# Low-momentum ring diagrams of neutron matter at and near the unitary limit

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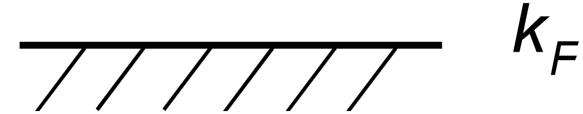
# Many-body problems with tunable interaction

$$H\Psi_0(A) = E_0\Psi_0(A),\ A \to \infty$$

$$H = H_0 + V, \qquad V \text{ is tunable}$$

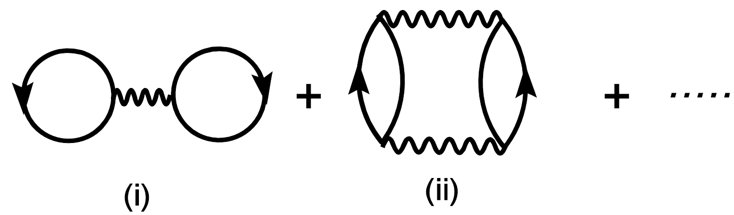
$$H_0\Phi_0 = E_0^{free}\Phi_0$$

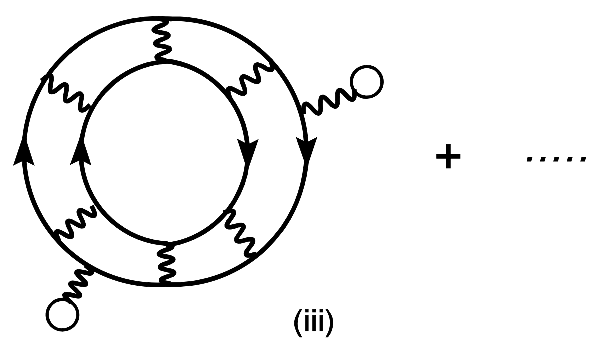
$\Phi_0$ =




Goldstone expansion for the ground-state energy shift

$$\Delta E_0 = (E_0 - E_0^{free}) =$$



# Unitary limit:

$a_s$ = scattering length of $V$, $V$ is <u>*tuned*</u> such that:

$$a_s \rightarrow \pm\infty, \text{ or } 1/a_s \rightarrow 0$$

At unitary limit ( Bertsch problem), expect

$$\xi = \frac{E_0}{E_0^{free}} = \text{ universal ratio } \approx 0.44$$

for two-species fermionic systems (e.g. neutron matter)

How to tune interaction $V$?

- **Cold Fermi gas** (laser trapped)

  Much progress made past few years

  Tune atomic-$V$ by external magnetic field

  Observed BCS-BEC cross-over near

  Feshbach resonance ($a_s = \pm\infty$)

- **Nuclear systems**

  Tune $V_{\mathrm{NN}}$ by master (Machleidt) !!

  Tune $V_{\mathrm{NN}}$ by Brown-Rho scaling

  (meson mass is medium dependent)

  ???????

# CD-Bonn $V_{NN}$ ($^1S_0$) of different $a_s$

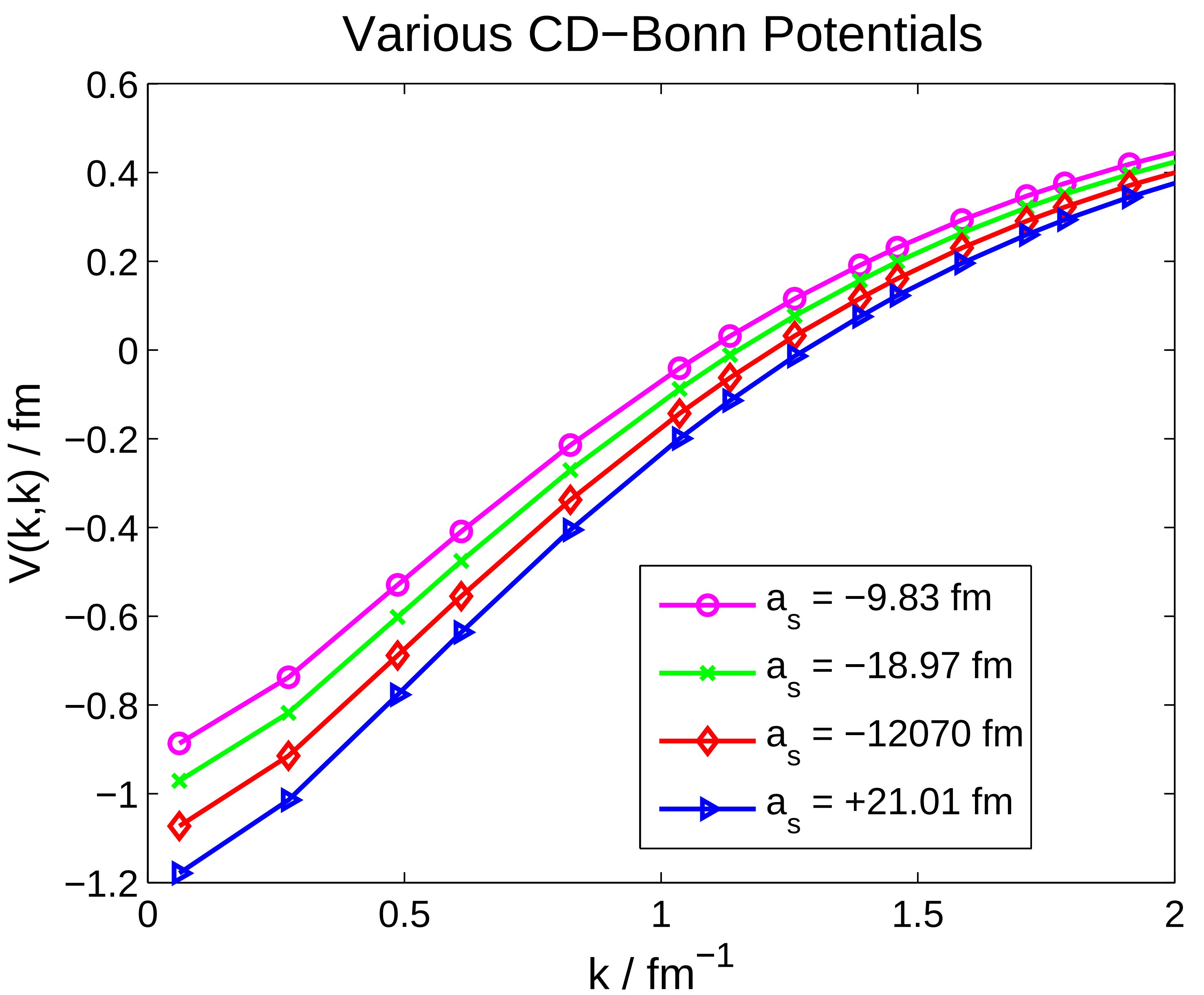
| | $m_\sigma$ [MeV] | $a_s$[fm] | $r_e$[fm] |
|---|---|---|---|
| original | 452.0 | -18.97 | 2.82 |
| tuned | 475.0 | -4.949 | 3.77 |
| | 447.0 | -42.52 | 2.66 |
| | 442.85 | -∞ (-12070) | 2.54 |
| | 442.80 | +∞ | 2.54 |
| | 434 | +21.01 | 2.31 |

one bound state exist

We tuned only $m_\sigma$

attraction in $^1S_0$ mainly from $\sigma$-exchange

$a_s$ depends sensitively on $m_\sigma$



Various CD−Bonn Potentials
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
V(k,k) / fm
0
0.5
1
1.5
2
k / fm−1
a_s = −9.83 fm
a_s = −18.97 fm
a_s = −12070 fm
a_s = +21.01 fm

# Model-space approach:

- Space $\{k > \Lambda\}$ is integrated out:

  $V_{bare}$ renormalized to $V_{low\text{-}k}$
  $V_{bare}$ has strong short range repulsion
  $V_{low\text{-}k}$ is smooth, and energy independent

- Space $\{k \leq \Lambda\}$:
  use $V_{low\text{-}k}$ to calculate all-order sum of ring diagrams

Note we need $V_{low\text{-}k}$ of specific $a_s$ , including $a_s \rightarrow \pm\infty$.

# $V^a_{low-k}$ of specific scattering length $a$

Start from $V^a$, a bare CD-Bonn potential of scat. length $a$. $V^a_{low-k}$ (of same $a$) given by:

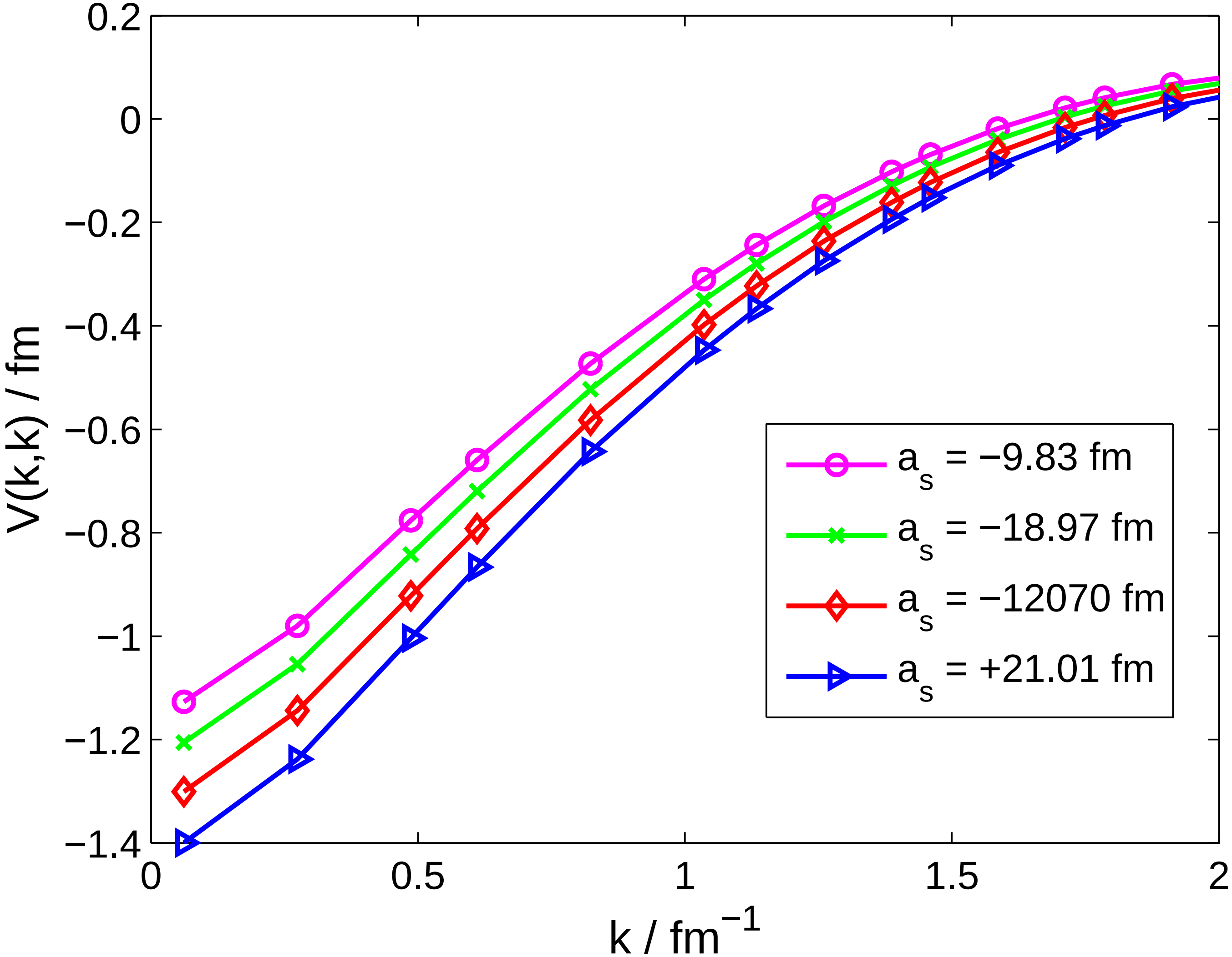
$$T(k',k,k^2) = V^a(k',k) + \int_0^\infty q^2 dq \frac{V^a(k',q)T(q,k,k^2)}{k^2-q^2+i0^+},$$

$$\begin{aligned} T_{low-k}(p',p,p^2) &= V^a_{low-k}(p',p) \\ &+ \int_0^\Lambda q^2 dq \frac{V^a_{low-k}(p',q)T_{low-k}(q,p,p^2)}{p^2-q^2+i0^+}, \end{aligned}$$

$$T(p',p,p^2) = T_{low-k}(p',p,p^2);\ (p',p) \le \Lambda.$$

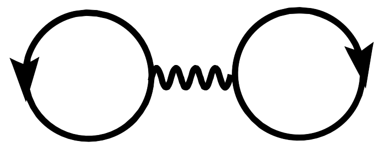
$V^a_{low-k}$ obtained from solving the above $T$-matrix equivalence equations using the iteration method of Lee-Suzuki-Andreozzi

# $V_{low\text{-}k}$ of potentials with various scattering lengths



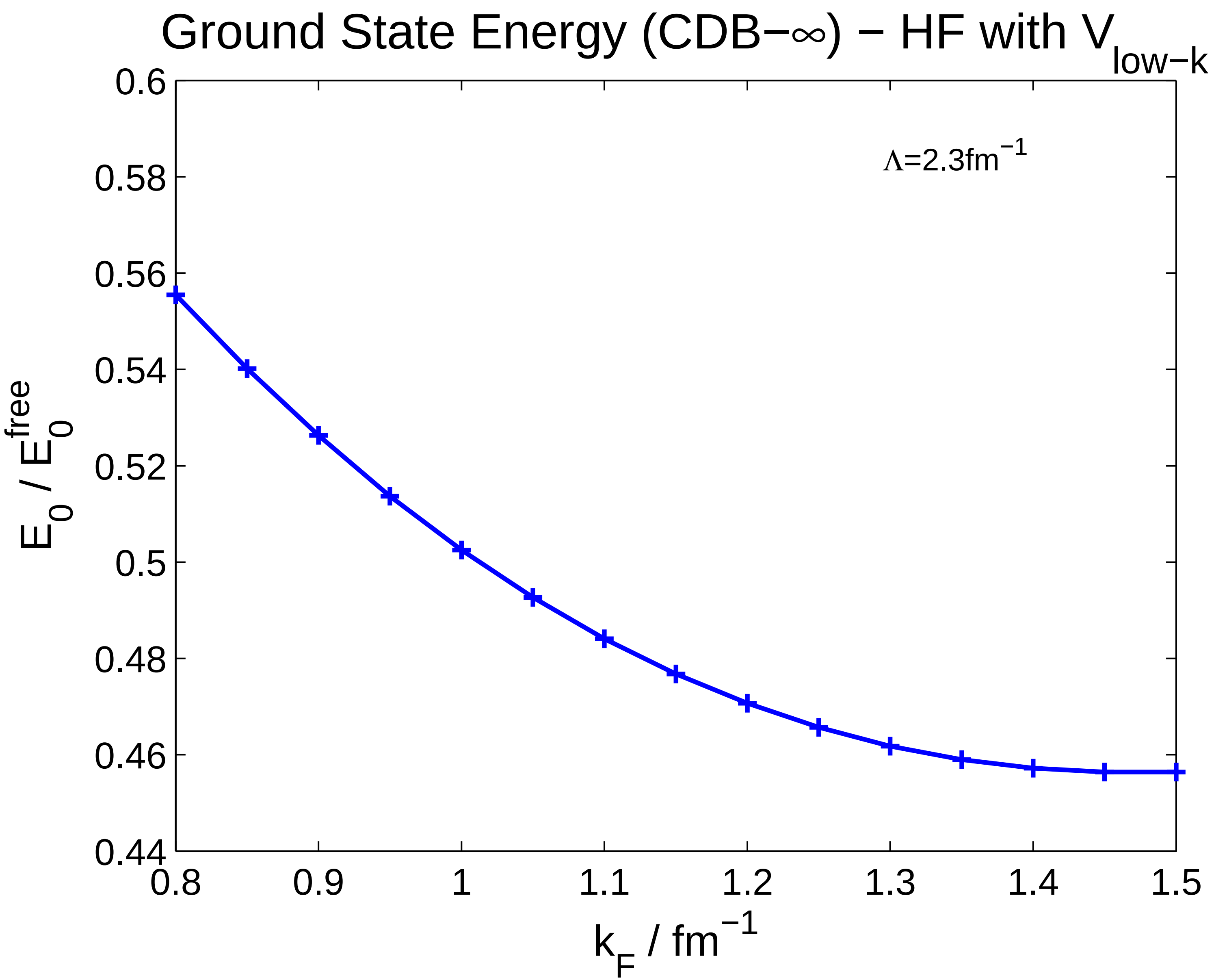
We first do HF calculation using $V_{low\text{-}k}$:

$$\Delta E_0^{HF} = \Sigma_{ij<k_F} \langle ij | V_{low-k} | ij \rangle$$

$=$ 

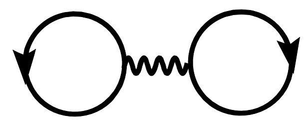
With this approximation,

$$\xi = \frac{E_0^{free} + \Delta E_0^{HF}}{E_0^{free}}$$


Ground State Energy (CDB−∞) − HF with V
low−k
Λ=2.3fm−1
0.6
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
E0 / E0free
kF / fm−1

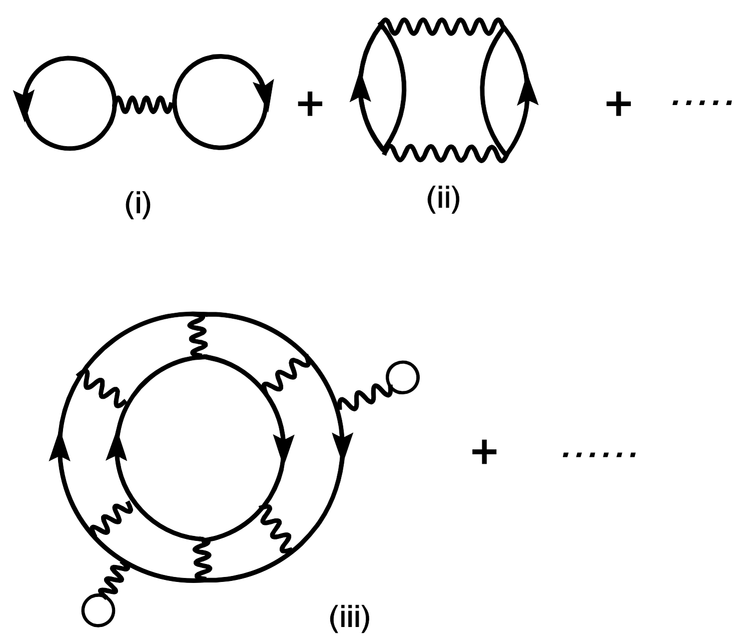
# To do better than HF:

HF includes only the lowest-order diagram:



We want to sum the *pphh* ring diagrams to all orders such as

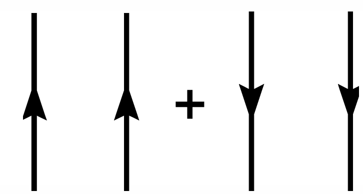
$$\Delta E_0 =$$



# Summation of ring diagrams:

## *pphh* free Green function:

$$F_{ab}(\omega) = \frac{\bar{n}_a \bar{n}_b}{\omega - (\epsilon_a + \epsilon_b) + i0^+} - \frac{n_a n_b}{\omega - (\epsilon_a + \epsilon_b) - i0^+}$$

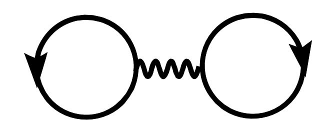
= 

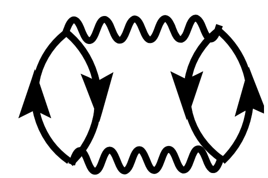
$$\Delta E_0^{pp} = \frac{-1}{2\pi i} \int_{-\infty}^{\infty} d\omega e^{i\omega 0^+} tr_{<\Lambda}[F(\omega)V_{low-k}$$

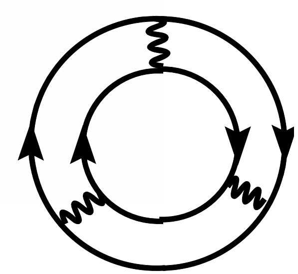
$$+\frac{1}{2}(F(\omega)V_{low-k})^2$$

$$+\frac{1}{3}(F(\omega)V_{low-k})^3$$

$$+\cdots]$$

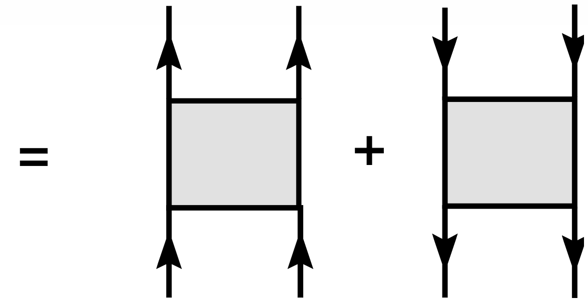






in terms of *pphh* true Green function

$$G^{pp}(\omega, \lambda) = F(\omega) + \lambda F(\omega) V_{low-k} G^{pp}(\omega, \lambda)$$



$$\Delta E_0^{pp} = \frac{-1}{2\pi i} \int_0^1 d\lambda \int_{-\infty}^{\infty} e^{i\omega 0^+} tr_{<\Lambda}[G^{pp}(\omega, \lambda) V_{low-k}]$$

Using Lehmann's representation of $G^{pp}$, $\Delta E_0^{pp}$ (all order) becomes

$$\Delta E_0^{pp} = \int_0^1 d\lambda \Sigma_m \Sigma_{ijkl<\Lambda} Y_m(ij, \lambda) Y_m^*(kl, \lambda) \times \langle kl | V_{low-k} | ij \rangle$$

The transition amplitudes $Y_m$ given by RPA equation; $m$ denotes states dominated by hole-hole components.

$pphh$-RPA equation:

$$\Sigma_{ef} R(ij, ef, \lambda) Y_m(ef, \lambda) = \omega_m Y_m(ij, \lambda);$$

$$(i, j, e, f) < \Lambda$$

$$R(ij, ef, \lambda)$$
$$= [(\epsilon_i + \epsilon_j)\delta_{ij,ef} + \lambda(1 - n_i - n_j)\langle ij|V_{low-k}|ef\rangle]$$

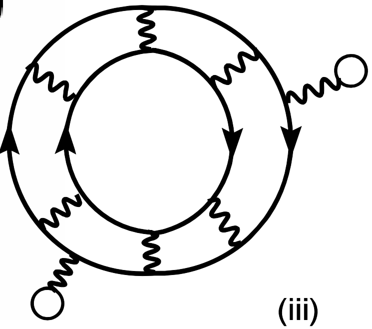
$m$ denotes states dominated by hole-hole components, namely $\langle Y_m|\frac{1}{Q}|Y_m\rangle = -1$ and $Q(i,j) = (1 - n_i - n_j)$. $n_i = (1, 0)$ for $k_i$ $(\leq, >)k_F$

We use HF s.p. spectrum, namely

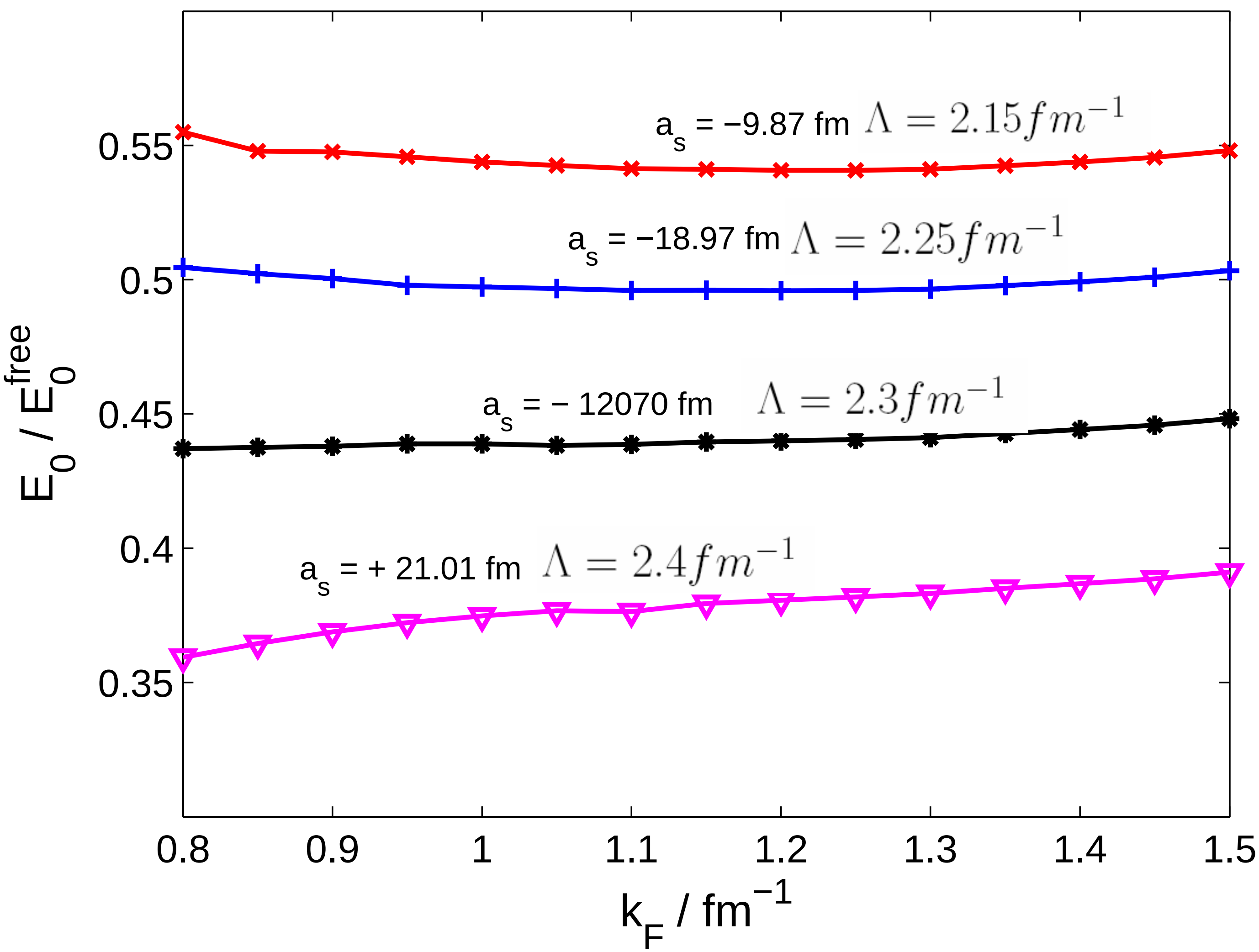
$$\epsilon_i = \frac{\hbar^2 k_i^2}{2m} + \Sigma_{h \leq k_F}\langle ih|V_{low-k}|ih\rangle$$

# Brief Summary

- By slightly tunning V(cdbonn), get $V^a$ (modified cdbonn of various $a_s$)
- Choose $\Lambda$ to define a momentum model space with $(k \leq \Lambda)$
- Integrate out $k > \Lambda$ to get model-space effective interaction $V^a_{low-k}$, which is E-indep. and of scat. length $a_s$.
- Nuclear matter ground st. energy $E_0$ calculated from the all-order sum of the *pphh* ring diagrams $(k \leq \Lambda)$ such as



# Results of low-momentum ring-diagram summation



choice of $\Lambda$:

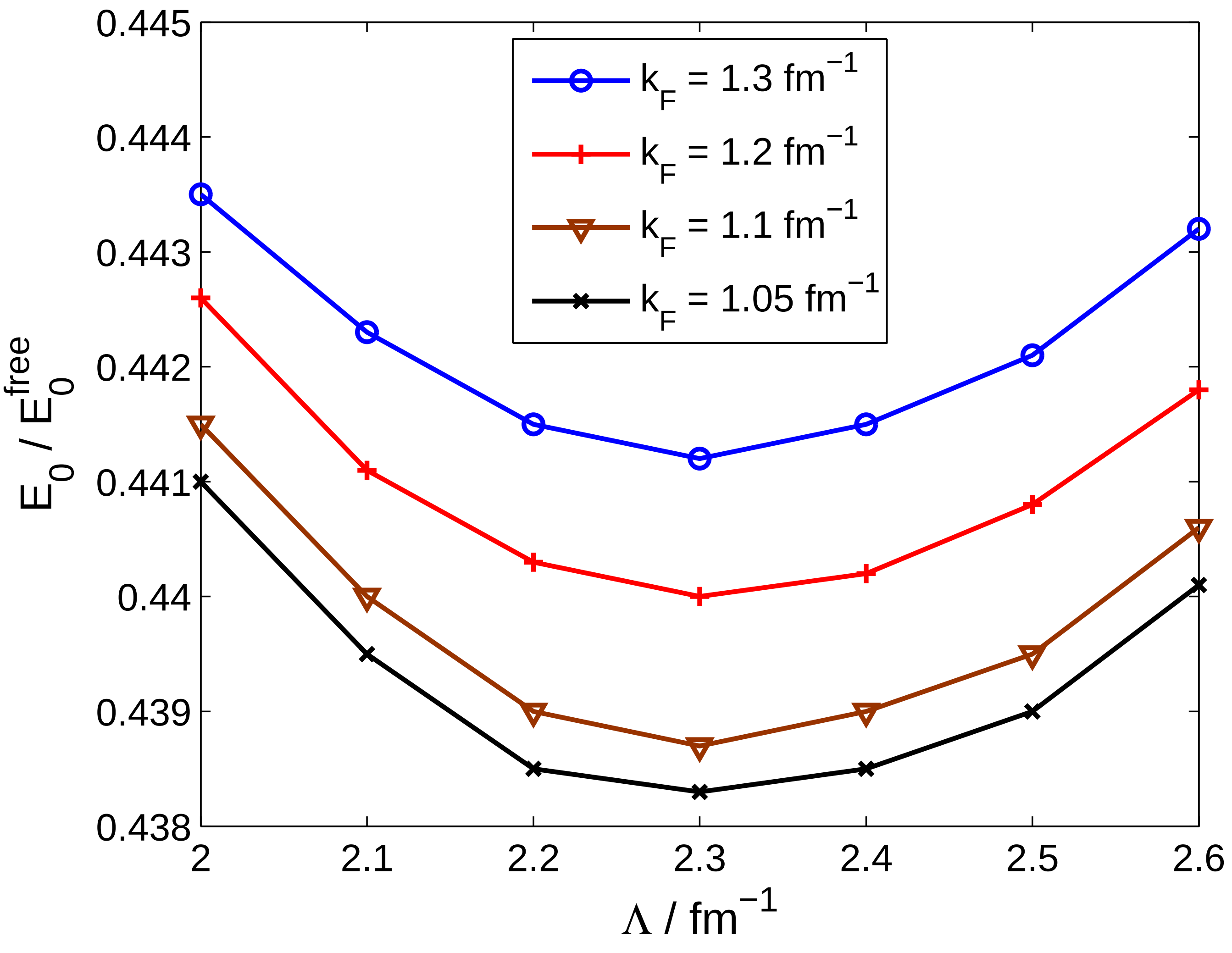
(1) $V_{NN}$ constrained by

$NN$ scattering with $E_{lab} \leq 300 MeV$

$\Rightarrow \Lambda \approx 2.0 fm^{-1}$

(2) Fix point : $\frac{\partial \xi}{\partial \Lambda} = 0$

# Determination of the fixed-point (CD Bonn-∞)



Integrating out $\{k > \Lambda\}$ leads to
two types of renormalized interactions:

(1) Energy-independent $V_{low-k}$

(2) Energy-dependent $G^M(\omega)$,
it is energy dependent,
rather complicated for computation

We shall use both for ring diagrams.

Formally these two approaches are equivalent;
just like "Bloch-Horowitz" vs "Rayleigh-Schroedinger"

In energy-dependent approach,
we first calculate model-space $G^M$ matrix:

$$G^M_{ijkl}(\omega) = V_{ijkl} + \Sigma_{rs} V_{ijrs} \frac{Q^M(rs)}{\omega - \hbar^2 k_r^2/2m - \hbar^2 k_s^2/2m + i0^+} G^M_{rskl}(\omega)$$

$Q^M$ is Pauli operator, assuring intermediate states outside momentum model space $\{k \leq \Lambda\}$:

$G^M$ is energy-dependent; note $\omega$ is determined self-consistently (not free parameter).

Ring-diagram all-order sum in energy-dep. formalism:

$$\Delta E_0^{pp} = \int_0^1 d\lambda \Sigma_m \Sigma_{ijkl<\Lambda} Y_m(ij,\lambda) Y_m^*(kl,\lambda) G_{kl,ij}^M(\omega_m^-)$$

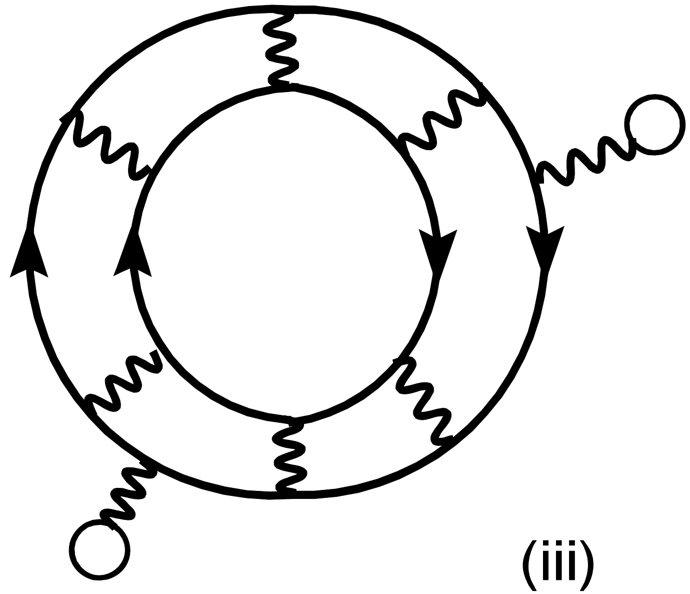
$Y$ and $\omega$ given by RPA equation:

$$\Sigma_{ef}[(\epsilon_i + \epsilon_j)\delta_{ij,ef} + \lambda(1 - n_i - n_j)L_{ij,ef}(\omega)]Y_m(ef,\lambda)$$
$$= \mu_m(\omega,\lambda)Y_m(ij,\lambda); \quad (i,j,e,f) < \Lambda$$

with self-consistent condition

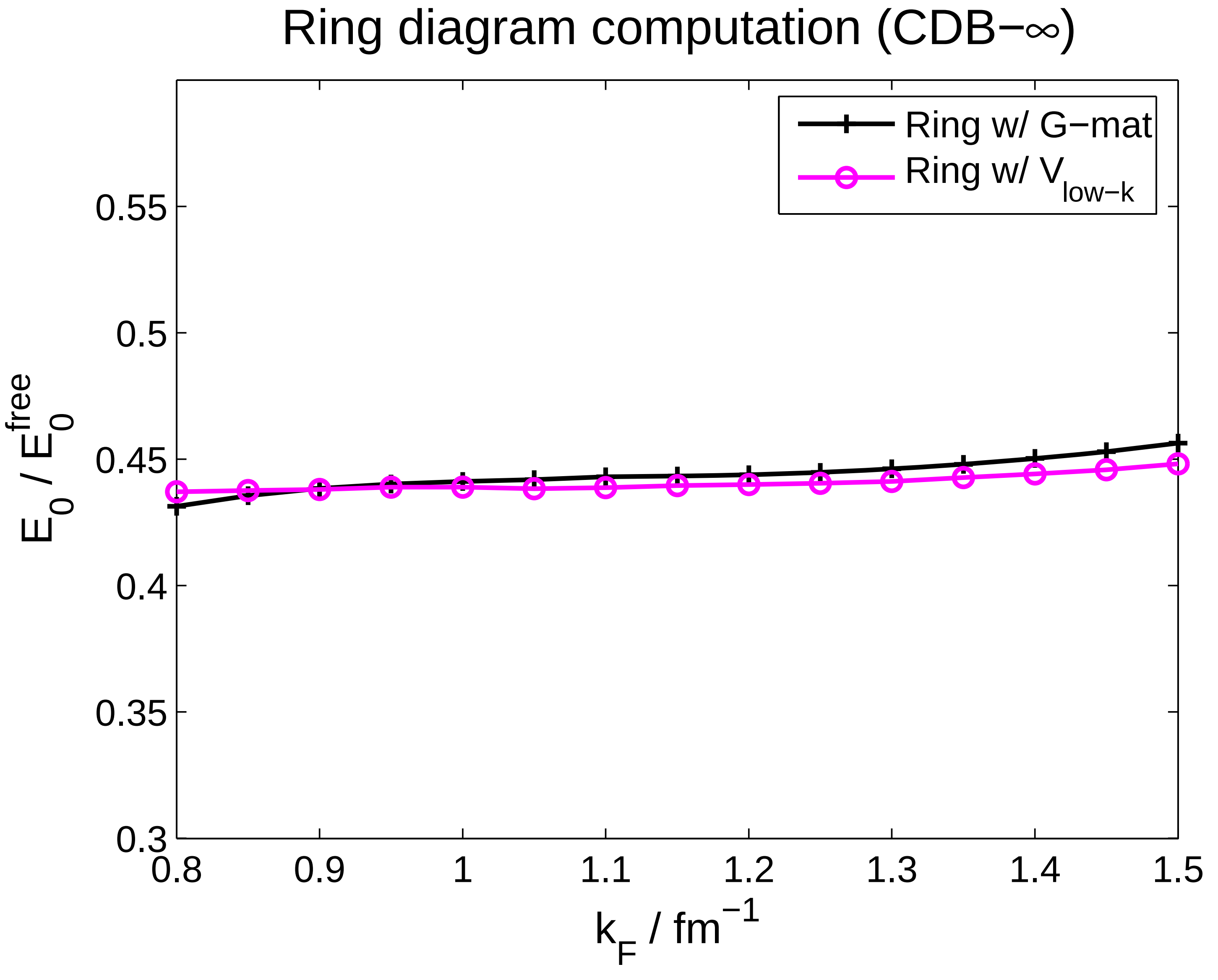
$$\omega = \mu_m(\omega,\lambda) \equiv \omega_m^-(\lambda).$$

# 2 ring diagram methods



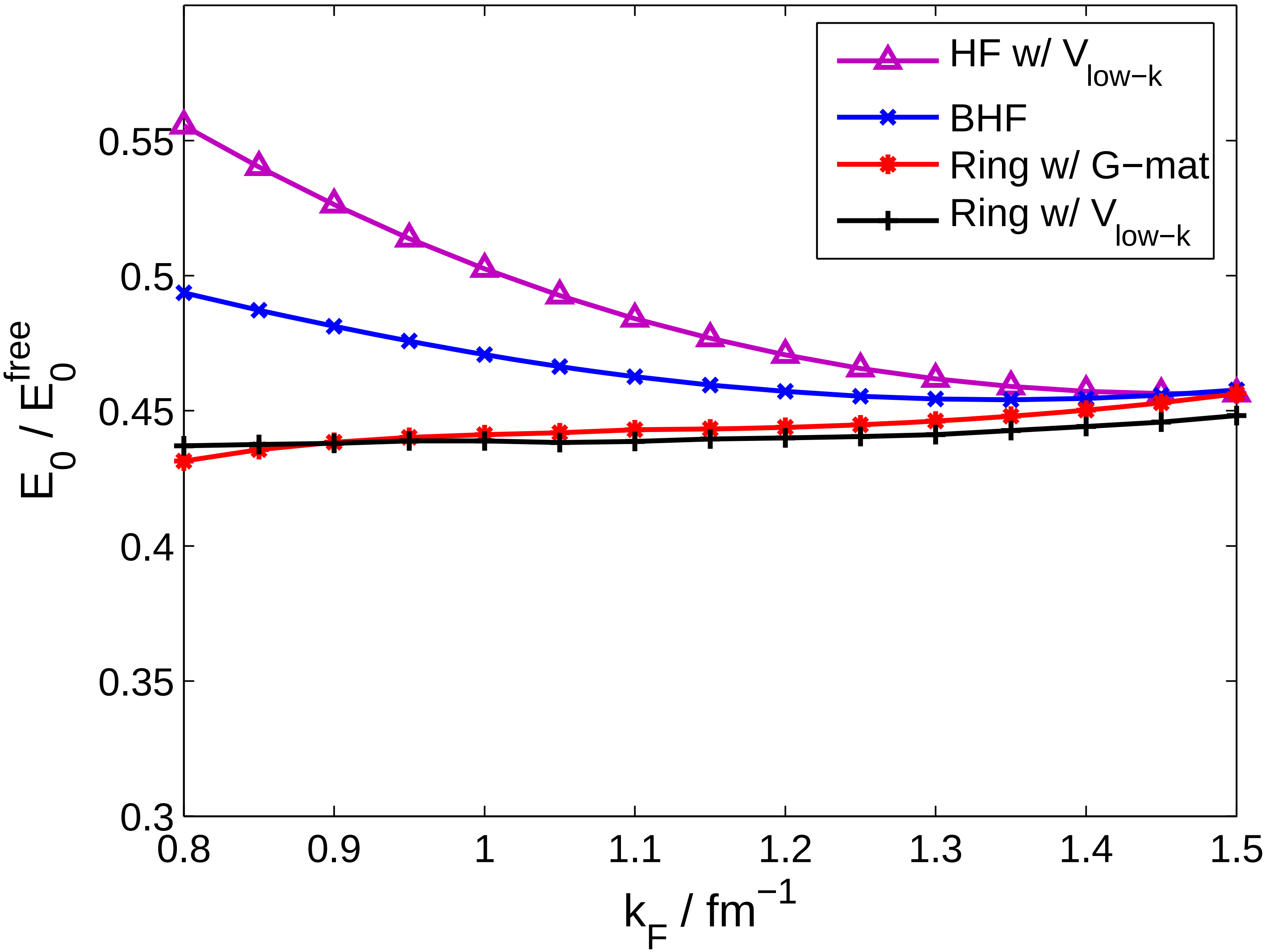

(i) each vertex = $V_{low-k}$

(ii) each vertex = $G$-matrix

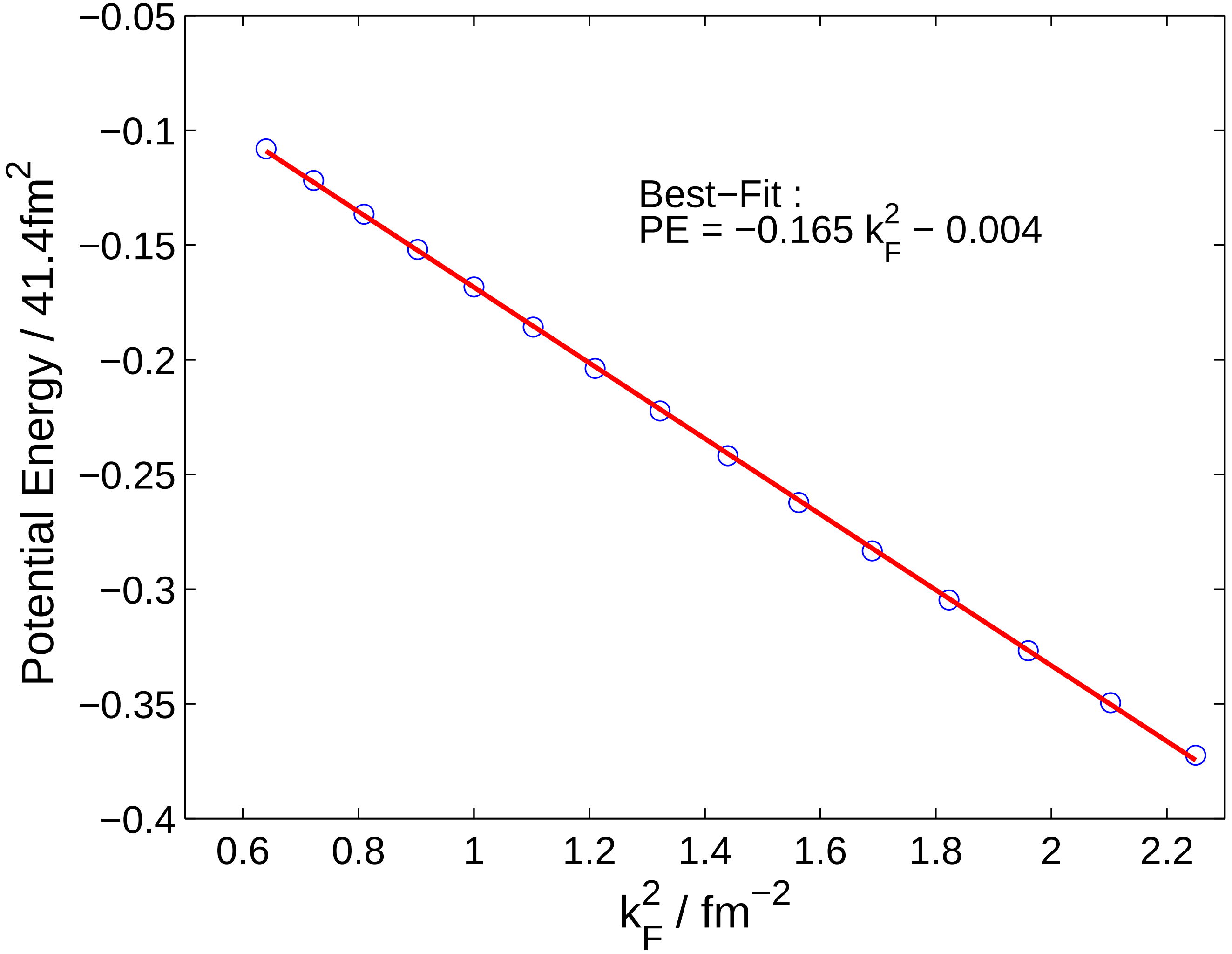


Ring diagram computation (CDB−∞)
Ring w/ G−mat
Ring w/ V_low−k
0.55
0.5
0.45
0.4
0.35
0.3
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
E_0 / E_0^free
k_F / fm^−1

# Ground state energy of neutron matter at unitary limit ( CDB-∞)



# CD-Bonn-∞ ($a_s$ = -12070fm)



At $a_s \to \pm\infty$, our ring results give

$\Delta E_0 = \alpha k_F^2 + \beta$; $\alpha = -0.165$, $\beta = -0.004$

Suppose we take

$\alpha = -\frac{1}{6} = -0.166666...$ and $\beta = 0$,

Then the universal ratio is

$$\xi = \frac{E_0^{free} + \Delta E_0}{E_0^{free}} = \frac{4}{9} = 0.4444..$$

# Experimental Values :

| $\xi$ | Authors |
| --- | --- |
| 0.36(15) | Bourdel *el. al* |
| 0.51(4) | Kinast *et. al.* |
| 0.46(5) | Partridge *et. al.* |
| $0.46^{+0.05}_{-0.12}$ | Stewart *et. al.* |

# Summary and Discussion

By slightly tuning $m_\sigma$ of V(CD-Bonn), we obtain neutron potentials of $a_s = \pm\infty$, -10, +20,...

$V^a_{low-k}$ obtained by integrating out $\{k > \Lambda\}$

All-order sum of ring diagrams calculated using $V^a_{low-k}$

Fix point is at $\Lambda \approx 2.3 fm^{-1}$

Our results indicate $\xi = \frac{4}{9}$ at the unitary limit