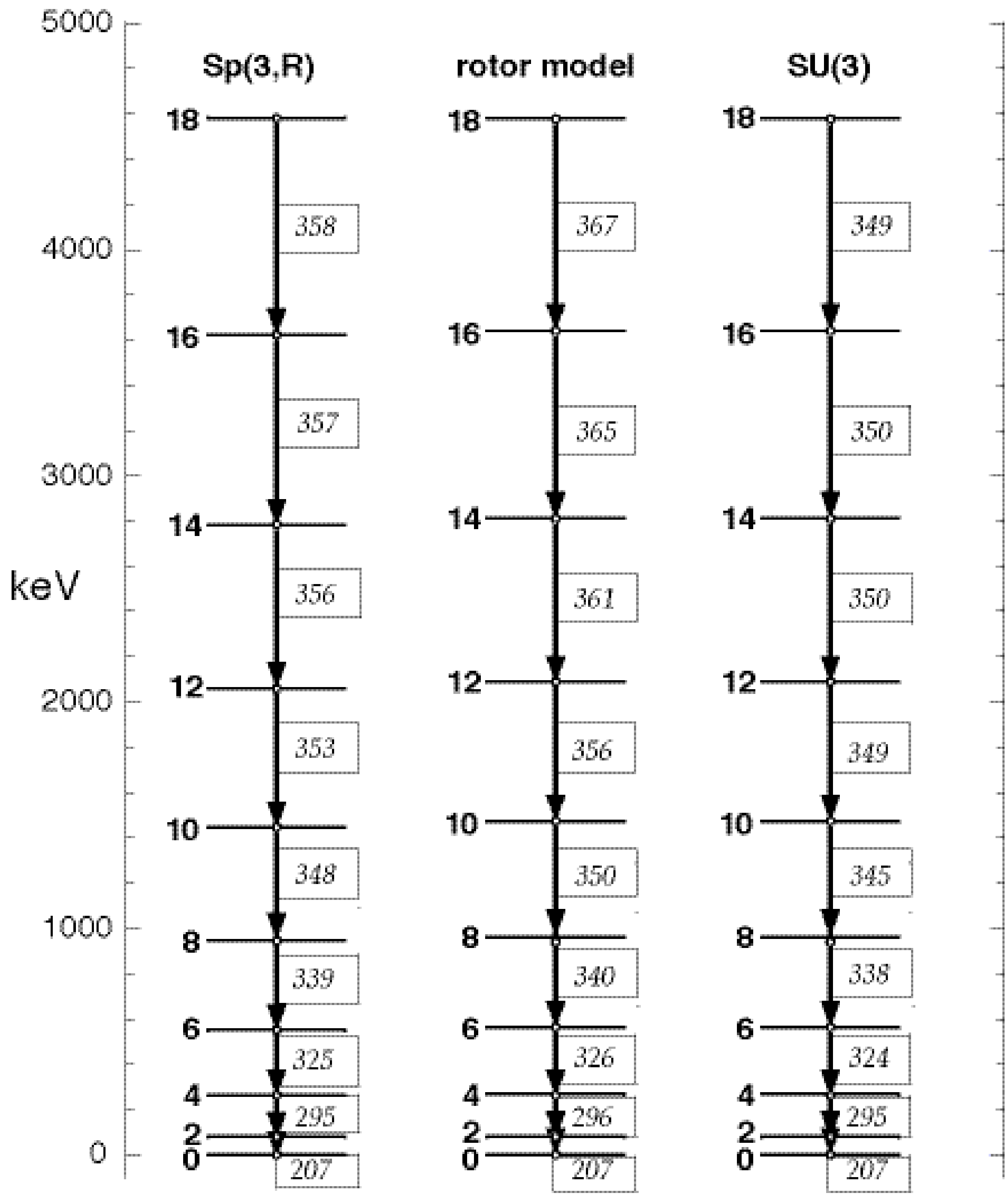
Projection:

Sp(3,R) 12$\hbar\omega$

↓

SU(3) 0$\hbar\omega$

Associated effective SU(3) interaction is simple to derive as is the effective charge



Implications for an effective shell-model space

In heavy deformed nuclei, an ideal effective shell model would be the span of lowest-weight U(3) irreps for the set of all shell-model Sp(3,R) irreps. (This includes irreps with non-zero intrinsic spins)

Carvalho et al. Nucl. Phys. A452, 240 (1986),
Le Blanc et al. Nucl. Phys. A452, 263 (1986).

This space is much to large but we can pick a subset of U(3) irreps. How?

- **The experimental approach: B(E2) values indicate (approximately) the Sp(3,R) irreps needed to describe given rotational bands.**

  M. Jarrio et al. Nucl. Phys. A528, 409 (1991)
- **Use the asymptotic Nilsson model**
- **Pick lowest-energy U(3) irreps w.r.t. a Hamiltonian**

$$H = N\hbar\omega - \kappa \mathrm{Cas}$$

Irreps with large deformations lie lower in energy

Ordering Sp(3,R) irreps

List Sp(3,R) irreps by harmonic oscillator energy.

For each Sp(3,R) evaluate $\langle \phi | H | \phi \rangle$

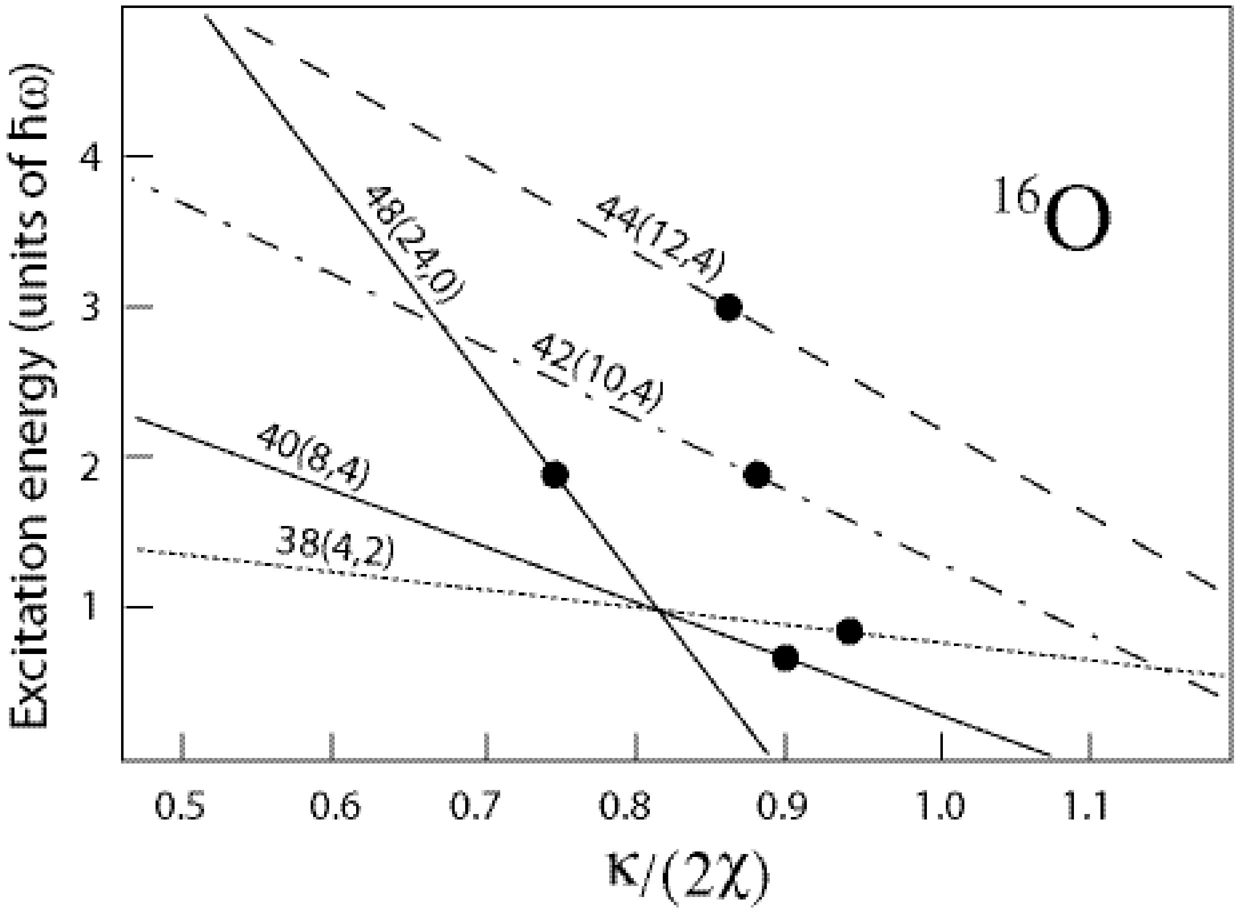
and reorder by increasing energy.

Results: For each $N\hbar\omega$ **the most deformed SU(3) irreps lie lowest.**

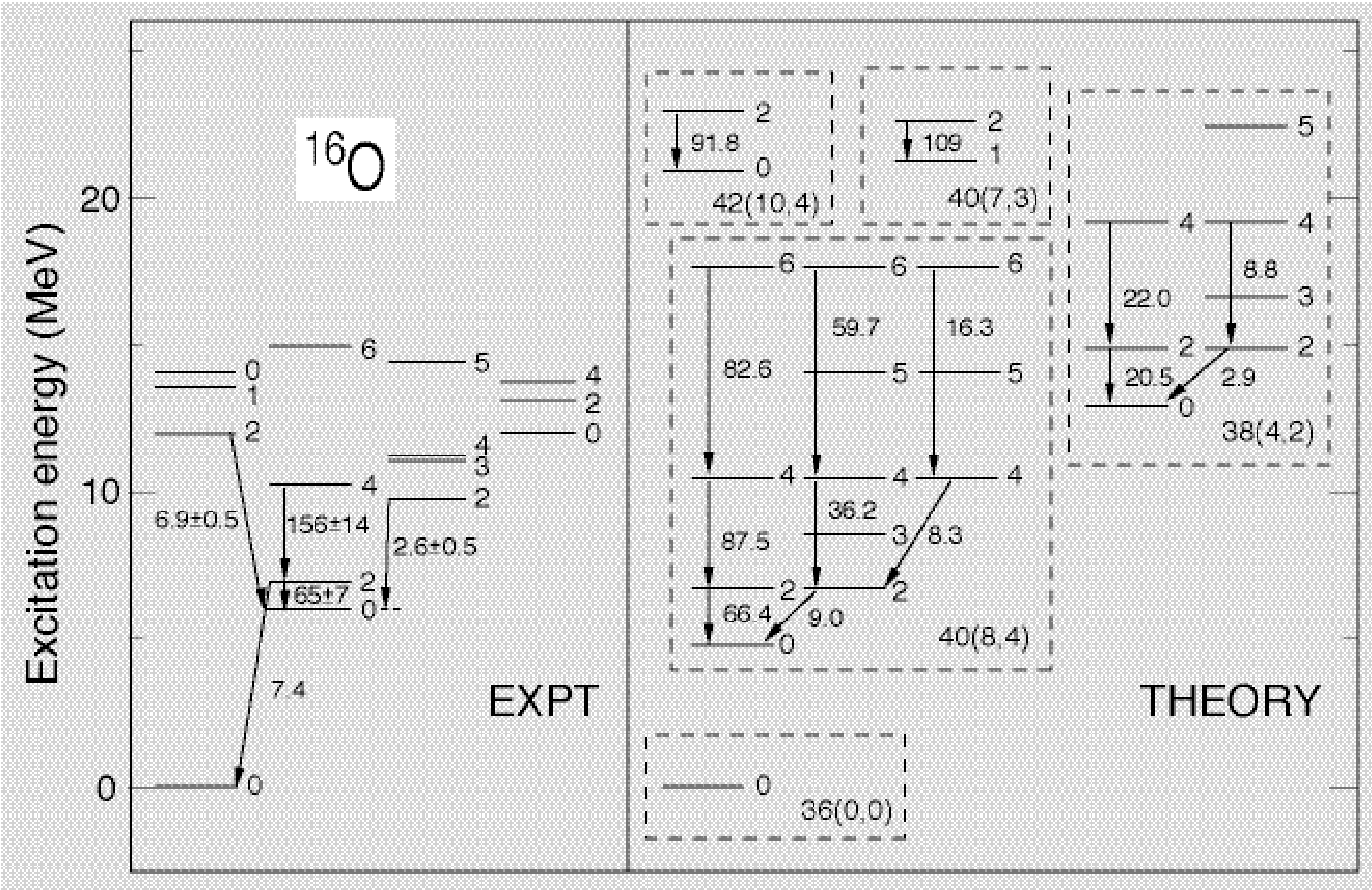
But irreps from different spherical shells cross.

Ordering of U(3) irreps for ^{16}O (simple SC approach)

$$H = N\hbar\omega - \kappa \mathrm{Cas}$$

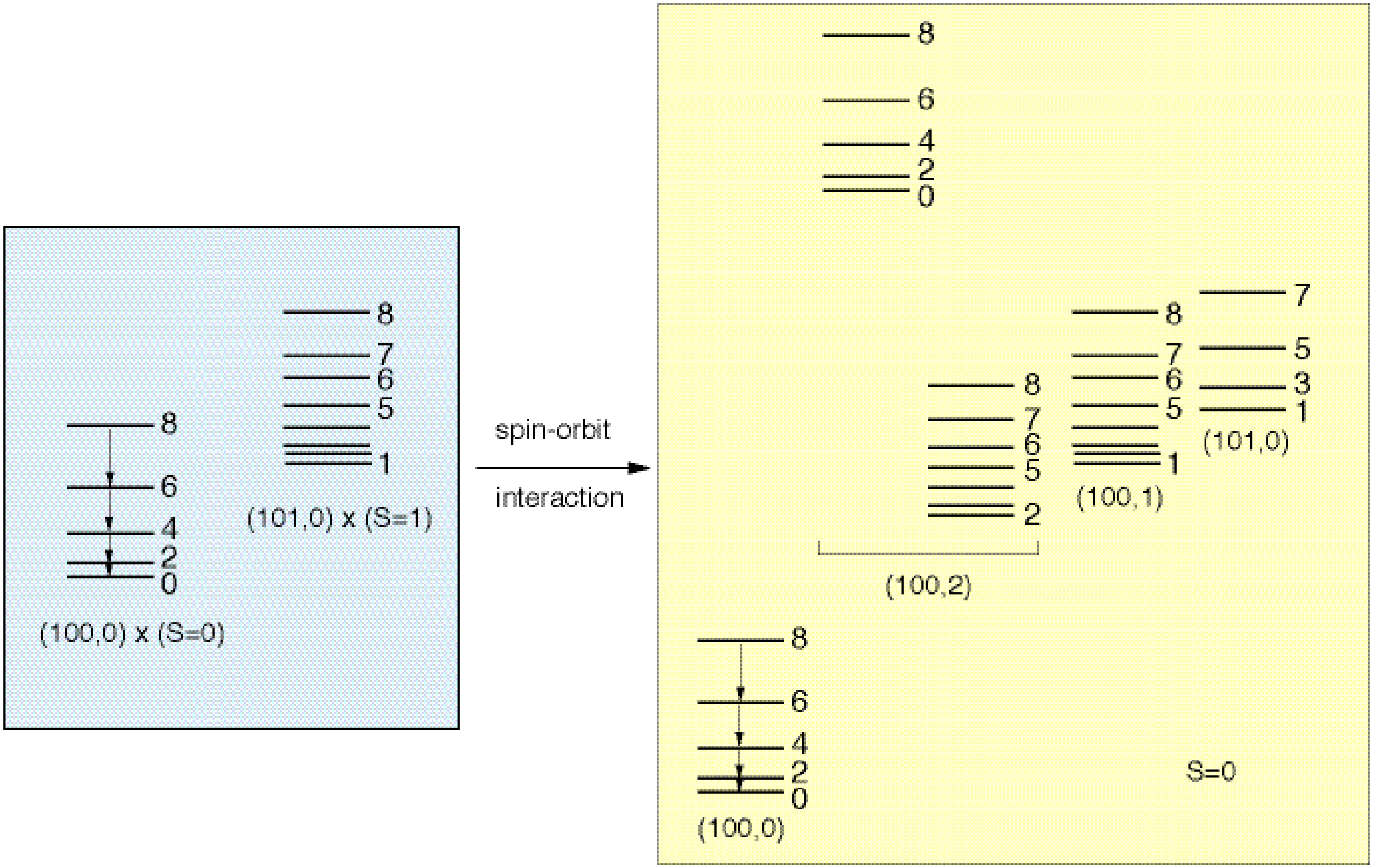


Rowe, Thiamova and Wood PRL 97, 202501 (2006)



16O
Excitation energy (MeV)
EXPT
THEORY
6.9±0.5
156±14
2.6±0.5
65±7
7.4
91.8
42(10,4)
109
40(7,3)
38(4,2)
22.0
8.8
20.5
2.9
82.6
59.7
16.3
87.5
36.2
8.3
66.4
9.0
40(8,4)
36(0,0)

SU(3) mixing with a spin-orbit interaction



The spin-orbit mixed irreps are indistinguishable from pure irreps at the 1% level

Rochford & Rowe Phys. Lett. B210, 5 (1988)

Model of a spherical superconductor to deformed rotor phase transition

Many-fermion model

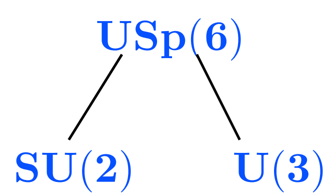
$$H(\alpha) = (1-\alpha)V_{\mathrm{SU(2)}} + \alpha V_{\mathrm{SU(3)}}$$

$H(\alpha\!=\!0)$ **has the SU(2) dynamical symmetry of a spherical superconductor**

$H(\alpha\!=\!1)$ **has the SU(3) dynamical symmetry of a deformed rotor**

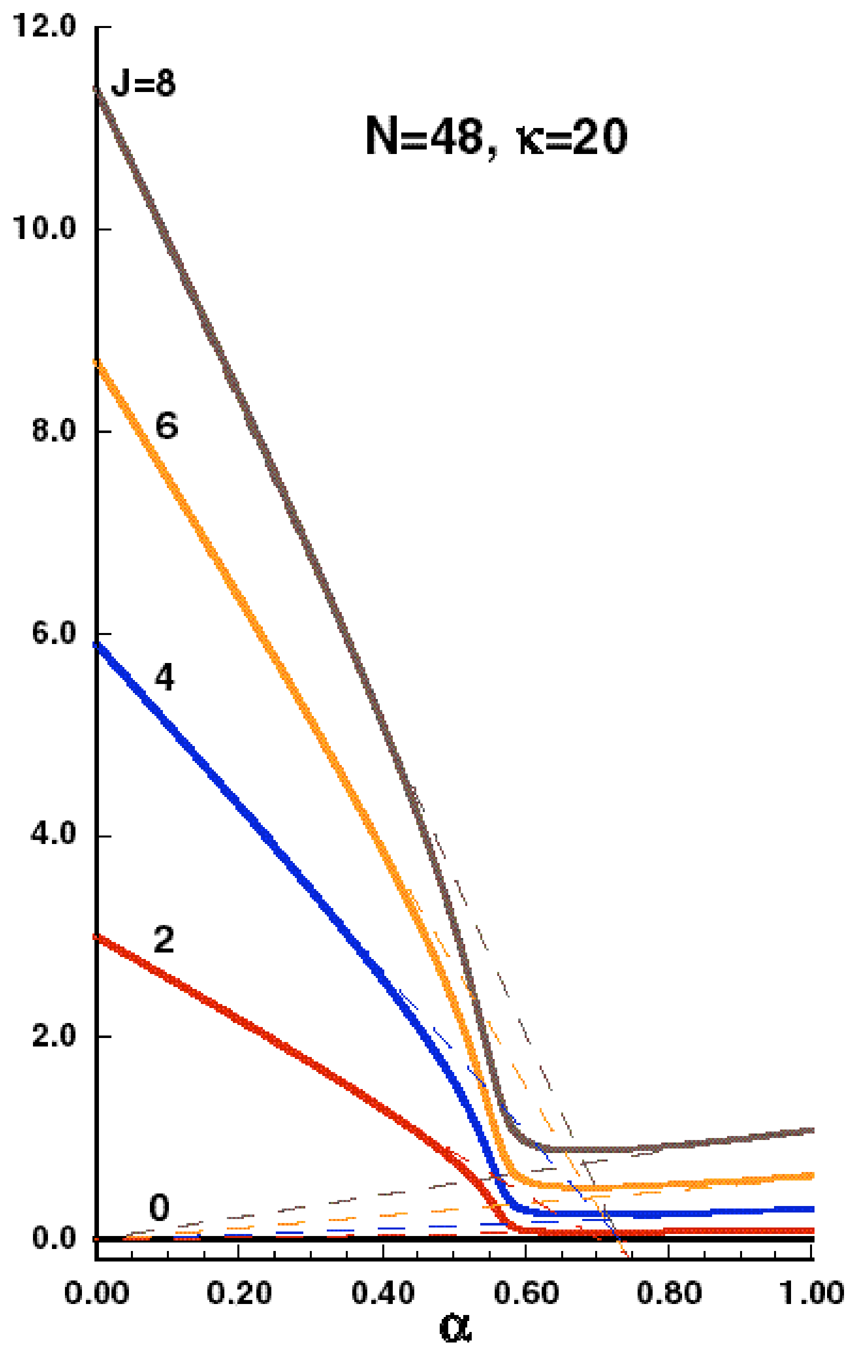
For $0 < \alpha < 1$**, solutions are generally too complicated to solve because there is no simple subgroup that contains both SU(2) and SU(3).**

A model problem: suppose SU(2) and SU(3) are non-commuting subgroups of USp(6)



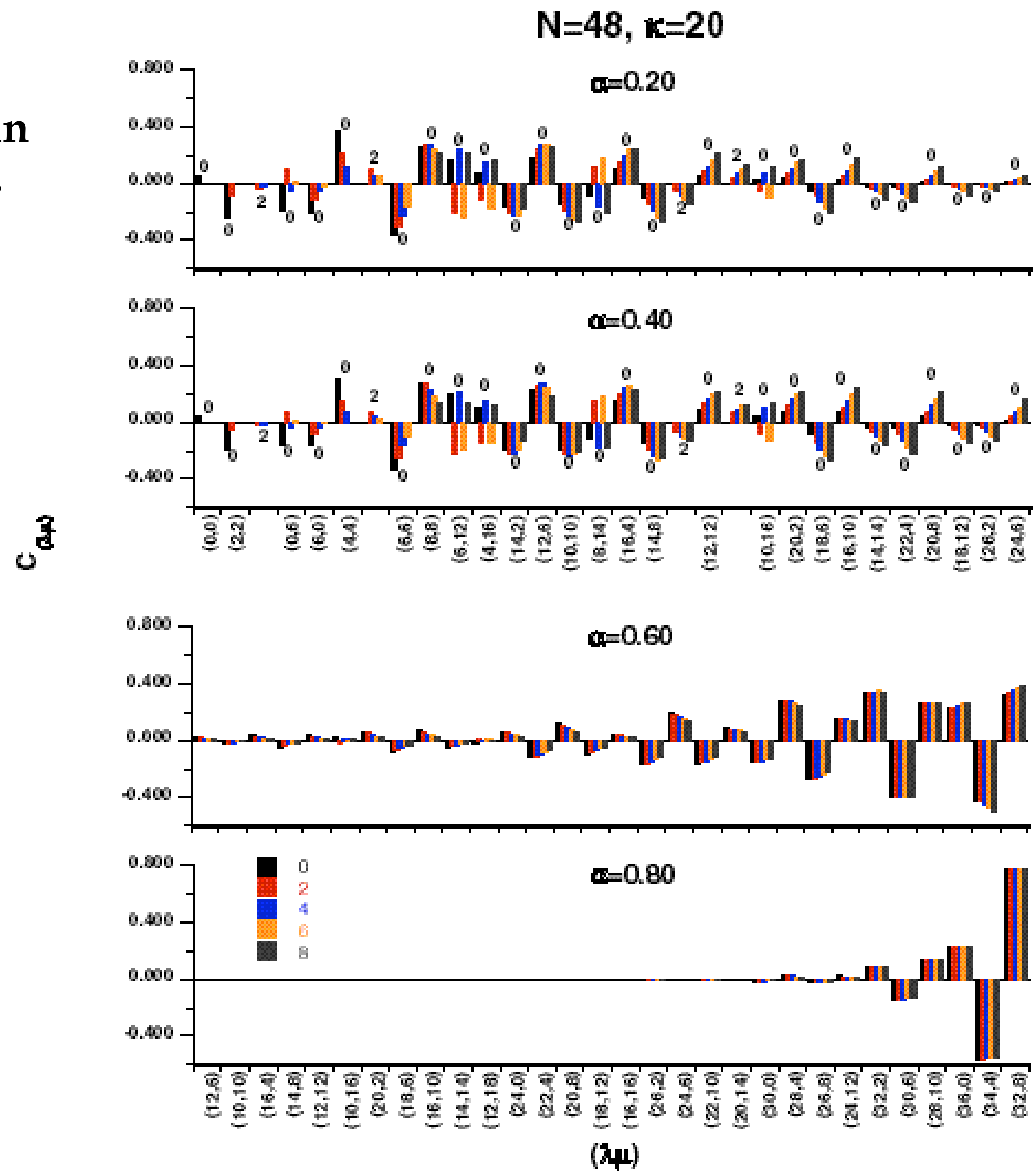
$$H_{\mathrm{SU(2)}} = \chi_1 S_+ S_-$$

$$H_{\mathrm{SU(3)}} = \chi_2 Q \cdot Q$$



Bahri, Rowe and Wijesundera
Phys. Rev. C58, 1539 (1998)

Wave functions in an SU(3) > SO(3) basis



Two LS coupling schemes

L=0 pair-coupling model

$$\mathrm{SU}(2)_S \times \mathrm{SU}(2)_T \times \mathrm{U}(\sum 2l+1) \supset \mathrm{O}(\sum 2l+1) \supset \mathrm{SO}(3)_L$$

$$S \qquad T \qquad (f) \qquad [\kappa] \qquad L$$

OR

$$\mathrm{SO}(3)_L \times \mathrm{O}(8) \supset \mathrm{U}(4) \supset \mathrm{SU}(2)_S \times \mathrm{SU}(2)_T$$

$$L \qquad [\kappa] \qquad (f) \qquad S \qquad T$$

SU(3) coupling

$$\mathrm{SU}(2)_S \times \mathrm{SU}(2)_T \times \mathrm{U}(\sum 2l+1) \supset \mathrm{SU}(3) \supset \mathrm{SO}(3)_L$$

$$S \qquad T \qquad (f) \qquad (\lambda\mu) \qquad L$$

OR

$$\mathrm{SU}(3) \times \mathrm{U}(4) \supset \mathrm{SO}(3)_L \times \mathrm{SU}(2)_S \times \mathrm{SU}(2)_T$$

$$(\lambda\mu) \qquad (f) \qquad L \qquad S \qquad T$$

For the 2s1d shell

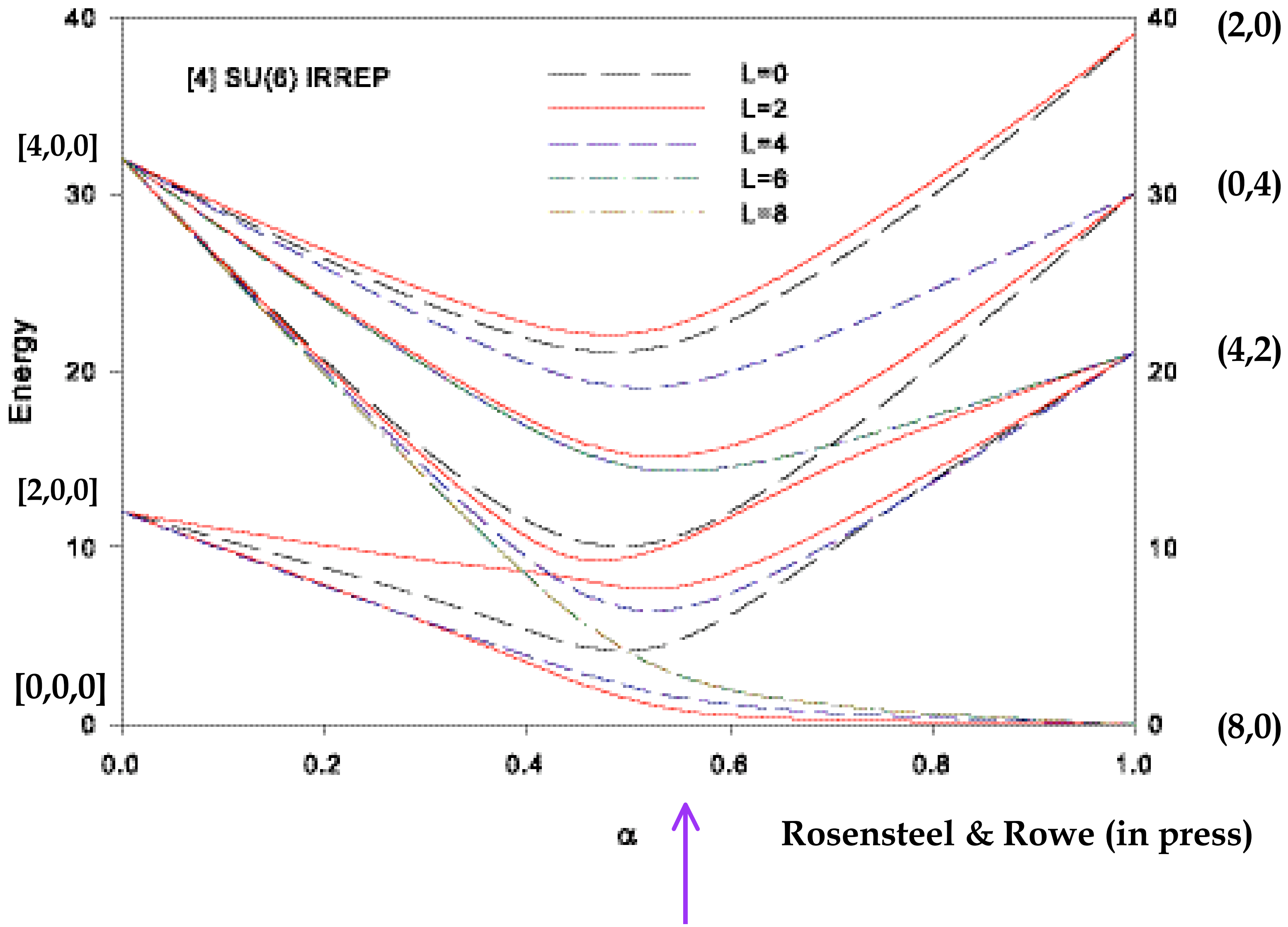
Pairing model

$$U(6) \supset O(6) \supset SO(3)$$

SU(3)-rotor model

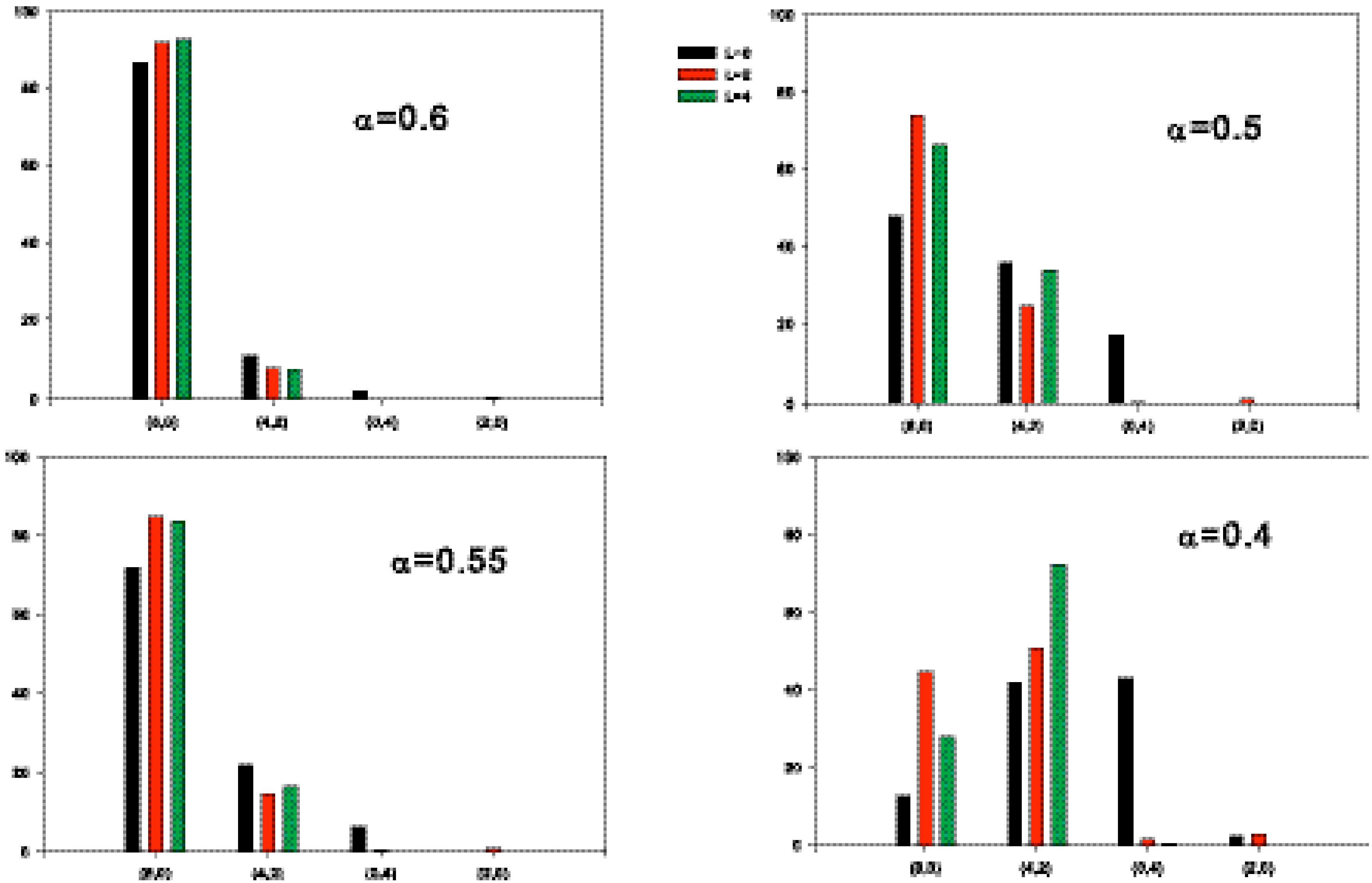
$$U(6) \supset SU(3) \supset SO(3)$$

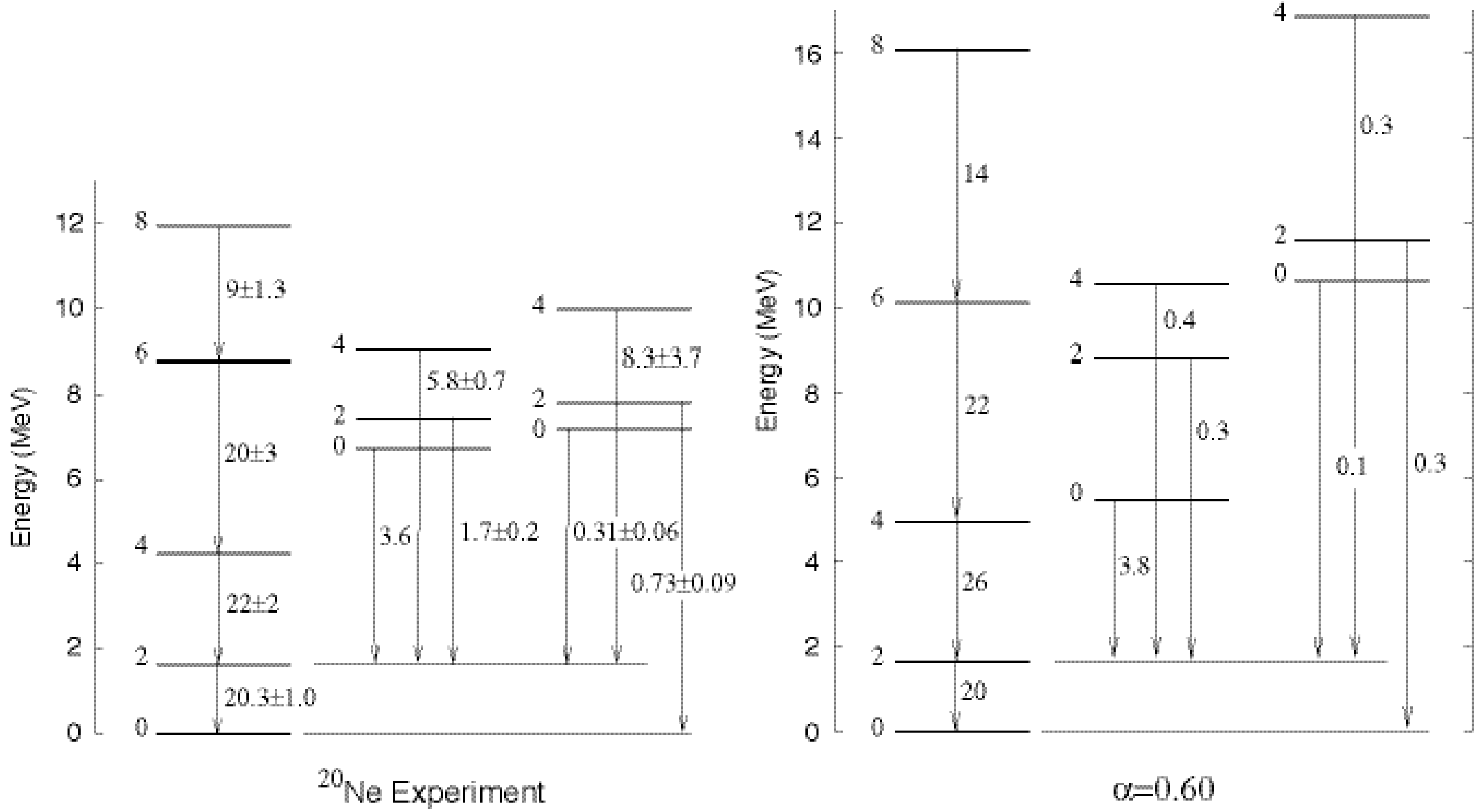
Mixed Q.Q + L=0 pairing



Rosensteel & Rowe (in press)

20 Ne pairing plus quadrupole wave functions





Energy (MeV)
0
2
4
6
8
10
12
8
6
4
2
0
9±1.3
20±3
22±2
20.3±1.0
4
2
0
5.8±0.7
3.6
1.7±0.2
4
2
0
8.3±3.7
0.31±0.06
0.73±0.09
^{20}Ne Experiment
Energy (MeV)
0
2
4
6
8
10
12
14
16
8
6
4
2
0
14
22
26
20
4
2
0
0.4
0.3
3.8
4
2
0
0.3
0.1
0.3
α=0.60

Effective shell-model spaces spanned by a finite number of Sp(3,R) irreps

In model calculations for nuclei in which rotational dynamics is dominant states are highly coherent mixtures of relatively few irreps.

Why is this? A similar question: Why are lowlying states of nuclei dominated by quadrupole collective states?

By embedding collective models in the shell model, we can use the information gained from model explanations of the data to design a suitable effective space for a microscopic shell model calculation.

This is not 100% predictive. But I doubt that nuclear physics will ever be 100% predictable. Nuclear physics is rich in emergent phenomena. By using experimental data and model interpretations to guide shell model descriptions, we can build up an understanding and progress towards a more predictive theory.

Note that, for intruder states and for well-deformed nuclei, we need to include Sp(3,R) irreps with lowest- weight SU(3) irreps from higher shells.