2-nucleon transfer reactions
and
shape/phase transitions in nuclei

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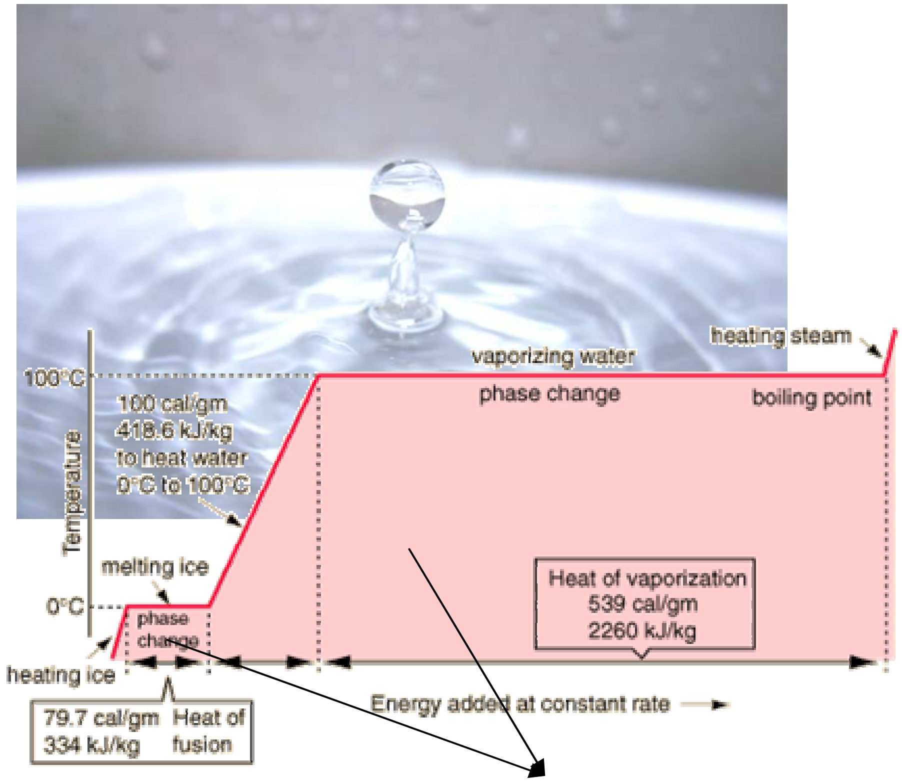
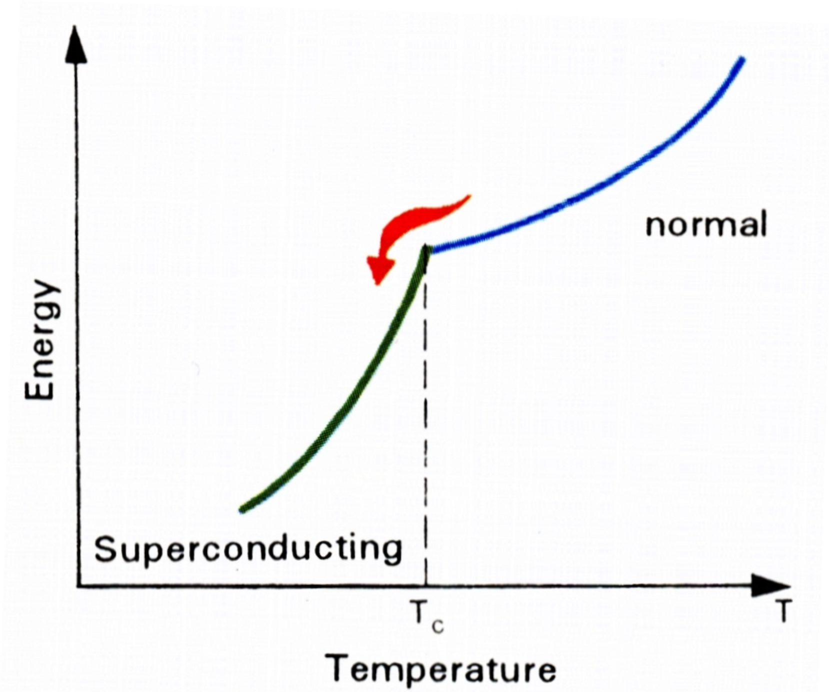
Phase transitions in **macroscopical systems**

First order
ex. Ice – water – steam

Second order
ex. Normal to superconductive phase



Coexistence between two phases



No coexistence

Phase/shape transitions in **nuclei**

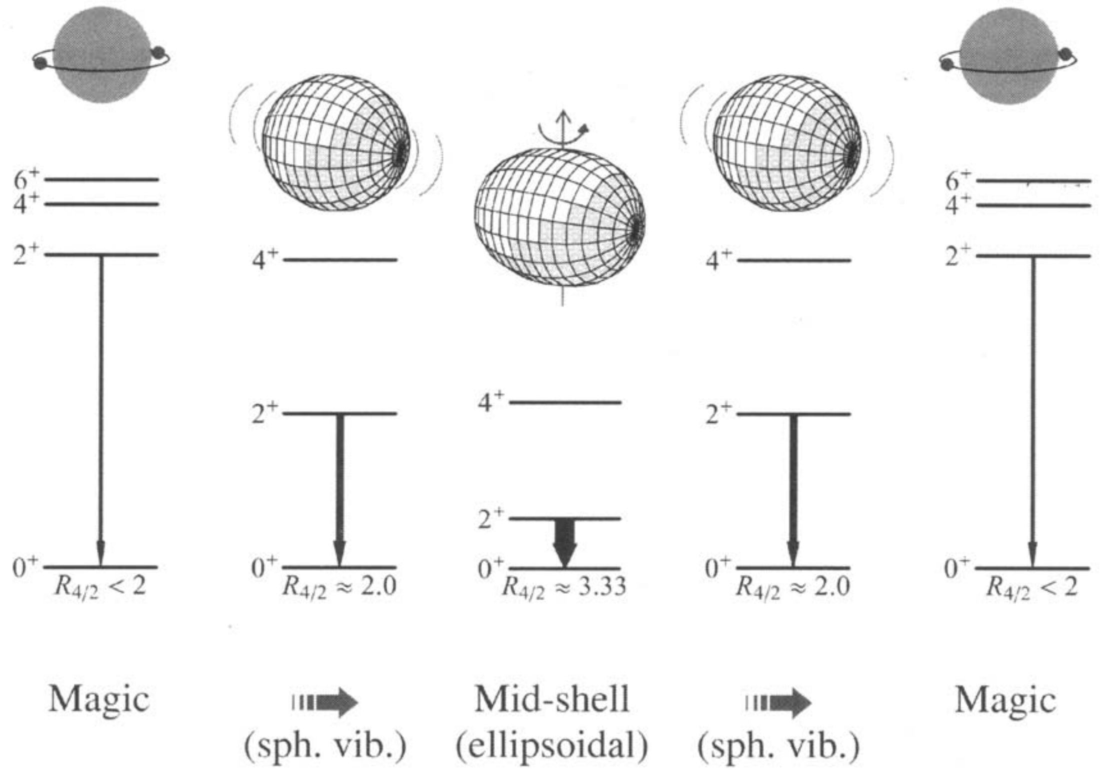


Fig. from R. Casten *"Nuclear Physics from a Simple Perspective"*

- **Finite number problem**: effects of the phase transition will be muted
- The **CONTROL PARAMETER** should be continuous but the **nucleon number changes discretely**
- The **ORDER PARAMETER** will be related to the shape of the nucleus: **ellipsoidal deformation parameter** $\boldsymbol{\beta}$ or related observables as $R_{4/2}$

nuclear shape

within nuclear models

- Mean-field models

- Bohr-Mottelson collective Hamiltonian

$$H_{BM} = \sum_{\kappa=1}^{3} \frac{\hat{M}_\kappa^2}{2\mathcal{I}_\kappa(\beta,\gamma)} + -\frac{\hbar^2}{2B_2}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2}\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}\right) + V(\alpha_{2\mu})$$

decides whether the nucleus will be vibrational, rotational,…

- Interacting Boson Model (IBM)

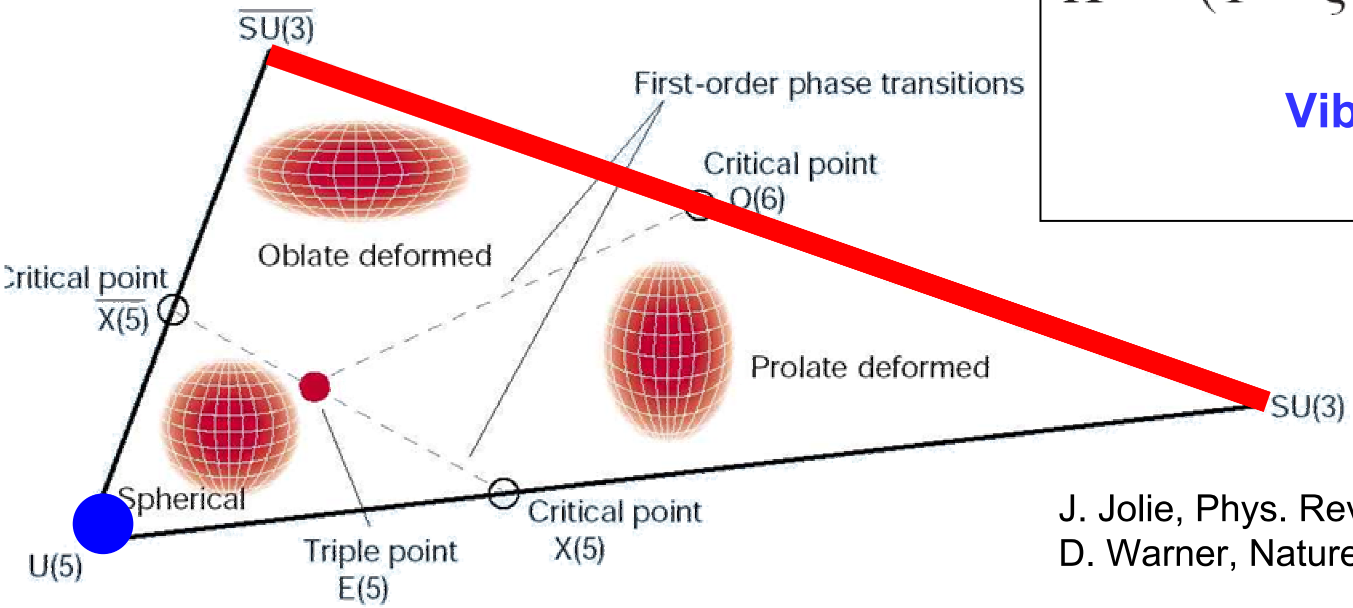
- …

Shape/Phase transitions in the

Interacting Boson Model (IBM)

Geometrical interpretation
using boson coherent states

$$E(\beta) = (1-\xi)\left(N\frac{\beta^2}{1+\beta^2}\right) - \xi\left[\frac{1}{1+\beta^2}(5+(1+\chi^2)\beta^2) + \frac{N-1}{(1+\beta^2)^2}\left(\frac{2}{7}\chi^2\beta^4 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos 3\gamma + 4\beta^2\right)\right].$$



U(6) group built from *s* and *d* bosons, with three dynamical limits **U(5), SU(3) and O(6)**

Transitional IBM hamiltonian with continuous control parameter ξ (or χ)

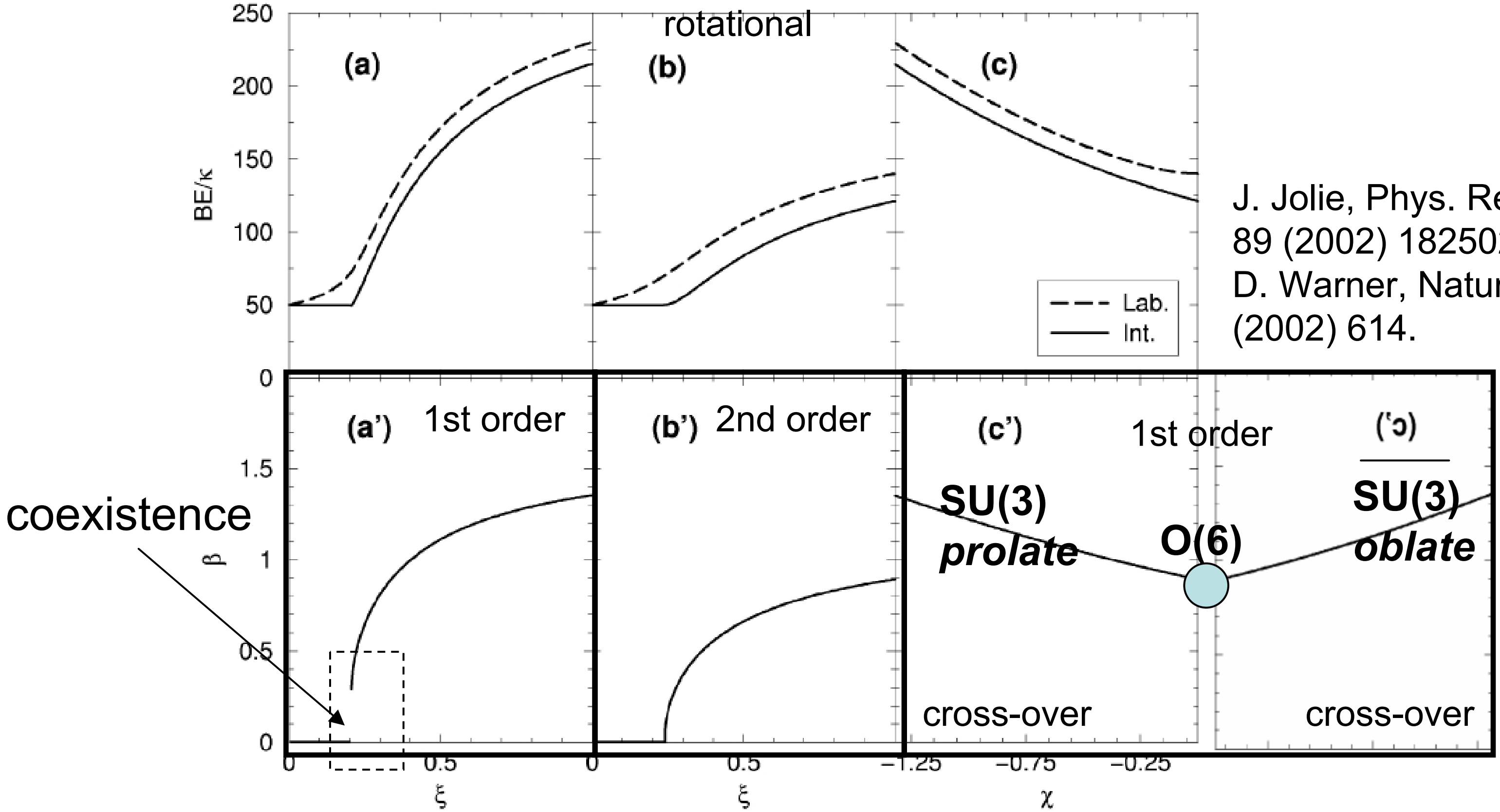
$$\hat{H} = (1-\xi)\hat{n}_d - \frac{\xi}{N}\hat{Q}(\chi)\cdot\hat{Q}(\chi)$$

Vibrational term **Rotational term**

J. Jolie, Phys. Rev. Lett. 89 (2002) 182502
D. Warner, Nature 420 (2002) 614.

1st and 2nd order shape/phase transitions and cross-over in the IBM

U(5)-SU(3)	U(5)-O(6)	SU(3)-O(6)
vibrational to axial-symmetric rotational	vibrational to γ-unstable rotational	Axial-symmetric rotational to γ-unstable



J. Jolie, Phys. Rev. Lett. 89 (2002) 182502
D. Warner, Nature 420 (2002) 614.

J.E. Garcìa-Ramos, C. De Coster, R. Fossion and K. Heyde, Nucl. Phys. A688 (2001) 735.

Observables

to study phase/shape transitions in nuclei

Crucial fingerprints

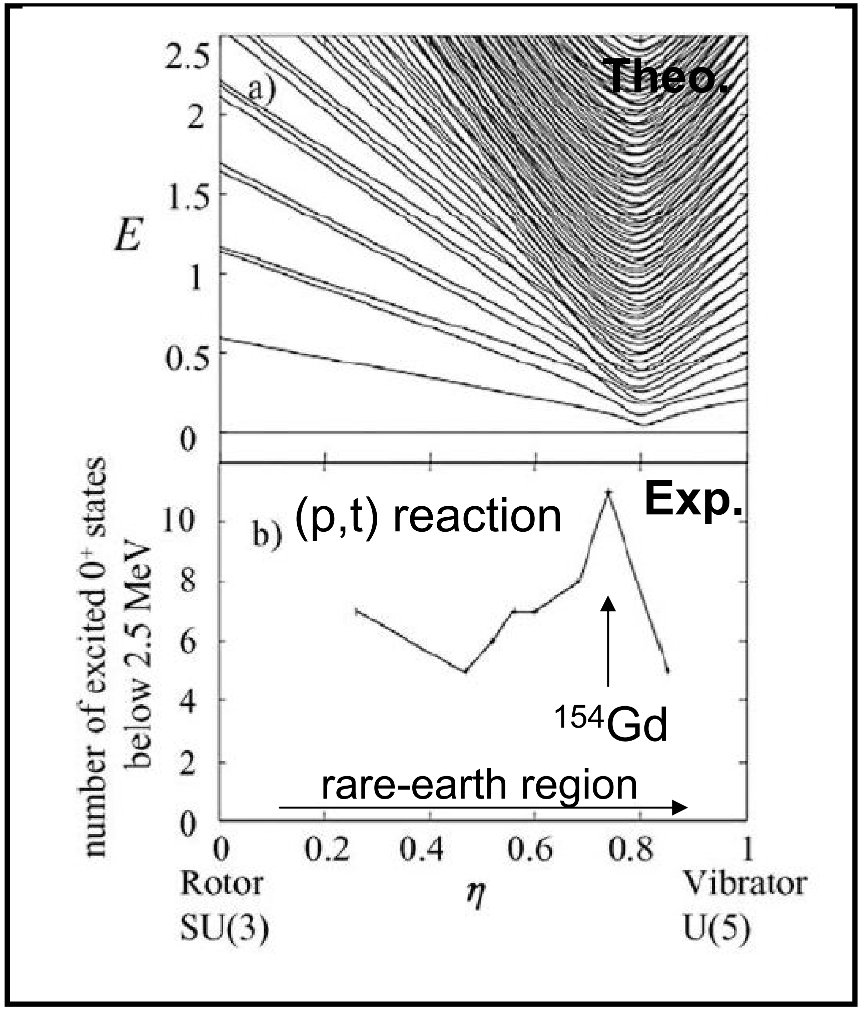
- Two-neutron separation energies
- $B(E2;2_1^+\text{->}0_1^+)$
- $B(E0;0_2^+\text{->}0_1^+)$
- Isomer shifts

Other quantities

- $R=E_{4(1)}/E_{2(1)}$
- Isotope shift
- **Intensities of two-nucleon transfer reactions**
- $B(E2;2_2^+\text{->}0_1^+)/B(E2;2_2^+\text{->}2_1^+)$

Enhanced density of low-lying 0+ states:

a corrobation of shape/phase transitional behaviour



THEO

(a) P. Cejnar and J. Jolie, Phys. Rev. E61 (2000) 6237, *"Quantum Phase Transitions Studied within the IBM"*

EXP (towards a complete levelspectrum for 0^+ up to ~3MeV)

(b) D.A. Meyer et al., Phys. Lett. B638 (2006) 44, "*(p,t) study of 8 nuclei in the rare-earth region"*

- D.A. Meyer et al., Phys. Rev. C74 (2006) 044309, *"extensive investigation of 0+ states in the rare-earth region"*
- D. Bucurescu et al., Phys. Rev. C73 (2006) 064309, *"study of 0+ and 2+ states with high-resolution (p,t) reactions in Er-168"*

2-particle transfer reactions
in the IBM, L=0

Total cross section for two-particle transfer

$$\sigma_{\rm exp}(\theta, E_x^i) \sim \epsilon(E_x^i)\sigma_{\rm DWBA}(\theta, E_x^i)$$

With: - geometric part σ_{DWBA}
- structure part $\varepsilon(E^i_x)$

Describing complete levelspectrum

- quasipart. phonon model (QPM)
- projected shell model (PSM)

In context of phase transitions

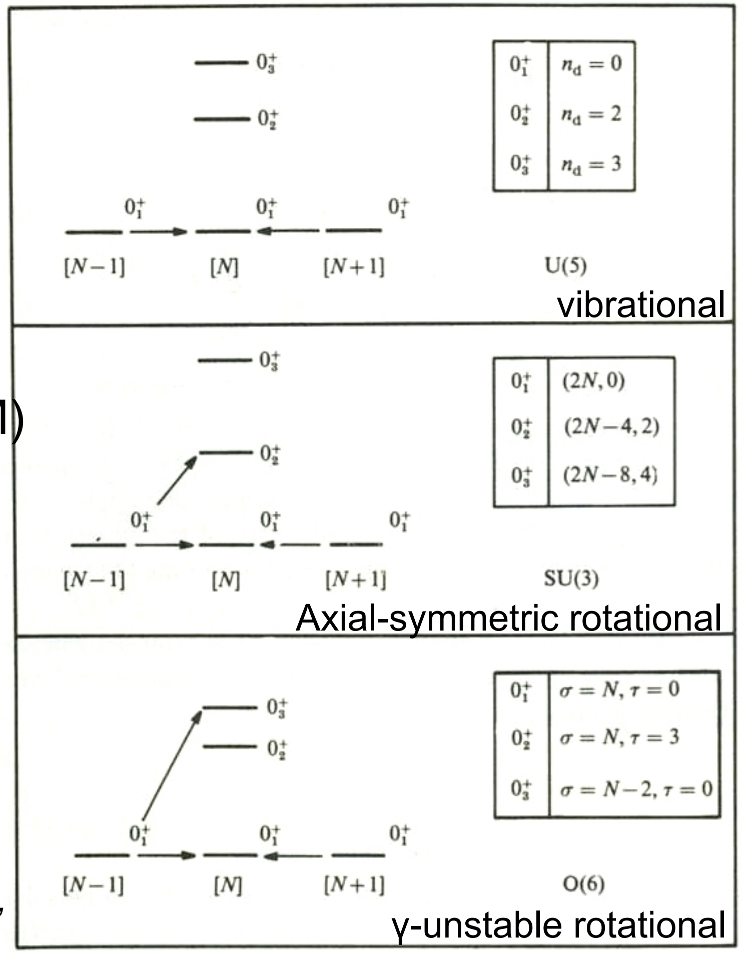
Structure part within the IBM

L=0 two-particle transfer

$$P_{+,0}^{(0)} = p_0 s^\dagger + q_0[[s^\dagger \times s^\dagger]^{(0)} \times \tilde{s}]_0^{(0)} + q_0'[[d^\dagger \times d^\dagger]^{(0)} \times \tilde{s}]_0^{(0)} + q_2[[d^\dagger \times s^\dagger]^{(2)} \times \tilde{d}]_0^{(0)} + q_2'[[d^\dagger \times d^\dagger]^{(2)} \times \tilde{d}]_0^{(0)}.$$

F. Iachello and A. Arima, *"The Interacting Boson Model"*

Selection rules for s-boson transfer



2-particle transfer reactions
in the IBM, L=0

F. Iachello and A. Arima,
"The Interacting Boson Model"

Analytical results within the limits

gs→gs $(0^+_1 \to 0^+_1)$

$$I^{U(5)}_{gsgs} = N+1,$$
$$I^{SU(3)}_{gsgs} = \frac{N+2}{3} + \frac{1}{3(2N+1)}$$
$$I^{O(6)}_{gsgs} = \frac{N+3}{2} - \frac{1}{N+2},$$

gs→bv $(0^+_1 \to 0^+_\beta)$

$$I^{U(5)}_{gsbv} = 0,$$
$$I^{SU(3)}_{gsbv} = \frac{2}{3} + \frac{2}{3(4N^2-1)},$$
$$I^{O(6)}_{gsbv} = \frac{1}{2} - \frac{1}{(N+1)(N+2)}$$

Between the limits

- Only numerical calculations
- Excitation of several exctited 0+ states possible

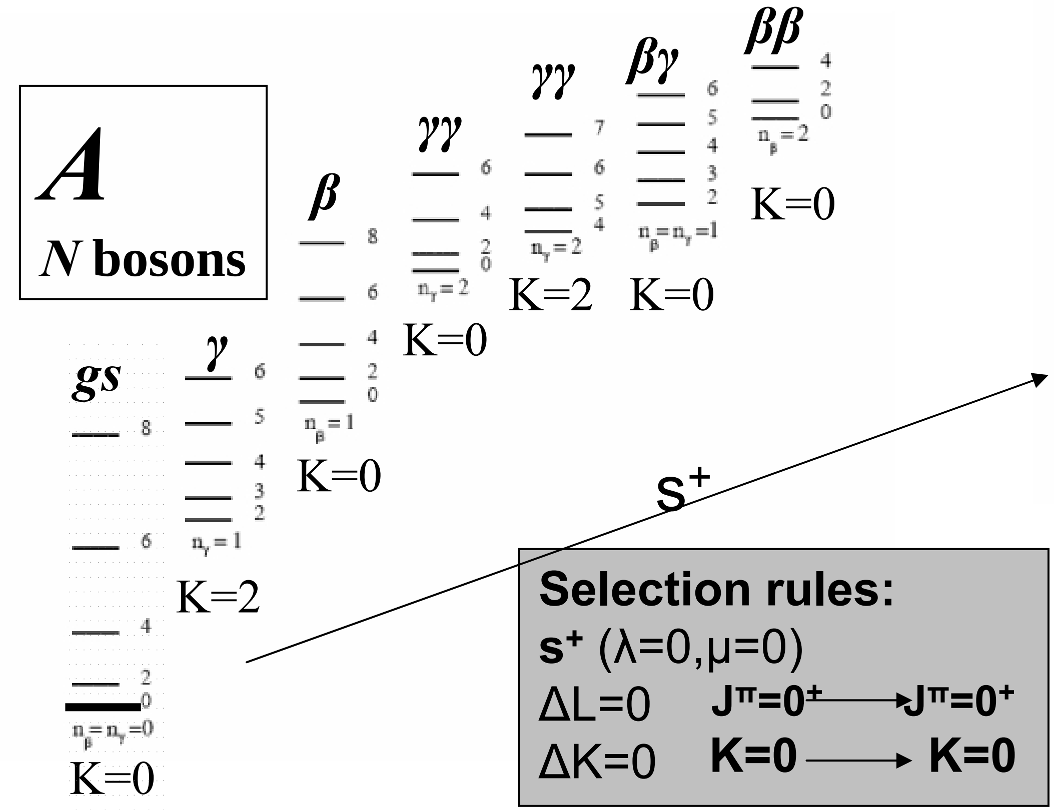
2-particle transfer reactions

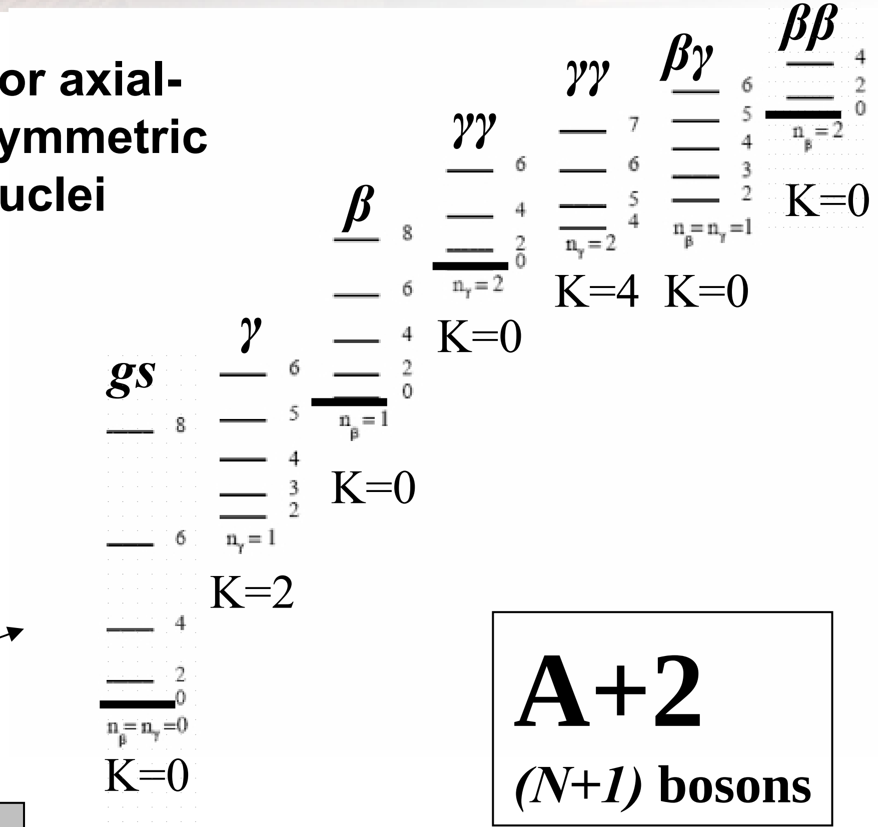
in the Boson Coherent-State formalism

$$|N;gs(\beta,\gamma)\rangle = \frac{1}{\sqrt{N!}}\frac{1}{(1+\beta^2)^{N/2}}\left(s^\dagger+\beta\cos\gamma d_0^\dagger+\frac{\beta}{\sqrt{2}}\sin\gamma\left(d_2^\dagger+d_{-2}^\dagger\right)\right)^N|0\rangle$$

$$|N;bv(\beta,\gamma)\rangle = \frac{1}{\sqrt{(N-1)!}}\frac{1}{(1+\beta^2)^{N/2}}\left(-\beta s^\dagger+\cos\gamma d_0^\dagger+\frac{1}{\sqrt{2}}\sin\gamma\left(d_2^\dagger+d_{-2}^\dagger\right)\right)\left(s^\dagger+\beta\cos\gamma d_0^\dagger+\frac{\beta}{\sqrt{2}}\sin\gamma\left(d_2^\dagger+d_{-2}^\dagger\right)\right)^{N-1}|0\rangle .$$

For axial-symmetric nuclei





Selection rules:

s⁺ (λ=0,μ=0)

ΔL=0 **$J^\pi=0^+ \rightarrow J^\pi=0^+$**

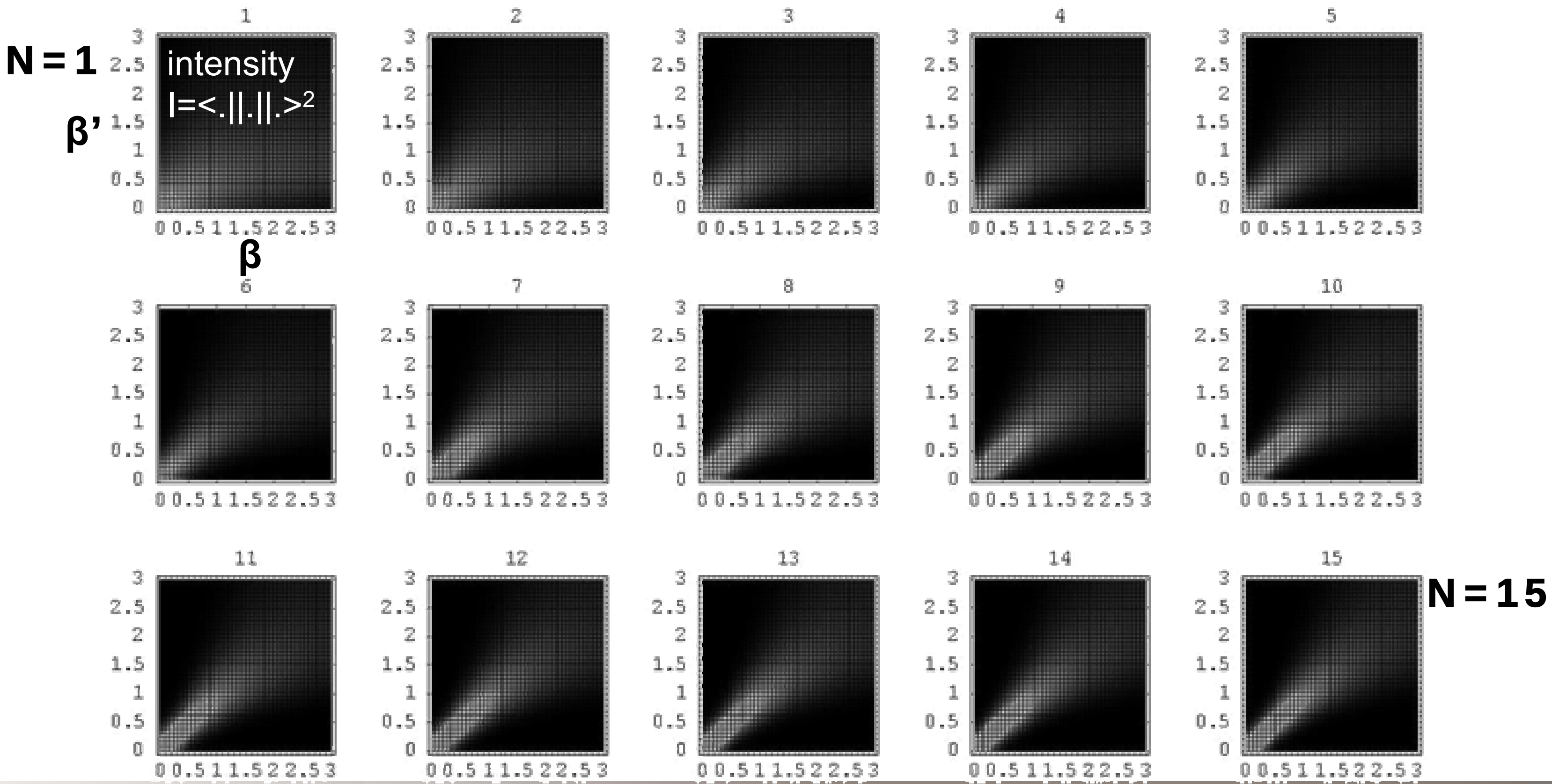
ΔK=0 **K=0 → K=0**

2-particle L=0 transfer reactions

in the Boson Coherent-State formalism

gs -> gs transfer with as input only the gs. quadr. deform. of the initial (β,γ) and final nucleus (β',γ')

$$\langle N+1; gs(\beta',\gamma')||s^\dagger||N; gs(\beta,\gamma)\rangle = \sqrt{N+1}\frac{(1+\beta\beta'\cos(\gamma-\gamma'))^N}{\sqrt{(1+\beta'^2)^{N+1}(1+\beta^2)^N}}$$



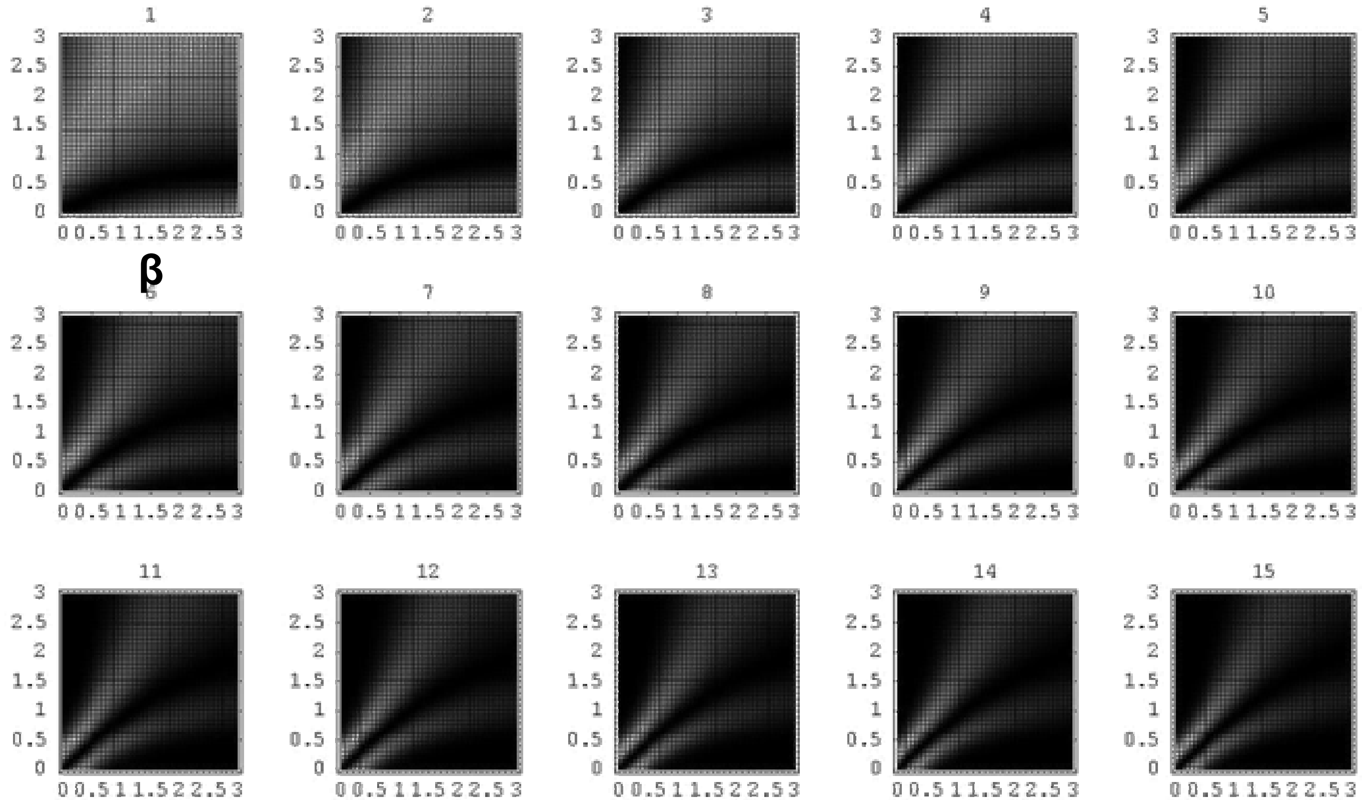
2-particle L=0 transfer reactions

in the Boson Coherent-State formalism

gs -> bv
transfer

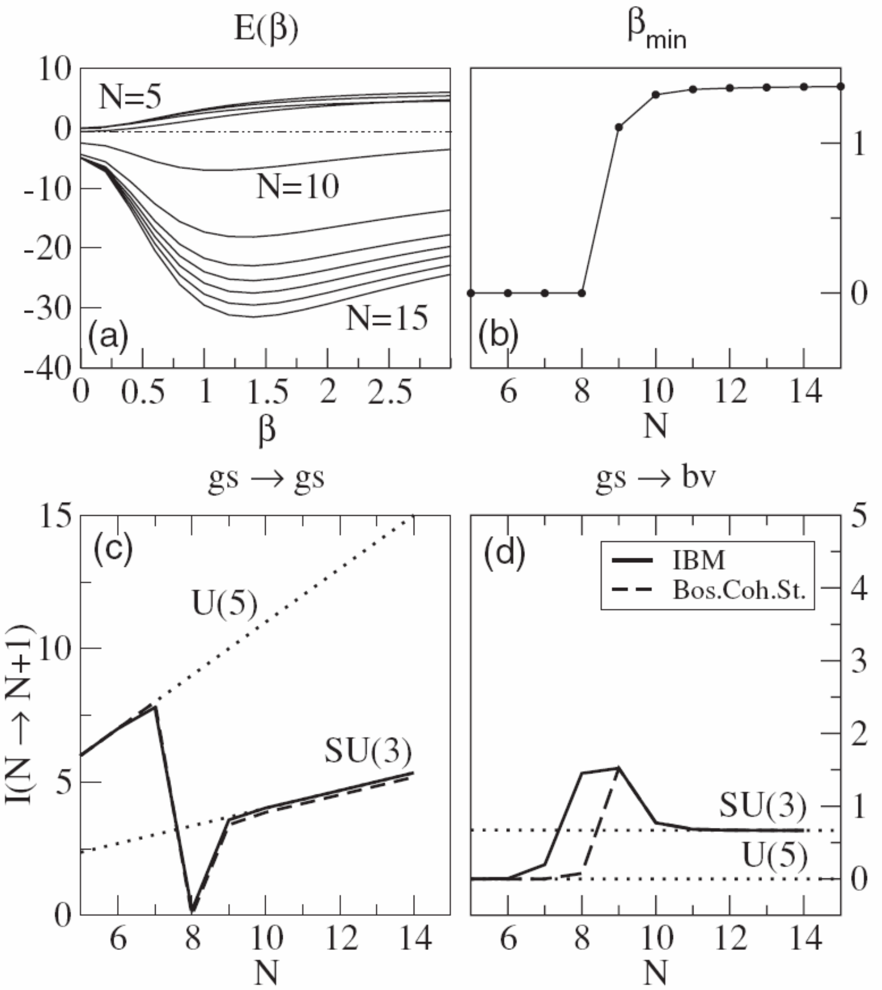
$$\langle N+1; bv(\beta',\gamma')||s^{\dagger}||N; gs(\beta,\gamma)\rangle = \frac{(1+\beta\beta'\cos(\gamma-\gamma'))^{N-1}}{\sqrt{(1+\beta'^2)^{N+1}(1+\beta^2)^N}}\Big(N(\beta\cos(\gamma-\gamma')-\beta')-\beta'(1+\beta\beta'\cos(\gamma-\gamma'))\Big)$$

dependence
on gs.
deformations
β, β'



U(5) to O(6)

vibrational to γ-unstable rotational

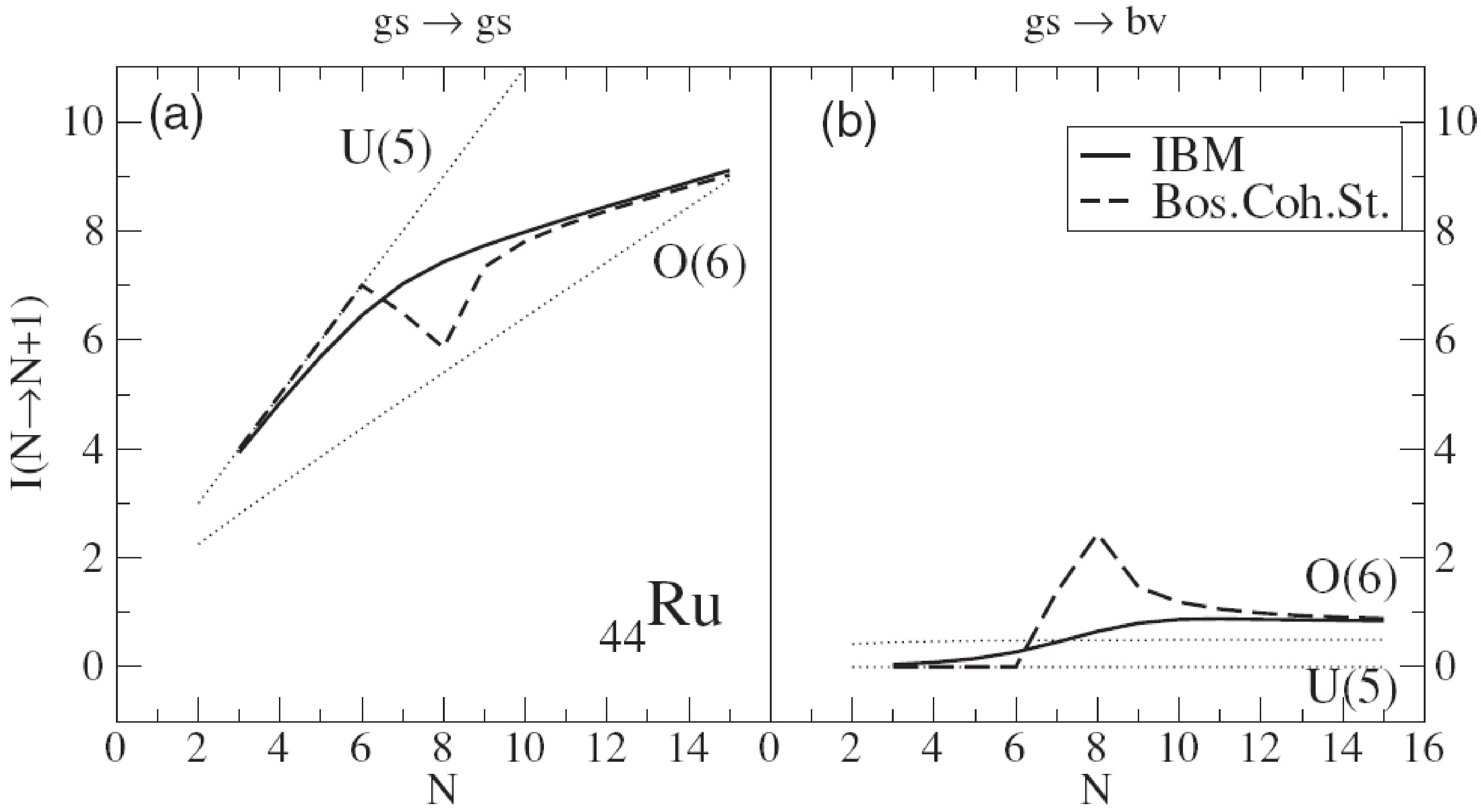


R. Fossion, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C76 (2007) 014316.

U(5) to O(6)

vibrational to γ-unstable rotational – 1st order trans.

Application to the Ru isotope series

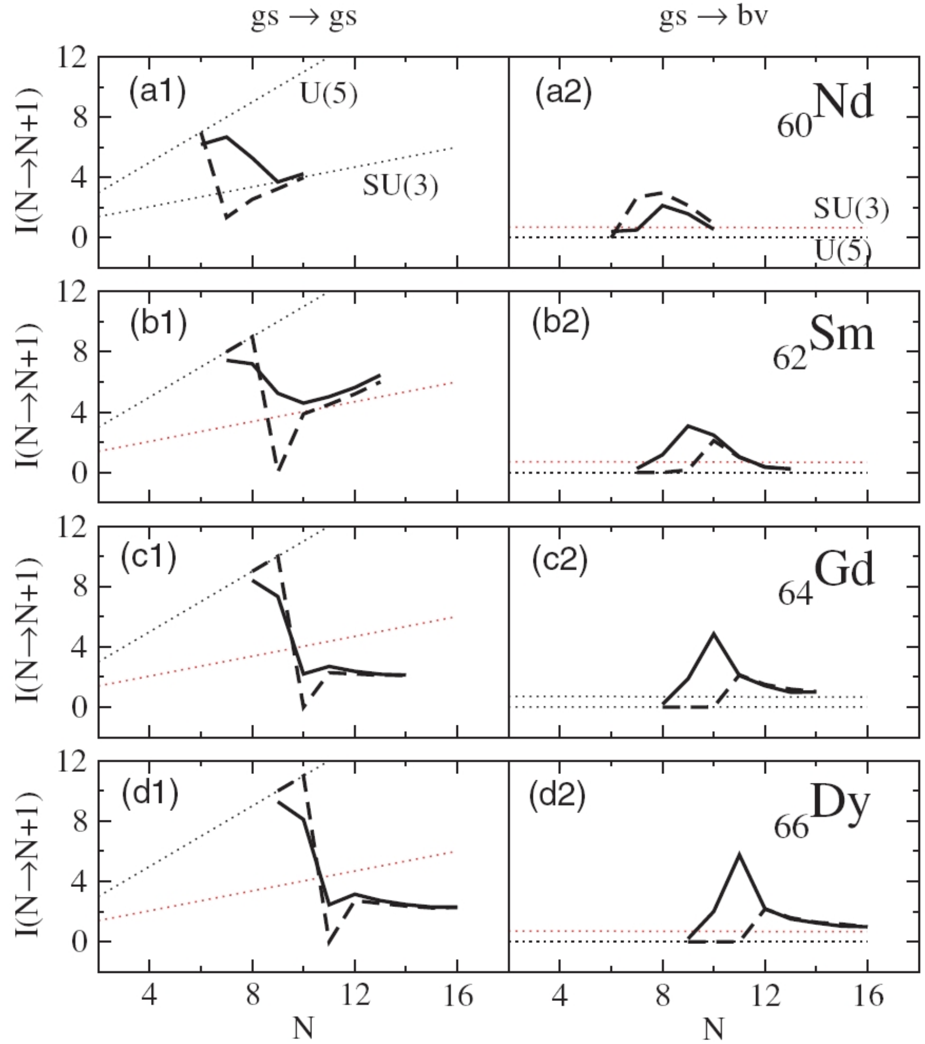


R. Fossion, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C76 (2007) 014316.

U(5) to SU(3)

Spherical to axial-symmetric deformed – 2nd order tr.

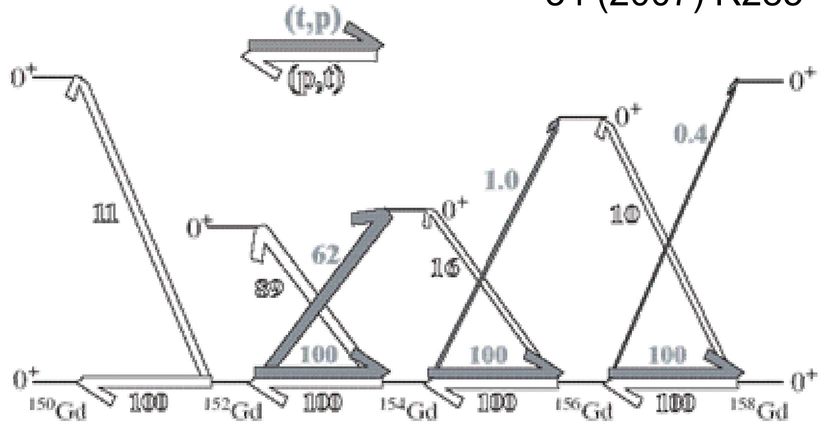
Application to the rare-earth isotopes



IBM

bos.coh.st

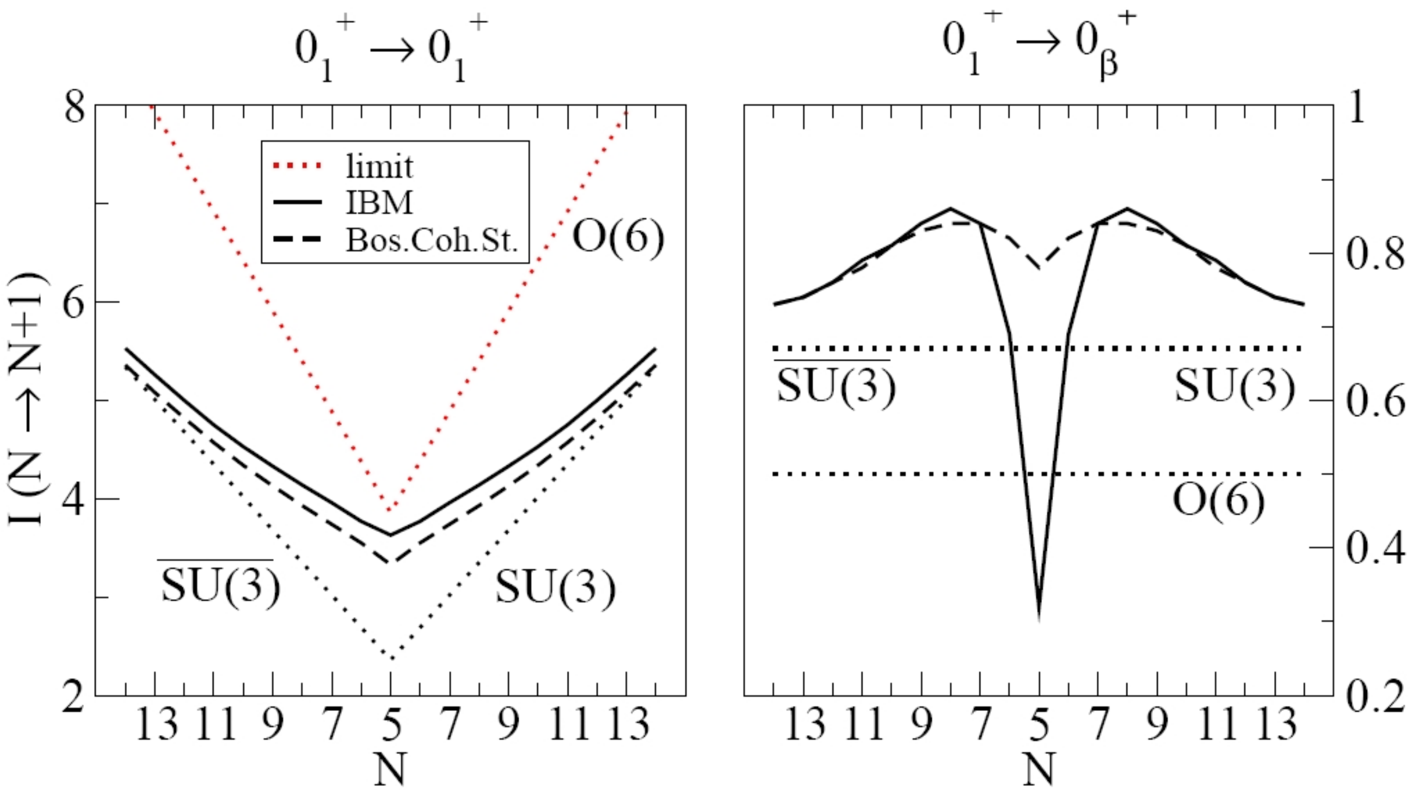
R. Casten and E.A. McCutchan, J. Phys. G, Nucl. Part. Phys. 34 (2007) R285



R. Fossion, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C76 (2007) 014316.

SU(3) to O(6) to SU(3)

Axial-symmetric rotational (oblate) to γ-unstable rotational to axial-symmetric rotational (prolate)



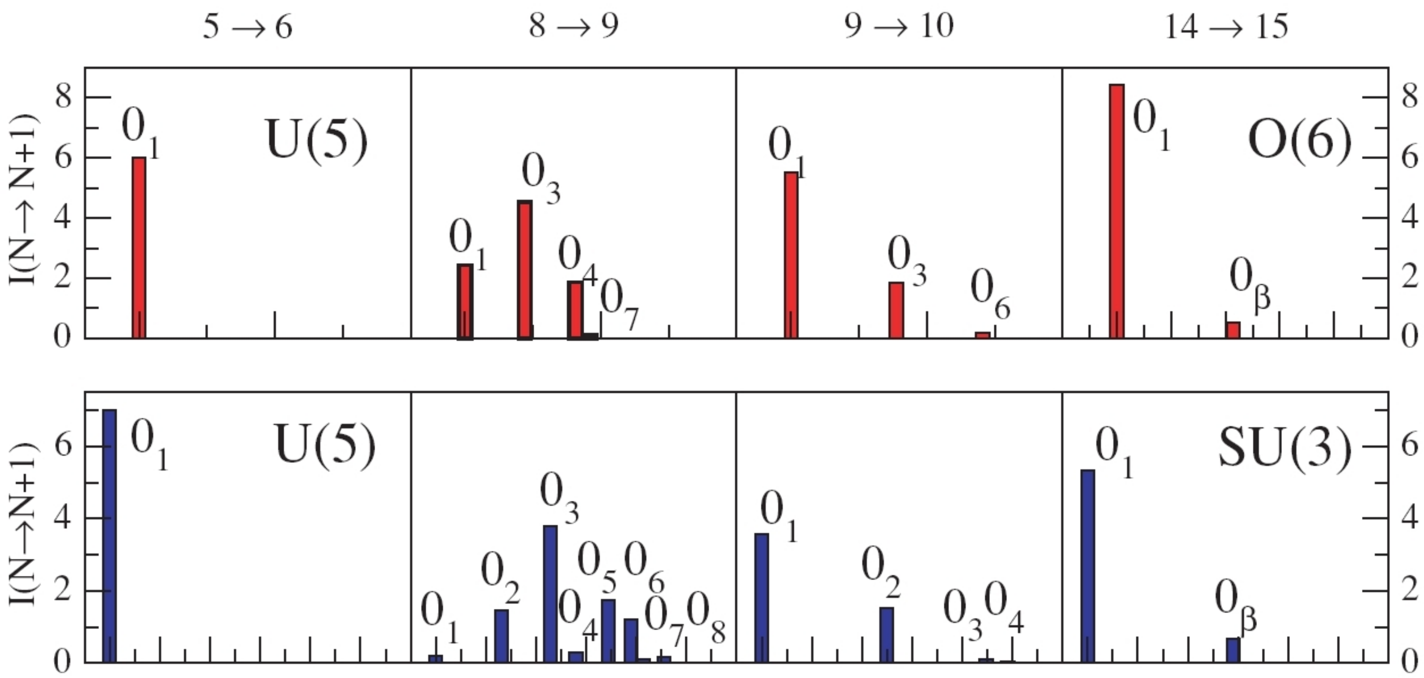
R. Fossion, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, INPC2007 conference proceedings, Tokyo

Fragmentation of the transfer strength

in the transition region

U(5) -> O(6)
2nd order

U(5) -> SU(3)
1st order



R. Fossion, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C76 (2007) 014316.

Conclusion

Possible signatures for nuclear phase/shape transitions in two-particle transfer reactions

• the appreciable **population of excited 0+ states in transfer processes,** in correspondence with **a loss of intensity in the transfer to the ground state**

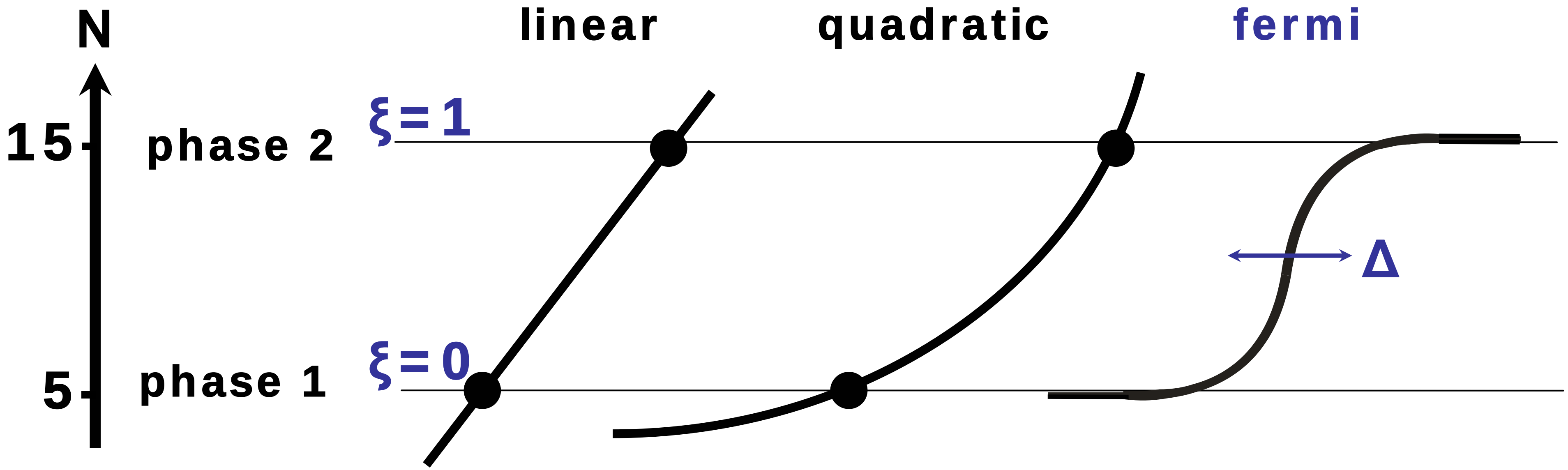
• **a fragmentation of the transfer strength to a large number of excited 0+ states**

Collaborators

C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi

The transition path

functional ξ(N) or χ(N)



$$\xi = 0.1N - 0.5,$$

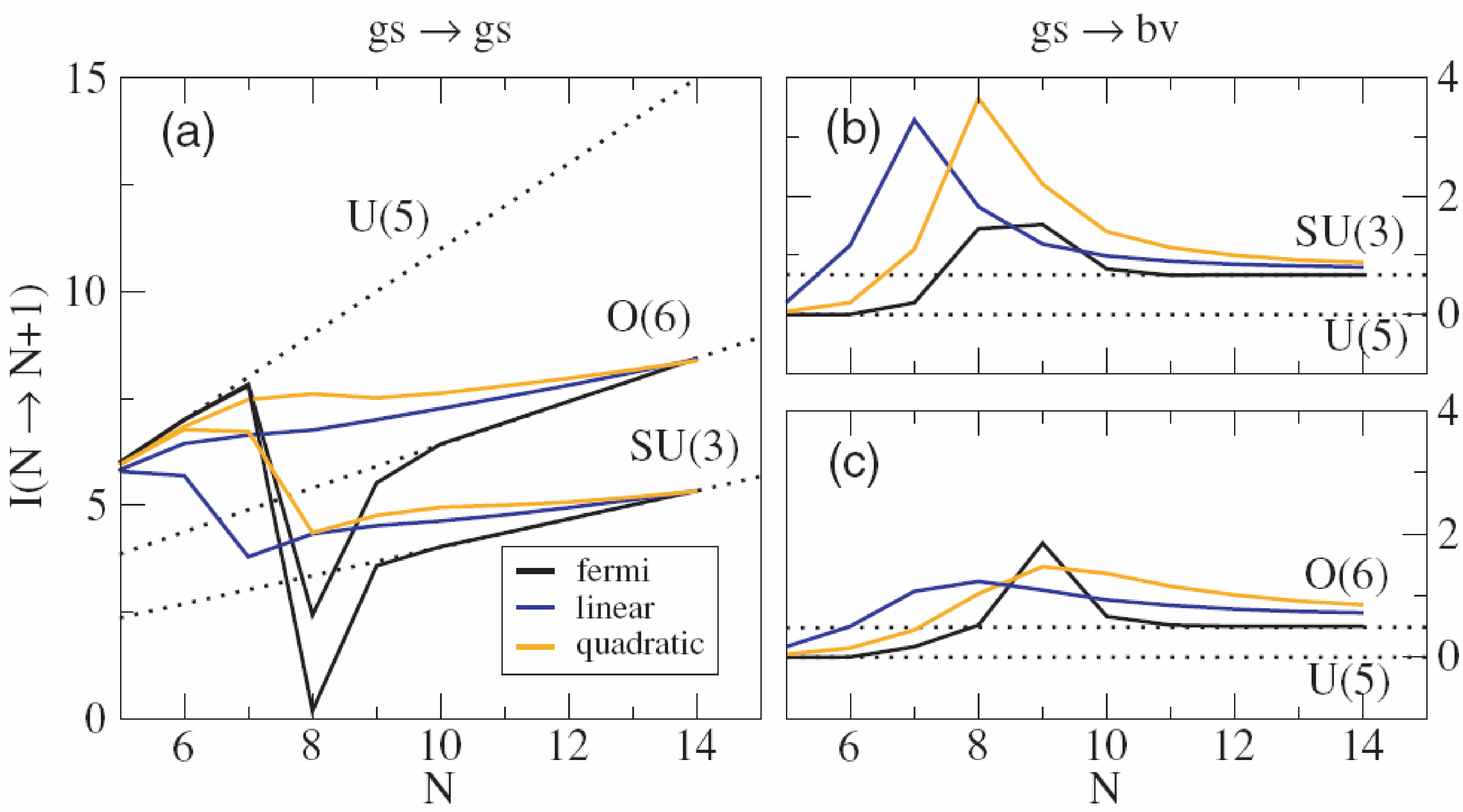
$$\xi = 0.005N^2 - 0.125,$$

$$\xi = \frac{1}{1 + \exp\left(\frac{N_0 - N}{\Delta}\right)}.$$

$$\chi(N) = \frac{-\sqrt{7}/2}{1 + \exp\left(\frac{N_0 - N}{\Delta}\right)}$$

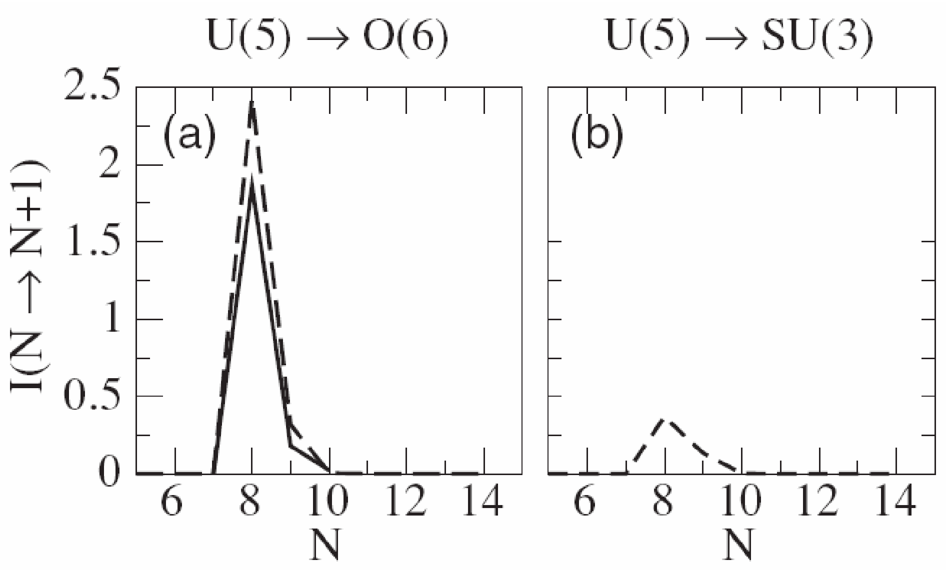
The transition path

functional ξ(N)



R. Fossion, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C76 (2007) 014316.

Transfer to the double-beta vibrational band



$\overline{SU(3)}$ N=15 → O(6) N=5 → SU(3) N=15