Recent Results with the Lorentz Integral Transform Method

in collaboration with

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Outline of the talk

- Motivation of LIT method
- Inclusive reactions
- Exclusive reactions
- Results on reactions with electromagnetic probes on:
  - 3-body system (e,e')
  - 7-body system (^{7}Be photoabsorption, radiative capture of ^{3}He-^{4}He)
  - 4-body system (test: consistence of incl. and excl. LIT-results)
- Conclusions and outlook

Motivation of LIT method

Aim: calculation of reactions involving A-body systems in the continuum

Well known: calculation of A-body continuum state tremendously more difficult than A-body bound state calculation

??? is it possible to calculate continuum observables without explicit knowledge of the corresponding continuum wave function ???

YES, via the LIT method!

Continuum state problem → bound-state like problem

LIT for Inclusive Reactions

Cross section described by response functions R(ω)

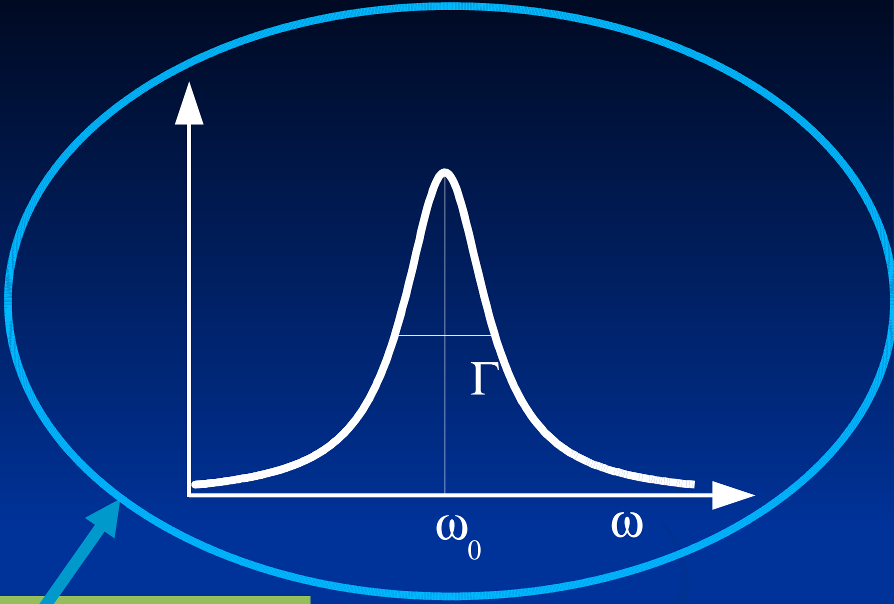
$$R(\omega) = \sum_n \left| \langle n | \Theta | 0 \rangle \right|^2 \delta(\omega - E_n + E_0)$$

steps:

1. Solve for many ω_0 and fixed Γ

$$(H - E_0 - \omega_0 + i\Gamma)\, \tilde{\Psi} = \Theta\, |0\rangle$$

2. Calculate



$$\left\langle \tilde{\Psi} \middle| \tilde{\Psi} \right\rangle = \int R(\omega) L(\omega, \omega_0, \Gamma) d\omega$$

for a Theorem based on closure

3. Invert transform

$$\int_{E^-_{th}}^{\infty} d\omega \frac{R(\omega)}{(\omega-\omega_0)^2+\Gamma^2} = \int_{E^-_{th}}^{\infty} d\omega \frac{R(\omega)}{(\omega-\omega_0-i\Gamma)\,(\omega-\omega_0+i\Gamma)}$$

$$= \int_{E^-_{th}}^{\infty} d\omega \frac{\int dn\ \langle 0|\Theta^\dagger|n\rangle\langle n|\Theta|0\rangle\ \delta(\omega - E_n - E_0)}{(\omega-\omega_0-i\Gamma)\,(\omega-\omega_0+i\Gamma)}$$

$$= \int dn\ \langle 0|\Theta^\dagger (E_n-E_0-\omega_0-i\Gamma)^{-1}|n\rangle\langle n|(E_n-E_0-\omega_0+i\Gamma)^{-1}\,\Theta\,|0\rangle$$

($E_n \to H$, $E_n \to H$)

$$= \langle 0|\Theta^\dagger (H-E_0-\omega_0-i\Gamma)^{-1}\ (H-E_0-\omega_0+i\Gamma)^{-1}\,\Theta\,|0\rangle$$

$$= \langle\tilde{\psi}|\tilde{\psi}\rangle \quad \text{with} \quad (H-E_0-\omega_0+i\Gamma)|\tilde{\psi}\rangle = \Theta\,|0\rangle$$

For ***exclusive reactions*** one has to calculate

$$\langle \Psi(E) | \Theta | 0 \rangle$$

transition matrix elements T

steps:

1. Solve for many ω_0 and fixed Γ

$$(H - E_0 - \omega_0 + i\Gamma)\, \tilde{\Psi}_1 = \Theta\, |0\rangle$$

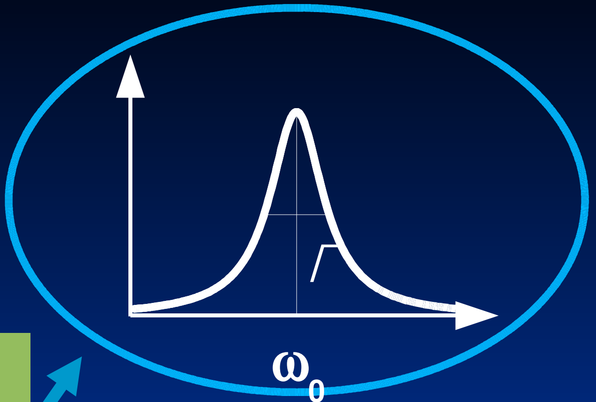
"inclusive" equation

$$(H - E_0 - \omega_0 + i\Gamma)\, \tilde{\Psi}_2 = V\, |PW\rangle$$

new equation

2. Calculate overlap

$$\left\langle \tilde{\Psi}_1 | \tilde{\Psi}_2 \right\rangle = \int F(E) L(E, E_0, \Gamma) dE$$



for a theorem based on closure

3. Invert transform and find *F(E)*

4. Calculate transition matrix element: $T = T_{BORN} + T_{FSI}$

$$T_{FSI} = \langle \Psi(E) | \Theta | 0 \rangle_{FSI} = \int dE' \frac{F(E')}{(E - E' + i\epsilon)}$$

LIT:

- 1994, proposed by V.Efros, W.L., G.Orlandini , applied for d(e,e')
- 1997, application for A>2 : ^{4}He(e,e') with semirealistic NN force
- 2000, application with realistic NN + 3N forces, total photoabsorption cross section of ^{3}H/^{3}He (V. Efros, W.L., G. Orlandini, E.L. Tomusiak)
- 2000, test for exclusive case: d(e,e'p)n (A. La Piana, W.L.)
- 2004, application for exclusive case with semirealistic NN force (S. Quaglioni, N. Barnea, V. Efros, W.L., G. Orlandini)
- 2006, first calculation of a 4-body reaction in the many-body continuum with realistic NN and 3N forces, ^{4}He total photoabsorption cross section (D. Gazit, S. Bacca, N. Barnea, W.L., G. Orlandini)
- many more application for A=3-7 also thanks to EIHH method (N. Barnea, W.L., G. Orlandini)
- 2007, review paper (V. Efros, W.L., G. Orlandini, N. Barnea), J. of Physics G in press (arXiv:0708.2803 (nucl-th))

main point of the LIT :

Schrödinger-like equation with a source

$$\left(H - E_0 - \omega_0 + i\Gamma\right) \tilde{\Psi} = S$$

The $\tilde{\Psi}$ solution is unique and has ***bound state like*** asymptotic behavior

→ one can apply ***bound state methods***

Our method for calculation of bound states

Hyperspherical **H**armonics Expansions (HH): **CHH** and **EIHH**

CHH: Additional two-body correlation functions are introduced

EIHH: Effective Interaction is constructed via Lee-Suzuki transformation

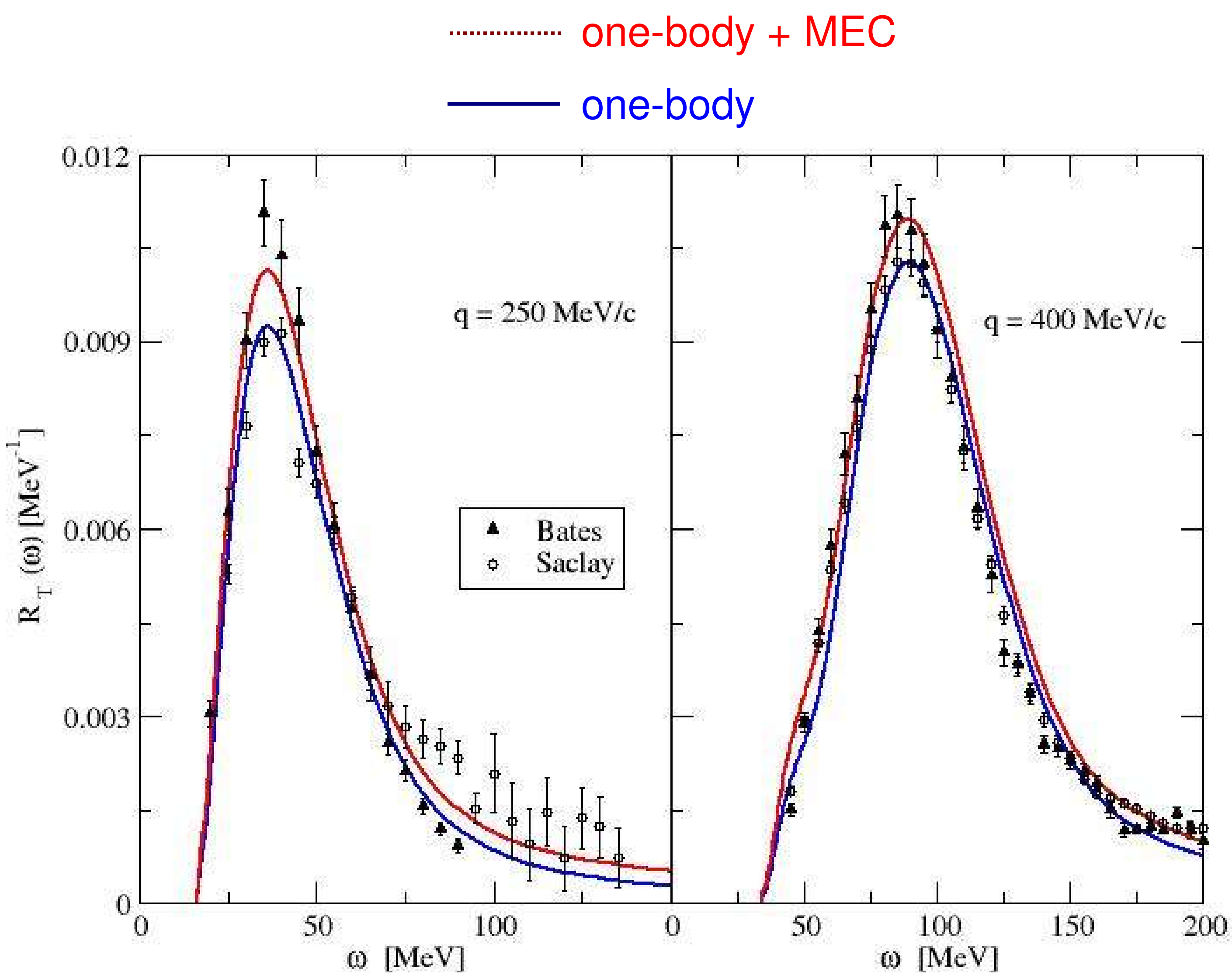
RESULTS

3 - body

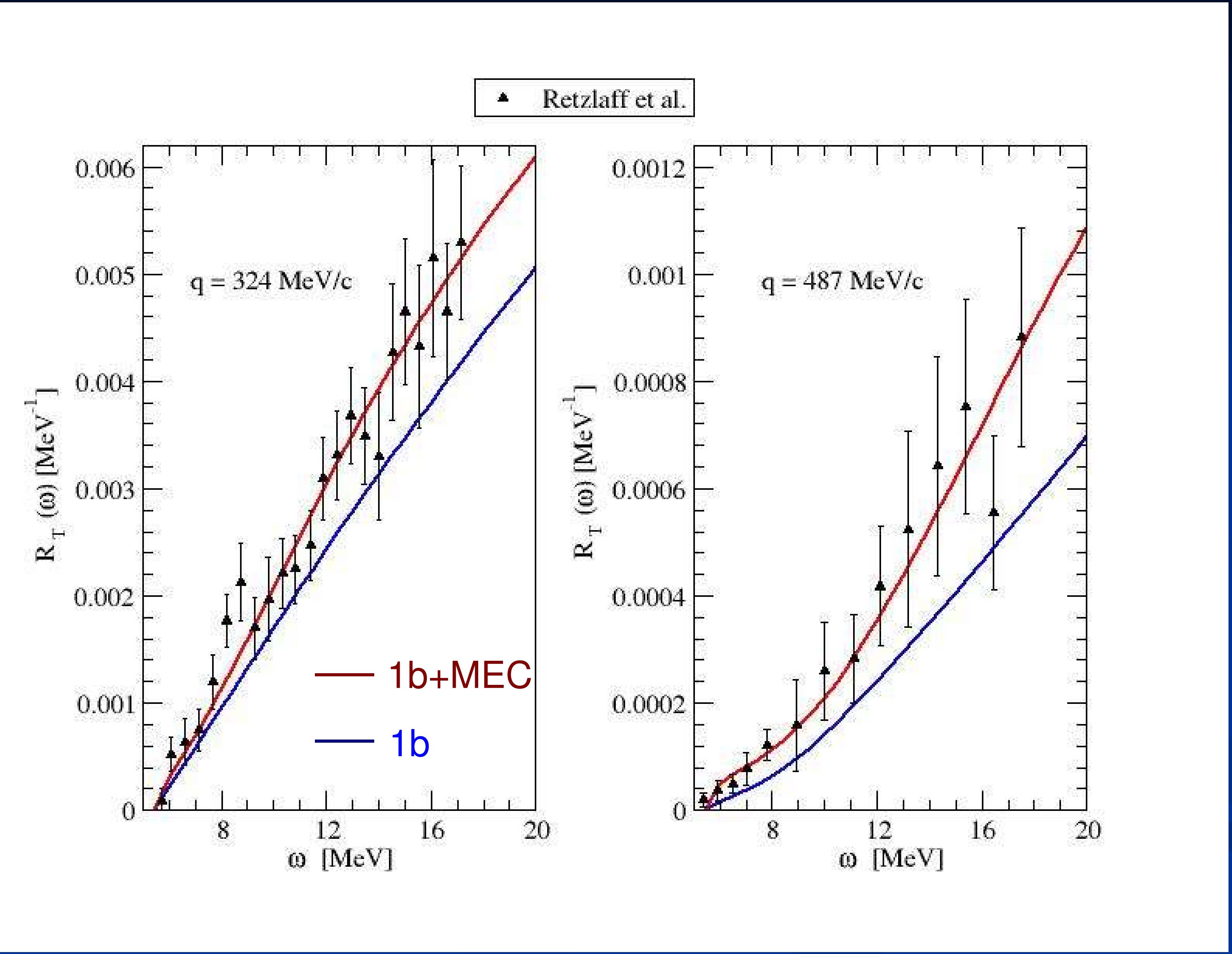
Transverse form factor $R_T(q,\omega)$ of ^{3}He

Nuclear force: Bonn-RA NN-potential +
Tucson-Melbourne 3N-force

Current operators: non-relativistic one-body operators
implicit MEC: Siegert operator
explicit MEC: π- and ρ-exchange



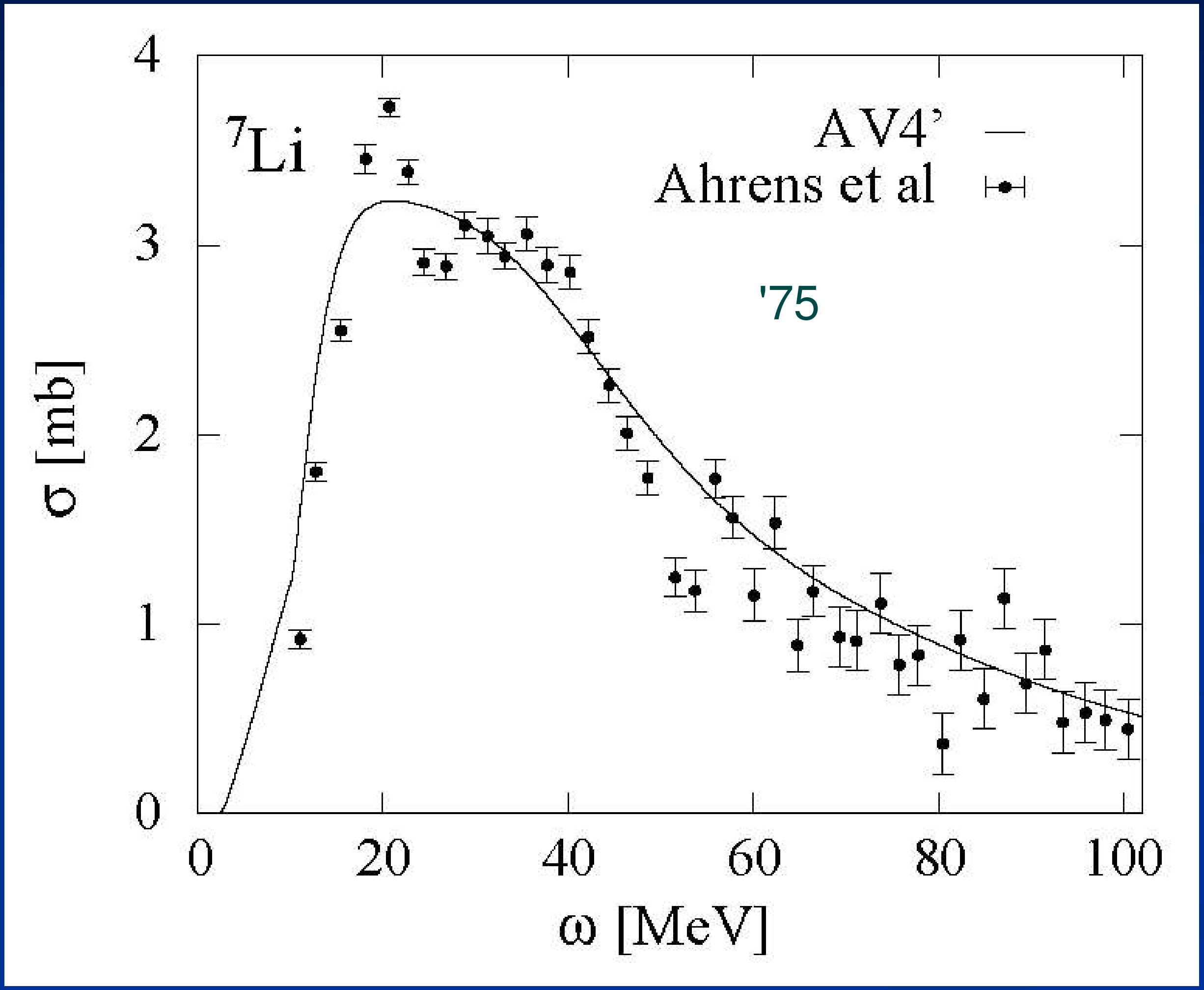
one-body + MEC
one-body
q = 250 MeV/c
q = 400 MeV/c
Bates
Saclay
$R_T(\omega)$ [MeV^{-1}]
0.012
0.009
0.006
0.003
0
0
50
100
0
50
100
150
200
ω [MeV]
ω [MeV]



Retzlaff et al.
q = 324 MeV/c
q = 487 MeV/c
1b+MEC
1b
R_T (ω) [MeV^-1]
ω [MeV]
0.006
0.005
0.004
0.003
0.002
0.001
0
8
12
16
20
0.0012
0.001
0.0008
0.0006
0.0004
0.0002

$A > 4$

7-Body total photodisintegration



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EIHH

^{7}Be total photoabsorption cross section

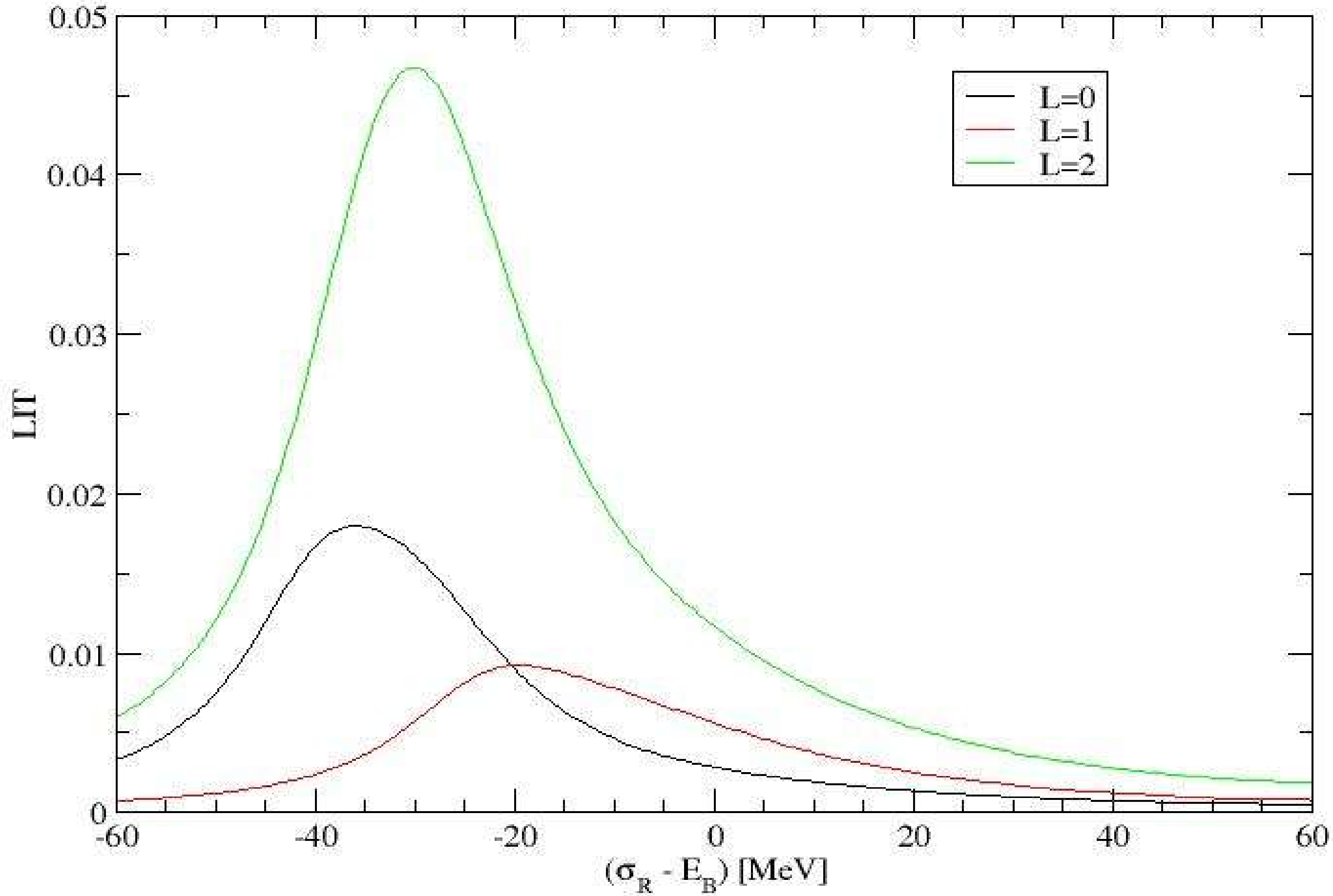
with semirealistic AV4' NN potential

6 different channels according to final state quantum numbers for isospin T and angular momentum L

1) T=1/2   L=0
2) T=1/2   L=1
3) T=1/2   L=2
4) T=3/2   L=0
5) T=3/2   L=1
6) T=3/2   L=2

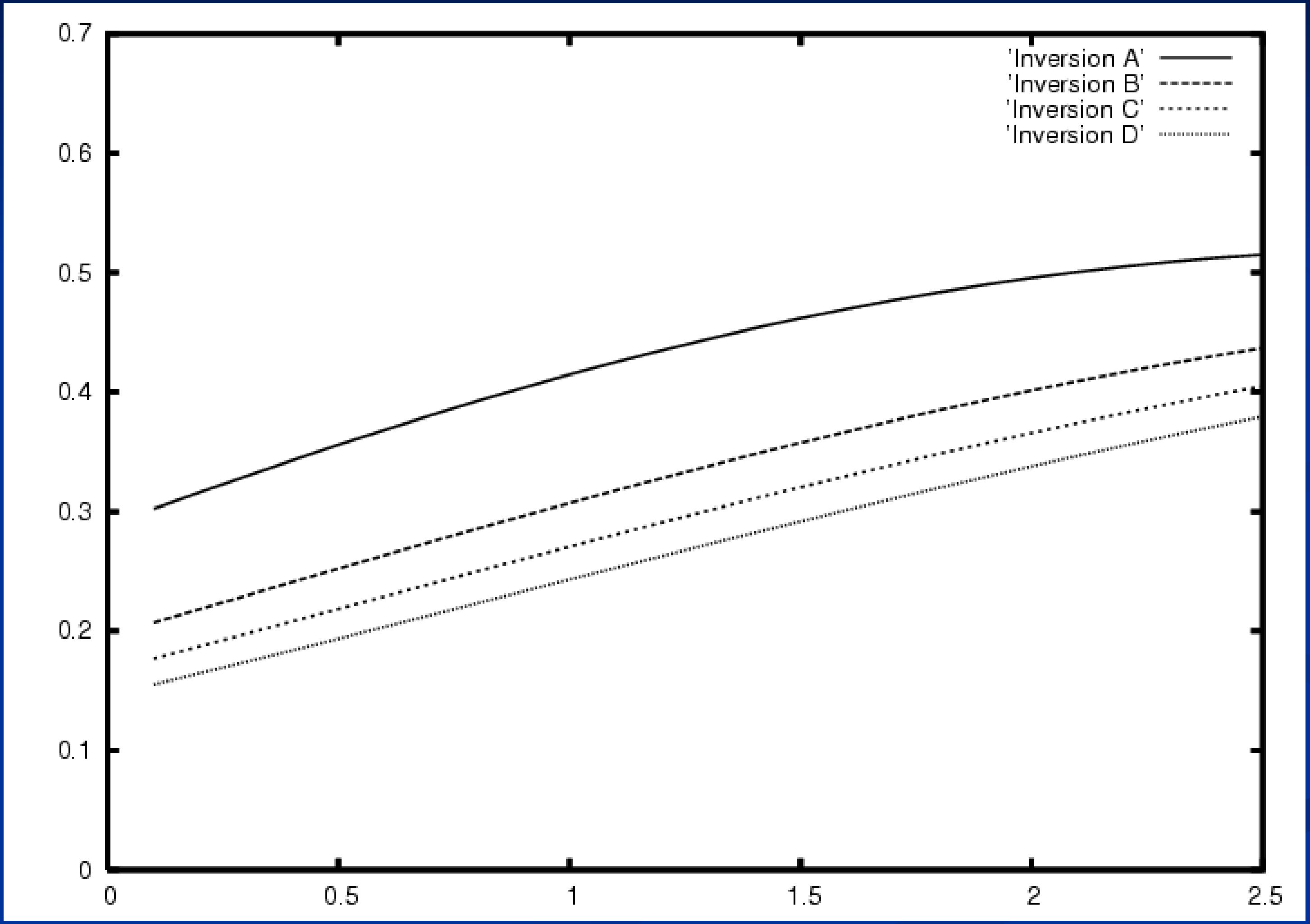
channels can have different thresholds

LIT $^{7}Be + \gamma \longrightarrow X\ (T=1/2)$



S-factor $^3He + ^4He$

Various inversion results for channel T=1/2, L=0



4 - body

Test of LIT for ^{4}He total photoabsorption cross section

Calculation in unretarded dipole approximation in two different ways:

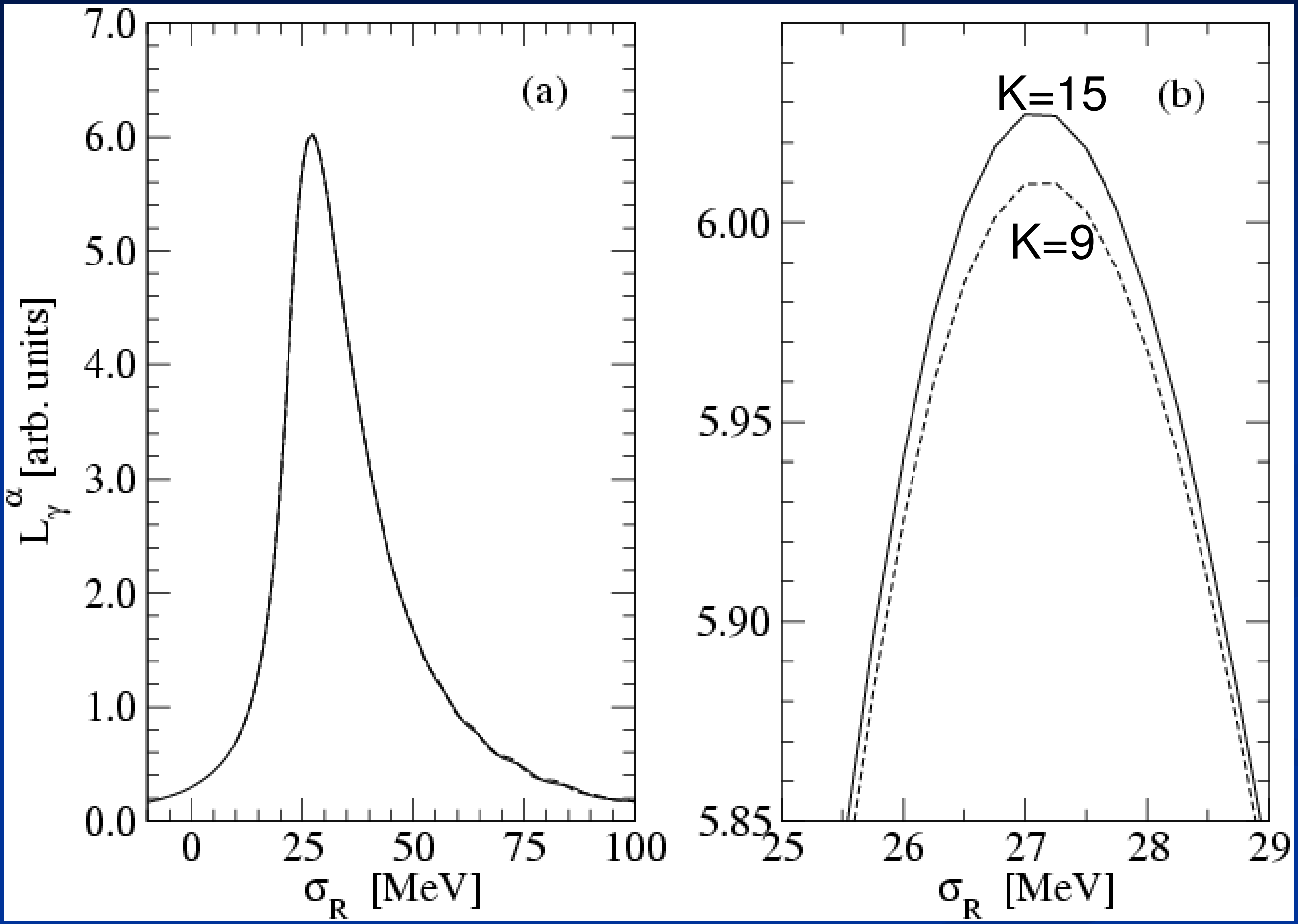
A) Direct calculation via LIT for inclusive reactions: σ_γ^{incl}

B) Calculation of sum of cross sections for the two-body break-up channels ^{4}He(γ,p)^{3}H and ^{4}He(γ,n)^{3}He via LIT for exclusive reactions: σ_γ^{TB}

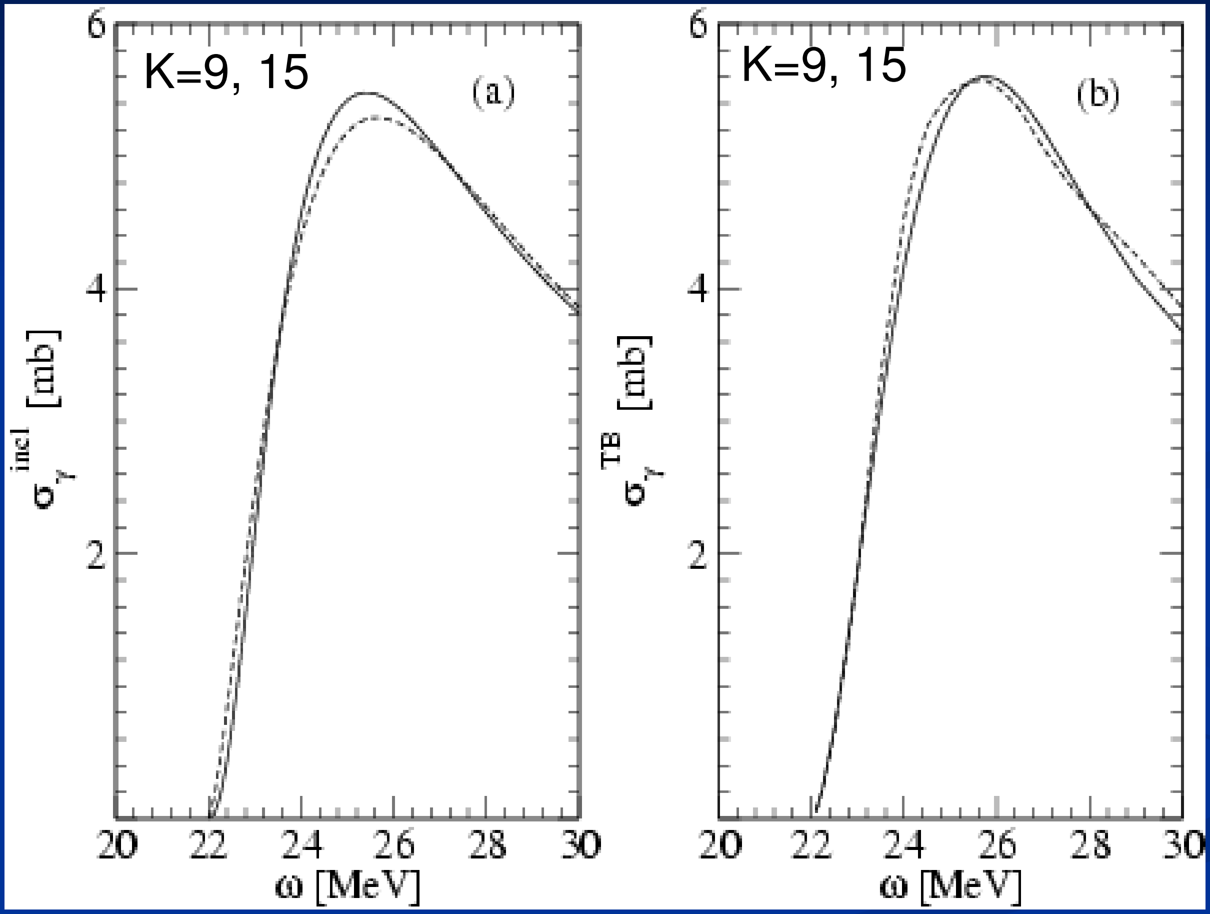
Below 3-body break-up threshold important check: $\sigma_\gamma^{incl} = \sigma_\gamma^{TB}$

Calculation made with semirealistic Volkov NN potential and without Coulomb force

HH convergence of LIT in our EIHH calculation



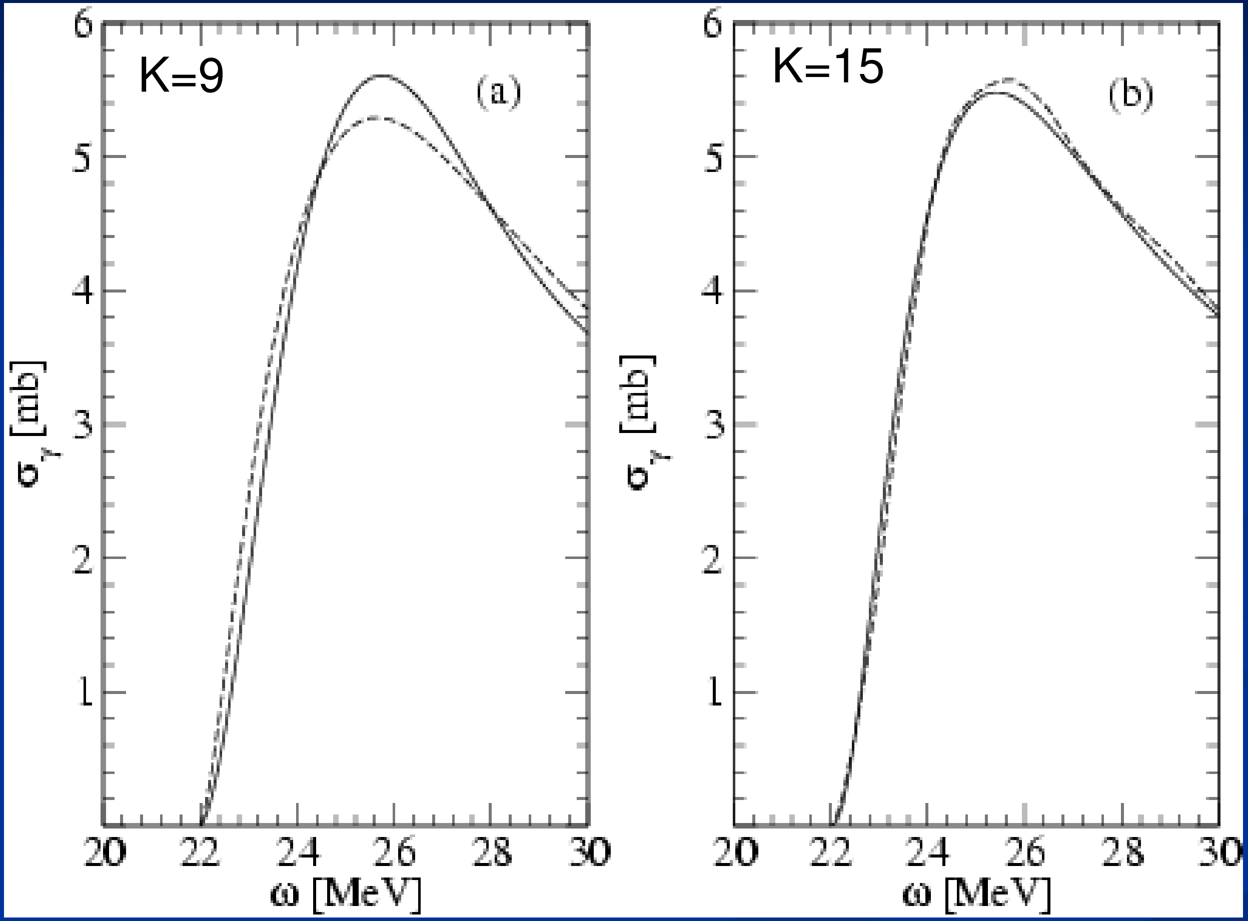
HH convergence of cross sections



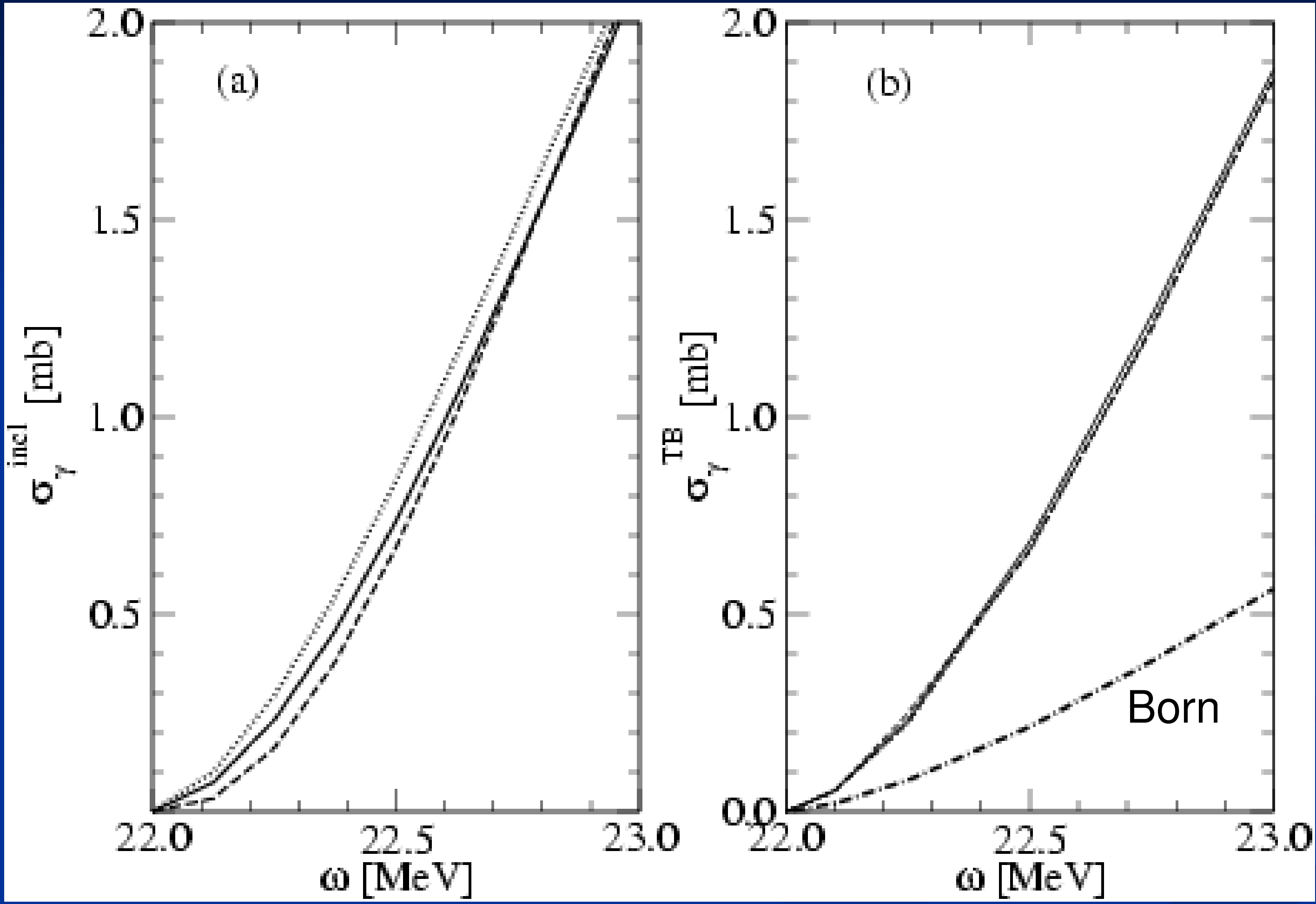
inclusive case

exclusive case

Comparison of σ_γ^{incl} and σ_γ^{TB}



various HH inversion results close to threshold



inclusive case

exclusive case

Conclusions

- the **LIT** opens up the possibility to carry out ab-initio calculations of reactions into the A-body continuum for A > 2

- only **bound states** techniques are needed