No-core shell model with JISP16 *NN* interaction: spectroscopy of light nuclei and neutron-nucleus scattering

Seattle, INT-07-03

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Highlights

JISP = *J*-matrix inverse scattering potential

PETs = phase-equivalent transformations

No-core shell model: *ab initio* ⇔ *ab exitu* approach ⇒ JISP16 *NN* interaction

No three-nucleon forces

Spectroscopy of nuclei with $A \leq 16$

N-nucleus scattering

JISP type *NN* interaction:

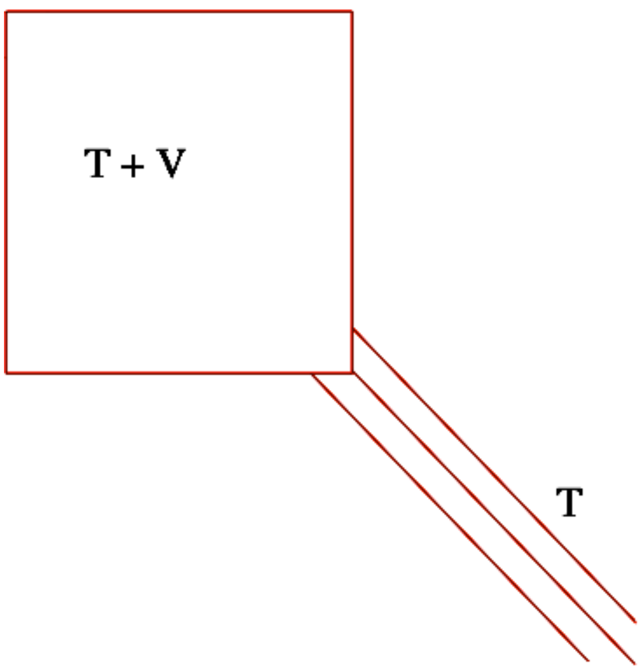
- Description of *NN* data
- Small matrix of the *NN* interaction in the oscillator basis:

$$V = \sum_{\Gamma,\Gamma'} \sum_{n=0}^{N_\Gamma} \sum_{n'=0}^{N_{\Gamma'}} |n\Gamma\rangle \, V_{nn'}^{\Gamma\Gamma'} \, \langle n'\Gamma'|$$

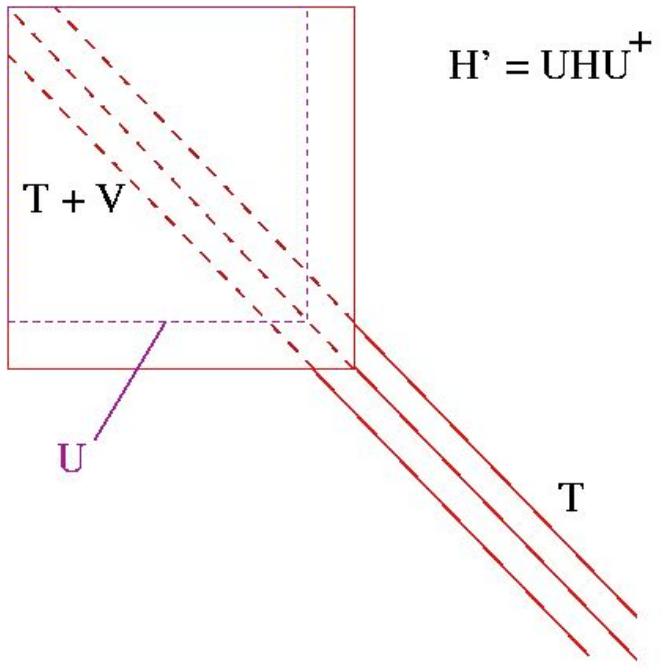
- Description of (light) nuclei

J-matrix formalism:

scattering in the oscillator basis



PETs



ab initio ⇔ ab exitu

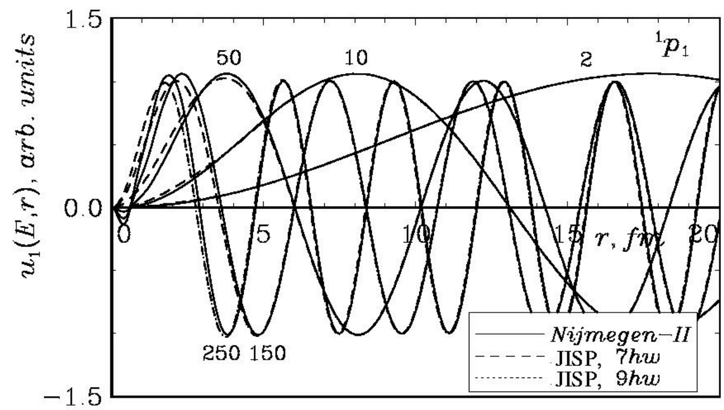
- JISP16: *J*-matrix inverse scattering 9hΩ *NN* potential with hΩ = 40 MeV fitted to nuclei up through ^{16}O
- Only simplest PETs generated by 2x2 unitary matrix *U* are used
- *Ab exitu* approach:
- PETs: *sd* wave - fitting deuteron properties (rms radius and quadrupole moment)
  various *p* and one of *d* waves - fitting few levels of ^{6}Li and binding energy of ^{16}O in relatively small model spaces
- All the rest NCSM results (other nuclei, larger model spaces) are *ab initio*

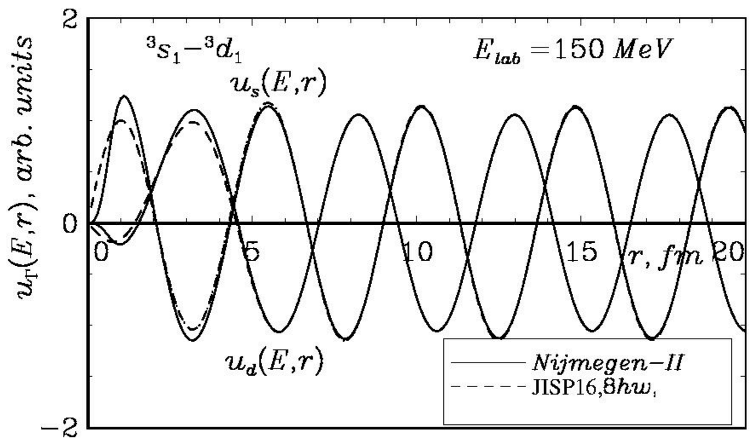
JISP16 properties

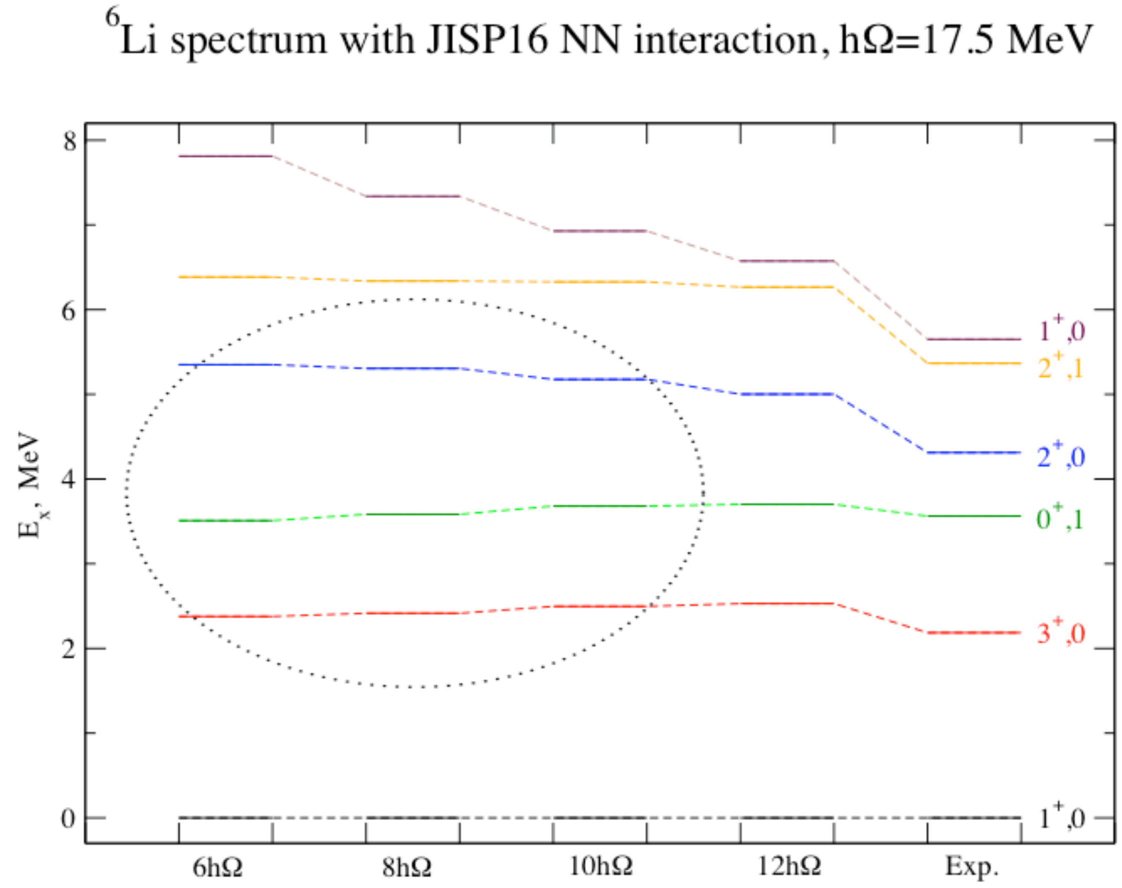
- 1992 *np* data base (2514 data): χ^2/datum = 1.03
- 1999 *np* data base (3058 data): χ^2/datum = 1.05

Table I: Deuteron properties.

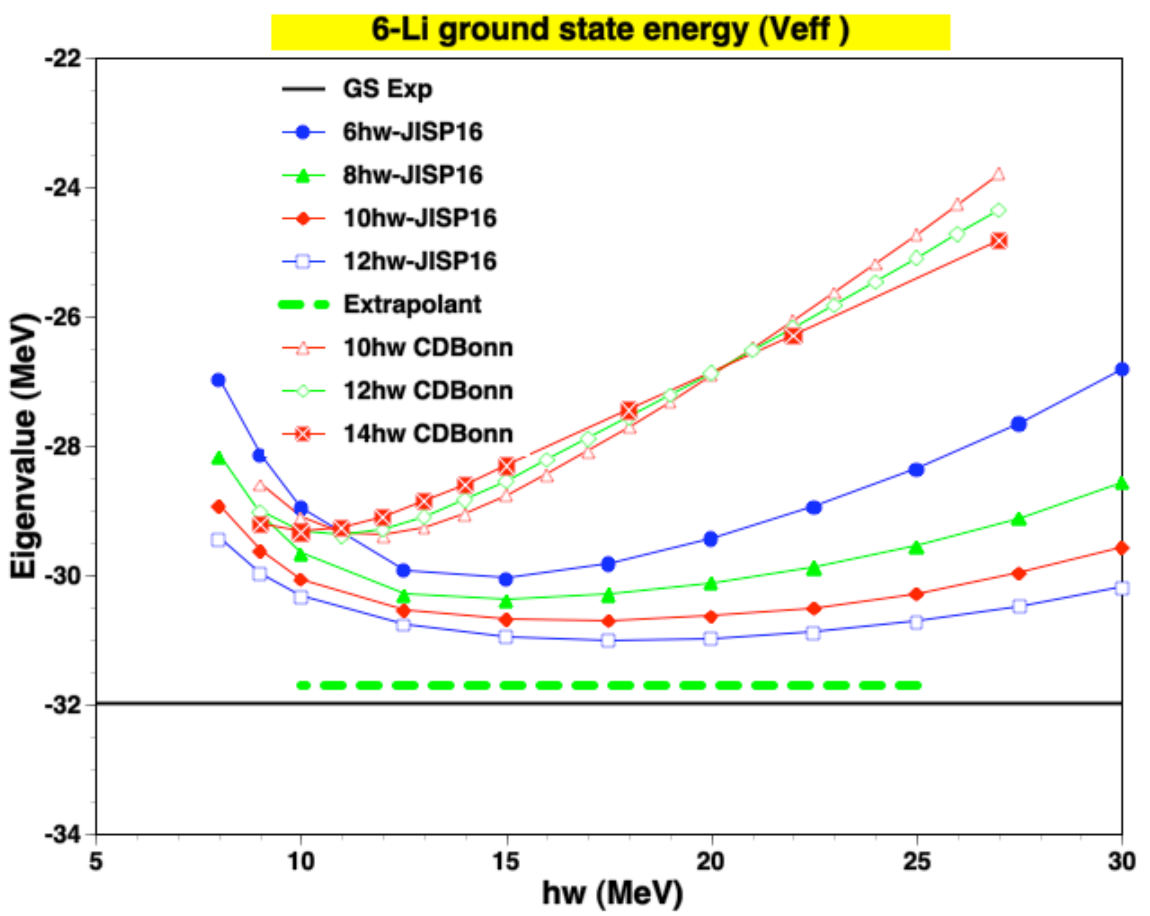
Potential	E_d, MeV	d state probability, %	rms radius, fm	Q, fm^2	As. norm. const. $\mathscr{A}_s$, fm$^{-1/2}$	$\eta = \frac{\mathscr{A}_d}{\mathscr{A}_s}$
JISP16	−2.224575	4.1360	1.9643	0.2886	0.8629	0.0252
Nijmegen-II	−2.224575	5.635	1.968	0.2707	0.8845	0.0252
AV18	−2.224575	5.76	1.967	0.270	0.8850	0.0250
CD–Bonn	−2.224575	4.85	1.966	0.270	0.8846	0.0256
Nature	−2.224575(9)	—	1.971(6)	0.2859(3)	0.8846(9)	0.0256(4)



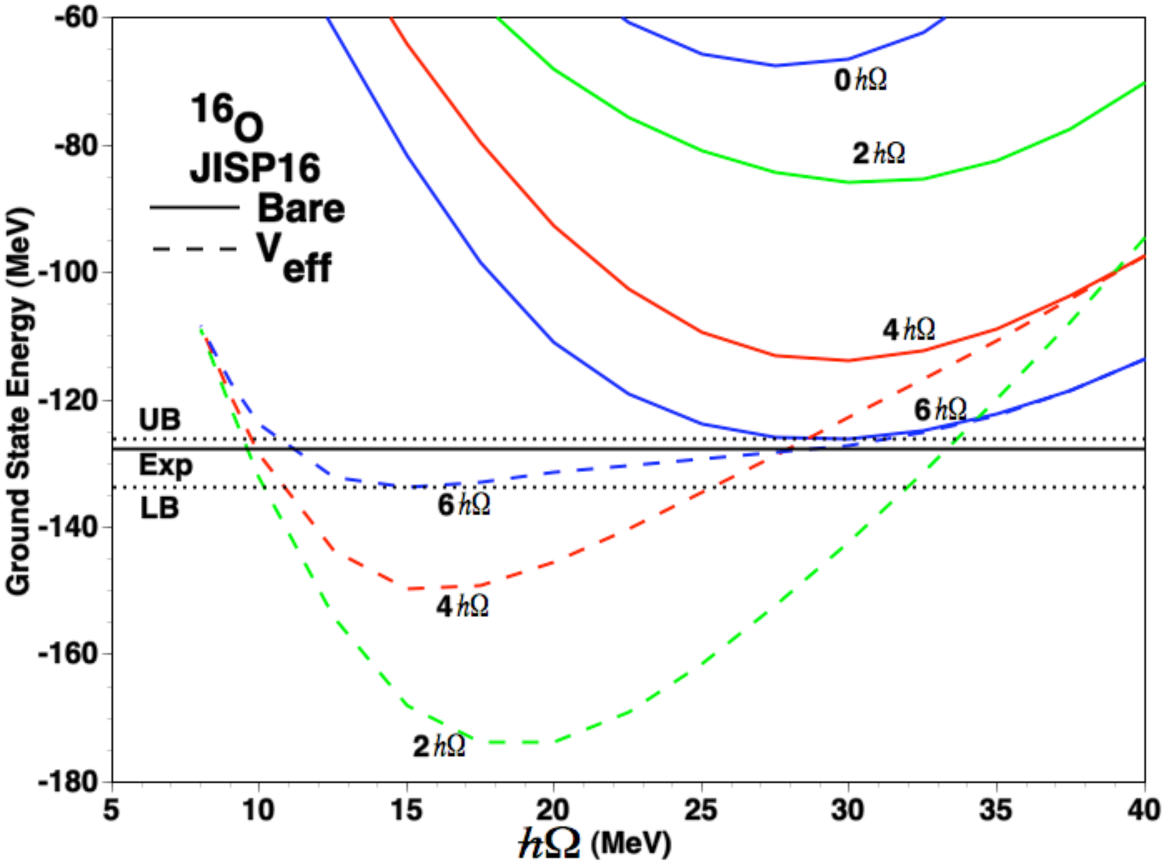




^{6}Li spectrum with JISP16 NN interaction, hΩ=17.5 MeV
E_x, MeV
8
6
4
2
0
6hΩ
8hΩ
10hΩ
12hΩ
Exp.
1^+,0
2^+,1
2^+,0
0^+,1
3^+,0
1^+,0

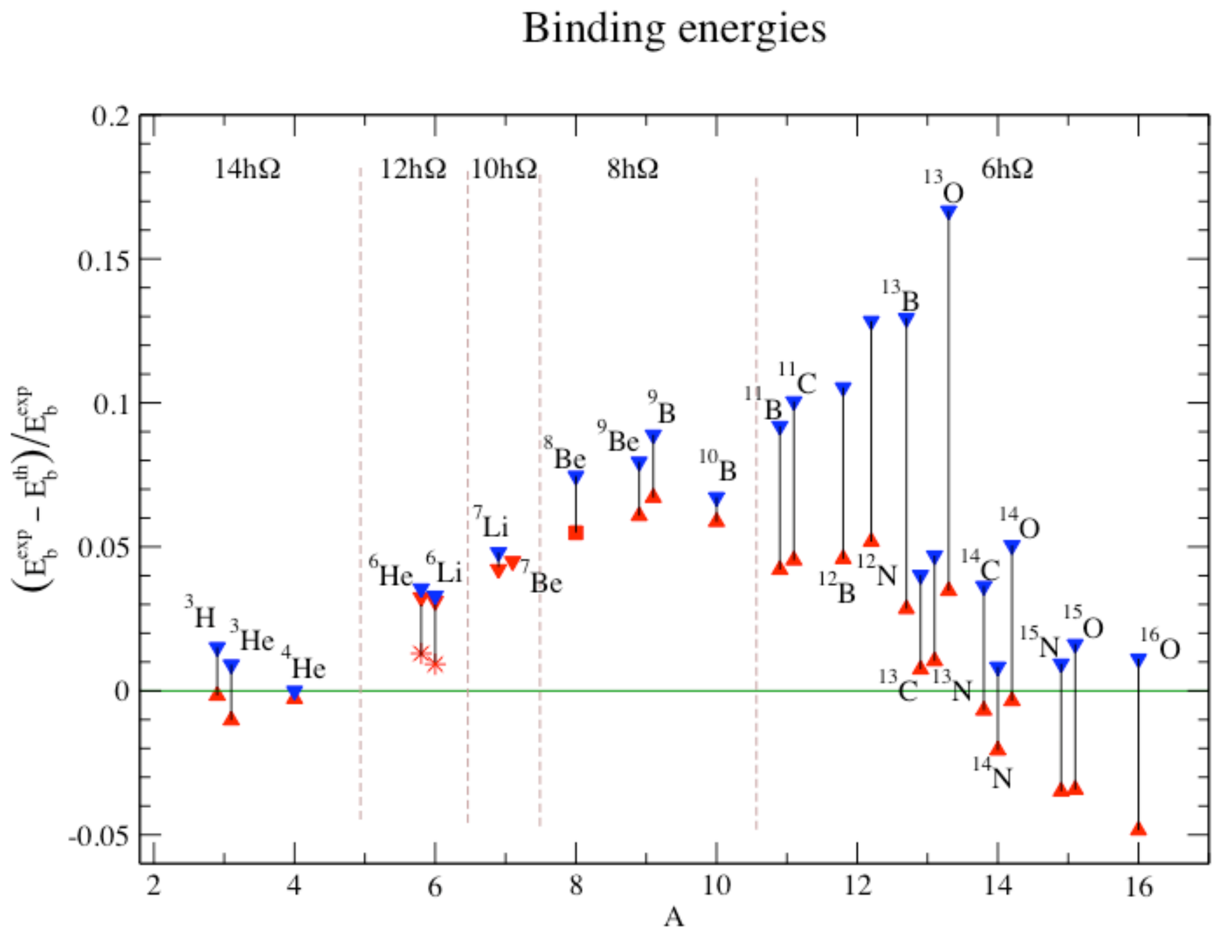


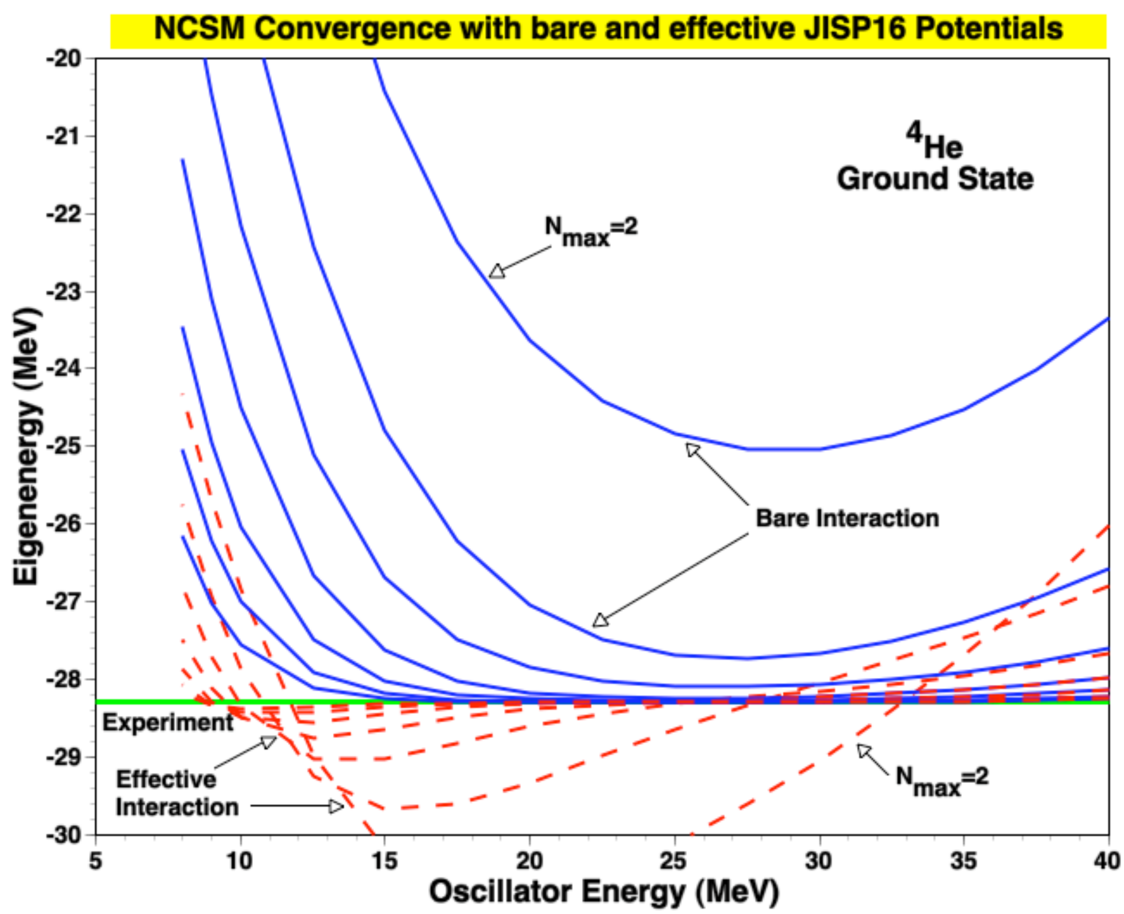
6-Li ground state energy (Veff)
Eigenvalue (MeV)
hw (MeV)
GS Exp
6hw-JISP16
8hw-JISP16
10hw-JISP16
12hw-JISP16
Extrapolant
10hw CDBonn
12hw CDBonn
14hw CDBonn
-22
-24
-26
-28
-30
-32
-34
5
10
15
20
25
30

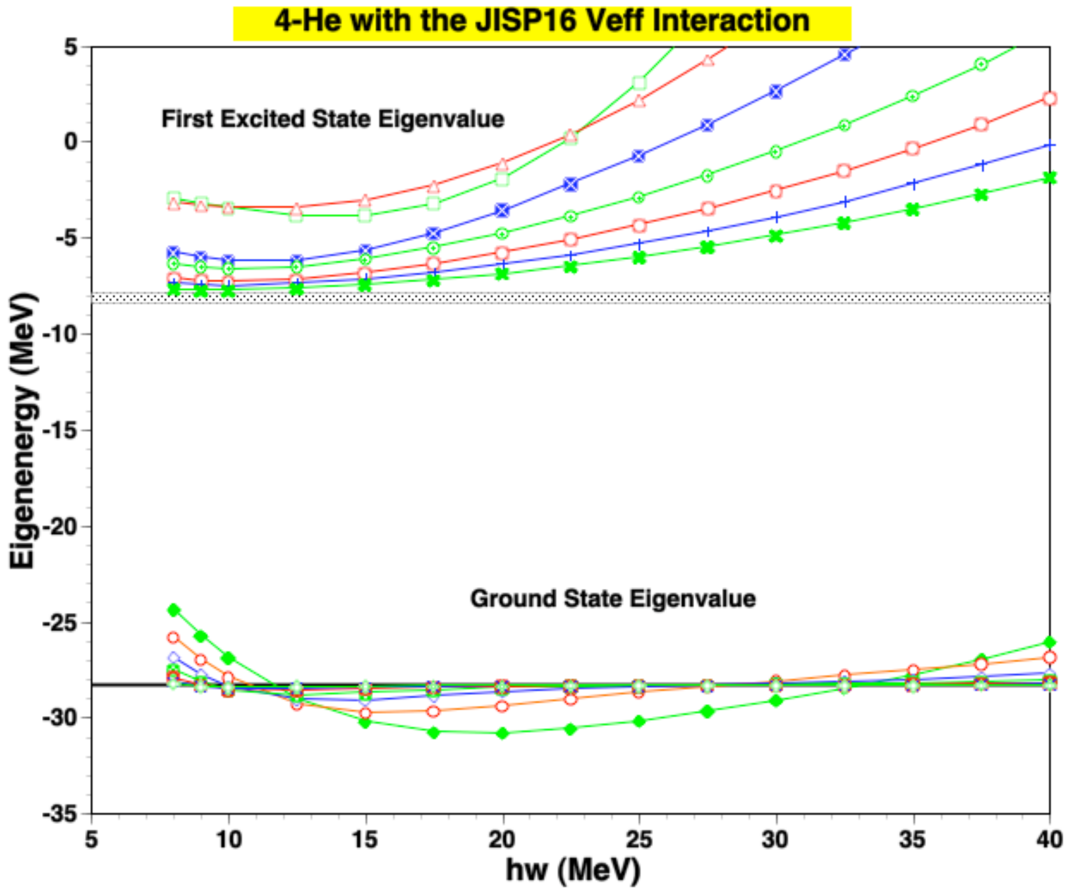


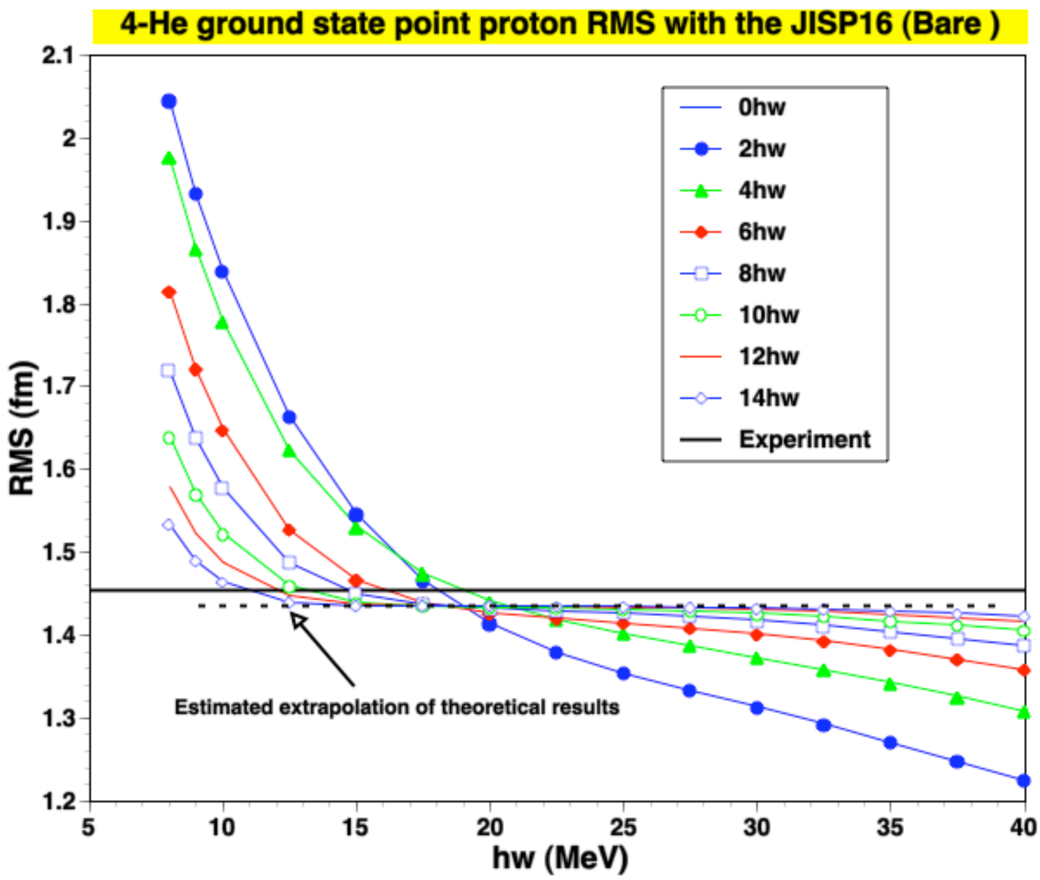
Nucleus	Nature	Bare	Effective	$\hbar\omega$ (MeV)	Model space
^{3}H	8.482	8.354	8.496(20)	7	$14\hbar\omega$
^{3}He	7.718	7.648	7.797(17)	7	$14\hbar\omega$
^{4}He	28.296	28.297	28.374(57)	10	$14\hbar\omega$
^{6}He	29.269		28.32(28)	17.5	$12\hbar\omega$
^{6}Li	31.995		31.00(31)	17.5	$12\hbar\omega$
^{7}Li	39.245		37.59(30)	17.5	$10\hbar\omega$
^{7}Be	37.600		35.91(29)	17	$10\hbar\omega$
^{8}Be	56.500		53.40(10)	15	$8\hbar\omega$
^{9}Be	58.165	53.54	54.63(26)	16	$8\hbar\omega$
^{9}B	56.314	51.31	52.53(20)	16	$8\hbar\omega$
^{10}Be	64.977	60.55	61.39(20)	19	$8\hbar\omega$
^{10}B	64.751	60.39	60.95(20)	20	$8\hbar\omega$
^{10}C	60.321	55.26	56.36(67)	17	$8\hbar\omega$
^{11}B	76.205	69.2	73.0(31)	17	$6\hbar\omega$

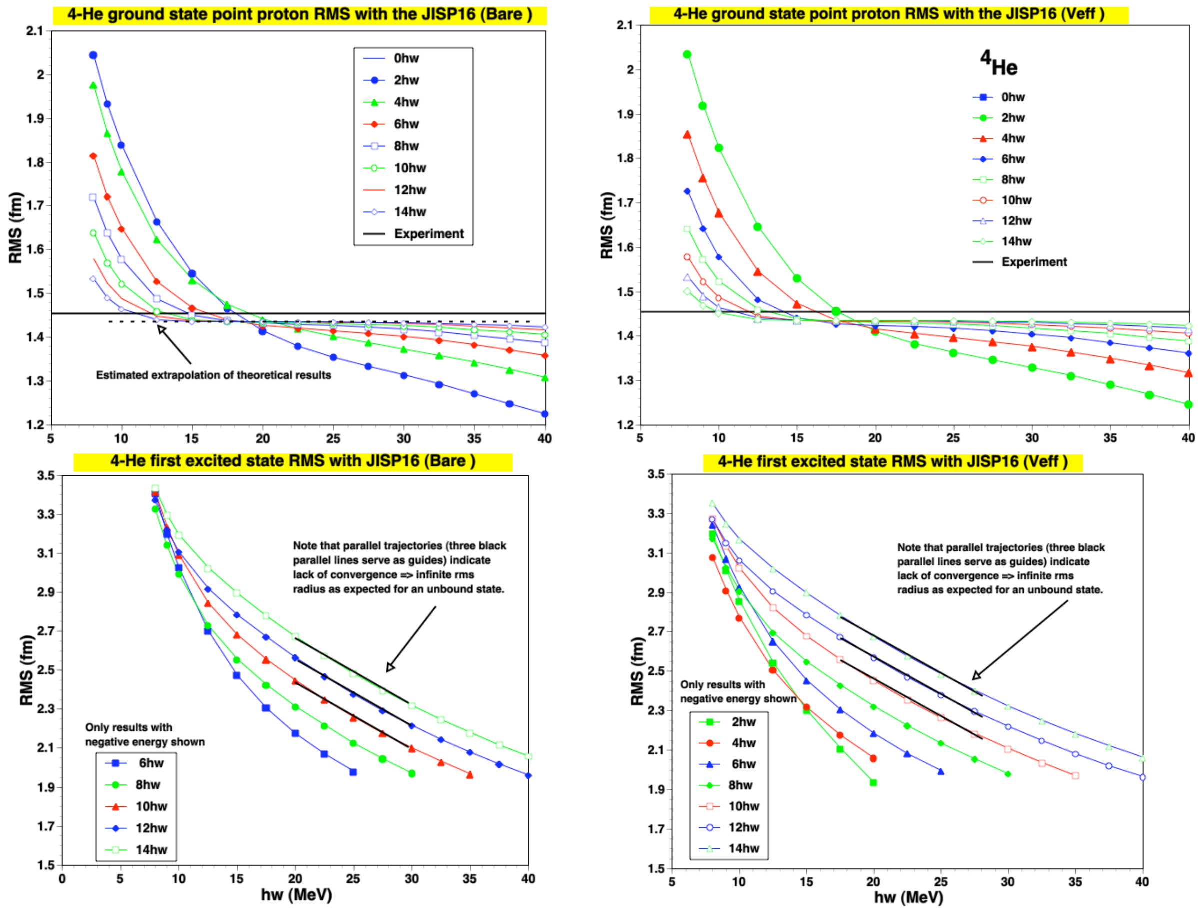
Nucleus	Nature	Bare	Effective	$\hbar\omega$ (MeV)	Model space
^{11}C	73.440	66.1	70.1(32)	17	$6\hbar\omega$
^{12}B	79.575	71.2	75.9(48)	15	$6\hbar\omega$
^{12}C	92.162	87.4	91.0(49)	17.5	$6\hbar\omega$
^{12}N	74.041	64.5	70.2(48)	15	$6\hbar\omega$
^{13}B	84.453	73.5	82.1(67)	15	$6\hbar\omega$
^{13}C	97.108	93.2	96.4(59)	19	$6\hbar\omega$
^{13}N	94.105	89.7	93.1(62)	18	$6\hbar\omega$
^{13}O	75.558	63.0	72.9(62)	14	$6\hbar\omega$
^{14}C	105.285	101.5	106.0(93)	17.5	$6\hbar\omega$
^{14}N	104.659	103.8	106.8(77)	20	$6\hbar\omega$
^{14}O	98.733	93.7	99.1(92)	16	$6\hbar\omega$
^{15}N	115.492	114.4	119.5(126)	16	$6\hbar\omega$
^{15}O	111.956	110.1	115.8(126)	16	$6\hbar\omega$
^{16}O	127.619	126.2	133.8(158)	15	$6\hbar\omega$


Binding energies
14hΩ
12hΩ
10hΩ
8hΩ
6hΩ
$(E_b^{exp} - E_b^{th})/E_b^{exp}$
A


NCSM Convergence with bare and effective JISP16 Potentials
4He
Ground State
N max=2
Bare Interaction
Experiment
Effective
Interaction
N max=2
Eigenenergy (MeV)
Oscillator Energy (MeV)

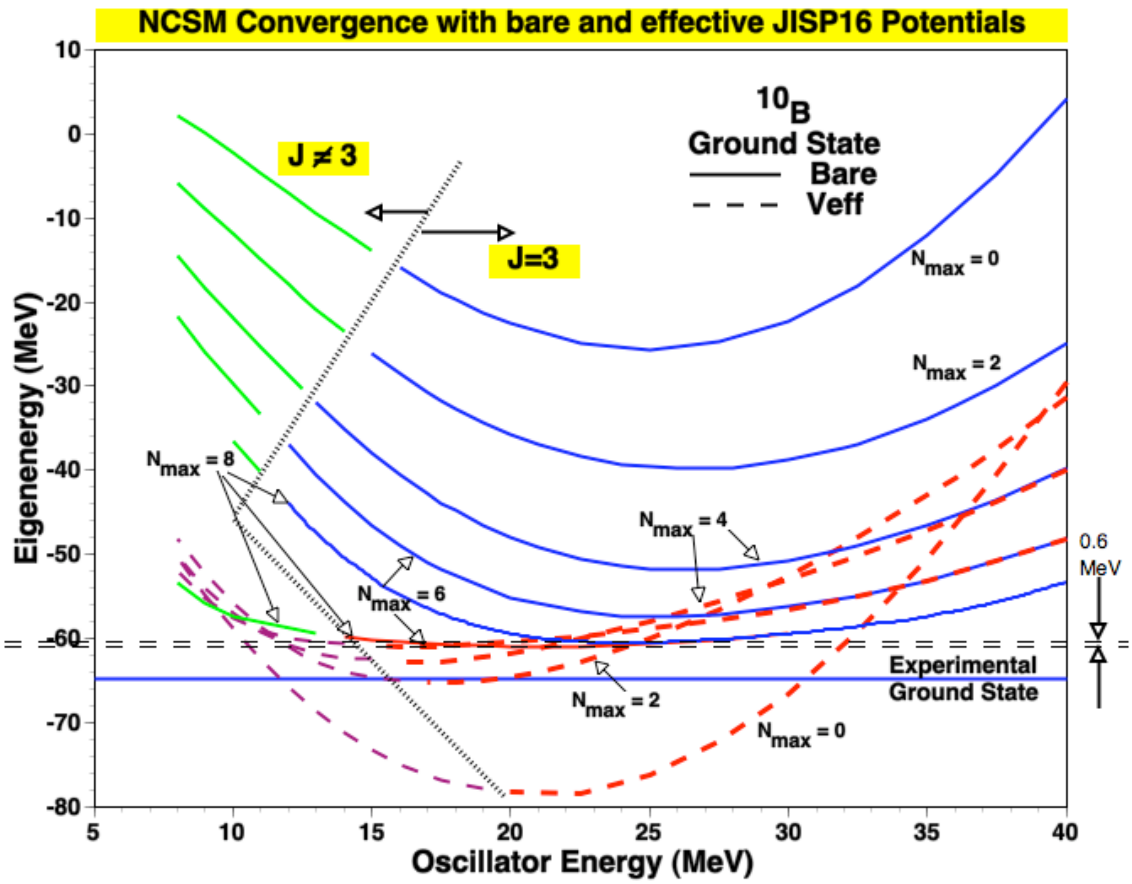

4-He with the JISP16 Veff Interaction
First Excited State Eigenvalue
Ground State Eigenvalue
Eigenenergy (MeV)
hw (MeV)

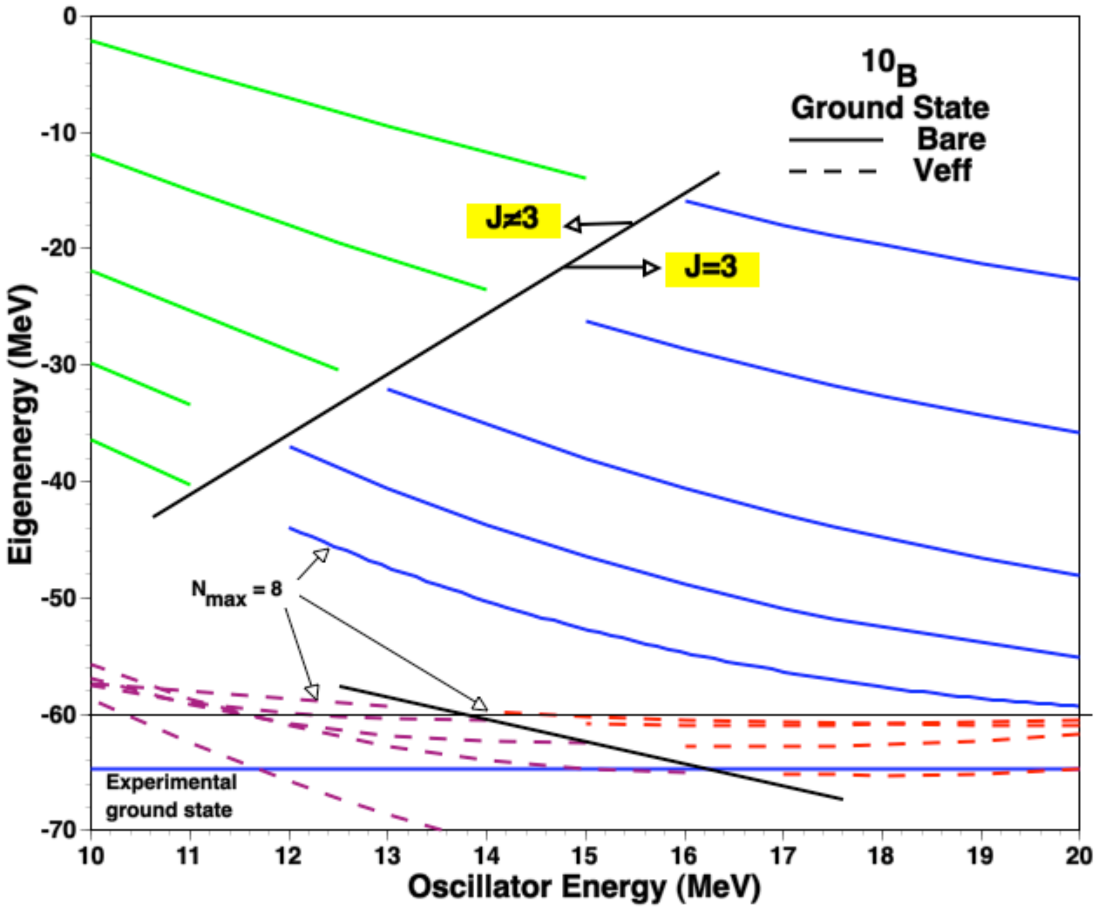

4-He ground state point proton RMS with the JISP16 (Bare)
0hw
2hw
4hw
6hw
8hw
10hw
12hw
14hw
Experiment
Estimated extrapolation of theoretical results
RMS (fm)
hw (MeV)



Ground state energy E_{gs} and excitation energies E_x (in MeV), ground state point-proton rms radius r_p (in fm) and quadrupole moment Q (in $e \cdot \text{fm}^2$) of the ^{6}Li nucleus; $\hbar\omega = 17.5$ MeV.

Interaction Method	Nature	JISP6 NCSM, $10\hbar\omega$ [6]	JISP16 NCSM, $12\hbar\omega$	AV8'+TM' NCSM, $6\hbar\omega$ [2]	AV18+UIX GFMC [8,15]	AV18+IL2 GFMC [10,15]
$E_{gs}(1_1^+,0)$	-31.995	-31.48	-31.00	-31.04	$-31.25(8)$	$-32.0(1)$
r_p	2.32(3)	2.083	2.151	2.054	2.46(2)	2.39(1)
Q	$-0.082(2)$	-0.194	-0.0646	-0.025	$-0.33(18)$	$-0.32(6)$
$E_x(3^+,0)$	2.186	2.102	2.529	2.471	2.8(1)	2.2
$E_x(0^+,1)$	3.563	3.348	3.701	3.886	3.94(23)	3.4
$E_x(2^+,0)$	4.312	4.642	5.001	5.010	4.0(1)	4.2
$E_x(2^+,1)$	5.366	5.820	6.266	6.482		5.5
$E_x(1_2^+,0)$	5.65	6.86	6.573	7.621	5.1(1)	5.6

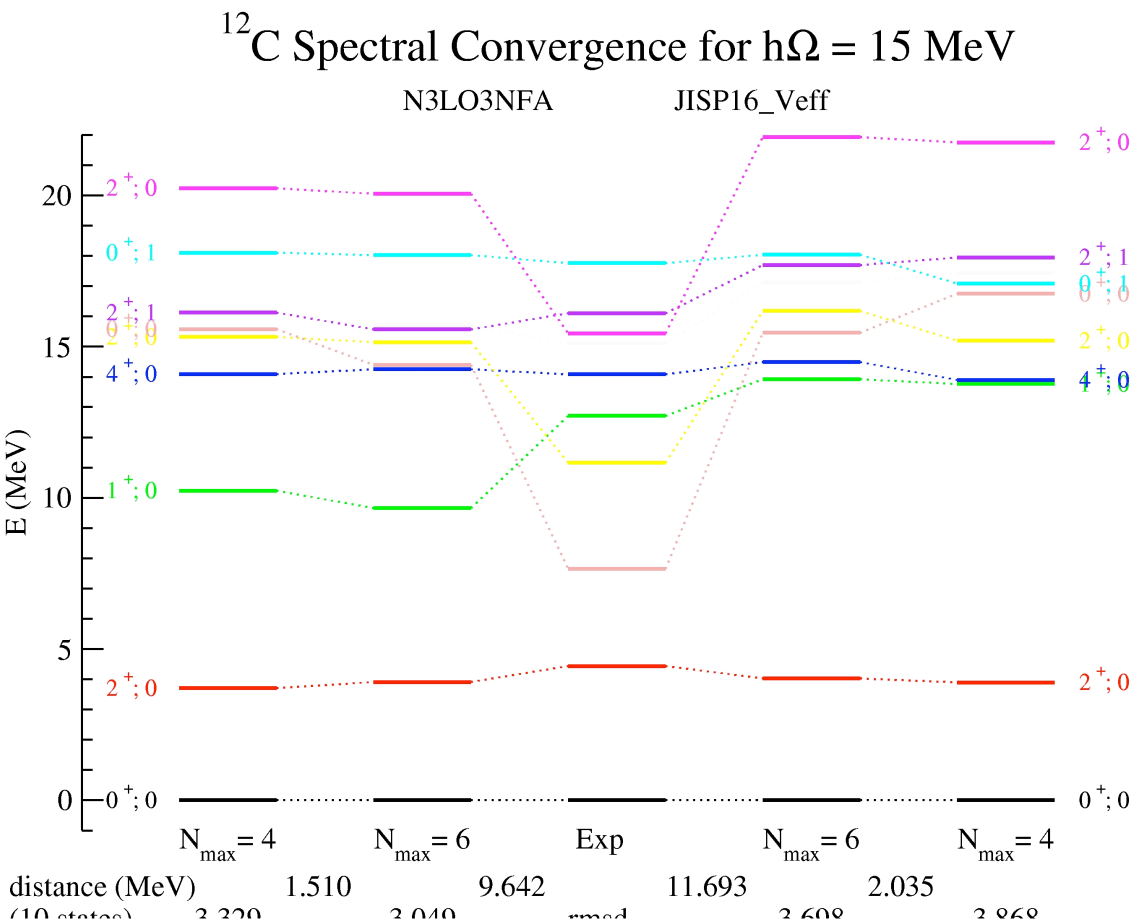

NCSM Convergence with bare and effective JISP16 Potentials
10B
Ground State
Bare
Veff
J ≠ 3
J=3
Nmax = 0
Nmax = 2
Nmax = 4
Nmax = 6
Nmax = 8
0.6
MeV
Experimental
Ground State
Eigenenergy (MeV)
Oscillator Energy (MeV)

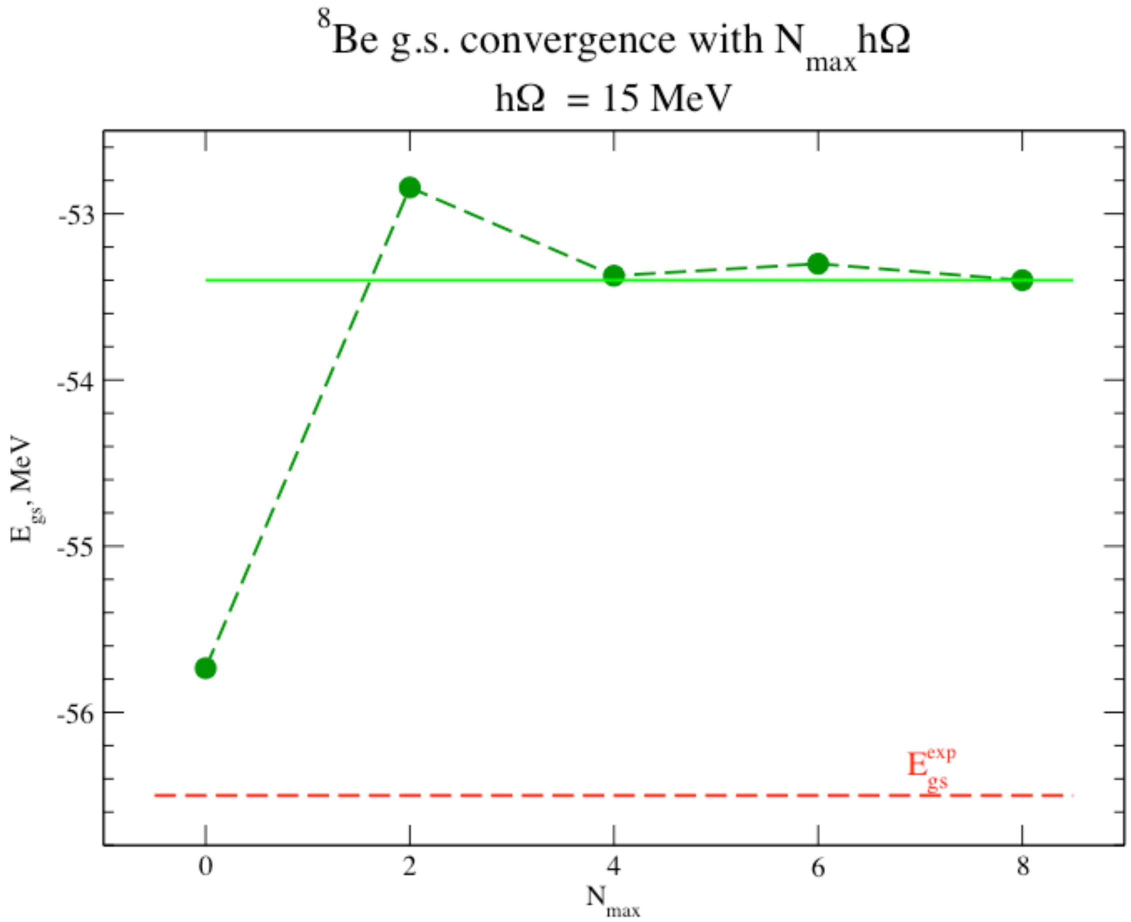

10B
Ground State
Bare
Veff
J≠3
J=3
Nmax = 8
Experimental
ground state
Eigenenergy (MeV)
Oscillator Energy (MeV)

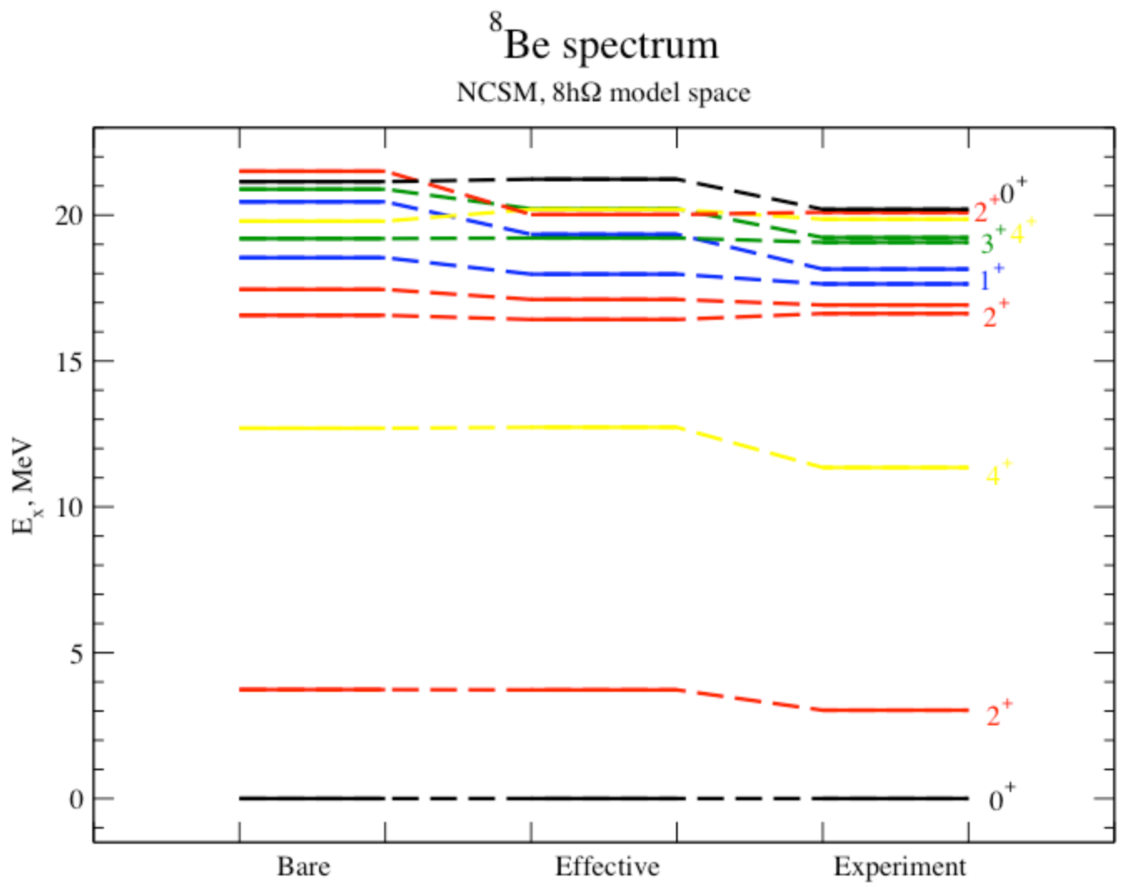
^{10}B

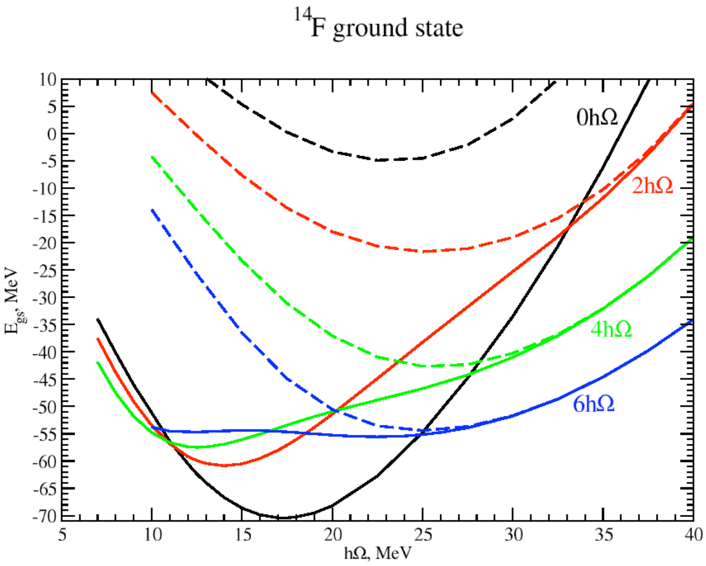
Potential Approach	Nature	JISP16 NCSM, $8\hbar\omega$[a]	AV8'+TM' NCSM, $4\hbar\omega$[b]	AV18+IL2 GFMC[c]	ChPT NCSM, $6\hbar\omega$[d]
$E_{gs}(3_1^+,0)$	-64.751	-60.14	-60.57	$-65.6(5)$	-64.78
r_p	2.30(12)	2.168	2.168	2.33(1)	2.197
Q	+8.472(56)	6.484	+5.682	+9.5(2)	+6.327
$E_x(1_1^+,0)$	0.718	0.555	0.340	0.9	0.523
$E_x(0^+,1)$	1.740	1.202	1.259		1.279
$E_x(1_2^+,0)$	2.154	2.379	1.216		1.432
$E_x(2_1^+,0)$	3.587	3.721	2.775	3.9	3.178
$E_x(3_2^+,0)$	4.774	6.162	5.971		6.729
$E_x(2_1^+,1)$	5.164	5.049	5.182		5.315
$E_x(2_2^+,0)$	5.92	5.548	3.987		4.835
$E_x(4^+,0)$	6.025	5.775	5.229	5.6	5.960
$E_x(2_2^+,1)$	7.478	7.776	7.491		7.823
$B(E2;1_1^+0\rightarrow 3_1^+0)$	4.13(6)	3.317	1.959		3.05
$B(E2;1_2^+0\rightarrow 3_1^+0)$	1.71(26)	0.627	1.010		0.50
$B(\mathrm{GT};3_1^+0\rightarrow 2_1^+1)$	0.083(3)	0.042	0.066		0.07
$B(\mathrm{GT};3_1^+0\rightarrow 2_2^+1)$	0.95(13)	1.652	1.291		1.22

[a] A.M.Shirokov, J.P.Vary, A.I.Mazur, T.A.Weber, Phys. Lett. **B644**, 33 (2007).
[b] P. Navrátil, W. E. Ormand, Phys. Rev. **C 68**, 034305 (2003).
[c] S. C. Pieper, K. Varga, R. B. Wiringa, Phys. Rev. **C 66**, 044310 (2002).
[d] P. Navrátil, V. G. Gueorguiev, J. P. Vary, W. E. Ormand, A. Nogga, Phys. Rev. Lett. **99**, 042501 (2007).

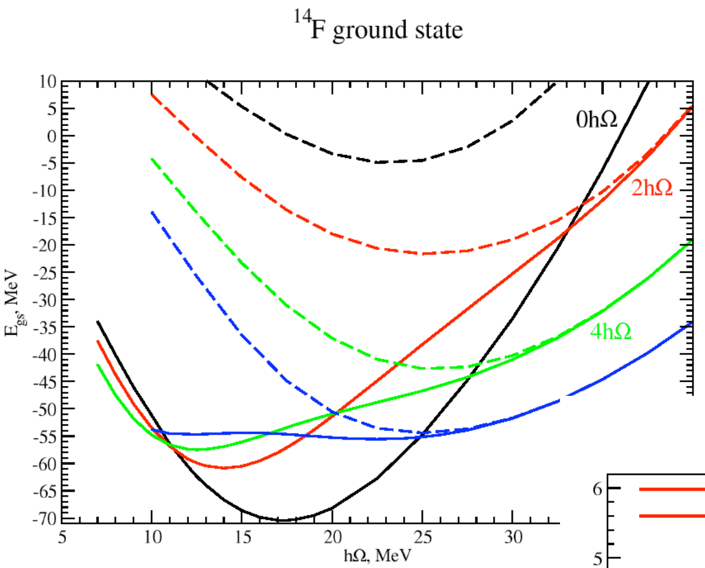



^{8}Be g.s. convergence with N_{max}hΩ
hΩ = 15 MeV
-53
-54
-55
-56
E_{gs}, MeV
E_{gs}^{exp}
0
2
4
6
8
N_{max}

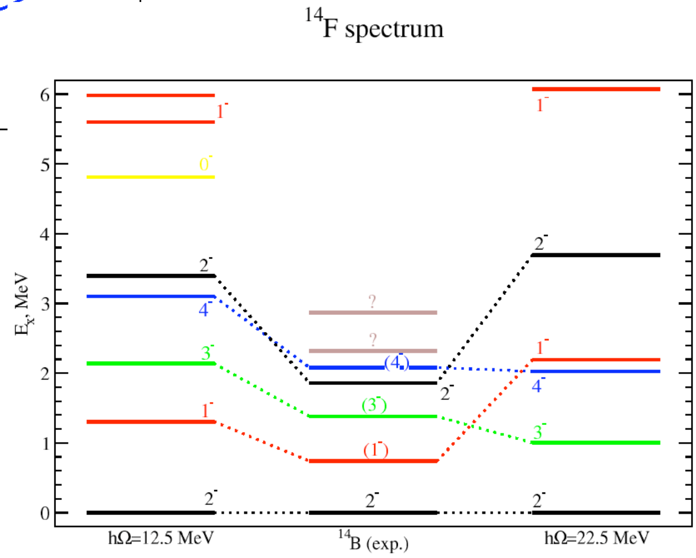

^{8}Be spectrum
NCSM, 8hΩ model space
20
15
10
5
0
E_x, MeV
0^+
2^+
3^+
4^+
1^+
2^+
4^+
2^+
0^+
Bare
Effective
Experiment



14F ground state
0hΩ
2hΩ
4hΩ
6hΩ
E_gs, MeV
hΩ, MeV



14F ground state
0hΩ
2hΩ
4hΩ
E_gs, MeV
hΩ, MeV



14F spectrum
E_x, MeV
hΩ=12.5 MeV
14B (exp.)
hΩ=22.5 MeV

Role of *NNN* force?

- W. Polyzou and W. Glöckle theorem (Few-body Syst. **9**, 97 (1990)):

  $$H=T+V_{ij} \Longrightarrow H'=T+V'_{ij}+V_{ijk},$$

  where V_{ij} and V'_{ij} are phase-equivalent, H and H' are isospectral.

  Hope:

  $$H'=T+V'_{ij}+V_{ijk} \Longrightarrow H=T+V_{ij}$$

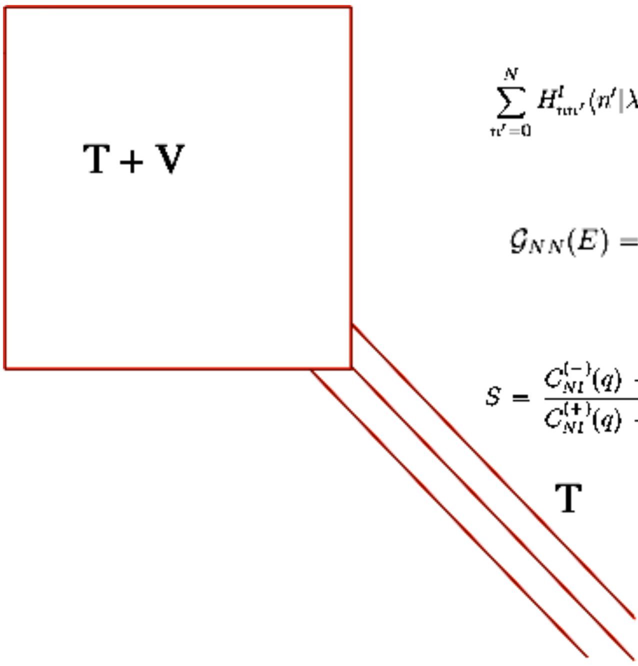
  with (approximately) isospectral H and H' .

  JISP16 seems to be *NN* interaction minimizing *NNN* force.

  Without *NNN* force calculations are simpler, calculations are faster, larger model spaces become available.

J-matrix formalism:

scattering in the oscillator basis

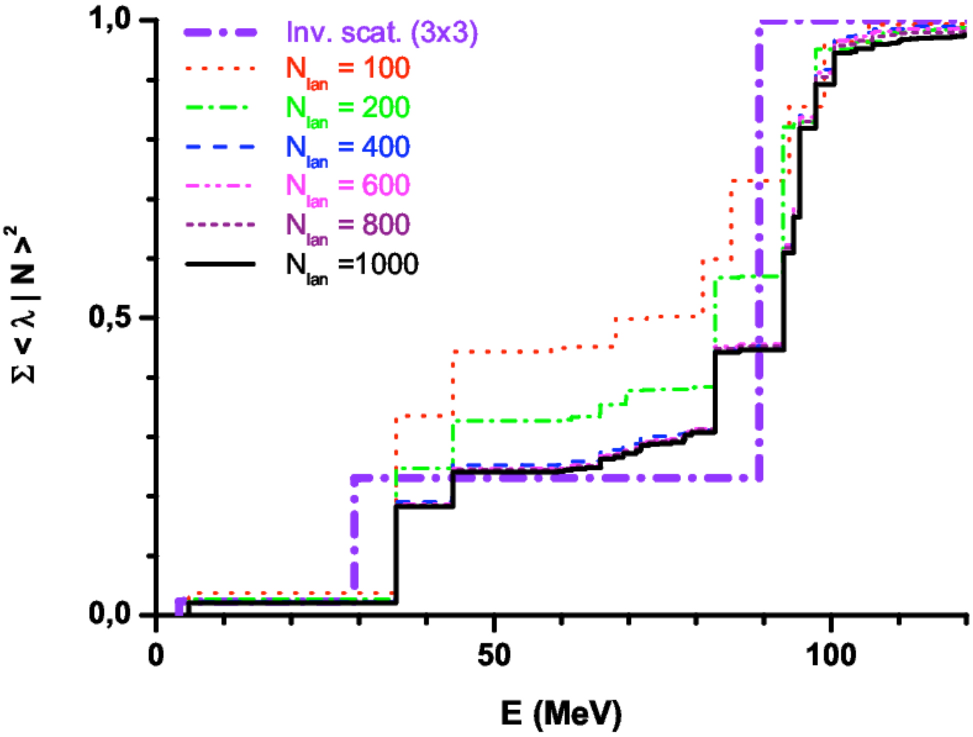


$$\sum_{n'=0}^{N} H^l_{nn'}\langle n'|\lambda\rangle = E_\lambda\langle n|\lambda\rangle, \qquad n\le N.$$

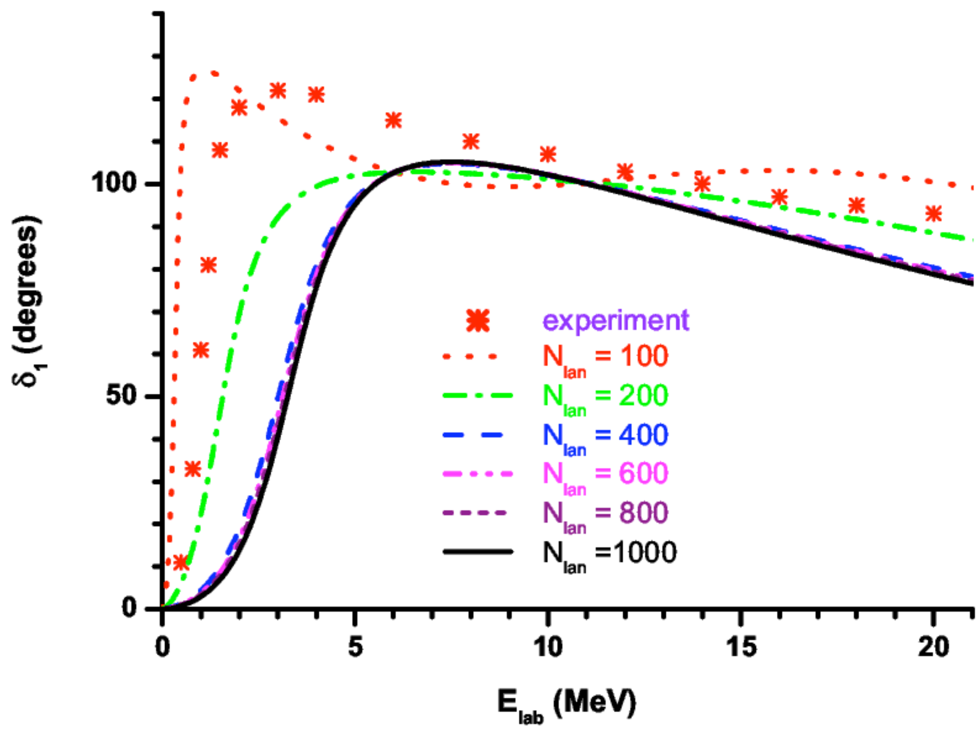
$$\mathcal{G}_{NN}(E) = -\sum_{\lambda=0}^{N}\frac{\langle N|\lambda\rangle^2}{E_\lambda - E},$$

$$S = \frac{C^{(-)}_{Nl}(q) - \mathcal{G}_{NN}(E)T^l_{N,N+1}C^{(-)}_{N+1,l}(q)}{C^{(+)}_{Nl}(q) - \mathcal{G}_{NN}(E)T^l_{N,N+1}C^{(+)}_{N+1,l}(q)},$$

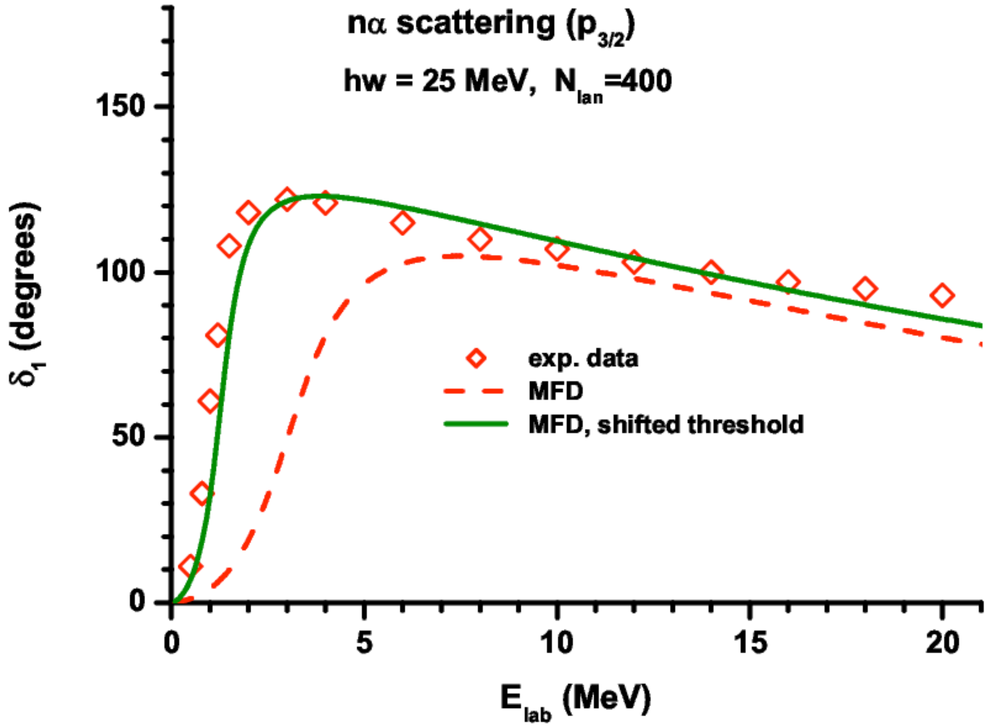
$n\alpha$ scattering



$n\alpha$ scattering

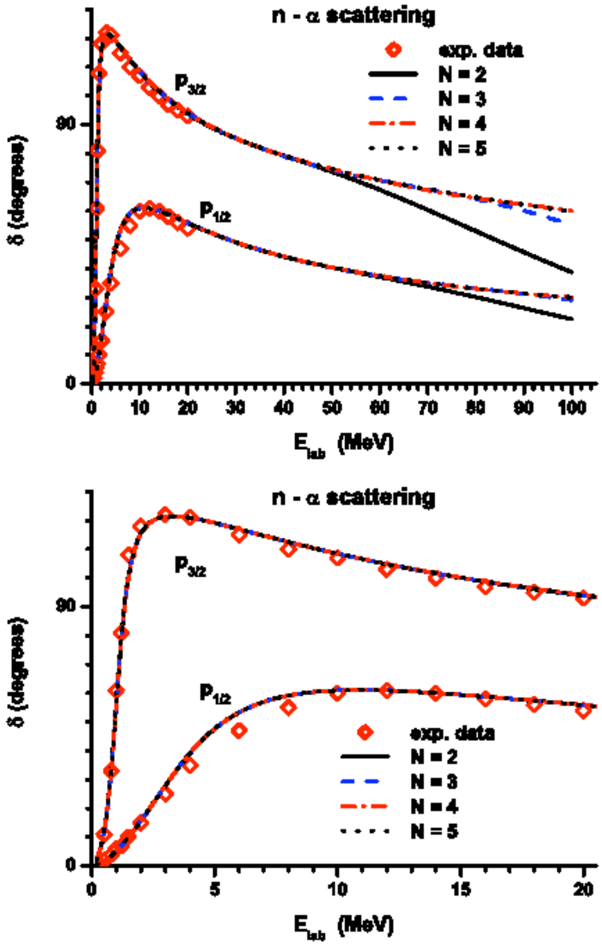


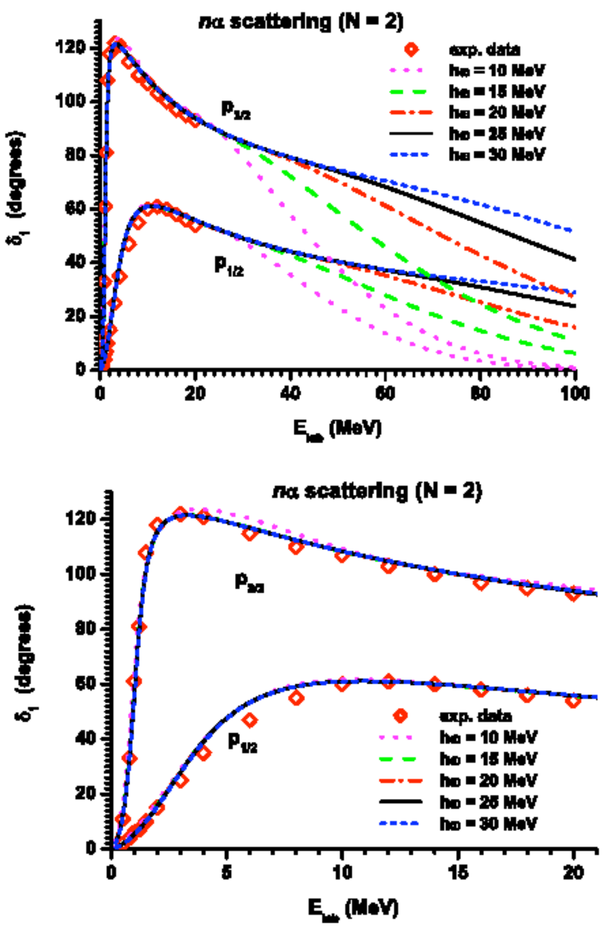
nα scattering



nα inverse scattering

- *J*-matrix inverse *N*-nucleus scattering analysis can be used to derive resonance parameters (position, width)
- *J*-matrix inverse *N*-nucleus scattering analysis suggests values for resonant and non-resonant states that should be compared with obtained in NCSM


n - α scattering
exp. data
N = 2
N = 3
N = 4
N = 5
P3/2
P1/2
δ (degrees)
E_lab (MeV)
n - α scattering
P3/2
P1/2
exp. data
N = 2
N = 3
N = 4
N = 5
δ (degrees)
E_lab (MeV)


nα scattering (N = 2)
exp. data
hω = 10 MeV
hω = 15 MeV
hω = 20 MeV
hω = 25 MeV
hω = 30 MeV
P3/2
P1/2
δ1 (degrees)
E_lab (MeV)
nα scattering (N = 2)
P3/2
P1/2
exp. data
hω = 10 MeV
hω = 15 MeV
hω = 20 MeV
hω = 25 MeV
hω = 30 MeV
δ1 (degrees)
E_lab (MeV)

$n\alpha$ inverse scattering

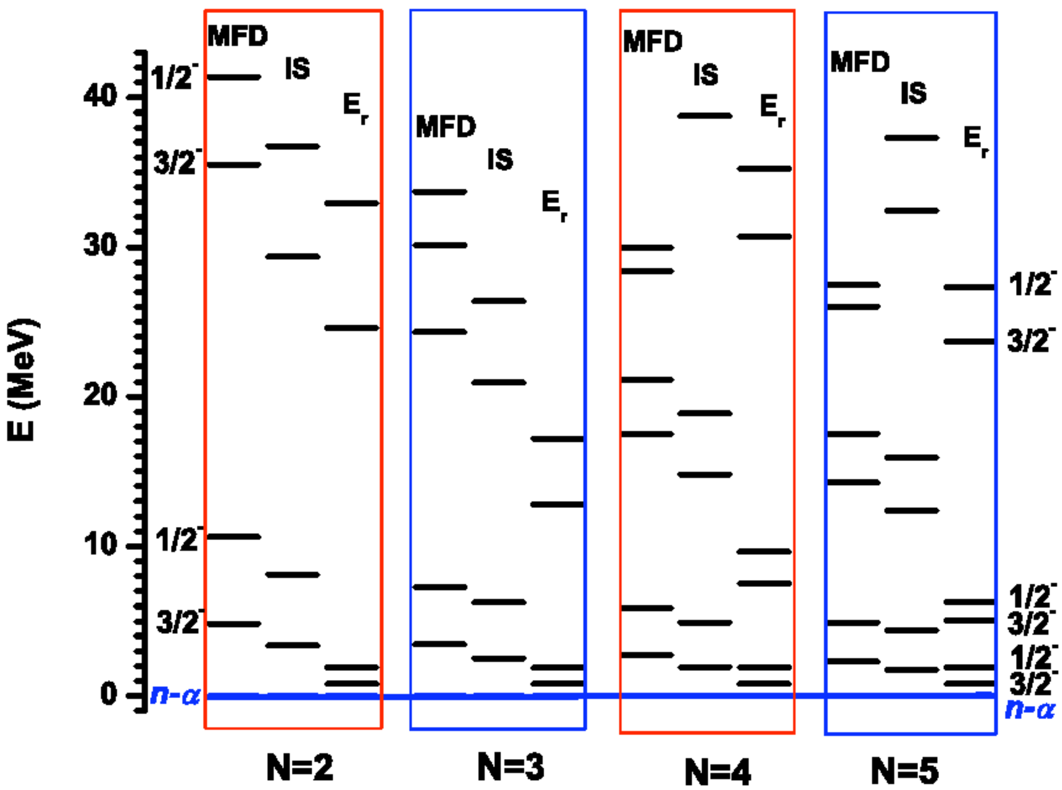
Table 1: S-matrix poles for $3/2^-$ states (energies and widths are in MeV).

	$N=2$		$N=3$		$N=4$		$N=5$	
	E_r	Γ	E_r	Γ	E_r	Γ	E_r	Γ
1	0.797	0.746	0.800	0.700	0.800	0.700	0.801	0.700
2	24.564	60.728	12.805	53.480	7.520	44.682	5.041	42.316
3	—	—	49.584	72.358	30.687	62.910	23.693	59.476
R-matrix (A. Csótó, G.M. Hale. Phys. Rew. C55 (1997) 536): $E_r=0.80$, $\Gamma=0.65$								

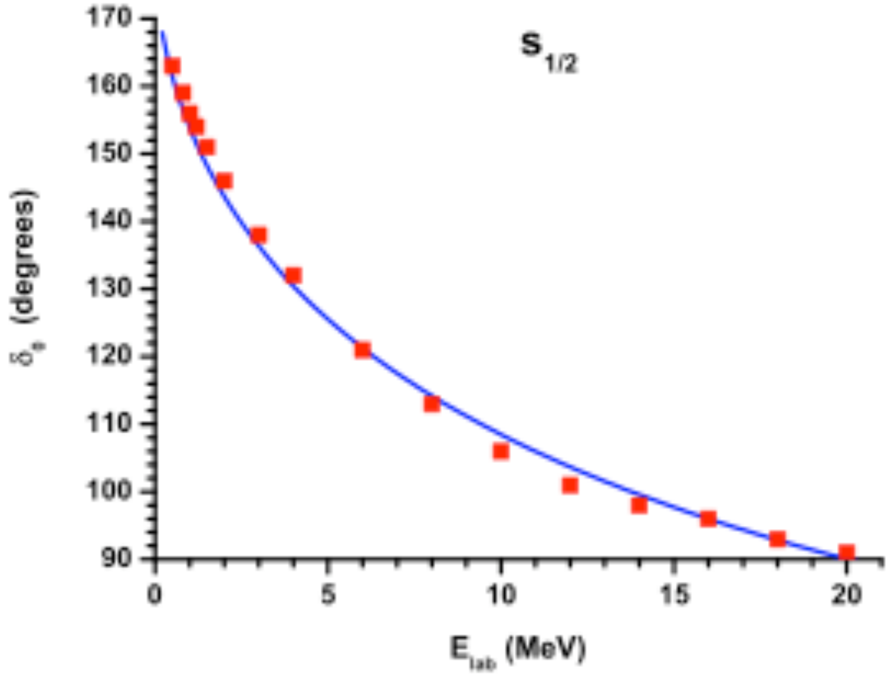
Table 2: S-matrix poles for $1/2^-$ states (energies and widths are in MeV).

	$N=2$		$N=3$		$N=4$		$N=5$	
	E_r	Γ	E_r	Γ	E_r	Γ	E_r	Γ
1	1.894	5.254	1.898	5.264	1.901	5.266	1.902	5.266
2	32.950	80.408	17.185	67.992	9.651	55.774	6.299	52.252
3	—	—	—	—	35.231	75.304	27.297	70.006
R-matrix (A. Csótó, G.M. Hale. Phys. Rew. C55 (1997) 536): $E_r=2.07$, $\Gamma=5.57$								

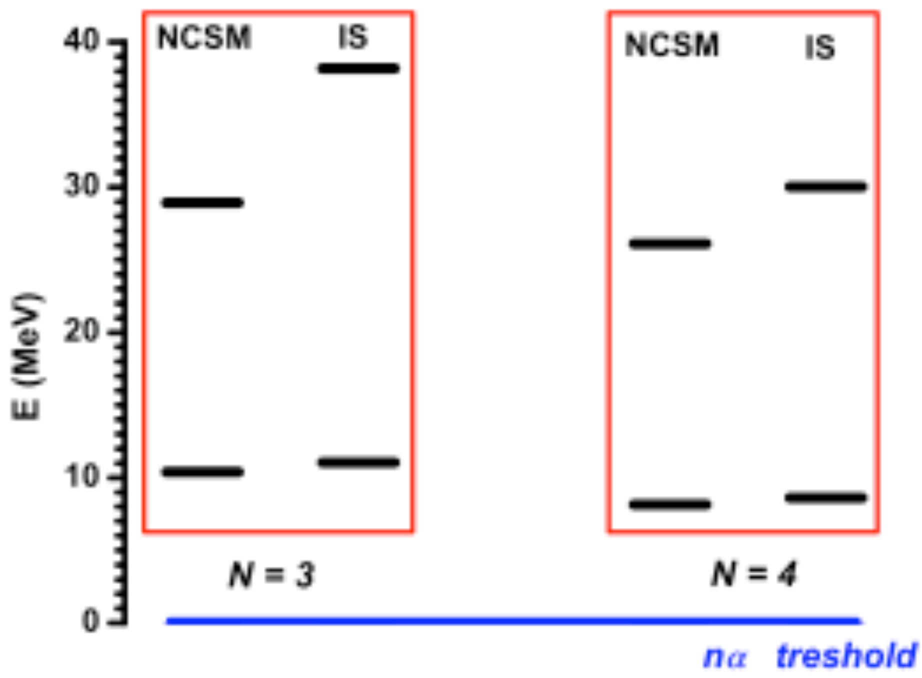
$n\alpha$ inverse scattering and NCSM



$n\alpha$ inverse scattering



$n\alpha$ inverse scattering and NCSM



Conclusions

- JISP16 provides a realistic description of two-body and many-body properties, comparable with modern realistic *NN* + *NNN* forces
- Convergence of NCSM calculations with JISP16 is faster, even the bare JISP16 calculation convergence is reasonable, i.e. the results are more reliable. A confidence region of the binding energy predictions can be obtained for many nuclei by comparing the bare and effective interaction results
- Combining *J*-matrix formalism and NCSM one can perform scattering calculations with bare *NN* interaction; using JISP16 we describe well $n\alpha$ scattering. *J*-matrix inverse *N*-nucleus scattering analysis suggests values for resonant and non-resonant states that should be compared with obtained in NCSM

Plans

- JISP16 improvement by the fit to the same nuclei
- Charge-dependent JISP16
- Extending the calculations to the *sd* shell
- NCSM + J-matrix: Scattering calculations