SUB-BARRIER FUSION of HEAVY IONS

Şerban Mişicu

Horia Hulubei National Institute of Physics and Nuclear Engineering
Bucharest-Magurele

(In collaboration with Henning Esbensen, ANL)

Seattle, Institute for Nuclear Theory, 9.XI.2007

WHAT CAN BE LEARNED ?

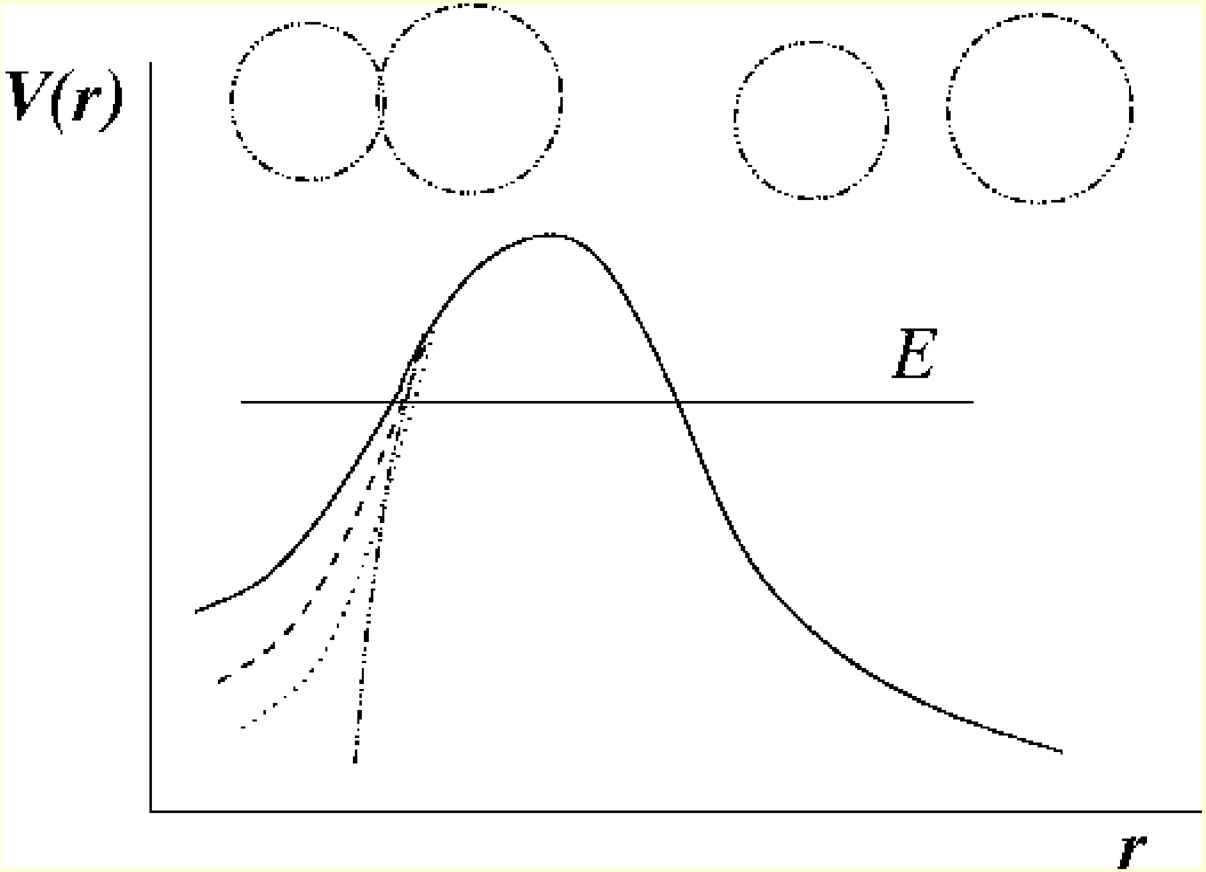
- Test the nuclear potential inside the barrier

- Enhancement of cross-sections due to nuclear structure :
  vibrational nuclei(sensitivity to multi-phonon excitations) ,
  rotational excitations (effect of higher multipole defomations)
  neutron transfer

- Gate to the synthesis of heavy and *superheavy* nuclei
  with a minimum of excitation energy

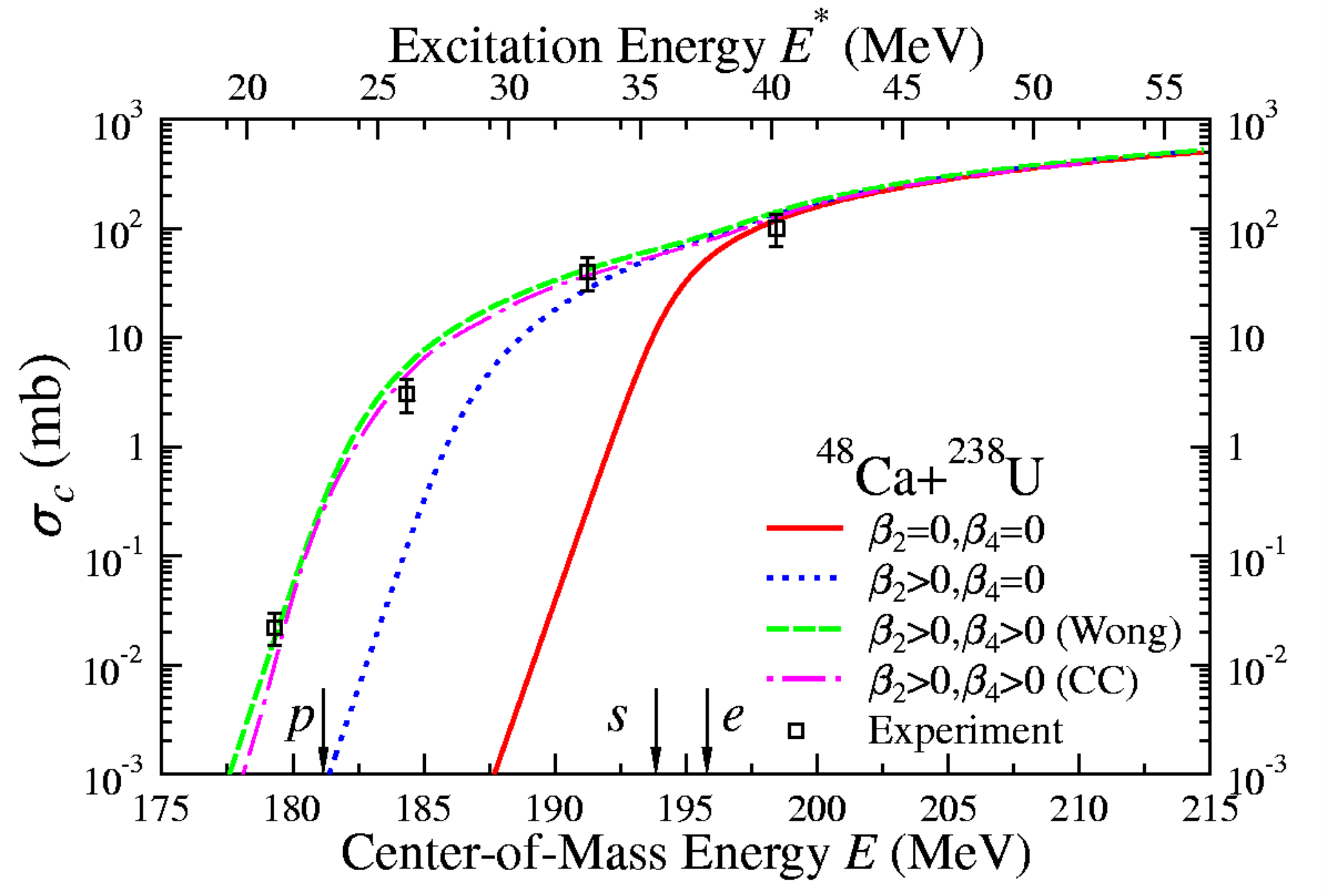
- Evolution of Stars – extrapolation of near-barrier data on
  σ_F of C and O down to astrophysical energies
  ---->*reaction rates in stellar environment*

- Test the nuclear potential inside the barrier



Enhancement of cross-sections due to nuclear structure

Rotational excitations (effect of higher multipole defomations)



S.Misicu &W.Greiner, PRC69, 054601(2004)

COUPLED-CHANNEL ANALYSIS

Vibrational Coupled-Channel Equations – for each *(LM)*

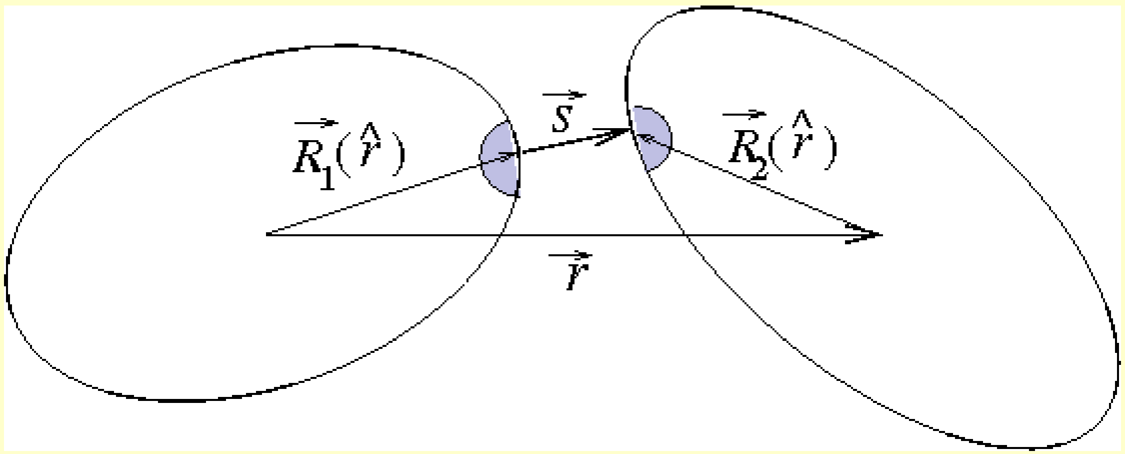
$$\left(\frac{\hbar^2}{2M_0}\left[-\frac{d^2}{dr^2}+\frac{L(L+1)}{r^2}\right]+\frac{Z_1Z_2e^2}{r}+V(r)+\sum_{n_1,n_2}\varepsilon_{n_1,n_2}-E\right)u_{n_1n_2}(r)$$

$$=-\sum_{m_1m_2}\langle n_1n_2 \mid \delta V_C+\delta V_N \mid m_1m_2\rangle u_{m_1m_2}(r),$$

Vibrational channels : $n_{1,2}$ – phonon quantum numbers

POTENTIAL

Proximity approximation



$$V(\boldsymbol{r}) = V(s), \quad s = \min|r - R_1(\hat{r}_1) - R_1(\hat{r}_2)|$$

$$R_i(\hat{r}_i) = R_i + \delta R_i(\hat{r}_i), \quad \delta R_i(\hat{r}_i) = \sum_{\lambda\mu} \alpha^{(i)}_{\lambda\mu} R_i Y_{\lambda\mu}(\hat{r}_i)$$

SPHERICAL POTENTIAL

Spherical Proximity ion-ion Potential (Akyüz-Winther)

$$V_N(r) = -\frac{16\pi\gamma a R_1 R_2}{R_1 + R_2} \frac{1}{1 + \exp(r - R_1 - R_2 - \Delta R)}$$

- a - potential diffuseness
- R_i - nuclear radii
- γ - nuclear surface tension
- ΔR - adjustable parameter

DEFORMED POTENTIAL

Linear+quadratic fluctuations for *N* and linear for *C*

$$\delta V_N = -\frac{\partial V}{\partial r}\delta R + \frac{1}{2}\frac{\partial^2 V}{\partial r^2}\left[(\delta R)^2 - \langle 0 \mid (\delta R)^2 \mid 0\rangle\right]$$

$$\delta V_C = -\frac{\partial V}{\partial r}\delta R = \sum_{\lambda\mu}\frac{3Z_1Z_2e^2}{(2\lambda+1)r^{\lambda+1}}\left[\alpha^{(1)}_{\lambda\mu}R^{\lambda}_{C1}Y_{\lambda\mu}(\hat{r}_1) + \alpha^{(2)}_{\lambda\mu}R^{\lambda}_{C2}Y_{\lambda\mu}(-\hat{r}_2)\right]$$

Double folding Heavy Ion Potential

Fold a n-n effective interaction veff with the projectile and traget density distributions

$$V(\boldsymbol{R}) = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2)v(\boldsymbol{r}_{12})$$

Ground state one-body densities

$$\rho(\boldsymbol{r}) = \rho_0 \left[1 + \exp\frac{1}{a}\left(r - R_0\left(1 + \sum_{\lambda=2,3,4}\beta_\lambda Y_{\lambda 0}(\theta, 0)\right)\right)\right]^{-1}$$

Double folding Heavy Ion Potential

Fold a n-n effective interaction veff with the projectile and traget density distributions

$$V(\boldsymbol{R}) = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2)v(\boldsymbol{r}_{12})$$

From the exceedingly large number of exchange terms retain only the knock-on (KOE) term : two nuleons are interacting and in the same time are exchanged

$$v = v^{\mathrm{d}}(\boldsymbol{r}, \rho) + v^{\mathrm{ex}}(\boldsymbol{r}, \rho) P_{12}^{x}$$

$$v^{\mathrm{d}} = \sum_{S,T} v_{S,T} P_s^{\sigma} P_s^{\tau}, \quad v^{\mathrm{ex}} = \sum_{S,T} (-)^{S+T+1} v_{S,T} P_s^{\sigma} P_s^{\tau},$$

Double folding potential with M3Y forces

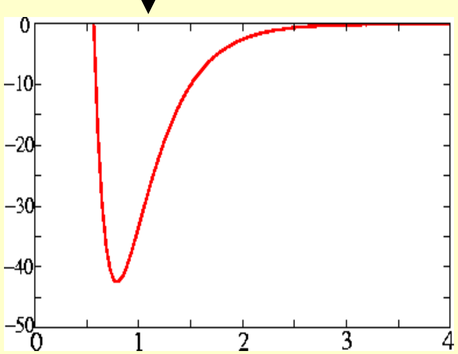
$$V(\boldsymbol{R}) = \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2)v(\boldsymbol{r}_{12})$$

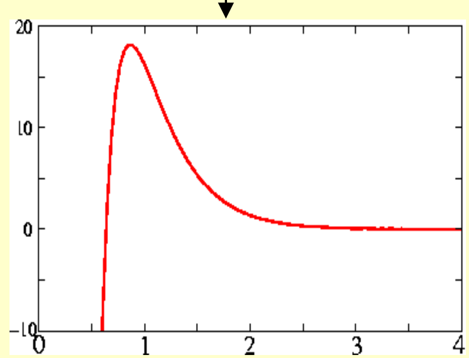
V_{ST} : from a fit of Yukawas m.e. to G-matrix in H.O. basis (Bertsch et al.)

Reid-Elliott

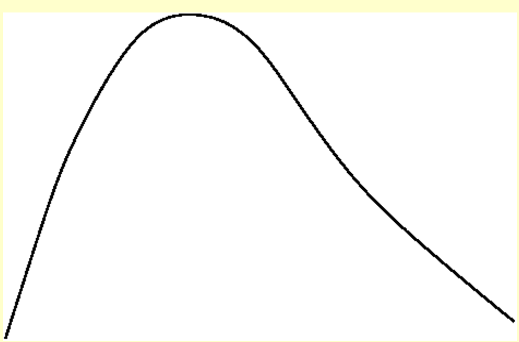
$$v^{\mathrm{d}}(\boldsymbol{r}) = 7999\frac{e^{-r/0.25}}{4r} - 2134.25\frac{-r/0.4}{2.5r} - \left(2692.25\frac{e^{-r/0.25}}{4r} + 478.75\frac{-r/0.4}{2.5r}\right)\boldsymbol{\tau}_1\boldsymbol{\tau}_2 + \ldots$$

$$v^{\mathrm{ex}}(\boldsymbol{r}) = 4631.375\frac{e^{-r/0.25}}{4r} - 1787.125\frac{-r/0.4}{2.5r} - 7.847\frac{e^{-r/0.7}}{1.43r} + \ldots$$





Heavy ion potential



Potential with Repulsive Core

Calculate the cost of overlapping completely the ions from the EOS
Equate the cost to the (increase in HI potential)/particle

$$\Delta V \approx 2A_p\left[\varepsilon(2\rho_0,\delta) - \varepsilon(\rho_0,\delta)\right]$$

EOS – Thomas-Fermi Model(Myers&Swiatecki)

$$\varepsilon(\rho,\delta) = \varepsilon_F\left[A(\delta)\left(\frac{\rho}{\rho_0}\right)^{2/3} + B(\delta)\left(\frac{\rho}{\rho_0}\right) + C(\delta)\left(\frac{\rho}{\rho_0}\right)^{5/3}\right]$$

Incompressibility of Cold Nuclear Matter at saturation

$$K = 9\left(\rho^2\frac{\partial^2\varepsilon}{\partial\rho^2}\right)_{\rho=\rho_0}$$

Potential with Repulsive Core

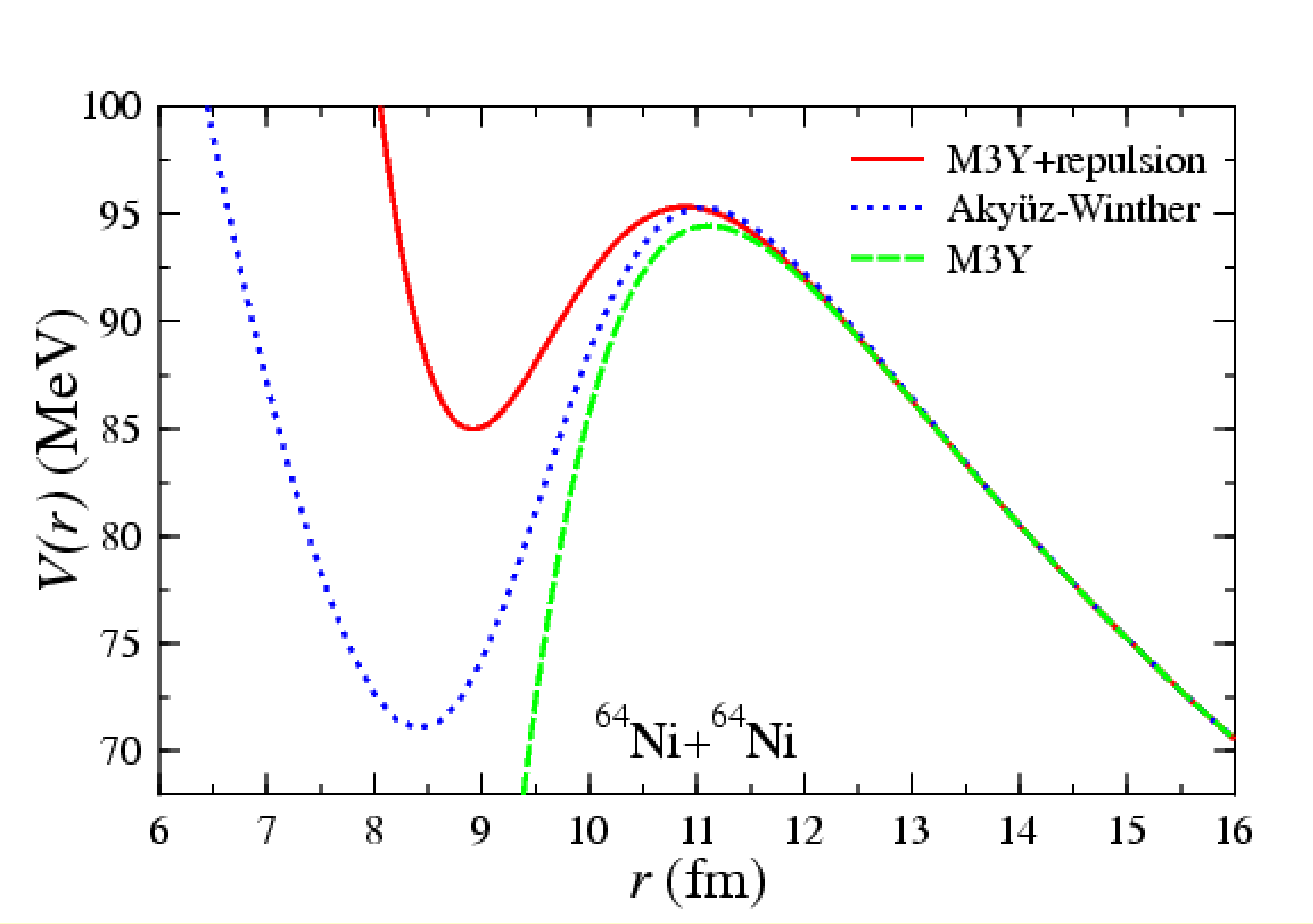
$$V_{\mathrm{rep}}(\boldsymbol{R}) = V_p \int d\boldsymbol{r}_1 \int d\boldsymbol{r}_2 \; \tilde{\rho}_1(\boldsymbol{r}_1)\tilde{\rho}_2(\boldsymbol{r}_2)\delta(\boldsymbol{r}_{12})$$

Approximations to calibrate the strength of the repulsion

$$\varepsilon(\rho, \delta) = \varepsilon(\rho_0, \delta) + \frac{K}{18\rho_0^2} \; (\rho - \rho_0)^2$$

$$\Delta V = V_N(0)$$

Potential with Repulsive Core



Matrix Elements

Assumption : Matrix elements of the nuclear amplitudes are identical to the corresponding electromagnetic m.e.

$$\beta_\lambda = \frac{\sqrt{4\pi(2\lambda+1)B(E_\lambda)}}{(\lambda+3)Z}$$

Linear matrix elements

$$\begin{aligned}
\langle \lambda\mu \mid \alpha_{\lambda\mu} \mid 0 \rangle &= \frac{\beta_\lambda}{\sqrt{2\lambda+1}} \\
\langle 20 \mid \alpha_{20} \mid 20 \rangle &= 0, && \text{vibrations} \\
&= -\sqrt{\frac{5}{16\pi}} \frac{4\pi Q_2}{3ZeR_c^2}, && \text{rotations}
\end{aligned}$$

Matrix Elements

Assumption : Matrix elements of the nuclear amplitudes are identical to the corresponding electromagnetic m.e.

$$\beta_\lambda = \frac{\sqrt{4\pi(2\lambda+1)B(E_\lambda)}}{(\lambda+3)Z}$$

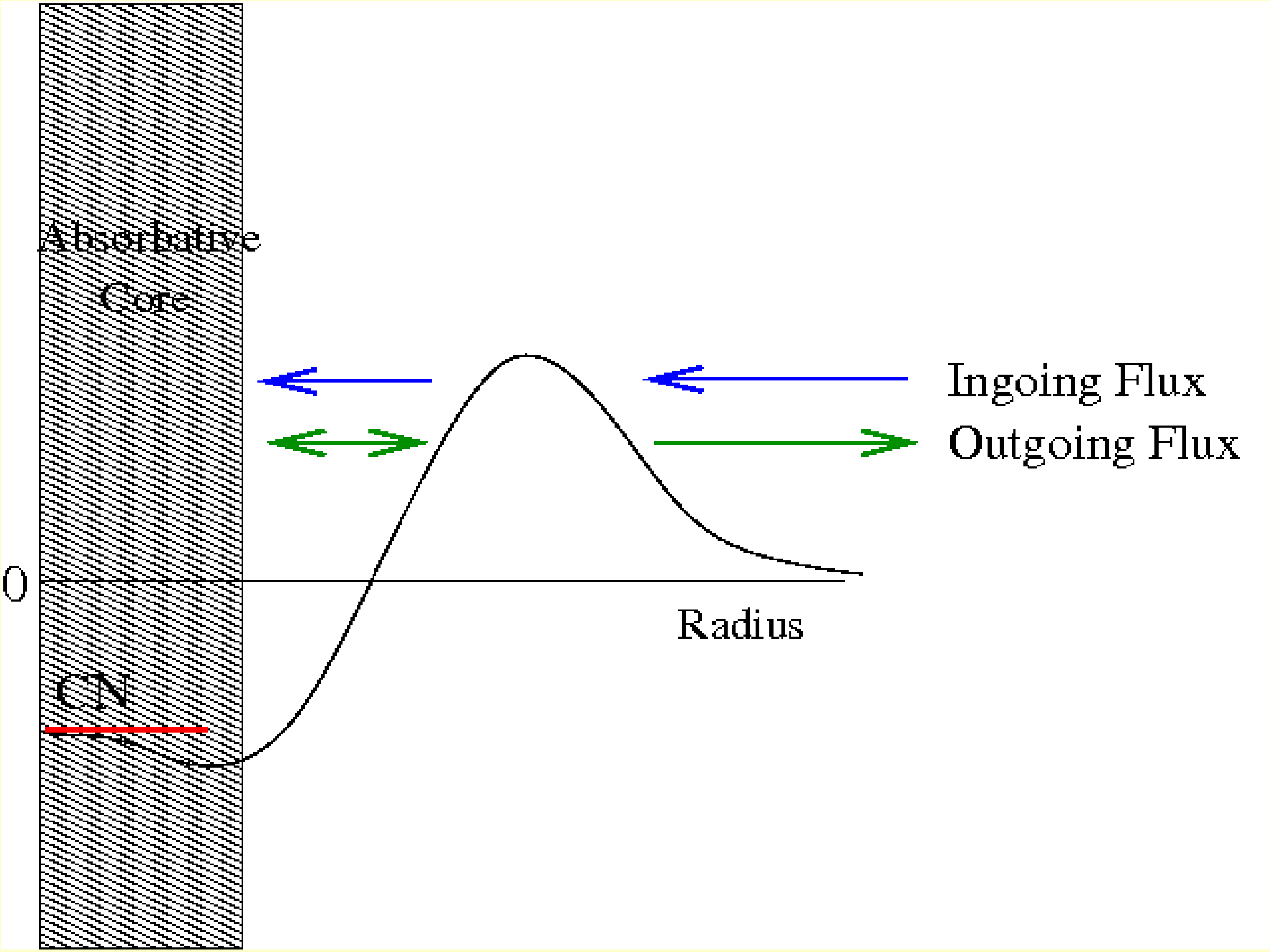
Quadratic matrix elements

$$\langle 2^+0 \mid \delta V_N^{(2)} \mid 2^+0\rangle = \frac{4}{7}\frac{\partial^2 U}{\partial r^2}\frac{(\beta_2 R)^2}{4\pi}, \quad \text{rotations}$$

$$\langle \lambda\; \mu \mid \delta V_N^{(2)} \mid \lambda\; \mu\rangle = \frac{\partial^2 U}{\partial r^2}\frac{(\beta_\lambda R)^2}{4\pi}, \quad \text{vibrations}$$

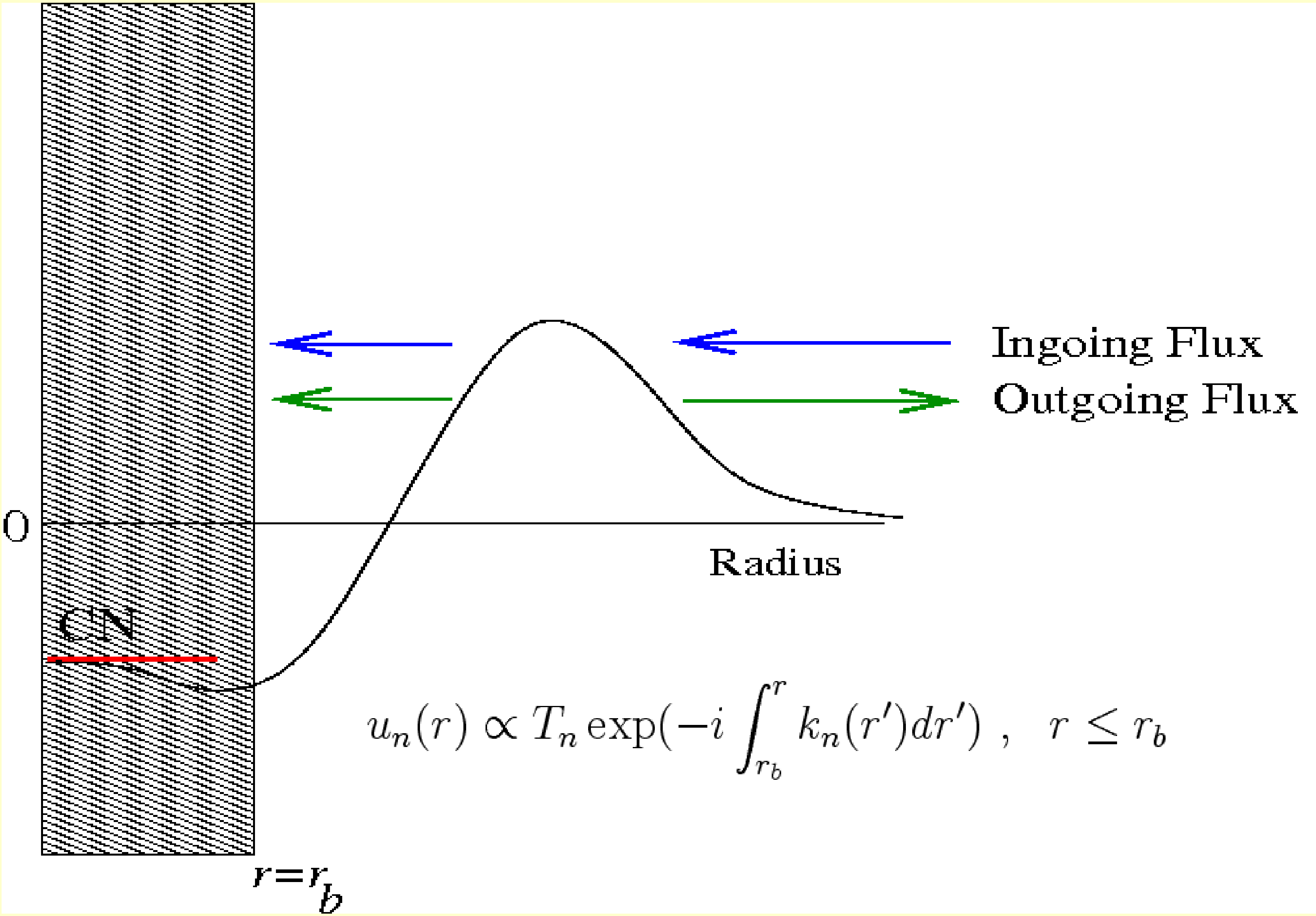
Non-diagonal quadratic m.e. are expressed as products of linear m.e. and have a large influence on subbarrier fusion than the diagonal.

REGULAR BOUNDARY CONDITIONS



$u_n(0)=0$

INCOMING WAVE BOUNDARY CONDITIONS
(Rawitscher 1963)



OUTGOING BOUNDARY CONDITIONS

Usual scattering conditions at large distances

$$u_n^{LM}(r) \longrightarrow \delta_{n0} F_L(k_n r) + T_n H_L^{(+)}(k_n r)$$

Reaction matrix

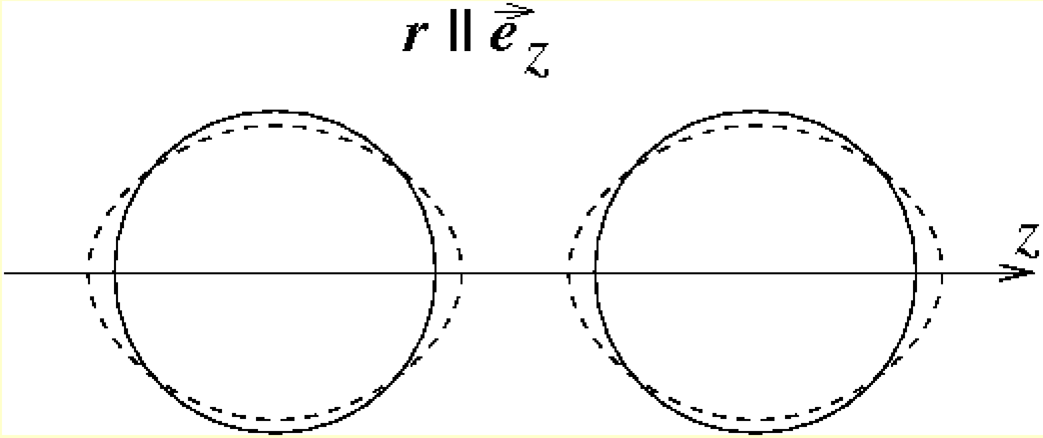
$$T_n = \frac{1}{2\pi}\left(\delta_{n0} - S_{n0}\right)$$

Transmission coefficient

$$T = 1 - \sum_n \mid S_{n0} \mid^2 \quad \left(T = e^{-\frac{2}{\hbar}\int_{R_{t1}}^{R_{t2}} \sqrt{2\mu(E-V(r'))}dr'} \quad -WKB\right)$$

ROTATING FRAME APPROXIMATION

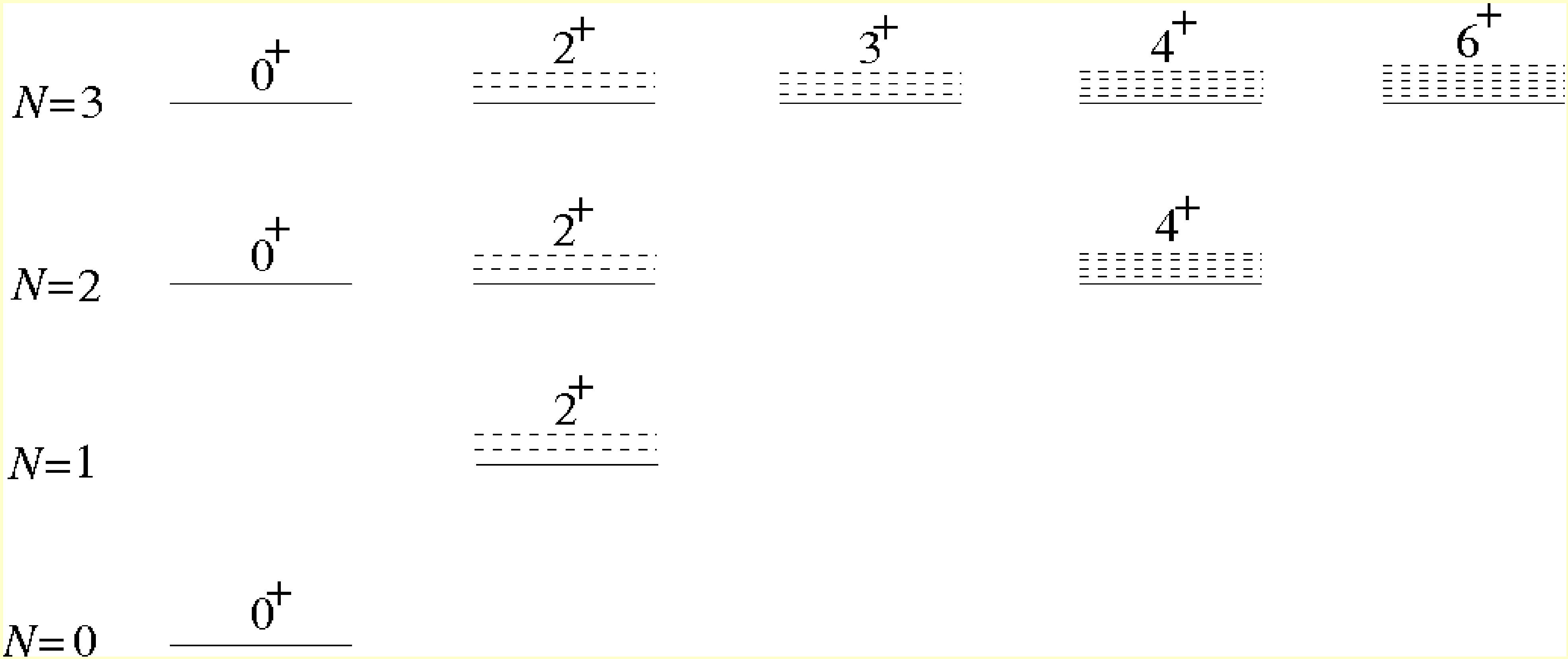
Vibrational excitations along *z*-axis : m=0



Isocentrifugal approximation : $\hbar^2 I^2/(2\mu r^2)$ is negligible around the barrier

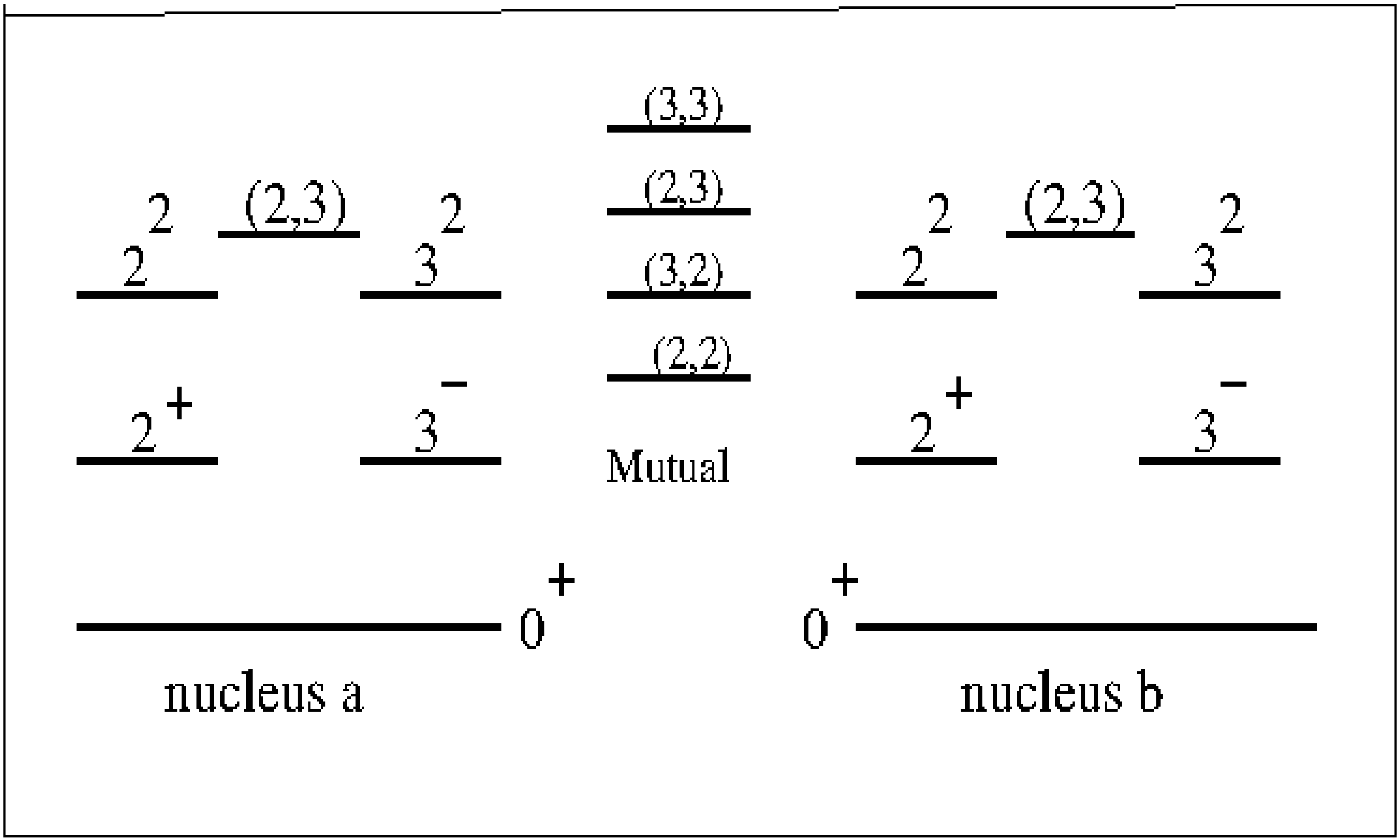
$$\frac{\hbar^2}{2\mu}\frac{L(L+1)}{r^2} \approx \frac{\hbar^2}{2\mu}\frac{J(J+1)}{r^2} \quad .$$

ROTATING FRAME APPROXIMATION



Full problem requires $\Sigma(I+1)$=33 channels
RFA selects only the m=0 state for each substate $\Sigma 1$=10 channels

Standard two-phonon calculation of fusion



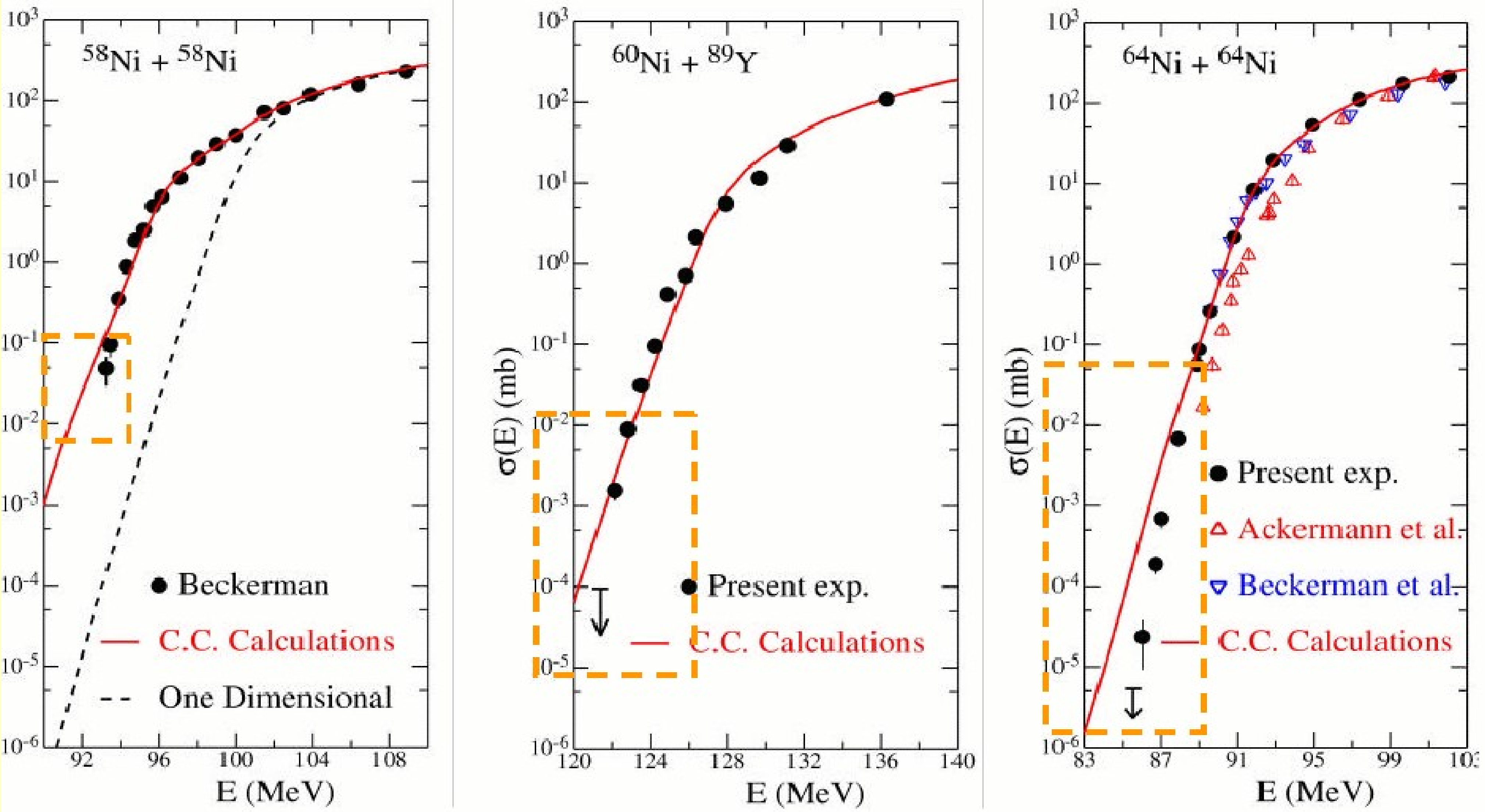
1 (GS) + 4 (1PH) + 4 (2PH) + 6 (Mutuals) = 15 channels.

FUSION CROSS-SECTIONS

$$\sigma_{\mathrm{F}}(E) = \frac{\pi}{k_0^2} \sum_L (2L+1) T_L$$

Hindrance deep under the barrier

E<E_s



C.L. Jiang et al.2002-2006

Diagnostic Tools: Astrophysical *S*-factor

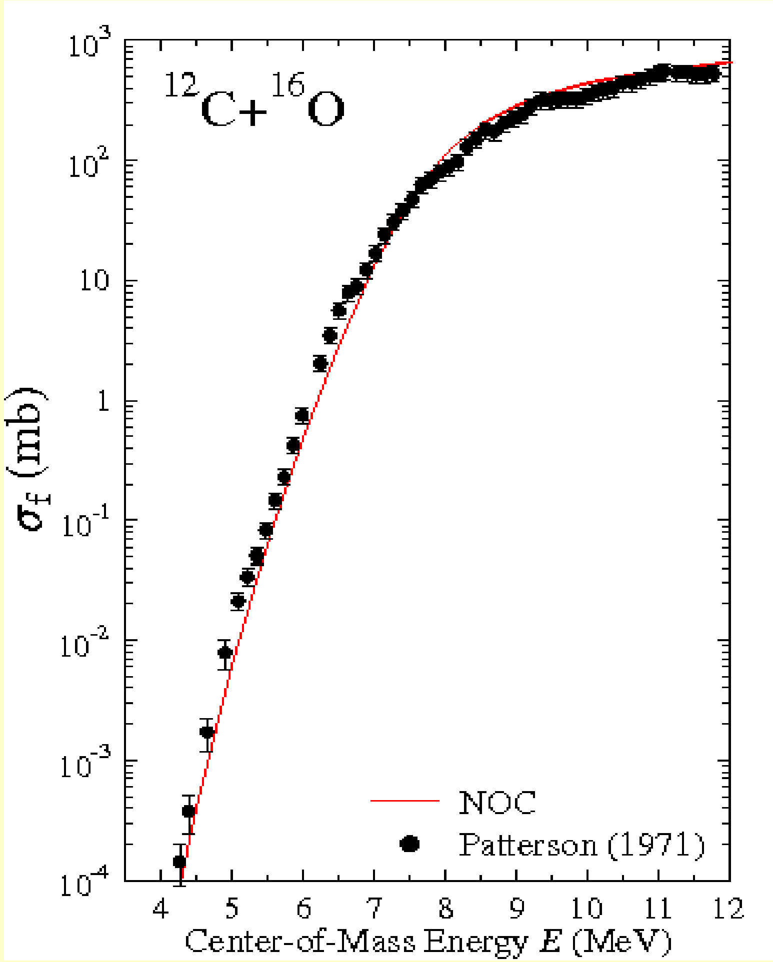
$$S = E\sigma_F(E)\exp(2\pi\eta), \qquad \eta = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0\hbar v}$$

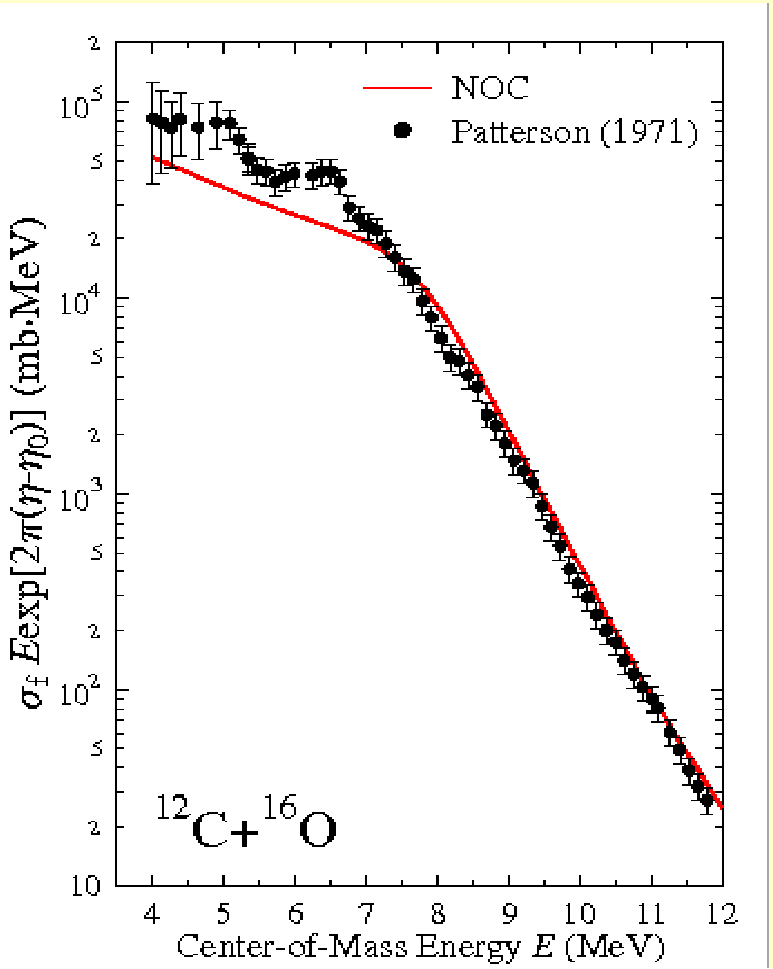
- Magnify the low-energy behavior of cross-sections

Diagnostic Tools: Astrophysical *S*-factor

Magnify the low-energy behavior of cross-sections

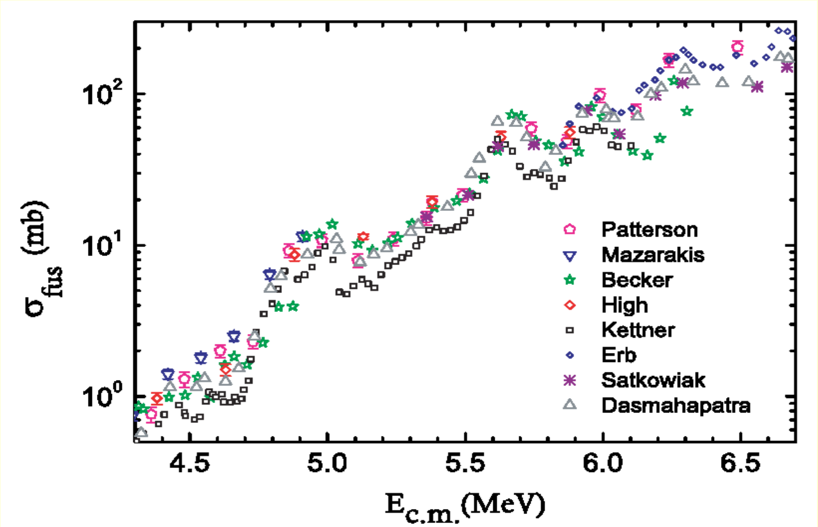
$$S = E\sigma_F(E)\exp(2\pi\eta), \qquad \eta = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar v}$$



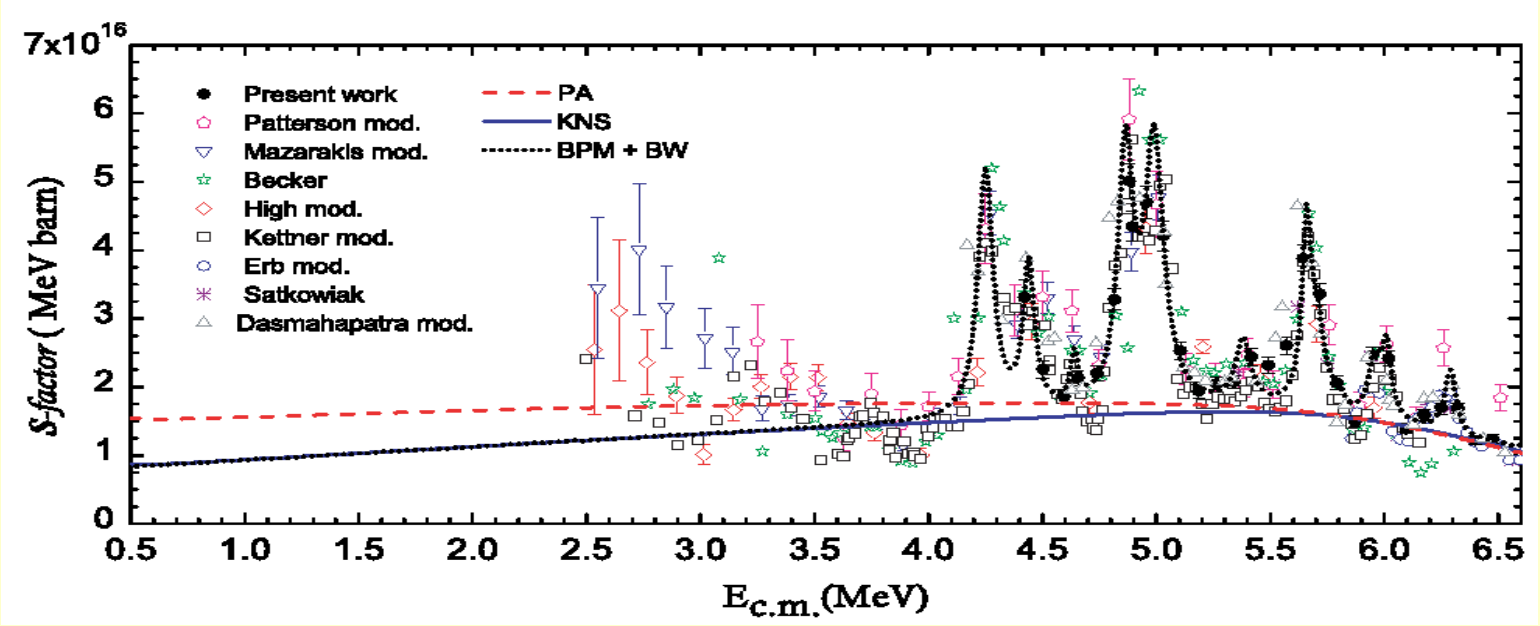


Diagnostic Tools: Astrophysical *S*-factor

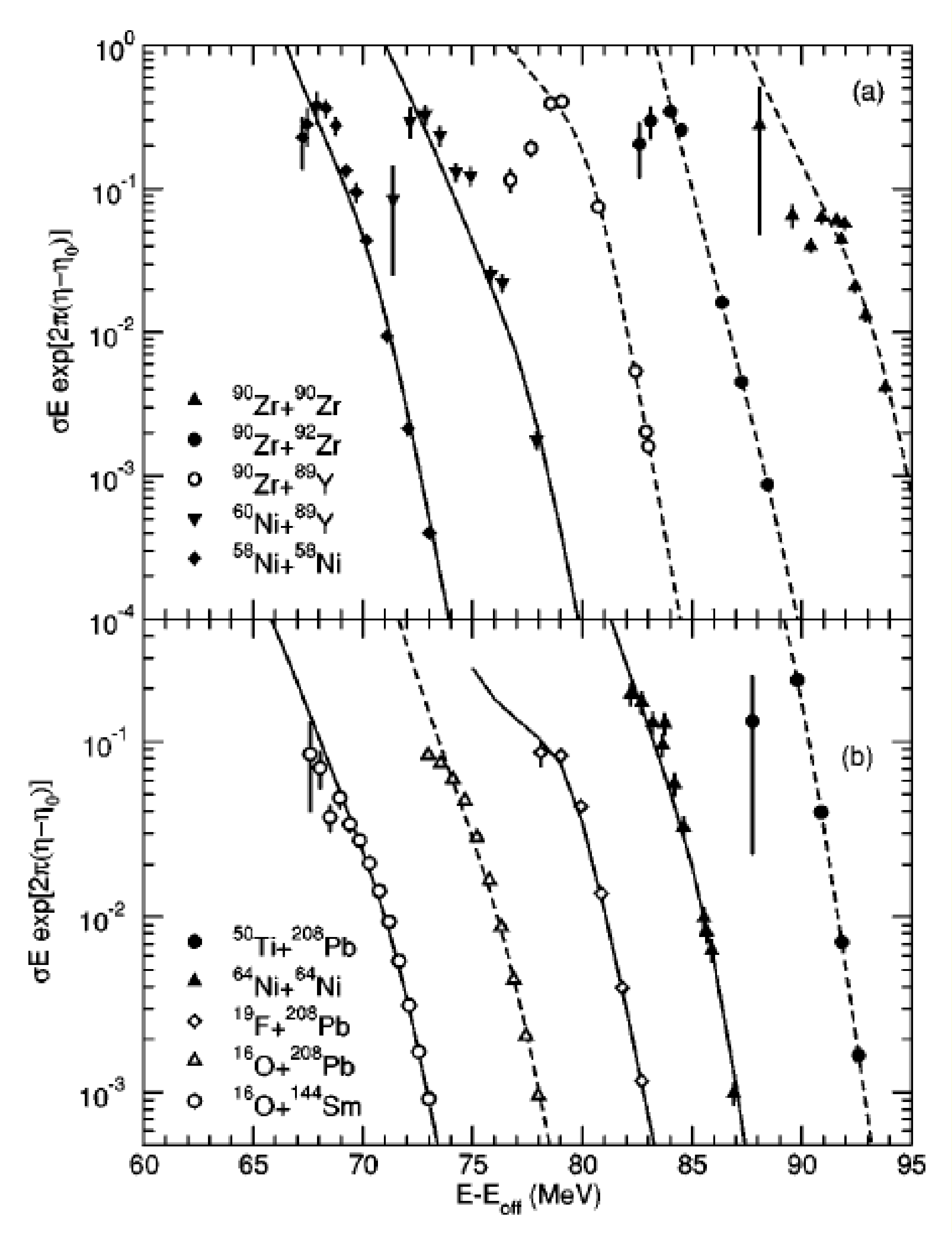
Unravel typical molecular resonant structures ($^{12}C+^{12}C$, ^{16}O, etc.)



$^{12}C+^{12}C$ Sub-barrier Fusion, Aguillera (2006)



Diagnostic Tools: Astrophysical *S*-factor

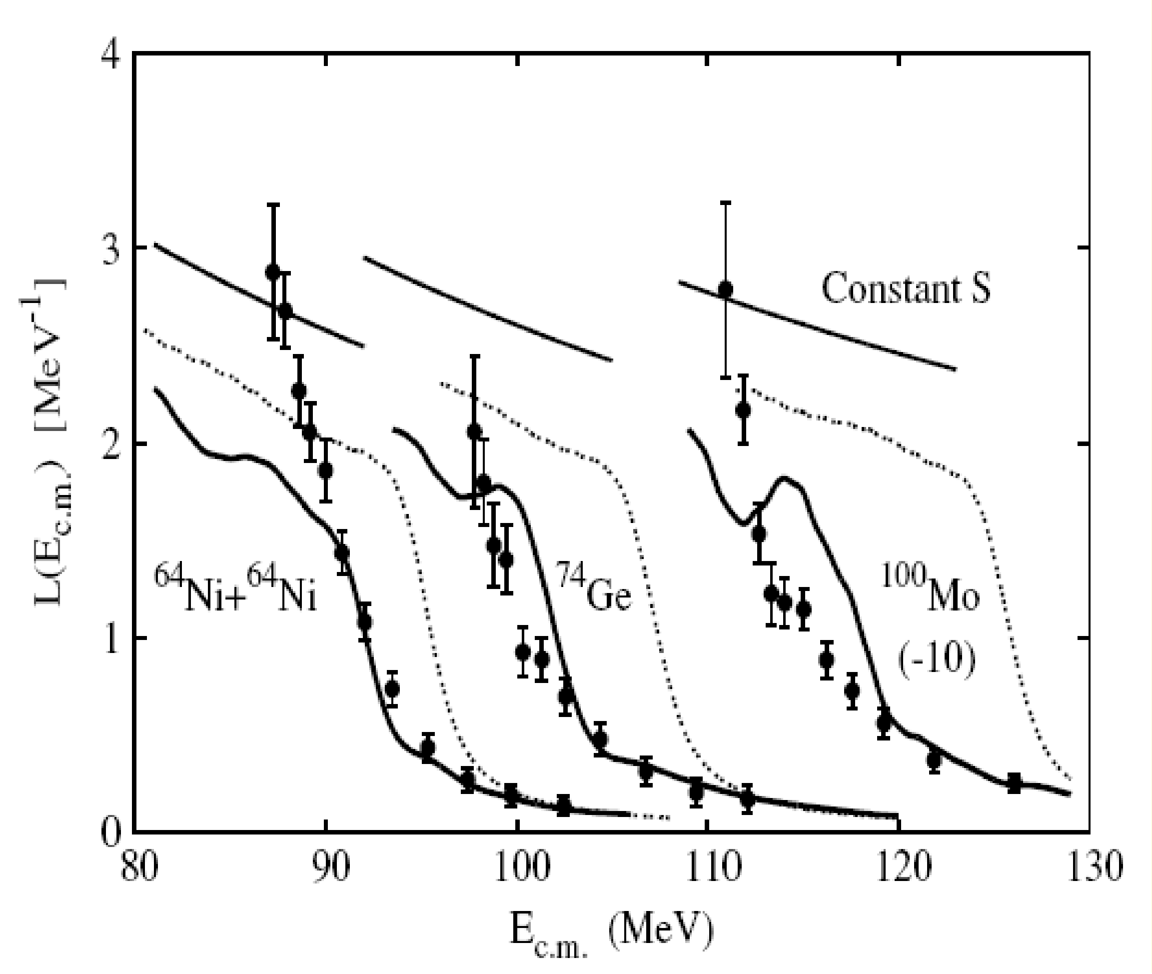


C.L.Jiang et al.
PRC69,014604

DIAGNOSTIC TOOLS IN SUB-BARRIER FUSION

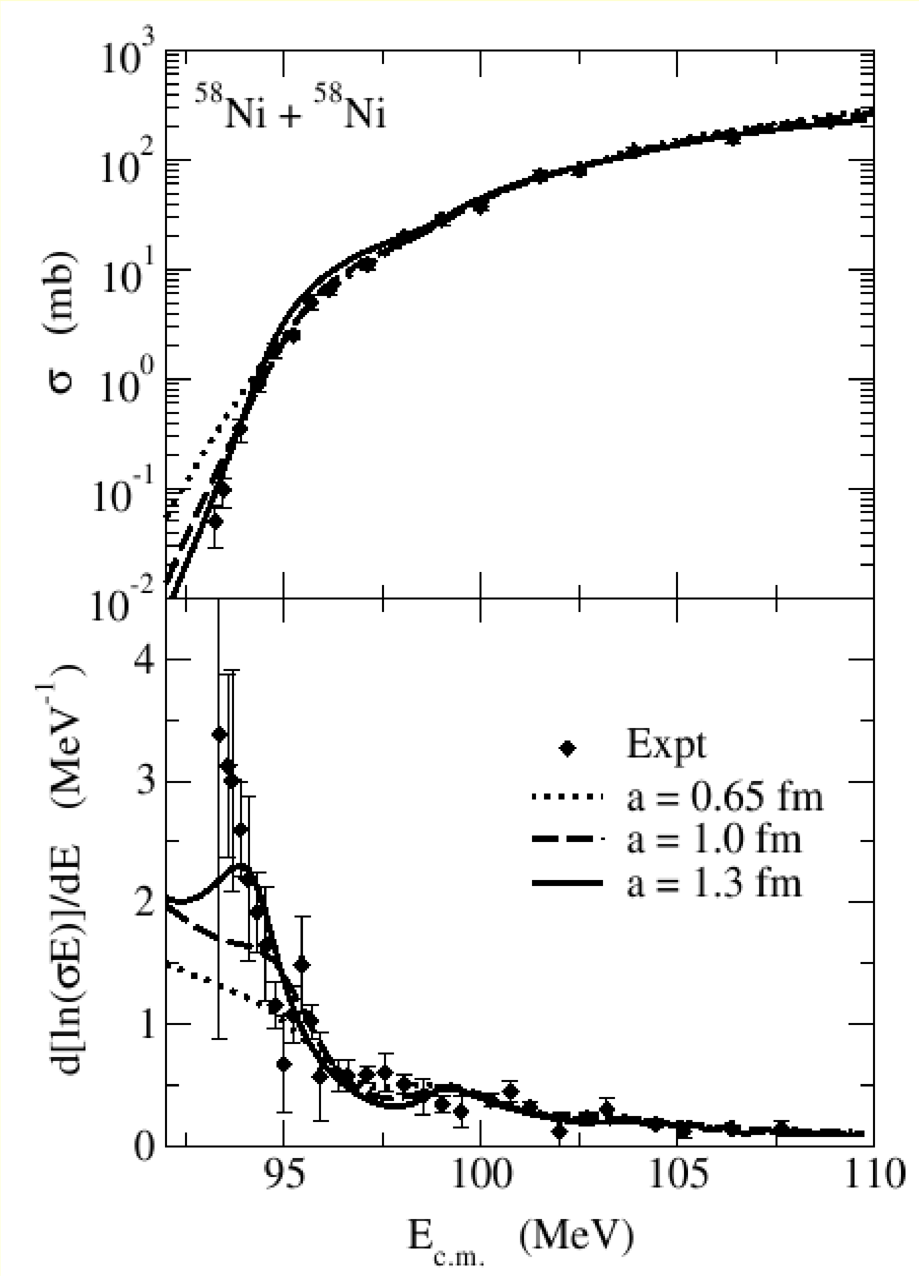
Logarithmic Derivative

$$L(E) = \frac{d[\ln(E\sigma)]}{dE} = \frac{1}{E\sigma}\frac{d(E\sigma)}{dE}$$



DIAGNOSTIC TOOLS IN SUB-BARRIER FUSION

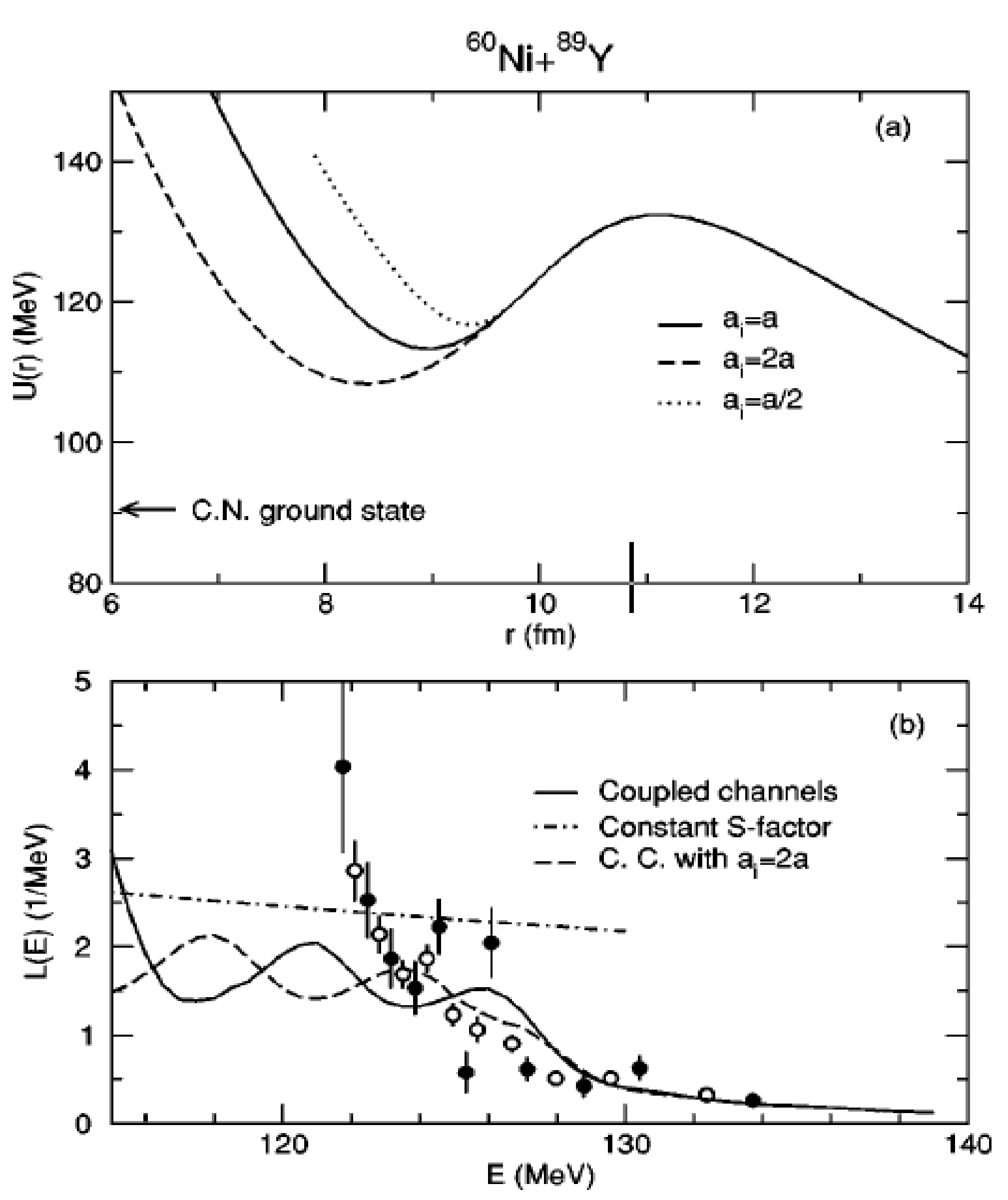
An attempt to solve the puzzle



Hagino, Rowley and Dasgupta
(PRC 67, 054603 (2006))

DIAGNOSTIC TOOLS IN SUB-BARRIER FUSION

Logarithmic Derivative Invalidate the large *a* scenario



C.L.Jiang et al.
PRC69,014604

DIAGNOSTIC TOOLS IN SUB-BARRIER FUSION

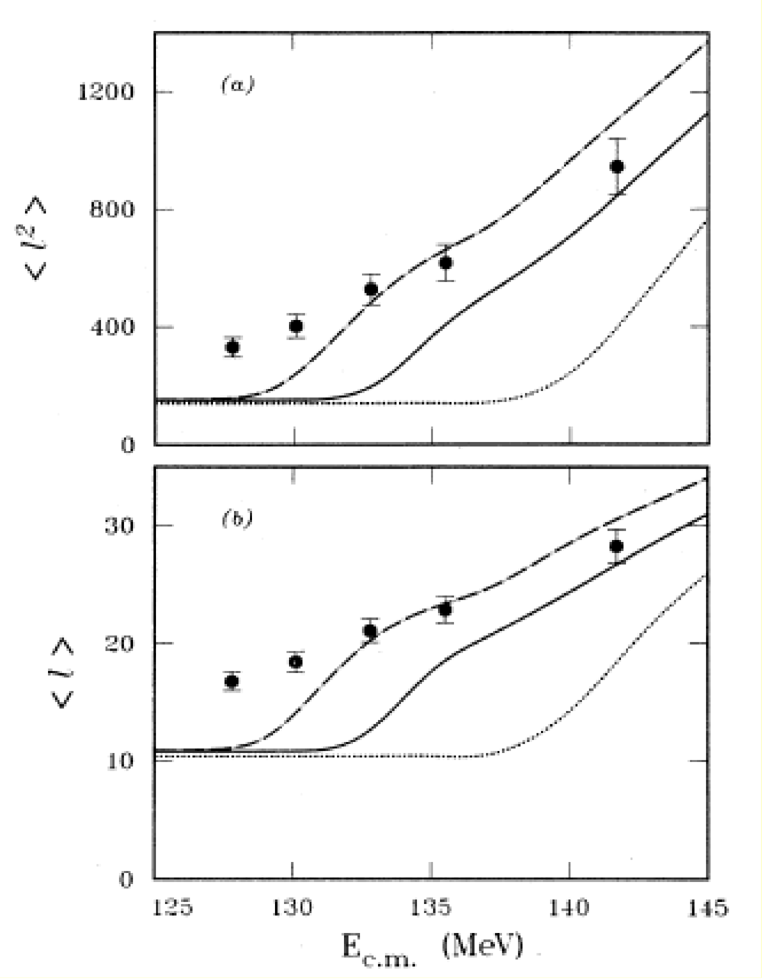
Spin distribution

Mean angular momentum $$\langle L \rangle = \frac{\sum_L L\sigma_L(E)}{\sum_L \sigma_L(E)}$$

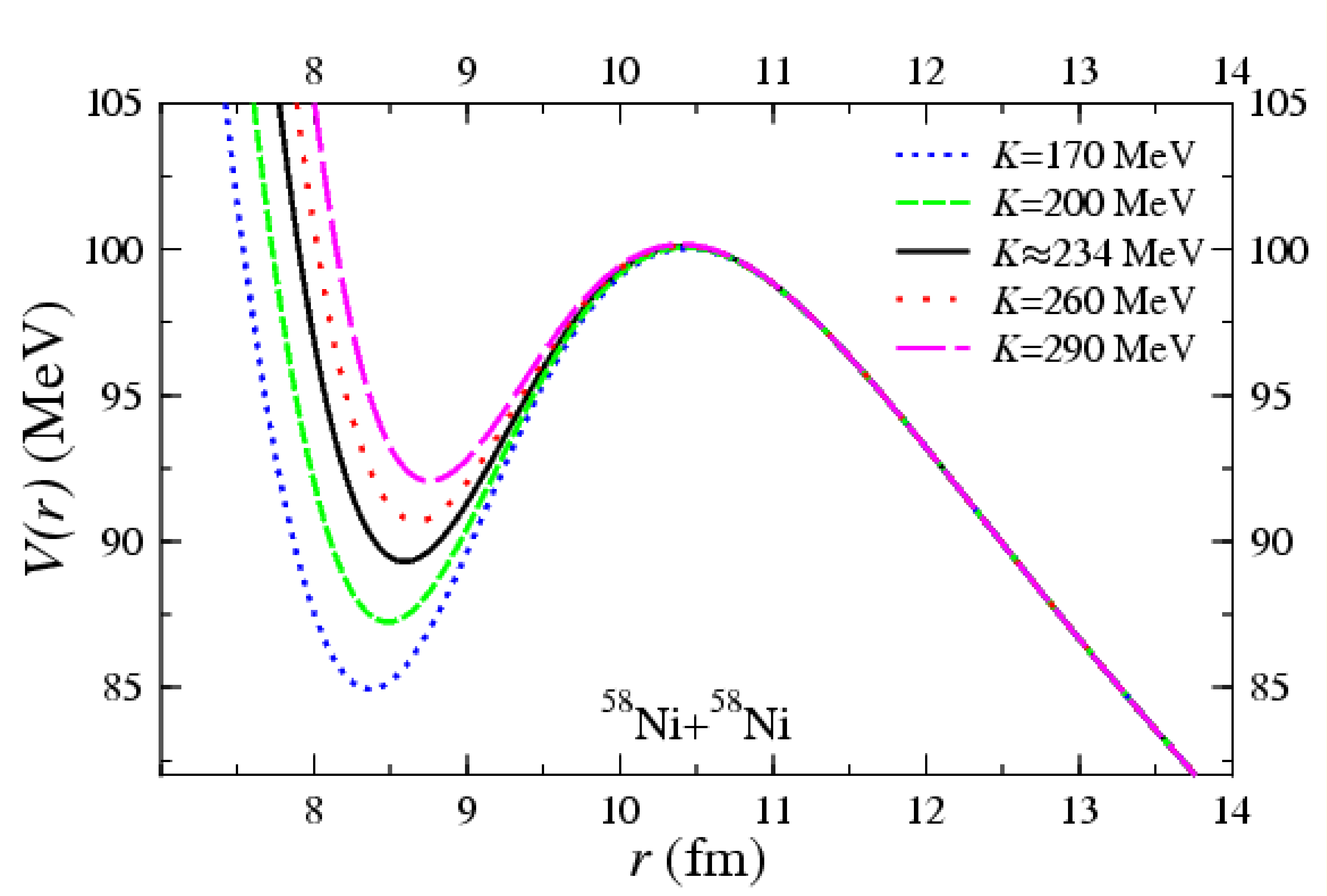
^{64}Ni+^{100}Mo

M.L.Halbert et al., PRC 40, 2558 (1989)

"80: Average angular momentum approach a constant value when *E*=0

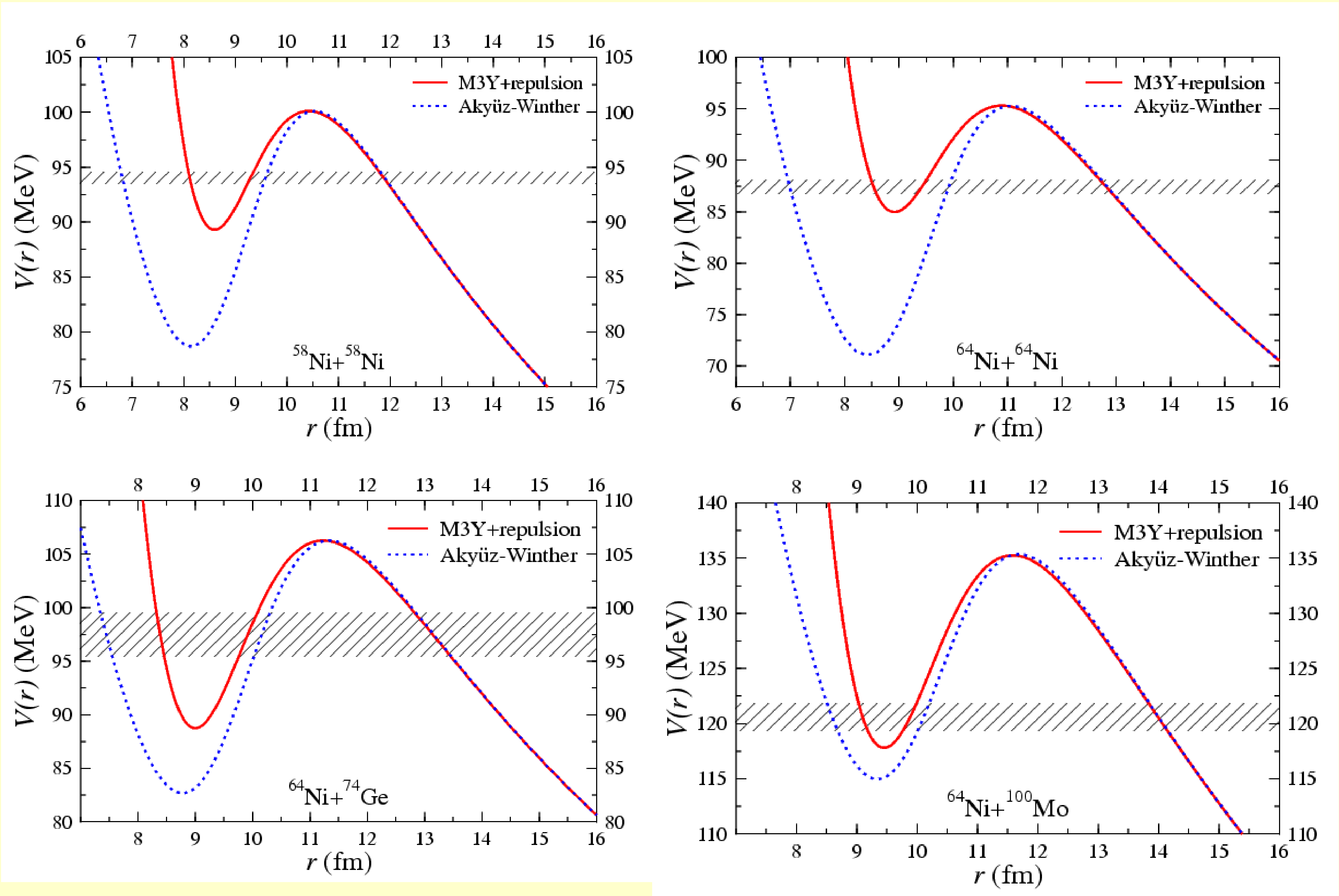


Potential with Repulsive Core

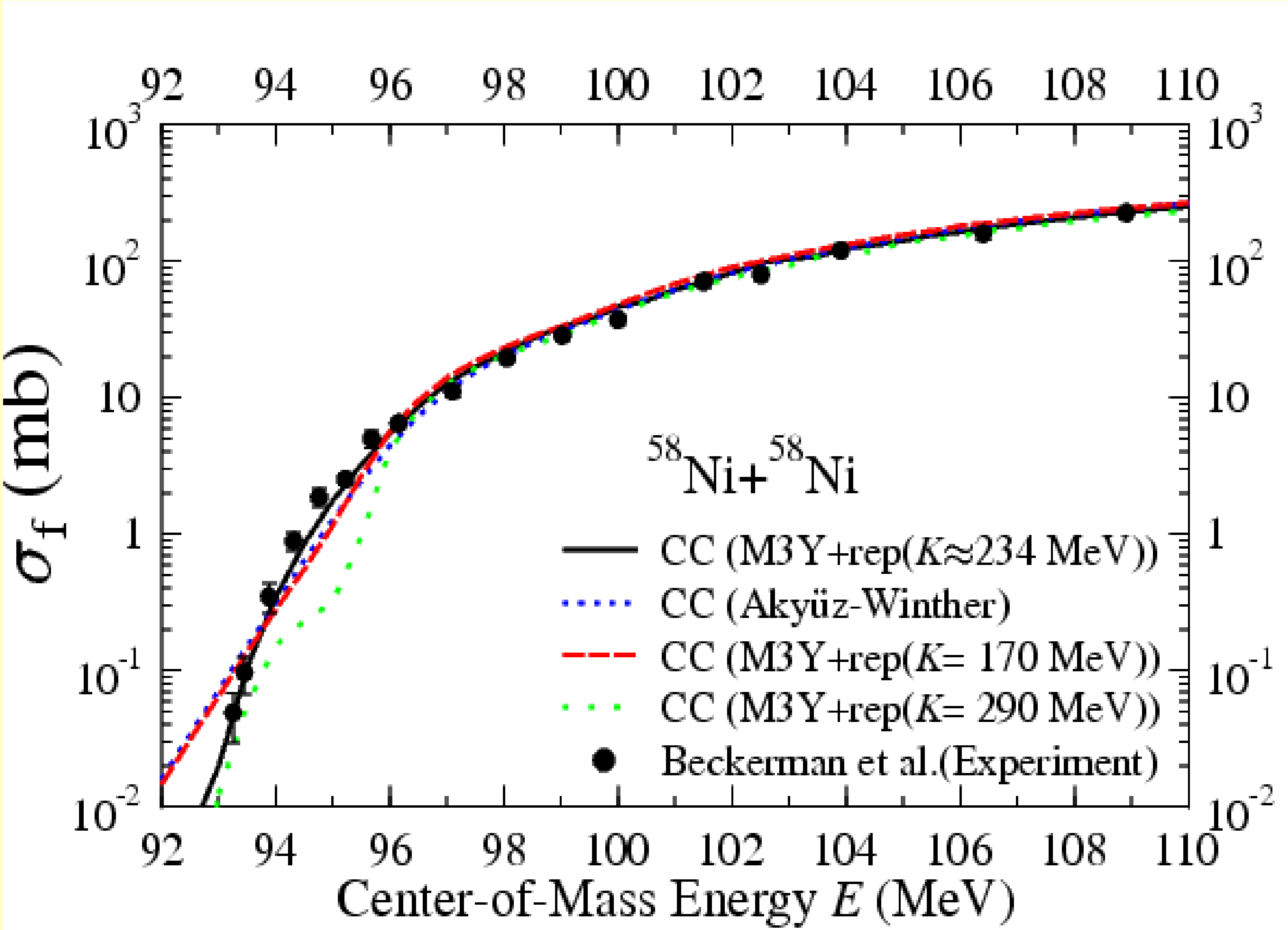


S.Misicu&H.Esbensen, PHY-11523-TH-2006, ANL

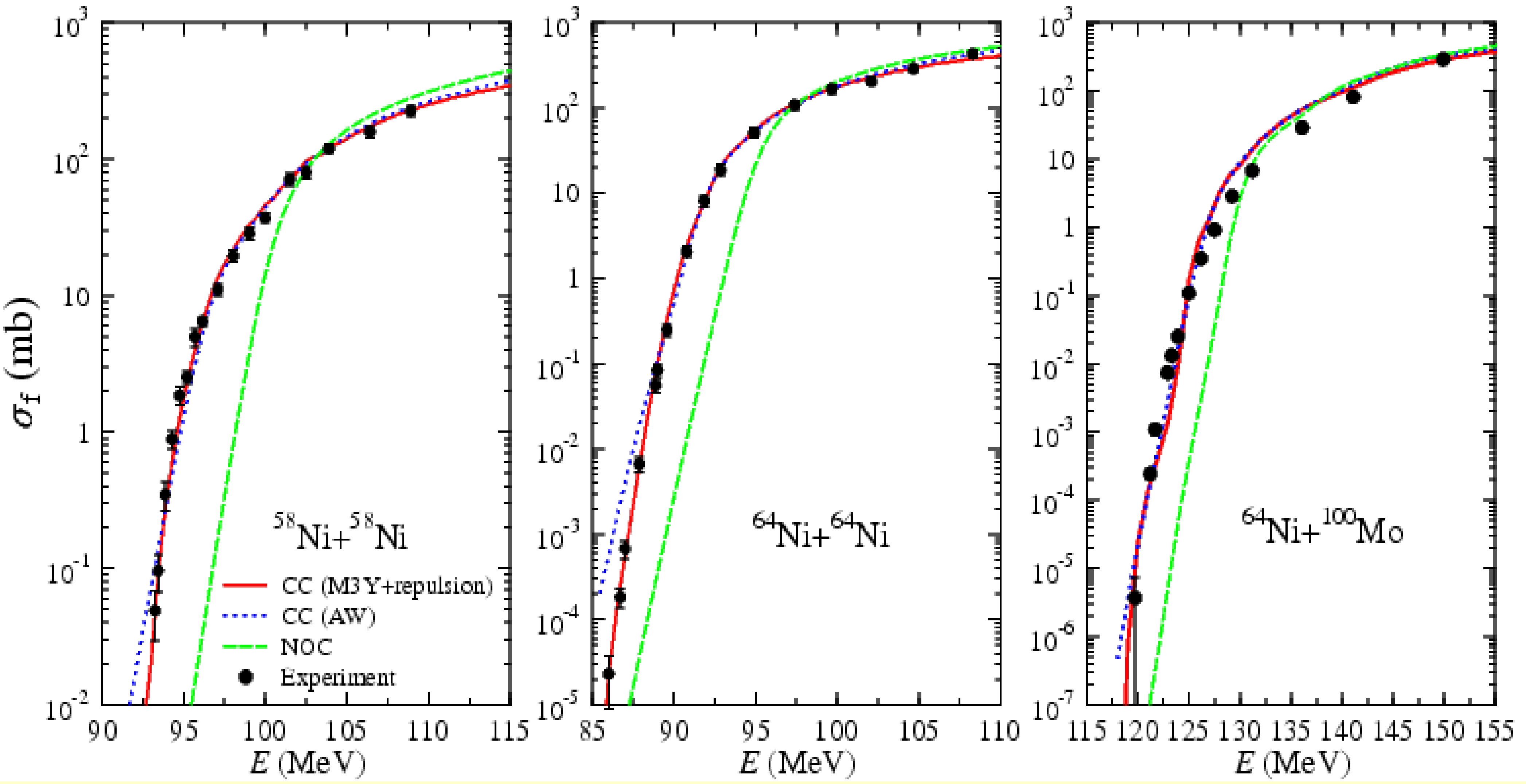
M3Y+repulsion vs. Akyüz-Winther



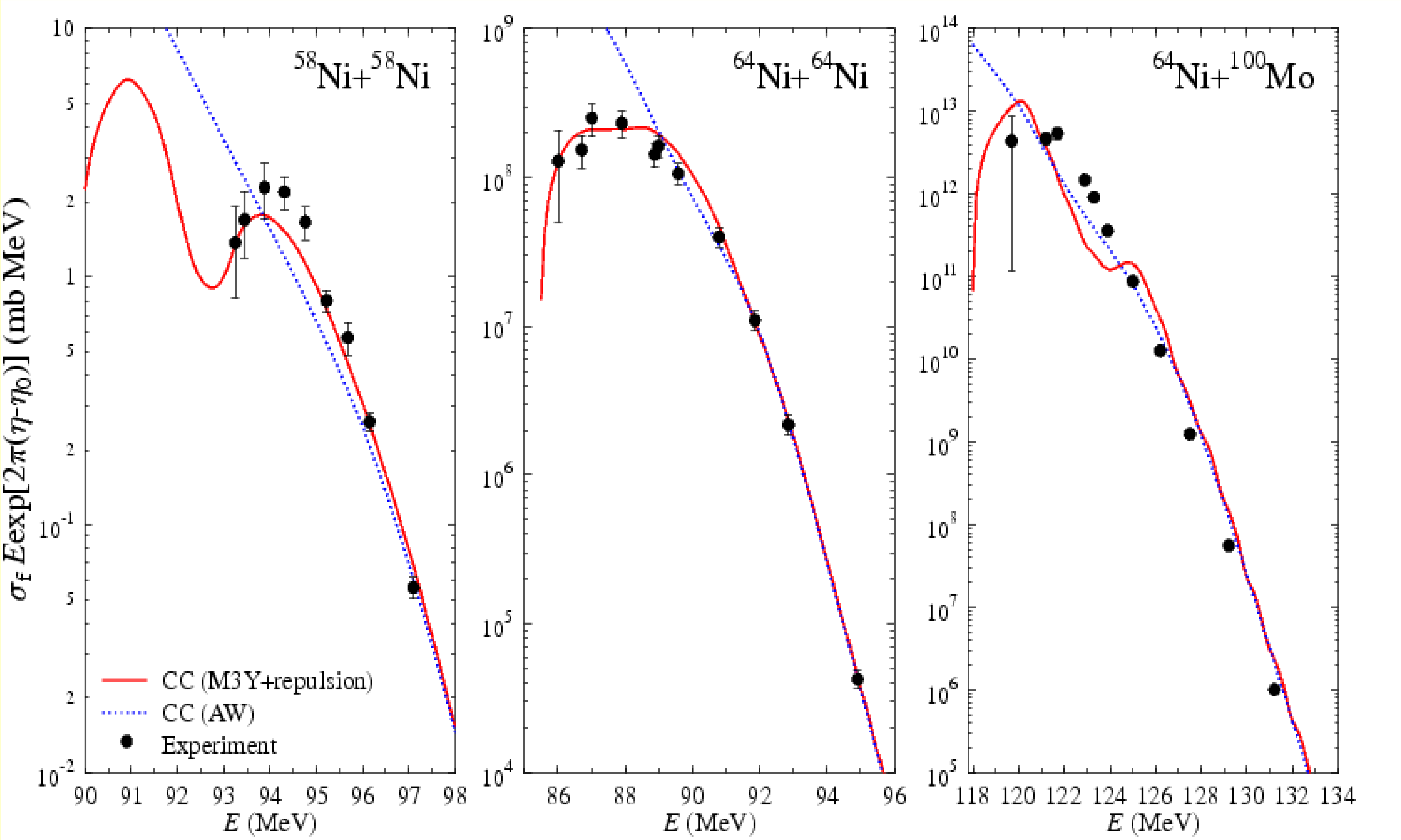
Fusion Cross-Sections vs. EOS



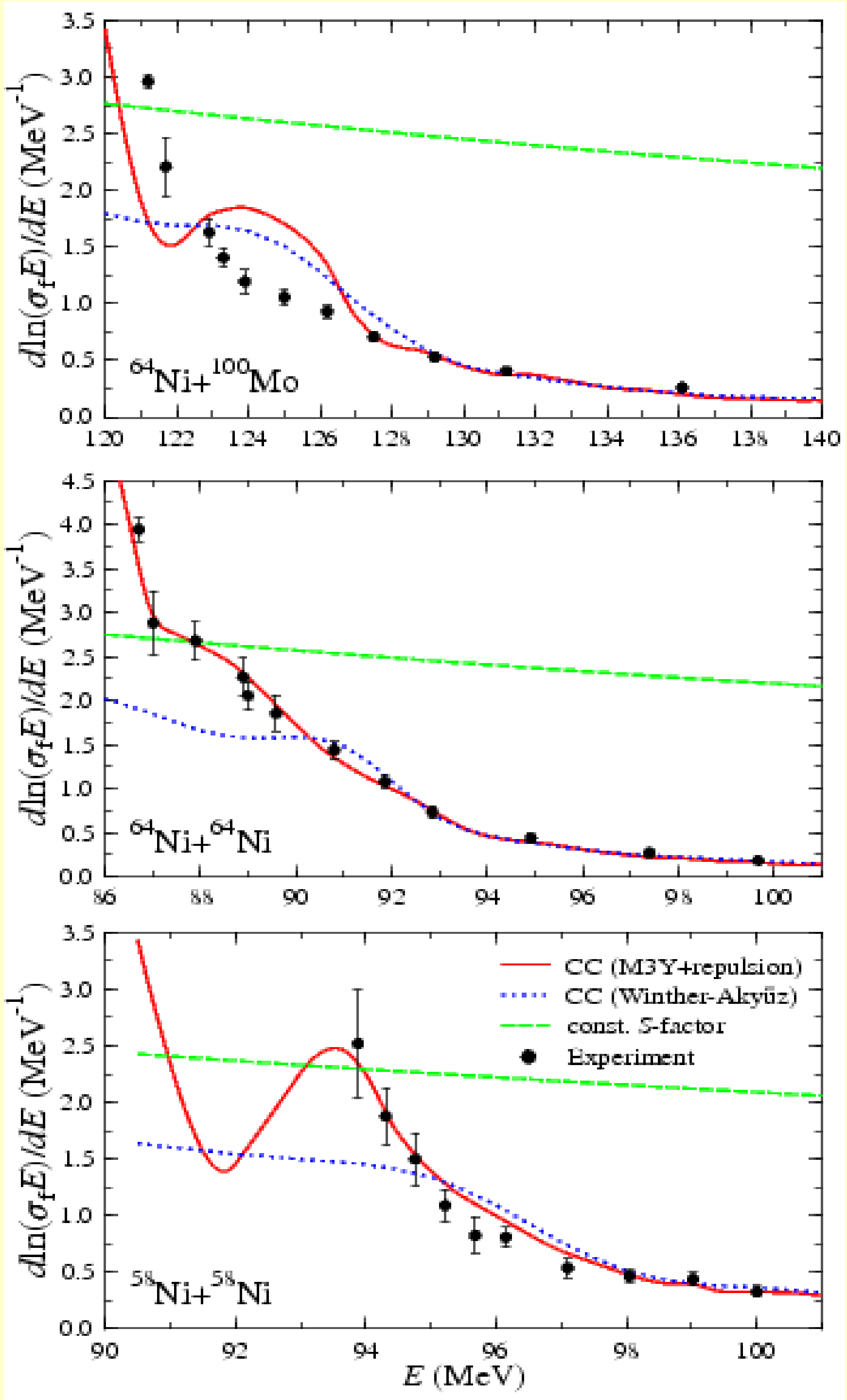
First Hindrance Cases : Fusion Cross-Sections



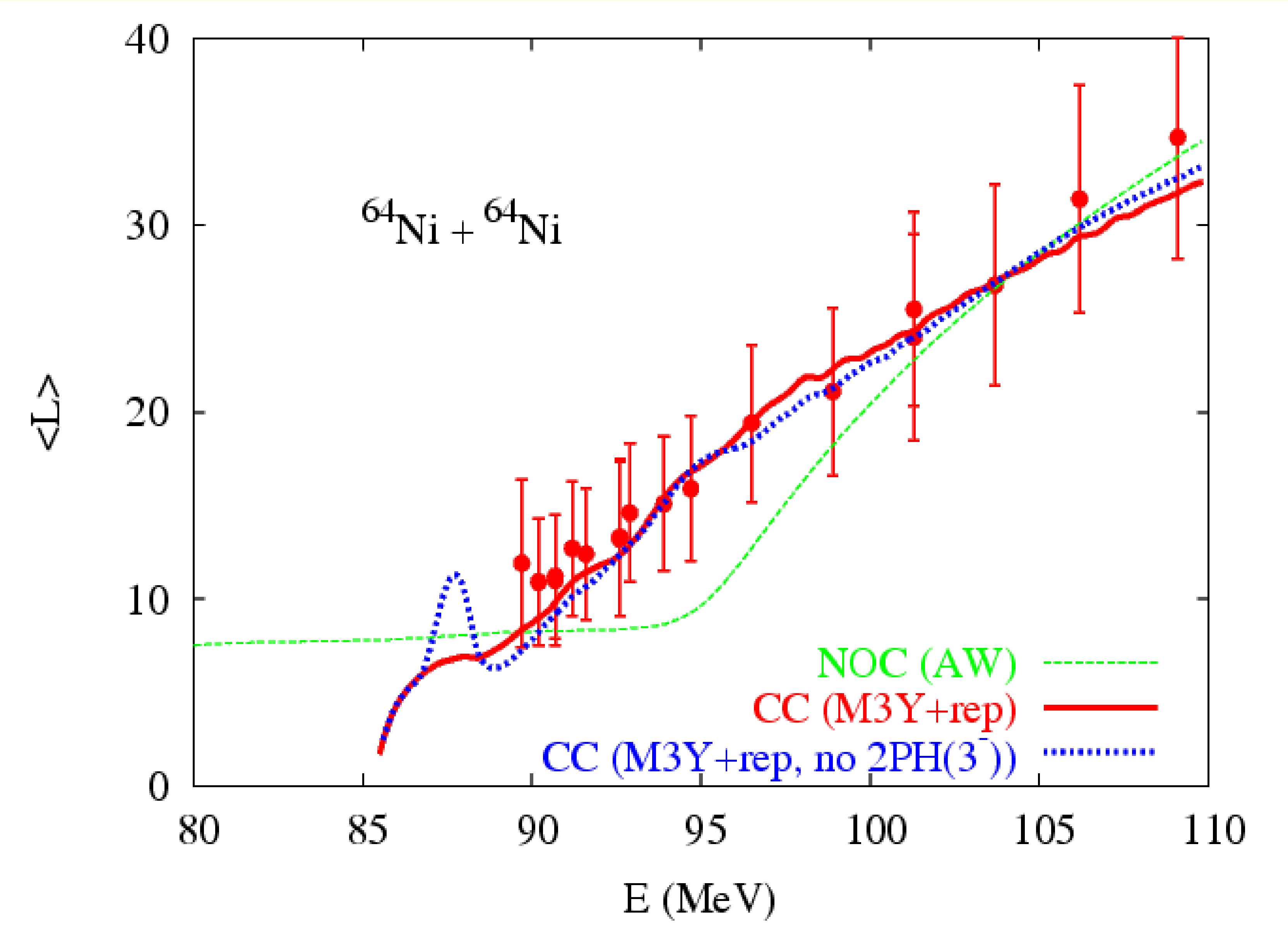
First Hindrance Cases: *S*-factors



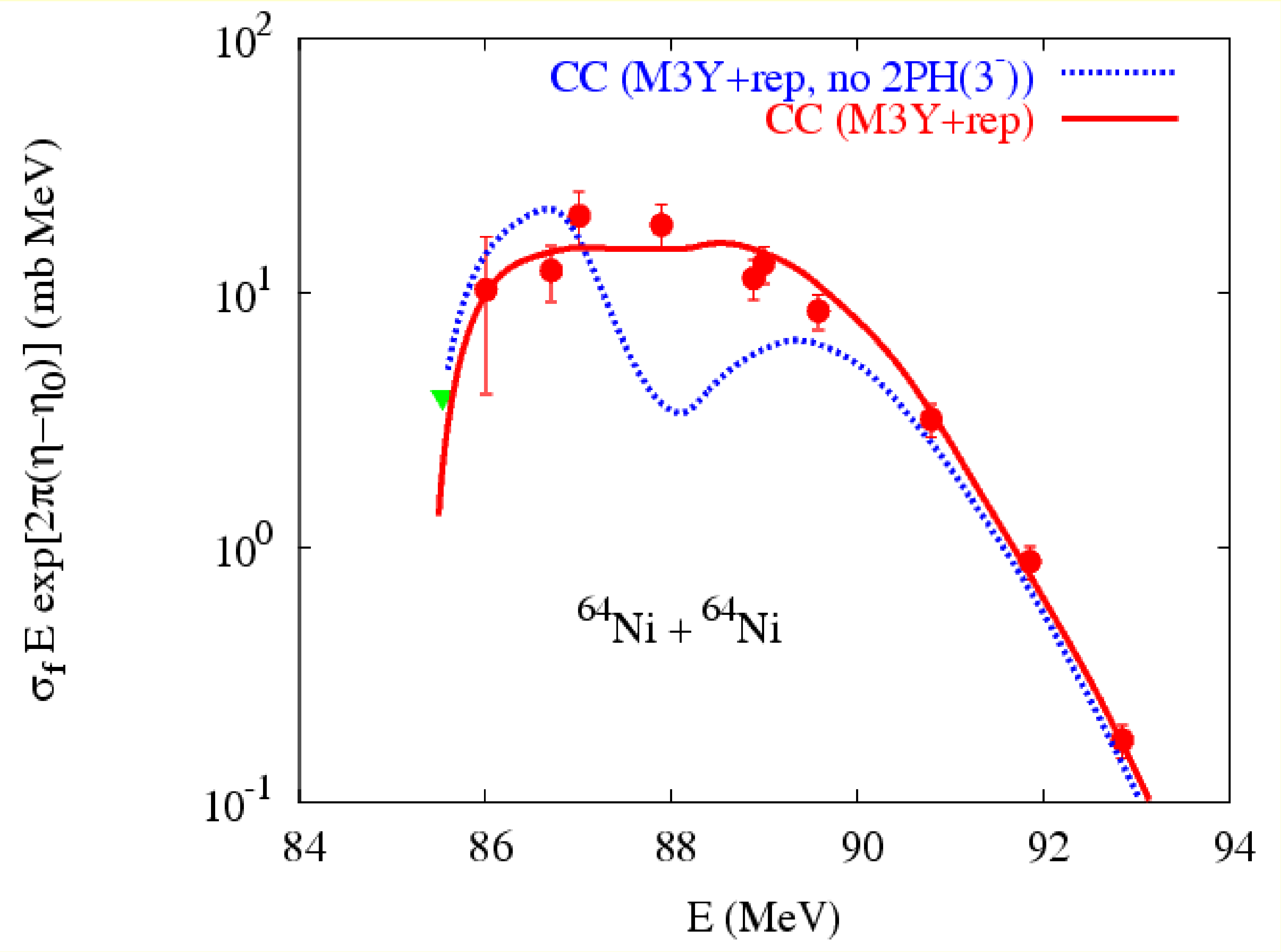
Hindrance Cases: Logarithmic derivatives



Hindrance Cases: Spin Distributions



Sensitivity to higher multiphonon excitations

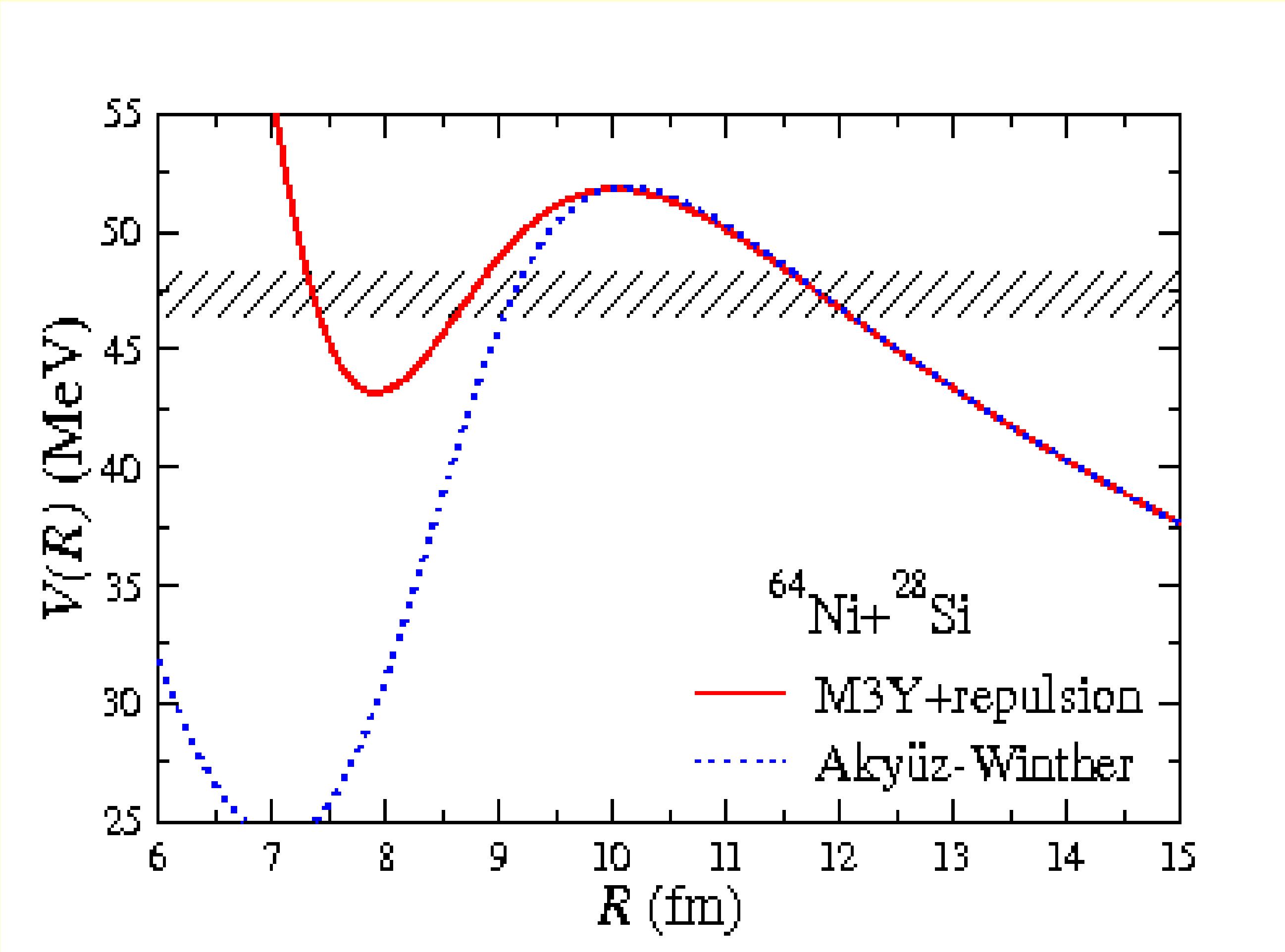


Necessary to include couplings to two-phonon octupole states !

Hindrance to Fusion for small Q

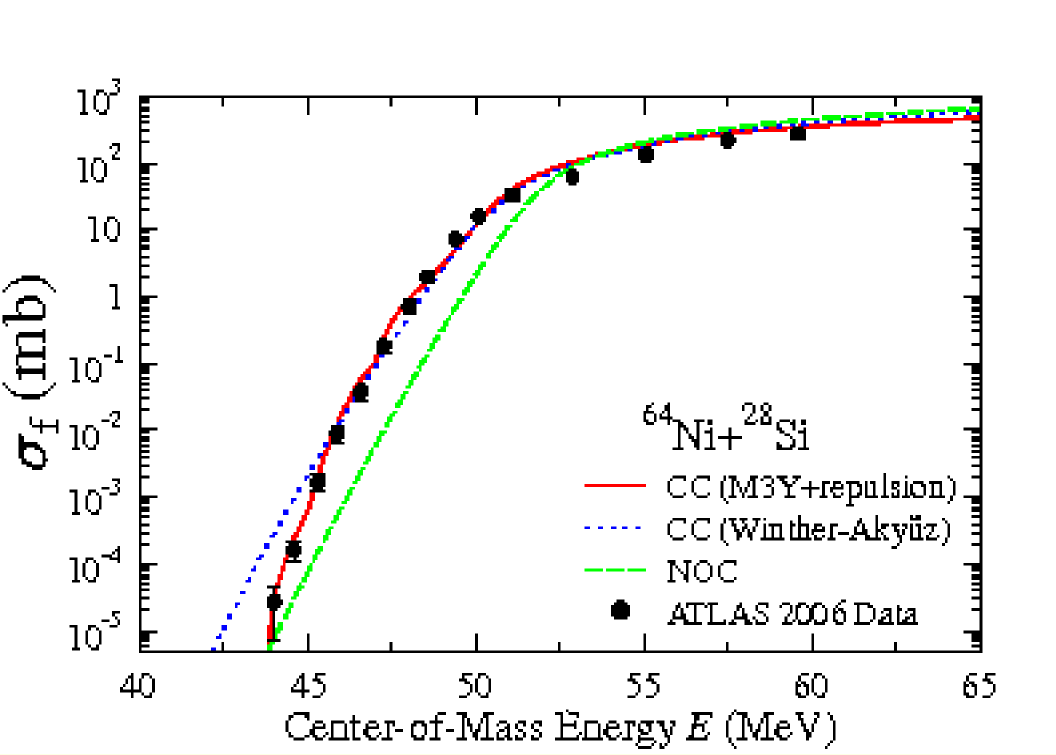
(C.L.Jiang et al., PLB 640, 18(2006))

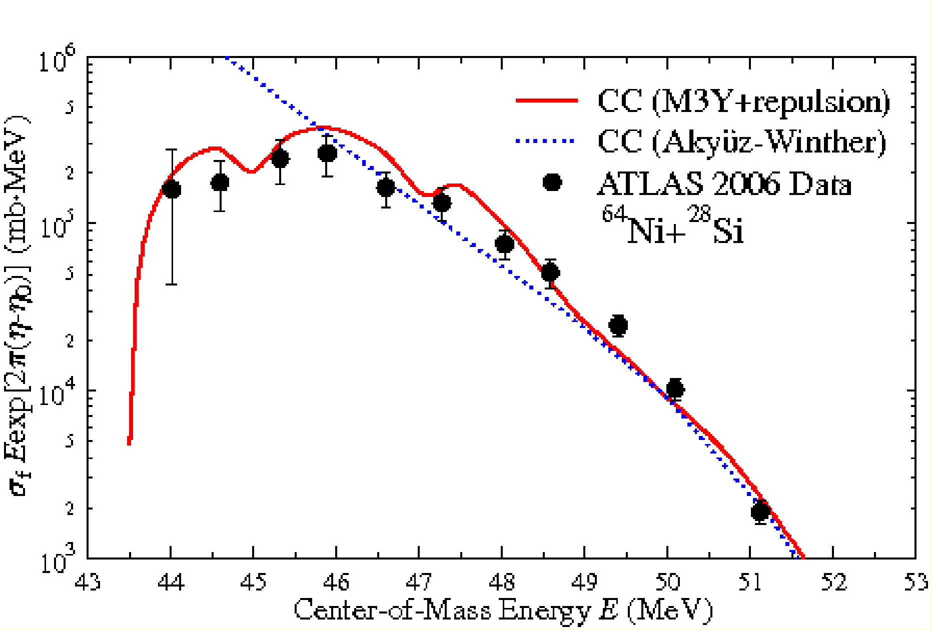
^{28}Si+^{64}Ni, Q=-1.78 MeV

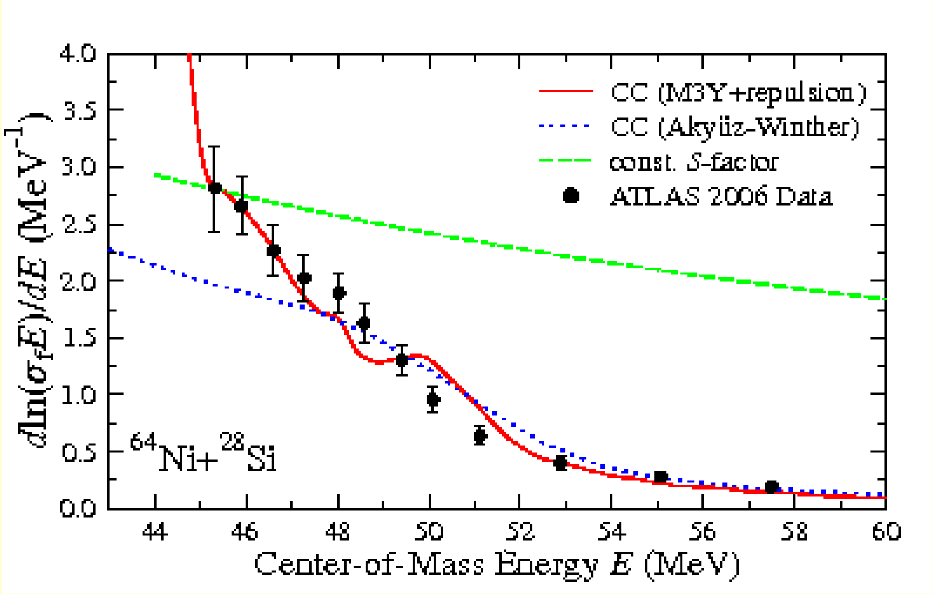


Hindrance to Fusion for small Q

$^{28}Si+^{64}Ni$, Q=-1.78 MeV

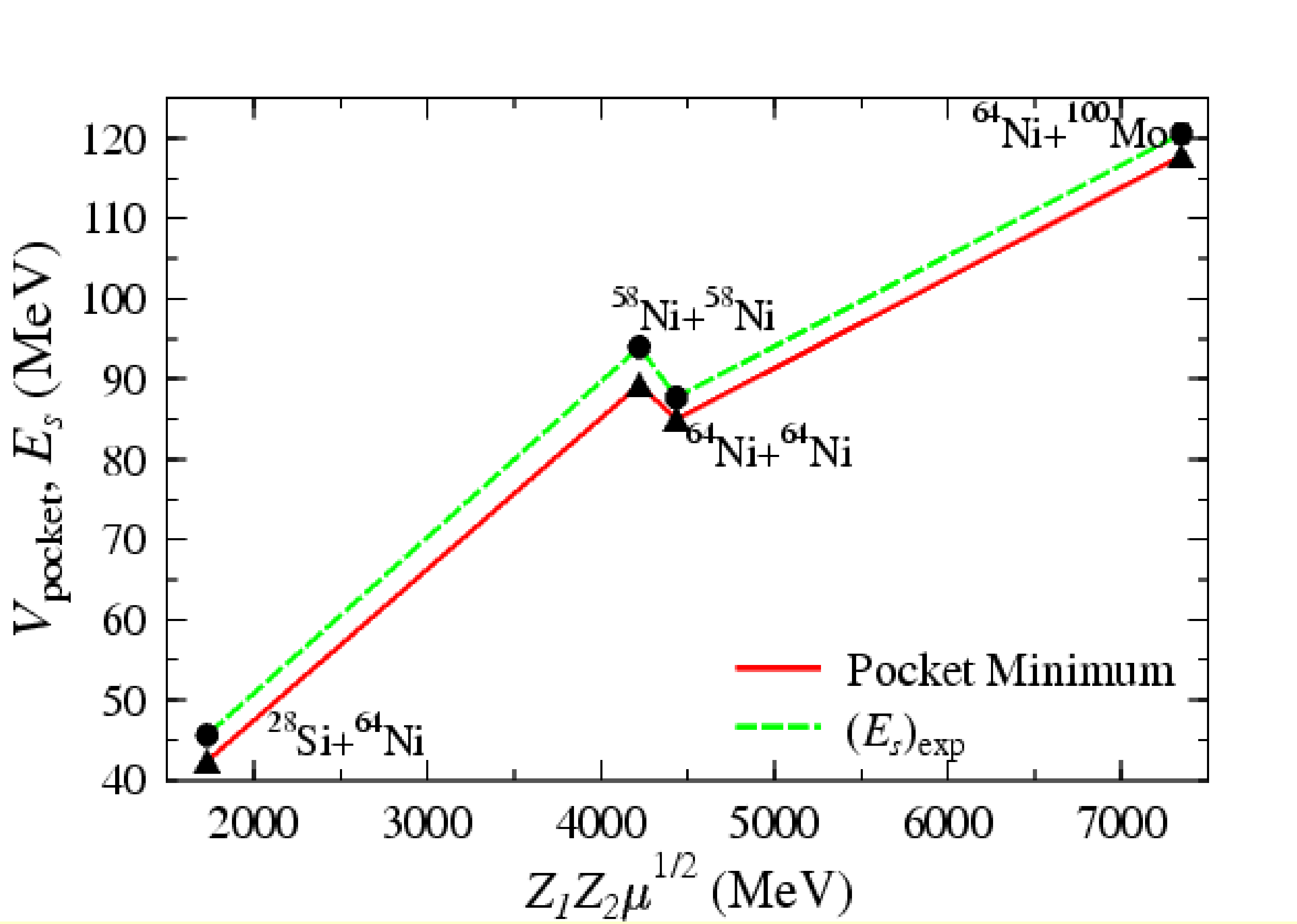






(C.L.Jiang et al., PLB 640, 18(2006))

Correlation V_{pocket}-E_s



Conclusions

The combination of a shallow potential , a thicker barrier and couplings to higher phonon states (and also to one-neutron channels for $^{16}O+^{208}Pb$) provides a good description of sub-barrier cross sections and thus of the hindrance phenomenon.

S-factor developes single or multiple peaks at the onset of hindrance and then will fall down.

Correlation between the onset of hindrance and minimum of the shallow potential

Narrowing of the spin distribution for fusion when bombarding energy approaches the minimum of the shallow potential

Outlook

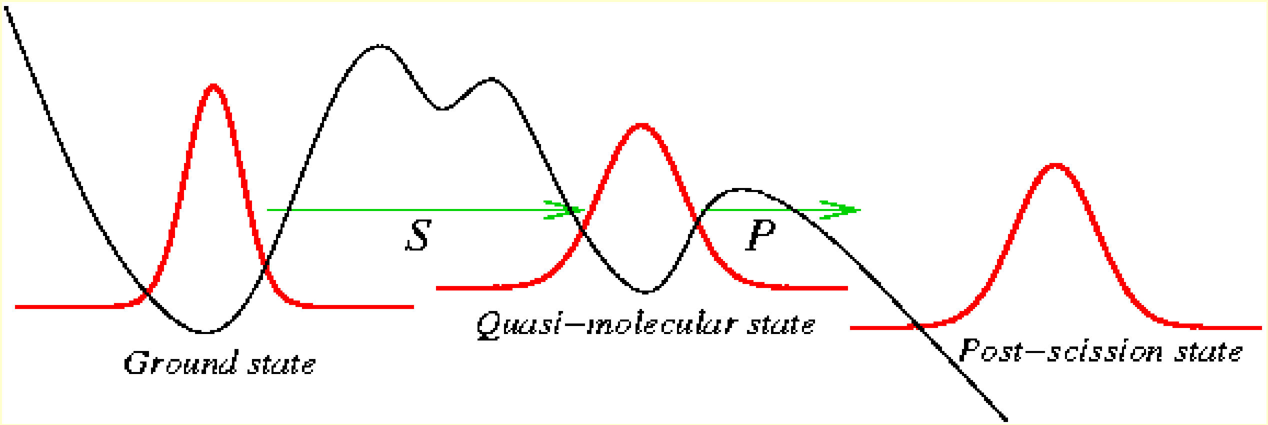
Investigate the extreme sub-barrier fusion for the Cluster Radioactivity combinations ($^{14}C+^{208}Pb$ and others) and extract the nuclear potential for the decay process and eventually the decay rates.

Use an effective *n-n* force that embodies in a natural way the saturation properties of the nuclear matter and is consistent with the nuclear structure input for the target and projectile.
Ex: Skyrme, Gogny

Cluster Radioactivity

- The residual nuclei are found in the ground state or in a low-lying excited state.
- Emitters - preactinides and actinides. Daughters - nuclei around ^{208}Pb. Clusters – from C up to Si.
- Decay Rates :

$$\lambda_C = SP$$



Acknowledgements

Work done at Argonne NATIONAL LABORATORY & IFIN-HH

Work supported by FULBRIGHT

U.S. DOE, Contract No.DE-AC02-06CH11357
Ministery for Education and Research, Romania