A Covariant, Chiral, Effective Field Theory for Nuclei

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- Outline
  - ∗ The fundamental theory of nuclei is Quantum ChromoDynamics. It contains "colored" quarks and gluons.
  - ∗ But we can't solve QCD for ordinary nuclei! (Nor do we really want to: it's more complicated than necessary.)
  - ∗ So how do we simplify the problem to make progress?
  - ∗ Use Effective Field Theory (EFT)
    - ∘ Basic principles are common to many areas of physics.
    - ∘ Include dynamics explicitly at large distances and parametrize short-range physics generically.
  - ∗ Use Density Functional Theory (DFT)
    - ∘ Concentrate on a subset of observables.
    - ∘ Compute them reliably without the many-body wave function or with a simple one.

Supported in part by NSF and DOE

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Why Use Hadrons?

- We focus on low-energy, long-range physics, and all observables are colorless.
- Hadrons (baryons and mesons) are the actual particles observed in experiments.
- Colored quarks and gluons participate only in intermediate states, and such "off-shell" behavior is unobservable.
- So pick the most efficient degrees of freedom! We have to parametrize the hamiltonian anyway, since we don't know its true form.
- We are interested primarily in the nuclear **many-body** problem, so include "collective" degrees of freedom like scalar and vector fields.
- Only nucleons and pions are "real" (stable) particles; other fields are always virtual and just parameterize the NN interaction (or EM form factors).

Why Impose Lorentz Covariance?

- The scalar and vector mean fields in nuclei are large (several hundred MeV). This is a new energy scale. The scalar and vector fields **cancel** to produce a small binding energy.
  - ∗ Consistent with QCD sum-rule results (size and density dependence).
  - ∗ Consistent with chiral power counting (two-body energy/nucleon is of order ρ_0/f_π^2).
- Large mean fields produce important relativistic interaction effects.
  - ∗ Velocity-dependent NN interaction provides a new saturation mechanism.
  - ∗ Scalar and vector mean fields **add** to produce correct spin-orbit force. (Compare "fine" structure in atoms and nuclei.)
  - ∗ Successful prediction of nucleon–nucleus spin observables in the RIA and energy dependence of the optical potential.
  - ∗ Explains pseudospin symmetry in nuclei.

There really is relativity in nuclei!

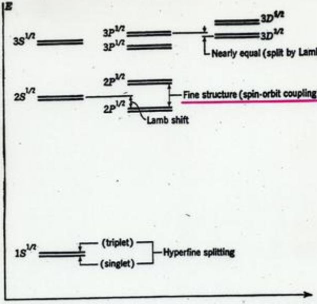


Fig. 4-2 Low-lying energy levels of atomic hydrogen. The diagram is not drawn to scale.

9

of $j = l + \frac{1}{2}$ are filled by one particle each and later the $2l$ states of $j = l - \frac{1}{2}$. The level sequence, as modified by strong spin-orbit coupling, is shown in Fig. IV.3.

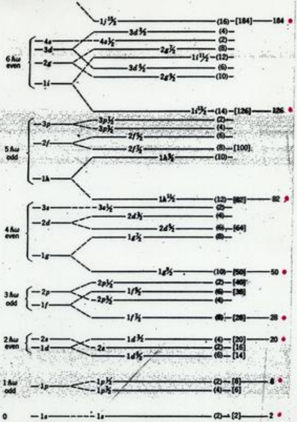


Fig. IV.3. Schematic diagram of nuclear level systems with spin-orbit coupling.

Why Use Effective Field Theory?

Lorentz-covariant hadronic field theories $\equiv$ Quantum HadroDynamics

- Interpret QHD lagrangians as nonrenormalizable $\mathcal{L}_{\text{EFT}}$'s
  - * known long-range interactions constrained by symmetries;
  - * a complete set of generic short-range interactions;
  - * the borderline is characterized by breakdown scale Λ of EFT. For QHD, $\Lambda \approx 600$ MeV (empirically).
- When based on a local, Lorentz-invariant lagrangian density, EFT is the most general way to parameterize observables consistent with the principles of quantum mechanics, special relativity, unitarity, cluster decomposition, microscopic causality, and the desired internal symmetries.
- It's not necessary to derive $\mathcal{L}$ from QCD
  - * Use a general $\mathcal{L}$ that respects the symmetries.
  - * By construction, this provides a general parametrization for energies $\lesssim \Lambda$ (remove redundancies).
- The freedom to redefine and transform the fields $\Longrightarrow$ infinitely many representations of low-energy QCD physics

Strategy

- Assign an index to each term in the lagrangian: $\nu = d + n/2 + b$.
  - $*$ $d =$ number of derivatives (except on nucleons).
  - $*$ $n =$ number of nucleon fields.
  - $*$ $b =$ number of non-Goldstone bosons.
- Organize $\mathcal{L}$ in powers of ν and truncate; this gives a reliable expansion in inverse powers of a "heavy" mass scale $\Lambda \approx M$.

Fields

- Nucleon (N), Lorentz scalar (ϕ = "sigma") [chiral scalar]
- Lorentz vector (V_μ = "omega" ; $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$) ["]
- Pion: $U \equiv \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi)\,, \quad \xi \equiv \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\,/2f_\pi)\,,$

  together with $a_\mu \equiv -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)\,,$

  $$v_\mu \equiv -\frac{i}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)\,, \quad v_{\mu\nu} \equiv -i[a_\mu, a_\nu]\,.$$
- Rho: $\rho_\mu \equiv \frac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\,, \quad D_\mu\rho_\nu \equiv \partial_\mu\rho_\nu + i[v_\mu, \rho_\nu]\,,$

  $$\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu + i\,\overline{g}_\rho[\rho_\mu, \rho_\nu]\,.$$

Modern EFT Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QHD}} &= \mathcal{L}_N + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_M \\
&= \overline{N}\left(i\gamma^\mu\left[D_\mu + ig_\rho \rho_\mu + ig_v V_\mu\right] + g_A \gamma^\mu \gamma_5 a_\mu - M + g_s\phi\right) N \\
&- \frac{f_\rho g_\rho}{4M}\overline{N}\rho_{\mu\nu}\sigma^{\mu\nu}N - \frac{f_v g_v}{4M}\overline{N}V_{\mu\nu}\sigma^{\mu\nu}N - \frac{\kappa_\pi}{M}\overline{N}v_{\mu\nu}\sigma^{\mu\nu}N \\
&+ \frac{4\beta_\pi}{M}\overline{N}N\,\mathrm{Tr}\left(a_\mu a^\mu\right) + \mathcal{L}_{\pi N}^{(4)} \\
&+ \tfrac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \tfrac{1}{4}f_\pi^2\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) \\
&- \tfrac{1}{2}\,\mathrm{Tr}\left(\rho_{\mu\nu}\rho^{\mu\nu}\right) - \tfrac{1}{4}\,V_{\mu\nu}V^{\mu\nu} \\
&- g_{\rho\pi\pi}\frac{2f_\pi^2}{m_\rho^2}\,\mathrm{Tr}\left(\rho_{\mu\nu}v^{\mu\nu}\right) + \tfrac{1}{2}\left(1 + \eta_1\frac{g_s\phi}{M} + \frac{\eta_2}{2}\frac{g_s^2\phi^2}{M^2}\right)m_v^2 V_\mu V^\mu \\
&+ \tfrac{1}{4!}\,\zeta_0 g_v^2 (V_\mu V^\mu)^2 + \left(1 + \eta_\rho\frac{g_s\phi}{M}\right)m_\rho^2\,\mathrm{Tr}\left(\rho_\mu\rho^\mu\right) \\
&- m_s^2\phi^2\left(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_s\phi}{M} + \frac{\kappa_4}{4!}\frac{g_s^2\phi^2}{M^2}\right).
\end{aligned}
$$

- $\mathcal{L}_{\mathrm{QHD}}$ contains all nonredundant terms through order $\nu = 4$.
- We see standard noninteracting hadron terms $\oplus$ Yukawa nucleon–meson couplings $\oplus$ anomalous-moment interactions $\oplus$ pion–nucleon and meson nonlinearities: nontrivial dynamics.

Transformation Laws

In our EFT (QHD) lagrangian:

- Chiral $SU(2)_L \times SU(2)_R$ symmetry is nonlinear.
- Isovector subgroup $SU(2)_V$ symmetry is linear.
- These are global symmetries.
- Vector transformations: $L = \exp(i\boldsymbol{\beta}\cdot\boldsymbol{\tau}/2) = R$
- Axial-vector transformations:
  $L = \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2)\,, \qquad R = \exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2)$
- Field transformations: (all objects are matrices)

$$
\begin{aligned}
U(x) &\rightarrow LU(x)R^\dagger\,, \\
\xi(x) &\rightarrow L\xi(x)h^\dagger(x) = h(x)\xi(x)R^\dagger \qquad \text{[defines } h(x)\text{]} \\
N(x) &\rightarrow h(x)N(x) \qquad \text{[generally, } h(x) \text{ is local]} \\
\rho_\mu(x) &\rightarrow h(x)\rho_\mu(x)h^\dagger(x)\,.
\end{aligned}
$$

- Chirally covariant derivatives:

$$
\begin{aligned}
D_\mu N &\equiv (\partial_\mu + iv_\mu)N \;: \qquad D_\mu N \rightarrow h(x)(D_\mu N)\,, \\
D_\mu\rho_\nu &\equiv \partial_\mu\rho_\nu + i[v_\mu, \rho_\nu] \;: \qquad D_\mu\rho_\nu \rightarrow h(x)(D_\mu\rho_\nu)h^\dagger(x)
\end{aligned}
$$

Discussion

- To realize the nonlinear $SU(2)_L \times SU(2)_R$ symmetry, the lagrangian must include pions explicitly.
- Note that U, ξ, and ρ_μ are 2×2 matrices.
- For isospin transformations, $L = R = h$ (constants); the transformations are linear.
- For general transformations: $L \neq R$. [Axial transformations have $L = R^\dagger$.]
  - Now $h(x)$ is nontrivial and contains pion fields.
  - So $h(x)N(x)$ mixes nucleons with any number of pions: the transformation is nonlinear.
- The only field or tensor that transforms inhomogeneously is $v_\mu \to h v_\mu h^\dagger - i h \partial_\mu h^\dagger$. This allows for the construction of chirally covariant derivatives.
- This is NOT the linear sigma model; the scalar field ϕ is a chiral scalar. It is NOT the chiral partner of the pion.

Important Things to Remember

- Off-shell behavior is not observable. Choose the dynamical variables that are most efficient.
- Vacuum dynamics involves short-range physics. Don't calculate it, but parametrize it in a few fitted constants. (Computation of hadronic loops $\Longrightarrow$ unnatural coefficients.) Use valence nucleons only.
- Although fields and couplings are local, nucleon substructure is also included:
  - ∗ Example: $\overline{N}N\sigma \rightarrow g(\sigma)\overline{N}N\sigma$
  - ∗ But define: $\phi \equiv g(\sigma)\sigma$, $[g(0) = 1]$; then invert for $\sigma(\phi)$.
  - ∗ Then: $g(\sigma)\overline{N}N\sigma + p(\sigma) = \overline{N}N\phi + a\phi^2 + b\phi^3 + c\phi^4 + \cdots$
- Nucleon EM structure included in a derivative expansion:

$$\begin{aligned}\mathcal{L}_{\rm EM} &= -\frac{e}{2}\overline{N}A^\mu\gamma_\mu(1+\tau_3)N - \frac{e}{4M}F^{\mu\nu}\overline{N}\{\lambda_s + \lambda_v\tau_3\}\sigma_{\mu\nu}N \\ &\quad - \frac{e}{2M^2}\partial_\nu F^{\mu\nu}\overline{N}(\{\widetilde{\beta}_s + \widetilde{\beta}_v\tau_3\}\gamma_\mu)N \\ &\quad - \frac{e}{M^4}\partial^2\partial_\nu F^{\mu\nu}\overline{N}(\{\widetilde{\delta}_s + \widetilde{\delta}_v\tau_3\}\gamma_\mu)N + \cdots + \text{VMD}\,,\end{aligned}$$

which generates e, λ, $r^{s,v}_{rms}$, ...

This works at long distances (low momenta).

(Naive) Dimensional Analysis: NDA

[Georgi & Manohar, 1984]

- Low-energy QCD is expected to contain two mass scales:

$$f_\pi \approx 93\,\mathrm{MeV}\,, \qquad \Lambda \approx 500 \text{ to } 800\,\mathrm{MeV}$$

- NDA rules for a generic term in the energy functional:

$$C\,[f_\pi^2\Lambda^2]\left[\left(\frac{\overline{N}N}{f_\pi^2\Lambda}\right)^{\ell}\frac{1}{m!}\left(\frac{\Phi}{\Lambda}\right)^{m}\frac{1}{n!}\left(\frac{W}{\Lambda}\right)^{n}\left(\frac{\partial}{\Lambda}\right)^{p}\right]$$

- "Naturalness" $\Longrightarrow$ dimensionless C is of order unity.
- Provides expansion parameters at finite density:

$$\frac{\Phi}{\Lambda} \approx \frac{W}{\Lambda} \approx 1/2\,, \qquad \frac{\rho_{\mathrm{s}}}{f_\pi^2\Lambda} \approx \frac{\rho_{\mathrm{B}}}{f_\pi^2\Lambda} \approx 1/5 \quad \text{at } \rho_{\mathrm{B}}^0$$

- Allows truncation and calibration with **quantitatively accurate** fits to bulk nuclear observables or "properties of nuclear matter" (plus "m_{s}" in nuclei).

Density Functional Theory

- Construct the ground-state energy functional from the lagrangian using a mean-field ("factorized") approximation:
  - * A functional of scalar (ρ_s) and baryon (ρ_B) densities.
  - * Lorentz scalar and vector fields are interpreted as Kohn–Sham single-particle potentials. Dirac (quasi)nucleons move in these local potentials.
- Kohn–Sham theorem [1965]: The exact ground-state scalar and vector densities, energy, and chemical potential for the fully interacting many-fermion system can be reproduced by a collection of (quasi)fermions moving in appropriately defined, self-consistent, local, classical fields.
- Mean-field energy functional provides a parametrization of the exact energy functional. Fit the parameters [define a χ^2] to (29) nuclear observables from ^{16}O, ^{40}Ca, ^{48}Ca, ^{88}Sr, and ^{208}Pb. There are more than enough parameters at the typical level of truncation. Parameters encode both short-range (vacuum, QCD) effects and long-range (many-body) effects.

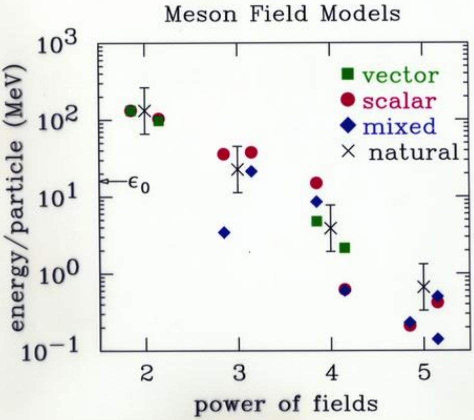
- Kohn–Sham quasi-particle orbitals are tailored to the generation of the ground-state density, so they include exchange, correlation, and short-range effects (approximately).
- Verify naturalness by examining the convergence of the truncation (and make predictions).
- Note the large scalar and vector fields! The scale of the lowest-order term in the energy/particle is given by

$$\rho_{\rm eq}/f_\pi^2 \approx 130\,{\rm MeV}$$

and is independent of Λ. This is a general result!

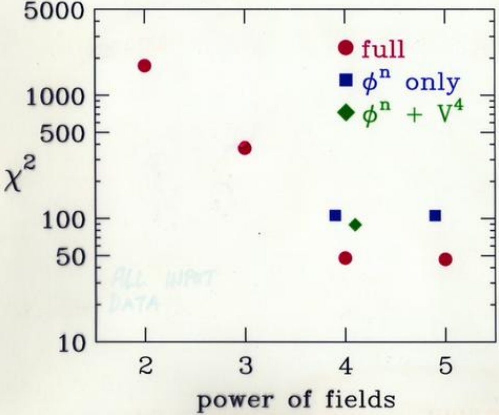
Table 1: Parameter sets from fits to finite nuclei. The parameters in the lower portion of the table are fitted to the (free) nucleon charge and magnetic form factors.

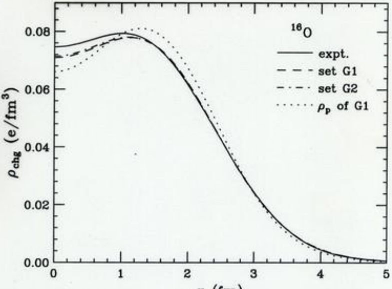
	ν	$W1$	$C1$	$Q1$	$Q2$	$G1$	$G2$
m_s/M	2	0.60305	0.53874	0.53735	0.54268	0.53963	0.55410
$g_s/4\pi$	2	0.93797	0.77756	0.81024	0.78661	0.78532	0.83522
$g_v/4\pi$	2	1.13652	0.98486	1.02125	0.97202	0.96512	1.01560
$g_\rho/4\pi$	2	0.77787	0.65053	0.70261	0.68096	0.69844	0.75467
η_1	3		0.29577			0.07060	0.64992
κ_3	3		1.6698	1.6582	1.7424	2.2067	3.2467
η_ρ	3					−0.2722	0.3901
η_2	4					−0.96161	0.10975
κ_4	4			−6.6045	−8.4836	−10.090	0.63152
ζ_0	4				−1.7750	3.5249	2.6416
α_1	5					1.8549	1.7234
α_2	5					1.7880	−1.5798
$f_v/4$	3					0.1079	0.1734
$f_\rho/4$	3	0.9332	1.1159	1.0332	1.0660	1.0393	0.9619
$\beta^{(0)}$	4	−0.38482	−0.01915	−0.10689	0.01181	0.02844	−0.09328
$\beta^{(1)}$	4	−0.54618	−0.07120	−0.26545	−0.18470	−0.24992	−0.45964



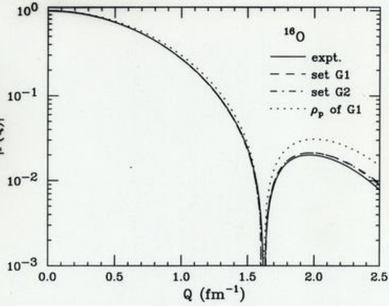
Meson Field Models
energy/particle (MeV)
10^3
10^2
10^1
10^0
10^-1
ε0
vector
scalar
mixed
natural
2
3
4
5
power of fields

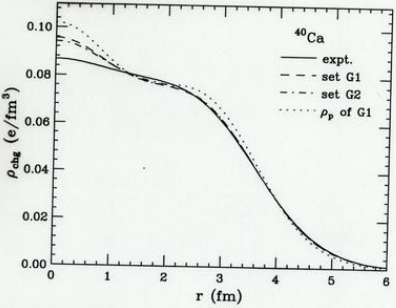
Meson Field Models



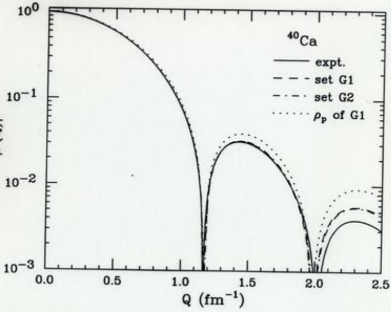


^{16}O
expt.
set G1
set G2
ρ_p of G1
ρ_{chg} (e/fm^3)
r (fm)
0.00
0.02
0.04
0.06
0.08
0
1
2
3
4
5

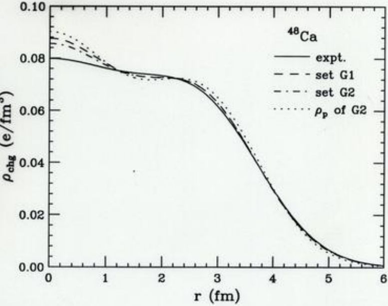

^{16}O
expt.
set G1
set G2
ρ_p of G1
|F(Q)|
Q (fm^{-1})
10^0
10^{-1}
10^{-2}
10^{-3}
0.0
0.5
1.0
1.5
2.0
2.5

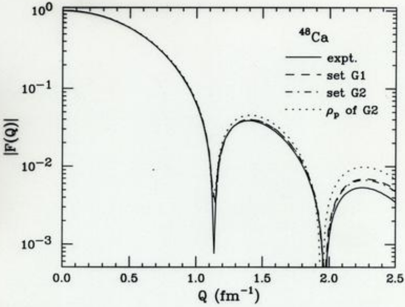


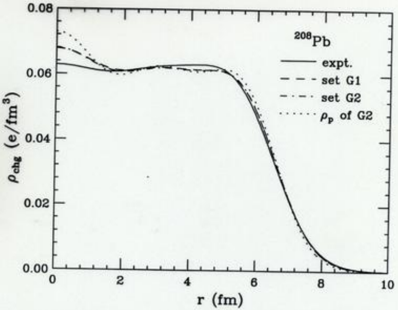
40Ca
expt.
set G1
set G2
ρp of G1
ρchg (e/fm3)
r (fm)
0.00
0.02
0.04
0.06
0.08
0.10
0
1
2
3
4
5
6



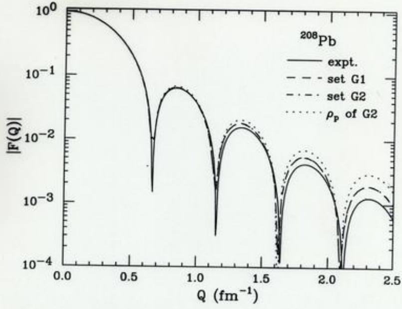
10^0
10^{-1}
10^{-2}
10^{-3}
|F(Q)|
0.0
0.5
1.0
1.5
2.0
2.5
Q (fm^{-1})
^{40}Ca
expt.
set G1
set G2
ρ_p of G1


0.10
0.08
0.06
0.04
0.02
0.00
ρ_{chg} (e/fm^3)
0
1
2
3
4
5
6
r (fm)
^{48}Ca
expt.
set G1
set G2
ρ_p of G2

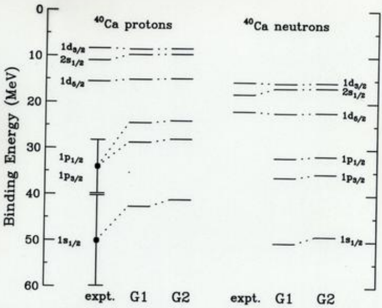

^{48}Ca
expt.
set G1
set G2
ρ_p of G2
10^0
10^{-1}
10^{-2}
10^{-3}
|F(Q)|
0.0
0.5
1.0
1.5
2.0
2.5
Q (fm^{-1})



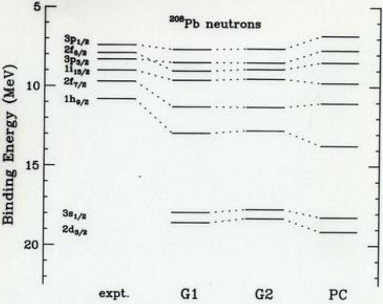
0.08
0.06
0.04
0.02
0.00
ρ_{chg} (e/fm^3)
0
2
4
6
8
10
r (fm)
^{208}Pb
expt.
set G1
set G2
ρ_p of G2



^{208}Pb
expt.
set G1
set G2
ρ_p of G2
$|F(Q)|$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.0
0.5
1.0
1.5
2.0
2.5
Q (fm^{-1})

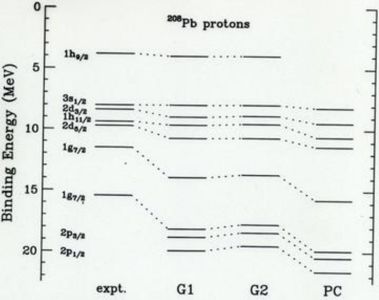


0
10
20
30
40
50
60
Binding Energy (MeV)
^{40}Ca protons
^{40}Ca neutrons
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
expt.
G1
G2



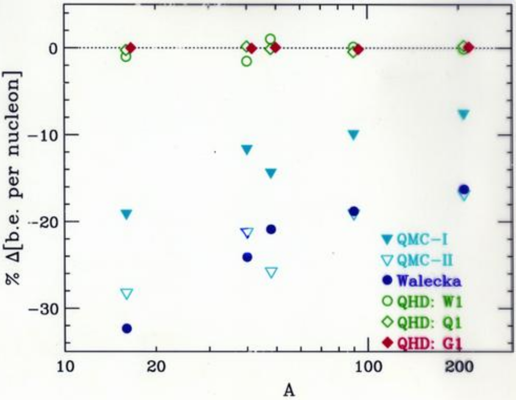
208Pb neutrons
Binding Energy (MeV)
5
10
15
20
3p1/2
2f5/2
3p3/2
1i13/2
2f7/2
1h9/2
3s1/2
2d3/2
expt.
G1
G2
PC

3.2

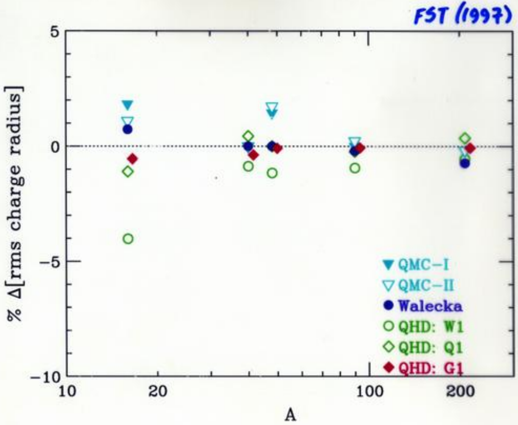


^{208}Pb protons
Binding Energy (MeV)
0
5
10
15
20
$1h_{9/2}$
$3s_{1/2}$
$2d_{3/2}$
$1h_{11/2}$
$2d_{5/2}$
$1g_{7/2}$
$1g_{7/2}$
$2p_{3/2}$
$2p_{1/2}$
expt.
G1
G2
PC

FST (1997)



0
−10
−20
−30
% Δ[b.e. per nucleon]
10
20
100
200
A
QMC−I
QMC−II
Walecka
QHD: W1
QHD: Q1
QHD: G1


FST (1997)
5
0
−5
−10
10
20
100
200
% Δ[rms charge radius]
A
QMC−I
QMC−II
Walecka
QHD: W1
QHD: Q1
QHD: G1
170

HUERTAS (2002)

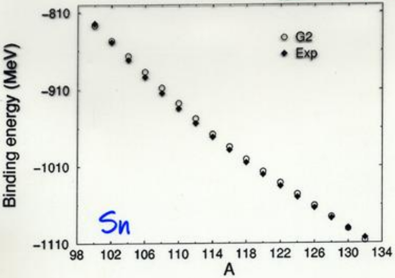


FIG. 3. Comparison between experimental and calculated total binding energies for Sn isotopes using the G2 parameter set.

PREDICTIONS!

HUERTAS (2002)

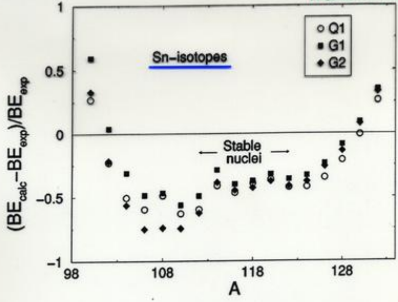


FIG. 4. Percentage deviation of the total binding energy for Sn isotopes using Q1, G1, and G2 parameter sets. The stable isotopes are indicated in the plot.

PREDICTIONS!

HUERTAS (2002)

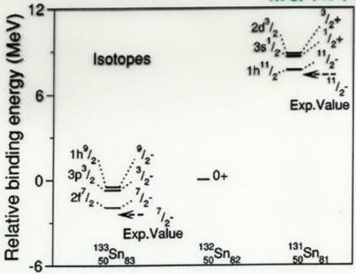


FIG. 5. Level spectrum of isotopes of $^{132}_{50}Sn_{82}$ differing by one neutron.

M. HUERTAS, PHYSICAL REVIEW C **66**, 024318 (2002)

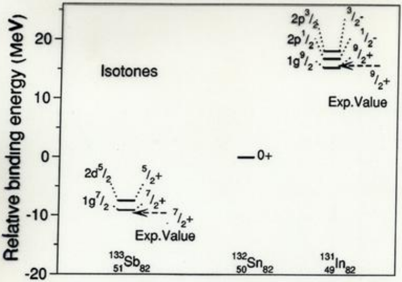


FIG. 6. Level spectrum of isotones of $^{132}_{50}Sn_{82}$ differing by one proton.

Summary

- We described a strong-coupling relativistic quantum field theory for nuclei that is manifestly Lorentz covariant and that embodies the symmetries of QCD.
- The primary focus of this QHD/EFT lagrangian is on the nuclear **many-body** problem.
- One can systematically expand and truncate the EFT lagrangian in powers of the fields and their derivatives.
- The mean-field approximation is really DFT, implemented through Kohn–Sham quasi-particle orbitals. The tested validity and accuracy of our truncation procedure (for fitted and predicted results) shows that we really know something about the energy functional for cold nuclear matter near equilibrium density!
- The energy functional can be extended beyond the mean-field parametrization using well-defined rules of EFT compute loops. And it has been [Hu, McIntire, BDS (2000, 2007)].
- The QHD/EFT/DFT/KS formalism provides a true representation of QCD in the low-energy nuclear domain.