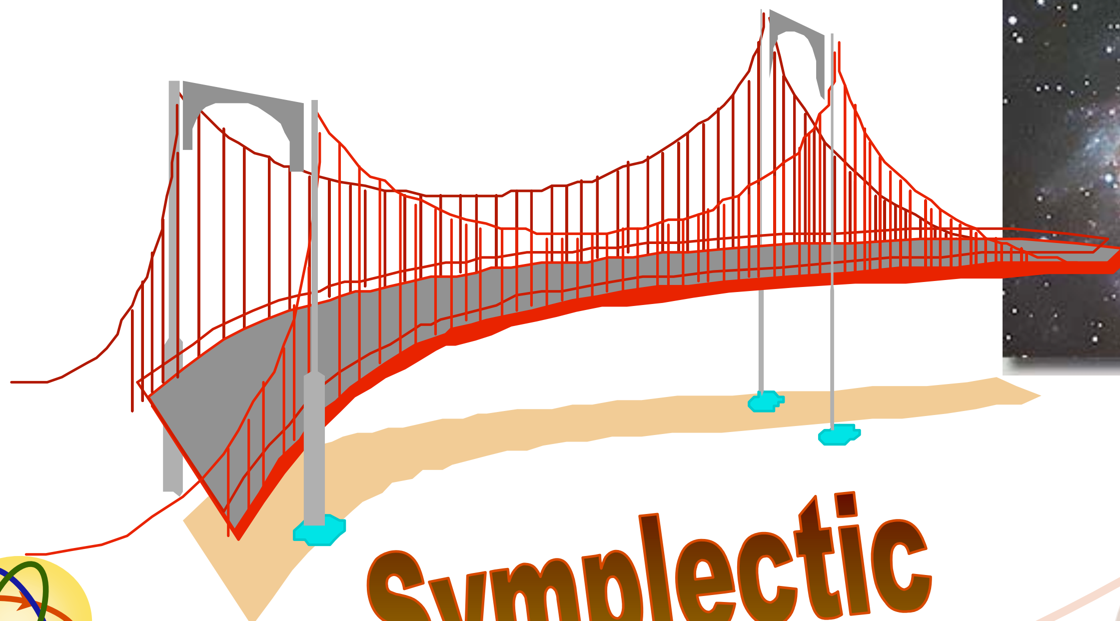
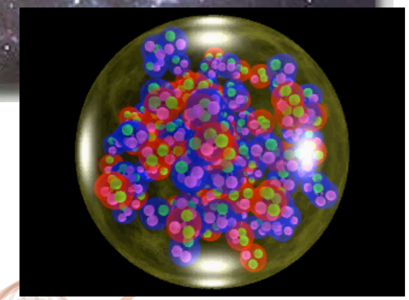
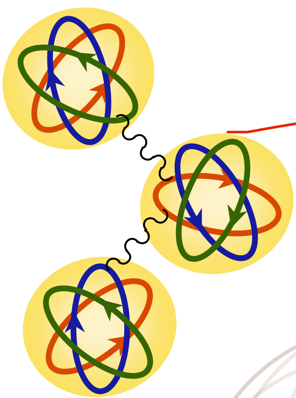






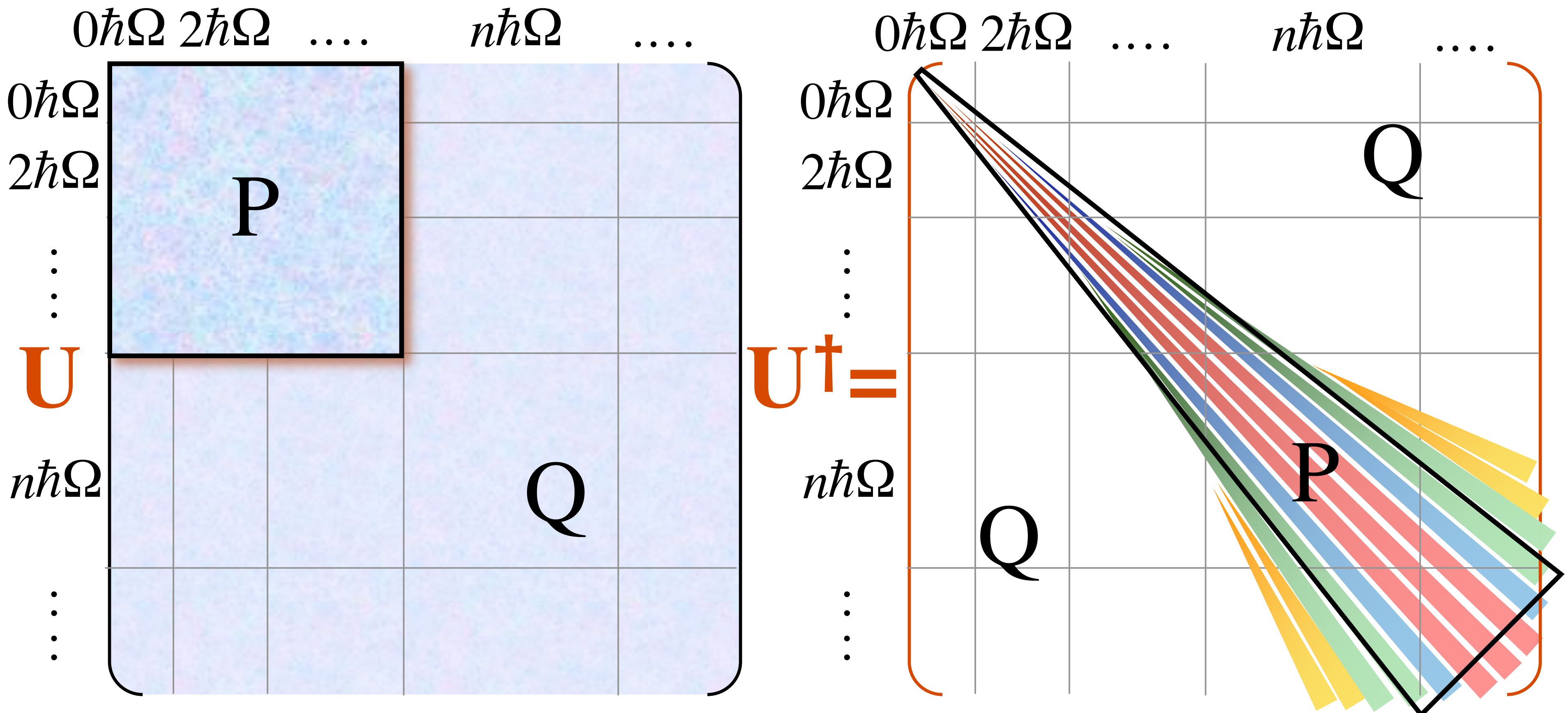
Symplectic Ab Initio No-Core Shell Model

Symplectic
Ab Initio No-Core Shell Model

...Taming the Model Space Dilemma

Tomas Dytrych, Kristina D. Sviratcheva, Jerry P. Draayer,
Chairul Bahri, James P. Vary

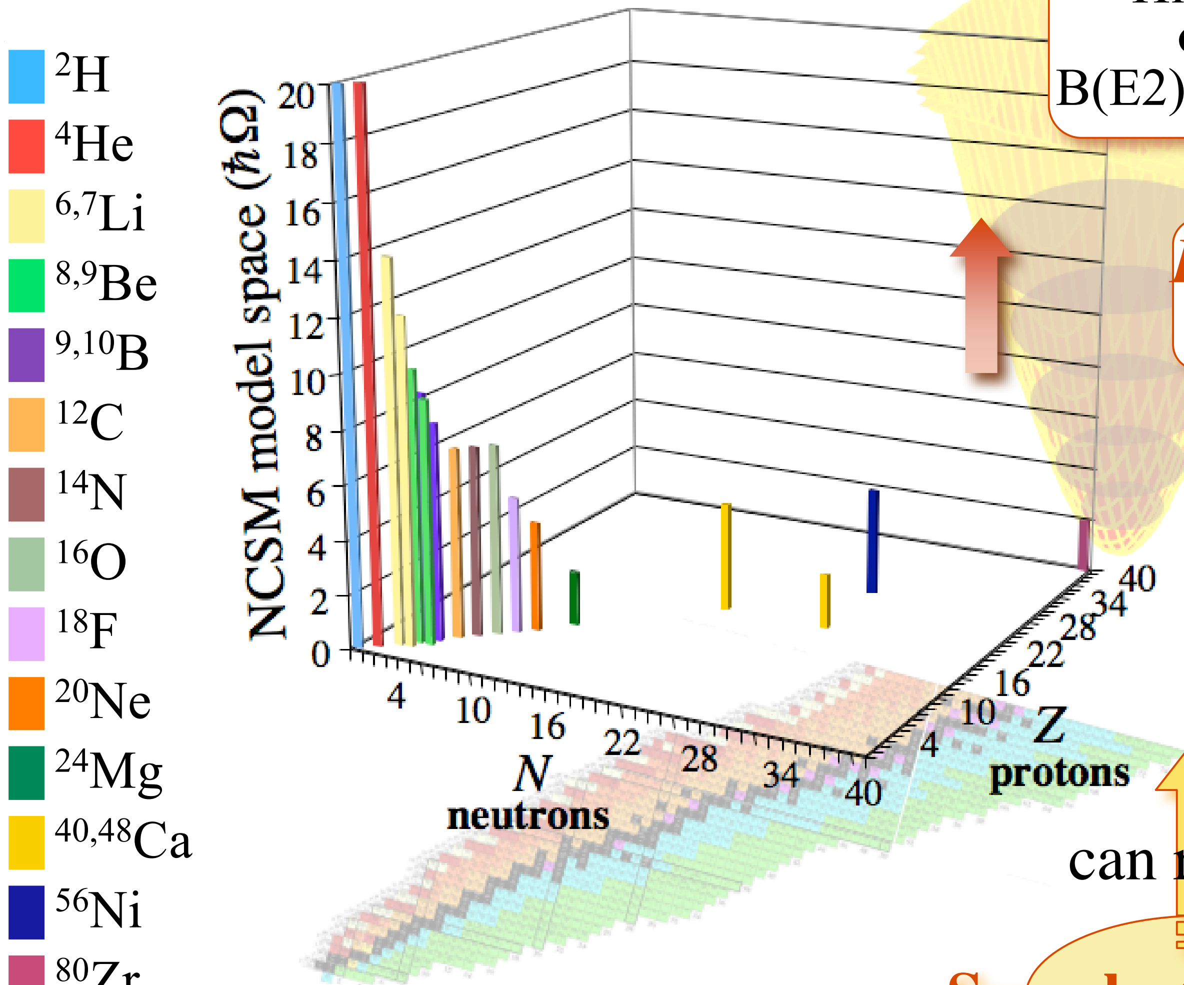
Symplectic No-Core Shell Model



Spherical harmonic oscillator basis

Symplectic basis

Achievements of No Core Shell Model (NCSM)



Highly deformed modes, α-cluster structures, B(E2) with NO effective charge

Larger model Spaces

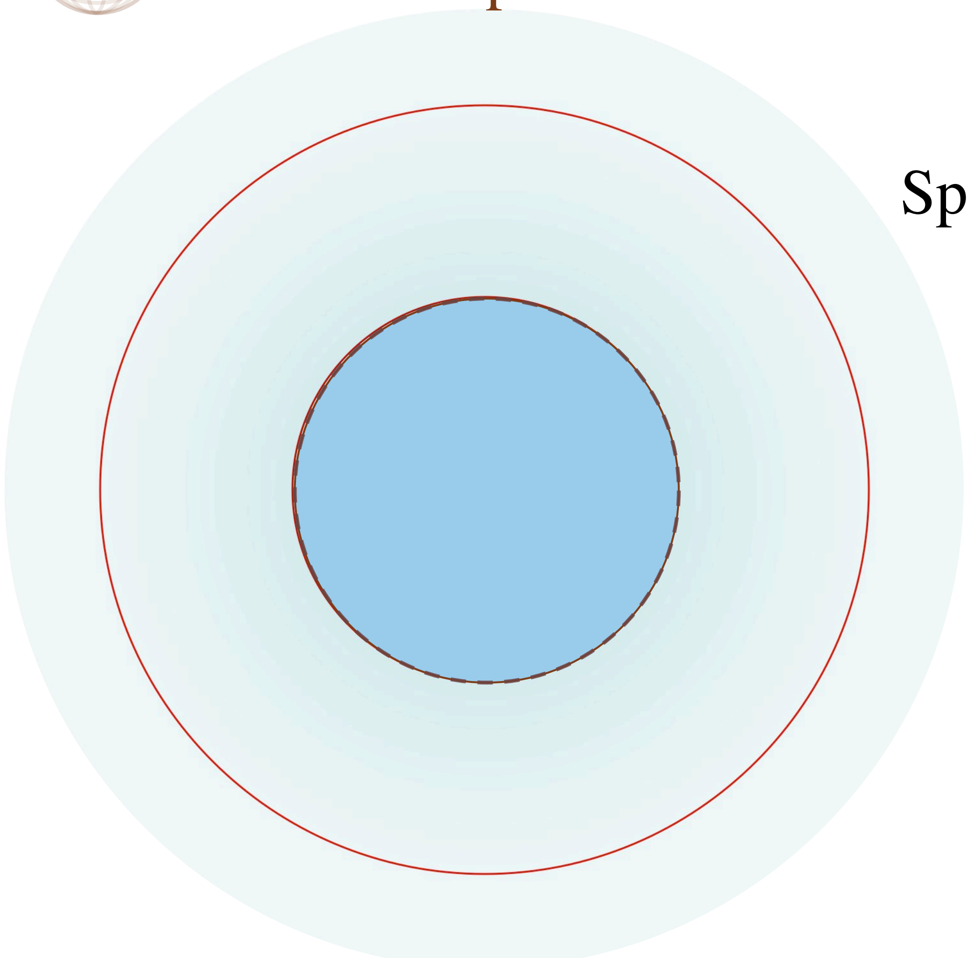
Heavier nuclei

can reach…

Symplectic-NCSM

Beyond What Computers Can Do: Choice of Basis

Spherical harmonic oscillator basis



No Core Shell Model

^{4}He

2 protons
2 neutrons

state in a Sp(3,$\mathbb{R}$) slice

$$|\, 2\hbar\Omega\,(2\,0)\; L{=}2\; S{=}0\; J{=}2\; M_J{=}2\rangle$$

$$\tfrac{1}{2}\,|\,00\tfrac{1}{2}\,{-\tfrac{1}{2}},00\tfrac{1}{2}\tfrac{1}{2};00\tfrac{1}{2}\,{-\tfrac{1}{2}},22\tfrac{5}{2}\tfrac{5}{2}\rangle$$

$$-\sqrt{\tfrac{1}{5}}\,|\,00\tfrac{1}{2}\,{-\tfrac{1}{2}},00\tfrac{1}{2}\tfrac{1}{2};00\tfrac{1}{2}\tfrac{1}{2},22\tfrac{3}{2}\tfrac{3}{2}\rangle$$

$$-\sqrt{\tfrac{1}{20}}\,|\,00\tfrac{1}{2}\,{-\tfrac{1}{2}},00\tfrac{1}{2}\tfrac{1}{2};00\tfrac{1}{2}\tfrac{1}{2},22\tfrac{5}{2}\tfrac{3}{2}\rangle$$

$$+\tfrac{1}{2}\,|\,00\tfrac{1}{2}\,{-\tfrac{1}{2}},22\tfrac{5}{2}\tfrac{5}{2};00\tfrac{1}{2}\,{-\tfrac{1}{2}},00\tfrac{1}{2}\tfrac{1}{2}\rangle$$

$$-\sqrt{\tfrac{1}{5}}\,|\,00\tfrac{1}{2}\tfrac{1}{2},22\tfrac{3}{2}\tfrac{3}{2};00\tfrac{1}{2}\,{-\tfrac{1}{2}},00\tfrac{1}{2}\tfrac{1}{2}\rangle$$

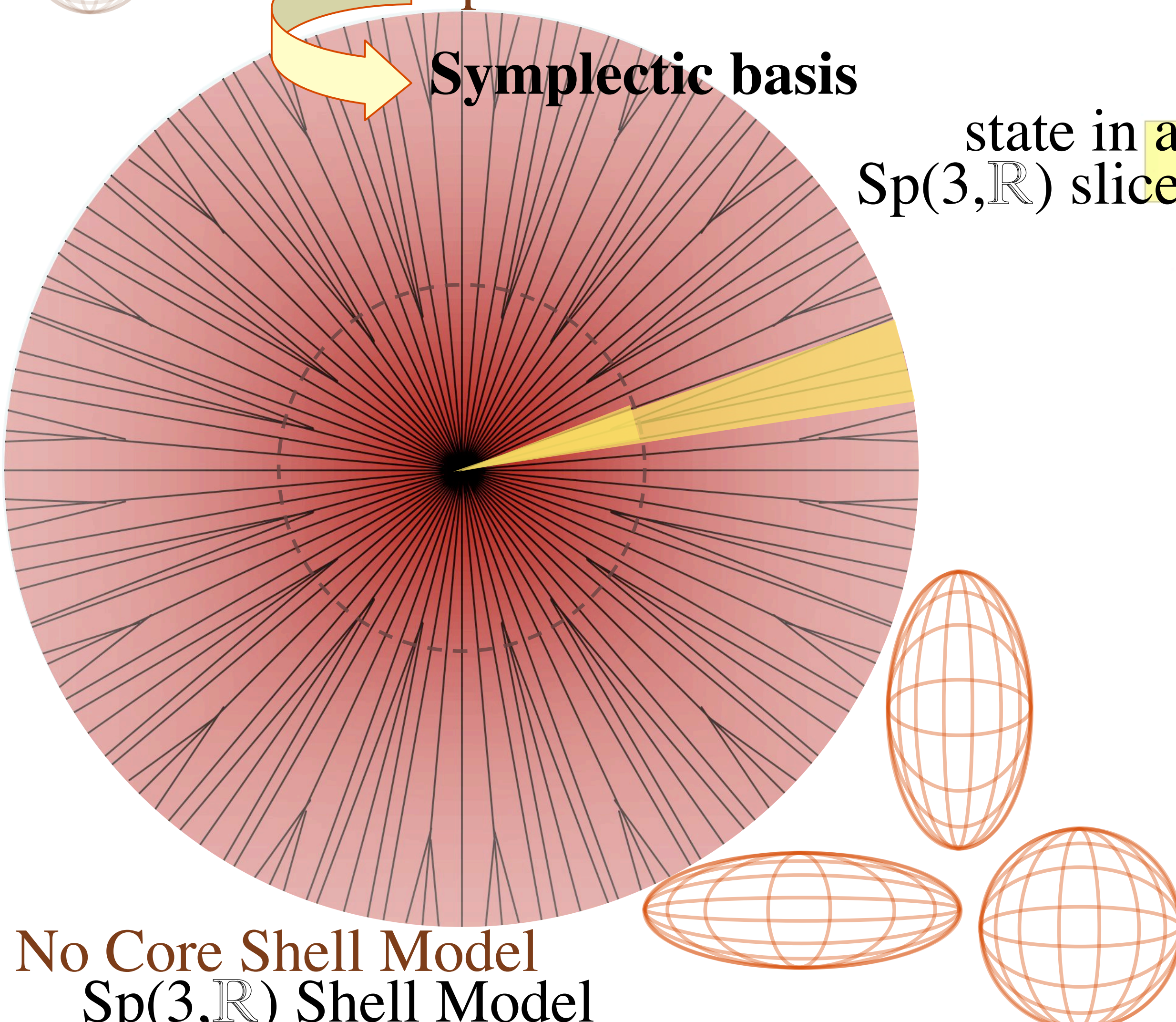
$$-\sqrt{\tfrac{1}{20}}\,|\,00\tfrac{1}{2}\tfrac{1}{2},22\tfrac{5}{2}\tfrac{3}{2};00\tfrac{1}{2}\,{-\tfrac{1}{2}},00\tfrac{1}{2}\tfrac{1}{2}\rangle$$

m-scheme… *n l j m*

Beyond What Computers Can Do: Choice of Basis

Spherical harmonic oscillator basis

Symplectic basis



No Core Shell Model

Sp(3,ℝ) Shell Model

state in a Sp(3,ℝ) slice

^{4}He

2 protons
2 neutrons

$$|\,2\hbar\Omega\,(2\,0)\,L{=}2\;S{=}0\;J{=}2\;M_J{=}2\rangle$$

$$
\begin{aligned}
&\tfrac{1}{2}\,|\,0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},0\,0\,\tfrac{1}{2}\,\tfrac{1}{2};0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},2\,2\,\tfrac{5}{2}\,\tfrac{5}{2}\rangle\\
&-\sqrt{\tfrac{1}{5}}\,|\,0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},0\,0\,\tfrac{1}{2}\,\tfrac{1}{2};0\,0\,\tfrac{1}{2}\,\tfrac{1}{2},2\,2\,\tfrac{3}{2}\,\tfrac{3}{2}\rangle\\
&-\sqrt{\tfrac{1}{20}}\,|\,0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},0\,0\,\tfrac{1}{2}\,\tfrac{1}{2};0\,0\,\tfrac{1}{2}\,\tfrac{1}{2},2\,2\,\tfrac{5}{2}\,\tfrac{3}{2}\rangle\\
&+\tfrac{1}{2}\,|\,0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},2\,2\,\tfrac{5}{2}\,\tfrac{5}{2};0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\rangle\\
&-\sqrt{\tfrac{1}{5}}\,|\,0\,0\,\tfrac{1}{2}\,\tfrac{1}{2},2\,2\,\tfrac{3}{2}\,\tfrac{3}{2};0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\rangle\\
&-\sqrt{\tfrac{1}{20}}\,|\,0\,0\,\tfrac{1}{2}\,\tfrac{1}{2},2\,2\,\tfrac{5}{2}\,\tfrac{3}{2};0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2},0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\rangle
\end{aligned}
$$

m-scheme… *n l j m*

Why Symplectic Symmetries?

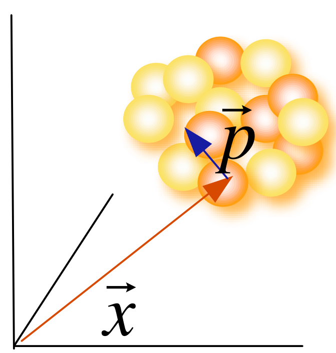
mass quadrupole moment

angular momentum

vorticity (from irrotational to rigid rotor flows)

many-particle kinetic energy

multi-shell monopole and quadrupole collective vibrations



Nucleus with A nucleons

$$\sum_s p_{si}p_{sj}, \sum_s x_{si}p_{sj} \pm x_{sj}p_{si}, \sum_s x_{si}x_{sj}$$

Symplectic {Sp(3,$\mathbb{R}$) ⊃ SU(3) ⊃ SO(3)} Model

microscopic		collective
x_i, p_i	⟺	$L^{(11)}_{1,M}$, $Q^{(11)}_{2,M}$, $N^{(00)}$, $A^{(20)}_{L,M}$, $B^{(02)}_{L,M}$

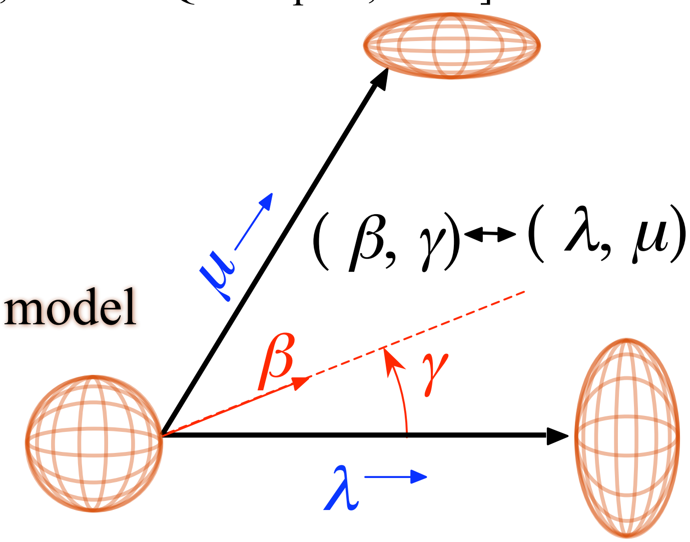
Elliott Model (single shell)

- Angular Momentum ……. $\vec{x} \times \vec{p}$ — SO(3)
- Quadrupole Moment ……… $x_i x_j$ — SU(3)
- Number Operator …… essentially HO Hamiltonian [Hamiltonian $H_0 = (p^2+x^2)/2$]
- Multi-shell Coupling … [Monopole, $L = 0$ & Quadrupole, $L = 2$]

- Higher-lying excitations
- Kinetic energy: $\dfrac{H^{(00)}_{00}}{2} - \dfrac{\sqrt{6}}{4}\left(A^{(20)}_{00} + B^{(02)}_{00}\right)$
- Microscopic formulation of the **Bohr-Mottelson** model

$$\frac{1}{12}[Q \otimes Q]^{L=0} = \frac{3}{20\pi} A^2 \left(\frac{R_0}{b}\right)^4 \beta^2$$

$$-\frac{1}{108}\sqrt{\frac{7}{2}}[Q \otimes Q \otimes Q]^{L=0} = \frac{1}{20\pi\sqrt{5\pi}} A^3 \left(\frac{R_0}{b}\right)^6 \beta^3 \cos 3\gamma$$



G. Rosensteel and D.J. Rowe, Phys. Rev. Lett. 38 (1977) 10

NCSM and Sp(3,$\mathbb{R}$) Shell Model

No-Core Shell-Model
(multi-$\hbar\Omega$)

horizontal slices

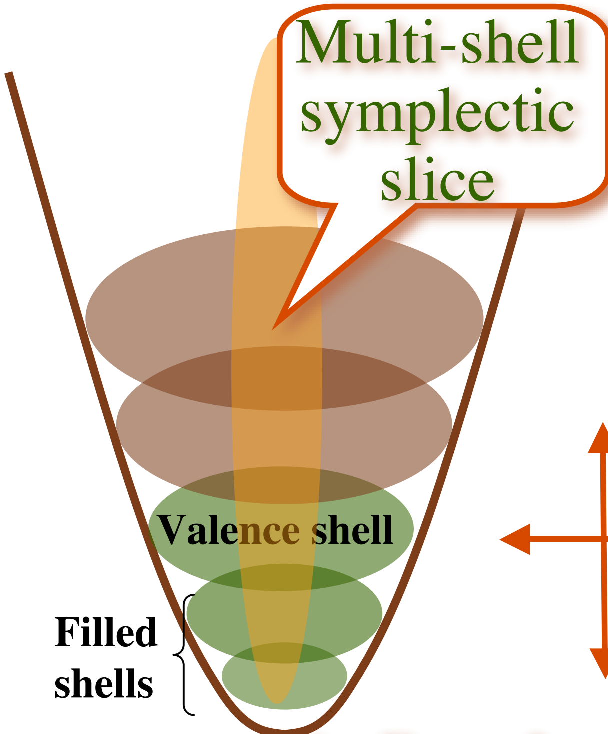
(0$\hbar\Omega$) limit

Symplectic Sp(3,$\mathbb{R}$) Shell-Model
(multi-$\hbar\Omega$)

vertical slices

monopole & quadrupole collective excitations

SU(3) limit (0$\hbar\Omega$)

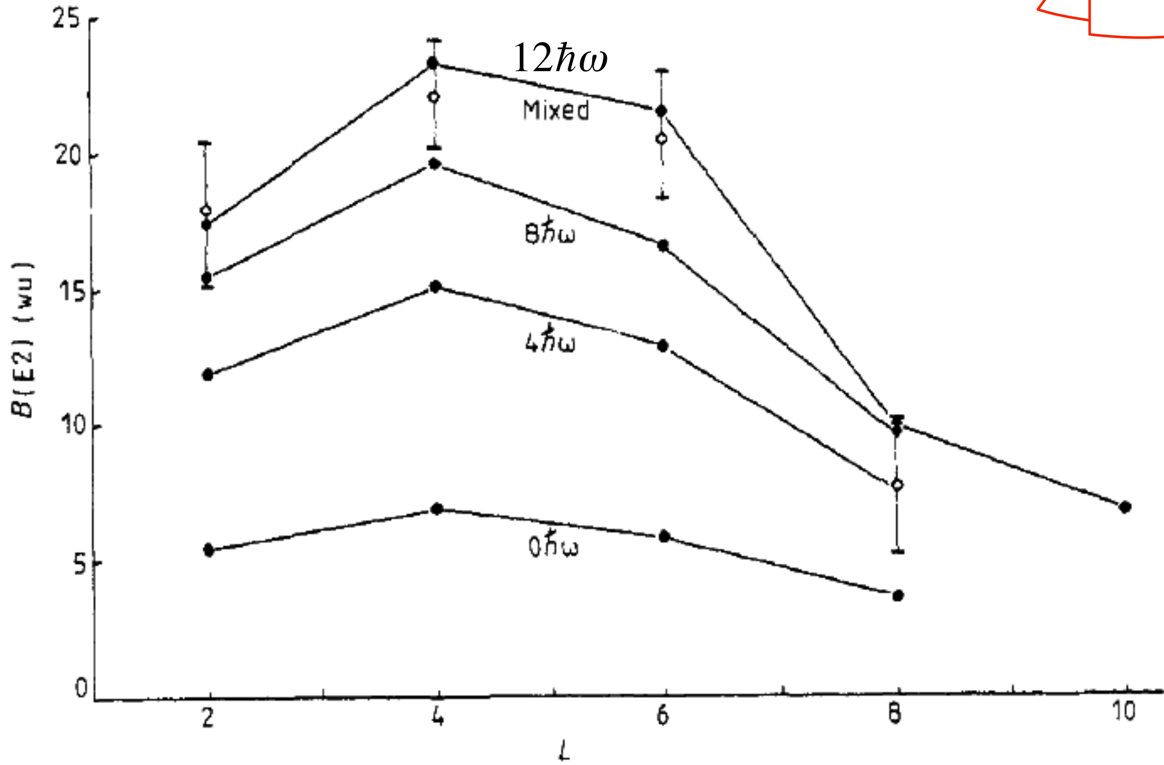


- Ab initio
- Realistic interaction (local/nonlocal;NN,NNN,…)
- Reproduction of binding energies and spectral features of light nuclei

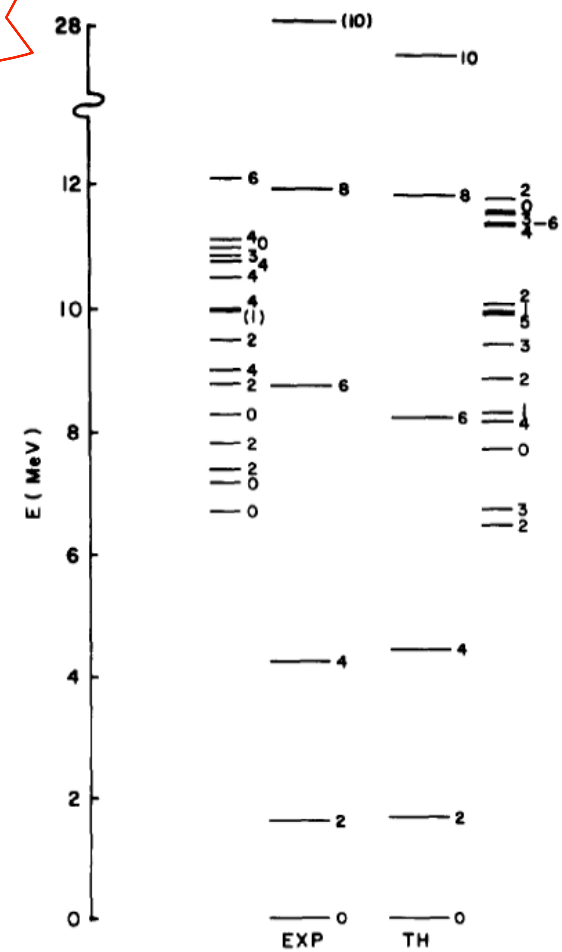
- **Free of spurious center-of-mass motion**
- Relation to NCSM basis: **microscopic**!
- Reproduction of rotational energy spectra and electromagnetic transitions without effective charge

Sp(3,ℝ)⊃SU(3) Model

^{20}Ne



G. Rosensteel and D.J. Rowe (1980)

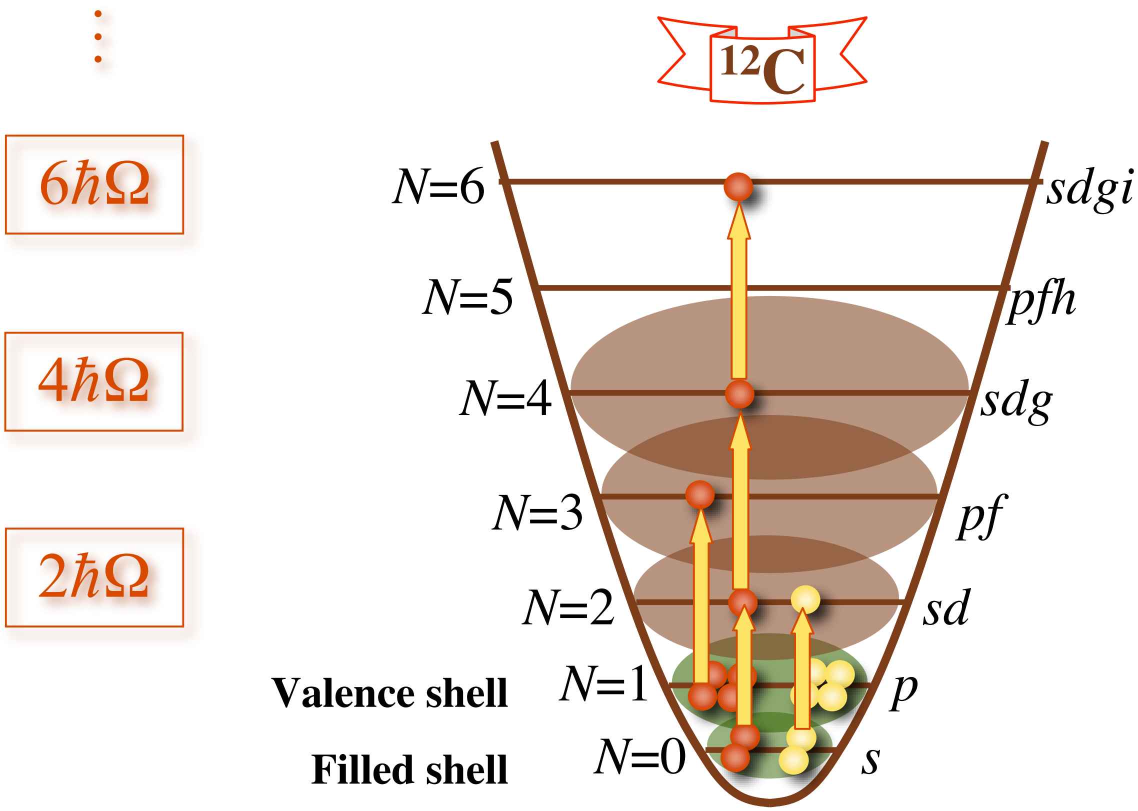


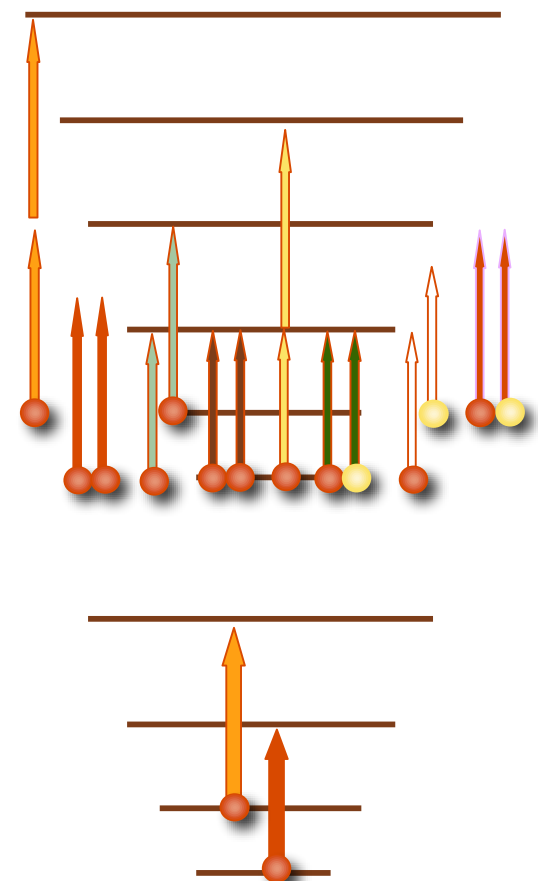
J.P. Draayer, K.J. Weeks, G. Rosensteel (1984)

$$H = \hbar\omega H_0 + b_2 Q\cdot Q + b_3 (Q\times Q)\cdot Q + b_4 (Q\cdot Q)^2 + \sum_j \varepsilon_j n_j + G_0 P$$

Symplectic {Sp(3,ℝ)} Basis

Vertical slices generated by $A^{(20)}_{L,M}$



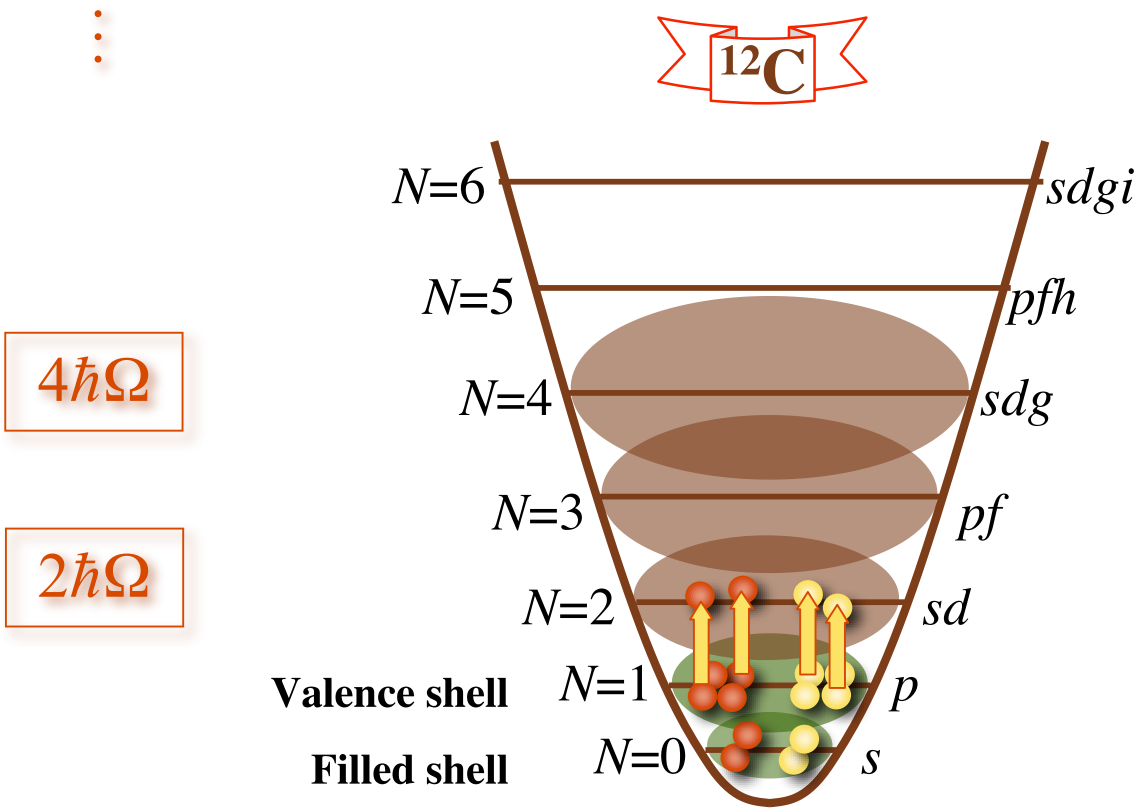


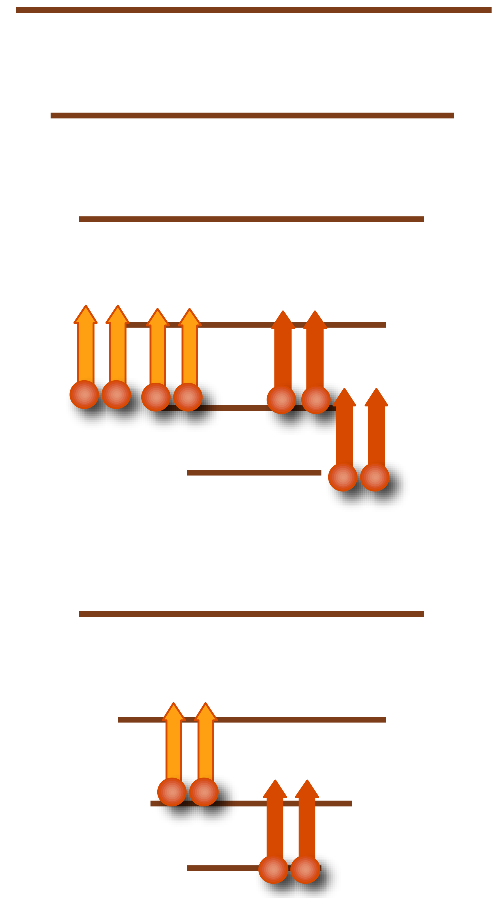
Examples for **protons** and **proton-neutron** excitations

In addition to the 2ħΩ 1p-1h excitations: small (~1/A) 2ħΩ 2p-2h correction for removing spurious center-of-mass motion

Missing Configurations within a Sp(3,ℝ) Slice

e.g., $2\hbar\Omega$ 2p-2h, $4\hbar\Omega$ 4p-4h, …





Examples for **proton** excitations

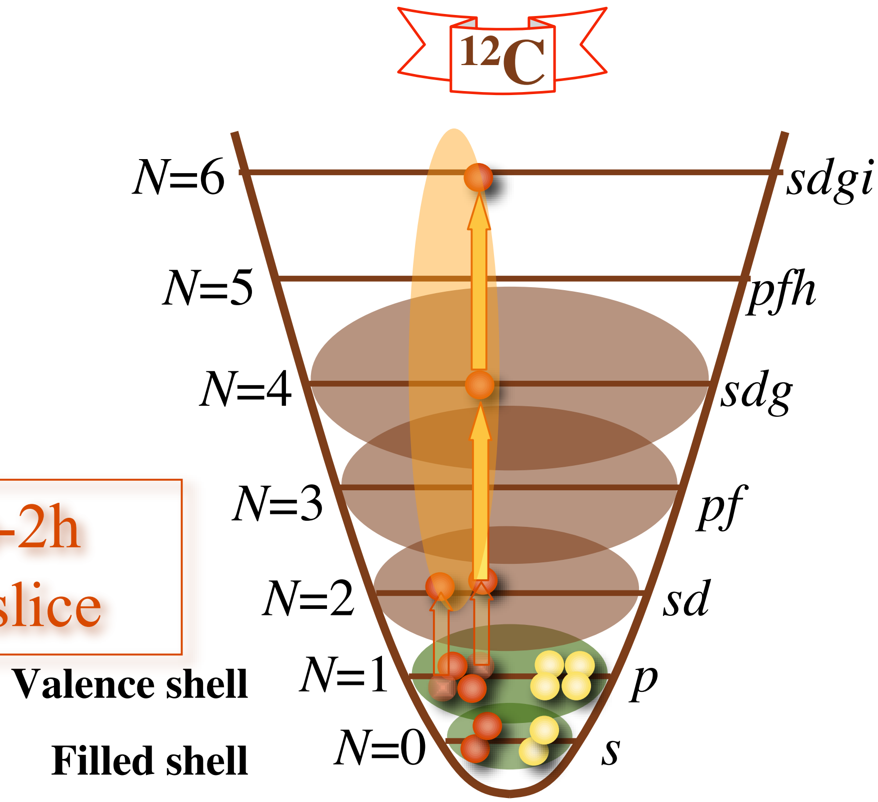
Multi-particle-multi-hole Sp(3,ℝ) Slices

⋮

$6\hbar\Omega$

$4\hbar\Omega$

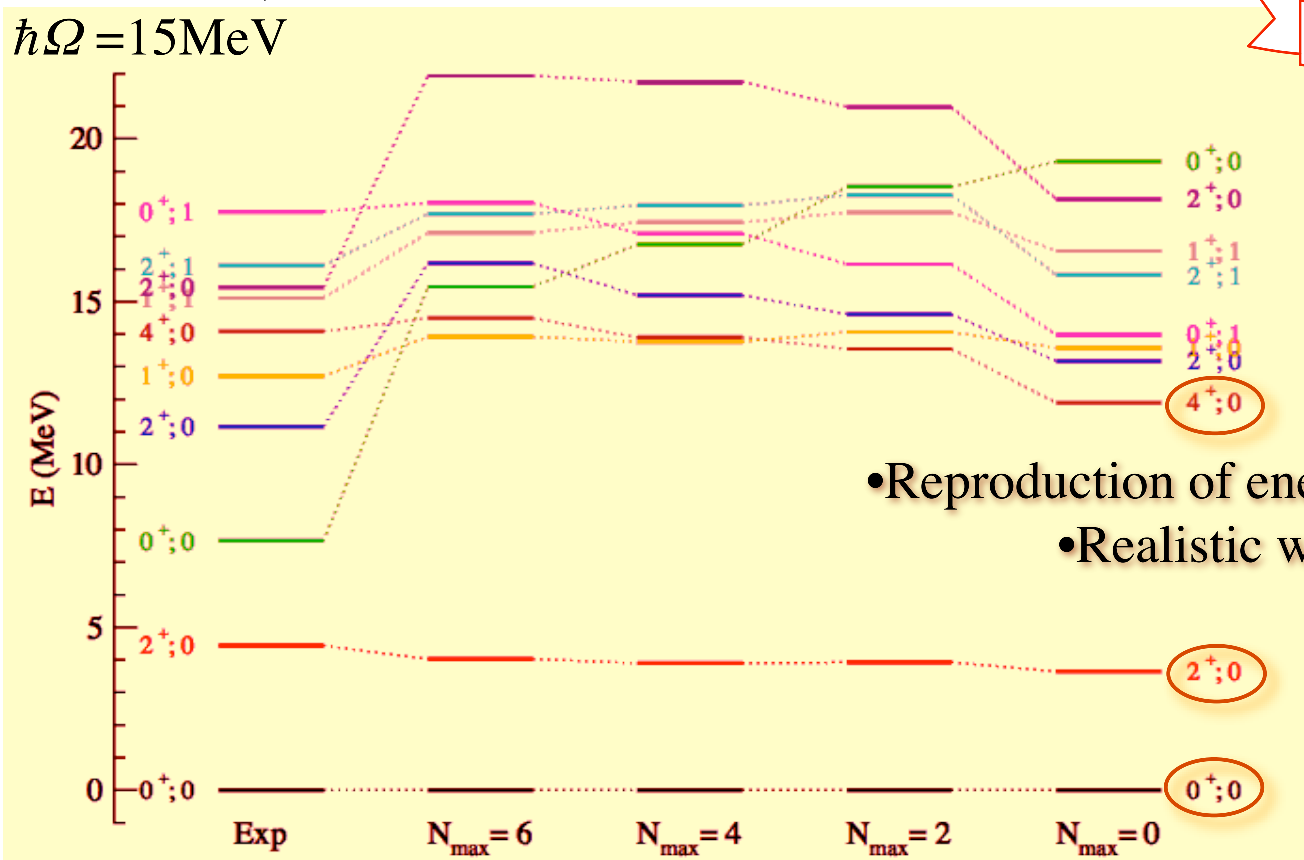
$2\hbar\Omega$ 2p-2h vertical slice



Model space of all possible Sp(3,ℝ) vertical slices = NCSM space

NCSM Results

^{12}C



NCSM:

•Reproduction of energy spectrum

•Realistic wave functions

"proof-of-principle" study

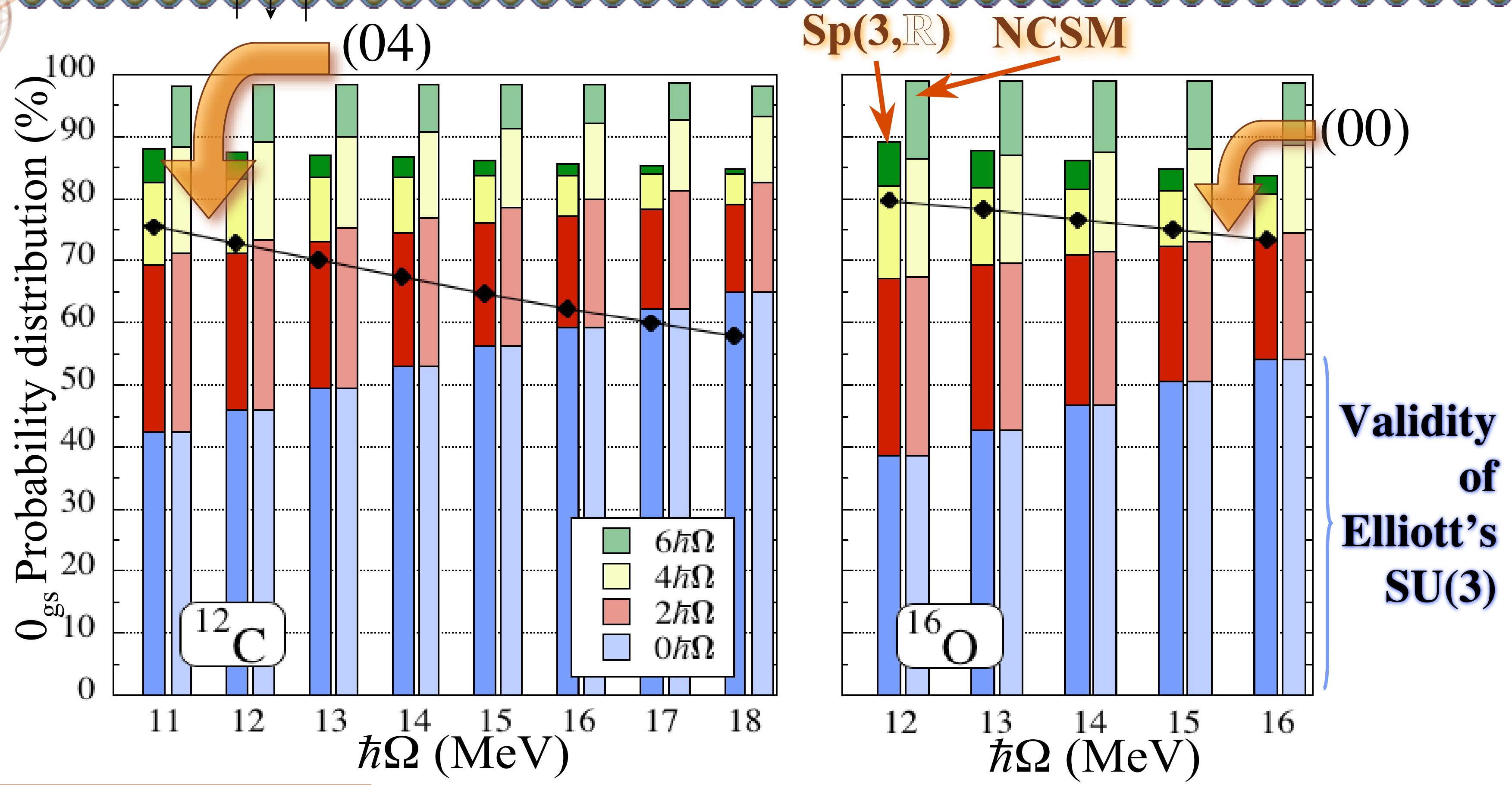
Projection of NCSM wave functions onto the symplectic Sp(3,ℝ) basis

Reproduction of NCSM results with only a few Sp(3,ℝ) states

Interaction: **JISP16**

A. M. Shirokov et al., Phys. Letts. B 621, 96(2005)

Probability Distribution: Ground State – 85-90%



Only **3** 0p-0h symplectic irreps: ~80%

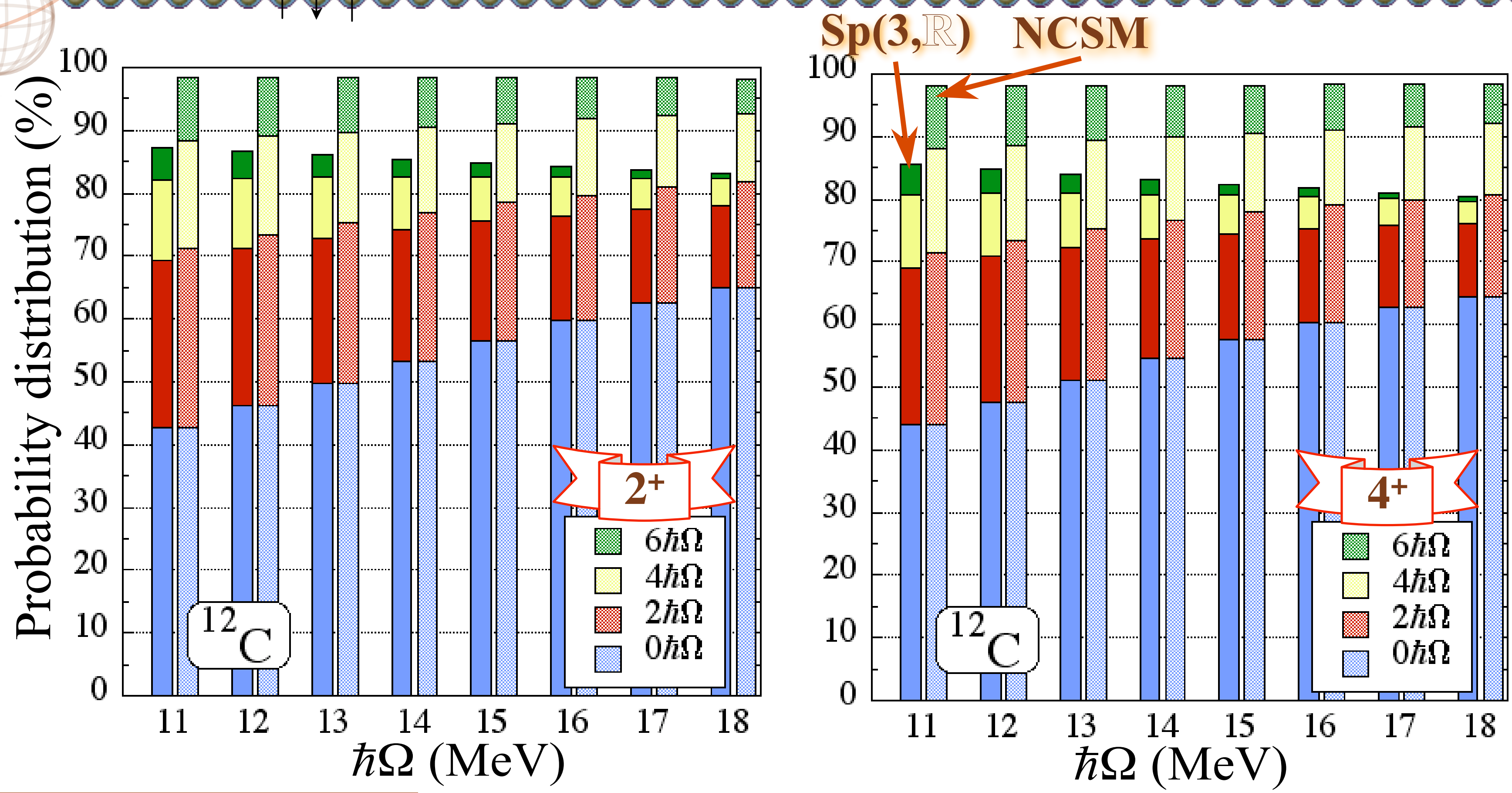
N_σ $(\lambda\mu)$

24.5(04): most deformed

$24.5(12)^2$

$2\hbar\Omega$ 2p-2h Sp(3,ℝ) irreps:
~4% (^{12}C)
~10% (^{16}O)

Probability Distribution: 2^+ and 4^+



Only **3** 0p-0h symplectic irreps: ~80%

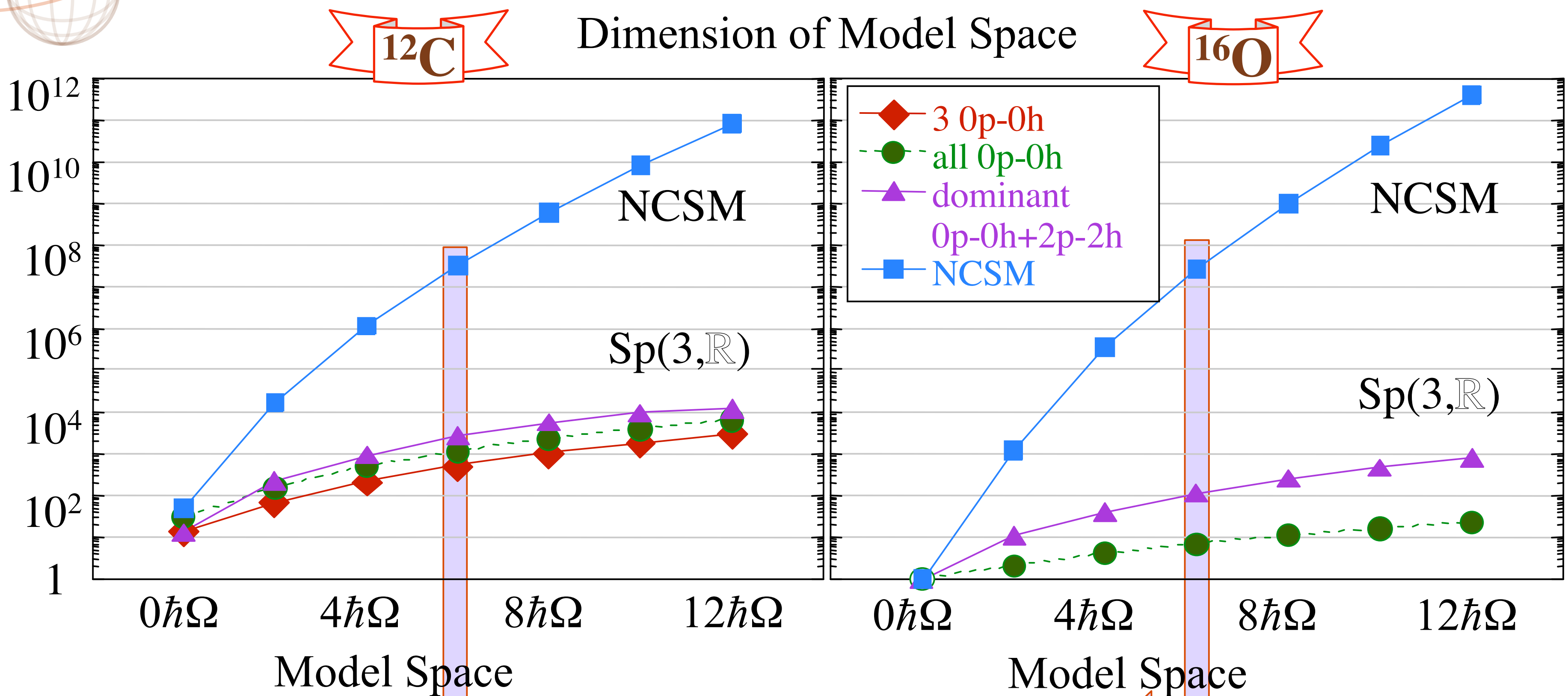
N_σ (λμ)

24.5(04): most deformed

$24.5(12)^2$

2ħΩ 2p-2h Sp(3,ℝ) irreps: ~4% (2^+ and 4^+)

Major Reductions in Model Space



Compared to NCSM

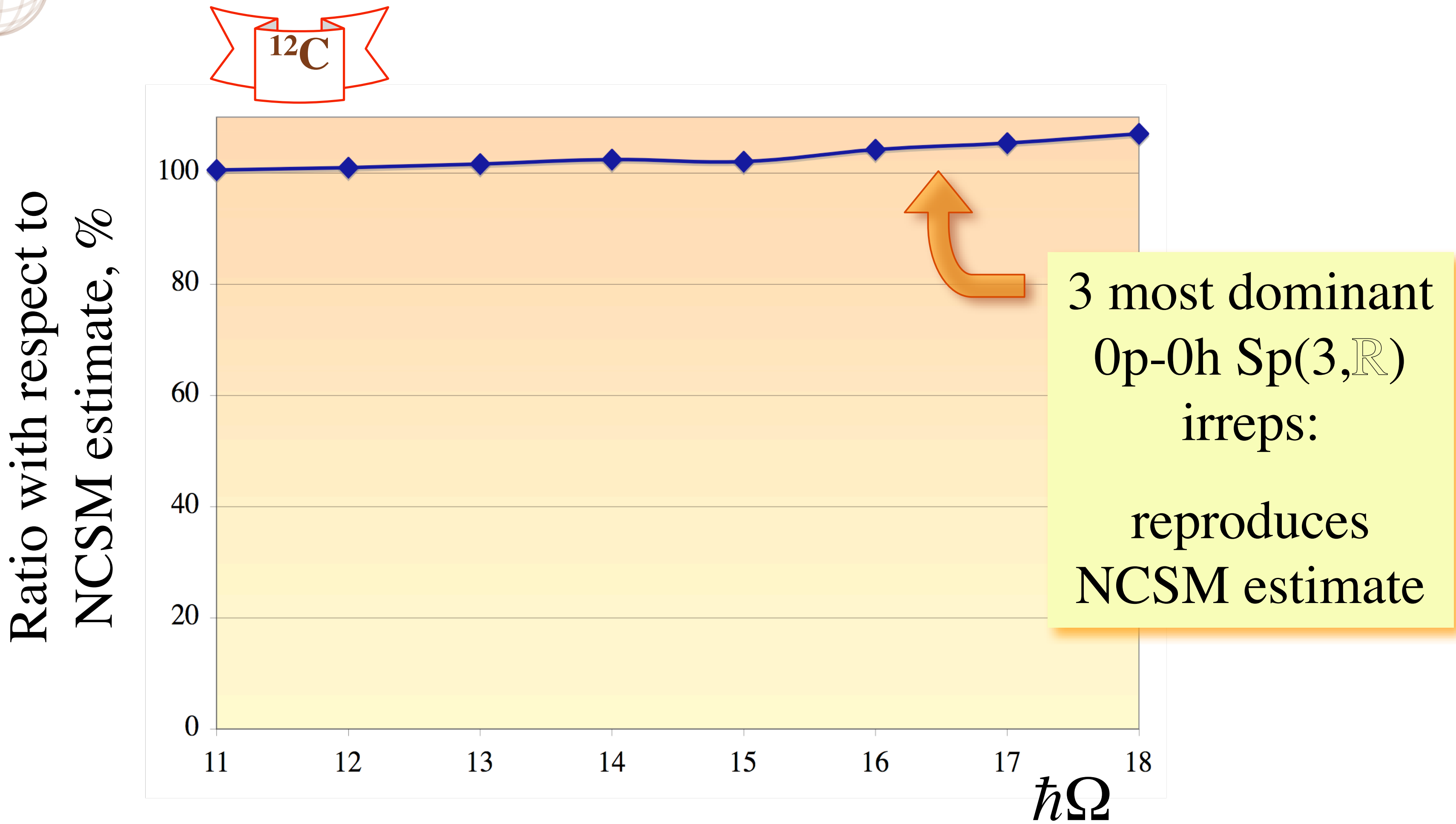
Dimension of model space

0.009% for ^{12}C

0.0004% for ^{16}O

T. Dytrych. K.D. Sviratcheva, C. Bahri, J.P. Draayer, J.P. Vary, Phys. Rev. Lett. 98 (2007) 162503

$B(E2: 2^+ \rightarrow 0^+)$

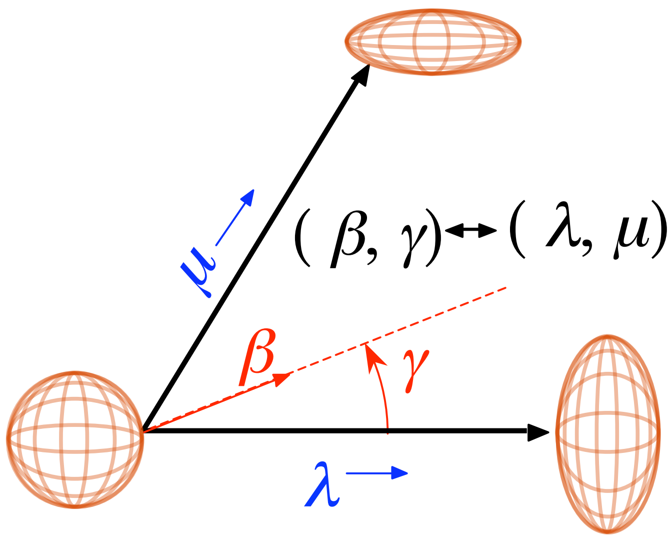


- CM spurious states eliminated
- NCSM model space… $6\hbar\Omega$

Matching "Dynamics" to "Geometry"

^{12}C

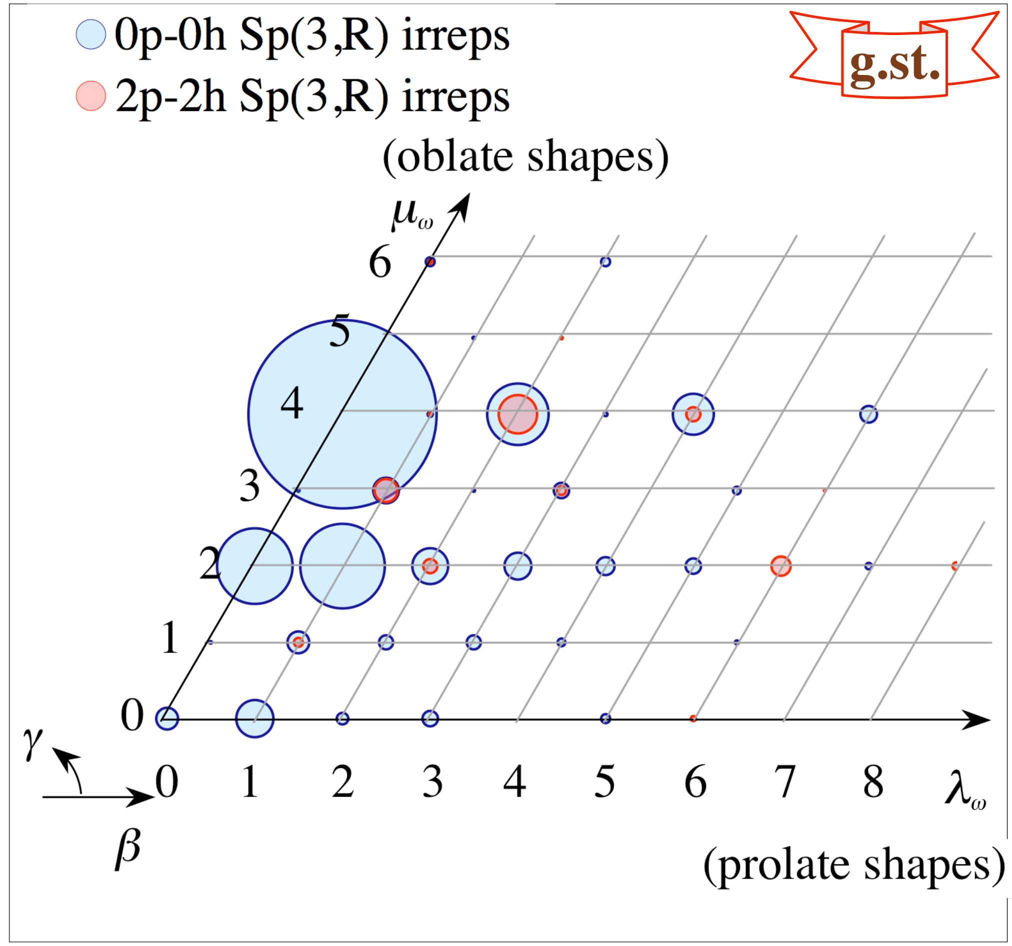
$\hbar\Omega = 15$ MeV



Dominant modes:

0p-0h: (0 4) [oblate]

2p-2h: (2 4)



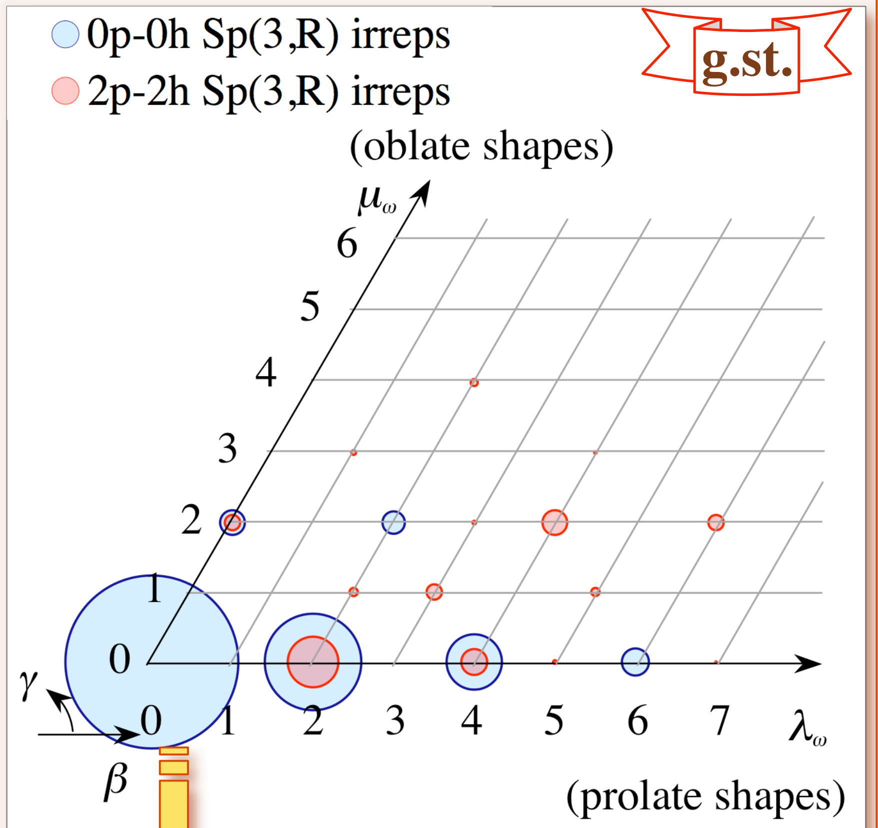
Area = Probability of dominant Sp(3,$\mathbb{R}$) irreps

Matching "Dynamics" to "Geometry"

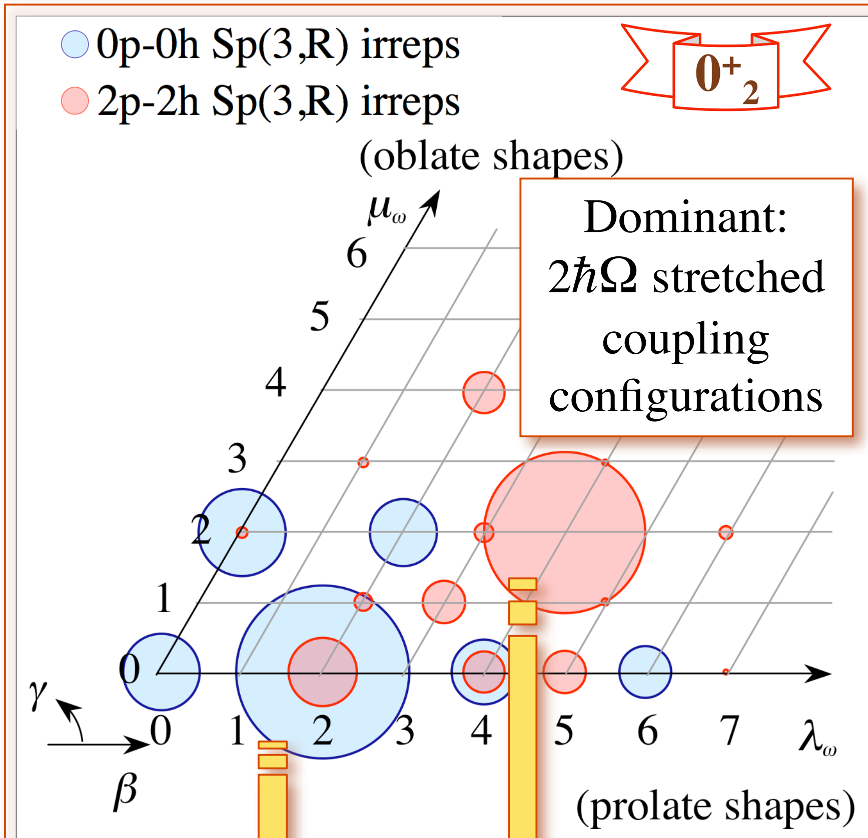
^{16}O

$\hbar\Omega = 15$ MeV

Reflect *NCSM results* with model spaces achieved on modern-day computers



(a)



(b)

Dominant: $2\hbar\Omega$ stretched coupling configurations

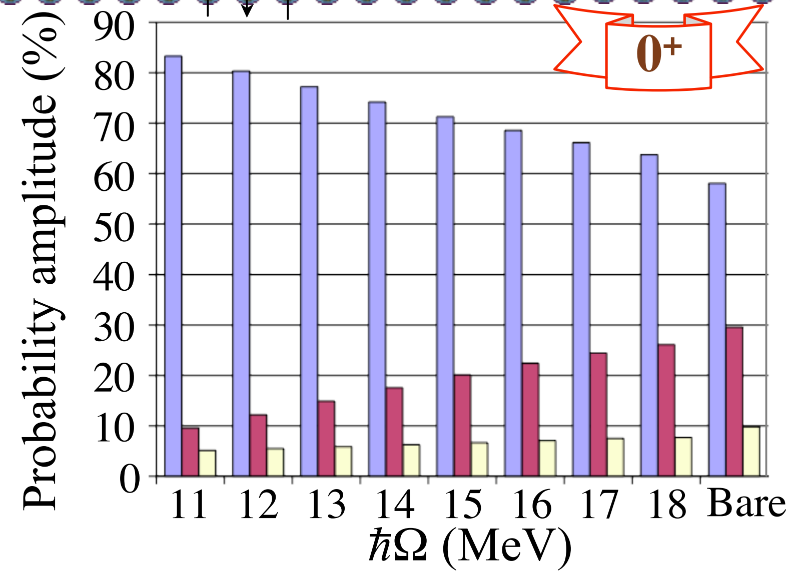
spherical

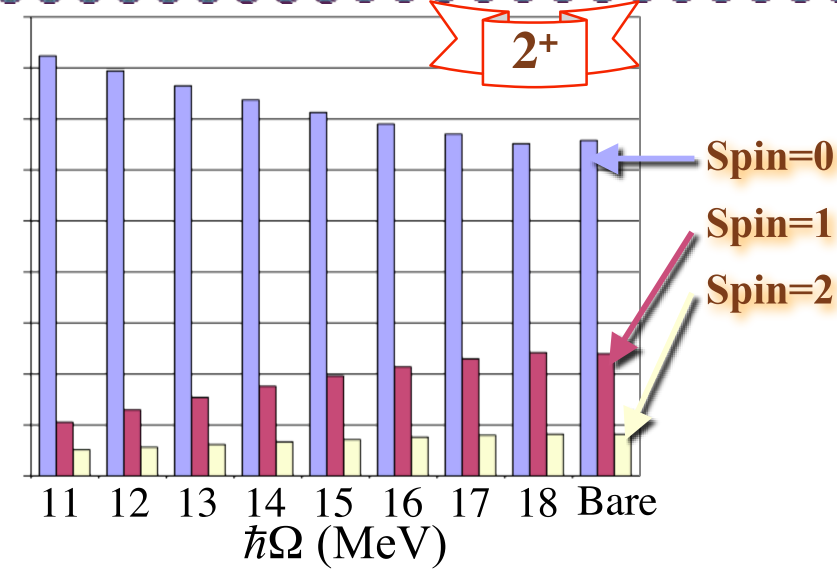
1 particle, 2 shells up

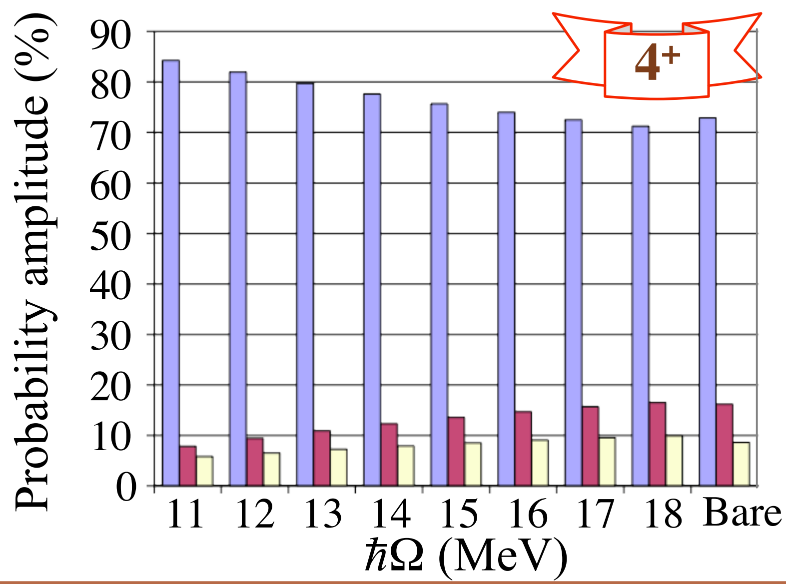
2 particles, a shell up

Spin Distribution in NCSM Eigenstates

^{12}C



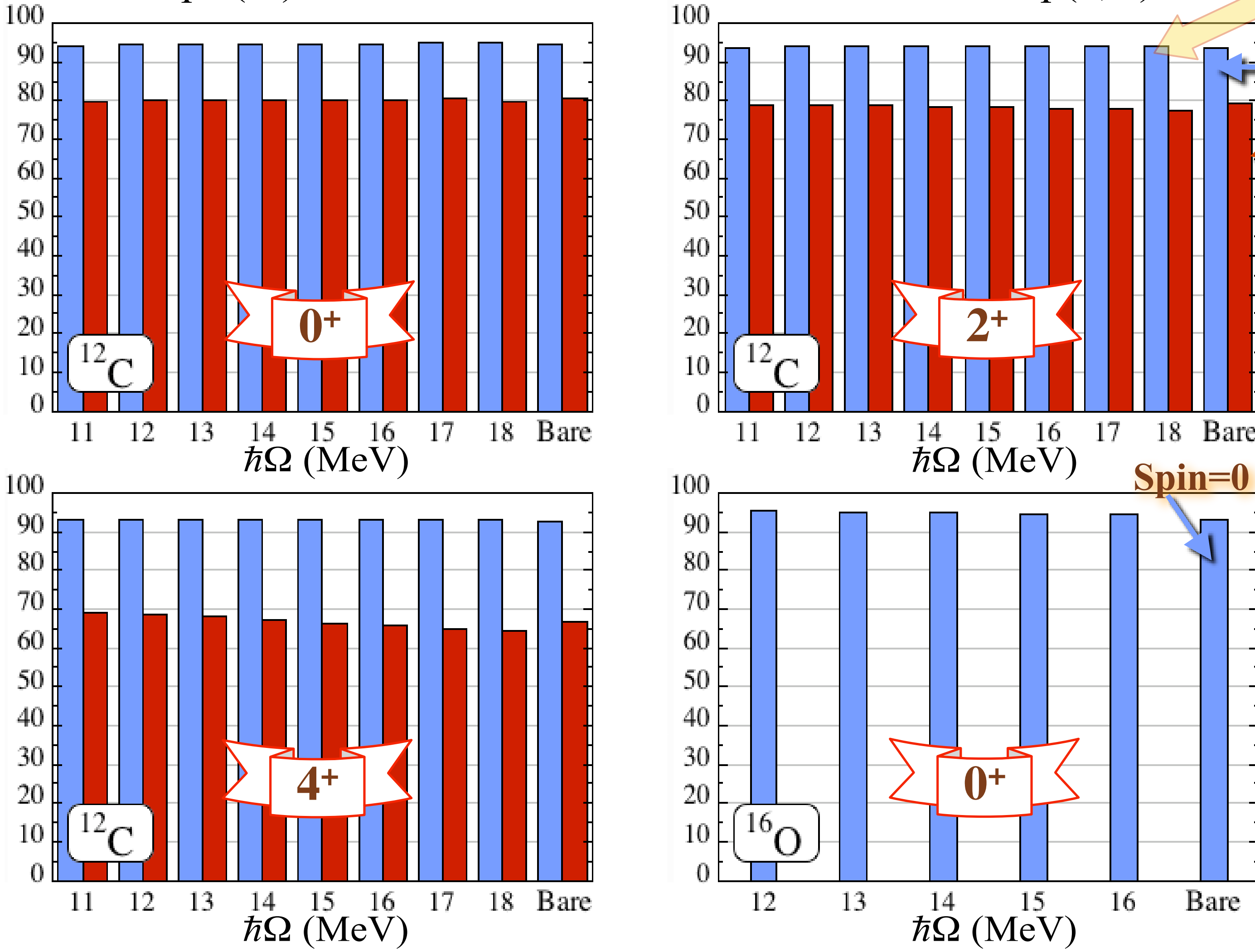




Independence of Oscillator Strength

Spin components of converged states

Overlaps (%) of NCSM wavefunctions with dominant Sp(3,ℝ) states



Only **6** Sp(3,ℝ) irreps (3 0p-0h and 3 2p-2h)

Spin=0

Spin=1

Symplectic structure **is not altered** by Lee-Suzuki transformation

Spatial wavefunctions: independent of whether bare or effective interaction is used

Origin of Symplectic Symmetry

Symplectic Sp(3,ℝ) Symmetry

Realistic interaction possesses
a Sp(3,ℝ) symmetry
+ the complementary
(spin-isospin) supermultiplet symmetry…

OR

…the nuclear many-body system acts as a filter:
propagates Sp(3,ℝ) symmetry in a coherent way;
reduces Sp(3,ℝ) symmetry-breaking effects.

Sp(3,ℝ) + Complementary (spin-isospin) symmetry

microscopic | collective

Elliott Model (single shell)

$x_i, p_i \Longleftrightarrow$

$L^{(11)}_{1,M}$	Angular Momentum	.... $\vec{x} \times \vec{p}$	SO(3), SU(3)
$Q^{(11)}_{2,M}$	Quadrupole Moment	.... $x_i x_j$	SU(3)
$N^{(00)}$	Number Operator	...... essentially HO Hamiltonian	
	[Hamiltonian $H_0 = (p^2+x^2)/2$]		
$A^{(20)}_{L,M}$	Multi-shell Coupling ...		
$B^{(02)}_{L,M}$	[Monopole, $L = 0$ & Quadrupole, $L = 2$]		

$$\mathrm{Sp}(3,\mathbb{R}) \otimes \mathrm{Spin}$$

$X \qquad S$

$$a\{X \times X\} + bL.S + cS^2$$

Symplectic Sp(3,ℝ) symmetry preserving Hamiltonian

(the most general)

Compare with ⇕

JISP16 realistic interaction

Matrix elements in fp

jr	js	jt	ju	J	T	Hsp4	GXPF1
p1/2	p1/2	p1/2	p1/2	1	0	-1.077001	-1.2431
p1/2	p1/2	p1/2	p1/2	0	1	-0.209086	-0.4469
p1/2	p1/2	p1/2	p3/2	1	0	0	-0.849
p1/2	p1/2	p3/2	p3/2	1	0	0	0.7675
p1/2	p1/2	p3/2	p3/2	0	1	-0.444876	-1.4928
p1/2	p1/2	p3/2	f5/2	1	0	0	0.8137
p1/2	p1/2	f5/2	f5/2	1	0	0	-0.3161
p1/2	p1/2	f5/2	f5/2	0	1	-0.54486	-0.8093
p1/2	p1/2	f5/2	f7/2	1	0	0	-0.1928
p1/2	p1/2	f7/2	f7/2	1	0	0	0.0271
p1/2	p1/2	f7/2	f7/2	0	1	-0.816667	-0.38
p1/2	p3/2	p1/2	p3/2	1	0	-1.077001	-2.5068
p1/2	p3/2	p1/2	p3/2	2	0	-1.077001	-2.3122
p1/2	p3/2	p1/2	p3/2	1	1	0.105489	-0.1594
p1/2	p3/2	p1/2	p3/2	2	1	0.105489	-0.2938
p1/2	p3/2	p1/2	f5/2	2	0	0	-0.69
p1/2	p3/2	p1/2	f5/2	2	1	0	0.249
p1/2	p3/2	p3/2	p3/2	1	0	0	-1.8059
p1/2	p3/2	p3/2	p3/2	2	1	0	0.634
p1/2	p3/2	p3/2	f5/2	1	0	0	0.993
p1/2	p3/2	p3/2	f5/2	2	0	0	-0.4885
p1/2	p3/2	p3/2	f5/2	1	1	0	-0.1076
p1/2	p3/2	p3/2	f5/2	2	1	0	0.4545
p1/2	p3/2	p3/2	f7/2	2	0	0	0.6228
p1/2	p3/2	p3/2	f7/2	2	1	0	0.4262
p1/2	p3/2	f5/2	f5/2	1	0	0	0.0337
p1/2	p3/2	f5/2	f5/2	2	1	0	-0.06
p1/2	p3/2	f5/2	f7/2	1	0	0	-1.4651
p1/2	p3/2	f5/2	f7/2	2	0	0	-0.7434
p1/2	p3/2	f5/2	f7/2	1	1	0	0.0552
p1/2	p3/2	f5/2	f7/2	2	1	0	-0.0153
p1/2	p3/2	f7/2	f7/2	1	0	0	-0.315
p1/2	p3/2	f7/2	f7/2	2	1	0	0.0367
p1/2	f5/2	p1/2	f5/2	2	0	-1.077001	-0.3174
p1/2	f5/2	p1/2	f5/2	3	0	-1.077001	-1.4023
p1/2	f5/2	p1/2	f5/2	2	1	0.105489	-0.1519
p1/2	f5/2	p1/2	f5/2	3	1	0.105489	0.2383
p1/2	f5/2	p1/2	f7/2	3	0	0	-0.4505
p1/2	f5/2	p1/2	f7/2	3	1	0	0.1586
p1/2	f5/2	p3/2	p3/2	3	0	0	0.115
p1/2	f5/2	p3/2	p3/2	2	1	0	-0.1923
p1/2	f5/2	p3/2	f5/2	2	0	0	-0.354
p1/2	f5/2	p3/2	f5/2	3	0	0	1.0151
p1/2	f5/2	p3/2	f5/2	2	1	0	-0.4043
p1/2	f5/2	p3/2	f5/2	3	1	0	-0.06
p1/2	f5/2	p3/2	f7/2	2	0	0	1.0933
p1/2	f5/2	p3/2	f7/2	3	0	0	0.7227
p1/2	f5/2	p3/2	f7/2	2	1	0	-0.803
p1/2	f5/2	p3/2	f7/2	3	1	0	-0.1814
p1/2	f5/2	f5/2	f5/2	3	0	0	-0.6276
p1/2	f5/2	f5/2	f5/2	2	1	0	-0.3208
p1/2	f5/2	f5/2	f7/2	2	0	0	-0.5447
p1/2	f5/2	f5/2	f7/2	3	0	0	-0.6262
p1/2	f5/2	f5/2	f7/2	2	1	0	0.1537
p1/2	f5/2	f5/2	f7/2	3	1	0	-0.1105
p1/2	f5/2	f7/2	f7/2	3	0	0	-0.1082
p1/2	f5/2	f7/2	f7/2	2	1	0	-0.1295
p1/2	f7/2	p1/2	f7/2	3	0	-1.638477	-1.6968
p1/2	f7/2	p1/2	f7/2	4	0	-1.638477	-1.0602
p1/2	f7/2	p1/2	f7/2	3	1	0.08819	0.4873
p1/2	f7/2	p1/2	f7/2	4	1	0.08819	-0.1347
p1/2	f7/2	p3/2	p3/2	3	0	0	-0.6411
p1/2	f7/2	p3/2	f5/2	3	0	0	0.0354
p1/2	f7/2	p3/2	f5/2	4	0	0	-1.3607
p1/2	f7/2	p3/2	f5/2	3	1	0	0.3891
p1/2	f7/2	p3/2	f5/2	4	1	0	0.6111
p1/2	f7/2	p3/2	f7/2	3	0	0	-1.685
p1/2	f7/2	p3/2	f7/2	4	0	0	-0.1706
p1/2	f7/2	p3/2	f7/2	3	1	0	0.1048
p1/2	f7/2	p3/2	f7/2	4	1	0	0.3351
p1/2	f7/2	f5/2	f5/2	3	0	0	0.2621
p1/2	f7/2	f5/2	f5/2	4	1	0	0.2248
p1/2	f7/2	f5/2	f7/2	3	0	0	-0.4252
p1/2	f7/2	f5/2	f7/2	4	0	0	-0.3789
p1/2	f7/2	f5/2	f7/2	3	1	0	0.3224
p1/2	f7/2	f5/2	f7/2	4	1	0	0.1907
p1/2	f7/2	f7/2	f7/2	3	0	0	-0.8883
p1/2	f7/2	f7/2	f7/2	4	1	0	0.2096
p3/2	p3/2	p3/2	p3/2	1	0	-1.077001	-0.6308
p3/2	p3/2	p3/2	p3/2	3	0	-1.077001	-2.289
p3/2	p3/2	p3/2	p3/2	0	1	-0.523662	-1.1165
p3/2	p3/2	p3/2	p3/2	2	1	0.105489	-0.0887
p3/2	p3/2	p3/2	f5/2	1	0	0	0.2373
p3/2	p3/2	p3/2	f5/2	3	0	0	0.2276
p3/2	p3/2	p3/2	f5/2	2	1	0	-0.4631
p3/2	p3/2	p3/2	f7/2	3	0	0	-0.4309
p3/2	p3/2	p3/2	f7/2	2	1	0	-0.3738
p3/2	p3/2	f5/2	f5/2	1	0	0	0.0483
p3/2	p3/2	f5/2	f5/2	3	0	0	-0.0546
p3/2	p3/2	f5/2	f5/2	0	1	-0.770548	-1.2457
p3/2	p3/2	f5/2	f5/2	2	1	0	0.0719
p3/2	p3/2	f5/2	f7/2	1	0	0	-0.8914
p3/2	p3/2	f5/2	f7/2	3	0	0	-0.6264
p3/2	p3/2	f5/2	f7/2	2	1	0	-0.0717
p3/2	p3/2	f7/2	f7/2	1	0	0	-0.4313
p3/2	p3/2	f7/2	f7/2	3	0	0	-0.3415
p3/2	p3/2	f7/2	f7/2	0	1	-1.154941	-0.7174
p3/2	p3/2	f7/2	f7/2	2	1	0	-0.2021
p3/2	f5/2	p3/2	f5/2	1	0	-1.077001	-2.7262
p3/2	f5/2	p3/2	f5/2	2	0	-1.077001	-1.511
p3/2	f5/2	p3/2	f5/2	3	0	-1.077001	-0.5859
p3/2	f5/2	p3/2	f5/2	4	0	-1.077001	-1.0882
p3/2	f5/2	p3/2	f5/2	1	1	0.105489	0.3284
p3/2	f5/2	p3/2	f5/2	2	1	0.105489	0.3608
p3/2	f5/2	p3/2	f5/2	3	1	0.105489	0.346
p3/2	f5/2	p3/2	f5/2	4	1	0.105489	-0.2584
p3/2	f5/2	p3/2	f7/2	2	0	0	1.2708
p3/2	f5/2	p3/2	f7/2	3	0	0	0.579
p3/2	f5/2	p3/2	f7/2	4	0	0	0.7103
p3/2	f5/2	p3/2	f7/2	2	1	0	-0.5436
p3/2	f5/2	p3/2	f7/2	3	1	0	-0.1836
p3/2	f5/2	p3/2	f7/2	4	1	0	-0.4546
p3/2	f5/2	f5/2	f5/2	1	0	0	0.477
p3/2	f5/2	f5/2	f5/2	3	0	0	0.32
p3/2	f5/2	f5/2	f5/2	2	1	0	-0.056
p3/2	f5/2	f5/2	f5/2	4	1	0	-0.3615
p3/2	f5/2	f5/2	f7/2	1	0	0	1.2721
p3/2	f5/2	f5/2	f7/2	2	0	0	-0.598
p3/2	f5/2	f5/2	f7/2	3	0	0	0.7716
p3/2	f5/2	f5/2	f7/2	4	0	0	-0.6408

… and more matrix elements

p3/2	f5/2	f5/2	f7/2	1	1	0	0.0521
p3/2	f5/2	f5/2	f7/2	2	1	0	0.4247
p3/2	f5/2	f5/2	f7/2	3	1	0	-0.0268
p3/2	f5/2	f5/2	f7/2	4	1	0	0.2699
p3/2	f5/2	f7/2	f7/2	1	0	0	-0.0907
p3/2	f5/2	f7/2	f7/2	3	0	0	0.0752
p3/2	f5/2	f7/2	f7/2	2	1	0	-0.1725
p3/2	f5/2	f7/2	f7/2	4	1	0	-0.2224
p3/2	f7/2	p3/2	f7/2	2	0	-1.638477	-0.5391
p3/2	f7/2	p3/2	f7/2	3	0	-1.638477	-1.0055
p3/2	f7/2	p3/2	f7/2	4	0	-1.638477	-0.3695
p3/2	f7/2	p3/2	f7/2	5	0	-1.638477	-2.967
p3/2	f7/2	p3/2	f7/2	2	1	0.08819	-0.6081
p3/2	f7/2	p3/2	f7/2	3	1	0.08819	0.1561
p3/2	f7/2	p3/2	f7/2	4	1	0.08819	-0.1398
p3/2	f7/2	p3/2	f7/2	5	1	0.08819	0.5918
p3/2	f7/2	f5/2	f5/2	3	0	0	0.166
p3/2	f7/2	f5/2	f5/2	5	0	0	0.0334
p3/2	f7/2	f5/2	f5/2	2	1	0	0.088
p3/2	f7/2	f5/2	f5/2	4	1	0	-0.2146
p3/2	f7/2	f5/2	f7/2	2	0	0	0.6381
p3/2	f7/2	f5/2	f7/2	3	0	0	-0.254
p3/2	f7/2	f5/2	f7/2	4	0	0	-0.1951
p3/2	f7/2	f5/2	f7/2	5	0	0	-0.6743
p3/2	f7/2	f5/2	f7/2	2	1	0	-0.0959
p3/2	f7/2	f5/2	f7/2	3	1	0	0.523
p3/2	f7/2	f5/2	f7/2	4	1	0	0.2486
p3/2	f7/2	f5/2	f7/2	5	1	0	0.481
p3/2	f7/2	f7/2	f7/2	3	0	0	-0.8807
p3/2	f7/2	f7/2	f7/2	5	0	0	-0.4265
p3/2	f7/2	f7/2	f7/2	2	1	0	-0.516
p3/2	f7/2	f7/2	f7/2	4	1	0	-0.2969
f5/2	f5/2	f5/2	f5/2	1	0	-1.077001	-0.8551
f5/2	f5/2	f5/2	f5/2	3	0	-1.077001	-0.5599
f5/2	f5/2	f5/2	f5/2	5	0	-1.077001	-2.2816
f5/2	f5/2	f5/2	f5/2	0	1	-0.838236	-1.2081
f5/2	f5/2	f5/2	f5/2	2	1	0.105489	-0.4621
f5/2	f5/2	f5/2	f5/2	4	1	0.105489	-0.1624
f5/2	f5/2	f5/2	f7/2	1	0	0	0.2735
f5/2	f5/2	f5/2	f7/2	3	0	0	-0.6378

f5/2	f5/2	f5/2	f7/2	5	0	0	-1.1302
f5/2	f5/2	f5/2	f7/2	2	1	0	0.5022
f5/2	f5/2	f5/2	f7/2	4	1	0	0.2709
f5/2	f5/2	f7/2	f7/2	1	0	0	0.6511
f5/2	f5/2	f7/2	f7/2	3	0	0	0.4358
f5/2	f5/2	f7/2	f7/2	5	0	0	0.1239
f5/2	f5/2	f7/2	f7/2	0	1	-1.414508	-1.3832
f5/2	f5/2	f7/2	f7/2	2	1	0	-0.2038
f5/2	f5/2	f7/2	f7/2	4	1	0	-0.0331
f5/2	f7/2	f5/2	f7/2	1	0	-1.638477	-4.5802
f5/2	f7/2	f5/2	f7/2	2	0	-1.638477	-3.252
f5/2	f7/2	f5/2	f7/2	3	0	-1.638477	-1.4019
f5/2	f7/2	f5/2	f7/2	4	0	-1.638477	-2.2583
f5/2	f7/2	f5/2	f7/2	5	0	-1.638477	-0.6084
f5/2	f7/2	f5/2	f7/2	6	0	-1.638477	-3.0351
f5/2	f7/2	f5/2	f7/2	1	1	0.08819	-0.0889
f5/2	f7/2	f5/2	f7/2	2	1	0.08819	-0.175
f5/2	f7/2	f5/2	f7/2	3	1	0.08819	0.6302
f5/2	f7/2	f5/2	f7/2	4	1	0.08819	0.4763
f5/2	f7/2	f5/2	f7/2	5	1	0.08819	0.7433
f5/2	f7/2	f5/2	f7/2	6	1	0.08819	-0.9916
f5/2	f7/2	f7/2	f7/2	1	0	0	-1.8998
f5/2	f7/2	f7/2	f7/2	3	0	0	-1.0917
f5/2	f7/2	f7/2	f7/2	5	0	0	-1.2853
f5/2	f7/2	f7/2	f7/2	2	1	0	-0.2167
f5/2	f7/2	f7/2	f7/2	4	1	0	0.4999
f5/2	f7/2	f7/2	f7/2	6	1	0	0.5643
f7/2	f7/2	f7/2	f7/2	1	0	-2.078472	-1.2838
f7/2	f7/2	f7/2	f7/2	3	0	-2.078472	-0.8418
f7/2	f7/2	f7/2	f7/2	5	0	-2.078472	-0.7839
f7/2	f7/2	f7/2	f7/2	7	0	-2.078472	-2.6661
f7/2	f7/2	f7/2	f7/2	0	1	-1.845204	-2.4385
f7/2	f7/2	f7/2	f7/2	2	1	0.062016	-0.9352
f7/2	f7/2	f7/2	f7/2	4	1	0.062016	-0.1296
f7/2	f7/2	f7/2	f7/2	6	1	0.062016	0.2783

Spectral Distribution Theory: Correlation coefficients

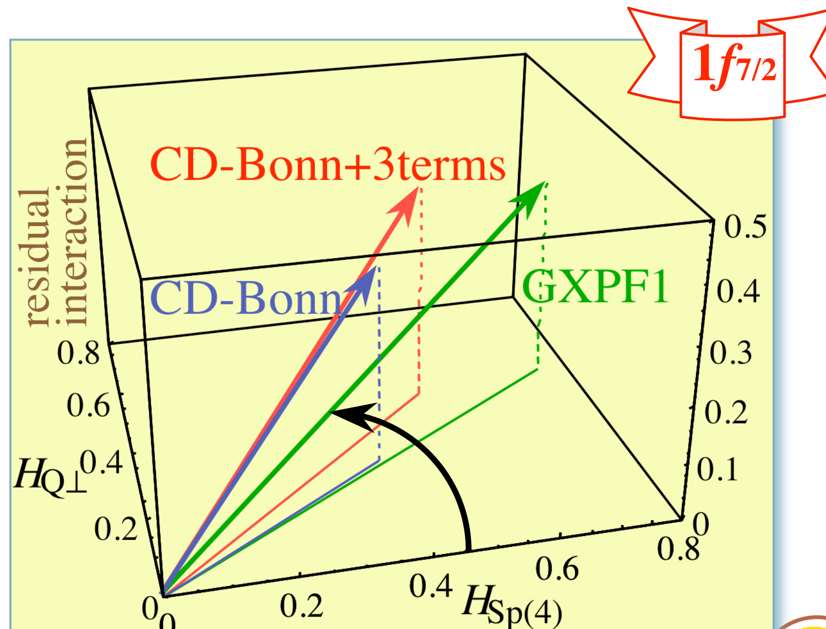
$$\zeta^{\alpha}_{H,H'} = \frac{\langle (H^\dagger - \langle H^\dagger\rangle^\alpha)(H' - \langle H'\rangle^\alpha)\rangle^\alpha}{\sigma_H \sigma_{H'}} = \frac{\langle H^\dagger H'\rangle^\alpha - \langle H^\dagger\rangle^\alpha \langle H'\rangle^\alpha}{\sigma_H \sigma_{H'}},$$

$$(\sigma^\alpha_H)^2 = \langle (H - \langle H\rangle^\alpha)^2\rangle^\alpha = \langle H^2\rangle^\alpha - (\langle H\rangle^\alpha)^2$$

Excellent method for comparing *any* two microscopic interactions

$$\cos\theta = \frac{\vec{v}\cdot\vec{v}'}{|\vec{v}||\vec{v}'|}$$

- Comparison of global properties
- Revealing underlying symmetries/ symmetry breaking patterns in realistic interactions
- Large correlation coefficients yield similar energy spectra

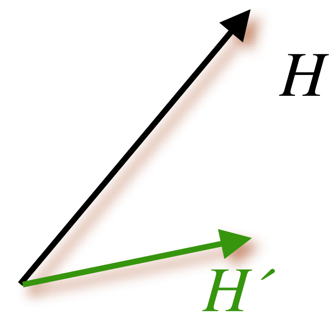


Correlations in Many-nucleon Systems

$$\zeta^{\alpha}_{H,H'} = \frac{\langle (H^\dagger - \langle H^\dagger \rangle^\alpha)(H' - \langle H' \rangle^\alpha) \rangle^\alpha}{\sigma_H \sigma_{H'}} = \frac{\langle H^\dagger H' \rangle^\alpha - \langle H^\dagger \rangle^\alpha \langle H' \rangle^\alpha}{\sigma_H \sigma_{H'}},$$

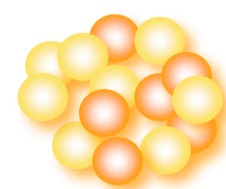
Nucleon-nucleon interaction

Correlation coefficients for 2 nucleons: **H.H´**



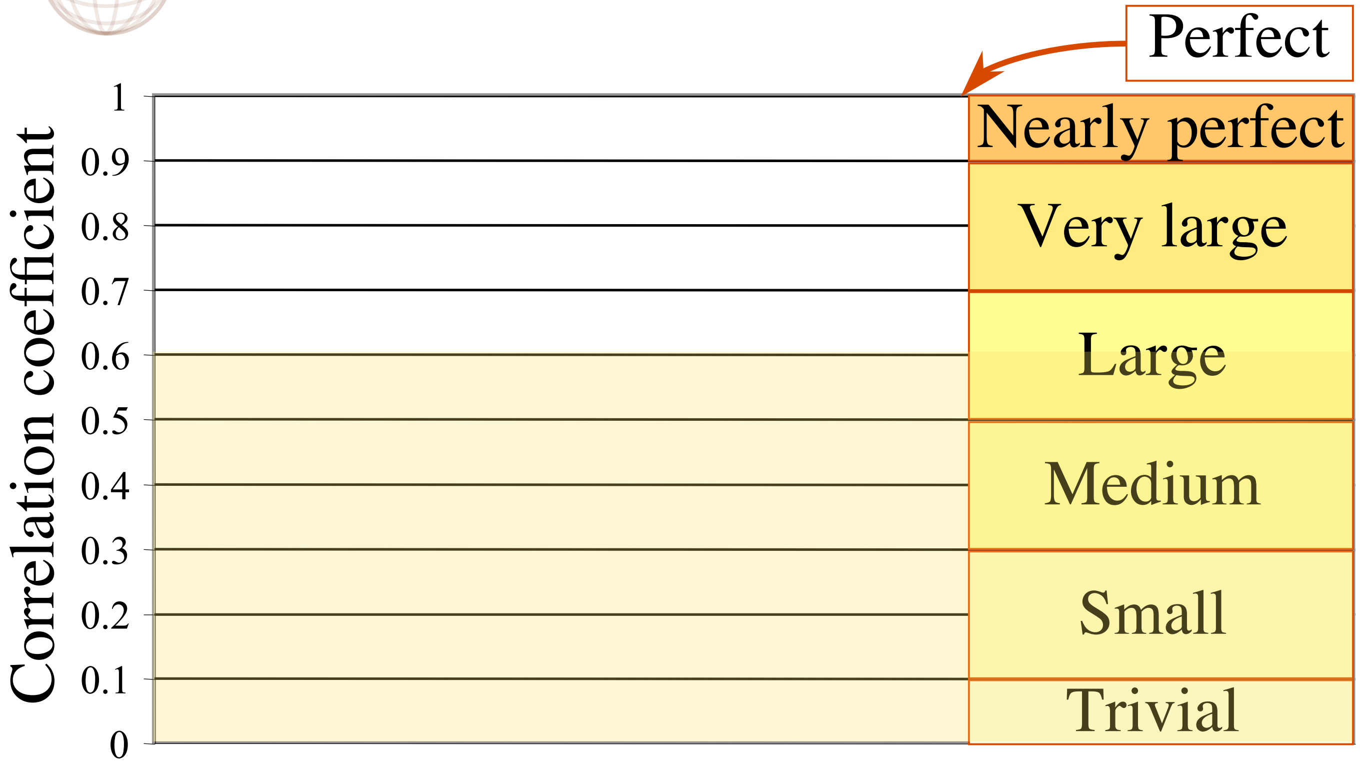
Information for 2-nucleon system

Model space dimension
Number of particles
(Isospin)

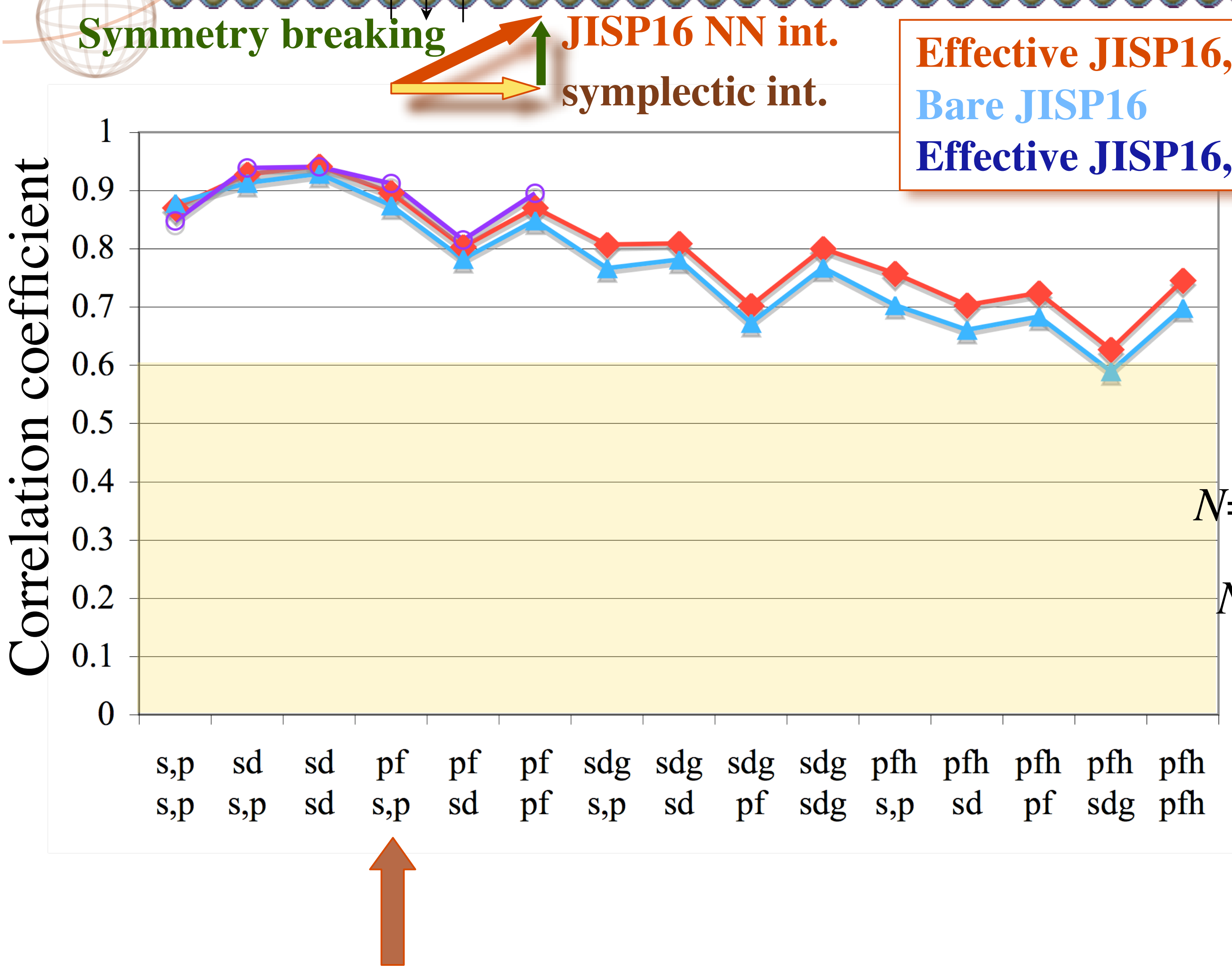


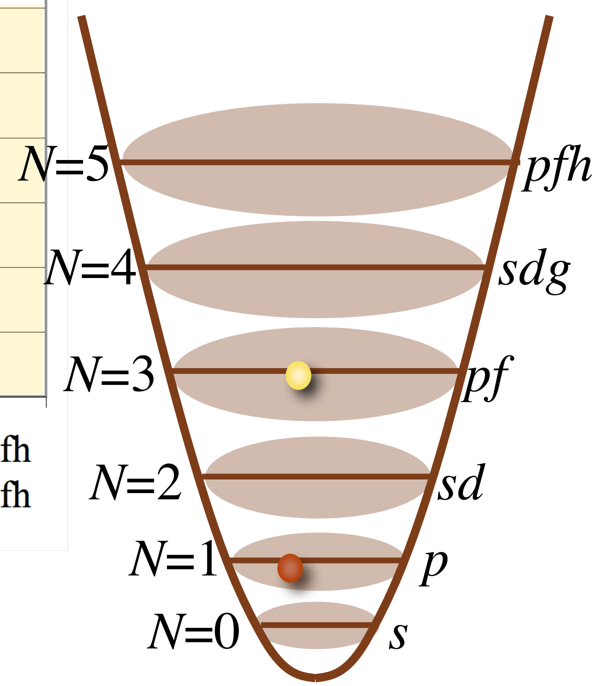
Correlation coefficients for *n* nucleons

Correlation Coefficients

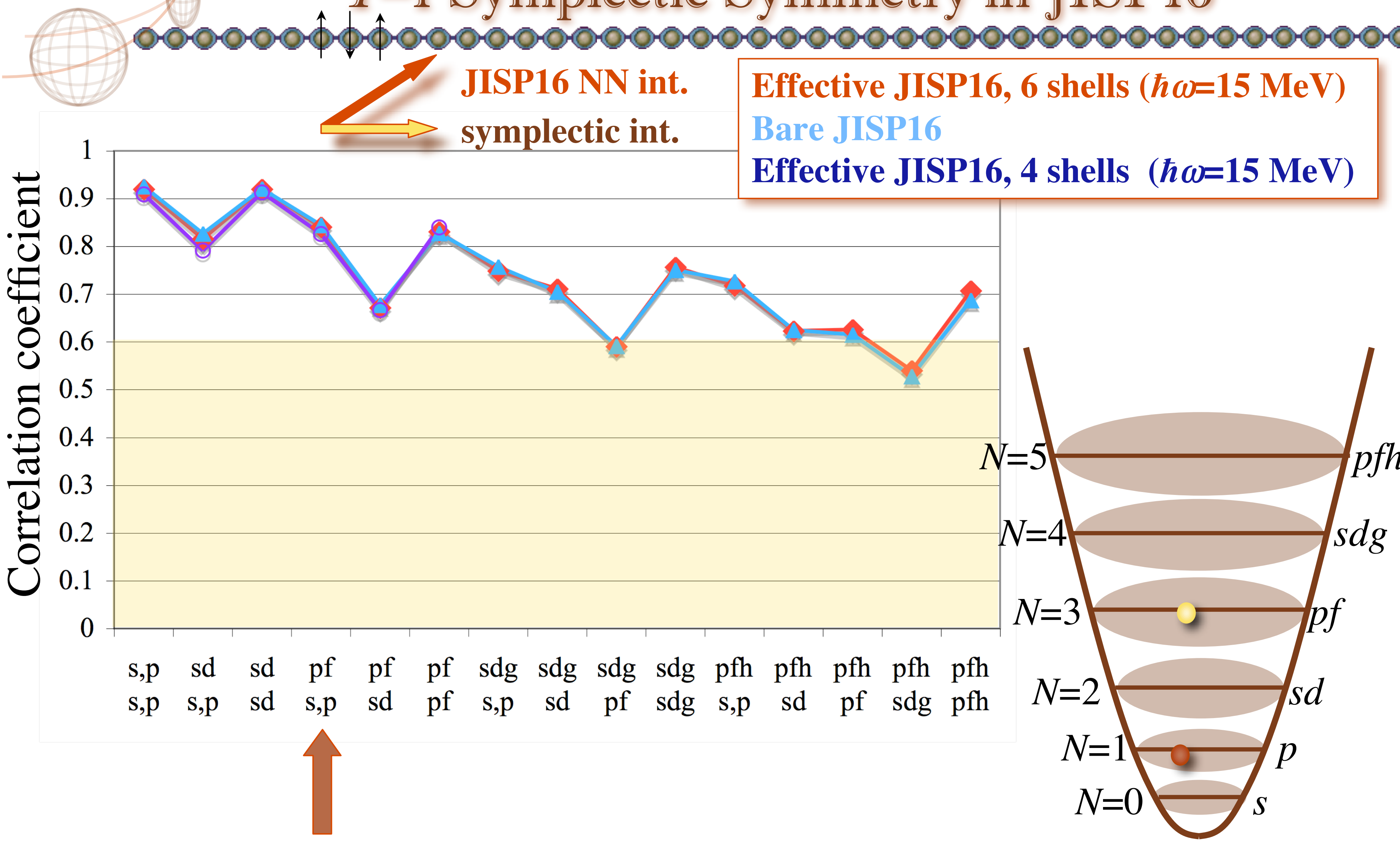


T=0 Symplectic Symmetry in JISP16





T=1 Symplectic Symmetry in JISP16



Summary

- *Ab-initio* No Core Shell Model: successfully reproduces (low-lying) features of the deuteron, alpha particle, ^{12}C and even ^{16}O
- Comparison of converged NCSM eigenstates with Sp(3,$\mathbb{R}$)-symmetric states shows:
  - Reproduction of NCSM results by a few Sp(3,$\mathbb{R}$) states–
    - ✓ 85%-90% overlap
    - ✓ 100% $B(E2: 2_1^+ \rightarrow 0_1^+)$
  - Dramatic reduction in model space (several orders of magnitude)
- Symplectic-NCSM: effective model space reduction scheme
- Sp(3,$\mathbb{R}$) symmetry found dominant in *ab initio* realistic solutions
- Symplectic-NCSM… simply matching "**geometry**" to "**dynamics**"