

# Halo Systems in Medium-Mass Nuclei : A New Analysis Method

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# Outline

- 1 Introduction
- 2 Current Situation in Medium-Mass Nuclei
  - Importance of Halo Configurations for the Nuclear EDF
  - Limitations of Current Approaches
- 3 A New Analysis Method
  - Properties of the Intrinsic One-Body Density
  - Model-Independent Definition of Nuclear Halos
  - Robust Criteria for Halo Formation
- 4 First Results
  - Cr Isotopes
  - Sn Isotopes
  - Systematics over Spherical Nuclei
  - Extensions and Limits
- 5 Conclusion

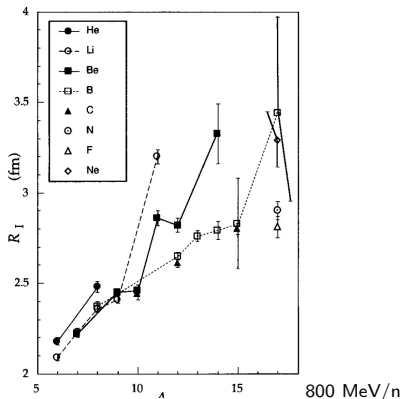
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# Halos in Light Nuclei

- Low density tail extending out to large distance
- Direct consequence of small nucleon separation energy / drip-line physics
- First observed experimentally in  $^{11}\text{Li}$  and  $^{11}\text{Be}$   
[I. Tanihata *et al*, Phys. Rev. Lett. **55** (1985) 2676 ; Phys. Lett. **206B** (1988) 592]
- Various experimental signatures

- Interaction radii/cross sections
- Momentum distribution
- ...



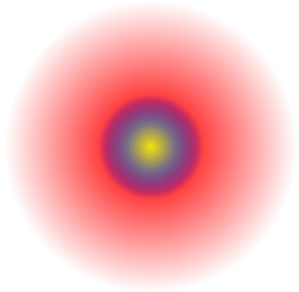
[I. Tanihata, Nucl. Phys. **A520** (1990) 411c-425c]

[B. Blank *et al.*, Z. Phys. **A343** (1992) 343-375]

[E. Arnold *et al.*, Phys. Lett. **B281** (1992) 16]

# Light Nuclei : Few-Body Models

- Halo degrees of freedom decoupled from the core
- Problem reduces to **2 or 3-body interacting clusters**
- Exact dynamics through Schrödinger or Faddeev equations



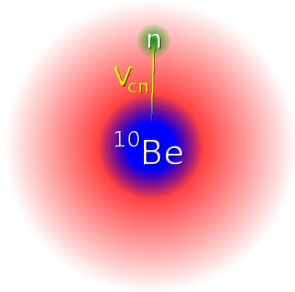
## Rule of thumb characterization for halo states :

[A.S. Jensen, M.V. Zhukov, Nucl. Phys. A693 (2001), 411]

- Halo extension:
  - > 50% probability in classically forbidden region
- Dominating cluster structure:
  - > 50% of the actual configuration

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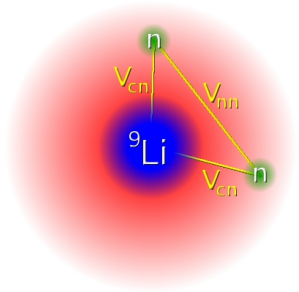
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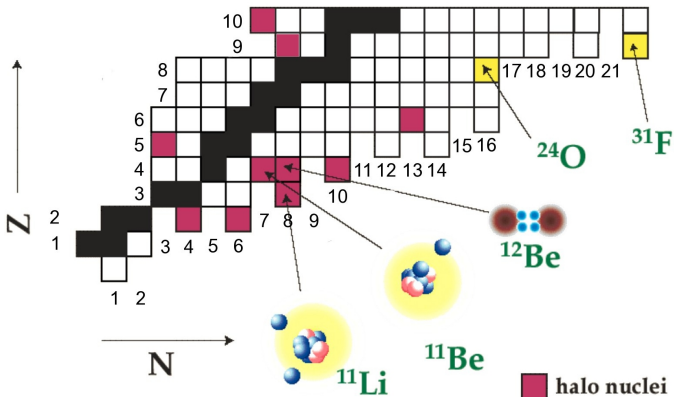
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# Known Light Halo Nuclei

- Experimental evidence for several proton and neutron light halo nuclei
- Ground and excited states



[RIA White Paper, 2005]

Cluster vision for light systems : valid for heavier masses ?

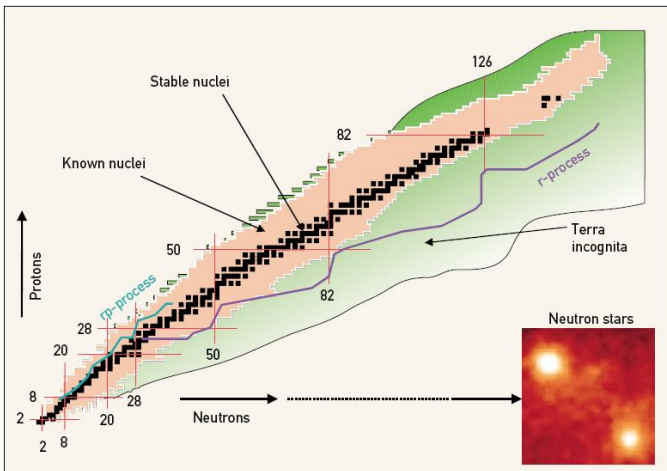


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# Challenges for Nuclear Energy Density Functional (EDF)

- EDF theory : appropriate for **mid- to heavy mass nuclei** ( $A > 40$ )
- EDF behavior at **small/surface density** / large asymmetry not under control





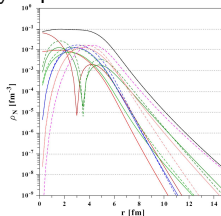
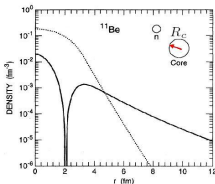
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- Potential use of halo structures to constrain current EDF ?
  - Surface physics: **low density configurations**
  - Surface physics : **gradient versus density dependence**
  - Drip-line phenomenon : **large isospin asymmetry**
  - Drip-line phenomenon : **shell evolution at low separation energy**
  - Pairing functional : constraints at low density/large asymmetry
- **Collective behaviors**: Cluster vision not really expected

Halo definition expected to change...

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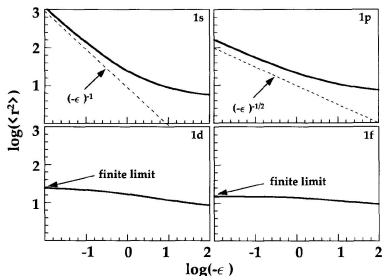
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# HFB calculations in spherical symmetry

- **LyHF spherical** HFB code [K. Bennaceur, INPL/ESNT, France]
- Discrete continuum in 40 fm spherical box
- **Even-even** nuclei : no time-reversal invariance breaking
  
- Particle-hole channel : **SLy4 functional**  
[E. Chabanat *et al.*, Nucl. Phys. **A635** (1998) 231-256]
- Particle-particle channel : DDDI functional

# Importance of low- $\ell$ states

- **Divergence of r.m.s. radii** for  $\ell = 0, 1$  weakly bound systems  
[K. Riisager *et al.*, Nucl. Phys. **A548** (1992) 393] - [T. Mitsu *et al.*, Nucl. Phys. **A614** (1997) 44]
- Focus on the evolution of the r.m.s. radius to predict halos
- Prerequisites : **presence + occupation of s/p orbitals**
- Higher order moments  $\langle r^n \rangle$  diverge for higher  $\ell$  in weak binding limit  $\epsilon \rightarrow 0$

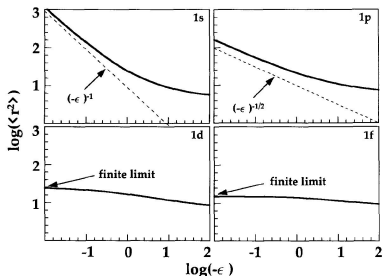


- $\langle r^n \rangle$  diverges as  $\epsilon^{\frac{2\ell-1-n}{2}}$  for  $n > 2\ell - 1$
- $\langle r^n \rangle$  diverges as  $\ln(\epsilon)$  for  $n = 2\ell - 1$
- $\langle r^n \rangle$  remains finite for  $n \leq 2\ell - 1$

Possible contributions from  $\ell > 1$  states to nuclear halos...

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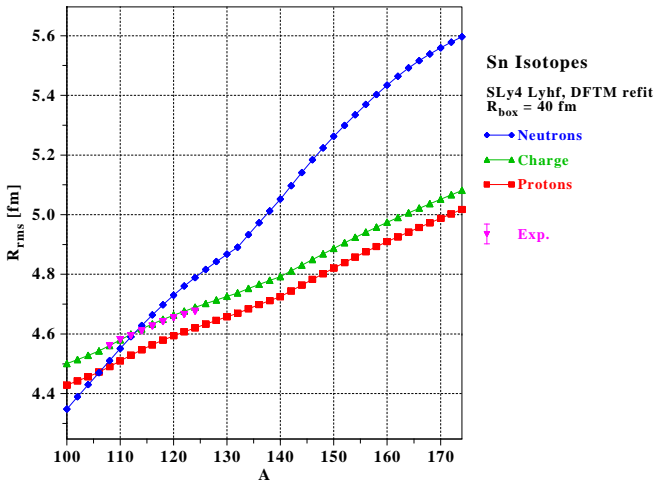
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# Results for Sn Isotopes ( $Z = 50$ )

- Root-mean-square radii



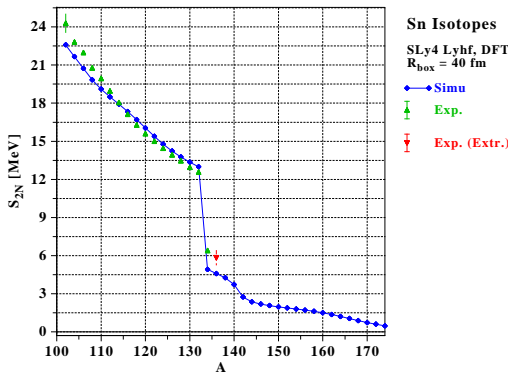
- Weak kink of neutron r.m.s.



# Results for Sn Isotopes ( $Z = 50$ )

- Kink of neutron r.m.s. : halo signature / shell effect ?
- Two-neutrons separation energy  $S_{2N}$  (drives asymptotic behavior)

- Drops at  $N = 82$
- No close from 0 for  $N > 82$

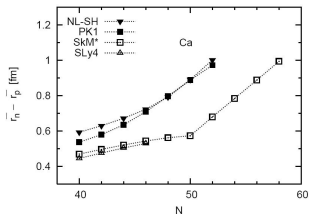


- Kink at  $N = 82$  may be due to shell effects only

# Detailed Analysis Methods

- Qualitative analysis misleading : "giant halo" ?

[M. Grasso *et al.*, Phys. Rev. **C74** (2006) 064317] - [J. Terasaki *et al.*, Phys. Rev. **C74** (2006) 054318]

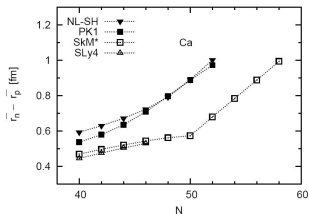


- Naive counting of nucleons
- Proton/neutron r.m.s. radii difference
- Missing part : halo neutrons decorrelated from
  - protons
  - core neutrons

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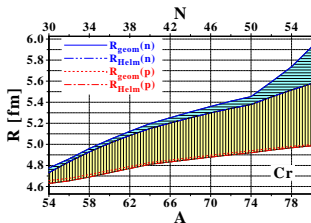
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- Naive counting of nucleons
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- Missing part : halo neutrons decorrelated from
  - protons
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- Quantitative analysis inadequate : Helm model

[S. Mizutori *et al.*, Phys. Rev. **C61** (2000) 044326]



- Ansatz for core density
- Extracts halo contribution to r.m.s. radius
- Model- and fit-dependent
- Halo in proton-rich / stable / doubly magic nuclei

# Roadmap

- Limits of existing methods
- **Shell effects** may explain part/all of neutron r.m.s. radii kinks
- Need of a **robust + quantitative** framework
- Lessons from previous attempts :
  - Halo region : **decorrelated from protons AND core neutrons**
  - **One-body density** : contains enough relevant information for characterization

## Goals

- Non ambiguous / model-independent definition of halos
- Extraction of **meaningful** criteria
- Separation of skin / shell / halo effects

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# Intrinsic One-Body Density

- **Self-bound system** : separation of center-of-mass and intrinsic d.o.fs.

$$\Psi_{i,\vec{K}}^N(\vec{r}_1 \dots \vec{r}_N) = e^{i\vec{K} \cdot \vec{R}_N} \Phi_i^N(\vec{r}_1 \dots \vec{r}_N) \equiv e^{i\vec{K} \cdot \vec{R}_N} \ddot{\Phi}_i^N(\vec{\xi}_1 \dots \vec{\xi}_{N-1})$$

- Uniform laboratory density : need to consider **intrinsic one-body density**
- Relevant degrees of freedom : **intrinsic spectroscopic amplitudes**

[D. Van Neck *et al.*, Phys. Rev. **C57** (1998) 2308] - [J. Escher *et al.*, Phys. Rev. **C64** (2001) 065801]

$$\varphi_i(\vec{r}) = \sqrt{N} \int d\vec{r}_1 \dots d\vec{r}_{N-1} \Phi_i^{N-1*}(\vec{r}_1 \dots \vec{r}_{N-1}) \delta(\vec{R}_{N-1}) \Phi_0^N(\vec{r}_1 \dots \vec{r}_{N-1}, \vec{r})$$

- Definition w/ respect to center-of-mass of  $(N - 1)$ -body frame
- Natural definition for knock-out
- Normalization : **spectroscopic factors**

$$S_i = \int d\vec{r} |\varphi_i(\vec{r})|^2 \quad \Leftrightarrow \quad \sum_i S_i = N$$

- Decomposition of one-body intrinsic density

$$\rho^{[i]}(\vec{r}, \vec{r}') = \sum_i \varphi_i^*(\vec{r}') \varphi_i(\vec{r})$$



# Asymptotic Behavior for $\varphi_i$

- **Asymptotic solution** (vanishing interaction) : free Schrödinger equation

$$\left( \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{l_i(l_i + 1)}{r^2} - \kappa_i^2 \right) \varphi_i^\infty(\vec{r}) = 0$$

- Asymptotic intrinsic overlap functions

$$\varphi_i^\infty(\vec{r}) = B_i h_{\ell_i}(\nu \kappa_i r) Y_{\ell_i}^{m_i}(\theta, \varphi)$$

- $B_i$ : Asymptotic Normalization Coefficient (ANC)
- $h_{\ell_i}$ : Hankel functions
- $\kappa_i$  related to one-nucleon separation energy

$$\kappa_i = \sqrt{\frac{2m\epsilon_i}{\hbar^2}} \quad \epsilon_i = E_i^{N-1} - E_0^N$$

# Asymptotic Behavior for $\rho$

- Asymptotics of one-body intrinsic density in spherical case

$$\rho^\infty(r) = \sum_i \frac{B_i^2}{4\pi} (2\ell_i + 1) |h_{\ell_i}(i\kappa_i r)|^2$$

- Leading order : **nucleon separation energy prevails for large  $r$ , regardless of  $\ell$**

$$|h_{\ell_i}(i\kappa_i r)|^2 \xrightarrow{r \rightarrow +\infty} \frac{e^{-2\kappa_i r}}{(\kappa_i r)^2}$$

$$\rho(r) \xrightarrow{r \rightarrow +\infty} \frac{B_0}{4\pi} (2\ell_0 + 1) \frac{e^{-2\kappa_0 r}}{(\kappa_0 r)^2}$$

- Energy ordering** of  $i$  components
- Corrections
  - $\ell$ -dep. of  $h_\ell$  : **centrifugal barrier**  
 $\Rightarrow$  favors low  $\ell$  states
  - $(2\ell + 1)$  **degeneracy factor**
  - Overall : **low  $\ell$  favored**

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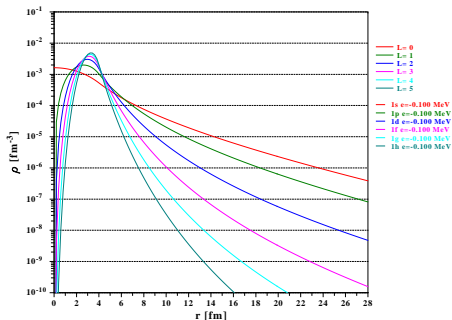
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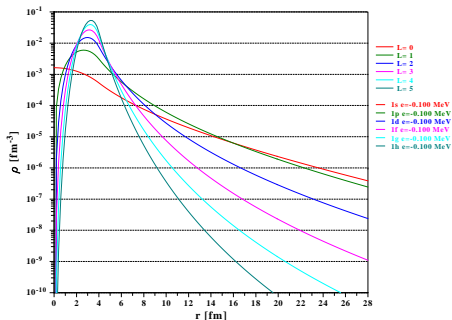
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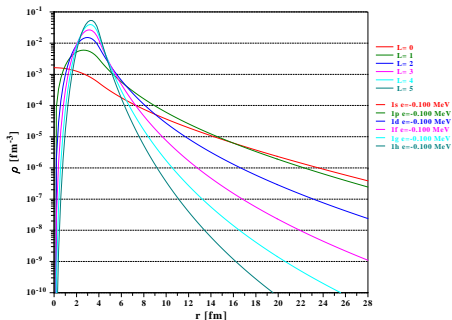
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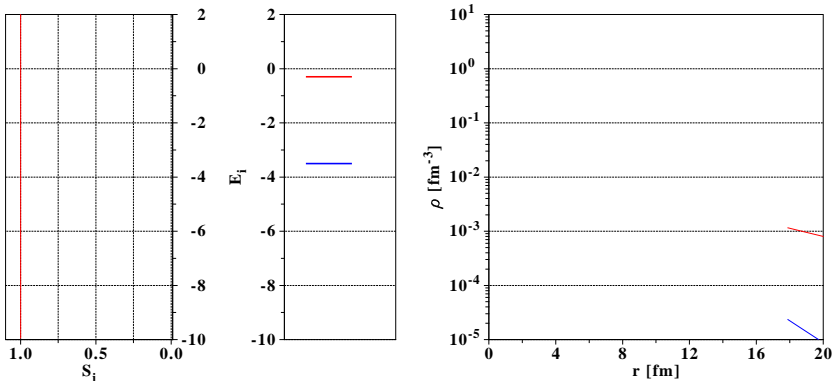
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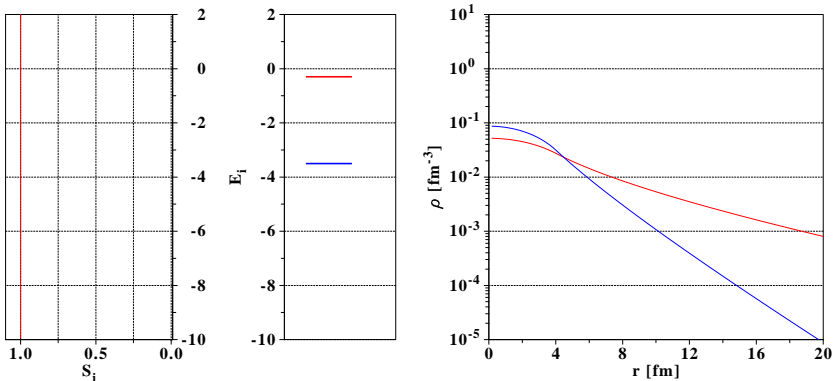
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- Density with normalized radial overlaps  $\psi_i(r) \Rightarrow \rho(\vec{r}) = \sum_i \frac{S_i}{4\pi} (2\ell_i + 1) |\psi_i(r)|^2$
- Assume (for now)  $S_i = 1$ 
  - Asymptotic ordering induces **crossings between normalized components**
  - Crossing sharpness depending on energy difference, angular momenta...
  - Crossing between  $i = 0$  and sum of higher components



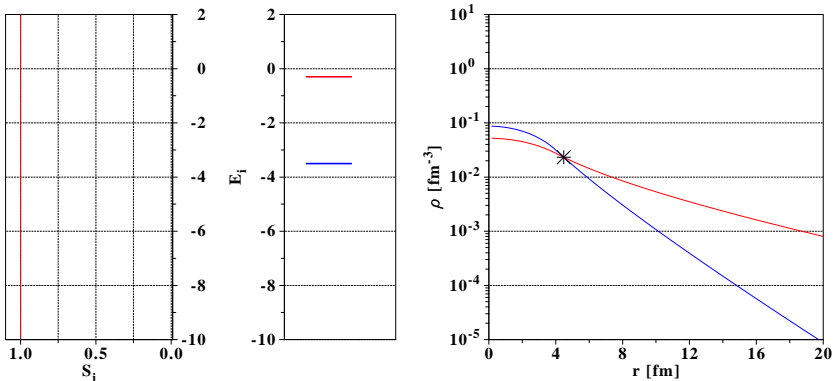
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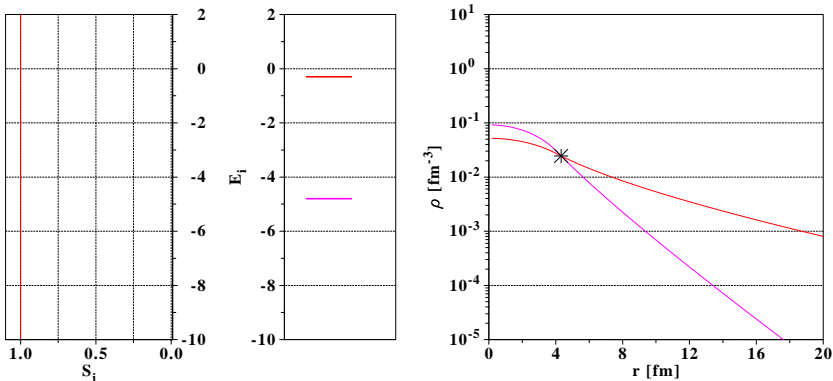
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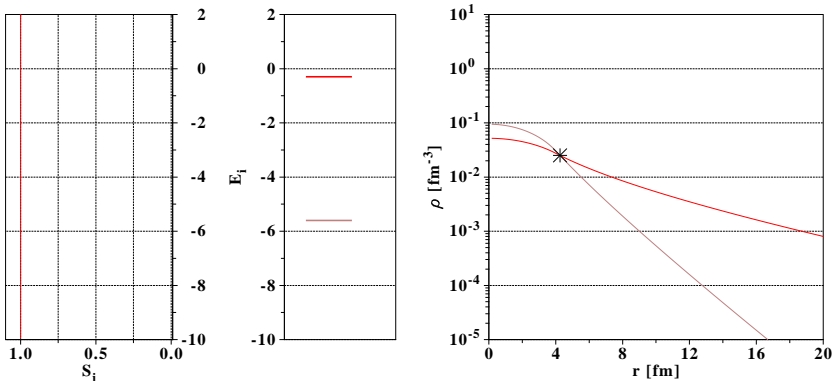
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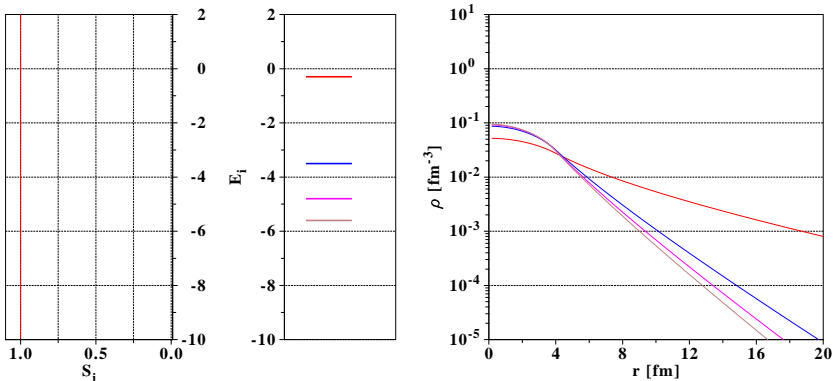
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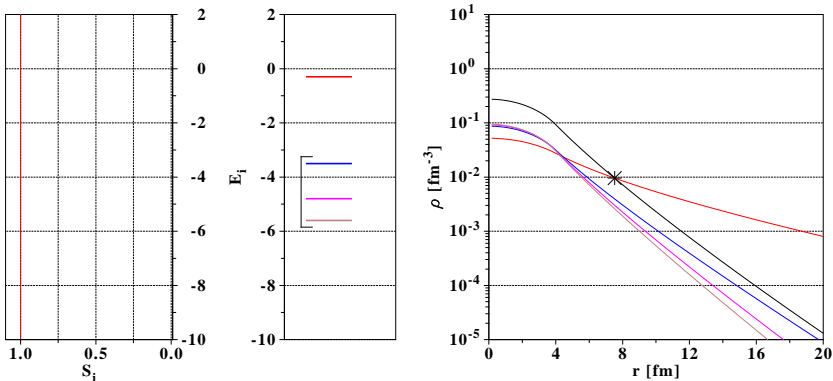
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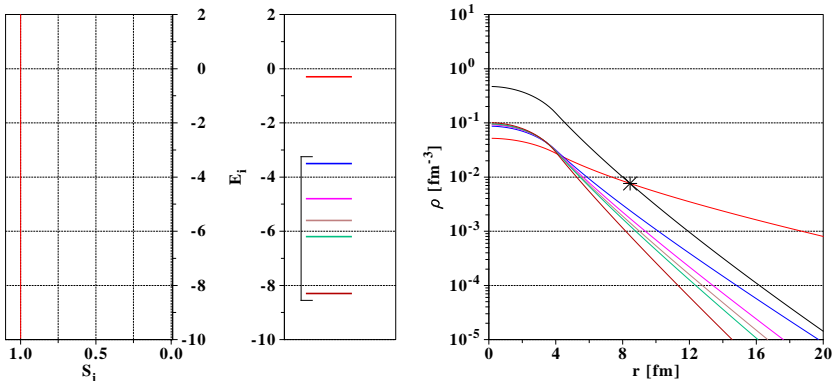
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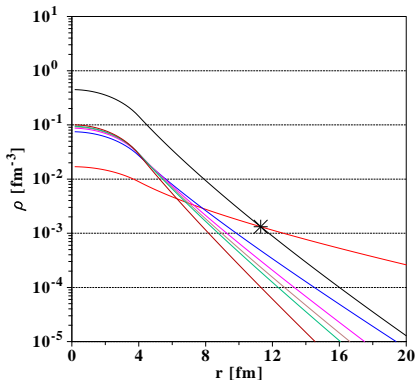
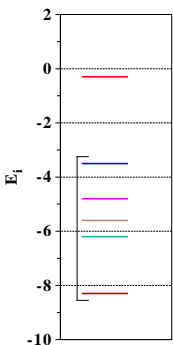
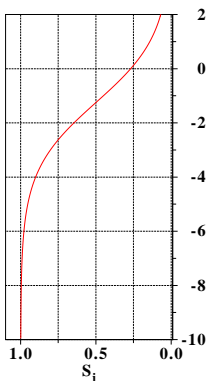
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- Density with normalized radial overlaps  $\psi_i(r) \Rightarrow \rho(\vec{r}) = \sum_i \frac{S_i}{4\pi} (2\ell_i + 1) |\psi_i(r)|^2$
- Assume (for now)  $S_i = 1$ 
  - Asymptotic ordering induces **crossings between normalized components**
  - Crossing sharpness depending on energy difference, angular momenta...
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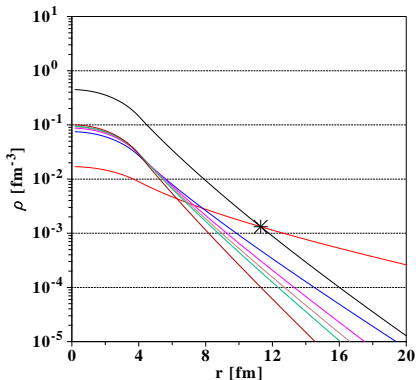
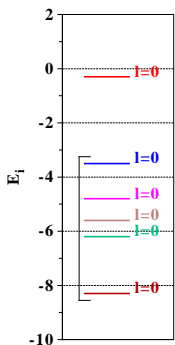
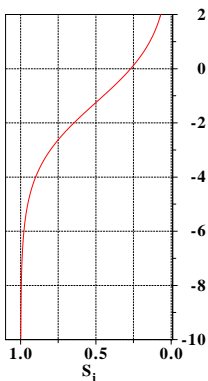
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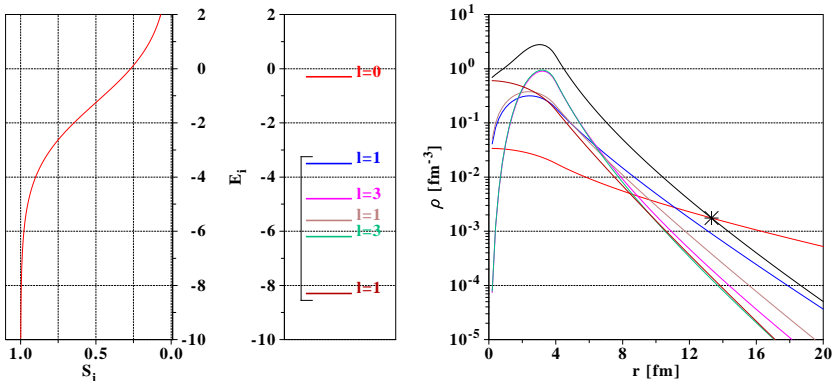
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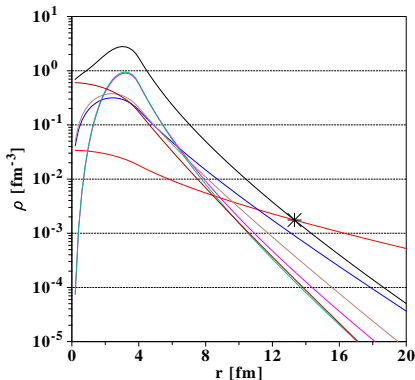
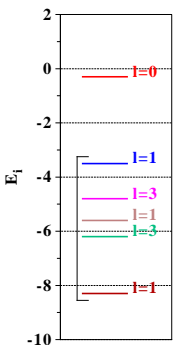
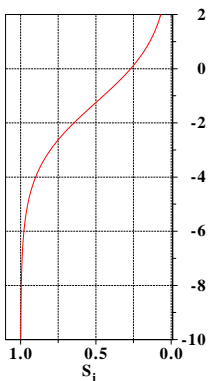
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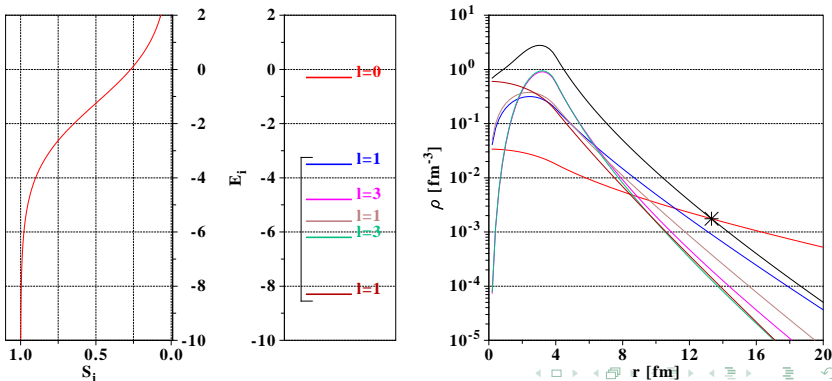


# Model-Independent Definition of Halos

Halo = region where nucleons are spatially decorrelated from the others

- Need to extend out : long tail (of course)
- Existence of sharp crossing between weakly bound states and remaining ones

A posteriori notion of **core and tail orbitals**

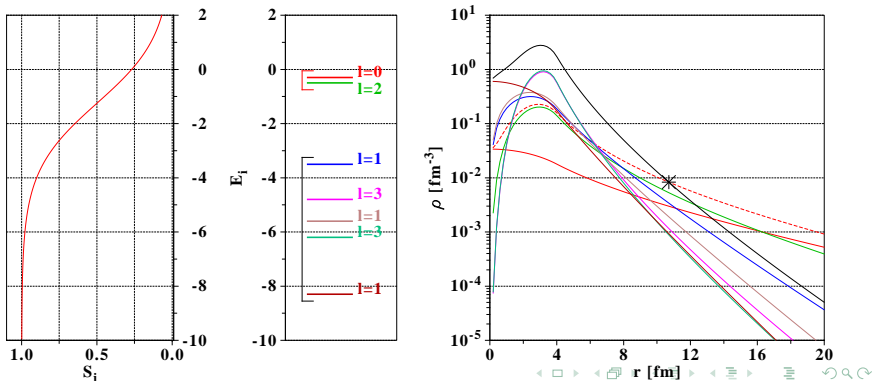


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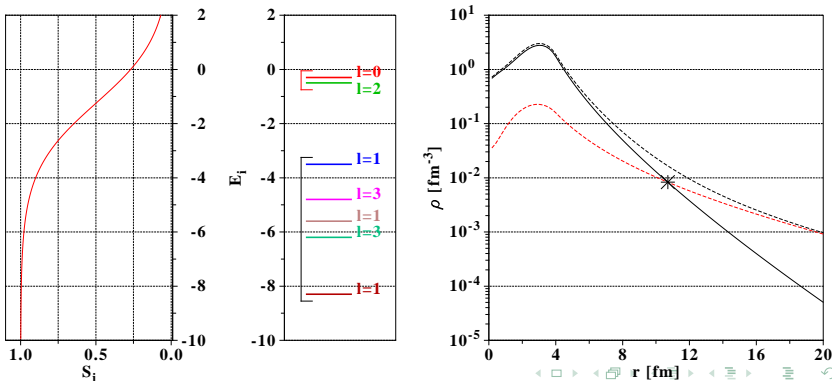


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# Energy Spectrum of Halo Systems

- Translation in terms of **excitation spectrum** of the  $(N - 1)$  system
- Long tail  
⇒  $\kappa_0 \ll 1$  : small separation energy / **low-lying states**
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⇒ **sharp crossing** in the density profile between core and halo components

## Halo energy scales

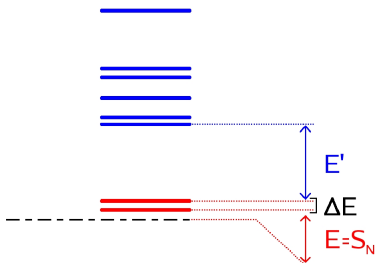
- Separation energy  $E$
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- Similar scales for "Halo EFT"  
[C. Bertulani *et al.*, Nucl. Phys. A712 (2002) 37]
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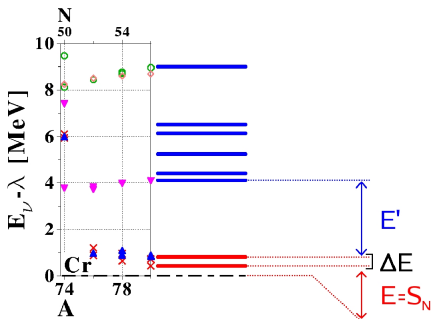


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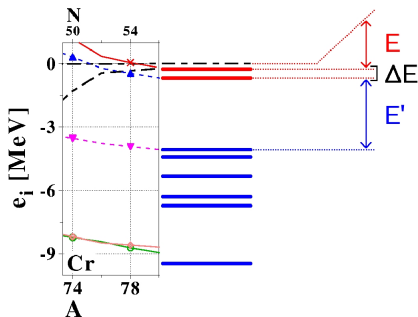


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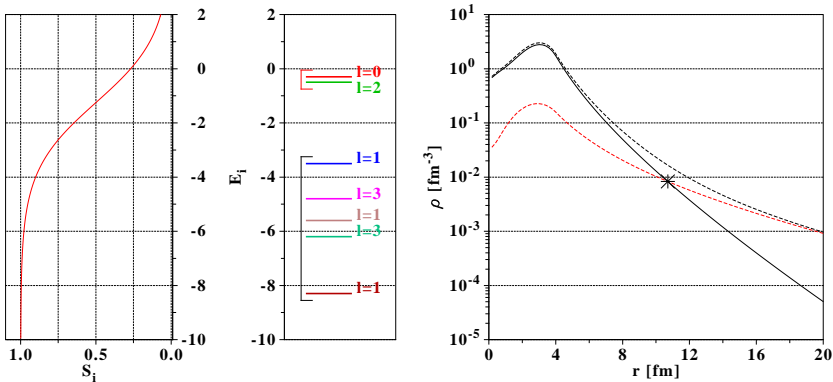


# Definition of Halo Region

- Pronounced crossings between core and tail components in the density
- Significant **curvature** of the log-density
- Second log-derivative separates regions where core/halo components dominate

## Halo Definition

Halo = region where tail components dominate by **at least one order of magnitude**



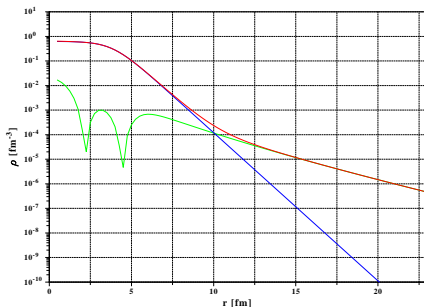


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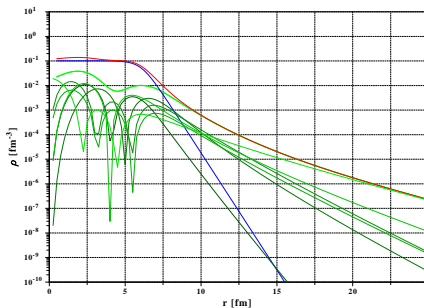
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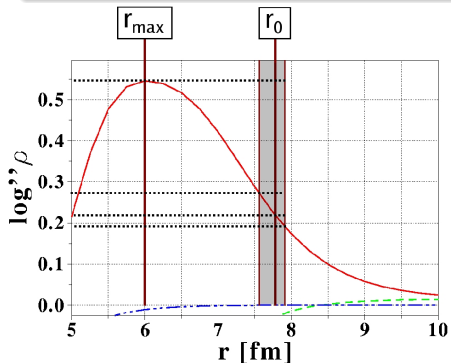
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- Model independent definition
- Error bars

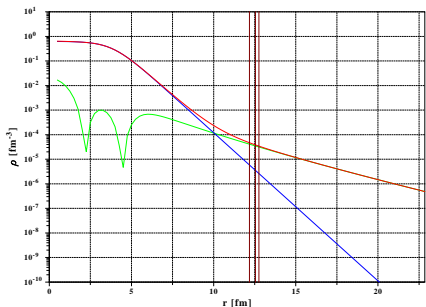
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# Quantitative Criteria for Halos

- Negligible core contribution in the outer  $r > r_0$  region
- Need of quantitative description of halo region

Average number of nucleons in the halo

$$N_{halo} = 4\pi \int_{r>r_0} r^2 \rho(r) dr$$

Effect of halo region on nuclear extension

$$\begin{aligned} \delta R_{halo} &= R_{rms,tot} - R_{rms,inner} \\ &= \sqrt{\frac{\langle r^2 \rangle}{\langle r^0 \rangle}} - \sqrt{\frac{\int_{r<r_0} r^4 \rho(r) dr}{\int_{r<r_0} r^2 \rho(r) dr}} \end{aligned}$$

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- Model-independent : regardless of where the one-body density comes from
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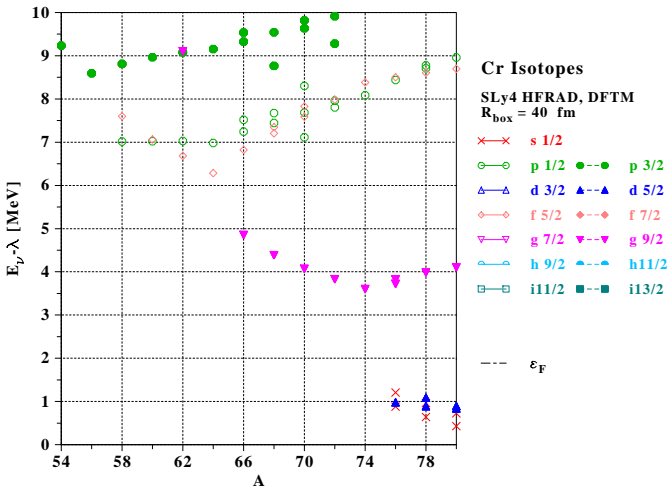


# Outline

- 1 Introduction
- 2 Current Situation in Medium-Mass Nuclei
  - Importance of Halo Configurations for the Nuclear EDF
  - Limitations of Current Approaches
- 3 A New Analysis Method
  - Properties of the Intrinsic One-Body Density
  - Model-Independent Definition of Nuclear Halos
  - Robust Criteria for Halo Formation
- 4 **First Results**
  - Cr Isotopes
  - Sn Isotopes
  - Systematics over Spherical Nuclei
  - Extensions and Limits
- 5 Conclusion

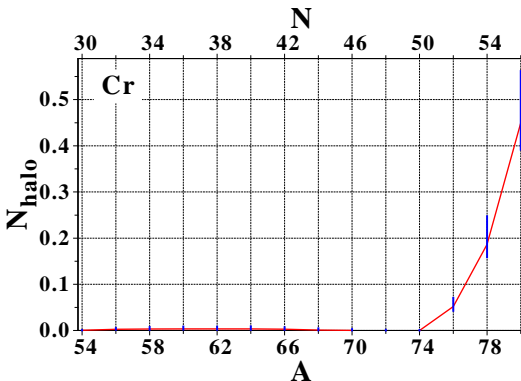
# Cr Isotopes

- QP energies: excitation spectrum of  $(N - 1)$ -body system
- Low-lying  $3s_{1/2}$  and  $2d_{5/2}$  states / **ideal conditions** regarding energy scales



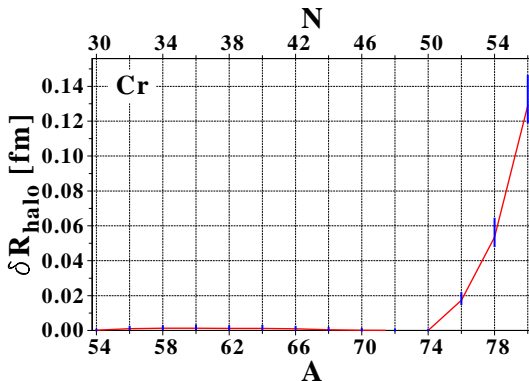
# Cr Isotopes

- Average number of nucleons in the halo region
- No effect before  $N = 50$  shell closure / sharp increase beyond
- Small value Vs  $N$  BUT same order as in light halo nuclei



# Cr Isotopes

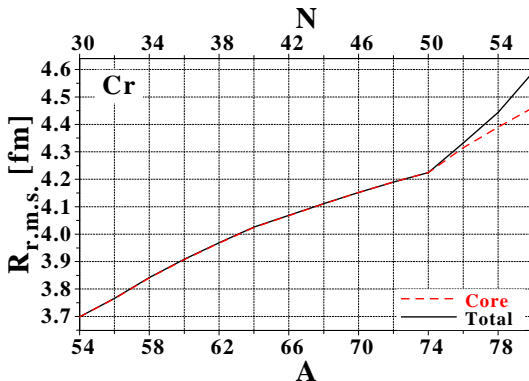
- Influence of the halo region on the nuclear extension
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- Important contribution to total r.m.s. radius + separation of shell effects



Evidence of halo formation

# Cr Isotopes

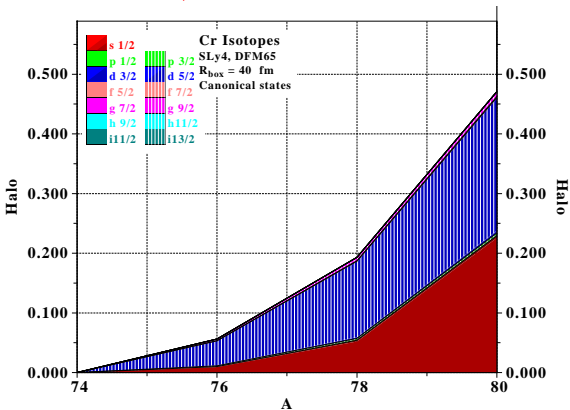
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Only relevant physics extracted

# Cr Isotopes

- Individual contributions to the halo (canonical basis)
- Only least-bound states contribute
- Major contribution from  $2d_{5/2}$  state (degeneracy +  $v^2$ )



Formation of a **Collective halo**

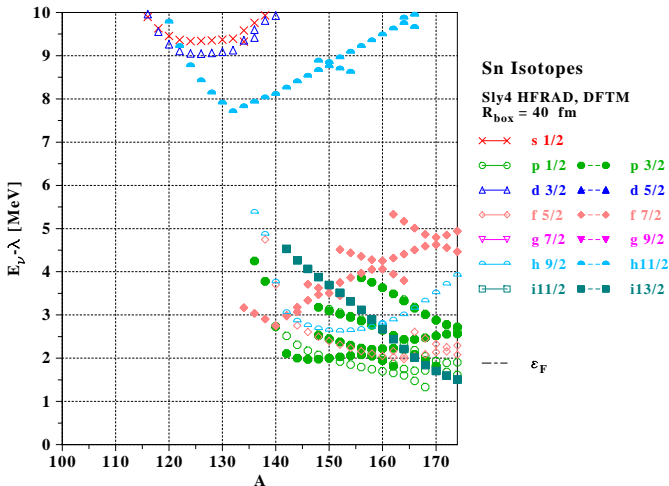
## Cr Isotopes : Conclusion

- **Small relative effect** :  $N_{halo} \sim 0.5$  for  $^{80}\text{Cr}$
- Significant contribution from halo to the nuclear extension
- Contributions from **multiple states**, including  $\ell = 2$
- Absolute values of  $N_{halo}$  comparable with situation in light halo nuclei  
 $\Rightarrow$  s-wave halo nucleus ( $^{11}\text{Be}$ ) :  $N_{halo} \approx .35$
- No “Giant” halo...

Converging leads for formation of collective halo in drip-line Cr isotopes

# Sn Isotopes

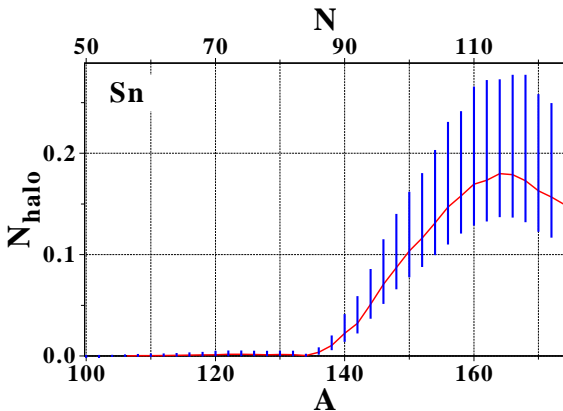
- Lot of collectivity / large bunch of high- $\ell$  states at drip-line
- Not favorable energy scales





# Sn Isotopes

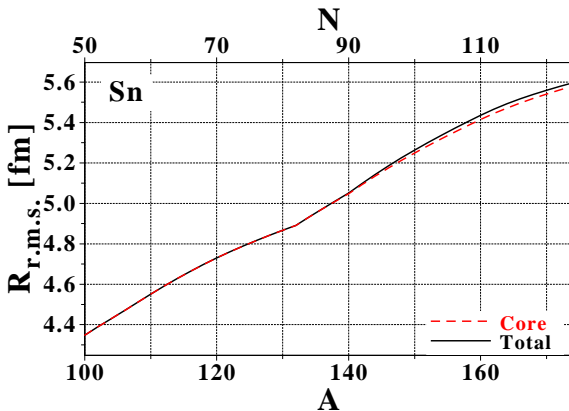
- **Number of nucleons** in the halo region
- No effect before  $N = 82$  shell closure
- **Small absolute contribution**: one third of Cr isotopes



- Hindrance from **filling high- $l$**  state at the drip-line

# Sn Isotopes

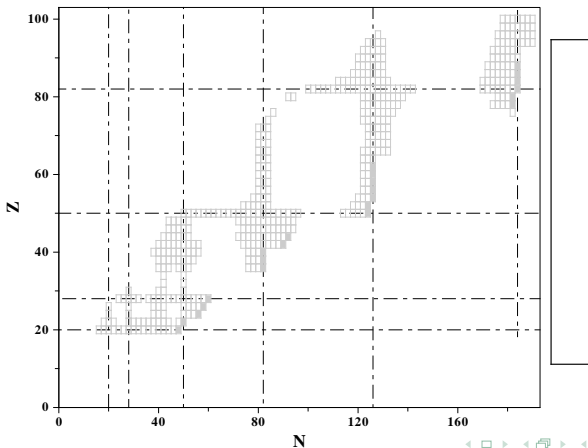
- Influence of the halo region on the nuclear extension
- Very small effect on nuclear extension



No halo seen

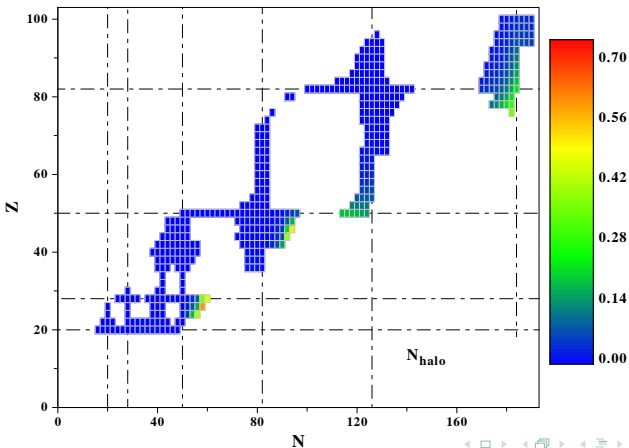
# Systematics

- Systematics over  $\sim 500$  spherical nuclei given by CEA-D1S online database



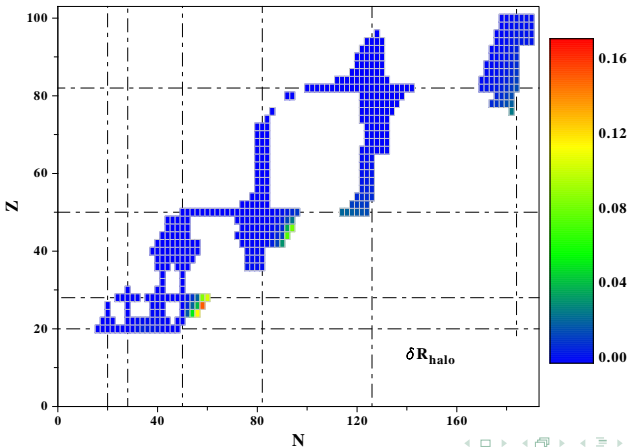
# Systematics

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- Number of nucleons in the halo region
- Decorrelated nucleons at the **very drip line** for several isotopes



# Systematics

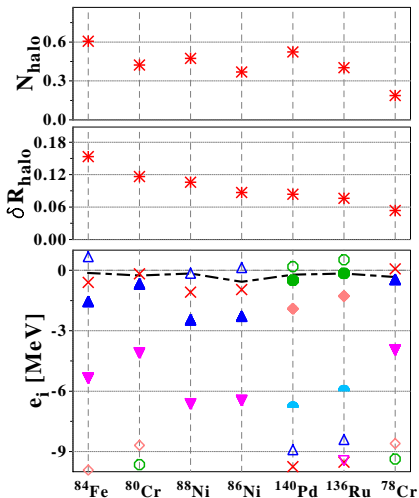
- Systematics over  $\sim 500$  spherical nuclei given by CEA-D1S online database
- Influence on total extension
- **Different information** from  $N_{\text{halo}}$ : reduced impact for heavy nuclei (collectivity)
- Best candidates: Fe, Cr, Ni, Pd, Ru



# Systematics

- Systematics over  $\sim 500$  spherical nuclei given by CEA-D1S online database

Common denominator :  
low-lying  $\ell = 0, 1$  states



# Influence of EDF characteristics on halo formation

## Role of pairing correlations

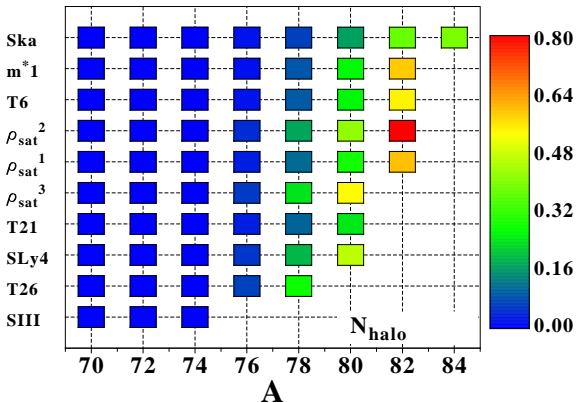
- Effect of surface Vs volume-type pairing on weakly bound states
- **Pairing anti-halo effect**  
[K. Bennaceur *et al.*, Phys. Lett. **B496** (2000) 154]
  - Extra **localization of QP states from pairing field**
  - Prevents divergences of r.m.s. radius from weakly bound states
- Particular role of *s*-waves  
[I. Hamamoto, B. Mottelson, Phys. Rev. **C68** (2003) 034312]
  - Extreme situations :  $\lambda \rightarrow 0$ ,  $e_s \rightarrow 0$
  - **Decoupling of *s*-wave from pairing field** : classical halo
  - Not encountered in realistic situations

## Influence of the particle-hole energy functional

- Role of **new terms**: tensor interaction. . .
- Influence of INM properties: effective mass, compressibility, saturation point. . .
- **Influence of the parametrization**

# Limits of current energy functionals

- Constrain low-density behavior of EDF using halo data ?



$N_{\text{halo}}$  :

Average number of nucleons

$\delta R_{\text{halo}}$  :

Influence on spatial extension

$S_{2N}$  :

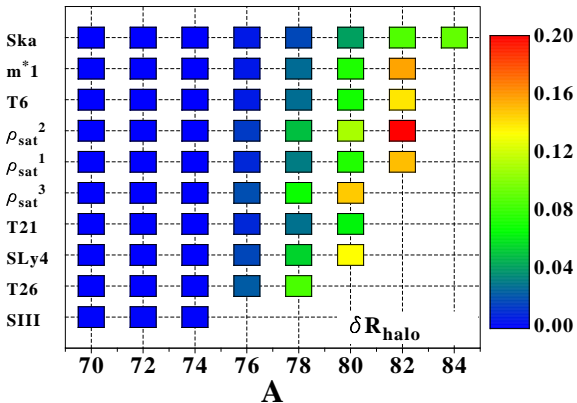
2n separation energy

- Strong dependence** of halo features on EDF parametrization
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- Predictivity** of current EDF models for exotic systems ?



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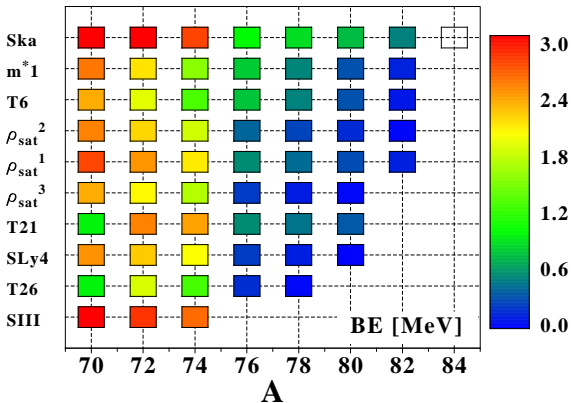
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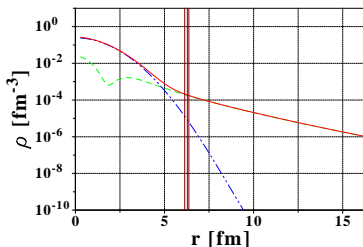
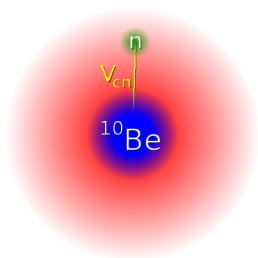
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# Application to Other Halo Systems

- **Model-independent analysis:** can be used for other systems
- **Light nuclei** from Coupled-Channels calculations (2-body clusters only)

[F. Nunes *et al.*, Nucl. Phys. **596** (1996) 171 - Nucl. Phys. **A609** (1996) 43]



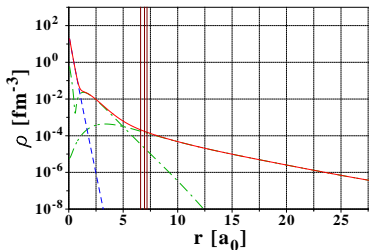
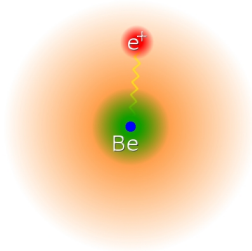
	$N_{halo}$	$\delta R_{halo}$ [fm]	$R_{rms}$ [fm]
$^{13}\text{C}$	$0.66 \cdot 10^{-3}$	$0.74 \cdot 10^{-3}$	2.487
$^{11}\text{Be}$	0.270	0.394	2.908

- Good separation between halo/non-halo systems
- **Absolute value for  $\delta R_{halo}$ :** much bigger Vs medium-mass nuclei (collectivity)

# Application to Other Halo Systems

- **Model-independent analysis:** can be used for other systems
- **Atom-positron complexes:**  $e^+$  binding to neutral atom by polarization potential

[J. Mitroy, Phys. Rev. Lett. **94** (2005) 033402]



- **Asymptotics:**  
 $e^+ + A$  or  $Ps + A^+$

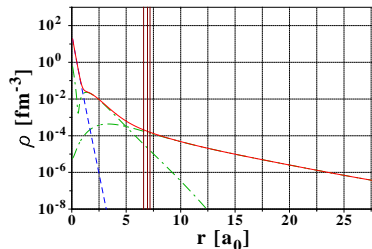
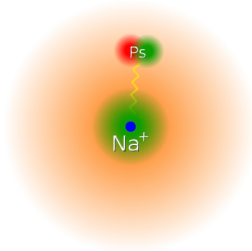
	$N_{e^-}$	$N_{halo}$	$R_{r.m.s.}$	$\delta R_{halo}$	$P_{e^+}$	$P_{e^-}$
Be	4	0.624	5.661	3.194	98.1	01.9
Mg	12	0.669	2.298	0.826	80.3	19.7
Cu	29	0.754	1.777	0.975	88.6	11.4
He	2	1.982	15.472	14.568	50.3	49.7
Li	3	1.972	7.781	7.088	50.8	49.2

- $P_X$  (%): proportion of  $X$  in halo outer region
- **Positron** ( $P_{e^+} \gg P_{e^-}$ ) and positronium ( $P_{e^+} \approx P_{e^-}$ ) **halos identified**

# Application to Other Halo Systems

- **Model-independent analysis:** can be used for other systems
- **Atom-positron complexes:**  $e^+$  binding to neutral atom by polarization potential

[J. Mitroy, Phys. Rev. Lett. **94** (2005) 033402]



- Asymptotics:  
 $e^+ + A$  or  $Ps + A^+$

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# Outline

- 1 Introduction
- 2 Current Situation in Medium-Mass Nuclei
  - Importance of Halo Configurations for the Nuclear EDF
  - Limitations of Current Approaches
- 3 A New Analysis Method
  - Properties of the Intrinsic One-Body Density
  - Model-Independent Definition of Nuclear Halos
  - Robust Criteria for Halo Formation
- 4 First Results
  - Cr Isotopes
  - Sn Isotopes
  - Systematics over Spherical Nuclei
  - Extensions and Limits
- 5 Conclusion

# Conclusion

- New analysis method based on analysis of intrinsic one-body density
- **Model-independent criteria** for halo formation
- Cr Isotopes
  - Small relative number of nucleons in halo region / Comparable with light systems
  - Large influence of halo region on nuclear extension
  - Contribution from several weakly bound states, including  $l = 2$
  - Notion of "giant halo" : meaningless...

Formation of a **collective halo in Cr isotopes**

- Systematics over spherical nuclei
  - **Good candidates** : drip-line Fe, Cr, Ni, Pd, Ru
  - Experimental validation in drip-line medium-mass region ?
- Successful application to **light nuclei and atom-positron complexes**
  - Correct extraction of halo factors
  - Proves model-independence

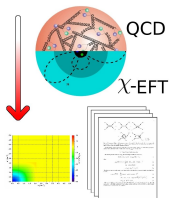
# Conclusion

- Extension of the method
    - **Deformed nuclei** : multipolar moments of the density
    - **Multi-reference EDF effects** : PNP, GCM on breathing modes...
    - Inclusion of cluster correlations: hindrance to halo formation?
    - Study of correlations: two-body density
  
  - **Link with experimental studies**: open question
    - Neutron drip-lines **beyond** reach for medium-mass nuclei
    - Robust method BUT **no robust predictions**
    - High sensitivity to EDF parametrization
- Fine tuning of EDF based on experimental data: not yet
- Halo: (very) rare exotic phenomenon
  - Missing terms in current functionals
- 
- **Lot of work needed** first on EDF used in single/multi- vacua calculations



# Long-range plan

- **Microscopic vertex** from  $\chi$ -EFT / low-momentum interactions + nuclei properties



Ground state

Mass, deformation



Spectroscopy

Excited states



Collective modes

RPA, QRPA, GCM



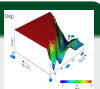
Symmetry restoration

$J$  Projection, PNP



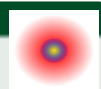
Heavy elements

Fission, superheavy



Exoticity

Drip-lines, halos



Astrophysics

r-process, SN



# Commercial break



V. R. and T. Duguet

*Halo phenomenon in medium-mass nuclei.*

*I. New analysis method and first applications*

[nucl-th/0702050](#)



V. R., K. Bennaceur and T. Duguet

*Halo phenomenon in medium-mass nuclei.*

*II. Impact of correlations and large scale analysis*

[arXiv:0711.1275](#)

● ???