Post-Modern Nuclear Structure Theory: First Steps

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Supported in part by US DOE

Outline

- Introduction
- Pionless Effective Field Theory
- Free-Particle Basis: $A \leq 4$
- No-Core-Shell-Model Basis
- Conclusion

Major unsolved problem:

derivation of nuclear structure within the Standard Model

Why bother?

Nuclei

- the simplest complex systems

non-relativistic
shallow
...

non-perturbative
many-body
...

- laboratories

neutron targets: nucleon properties
incubators for rare processes: beyond the SM

Deconstructing Nuclear Structure

Pre-modern theory

- use mean-field and other simplifying but uncontrolled approximations
- forget about connection with few-nucleon systems

Modern theory

- use NN potentials that fit data “perfectly all the way”
  ... and 3N potentials not nearly as good
- beat the hell out of your computer hoping for exact calculations
- give up and make uncontrolled truncations of “effective interactions”

Post-modern theory

- don’t work on few-nucleon systems harder than you need
- beat the hell out of your computer to reduce error in many-body calculation to level of error in few-body interactions

Crucial issue

As A grows, given computational power limits number of accessible one-nucleon states

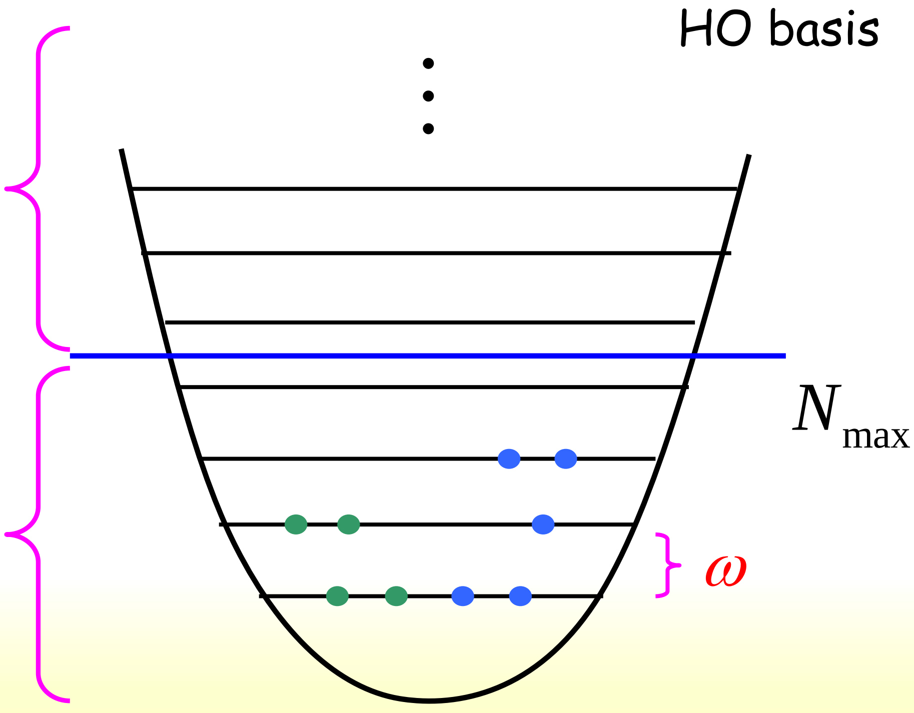
A-body problem: shell model

"excluded space"

$$Q = 1 - P$$

"model space"

$$P = \sum_{n,l}^{2n+l \leq N_{max}} |nl\rangle\langle nl|$$



What are the "effective interactions" in the model space?

Barrett, Vary + Zhang '93

The traditional no-core shell model approach

...

start with god-given (can be non-local!) potential, and run the RG in an HO basis

$$O_a \to PO_a^{\text{eff}}P = PO_aP + PHQ\frac{1}{E-QH_2Q}QO_aP + \ldots$$

Feshbach projection

$$= O_a' + O_{a+1}' + \ldots + O_{a+b}' + \ldots + O_A'$$

arbitrary truncation ("cluster approximation")

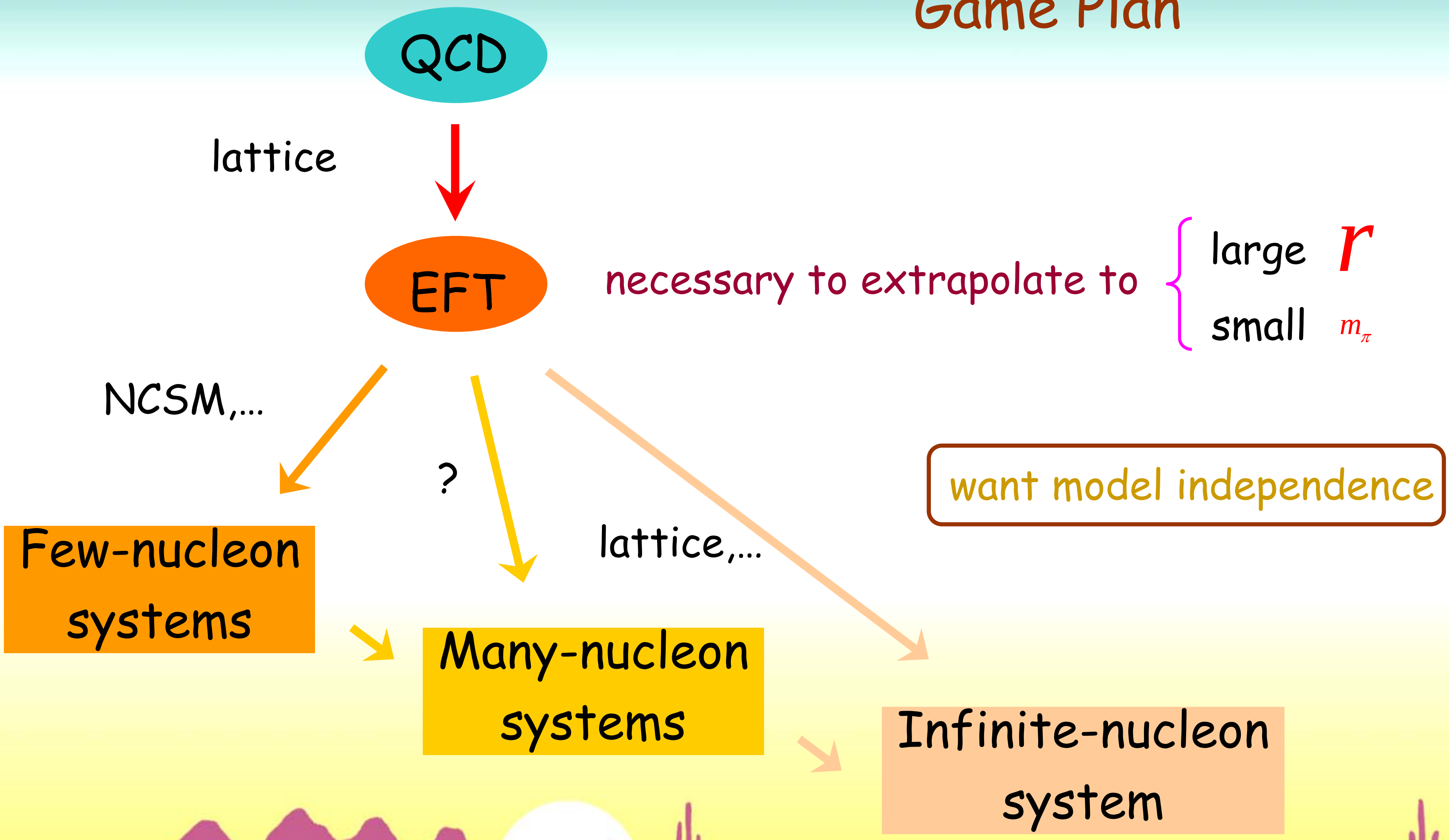
convergence:

$$\begin{cases} a+b \to A \text{ for fixed } P \\ P \to 1 \text{ for fixed } a+b \end{cases}$$

issues: systematic truncation error, consistent currents, *etc.*

EFT addresses just these issues!

Game Plan


QCD
lattice
EFT
necessary to extrapolate to
large r
small m_π
NCSM,...
?
lattice,...
want model independence
Few-nucleon systems
Many-nucleon systems
Infinite-nucleon system

Nuclear physics scales

"His scales are His pride", Book of Job

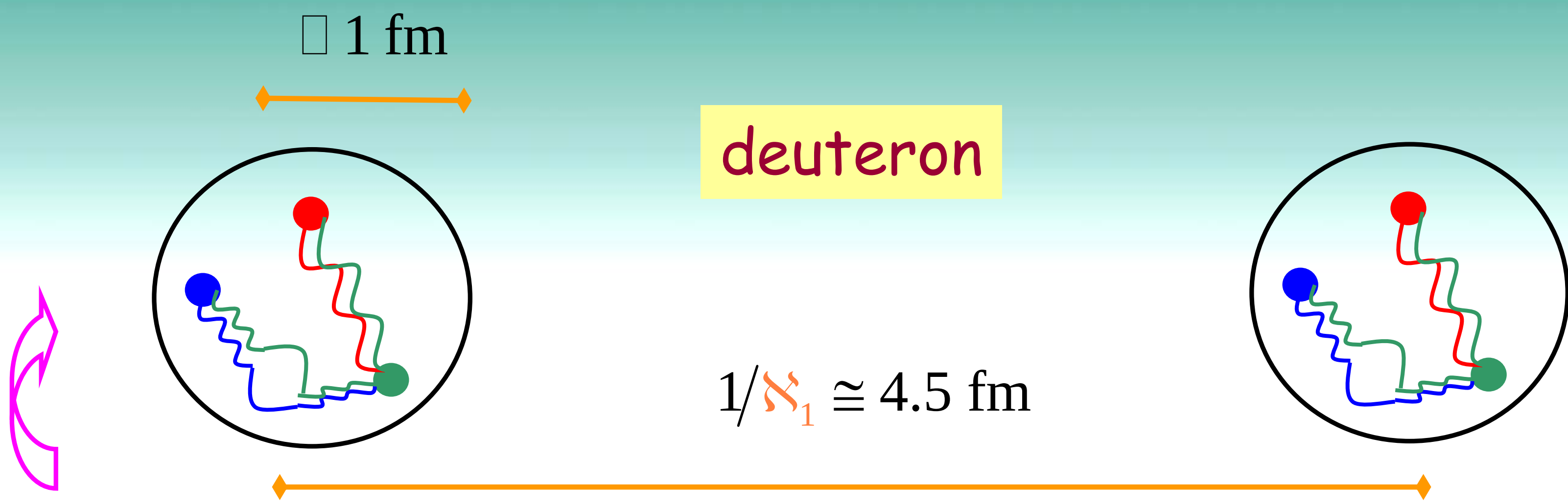
(according to J. Friar)

$\ln Q$

~1 GeV $M_{QCD} \sim m_N,\, m_\rho,\, 4\pi f_\pi,\, \ldots$

~100 MeV $M_{nuc} \sim f_\pi,\, 1/r_{NN},\, m_\pi,\, \ldots$

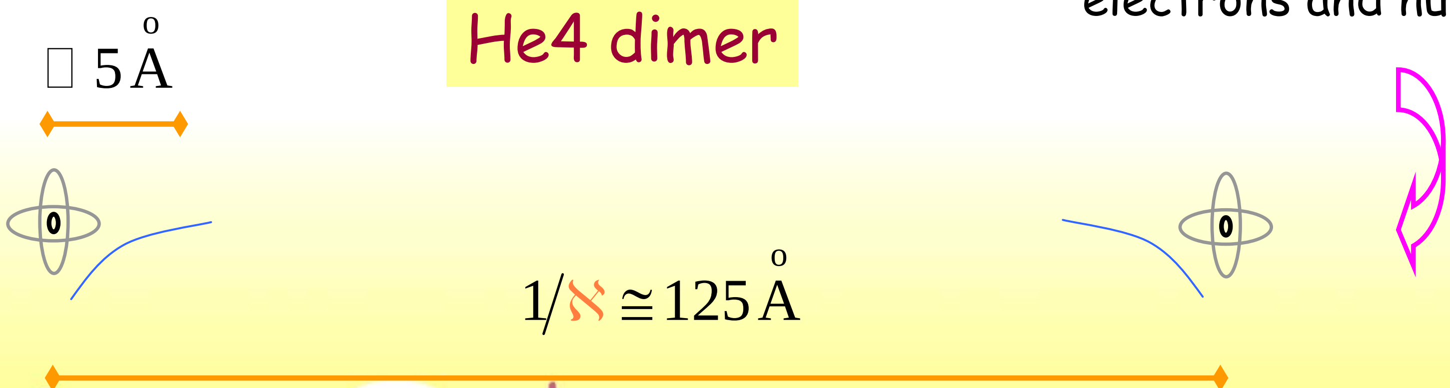
~30 MeV $\aleph \sim 1/a_{NN}$



QCD: $SU(3)$
gauge theory
of quarks

QED: $U(1)$
gauge theory of
electrons and nuclei

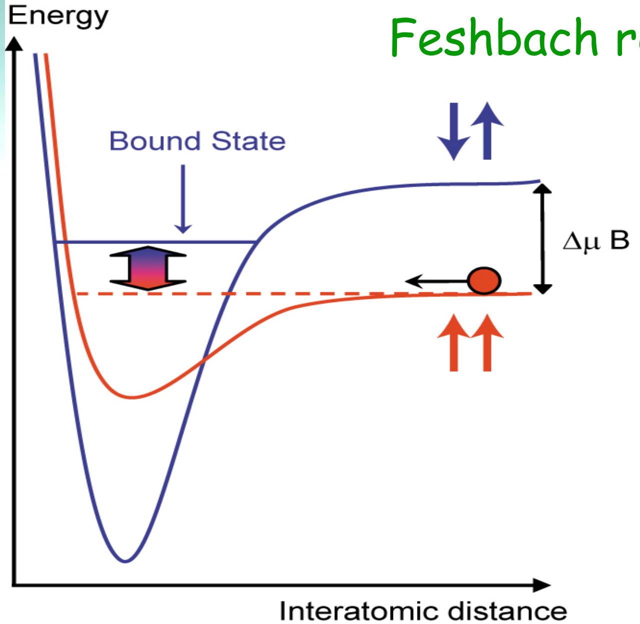
c.f.



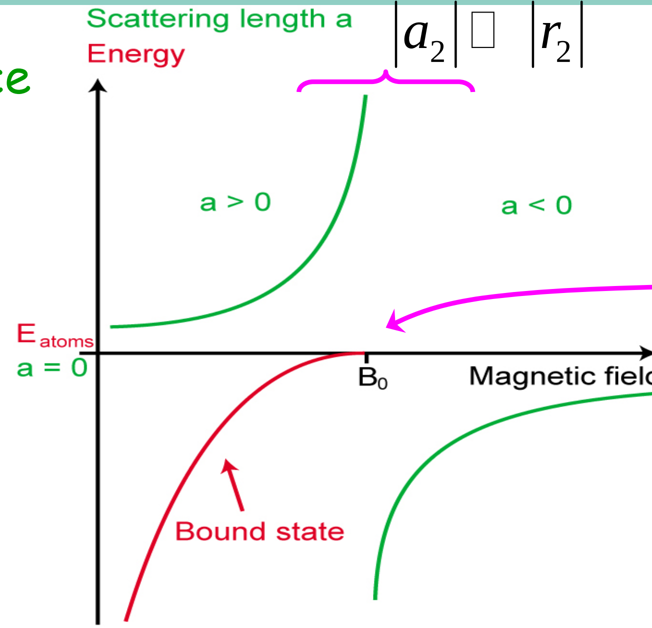
Trapped fermions

MIT group webpage

Feshbach resonance



$|a_2|$ □ $|r_2|$



unitarity limit

$|a_2| \to \infty$

optical trapping

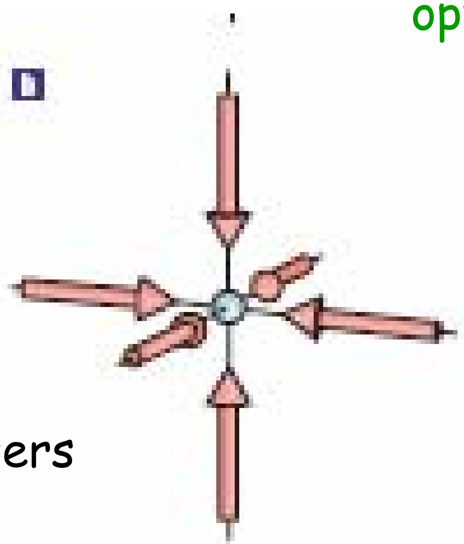
$$V(\vec{r}) \propto \alpha(\omega_L)\left|\vec{E}(\vec{r})\right|^2$$

$$\propto \sum_i \sin^2(k_L r_i)$$

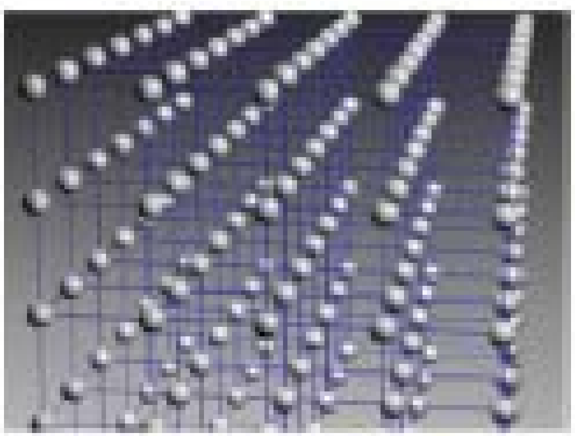
orthogonal standing waves

$$\approx k_L^2 \vec{r}^{\,2}$$

I. Bloch



lasers



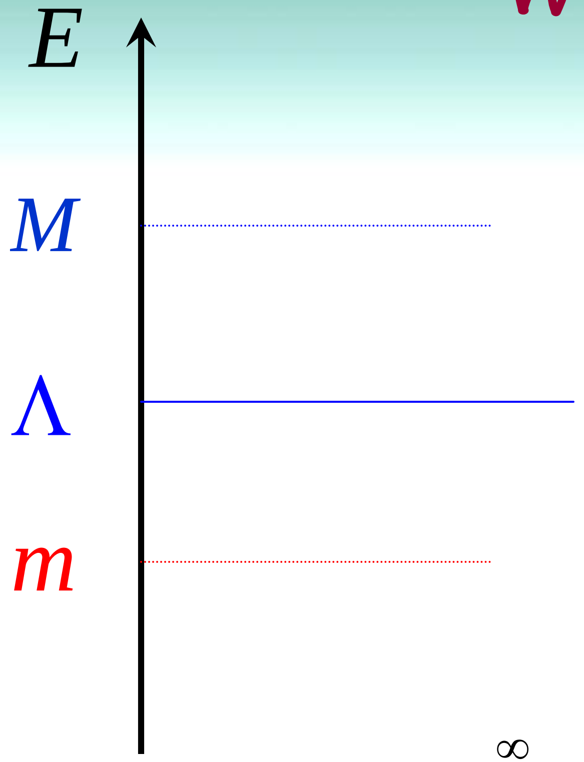
low-tunneling regime

(band insulator)

test of our method

What is Effective?

Weinberg, Wilson, etc.



$$Z = \int \mathcal{D}\phi_H \int \mathcal{D}\phi_L \exp\left(i \int d^4x\, \mathcal{L}_{und}(\phi_H, \phi_L) \right)$$

$$\times \int \mathcal{D}\varphi\, \delta\left(\varphi - f_\Lambda(\phi_L)\right)$$

$$= \int \mathcal{D}\varphi \exp\left(i \int d^4x\, \mathcal{L}_{EFT}(\varphi) \right)$$

$$\mathcal{L}_{EFT} = \sum_{d=0}^{\infty} \sum_{i(d,n)} c_i(M, \Lambda)\, O_i\left((\partial, m)^d \varphi^n \right)$$

renormalization-group invariance

$$\frac{\partial Z}{\partial \Lambda} = 0$$

local

underlying symmetries

$$\begin{cases} T = T^{(\infty)}(Q) \sim N(M) \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_i \tilde{c}_{\nu,i}(\Lambda)\left[\frac{Q}{M}\right]^{\nu} F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right) \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

normalization

non-analytic, from loops

$$\nu = \nu(d, n, \ldots)$$ "power counting"

↳ e.g. # loops L

For $Q \sim m$, truncate consistently with RG invariance

so as to allow systematic improvement (perturbation theory):

$$T = T^{(\nu_{\max})} + O\left(N\left(\frac{Q}{M}\right)^{\nu_{\max}+1}\right) \qquad \Lambda\frac{\partial T^{(\nu_{\max})}}{\partial \Lambda} = O\left(N\left(\frac{Q^{\nu_{\max}+1}}{M^{\nu_{\max}}\Lambda}\right)\right)$$

Nuclear physics scales

"His scales are His pride", Book of Job

(according to J. Friar)

$\ln Q$

perturbative QCD

~1 GeV — $M_{QCD} \sim m_N,\ m_\rho,\ 4\pi f_\pi,\ \ldots$ — brute force (lattice), ...?

hadronic th with chiral symm

Chiral EFT

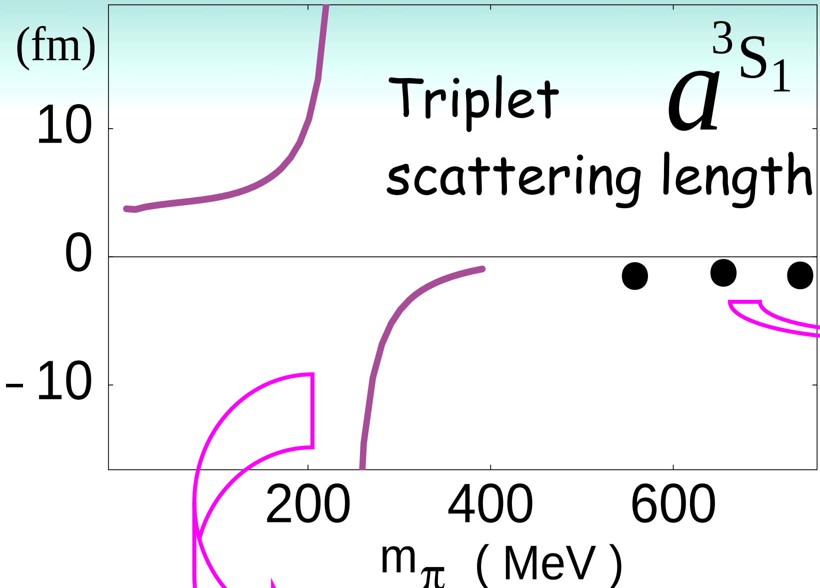
~100 MeV — $M_{nuc} \sim f_\pi,\ 1/r_{NN},\ m_\pi,\ \ldots$ — Q/M_{QCD}

~30 MeV — $\aleph \sim 1/a_{NN}$ — Q/M_{nuc}

this talk

no small coupling → expansion in ↑

Pion-mass dependence (from pionful EFT)



Lattice QCD: quenched

Fukugita et al. '95

cf. Beane, Bedaque, Orginos + Savage '06

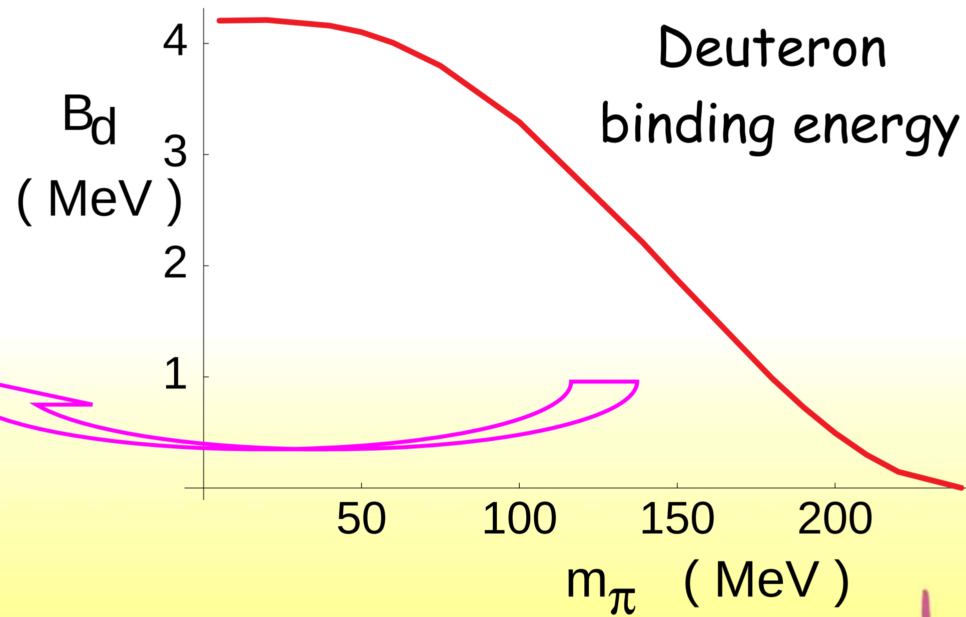
EFT: (incomplete) NLO

Beane, Bedaque, Savage + v.K. '02

c.f. Beane + Savage. '03

...

Epelbaum, Nogga, Hammer + Meissner '06



Large deuteron size consequence of pion mass being close to ~200 MeV

$Q \sim \aleph \ll M_{nuc}$

pionless EFT

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{P}$, $\not{T}$
- expansion in:

$$\frac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \cdots & \text{multipole} \end{cases}$$

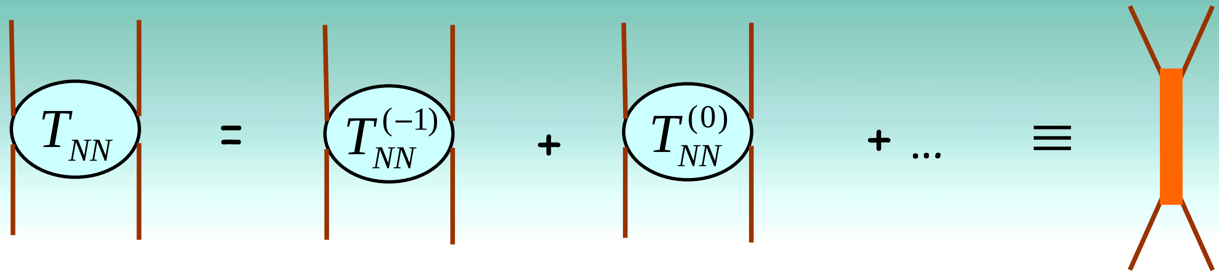
$$\mathcal{L}_{EFT} = N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + C_0\, N^+N\, N^+N$$

$$+ N^+\frac{\nabla^4}{8m_N^3}N + C_2\, N^+N\, N^+\nabla^2 N$$

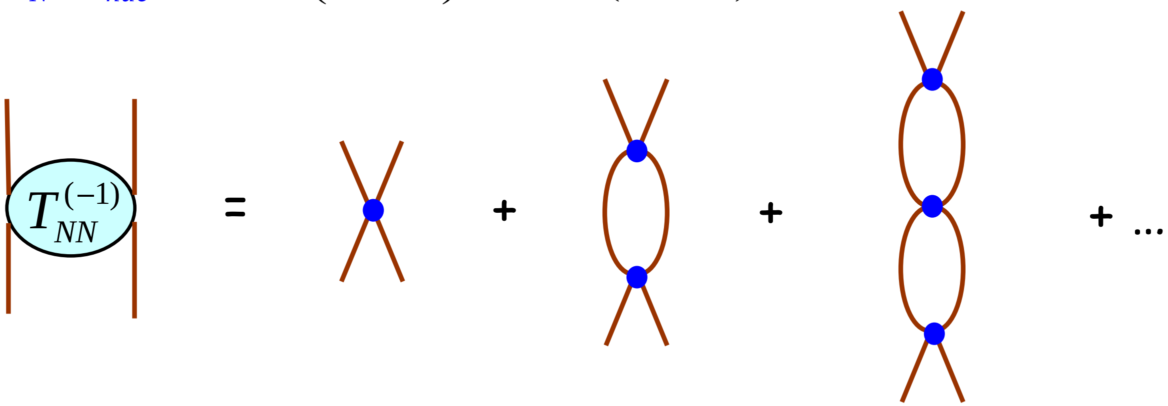
$$+ C_2'\, N^+\overleftarrow{\nabla}N \cdot N^+\overrightarrow{\nabla}N + \ldots$$

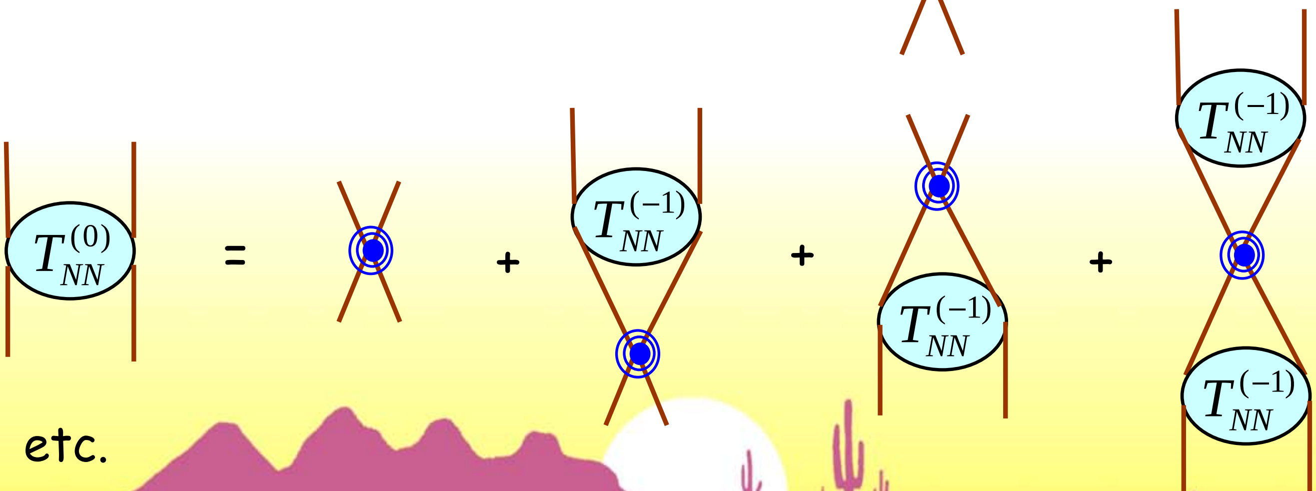
(omitting spin, isospin)

v.K. '97 '99
Kaplan, Savage + Wise '98
Gegelia '98
...



$$□ \frac{4\pi}{m_N M_{nuc}} \times \quad □ \left(\frac{ℵ}{M_{nuc}}\right)^{-1} \quad □ \left(\frac{ℵ}{M_{nuc}}\right)^{0}$$





etc.

$$C_0 = \frac{4\pi}{m_N ℵ}$$

⇨ $a_0 \sim 1/ℵ$

scattering length

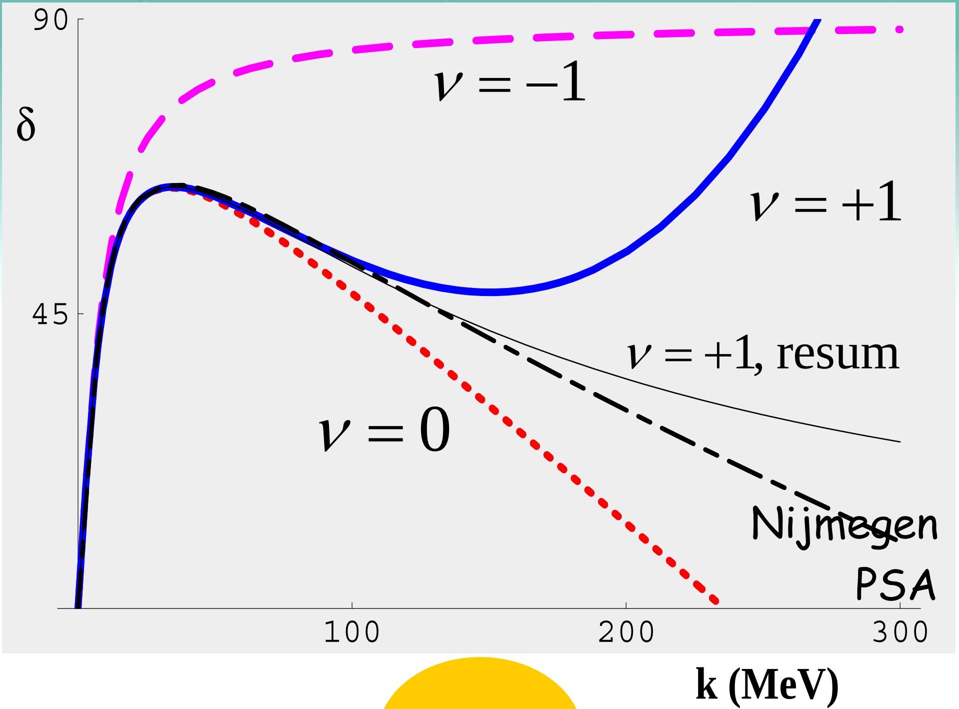
& b.s. at

$Q \sim i ℵ$

$$C_2 □ \frac{4\pi}{m_N ℵ^2 M_{nuc}}$$

⇨ $r_0 \sim 1/M_{nuc}$

effective range



Chen, Rupak + Savage '99

fitted

$C_0^{(1)} \Rightarrow a_1 = 5.42\ \text{fm (exp)}$

$C_2^{(1)} \Rightarrow r_1 = 1.75\ \text{fm (exp)}$

predicted

$B_d = 1.91\ \text{MeV}\ (\nu = 0)$

$B_d = 2.22\ \text{MeV (exp)}$

S_0

S_1

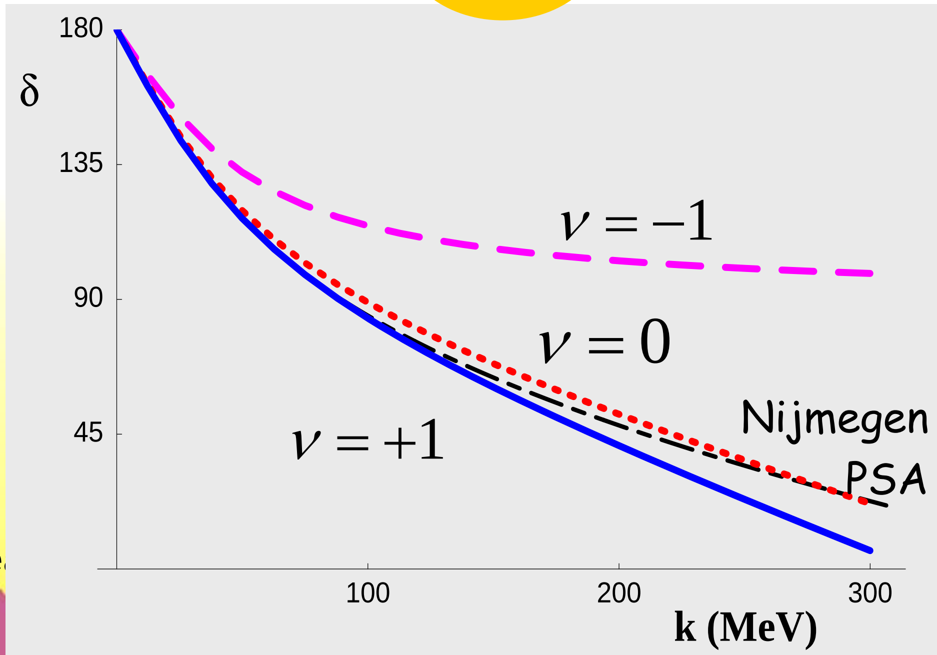
fitted

$C_0^{(0)} \Rightarrow a_0 = -20.0\ \text{fm (exp)}$

$C_2^{(0)} \Rightarrow r_0 = 2.78\ \text{fm (exp)}$

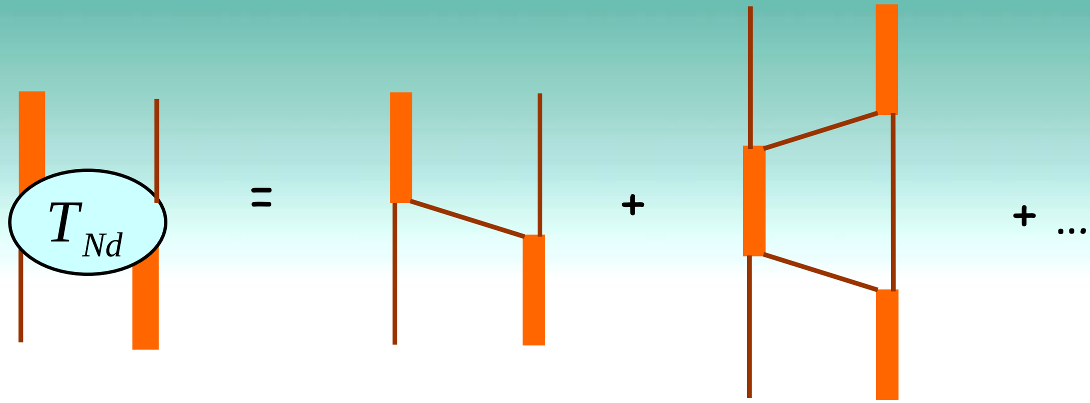
predicted

$B_{d^*} = 0.09\ \text{MeV}\ (\nu = 0)$

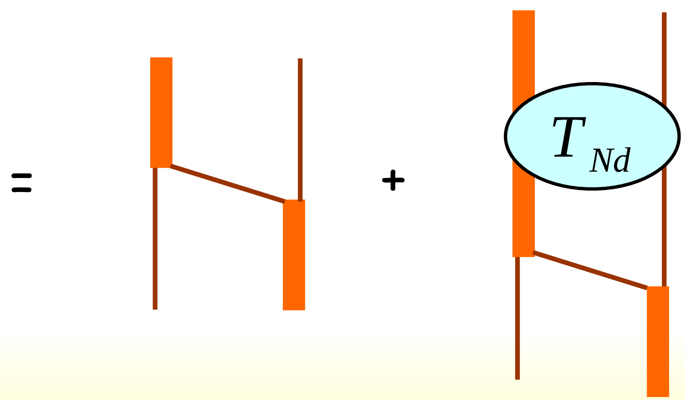


v. Kolck, Nucle

Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...
cf. Skornyakov + Ter-Martirosian '57
...

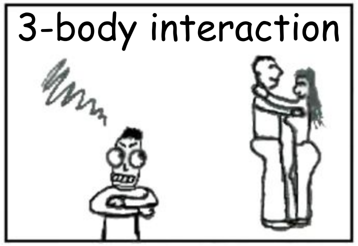


$$\square \left(\frac{4\pi}{m_N \aleph}\right)^2 \times \quad \sim \frac{1}{Q^2/m_N} \qquad \sim \frac{Q^3}{4\pi}\left(\frac{1}{Q^2/m_N}\right)^2 \frac{4\pi}{m_N \aleph} \quad \sim \frac{1}{Q^2/m_N}\,\frac{Q}{\aleph}$$



$$T_{Nd} = K_{ONE} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} \frac{K_{ONE}\, T_{Nd}}{D}$$

$$\mathcal{L}_{EFT} = \ldots + D_0\, N^+N\, N^+N\, N^+N + \ldots$$



Wikipedia

naïve dimensional analysis $D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{nuc}^4} \quad (\nu = +1)$

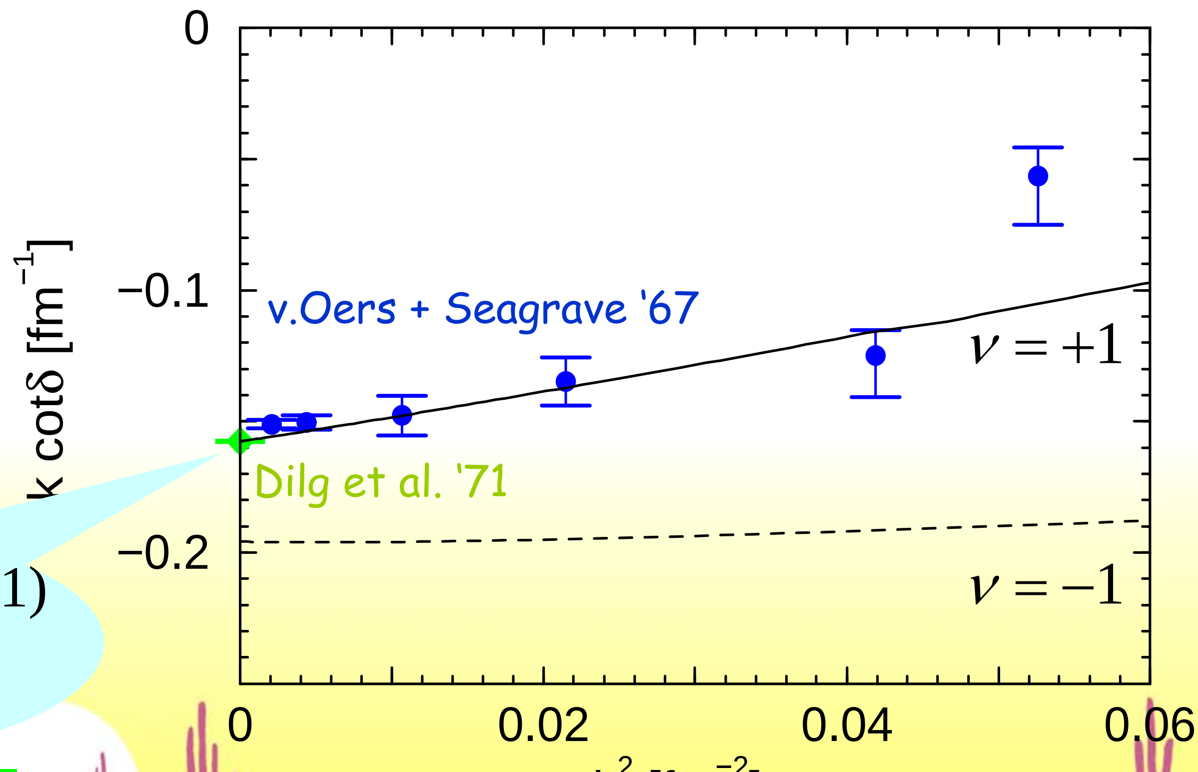
Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...

$S_{3/2}$ no three-body force up to $\nu = +3$

$$\lambda \le \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \ll \aleph} \frac{1}{p^2}$$

$$\Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} 0$$



predicted

$a_{3/2} = 6.33 \pm 0.10$ fm $(\nu = +1)$

$a_{3/2} = 6.35 \pm 0.02$ fm (exp)

QED-like precision!

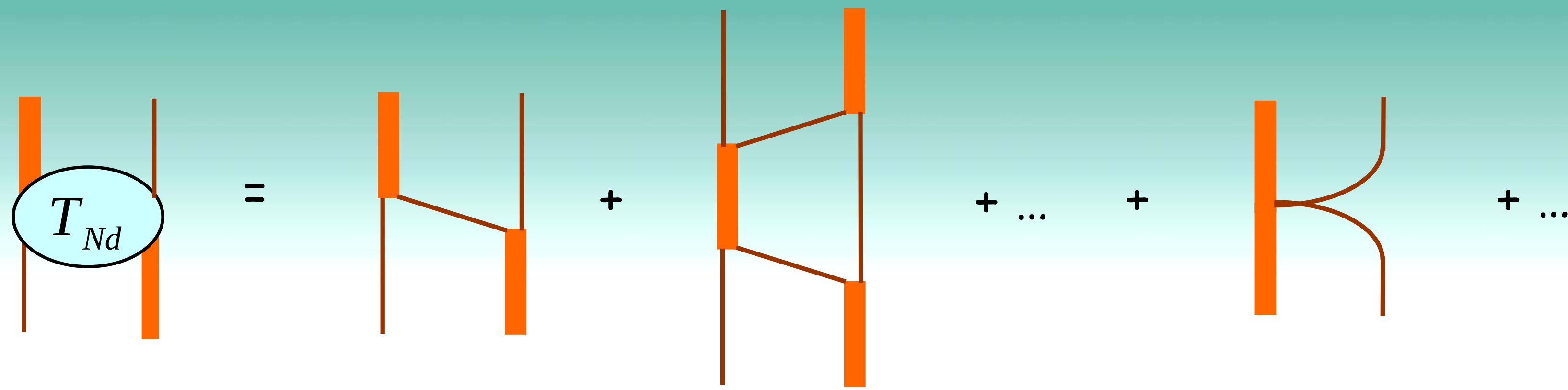
$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

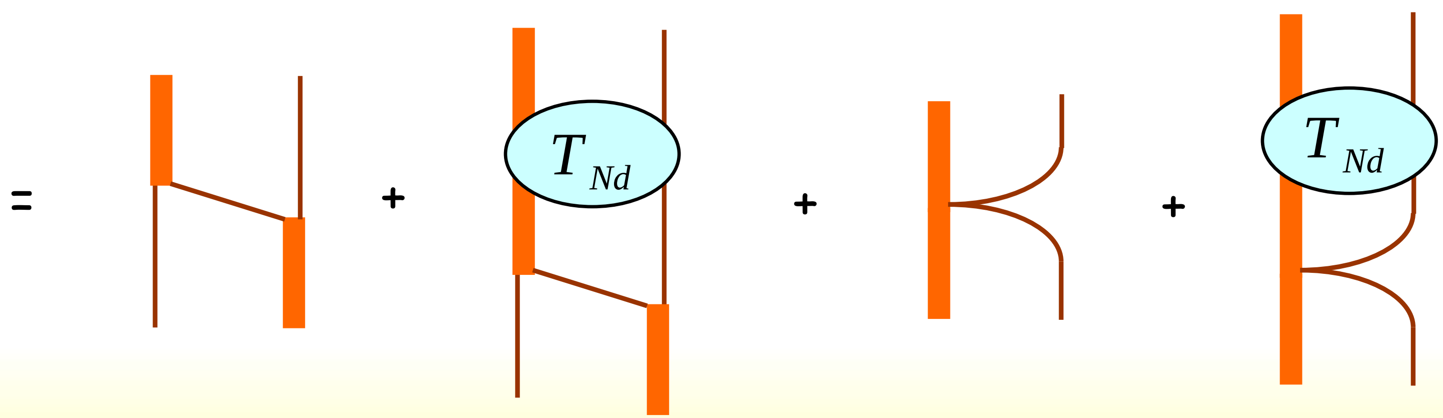
Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03

$$T_{Nd} \xrightarrow{p \ll \aleph} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta \right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} \neq 0$$ **unless**

$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{\aleph^4} \qquad (\nu = -1)$$



□ $\left(\frac{4\pi}{m_N \aleph}\right)^2 \times$ $\sim \frac{1}{Q^2/m_N}$ $\sim \frac{4\pi}{\aleph^2}$



$$T_{Nd} = K_{ONE} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} \frac{K_{ONE}\, T_{Nd}}{D} + K_{TBF} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} \frac{K_{TBF}\, T_{Nd}}{D}$$

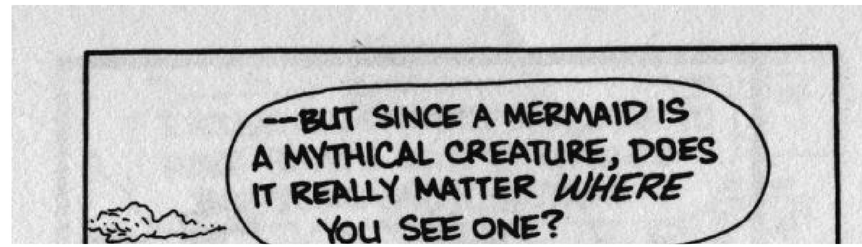
$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03
...

$$T_{Nd} \xrightarrow{p \ll \aleph} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta \right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} \neq 0 \quad \text{unless}$$

$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{\aleph^4} \qquad (\nu = -1)$$

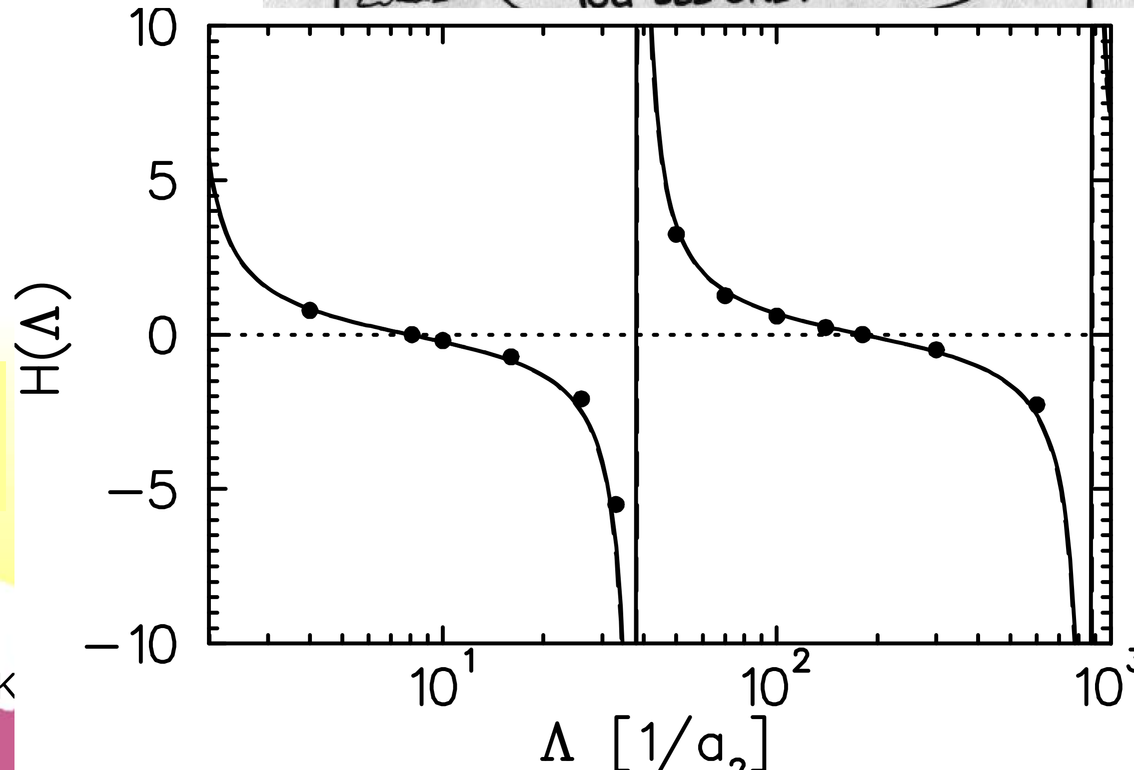


limit cycle!

$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{\aleph^4} H(\Lambda)$$

$$H(\Lambda) \propto \frac{\sin\left(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0)\right)}{\sin\left(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0)\right)}$$

dimensionful parameter
(dimensional transmutation)



Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03
...

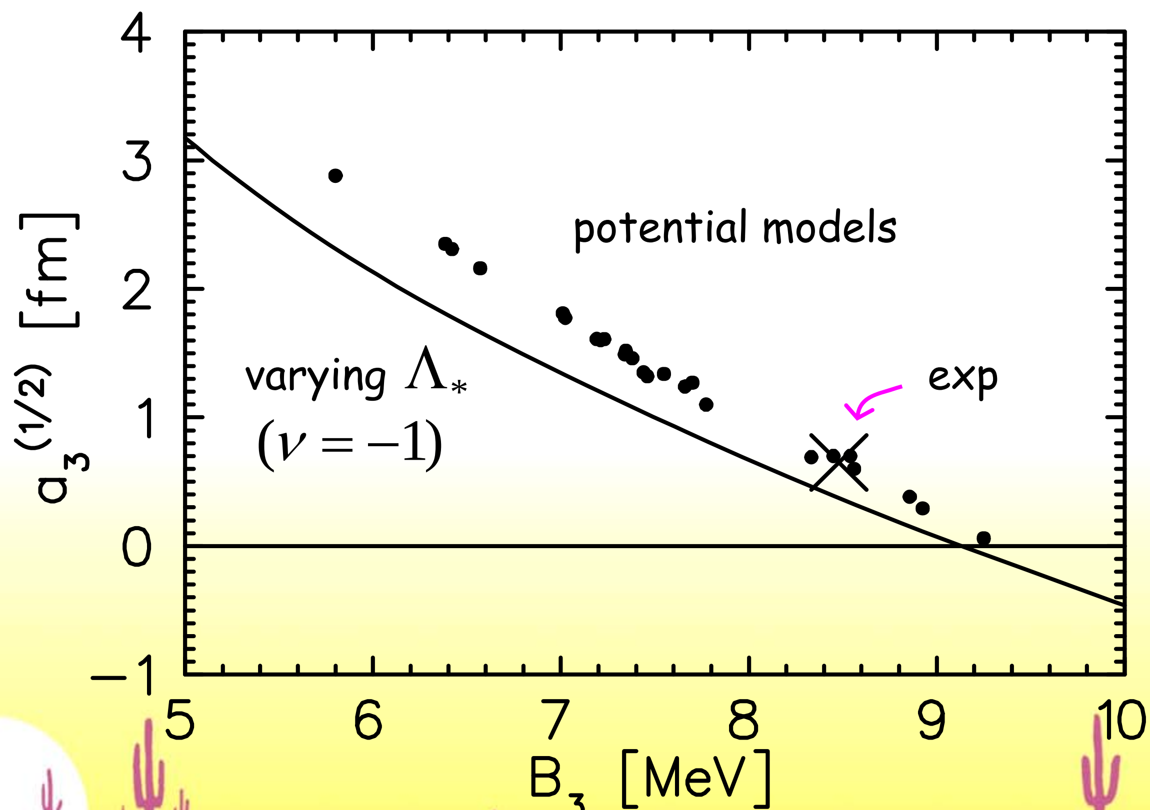
$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \gg \aleph} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} \neq 0$$ unless

$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{\aleph^4} \qquad (\nu = -1)$$

Phillips line



Bedaque, Hammer + v.K. '99 '00
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...

$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \ll \aleph} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta \right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} \neq 0$$ **unless**

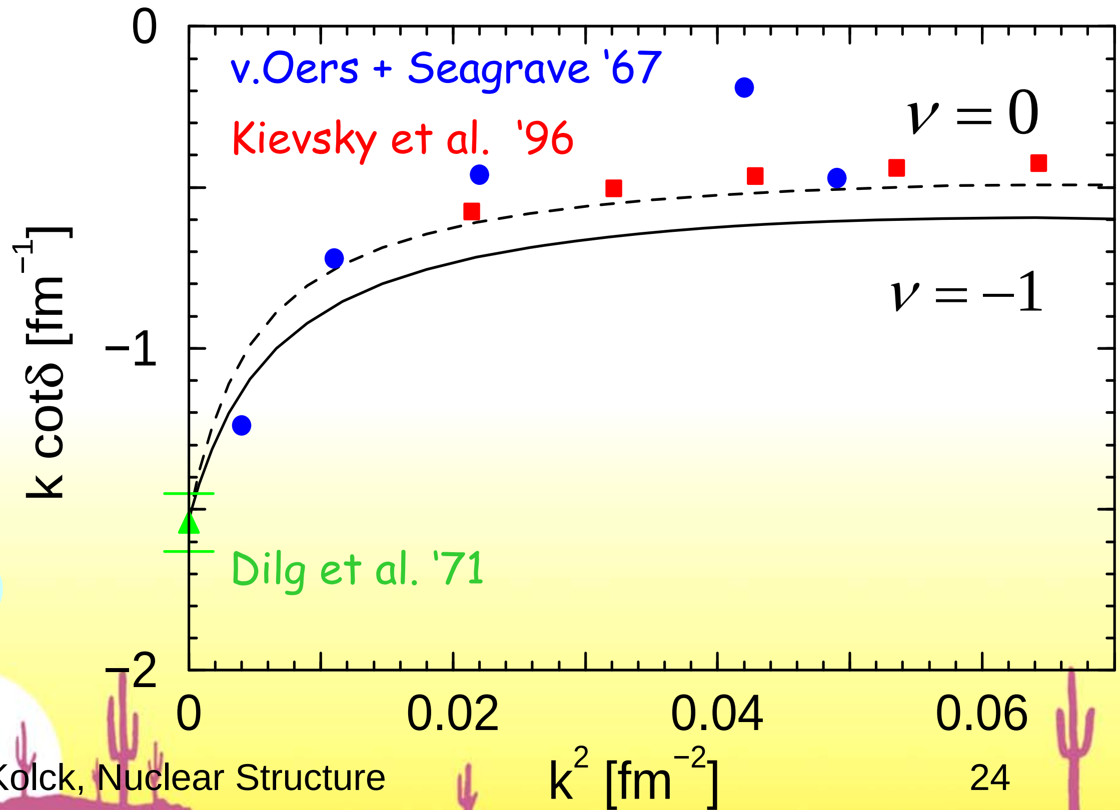
$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{\aleph^4} \qquad (\nu = -1)$$

fitted

$$a_{1/2} = 0.65 \,\text{fm (exp)}$$

predicted

$$B_t = 8.3 \,\text{MeV} \; (\nu = 0)$$
$$B_t = 8.48 \,\text{MeV (expt)}$$



Bedaque, Hammer + v.K. '99 '00
Bedaque, Braaten + Hammer '01
...

Similar for atomic/molecular systems with large scattering lengths

at B=0 or at Feshbach resonances   *e.g.* ^{4}He atoms

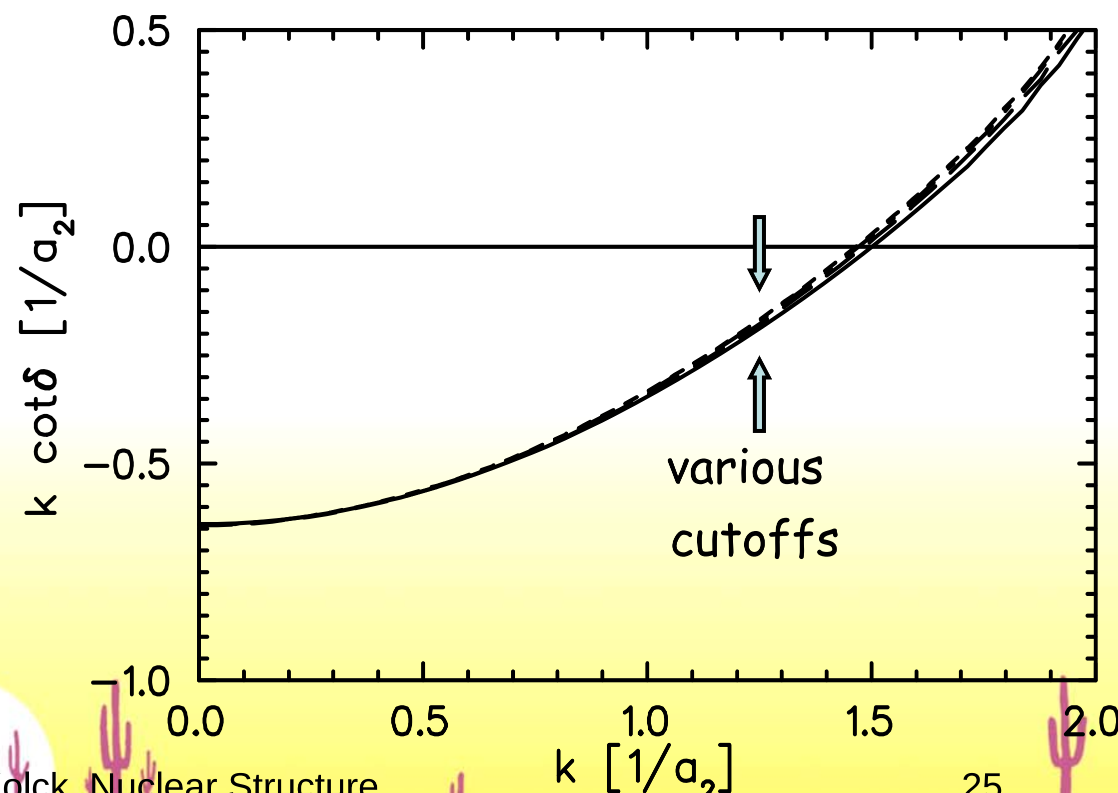
atom-dimer scattering

S_0

fitted $a_3 = 1.56\, a_2$

Uang+Stwalley '82, ...

predicted $r_3 = 0.57\, a_2$

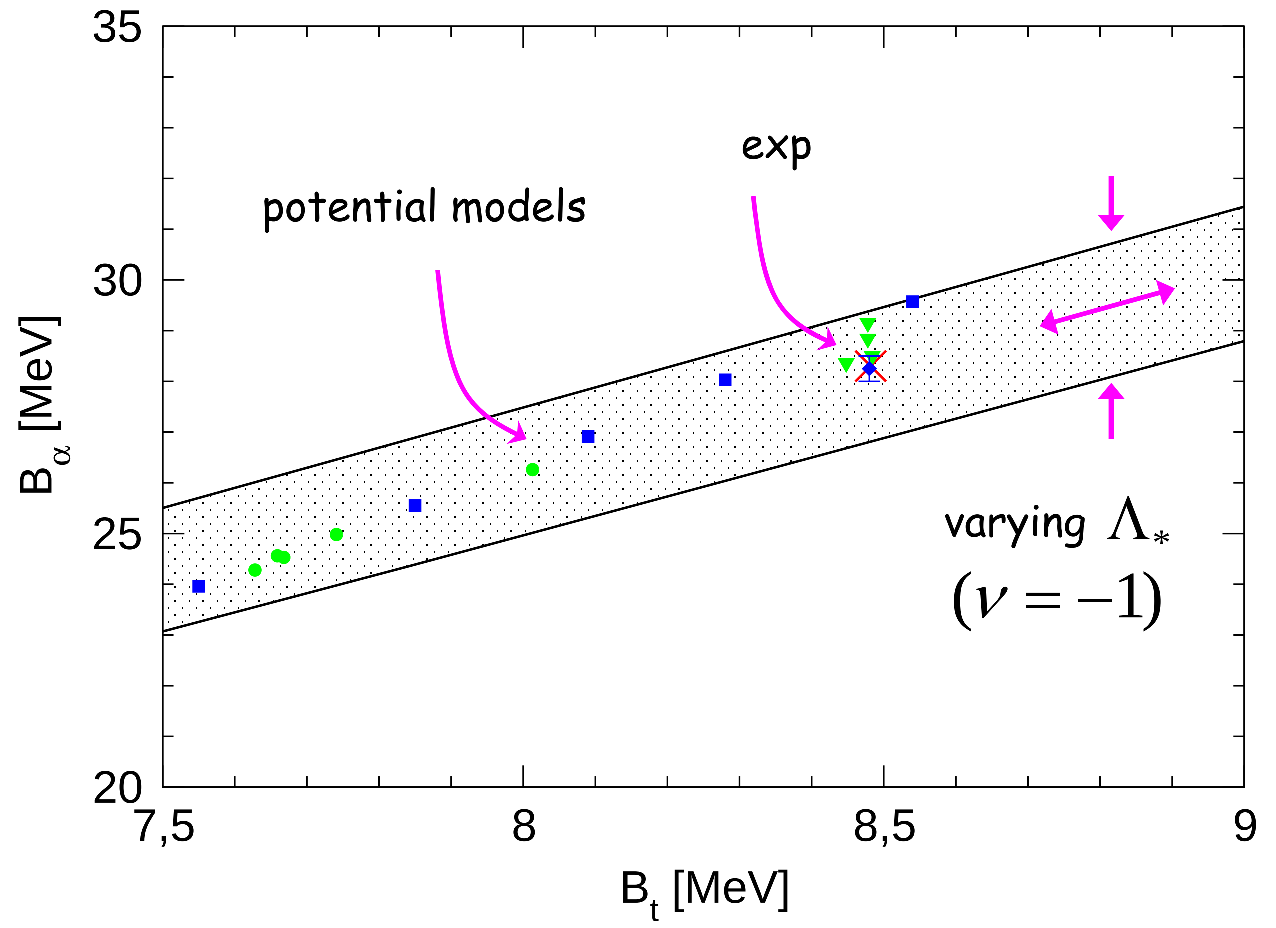


+ four-body bound state can be addressed similarly

⇨ no four-body force at $\nu = -1$

ter '04

Tjon line



~ larger nuclei?

No-Core Shell Model!

Stetcu, Barrett +v.K., '06
Stetcu, Barrett, Vary + v.K., '07
Vary + v.K., in progress

start with EFT in restricted space;
fit parameters in few-nucleon systems
for various ω and $N_{\max}\omega$;

and
predict larger nuclei

cutoffs — IR, UV

$$\lambda = \sqrt{m_N \omega} \qquad \Lambda = \sqrt{m_N \left(N_{\max} + 3/2\right)\omega}$$

strategies:

determine parameters from { light-nuclei binding energies; scattering phase shifts }

Trapped two-component fermions: $S = 1/2$

LO

$$H_A = \frac{\omega}{2}\left\{\sum_{i=1}^{A}\left[\frac{1}{2}b^2 p_i^2 + 2\frac{r_i^2}{b^2}\right] + 2\mu_2 C_0^{(0)} b^2 \sum_{[i<j]_0} \delta^{(3)}\left(\vec{r}_i - \vec{r}_j\right)\right\} = H_A^{(cm)} + H_A^{(rel)}$$

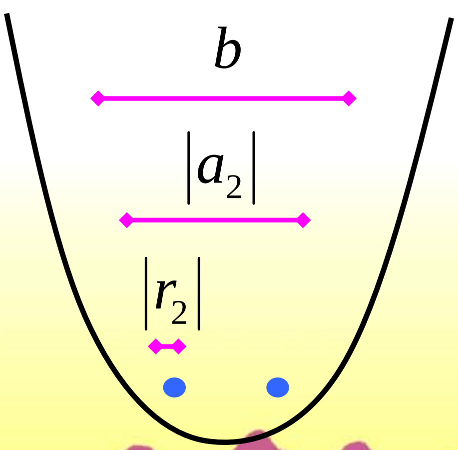
$S = 0$ pairs

two-body reduced mass $\mu_2 = \frac{m}{2}$

HO length $b = \frac{1}{\sqrt{\mu_2 \omega}}$

no { $S = 1$ two-body force; three-body force }

\+ HO is physics



$\frac{b}{|r_2|} \gg 1$ — universal behavior

$\frac{b}{|a_2|}$ { $\to \infty$ — untrapped limit; $\lesssim 1$ — significant trap effects; $\to 0$ — only low-energy scale given by b }

$$H_2^{(rel)} = \frac{\omega}{2}\left\{b^2 p^2 + \frac{r^2}{b^2} + 2\mu_2 C_0^{(0)} b^2 \delta^{(3)}(\vec{r})\right\}$$

S wave only

$$\psi(\vec{r}) = \sum_{n=0}^{N_{max}/2} A_n \phi_n(r) \qquad \phi_n(r) = \frac{1}{\sqrt{4\pi}}\left(\frac{2\,n!}{b^3\,\Gamma(n+3/2)}\right)^{\frac{1}{2}} e^{-r^2/2b^2} L_n^{(1/2)}\left(\frac{r^2}{b^2}\right)$$

HO wavefunction

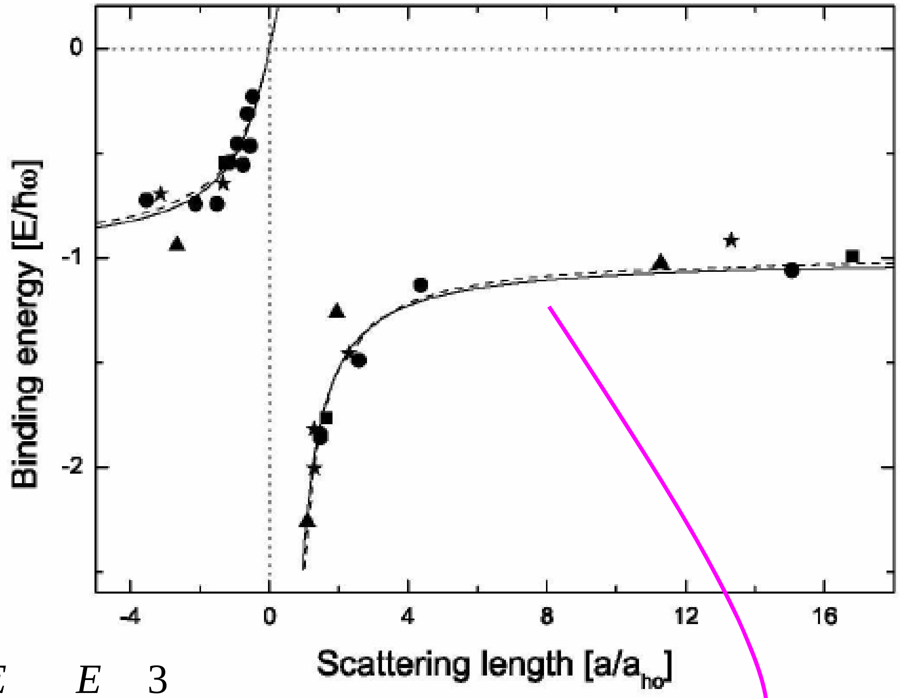
generalized Laguerre polynomial

$$H_2^{(rel)}\psi(\vec{r}) = E_2\psi(\vec{r}) \Rightarrow \frac{2\pi b}{\mu_2 C_0(N_{max},\omega)} = -\frac{2}{\pi^{1/2}} \sum_{n=0}^{N_{max}/2} \frac{L_n^{(1/2)}(0)}{2n+3/2-(E_2/\omega)}$$

measure $\frac{E_2}{\omega} = \frac{E_2}{\omega}\left(\frac{b}{a_2}\right) \Rightarrow$ determine $C_0(N_{max},\omega)$

e.g. lowest level

Stoeferle et al (ETH) '06



$$^{40}\mathrm{K}$$ atoms

$$|F = 9/2, m_F = -9/2\rangle$$

$$|F = 9/2, m_F = -7/2\rangle$$

F total angular momentum

m_F magnetic quantum number

$$\frac{E}{\omega} \to \frac{E}{\omega} - \frac{3}{2}$$

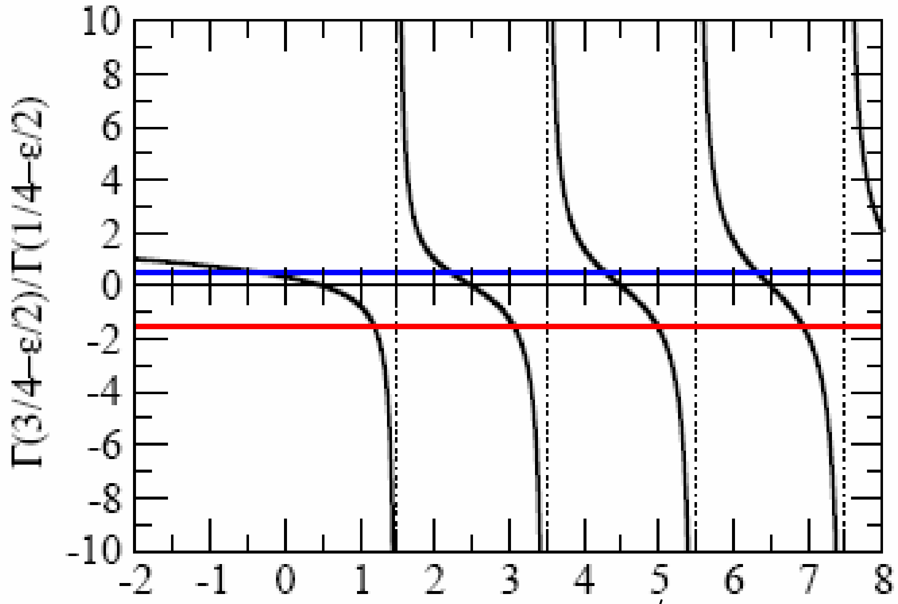
$$a_{ho} \to b$$

Busch, Englert, Rzazewski + Wilkens '98

pseudopotential in trap

$$\frac{\Gamma(3/4 - E_2/2\omega)}{\Gamma(1/4 - E_2/2\omega)} = \frac{b}{2a_2}$$

use, *e.g.* lowest level



$$\varepsilon = E_2/\omega$$

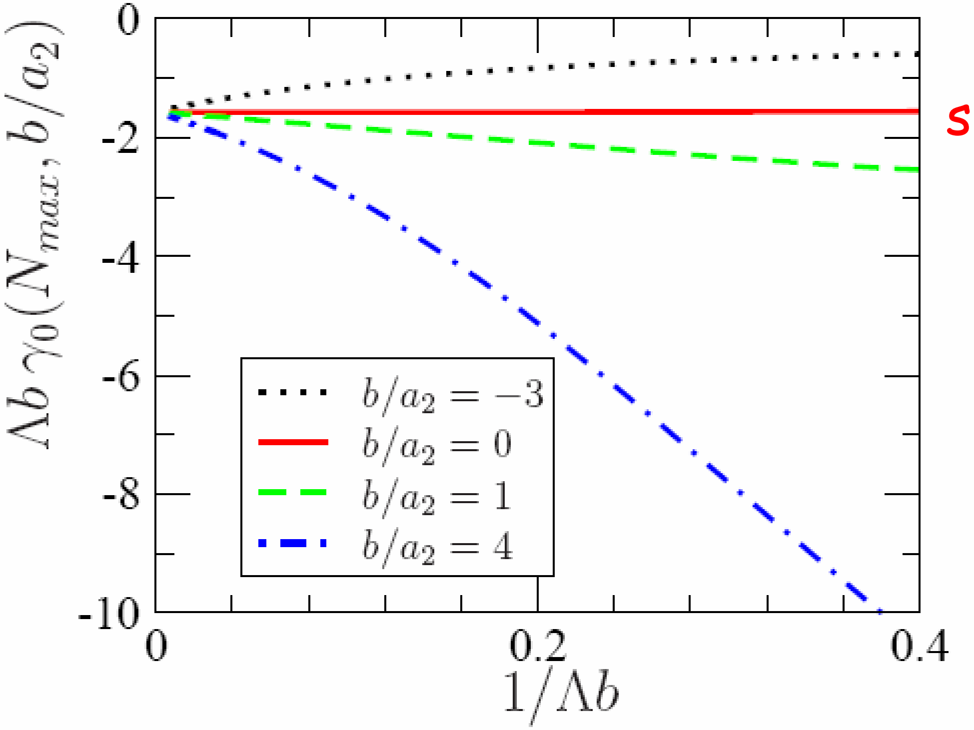
$$\frac{b}{2a_2}$$

RG running

$$\Lambda b\gamma_0\left(N_{\max},b/a_2\right)\equiv\frac{\mu_2\Lambda}{2\pi}C_0\left(N_{\max},\omega\right)$$

$$=-\left[\frac{\pi}{2}\left(N_{\max}+3/2\right)\right]^{1/2}\left\{\sum_{n=0}^{N_{\max}/2}\frac{L_n^{(1/2)}(0)}{2n+3/2-\left(E_2/\omega\right)}\right\}^{-1}$$

$$-\frac{\pi}{2}$$



simple!

For fixed *b*,
similar to free-particle basis:

- same limit $\Lambda\to\infty$
- approach controlled by a_2

$$\frac{1}{\Lambda b}=\left[2\left(N_{\max}+3/2\right)\right]^{-1/2}$$

$\frac{b}{a_2} \to -\infty$

$$\frac{E_{2,n}}{\omega} = \frac{3}{2} + 2n \quad (n = 0, 1, \ldots) \Longrightarrow \mu_2 C_0(N_{\max}, \omega) = 0$$

$$\Longrightarrow \frac{E_A}{\omega} = \text{filling of HO shells}$$

$\frac{b}{|a_2|} \to 0$

$$\frac{E_{2,n}}{\omega} = \frac{1}{2} + 2n \quad (n = 0, 1, \ldots) \Longrightarrow \mu_2 C_0(N_{\max}, \omega) = 2\pi b\, \gamma_0(N_{\max})$$

$$\simeq -\pi^2 b / \sqrt{2N_{\max}}$$

$$\Longrightarrow \frac{E_A}{\omega} = e(N_{\max})$$

expected, since only low-energy scale given by *b*

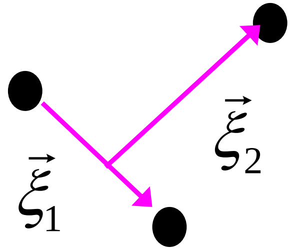
$\frac{b}{a_2} \to \infty$

$$\begin{cases} E_{2,0} = -\dfrac{1}{2\mu_2 a_2^2} & \text{untrapped bound state} \\[2ex] \dfrac{E_{2,n}}{\omega} = -\dfrac{1}{2} + 2n \quad (n = 1, 2, \ldots) & \text{scattering states} \end{cases}$$

$E_A = E_A(N_{\max}, \omega)$ in general case?

single particle $$\phi_{nl(s)j} = N_{nl} b^{-3/2} \left(\frac{r}{b}\right)^{l} \exp\left(-r^2/2b^2\right) L_n^{(l+1/2)}\left(r^2/b^2\right) \left[Y_l(\hat{r}) \otimes \chi_s\right]_j$$

$A \le 4$: relative coordinates



$$\psi\left(\vec{\xi}_1, \vec{\xi}_2\right) = \mathrm{A}\left[\phi_{nlj}\left(\vec{\xi}_1\right)\phi_{n'l'j'}\left(\vec{\xi}_2\right)\right]_{JI}$$

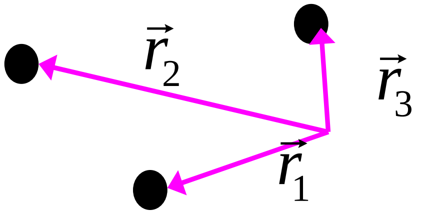
code `a la

Navratil, Kamuntavicius + Barrett '00

(reduced dimensions, but difficult antisymmetrization)

Basis

$A \ge 3$: Slater-determinant



$$\psi\left(\vec{r}_1, \vec{r}_2, \vec{r}_3\right) = \begin{vmatrix} \phi_{n_1 l_1 j_1}\left(\vec{r}_1\right) & \phi_{n_2 l_2 j_2}\left(\vec{r}_1\right) & \phi_{n_3 l_3 j_3}\left(\vec{r}_1\right) \\ \phi_{n_1 l_1 j_1}\left(\vec{r}_2\right) & \phi_{n_2 l_2 j_2}\left(\vec{r}_2\right) & \phi_{n_3 l_3 j_3}\left(\vec{r}_2\right) \\ \phi_{n_1 l_1 j_1}\left(\vec{r}_3\right) & \phi_{n_2 l_2 j_2}\left(\vec{r}_3\right) & \phi_{n_3 l_3 j_3}\left(\vec{r}_3\right) \end{vmatrix}$$

code: REDSTICK

Navratil + Ormand '03

matrix elements of delta-functions

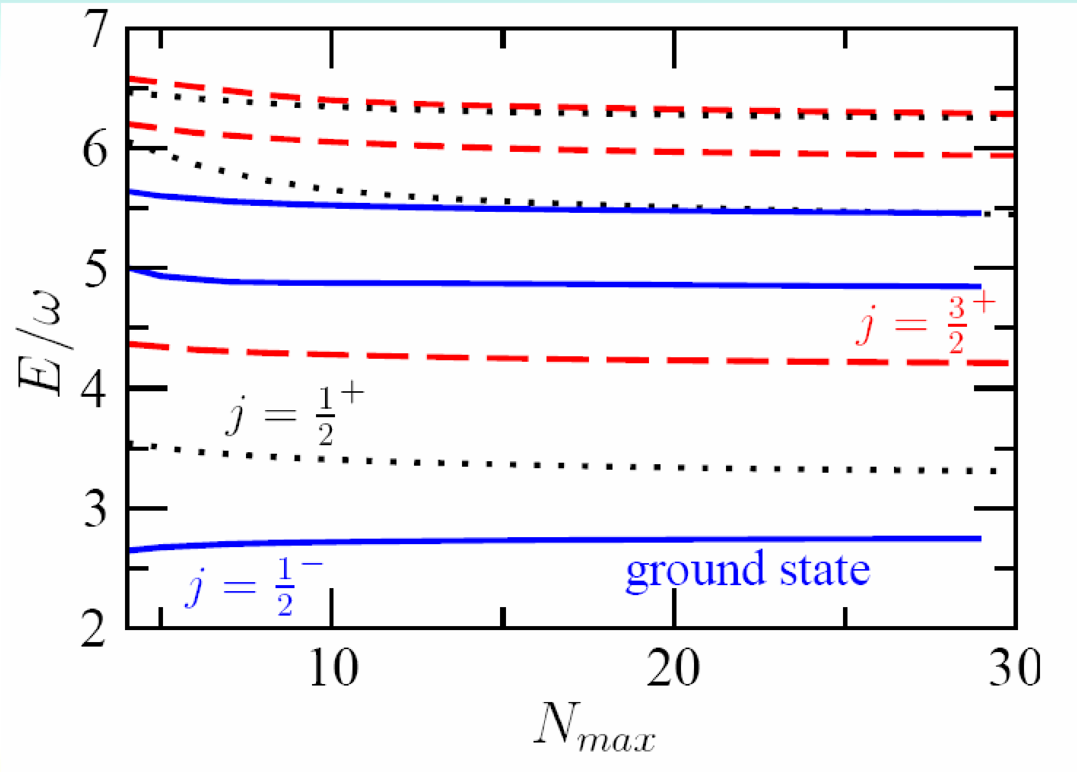
e.g.

$$\left\langle n_1, l=0 \middle| \delta \middle| n_2, l=0 \right\rangle \square \left(\frac{n_1!\,n_2!}{\Gamma\left(n_1 + 3/2\right)\Gamma\left(n_2 + 3/2\right)}\right)^{\frac{1}{2}} L_{n_1}^{(1/2)}(0)\, L_{n_2}^{(1/2)}(0)$$

$A = 3$

$$\frac{b}{|a_2|} = 0$$

Stetcu, Barrett, Vary + v.K., '07



$$E_3 = E_3^{(\infty)} + \frac{E_3^{(c)}}{\left(N_{\max} + 3/2\right)^{\alpha}}$$

$$\alpha \approx 1.5$$

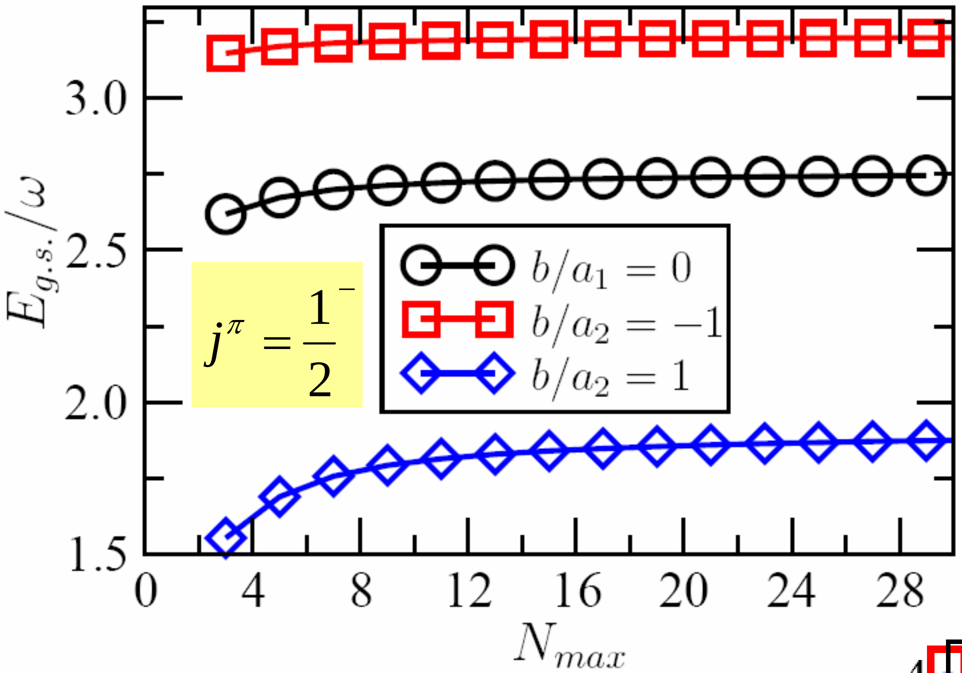
Find, within 10%,

$$\frac{E_3^{(\infty)}}{\omega} \cong 1 + s + 2q$$

semi-analytical result $\quad q = 0, 1, \ldots$

Werner + Castin '06 $\quad s = 1.77, 2.17, 3.10, \ldots$

Stetcu, Barrett, Vary + v.K., '07



repulsion

as in two-body system

attraction

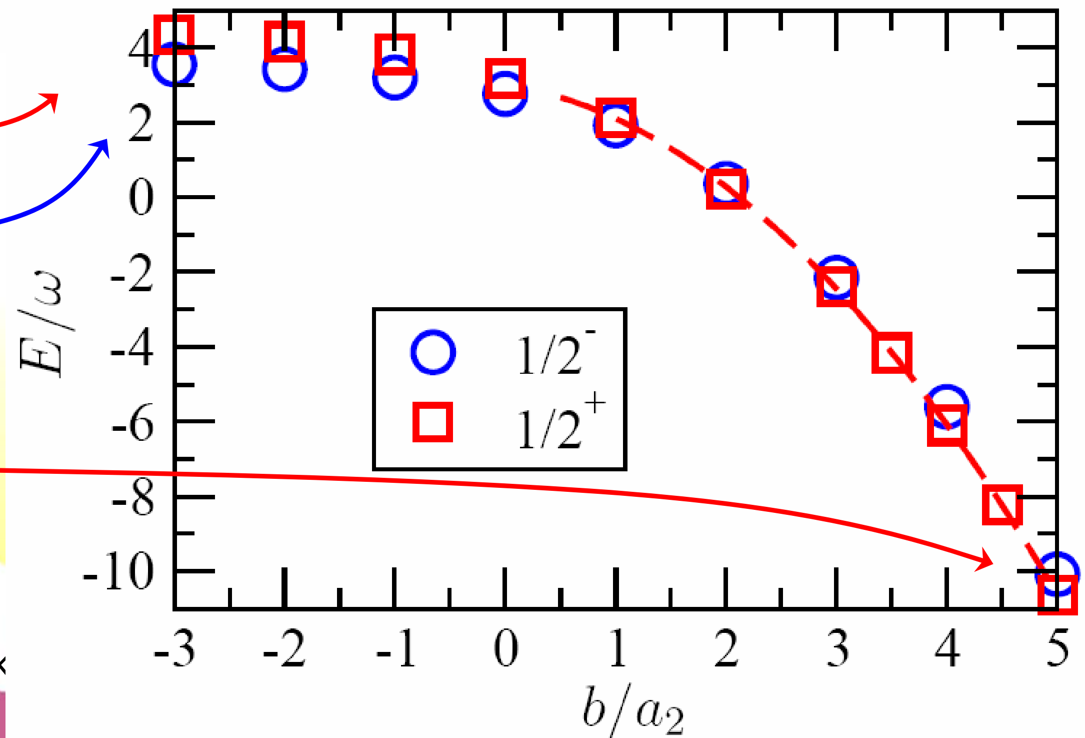
confirmed: Kestner + Duan '07

inversion of parity!

$$\frac{E_3}{\omega} = \begin{cases} 5 & 1S2P \\ 4 & 2S1P \end{cases}$$

$$E_3 \approx -\frac{1}{2\mu_2 a_2^2}$$

$(\text{fermion+dimer})_{S\text{ wave}}$

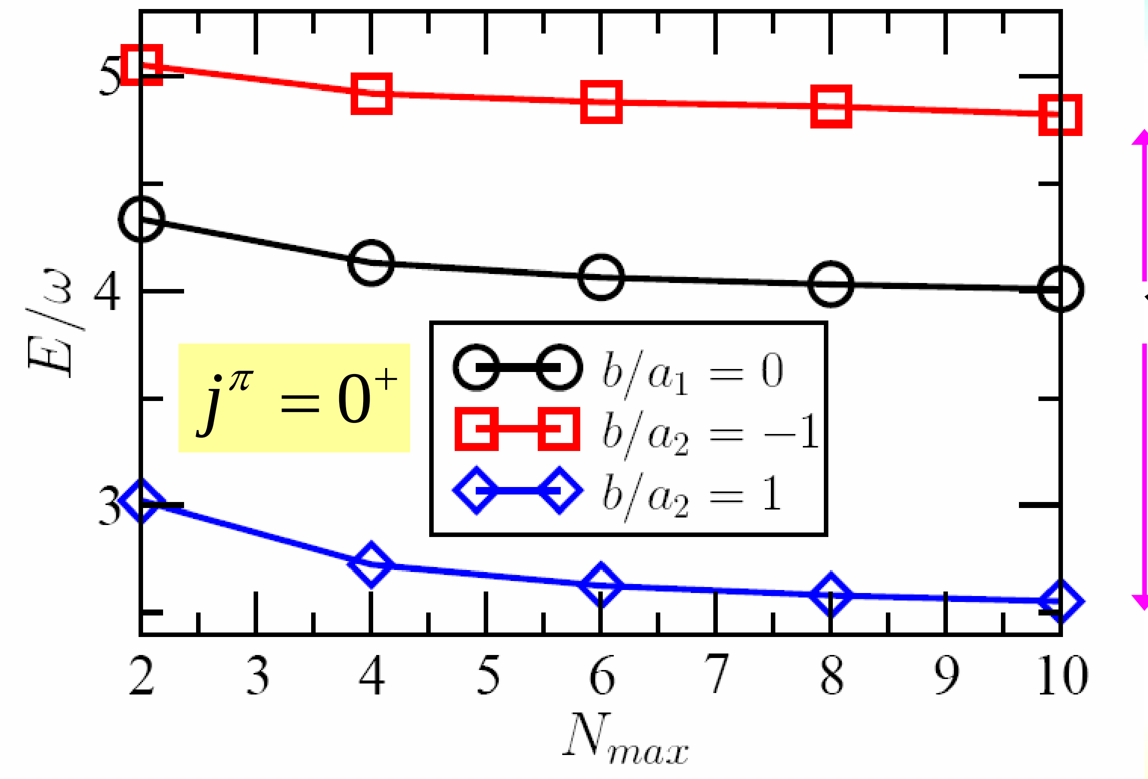


$A = 4$

Stetcu, Barrett, Vary + v.K., '07

larger errors…

$$E_4 = E_4^{(\infty)} + \frac{E_4^{(c)}}{\left(N_{\max} + 3/2\right)^{\alpha}}$$



repulsion

attraction

$\alpha \approx 0.5 - 1$

$$\frac{E_4^{(\infty)}}{\omega} \cong 3.6 - 3.9$$

cf. Chang + Bertsch '07

$$\frac{E_4^{(\infty)}}{\omega} \cong 5.1 \quad \text{GFMC}$$

$\frac{b}{a_2} \to -\infty$ $\quad \frac{E_4^{(\infty)}}{\omega} \cong \frac{13}{2} \quad 2S2P$

$\frac{b}{a_2} \to \infty$ $\quad E_4 \approx -\frac{1.4}{\mu_2 a_2^2}$ bound state?

cf. Petrov, Salomon + Shlyapnikov '05

Untrapped nucleons

LO

$$H_A = \frac{1}{2m_N A} \sum_{[i<j]} \left(\vec{p}_i - \vec{p}_j \right)^2 + C_0^{(0)} \sum_{[i<j]_0} \delta^{(3)}\left(\vec{r}_i - \vec{r}_j \right)$$

$$+ C_0^{(1)} \sum_{[i<j]_1} \delta^{(3)}\left(\vec{r}_i - \vec{r}_j \right) + D_0 \sum_{[i<j<k]} \delta^{(3)}\left(\vec{r}_i - \vec{r}_j \right) \delta^{(3)}\left(\vec{r}_j - \vec{r}_k \right)$$

$S = 1$ pairs $\qquad$ $S = 1/2$ triplets

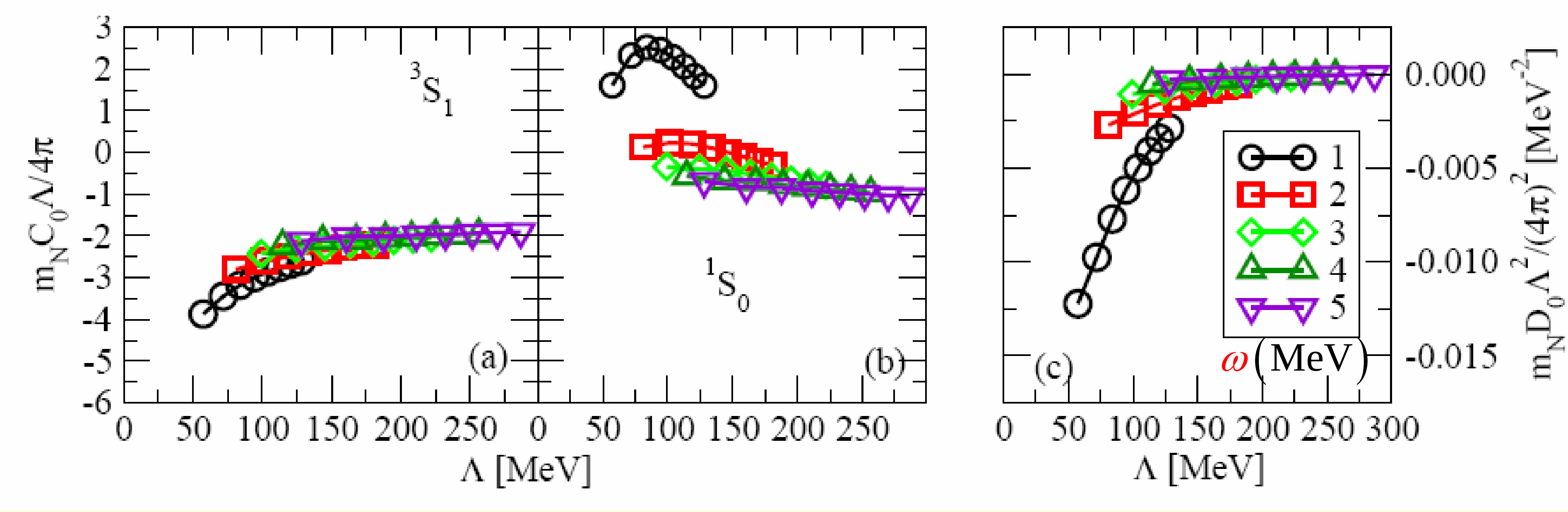
EFT PC effectively justifies (modified) cluster approximation

- LO parameters

$$C_0^{(0)}(\Lambda),\ C_0^{(1)}(\Lambda),\ D_0(\Lambda)$$

fitted to d, t, α ground-state binding energies

Stetcu, Barrett +v.K., '06



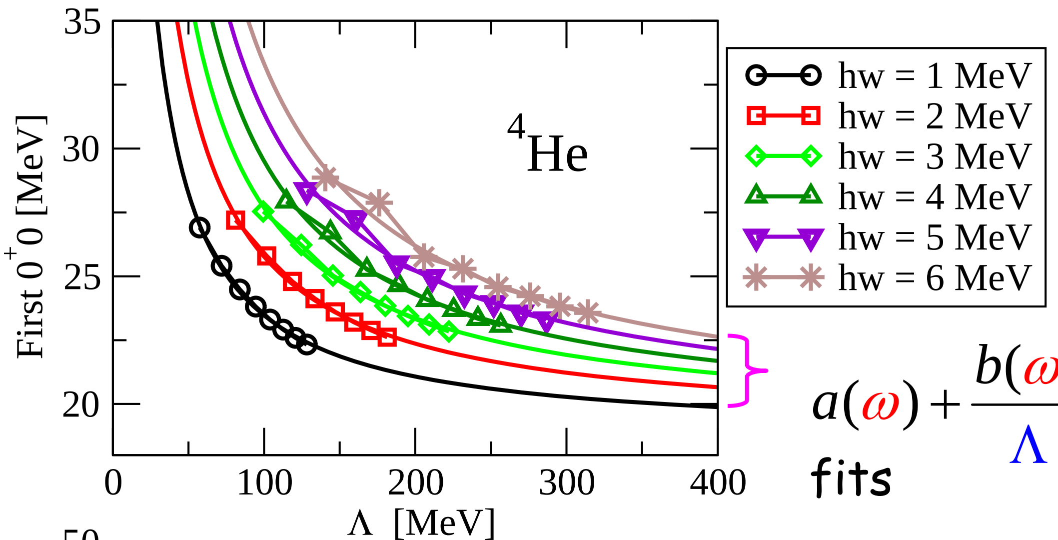
$$m_N C_0^s \Lambda / 4\pi \xrightarrow{\Lambda\to\infty} -\pi/2$$

limit cycle?

LO $(\nu = -1)$

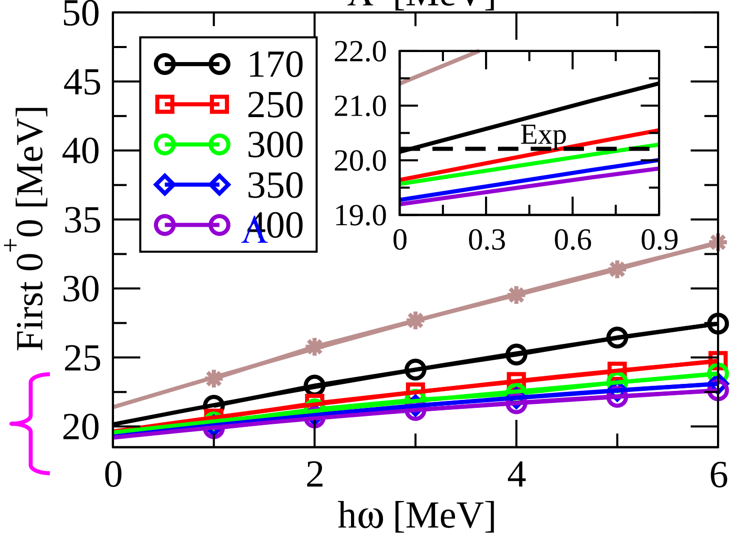
Stetcu, Barrett +v.K., '06

$N_{\max} \leq 16$



$a(\omega) + \dfrac{b(\omega)}{\Lambda}$

fits



$\alpha + \beta\omega + \gamma\omega^2$

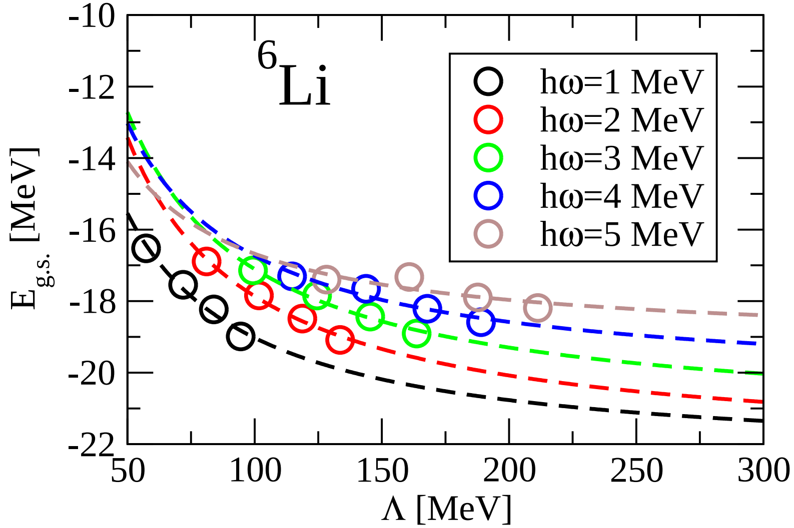
fits

works within ~10% !

LO $(\nu = -1)$

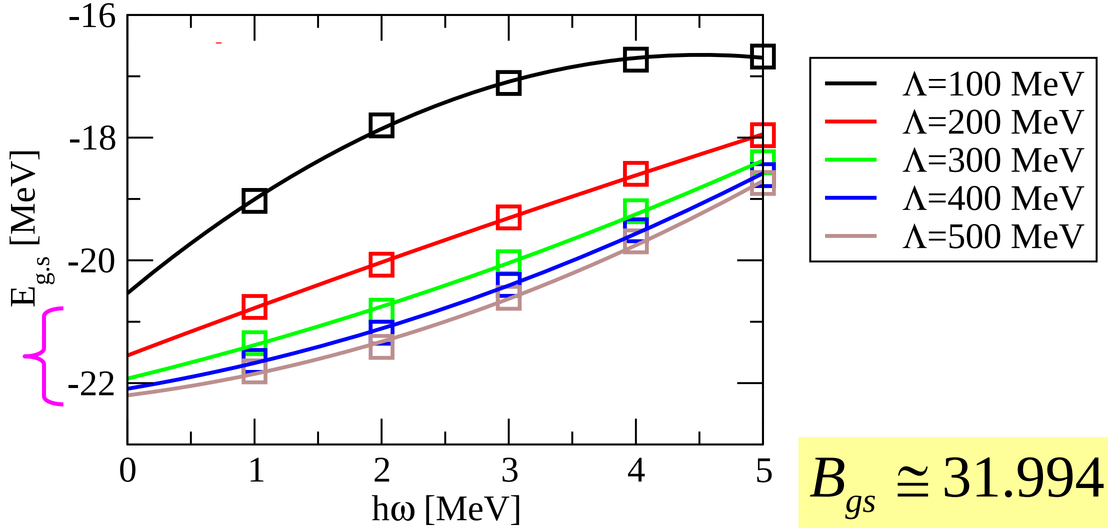
Stetcu, Barrett +v.K., '06

$N_{\max} \leq 8$



$a(\omega) + \frac{b(\omega)}{\Lambda}$

fits



$\alpha + \beta\omega + \gamma\omega^2$

fits

$B_{gs} \cong 31.994$ MeV (exp)

works within ~30%

→ works as expected!

next, to higher orders: perturbation theory

$$E_A = E_A^{(\infty)} + \frac{E_A^{(c)}}{\left(N_{\max} + 3/2\right)^{\alpha}}$$

smaller:
faster convergence,
heavier systems

cf. Alhassid, Bertsch + Fang '07

Conclusion

EFT the framework to describe nuclei within the SM

- ✓ is consistent with symmetries
- ✓ incorporates hadronic physics
- ✓ has controlled expansion

Many successes so far …

- $A \lesssim 4$: low-E scatt, bs's, probes
- $A \gtrsim 4$: bs's

but still much to do …

- ➢ grow to larger nuclei
- ➢ go to next order
- ➢ connect to scattering
- ➢ extend to pionful EFT
- ➢ …