The No-Core Shell Model and Shell Model Effective Interactions

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INT Workshop

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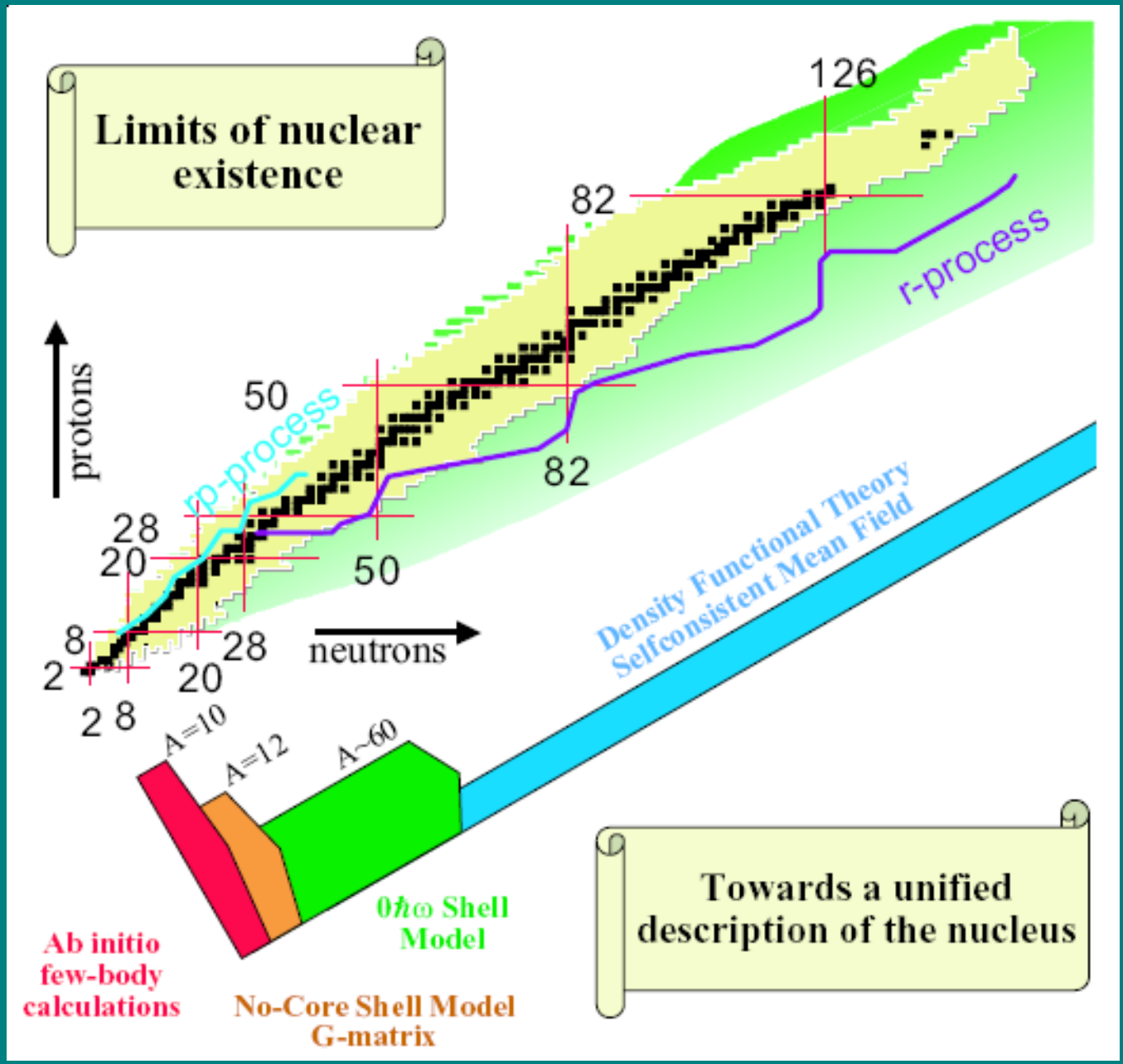
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Density Functional Theory
Selfconsistent Mean Field
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A=12
A~60
0ħω Shell Model
Ab initio few-body calculations
No-Core Shell Model
G-matrix
Towards a unified description of the nucleus

$$H\Psi = E\Psi$$

We cannot, in general, solve the full problem in the complete Hilbert space, so we must truncate to a finite model space

$\Longrightarrow$ We must use effective interactions and operators!

Some current shell-model references

1. B. A. Brown, "The Nuclear Shell Model towards the Drip Lines," *Progress in Particle and Nuclear Physics* **47**, 517 (2001)
2. E. Caurier, G. Martinez-Pinedo, F. Nowacki, A. Poves, and A. P. Zuker, "The Shell Model as a Unified View of Nuclear Structure," *Reviews of Modern Physics* **77**, 427 (2005)
3. I. Talmi, "Fifty Years of the Shell Model-The Quest for the Effective Interaction," *Advances in Nuclear Physics*, Vol. **27**, ed. J. W. Negele and E. Vogt (Plenum, NY, 2003)
4. B. R. B., "Effective Operators in Shell-Model Calculations," 10th Indian Summer School of Nuclear Physics:Theory of Many-Fermion Systems, *Czechoslovak Journal of Physics* **49**, 1 (1999)

No Core Shell Model

"*Ab Initio*" approach to microscopic nuclear structure calculations, in which all A nucleons are treated as being active.

Want to solve the A-body Schrödinger equation

$$H_A \Psi^A = E_A \Psi^A$$

Ref: P. Navrátil, J.P. Vary, B.R.B., PRC 62, 054311 (2000)

No-Core Shell-Model Approach

- Start with the purely intrinsic Hamiltonian

$$H_A = T_{rel} + \mathcal{V} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j=1}^{A} V_{NN}\left(+\sum_{i<j<k}^{A} V_{ijk}^{3b}\right)$$

Note: There are no phenomenological s.p. energies!

Can use any NN potentials

Coordinate space: Argonne V8', A V18
Nijmegen I, II

Momentum space: CD Bonn, EFT Idaho

No-Core Shell-Model Approach

- Next, add CM harmonic-oscillator Hamiltonian

$$H_{CM}^{HO} = \frac{\vec{P}^{\,2}}{2Am} + \frac{1}{2}Am\Omega^2\vec{R}^{\,2}; \quad \vec{R} = \frac{1}{A}\sum_{i=1}^{A}\vec{r}_i\,, \quad \vec{P} = Am\dot{\vec{R}}$$

To H_A, yielding

$$H_A^{\Omega} = \sum_{i=1}^{A}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i^{\,2}\right] + \underbrace{\sum_{i<j=1}^{A}\left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right]}_{V_{ij}}$$

Defines a basis (*i.e.* HO) for evaluating V_{ij}

Effective Interaction

- Must truncate to a finite model space $V_{ij} \longrightarrow V_{ij}^{effective}$
- In general, V_{ij}^{eff} is an A-body interaction
- We want to make an a-body cluster approximation

$$\mathcal{H} = \mathcal{H}^{(I)} + \mathcal{H}^{(A)} \quad \underset{a < A}{\gtrapprox} \quad \mathcal{H}^{(I)} + \mathcal{H}^{(a)}$$

Two-body cluster approximation (a=2)

$$\mathcal{H} \approx \mathcal{H}^{(1)} + \mathcal{H}^{(2)}$$

$$H_2^\Omega = \underbrace{H_{0_2} + H_2^{CM}}_{h_1+h_2} + V_{12} = \frac{\vec{p}^{\,2}}{2m} + \frac{1}{2} m\Omega^2 \vec{r}^{\,2} + H_2^{CM} + V(\sqrt{2}\vec{r}) - \frac{m\Omega^2}{A}\vec{r}^{\,2}$$

Carry out a unitary transformation on H_2^Ω

$$\mathcal{H}_2 = e^{-S^{(2)}} H_2^\Omega e^{S^{(2)}} \quad \text{where } S^{(2)} \text{ is anti Hermitian}$$

$S^{(2)}$ is determined from the decoupling condition

$$Q_2 e^{-S^{(2)}} H_2^\Omega e^{S^{(2)}} P_2 = 0$$

P_2 = model space, Q_2 = excluded space, $P_2 + Q_2 = 1$

with the restrictions $P_2 S^{(2)} P_2 = Q_2 S^{(2)} Q_2 = 0$

Two-body cluster approximation (a=2)

It is convenient to rewrite S(2) in terms of a new operator

$$S^{(2)} = \operatorname{arctanh}(\omega - \omega^\dagger) \quad \text{with} \quad Q_2 \omega P_2 = \omega$$

Then the Hermitian effective operator in the P_2 space can be expressed in the form

$$\mathcal{H}^{(2)}_{eff} = P_2 \mathcal{H}_2 P_2 = \frac{P_2 + P_2 \omega^\dagger Q_2}{\sqrt{P_2 + \omega^\dagger \omega}} H_2^\Omega \frac{P_2 + Q_2 \omega P_2}{\sqrt{P_2 + \omega^\dagger \omega}}$$

Analogously, any arbitrary operator can be written in the P_2 space

$$\mathcal{O}^{(2)}_{eff} = P_2 \mathcal{O}_2 P_2 = \frac{P_2 + P_2 \omega^\dagger Q_2}{\sqrt{P_2 + \omega^\dagger \omega}} O \frac{P_2 + Q_2 \omega P_2}{\sqrt{P_2 + \omega^\dagger \omega}}$$

Exact solution for ω:

Let E_k and $|k\rangle$ be the eigensolutions

$$H_2^{\Omega}\,|k\rangle = E_k\,|k\rangle$$

Let $|\alpha_P\rangle$ & $|\alpha_Q\rangle$ be HO states belonging to the model space P and the excluded space Q, respectively. Then ω is given by:

$$\langle\alpha_Q|k\rangle = \sum_{\alpha_P}\langle\alpha_Q|\omega|\alpha_P\rangle\langle\alpha_P|k\rangle$$

or

$$\langle\alpha_Q|\omega|\alpha_P\rangle = \sum_{k\in K}\langle\alpha_Q|k\rangle\langle\tilde{k}|\alpha_P\rangle$$

NCSM ROAD MAP

1. Choose a NN interaction (or NN + NNN interactions)

2. Solve $H^{\Omega}_{n}|k_n\rangle = E_n|k_n\rangle$ for E_n and $|k_n\rangle$ with n=2,3,...

3. Calculate $\langle \alpha^{n}_{Q} | \omega | \alpha^{n}_{P} \rangle = \sum_{k \in K} \langle \alpha_Q | k_n \rangle \langle \tilde{k}_n | \alpha_P \rangle$

4. Determine $\mathcal{H}^{\text{eff}}_{n}$ and $\mathcal{O}^{\text{eff}}_{n}$ in the given model space

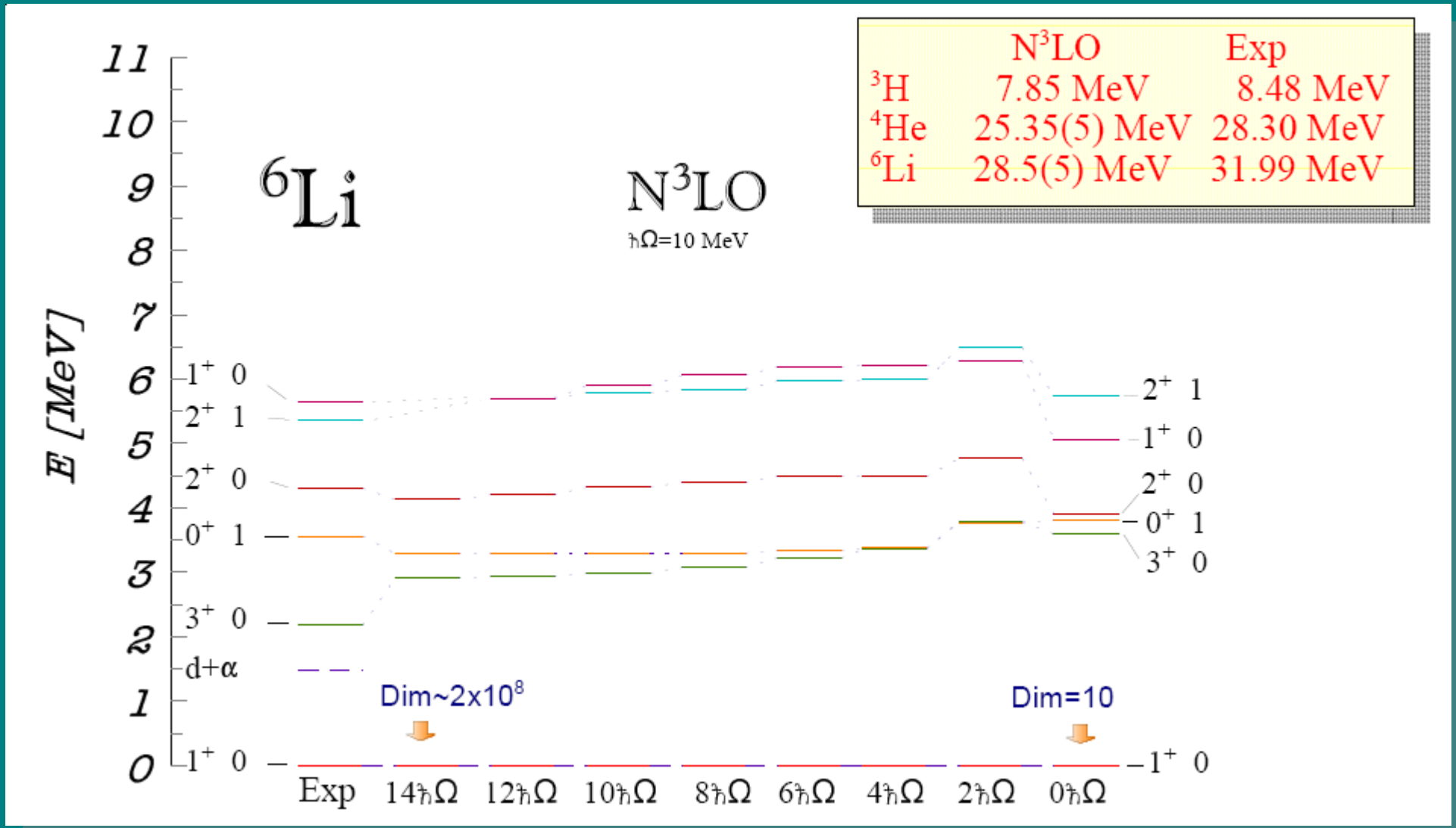
5. Diagonalize $\mathcal{H}^{\text{eff}}_{n}$ in the given model space, *i.e.*,

   $N_{max}\,\hbar\Omega$ = energy above the ground state

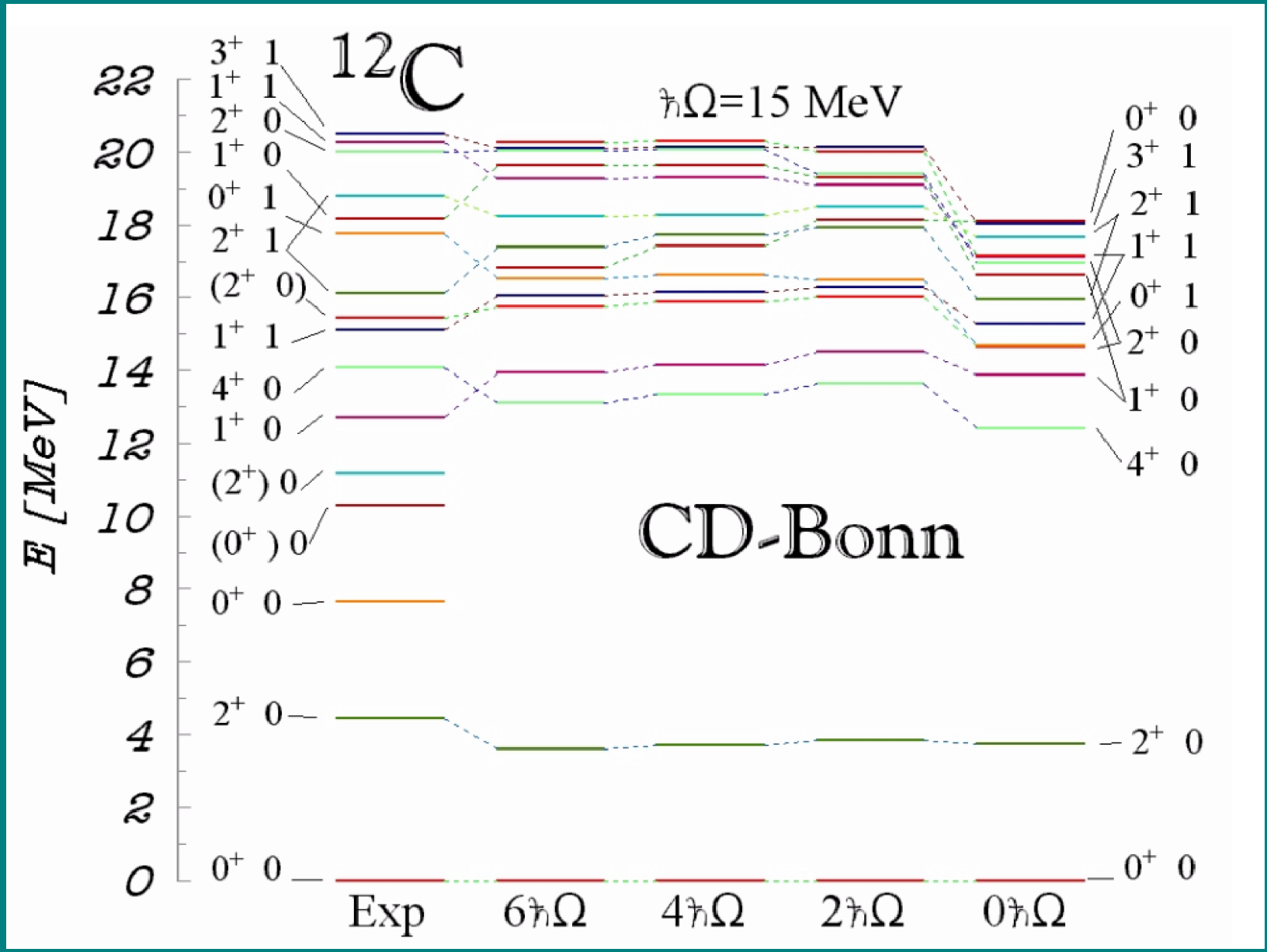
6. To check convergence of results repeat calculations

   for: *i)* increasing N_{max} and/or cluster level

   ii) several values of $\hbar\Omega$



P. Navrátil and E. Caurier, Phys. Rev. C **69**, 014311 (2004)



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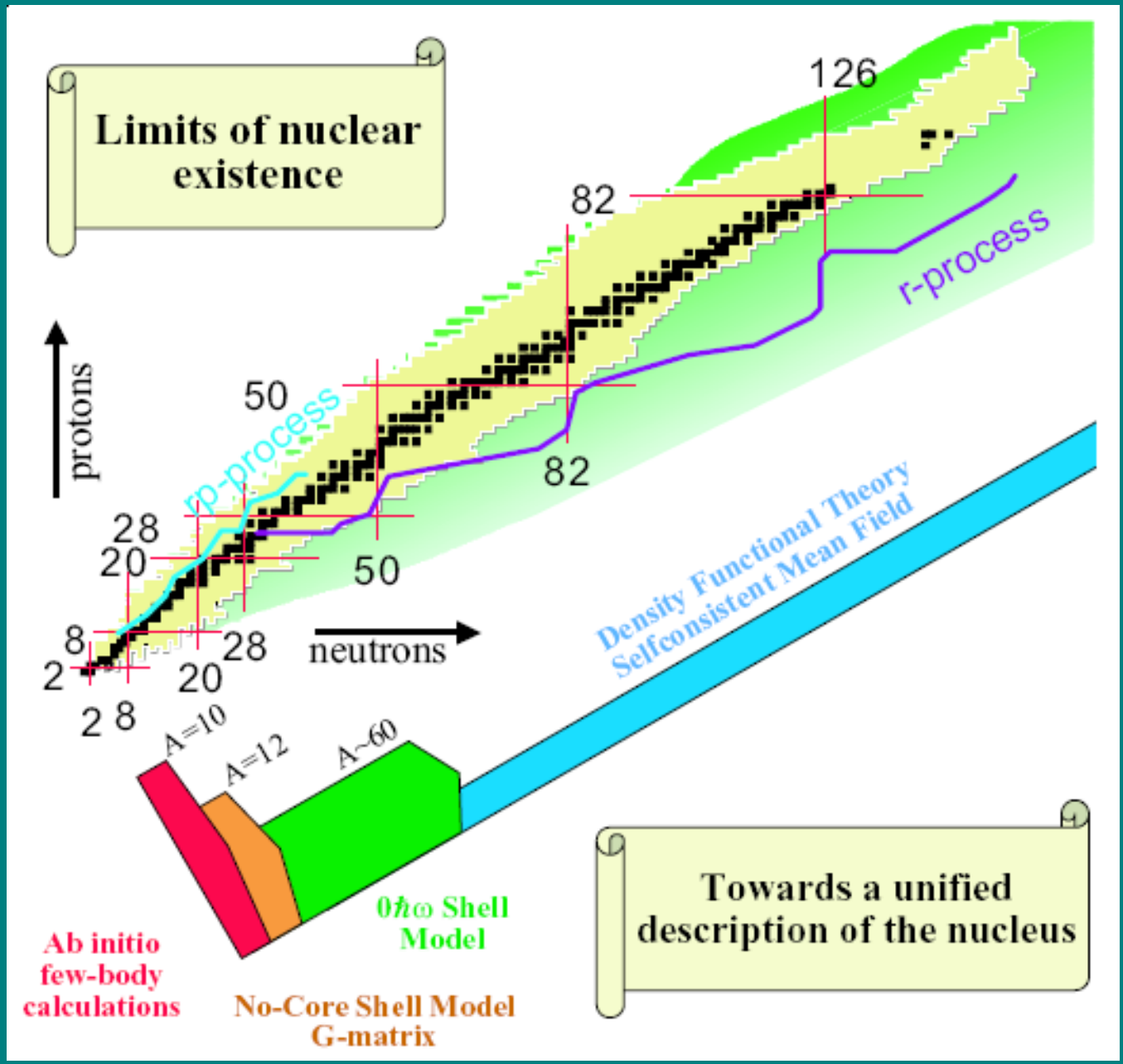
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