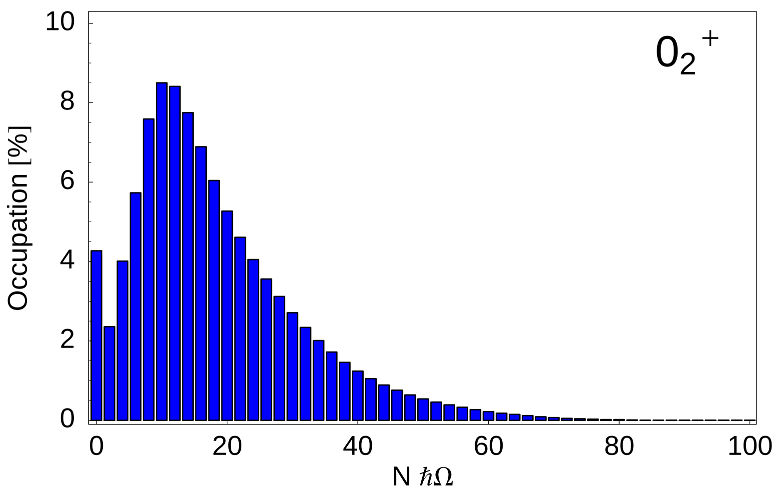
Cluster States and Shell Model Configurations in the Fermionic Molecular Dynamics Approach



Thomas Neff

INT Workshop on
'New Approaches in
Nuclear Many-Body Theory'
Seattle, USA
October 15-19, 2007

GSI

Overview

Unitary Correlation Operator Method

Fermionic Molecular Dynamics

Helium Isotopes

Cluster States in ^{12}C

Nucleon-Nucleon Interaction

Short-range Correlations

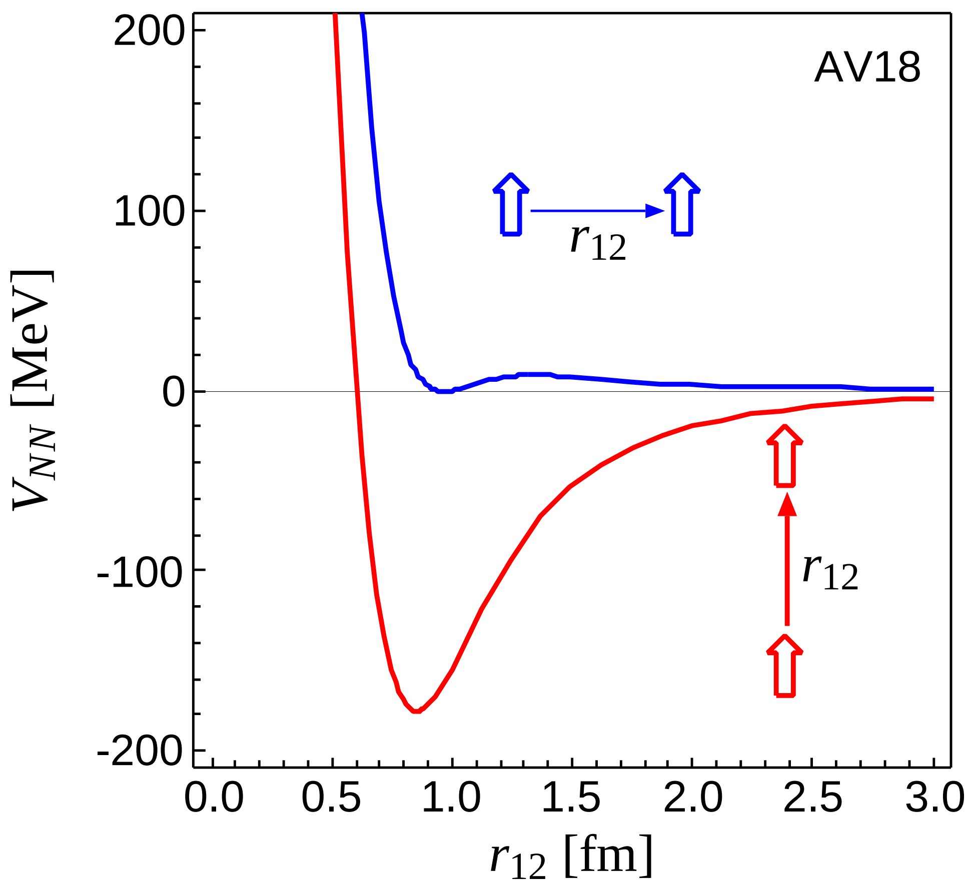
Unitary Correlation Operator Method

- **Correlation Operators**
- **Interaction in Momentum Space**

Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector



- strong repulsive core: nucleons can not get closer than $\approx 0.5\,\mathrm{fm}$

➨ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➨ **tensor correlations**

the nuclear force will induce **strong short-range correlations** in the nuclear wave function

UCOM

Central and Tensor Correlations

$$\underset{\sim}{C} = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r$$

$$\mathbf{p} = \mathbf{p}_r + \mathbf{p}_\Omega$$

$$\mathbf{p}_r = \frac{1}{2}\left\{\frac{\mathbf{r}}{r}\left(\frac{\mathbf{r}}{r}\mathbf{p}\right) + \left(\mathbf{p}\frac{\mathbf{r}}{r}\right)\frac{\mathbf{r}}{r}\right\}, \qquad \mathbf{p}_\Omega = \frac{1}{2r}\left\{\mathbf{l} \times \frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r} \times \mathbf{l}\right\}$$

Central Correlations

$$\underset{\sim}{c}_r = \exp\left\{-\frac{i}{2}\left\{p_r s(r) + s(r) p_r\right\}\right\}$$

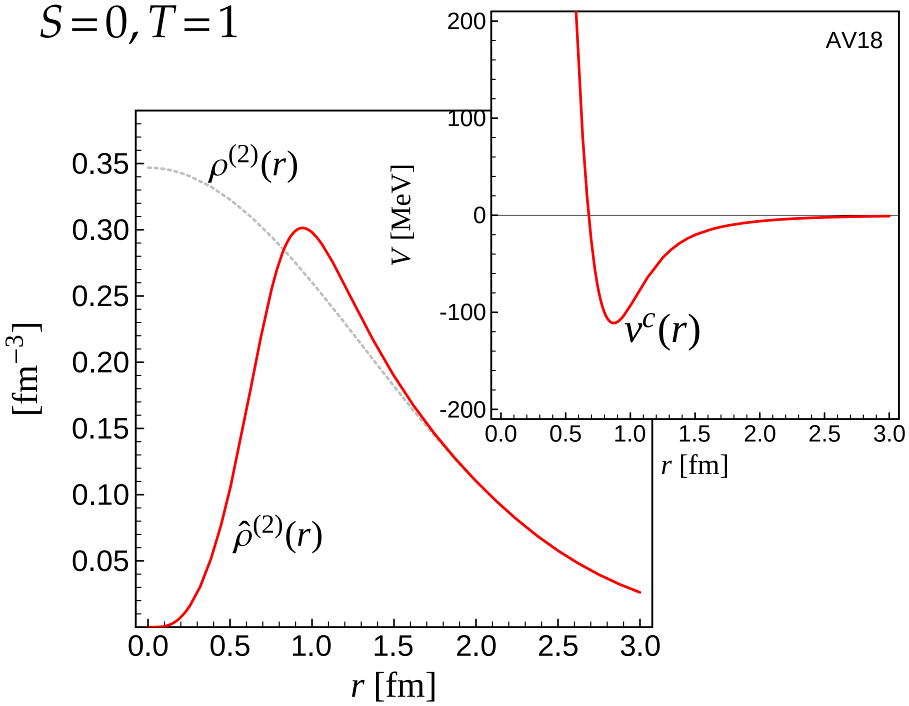
- probability density shifted out of the repulsive core

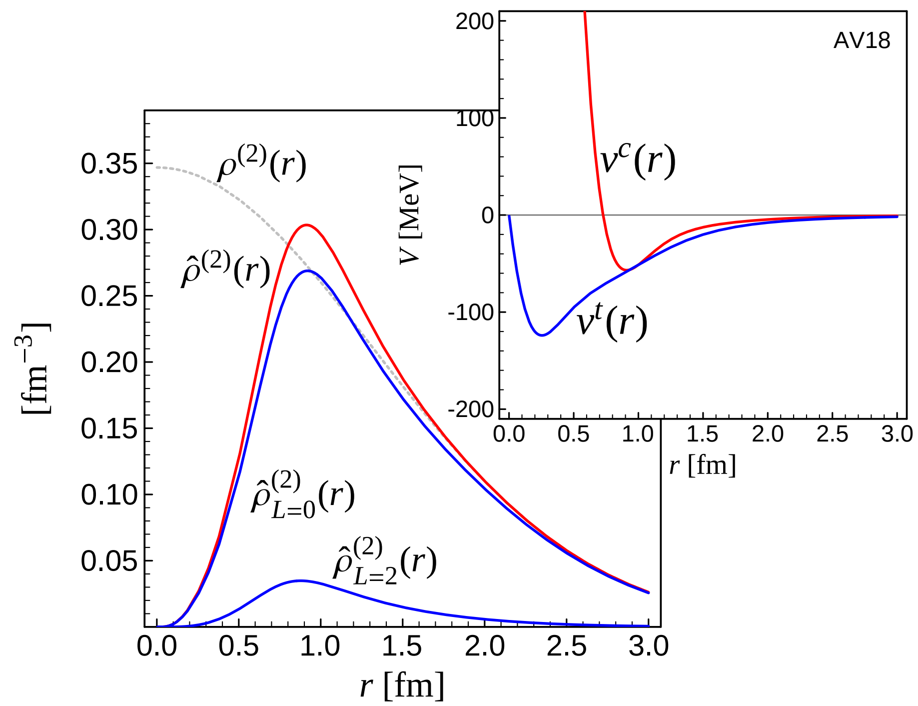
Tensor Correlations

$$\underset{\sim}{c}_\Omega = \exp\left\{-i\vartheta(r)\left\{\frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_\Omega)(\boldsymbol{\sigma}_2\cdot\mathbf{r}) + \frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_\Omega)\right\}\right\}$$

- tensor force admixes other angular momenta

$S=0, T=1$





Central and Tensor Correlations

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Central Correlations

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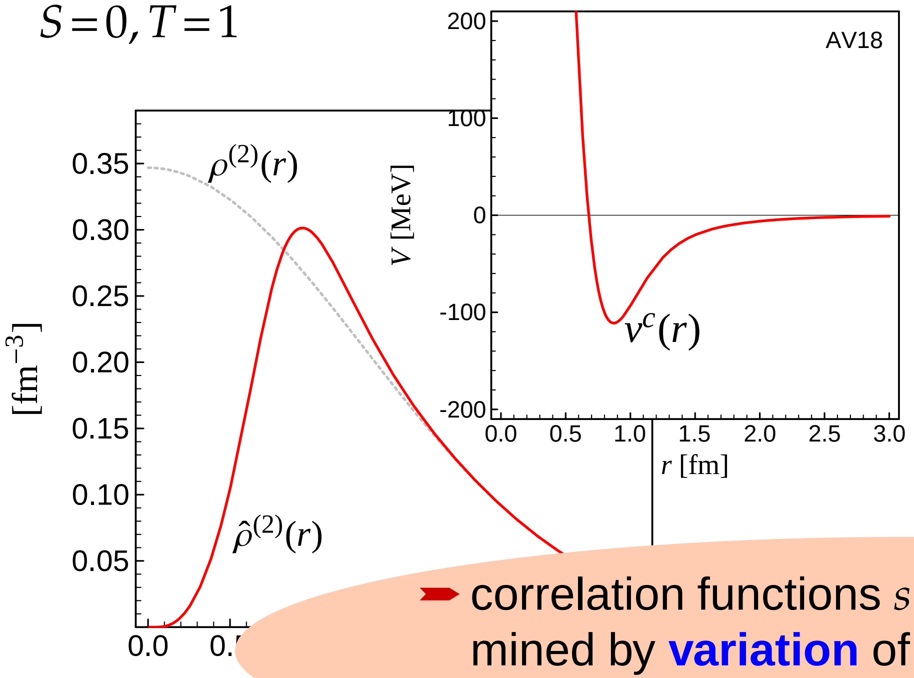

Tensor Correlations

$$\underset{\sim}{c}_\Omega = \exp\left\{-i\vartheta(r)\left\{\frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_\Omega)(\boldsymbol{\sigma}_2\cdot\mathbf{r}) + \frac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_\Omega)\right\}\right\}$$

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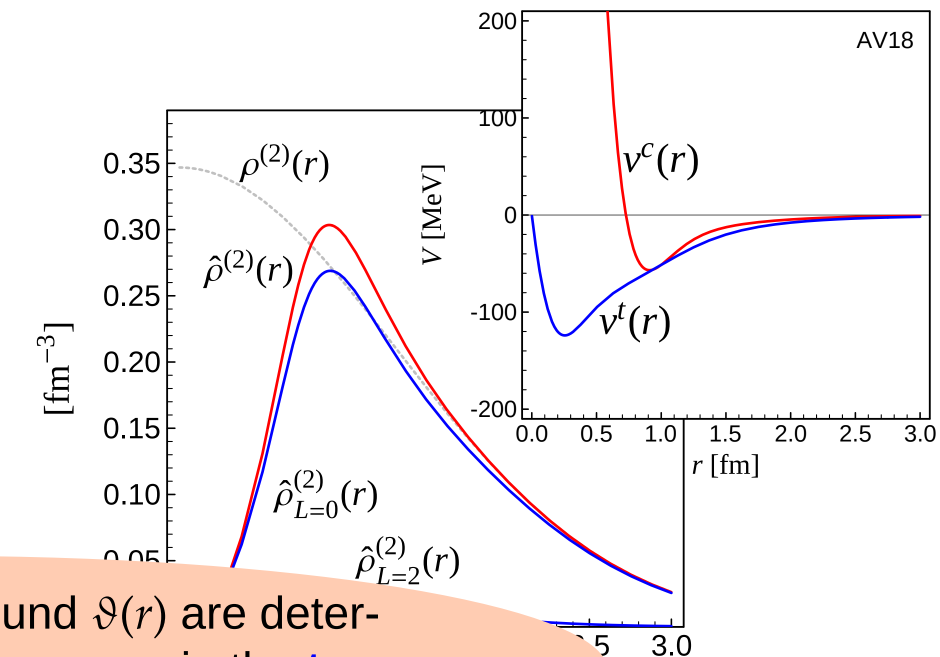
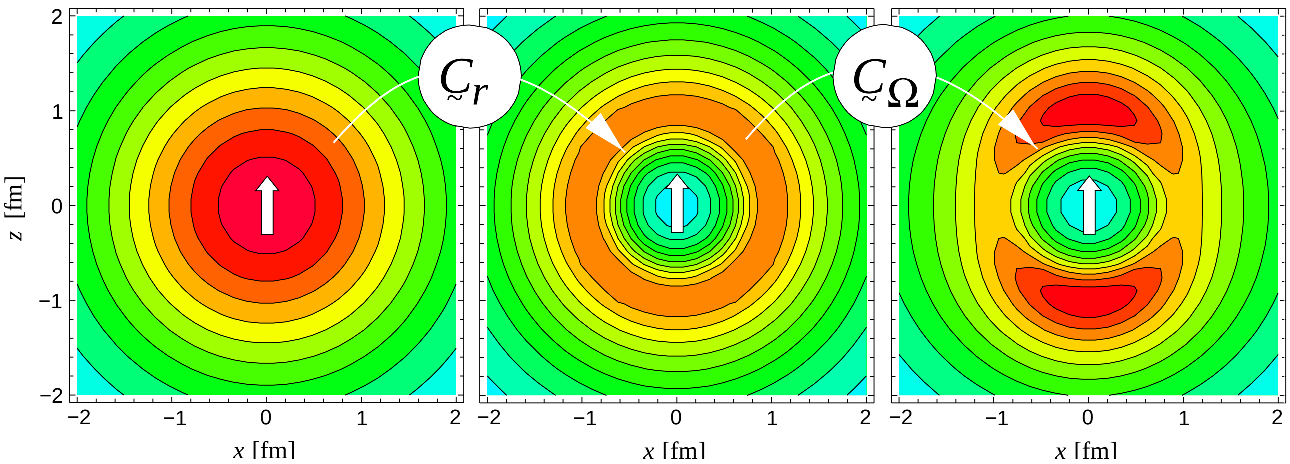


- correlation functions $s(r)$ und $\vartheta(r)$ are determined by **variation** of the energy in the **two-body system** for each S, T channel

Correlated Two-Body Densities and Energies

two-body densities



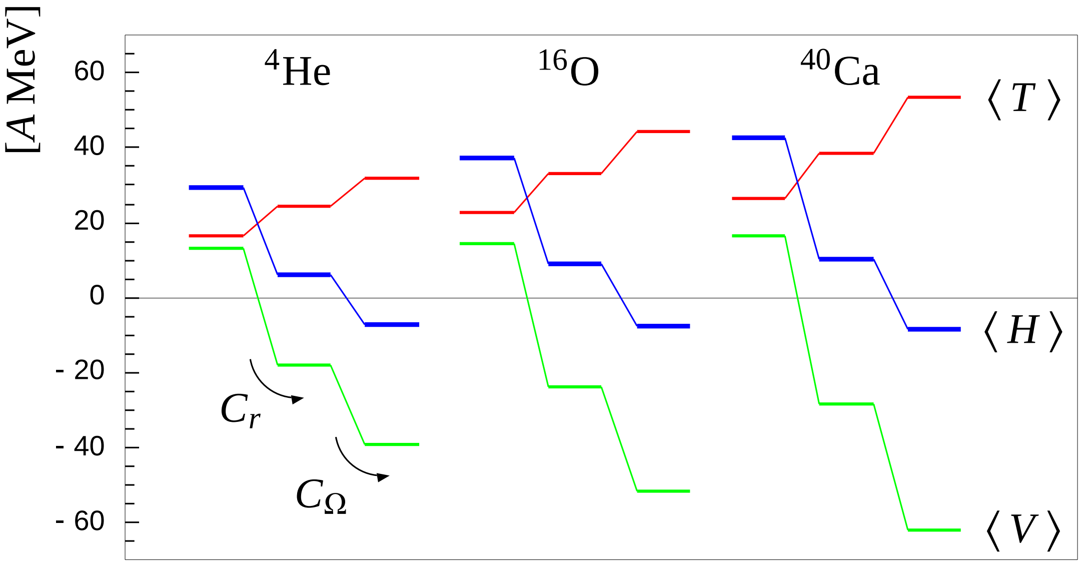
$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

central correlator $\underset{\sim}{C}_{\mathbf{r}}$ shifts density out of the repulsive core

tensor correlator $\underset{\sim}{C}_{\mathbf{\Omega}}$ aligns density with spin orientation

energies

$0\hbar\omega$ Harmonic Oscillator

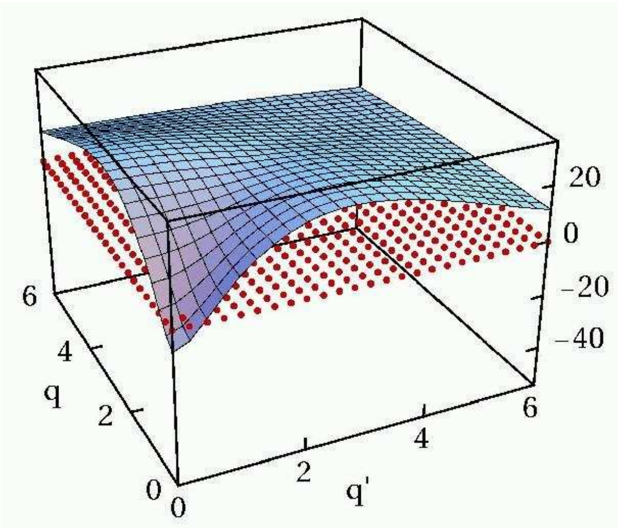


both central and tensor correlations are essential for binding

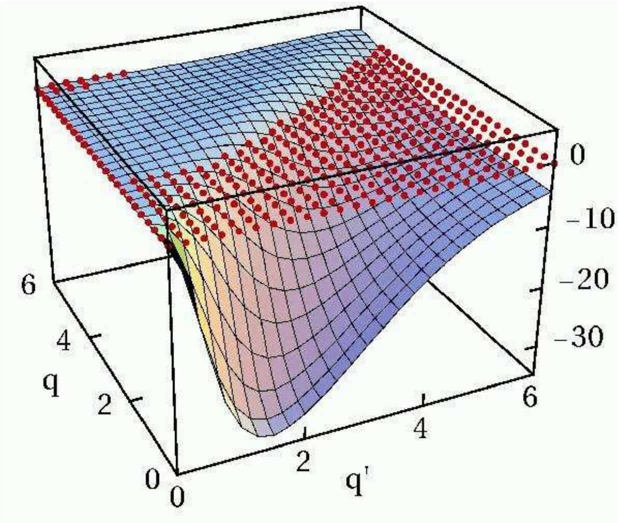
Nucl. Phys. **A713** (2003) 311

Correlated Interaction in Momentum Space

$^3S_1$ bare

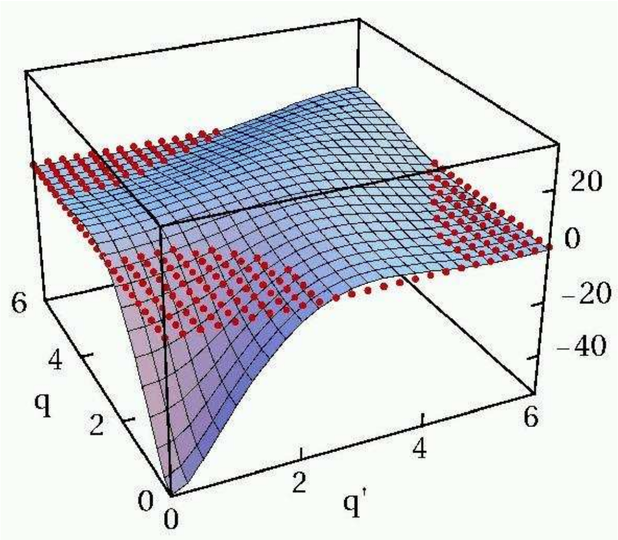


$^3S_1$ - $^3D_1$ bare

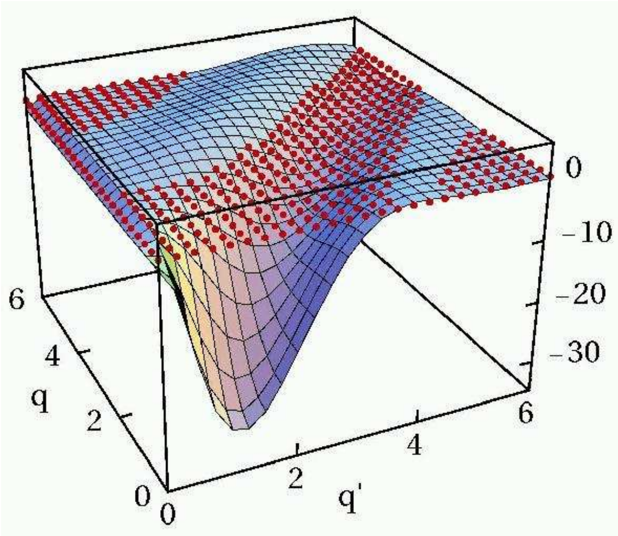


correlated interaction is **more attractive** at low momenta

$^3S_1$ correlated



$^3S_1$ - $^3D_1$ correlated



off-diagonal matrix elements connecting low- and high-momentum states are **strongly reduced**

Phys. Rev. **C72** (2005) 034002

Fermionic Molecular Dynamics

FMD Wave Functions

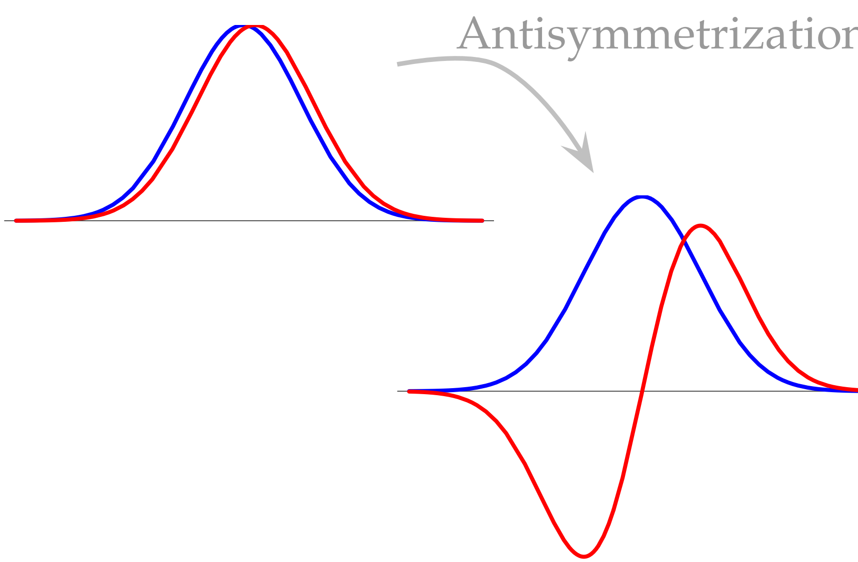
Nucleon-Nucleon Interaction

Mean-Field Calculations

Projection After Variation and Variation After Projection

Helium Isotopes

Fermionic Molecular Dynamics



Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}}\Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle \Big)$$

- antisymmetrized A-body state

Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \, \exp\left\{ -\frac{(\mathbf{x}-\mathbf{b}_i)^2}{2a_i} \right\} \otimes \left| \chi^{\uparrow}_i, \chi^{\downarrow}_i \right\rangle \otimes \left| \xi \right\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- superposition of two wave packets for each single particle state

Rev. Mod. Phys. **72** (2000) 655

Nucl. Phys. **A745** (2004) 3

Dynamics

Time-dependent

Time-dependent variational principle

$$\delta \int dt \frac{\left\langle Q \middle| i\frac{d}{dt} - \underset{\sim}{\hat{H}} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle} = 0$$

➨ describe heavy-ion reactions, thermodynamics with ergodic ensembles

Time-independent

Ritz variational principle

$$\delta \frac{\left\langle Q \middle| \underset{\sim}{\hat{H}} - \underset{\sim}{\hat{T}}_{\mathrm{cm}} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle} = 0$$

➨ minimize expectation value with respect to all the sp-parameters $q_k = \{c_k, a_k, \mathbf{b}_k, \chi_k\}$

➨ need analytical gradients

$$\frac{\partial}{\partial q_i^*} \frac{\left\langle Q \middle| \underset{\sim}{\hat{H}} - \underset{\sim}{\hat{T}}_{\mathrm{cm}} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle}$$

Rev. Mod. Phys. **72** (2000) 655

Nucl. Phys. **A745** (2004) 3

Evaluation of Matrix Elements

➨ non-orthogonal basis, use inverse overlap matrix

One-Body Operators

$$\frac{\langle Q | \underset{\sim}{T}^{[1]} | Q \rangle}{\langle Q | Q \rangle} = \sum_{k,l}^{A} \langle q_k | \underset{\sim}{T}^{[1]} | q_l \rangle \, \mathrm{o}_{lk}$$

Two-Body Operators

$$\frac{\langle Q | \underset{\sim}{V}^{[2]} | Q \rangle}{\langle Q | Q \rangle} = \frac{1}{2} \sum_{k,l,m,n}^{A} \langle q_k, q_l | \underset{\sim}{V}^{[2]} | q_m, q_n \rangle \, (\mathrm{o}_{mk}\mathrm{o}_{nl} - \mathrm{o}_{ml}\mathrm{o}_{nk})$$

$$\mathrm{o} = \mathrm{n}^{-1} = \left(\langle q_i | q_j \rangle \right)^{-1}$$

Interaction Matrix Elements

(One-body) Kinetic Energy

$$\left\langle q_k \middle| \underset{\sim}{T} \middle| q_l \right\rangle = \left\langle a_k \mathbf{b}_k \middle| \underset{\sim}{T} \middle| a_l \mathbf{b}_l \right\rangle \left\langle \chi_k \middle| \chi_l \right\rangle \left\langle \xi_k \middle| \xi_l \right\rangle$$

$$\left\langle a_k \mathbf{b}_k \middle| \underset{\sim}{T} \middle| a_l \mathbf{b}_l \right\rangle = \frac{1}{2m}\left(\frac{3}{a_k^\star + a_l} - \frac{(\mathbf{b}_k^\star - \mathbf{b}_l)^2}{(a_k^\star + a_l)^2} \right) R_{kl}$$

(Two-body) Potential

$$\alpha_{klmn} = \frac{a_k{}^\star a_m}{a_k{}^\star + a_m} + \frac{a_l{}^\star a_n}{a_l{}^\star + a_n}$$

$$\rho_{klmn} = \frac{a_m \mathbf{b}_k{}^\star + a_k{}^\star \mathbf{b}_m}{a_k{}^\star + a_m} - \frac{a_n \mathbf{b}_l{}^\star + a_l{}^\star \mathbf{b}_n}{a_l{}^\star + a_n}$$

$$R_{km} = \left\langle a_k \mathbf{b}_k \middle| a_m \mathbf{b}_m \right\rangle$$

➨ fit radial dependencies by (a sum of) Gaussians

$$G(\mathbf{x}_1 - \mathbf{x}_2) = \exp\left\{ -\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{2\kappa} \right\}$$

➨ perform Gaussian integrals

$$\left\langle a_k \mathbf{b}_k, a_l \mathbf{b}_l \middle| \underset{\sim}{G} \middle| a_m \mathbf{b}_m, a_n \mathbf{b}_n \right\rangle = R_{km} R_{ln} \left(\frac{\kappa}{\alpha_{klmn} + \kappa} \right)^{3/2} \exp\left\{ -\frac{\rho_{klmn}^2}{2(\alpha_{klmn} + \kappa)} \right\}$$

➨ analytical formulas for matrix elements

Operator Representation of V_{UCOM}

$$\underset{\sim}{C}^{\dagger}(\underset{\sim}{T} + \underset{\sim}{V})\underset{\sim}{C} = \underset{\sim}{T}$$

one-body kinetic energy

$$+ \sum_{ST} \hat{V}_{c}^{ST}(r) + \frac{1}{2}\left(\underset{\sim}{p_r}^2\, \hat{V}_{p^2}^{ST}(r) + \hat{V}_{p^2}^{ST}(r)\, \underset{\sim}{p_r}^2\right) + \hat{V}_{l^2}^{ST}(r)\, \underset{\sim}{\mathbf{l}}^2$$

central potentials

$$+ \sum_{T} \hat{V}_{ls}^{T}(r)\, \underset{\sim}{\mathbf{l}} \cdot \underset{\sim}{\mathbf{s}} + \hat{V}_{l^2ls}^{T}(r)\, \underset{\sim}{\mathbf{l}}^2 \underset{\sim}{\mathbf{l}} \cdot \underset{\sim}{\mathbf{s}}$$

spin-orbit potentials

$$+ \sum_{T} \hat{V}_{t}^{T}(r)\, \underset{\sim}{S}_{12}(\mathbf{r}, \mathbf{r}) + \hat{V}_{trp_\Omega}^{T}(r)\, \underset{\sim}{p_r}\, \underset{\sim}{S}_{12}(\mathbf{r}, \mathbf{p}_\Omega) + \hat{V}_{tll}^{T}(r)\, \underset{\sim}{S}_{12}(\mathbf{l}, \mathbf{l}) +$$
$$\hat{V}_{tp_\Omega p_\Omega}^{T}(r)\, \underset{\sim}{S}_{12}(\mathbf{p}_\Omega, \mathbf{p}_\Omega) + \hat{V}_{l^2tp_\Omega p_\Omega}^{T}(r)\, \underset{\sim}{\mathbf{l}}^2 \underset{\sim}{S}_{12}(\mathbf{p}_\Omega, \mathbf{p}_\Omega)$$

tensor potentials

bulk of tensor force mapped onto central part of correlated interaction

tensor correlations also change the spin-orbit part of the interaction

Nucl. Phys. **A745** (2004) 3

Phenomenological Correction to V_{UCOM}

Effective two-body interaction

- FMD model space can't describe correlations induced by residual medium-long ranged tensor forces

➨ use **longer ranged tensor correlator** to partly account for that

- no three-body forces, saturation with UCOM force not correct

➨ add phenomenological two-body correction term with a **momentum-dependend** central and (isospin-dependend) **spin-orbit** part
(about 15% contribution to potential)

➨ fit correction term to binding energies and radii of "closed-shell" nuclei
(^{4}He, ^{16}O, ^{40}Ca), (^{24}O, ^{34}Si, ^{48}Ca)

➨ **Todo:**
use **three-body** or **density dependent two-body force** instead of two-body correction term

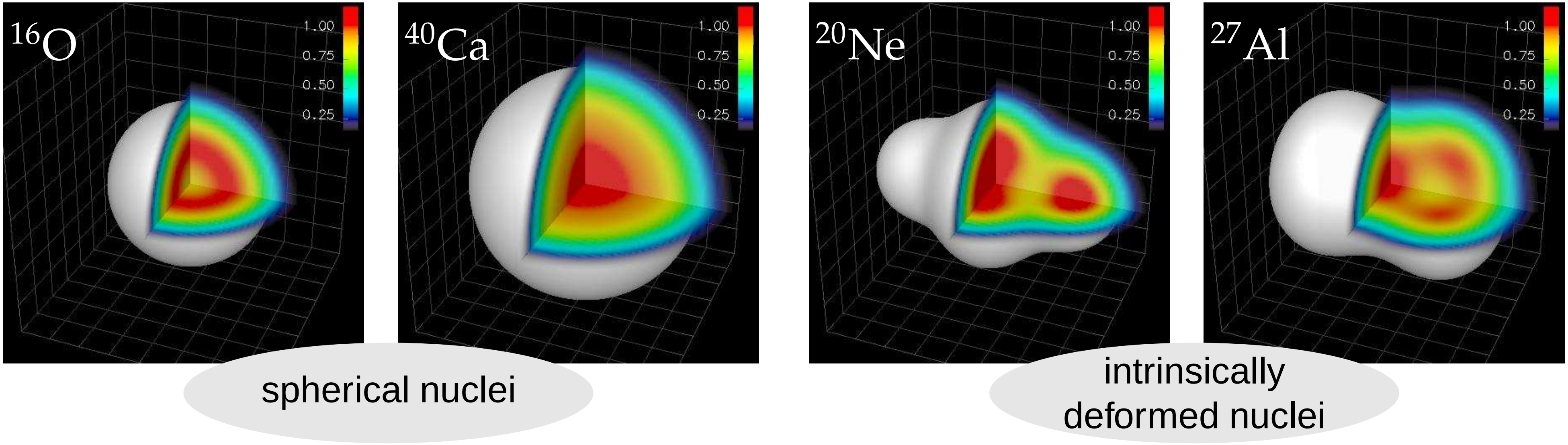
Perform Variation

Minimization

- minimize Hamiltonian expectation value with respect to all single-particle parameters q_k

$$\min_{\{q_k\}} \frac{\left\langle Q \middle| \underset{\sim}{\hat{H}} - \underset{\sim}{T}_{cm} \middle| Q \right\rangle}{\left\langle Q \middle| Q \right\rangle}$$

- this is a Hartree-Fock calculation in our particular single-particle basis
- the mean-field may break the symmetries of the Hamiltonian



PAV, VAP and Multiconfiguration

Projection After Variation (PAV)

- mean-field may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \; \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P}) \cdot \mathbf{X}\}$$

$$\underset{\sim}{P}^J_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \; {D^J_{MK}}^\star(\Omega) \, \underset{\sim}{R}(\Omega)$$

Variation After Projection (VAP)

- effect of projection can be large
- perform Variation after Parity Projection VAP$^\pi$
- Variation after Angular Momentum Projection (VAP) - expensive
- perform VAP in GCM sense by applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimize the energy in the projected energy surface

Multiconfiguration Calculations

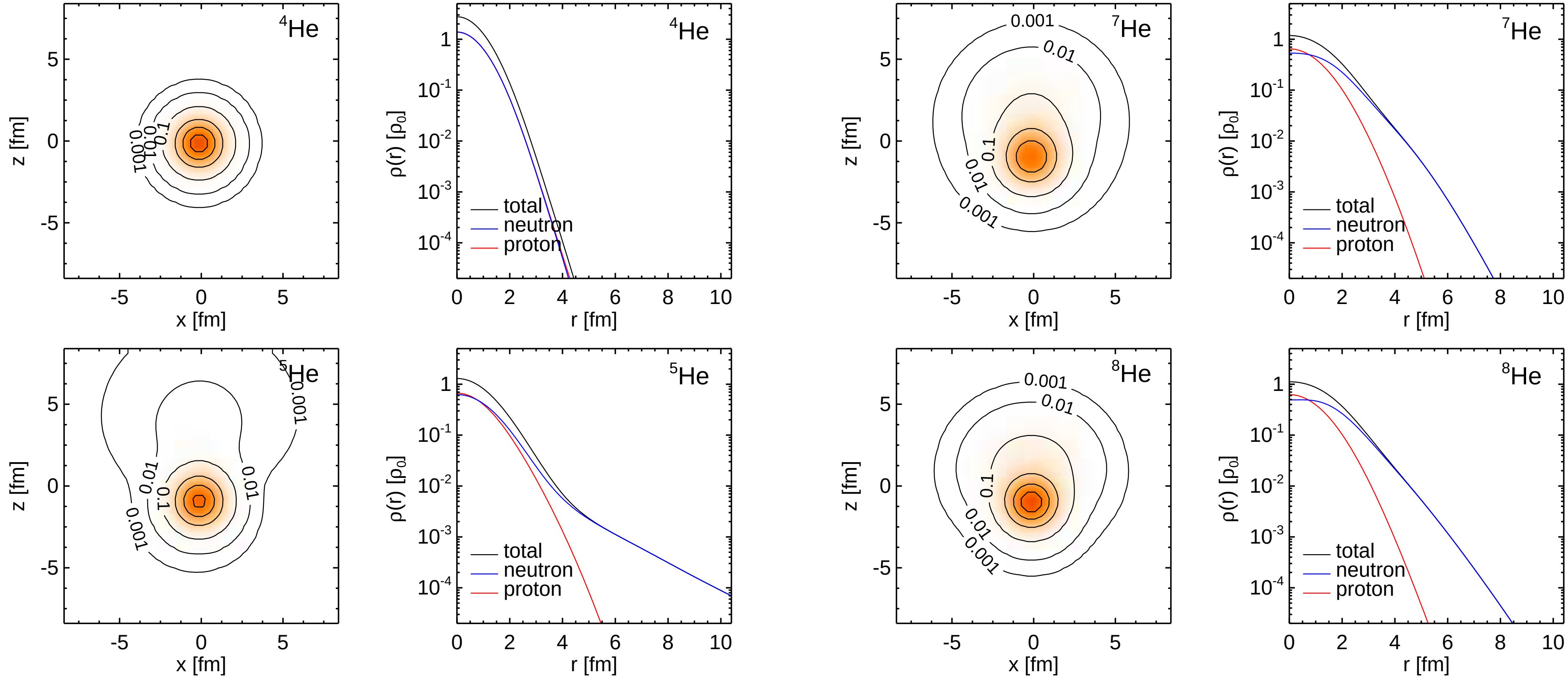
- **diagonalize** Hamiltonian in a set of projected intrinsic states

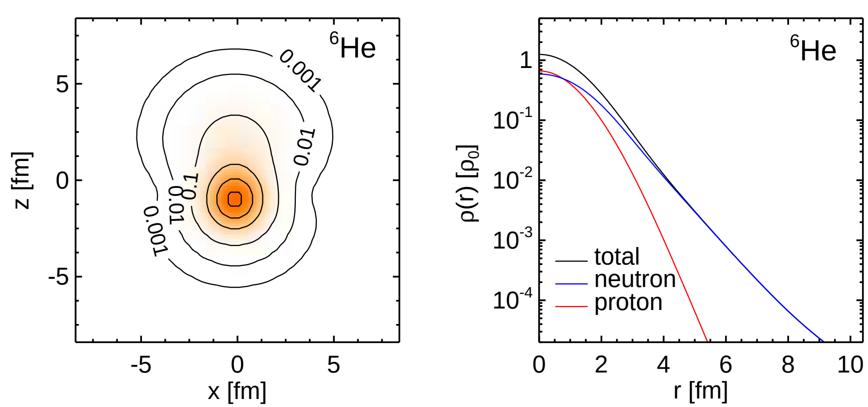
$$\left\{ \left| Q^{(a)} \right\rangle , \quad a = 1, \ldots, N \right\}$$

$$\sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{H} \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{(i)}_{K'b} = E^{J^\pi (i)} \sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{(i)}_{K'b}$$

Helium Isotopes

dipole and quadrupole constraints

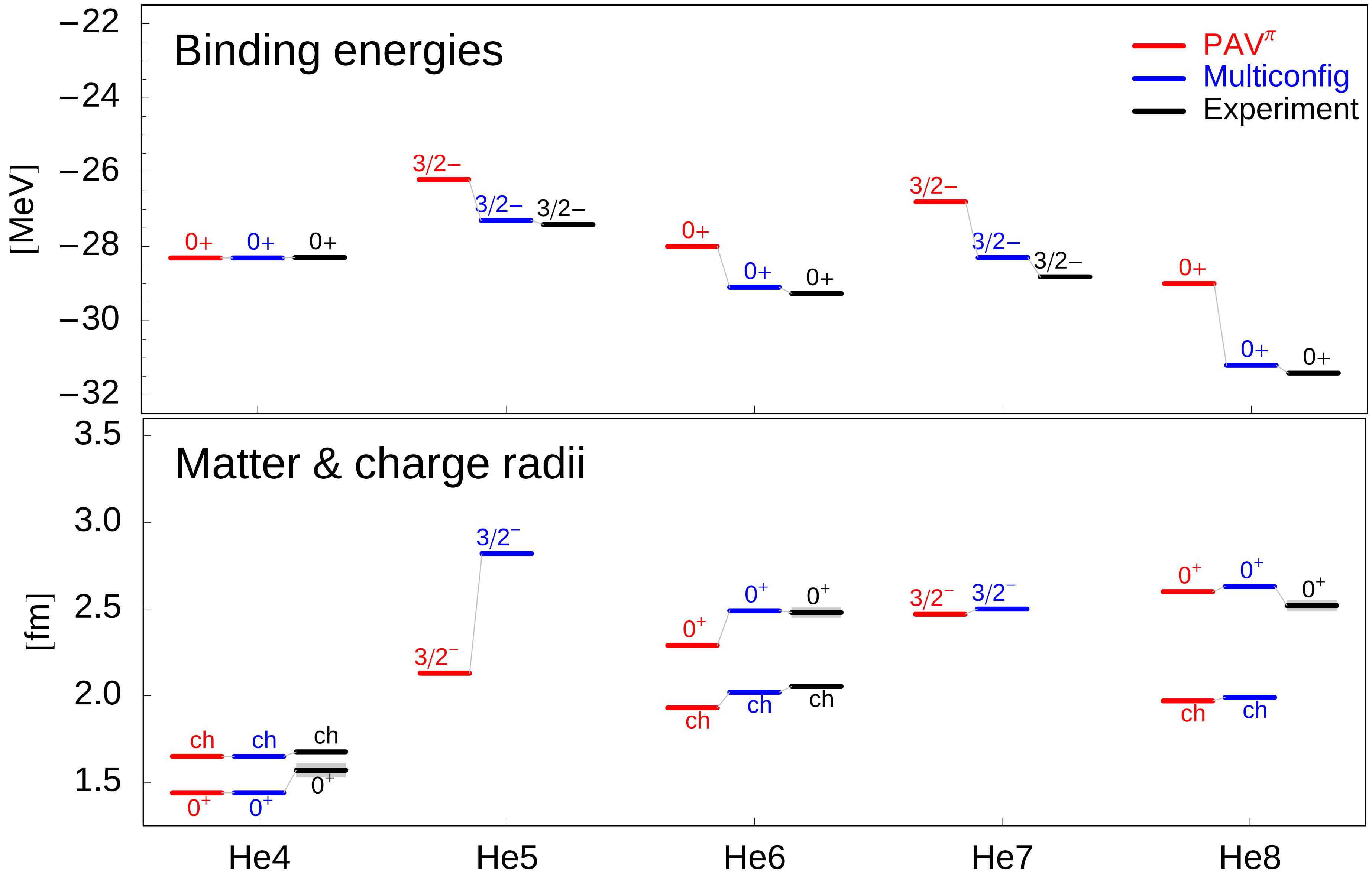




- intrinsic nucleon densities of VAP states
- radial densities from multiconfiguration calculations

FMD

Helium Isotopes



^{6}He charge radius: L.-B. Wang et al, Phys. Rev. Lett. **93** (2004) 142501

Cluster States in ^{12}C

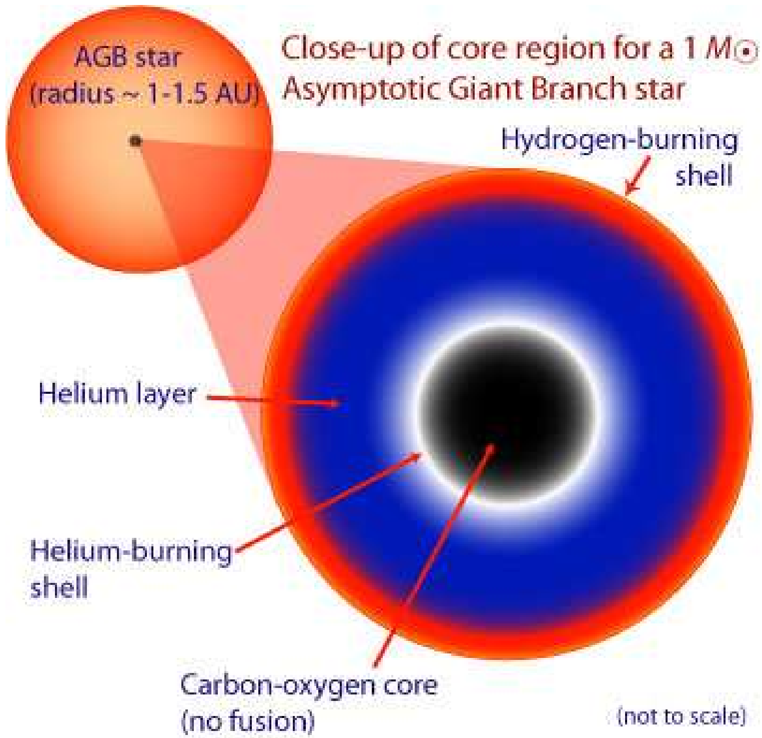
Astrophysical Motivation

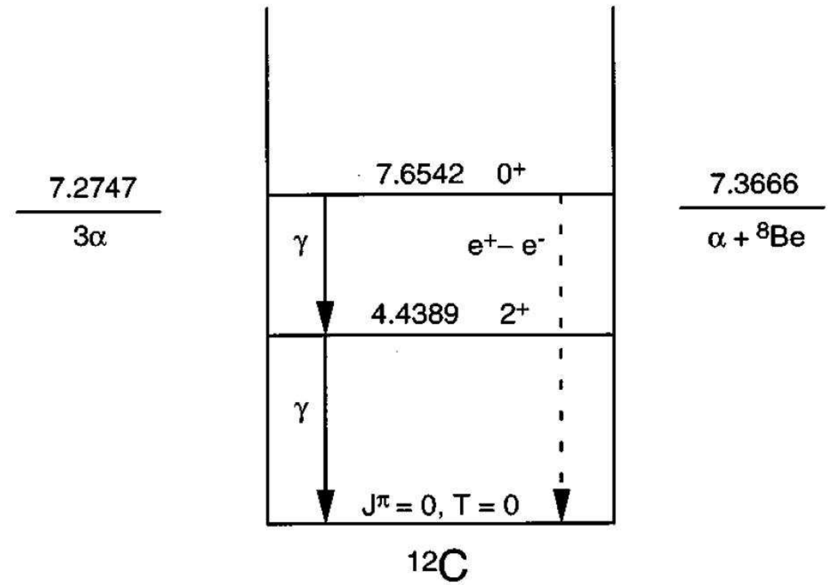
Structure

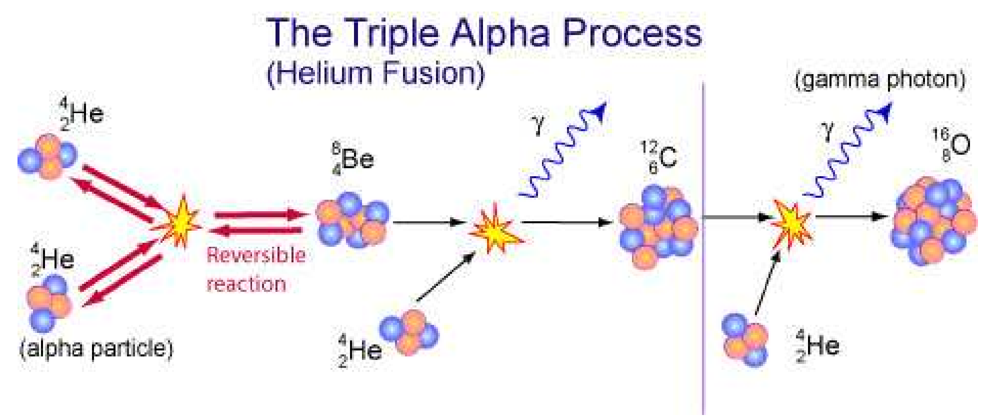
- What is the structure of the Hoyle state ?
- higher lying 0^+ and 2^+ states
- Compare to α-cluster model
- Analyze wave functions in harmonic oscillator basis
- No-Core Shell Model Calculations

Phys. Rev. Lett. 98, 032501 (2007)

Triple α Reaction

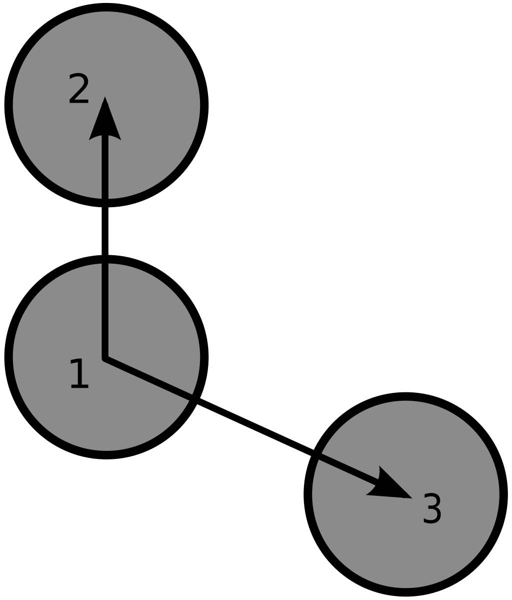






http://outreach.atnf.csiro.au/education/senior/astrophysics/stellarevolution_postmain.html

Microscopic α-Cluster Model



$$R_{12} = (2, 4, \ldots, 10)\ \text{fm}$$
$$R_{13} = (2, 4, \ldots, 10)\ \text{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \ldots, -1.0)$$

alltogether 165 configurations

Kamimura, Nuc. Phys. **A351**, 456 (1981)

Basis States

- describe Hoyle State as a system of 3 ^{4}He nuclei

$$\left| \Psi_{3\alpha}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3); JMK\pi \right\rangle = P^J_{MK} P^\pi \mathcal{A} \left\{ \left| \psi_\alpha(\mathbf{R}_1) \right\rangle \otimes \left| \psi_\alpha(\mathbf{R}_2) \right\rangle \otimes \left| \psi_\alpha(\mathbf{R}_3) \right\rangle \right\}$$

Volkov Interaction

- simple central interaction
- parameters adjusted to reproduce α binding energy and radius, $\alpha-\alpha$ scattering data and C12 ground state energy

✘ only reasonable for ^{4}He, ^{8}Be and ^{12}C nuclei

FMD

Basis States

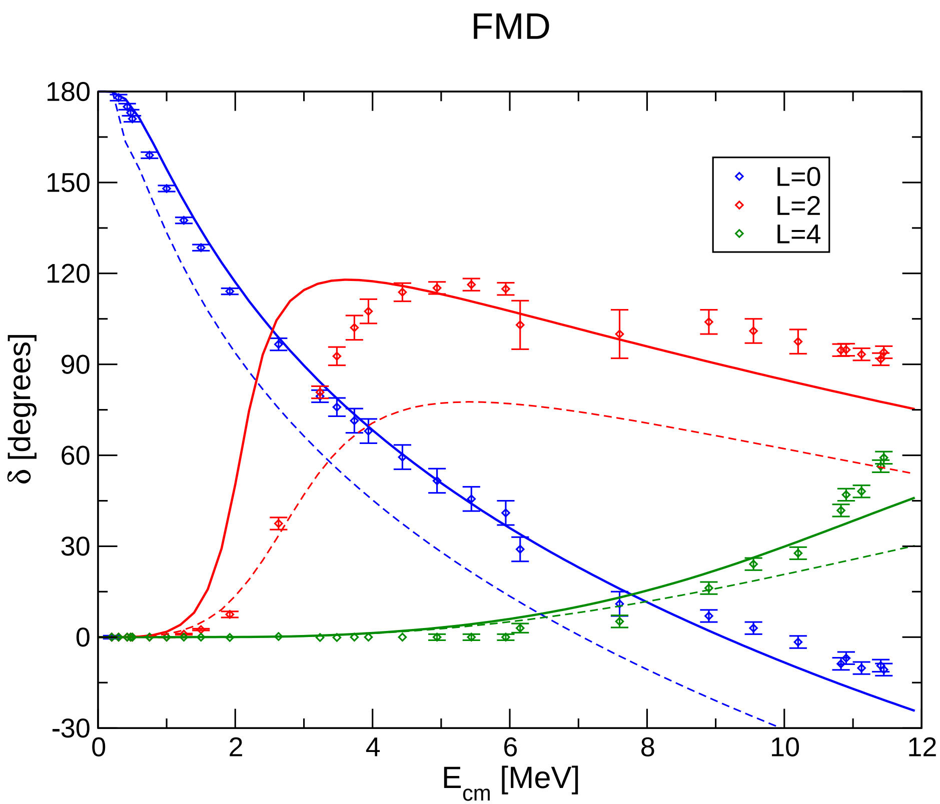
- 20 FMD states obtained in Variation after Projection on 0^+ and 2^+ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 α-cluster configurations

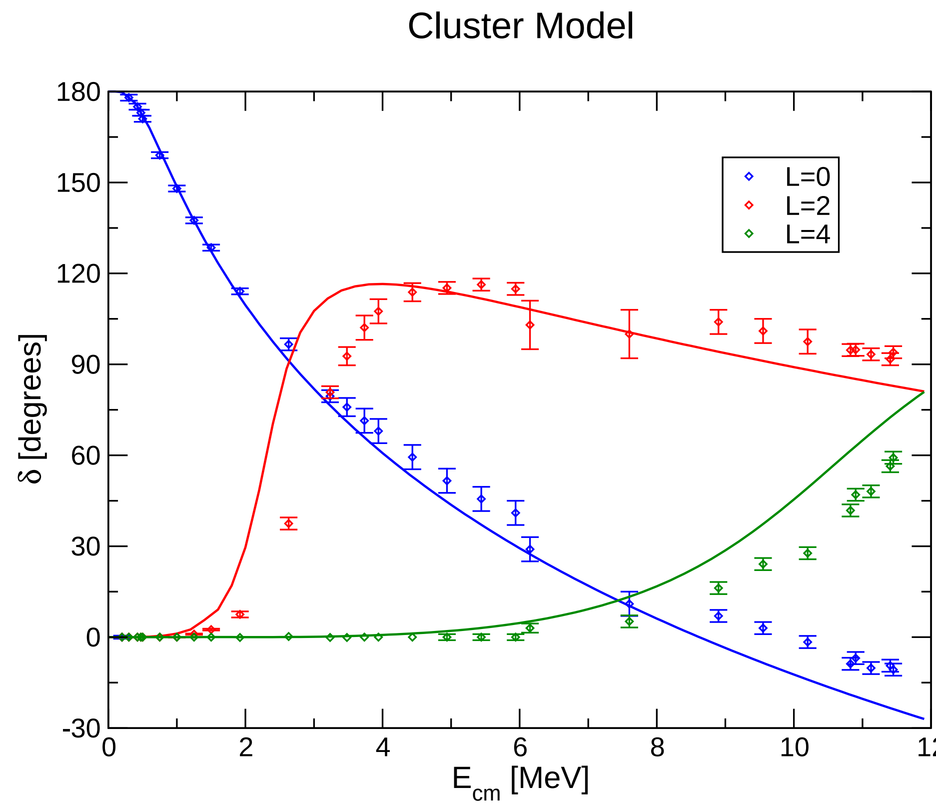
➨ projected on angular momentum and linear momentum

Interaction

- FMD interaction based on UCOM interaction with phenomenological two-body correction term fitted to energies and radii of doubly-magic nuclei
- not explicitly tuned for α-α scattering or ^{12}C properties

α-α Phaseshifts



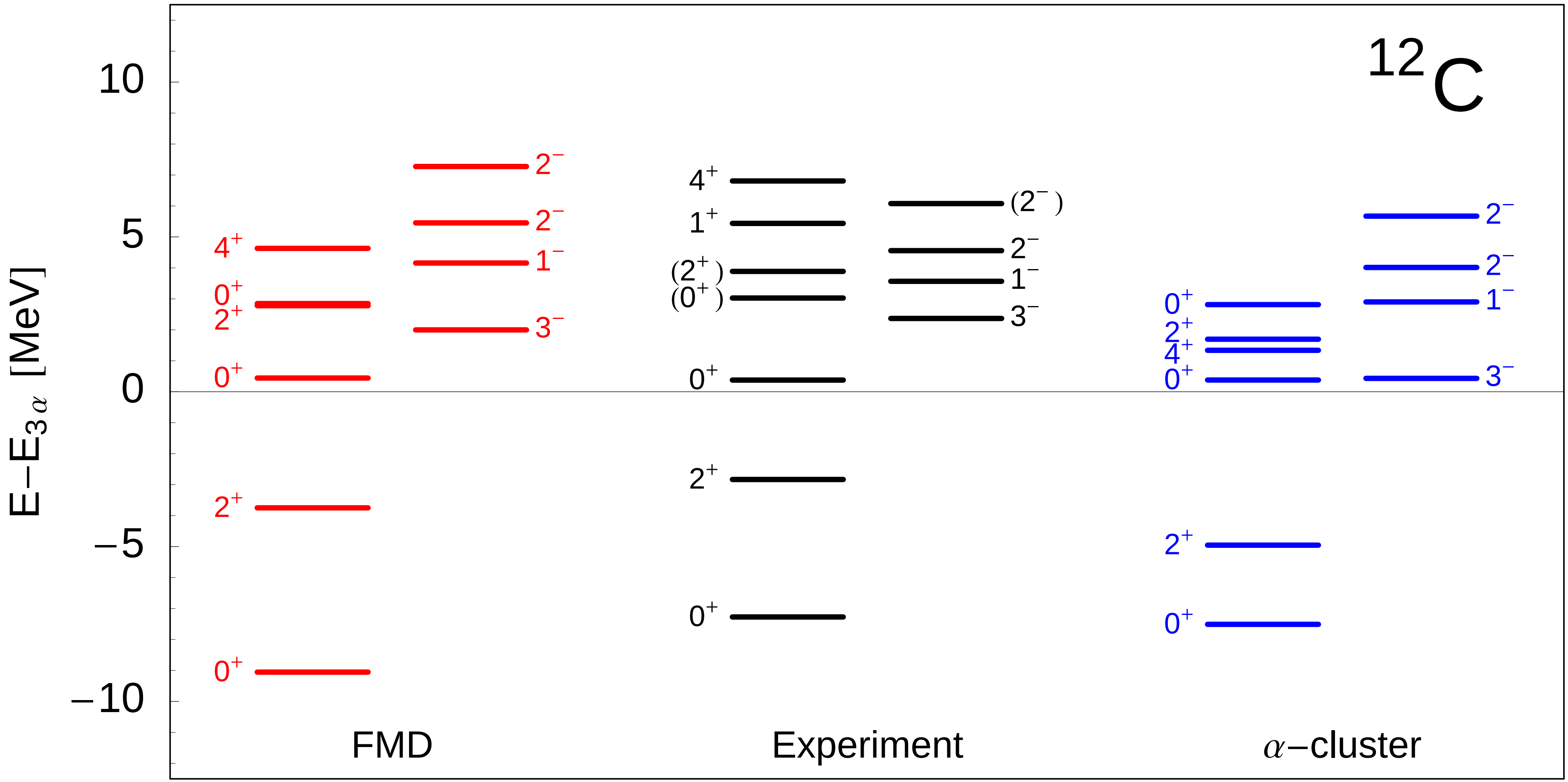


- Phaseshifts calculated with cluster configurations only (dashed lines)
- Phaseshifts calculated with additional FMD VAP configurations in the interaction region (solid lines)

- only cluster configurations included

➡ similar quality for description of α-α-scattering

Comparison



^{12}C
E−E$_{3\alpha}$ [MeV]
10
5
0
−5
−10
FMD
Experiment
α−cluster
4$^+$
0$^+$
2$^+$
0$^+$
2$^+$
0$^+$
2$^-$
2$^-$
1$^-$
3$^-$
4$^+$
1$^+$
(2$^+$)
(0$^+$)
0$^+$
2$^+$
0$^+$
(2$^-$)
2$^-$
1$^-$
3$^-$
0$^+$
2$^+$
4$^+$
0$^+$
2$^+$
0$^+$
2$^-$
2$^-$
1$^-$
3$^-$

Comparison

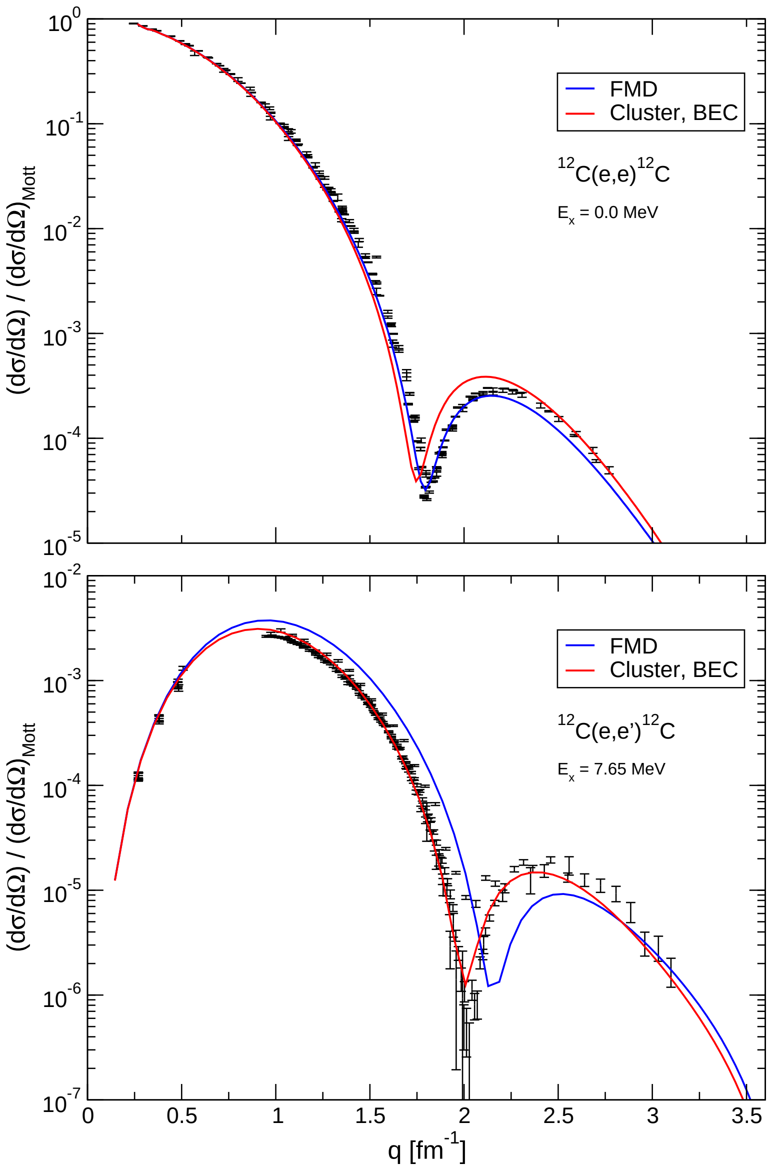
	Exp[1]	Exp[2]	Exp[3]	FMD	α-cluster	'BEC'[4]
$E(0^+_1)$	-92.16			-92.64	-89.56	-89.52
$E^*(2^+_1)$	4.44			5.31	2.56	2.81
$E(3\alpha)$	-84.89			-83.59	-82.05	-82.05
$E(0^+_2) - E(3\alpha)$	0.38			0.43	0.38	0.26
$E(0^+_3) - E(3\alpha)$	(3.0)	2.7(3)	3.96(5)	2.84	2.81	
$E(2^+_2) - E(3\alpha)$	(3.89)	2.6(3)	6.63(3)	2.77	1.70	
$r_{\text{charge}}(0^+_1)$	2.47(2)			2.53	2.54	
$r(0^+_1)$				2.39	2.40	2.40
$r(0^+_2)$				3.38	3.71	3.83
$r(0^+_3)$				4.62	4.75	
$r(2^+_1)$				2.50	2.37	2.38
$r(2^+_2)$				4.43	4.02	
$M(E0, 0^+_1 \rightarrow 0^+_2)$	5.4(2)			6.53	6.52	6.45
$B(E2, 2^+_1 \rightarrow 0^+_1)$	7.6(4)			8.69	9.16	
$B(E2, 2^+_1 \rightarrow 0^+_2)$	2.6(4)			3.83	0.84	

experimental situation for 0^+_3 and 2^+_2 states still unsettled

2^+_2 resonance at 1.8 MeV above treshold included in NACRE compilation

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Fynbo et al., Nature **433**, 137 (2005). Diget et al., Nuc. Phys. **A738**, 760 (2005)
[4] Funaki et al., Phys. Rev. C **67**, 051306(R) (2003)

Electron Scattering Data



- compare with precise electron scattering data up to high momenta in Distorted Wave Born Approximation
- use intrinsic density

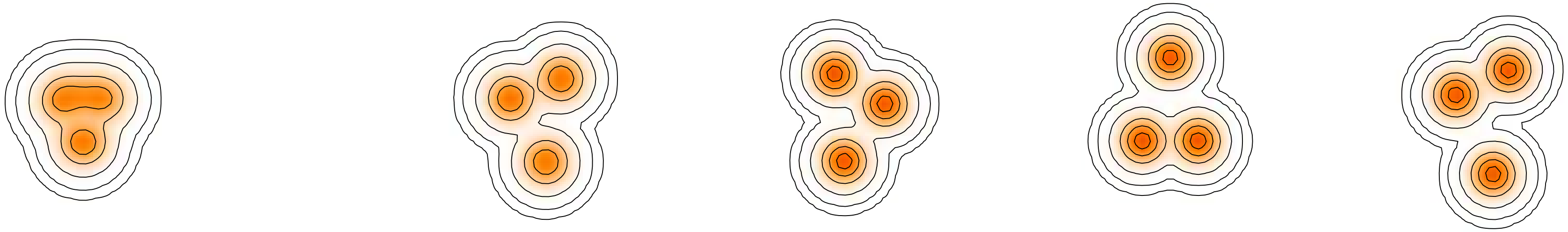
$$\rho(\mathbf{x}) = \sum_{k=1}^{A} \left\langle \Psi \,\middle|\, \delta(\underset{\sim}{\mathbf{x}}_k - \underset{\sim}{\mathbf{X}} - \mathbf{x}) \,\middle|\, \Psi \right\rangle$$

- ➨ elastic form factor described very well by FMD
- ➨ transition form factor better described by cluster model

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **98**, 032501 (2007)

Important Configurations

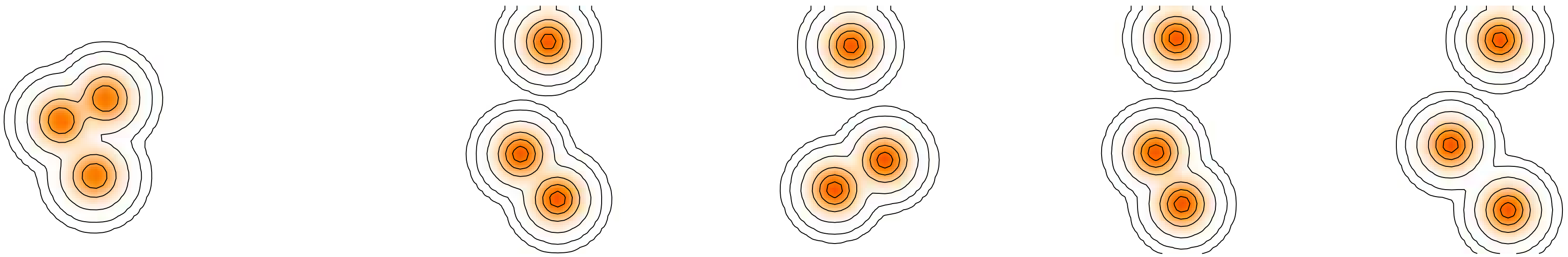
- Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state



$\left|\left\langle \cdot \mid 0_1^+ \right\rangle\right| = 0.94$

$\left|\left\langle \cdot \mid 2_1^+ \right\rangle\right| = 0.93$

$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.72$ $\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.71$ $\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$ $\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



$\left|\left\langle \cdot \mid 3_1^- \right\rangle\right| = 0.83$

$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.50$ $\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.49$ $\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.44$ $\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.41$

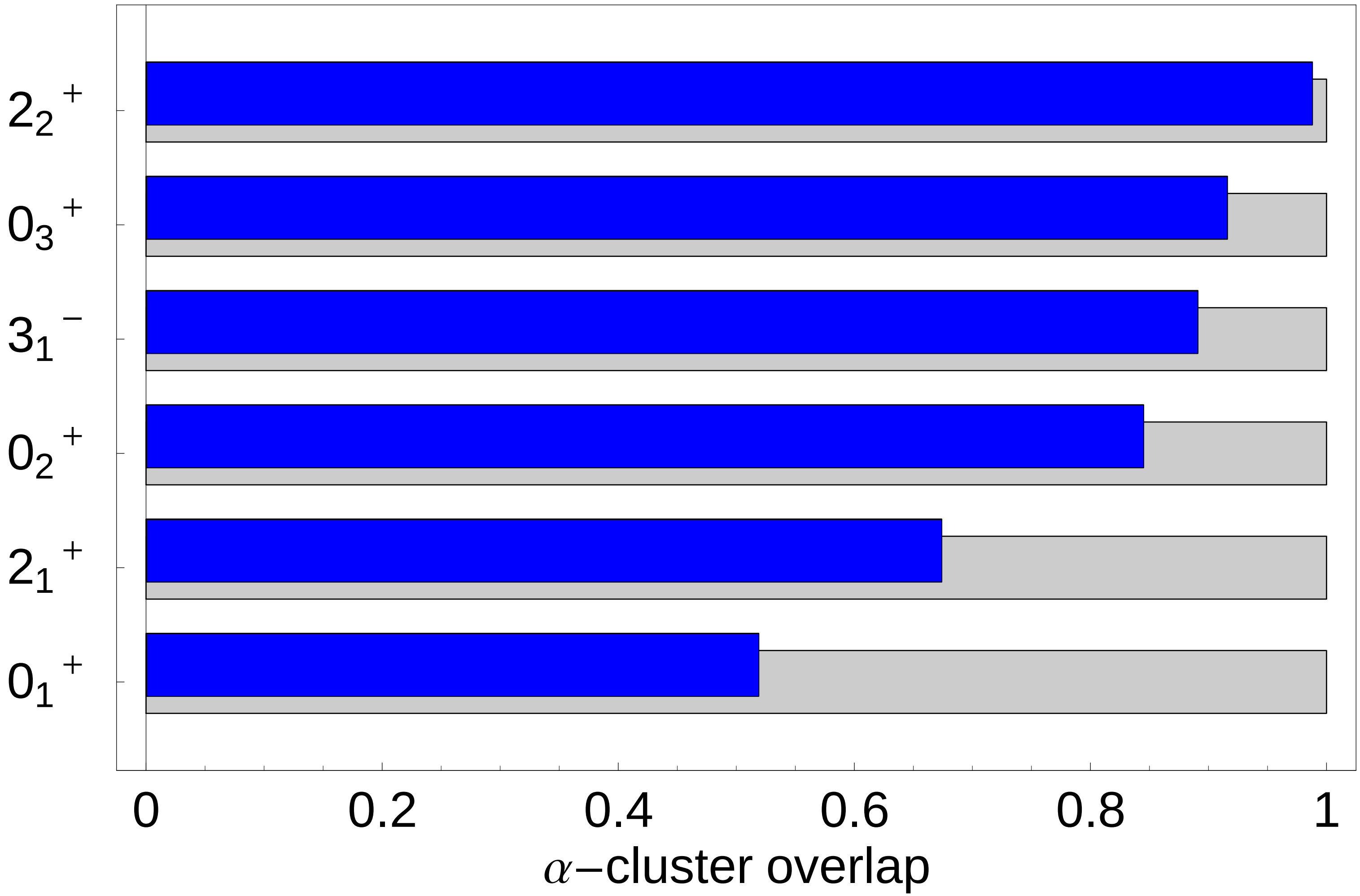
FMD basis states are not orthogonal!

loosely bound, gas-like states

Overlap with Cluster Model Space

Calculate the overlap of FMD wave functions with pure α-cluster model space

$$N_\alpha = \left\langle \Psi \middle| P_{3\alpha} \middle| \Psi \right\rangle$$

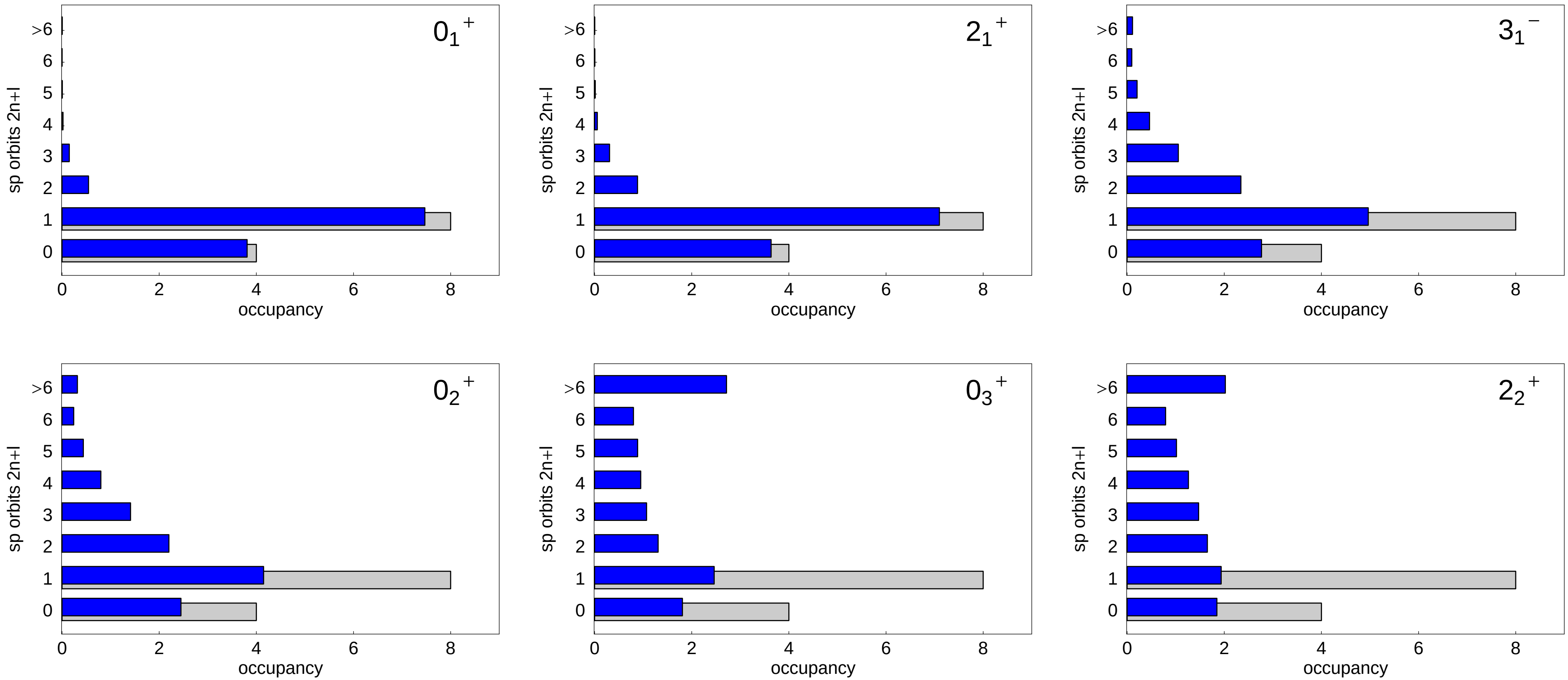


Harmonic Oscillator Occupation Numbers

calculate one-body density in harmonic oscillator basis

$$n_{nlj} = \sum_m \left\langle \Psi \middle| a^{\dagger}_{nljm} a_{nljm} \middle| \Psi \right\rangle$$

FMD

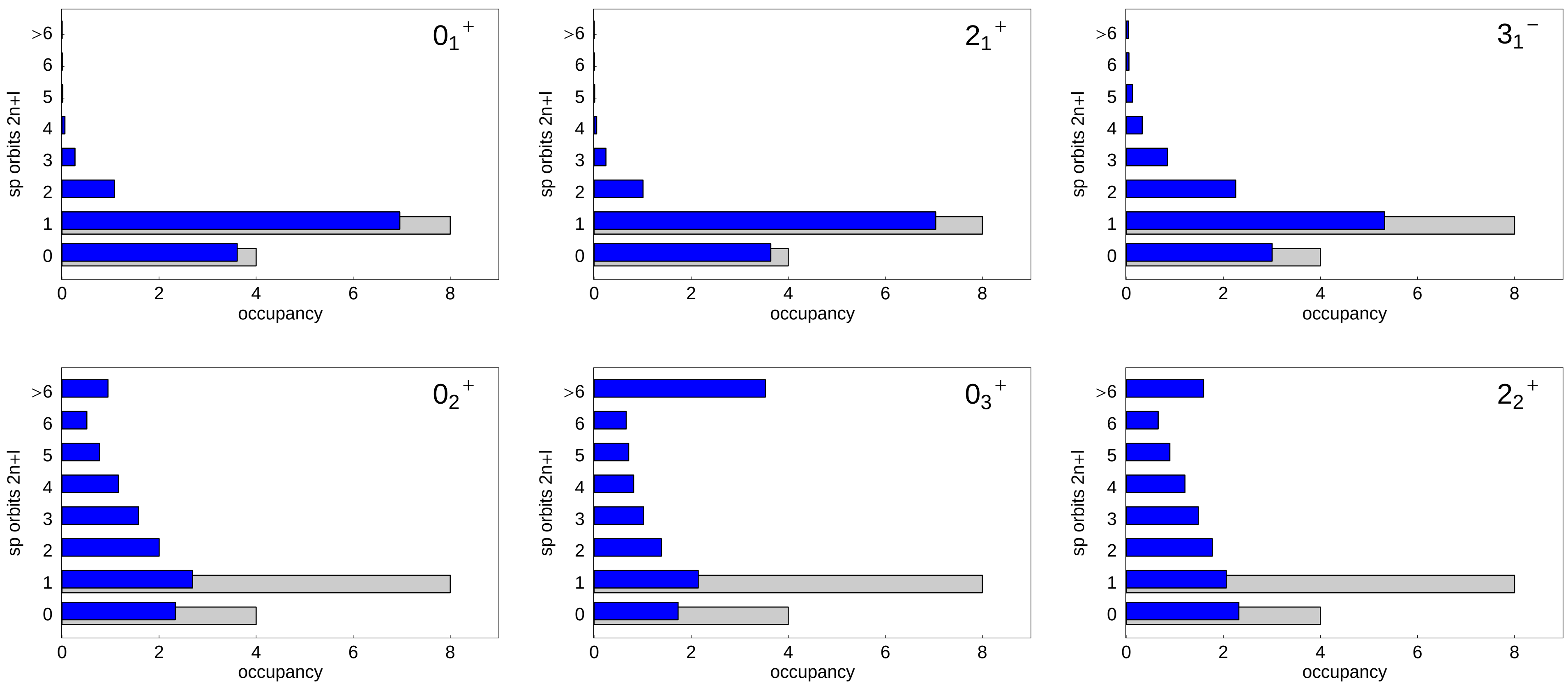


Harmonic Oscillator Occupation Numbers

calculate one-body density in harmonic oscillator basis

$$n_{nlj} = \sum_m \left\langle \Psi \left| a^\dagger_{nljm} a_{nljm} \right| \Psi \right\rangle$$

Cluster Model

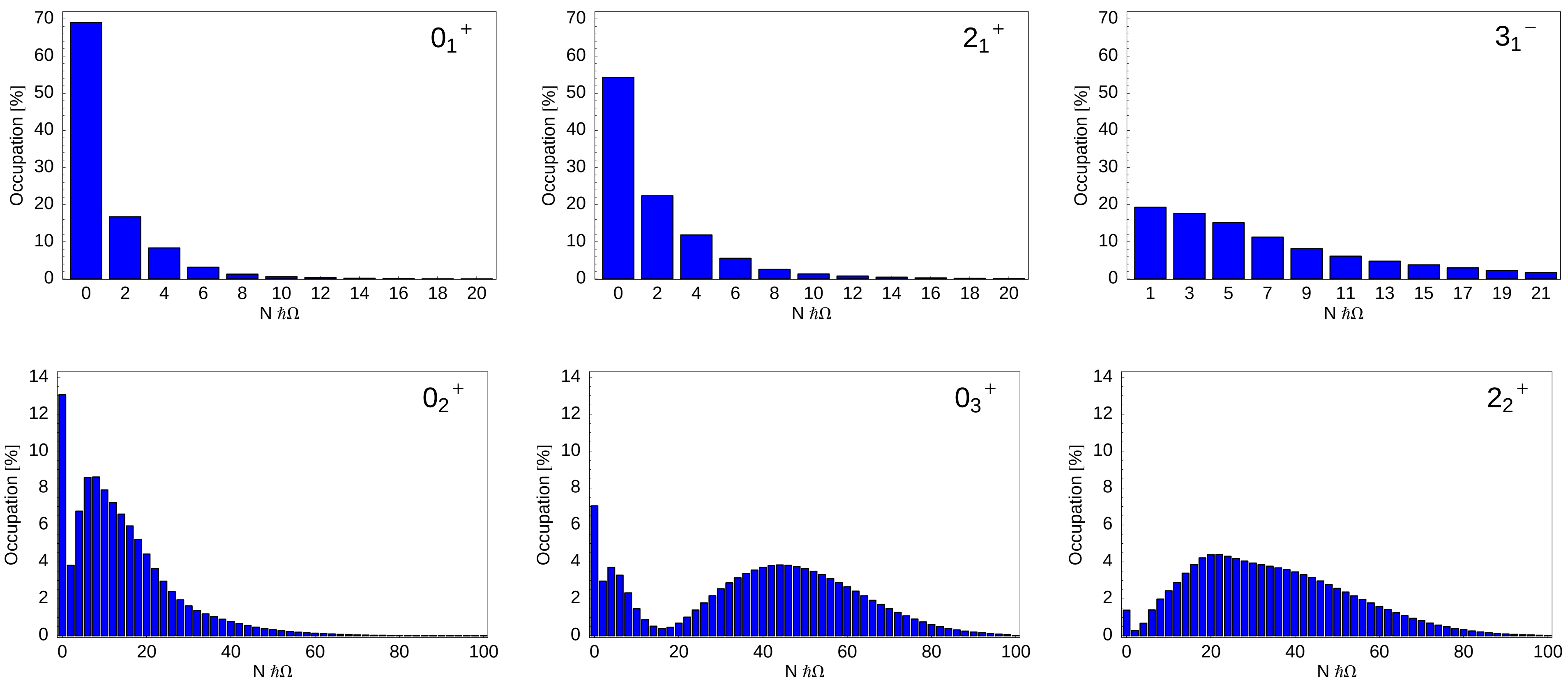


Harmonic Oscillator $N\hbar\Omega$ Excitations

Y. Suzuki *et al*, Phys. Rev. C **54**, 2073 (1996).

$$N_Q = \left\langle \Psi \middle| \delta\left(\textstyle\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - Q\right) \middle| \Psi \right\rangle$$

FMD



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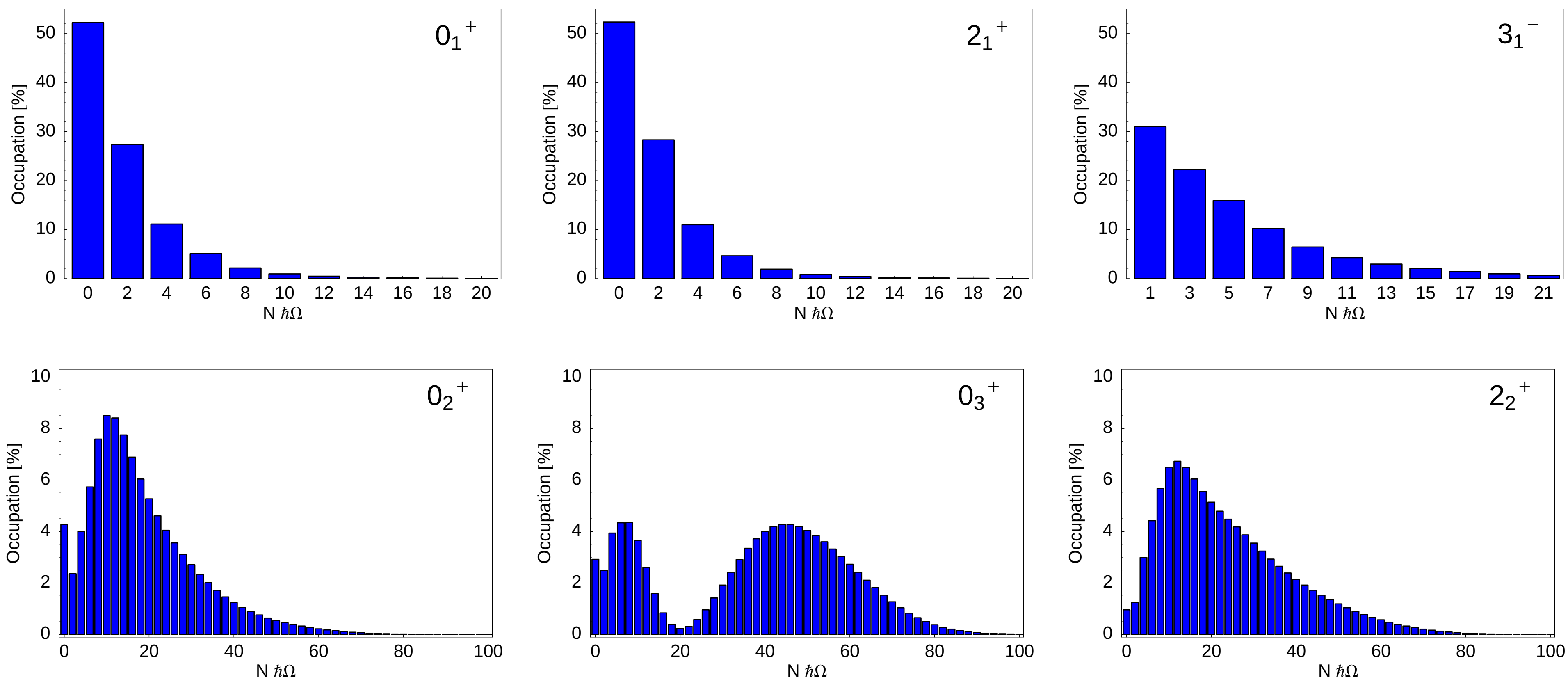
Cluster States

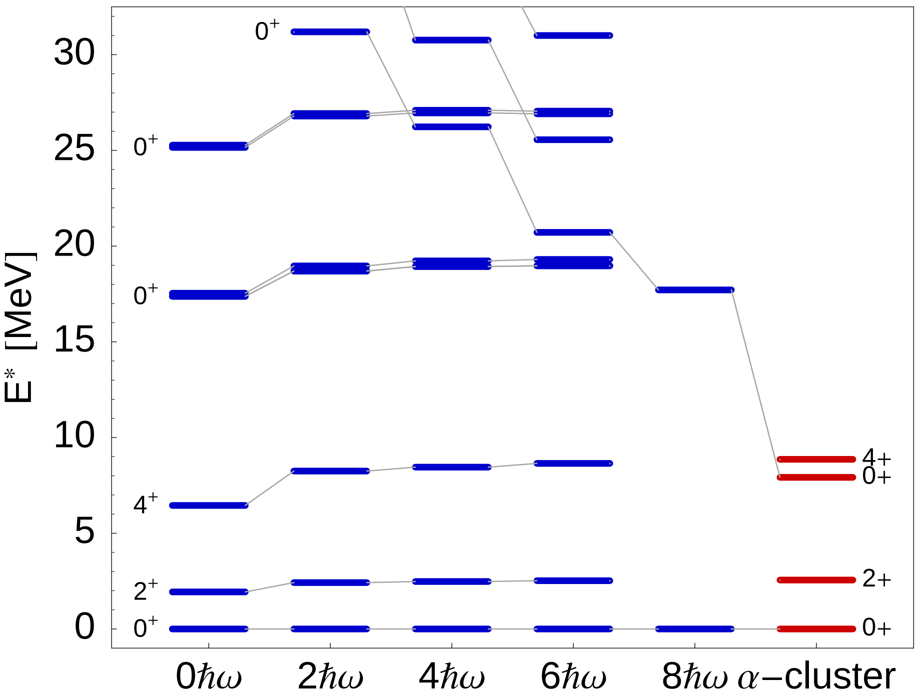


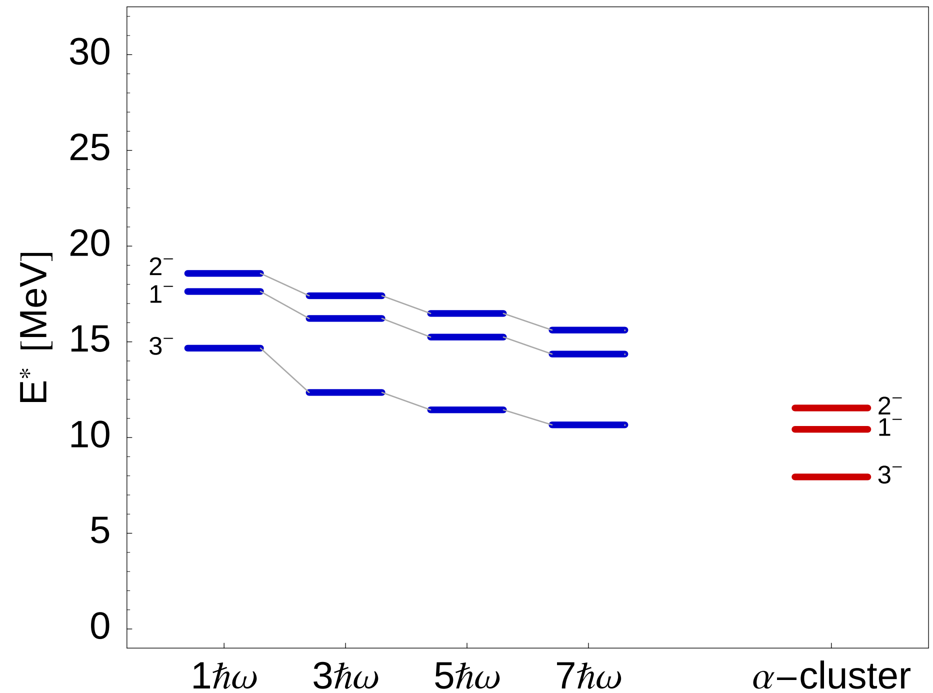
α-cluster states in the No-Core Shell Model ?

- compare spectra in NCSM and α-cluster model using the Volkov interaction
- bare interaction used in NCSM calculations
- ➨ good agreement for ground state band (0_1^+, 2_1^+, 4_1^+)
- ➨ very slow convergence for cluster states

Binding energies

	^{4}He	^{12}C
Cluster	-27.3 MeV	-89.6 MeV
NCSM	-28.3 MeV	-95.4 MeV





Summary

Unitary Correlation Operator Method

- explicit description of short-range central and tensor correlations
- phase-shift equivalent correlated interaction V_{UCOM}

Fermionic Molecular Dynamics

- Single-particle basis using Gaussian wave-packets
- Many-body wave functions projected on parity, angular momentum and linear momentum
- PAV, VAP and Multiconfiguration
- Allows to describe halos and clustering

Cluster States in ^{12}C

- Describe cluster states with FMD and cluster model
- Hoyle state has gas-like structure
- FMD and cluster model wave functions correspond to many $N\hbar\Omega$ shell model configurations

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