Relativistic pseudospin symmetry, the nucleon-nucleon interaction, and effective field theory

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Collaborators: A. Leviatan, D. Madland, P. von Neumann-Cosel

JNG, Physics Reports 414, 165 (2005)

Pseudospin Symmetry in Nuclei

More than thirty years ago a quasi-degeneracy was observed in single-nucleon doublets in nuclei with quantum numbers

$$(n, \ell, j), (n-1, \ell + 2, j)$$

$$j = \tilde{\ell} \pm \tilde{s}, \quad \tilde{s} = 1/2$$

$\tilde{\ell}$ pseudo-orbital angular momentum, $\tilde{s}$ pseudospin

$2s_{1/2}$, $1d_{3/2}$ — $(\tilde{1})_{3/2, 1/2}$

$2p_{3/2}$, $1f_{5/2}$ — $(\tilde{2})_{5/2, 3/2}$

$1d_{3/2}$, $0g_{5/2}$ — $(\tilde{3})_{7/2, 5/2}$

$1f_{7/2}$, $0h_{9/2}$ — $(\tilde{4})_{9/2, 7/2}$

Hence a quasi-degeneracy in pseudospin

Rotational bands

built on different

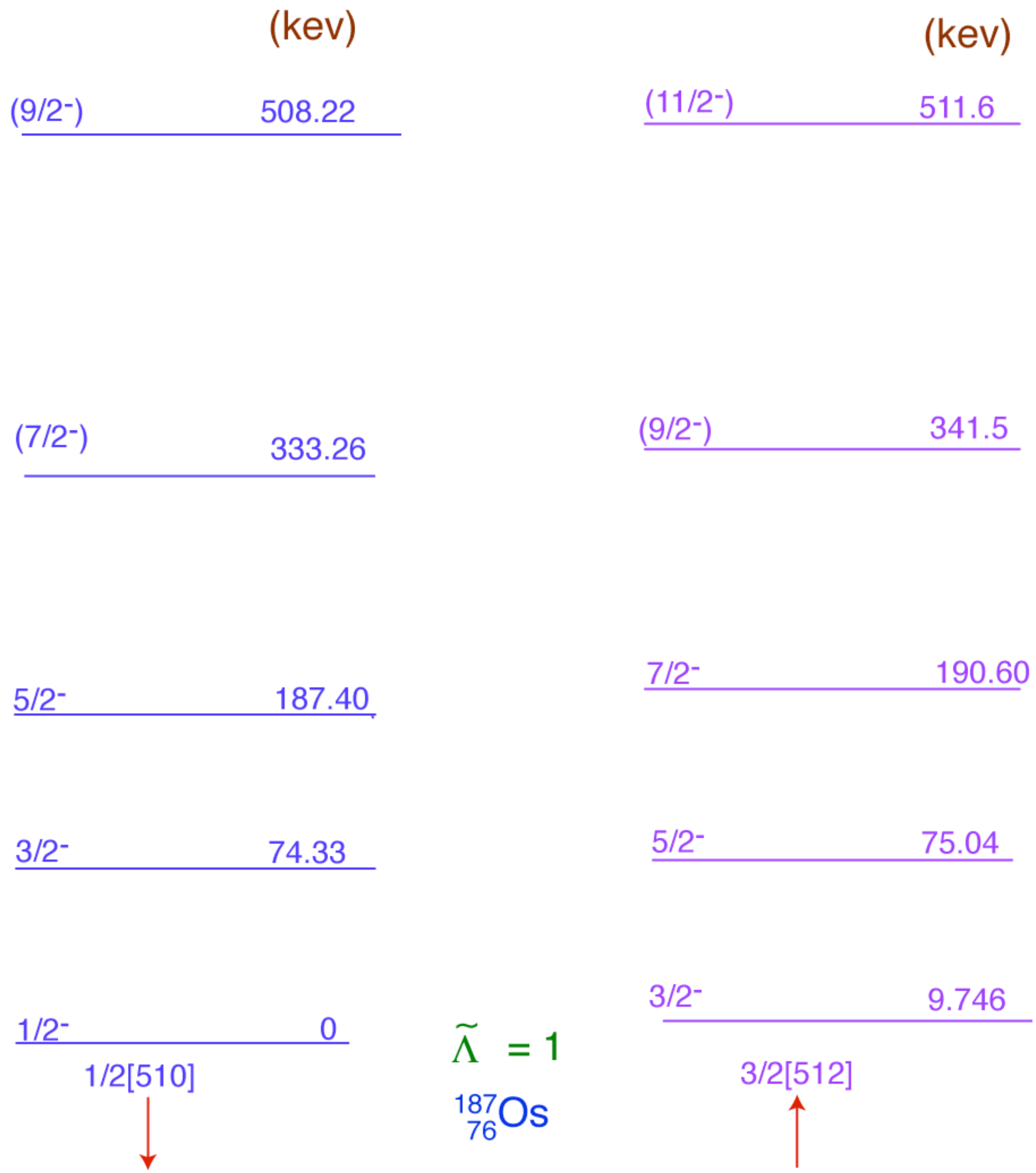
alignments of

pseudospin

along the body

fixed axis.

$\Omega[N\ n_3\ \Lambda]$



The Dirac Hamiltonian

$$H = [\ \vec{\alpha} \cdot \vec{p} + \beta(m + V_S(\vec{r})) + V_V(\vec{r})]$$

α, β are the usual Dirac matrices

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

σ_i = Pauli matrices.

Nucleons move in a scalar, $V_S(\vec{r})$, and vector, $V_V(\vec{r})$, mean fields.

The Dirac Hamiltonian has an invariant SU(2) symmetry for two limits:

Vs – Vv = Cs	**Spin Symmetry**
Vs + Vv = Cps	**P-Spin Symmetry**

Spin Symmetry occurs in the spectrum of a:

1) meson with one heavy quark (PRL 86, 204 (2001))
2) anti-nucleon bound in a nucleus (Phys. Rep. 315, 231 (1999))

Pseudospin Symmetry occurs in the spectrum of nuclei
PRL 78, 436 (1997)

Pseudospin Symmetry

$V_S(\vec{r}) + V_V(\vec{r}) = C_{ps}$, nuclear matter

$V_S(\vec{r})$ = - $V_V(\vec{r})$, finite nulei

$$[\tilde{S}_i, H_{ps}] = 0,$$

$$[\tilde{S}_i, \tilde{S}_j] = i\epsilon_{ijk}\ \tilde{S}_k,$$

$$\tilde{S}_i = \frac{\vec{\alpha}\cdot\vec{p}\ \hat{s}_i\ \vec{\alpha}\cdot\vec{p}}{p^2}\ \frac{(1-\beta)}{2} + \hat{s}_i\ \frac{(1+\beta)}{2},$$

$\hat{s}_i = \sigma_i/2$, σ_i the Pauli matrices.

Simplifies to:

$$\hat{\tilde{S}}_i = \begin{pmatrix} \hat{\tilde{s}}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}$$

$$\hat{\tilde{s}}_i = U_p \; \hat{s}_i \; U_p = \frac{2\hat{s} \cdot p}{p^2} \, p_i - \hat{s}_i .$$

$$U_p = \frac{\sigma \cdot p}{p}$$

U_p is the momentum - helicity unitary operator introduced in

A. L. Blokhin, C. Bahri, and J. P. Draayer, Physical Review Lett. **74**, 4149 (1995).

Degenerate Pseudospin Doublets

$$H_{\text{ps}} \Phi_{k,\mu}(\vec{r}) = E_k \Phi_{k,\mu}(\vec{r})$$

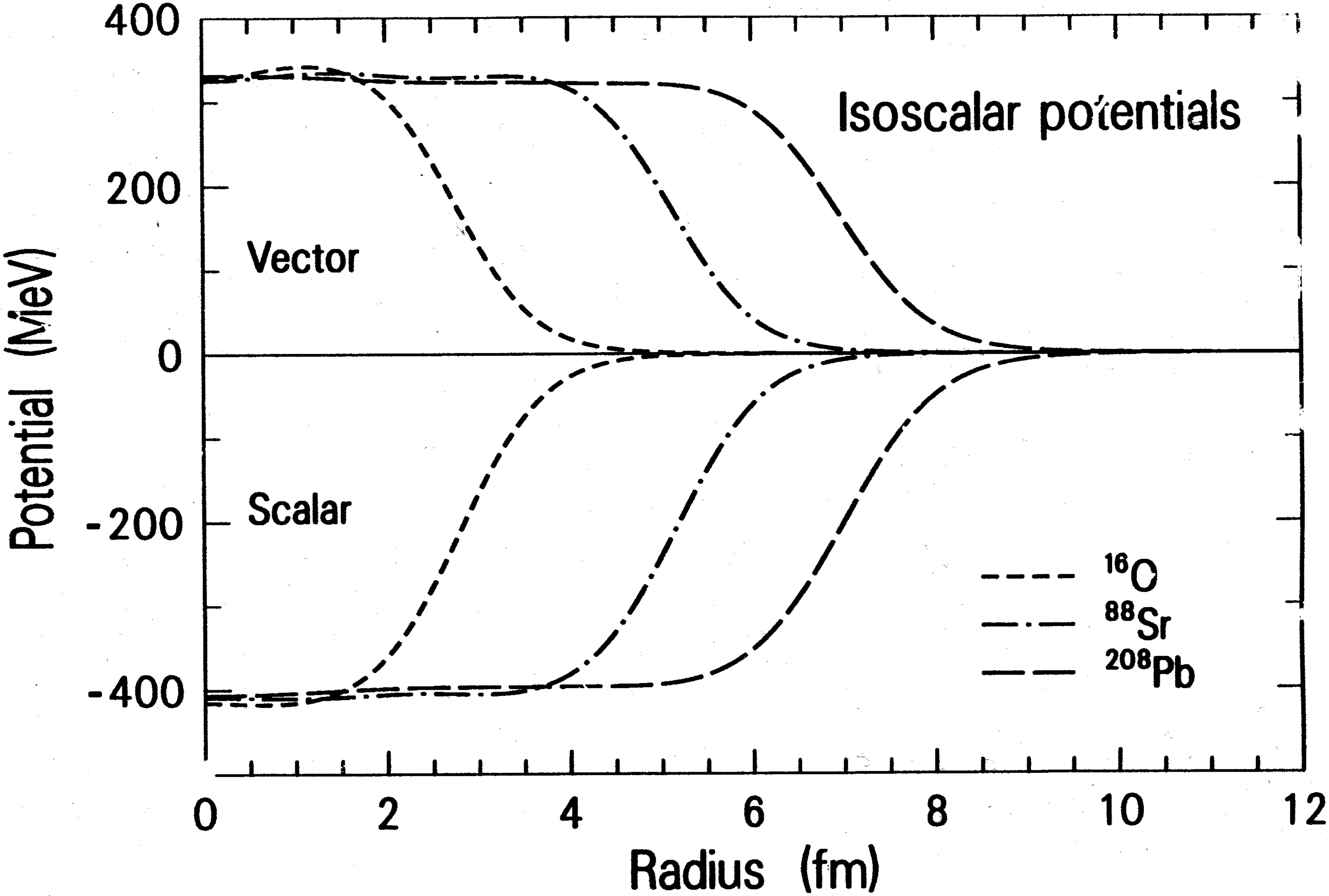
k all other quantum numbers

$$\tilde{S}_z \; \Phi_{k,\mu}(\vec{r}) = \mu \; \Phi_{k,\mu}(\vec{r})$$

$$\tilde{S}_\pm \; \Phi_{k,\mu}(\vec{r}) = \sqrt{\left(\frac{1}{2} \mp \mu\right)(\frac{3}{2} \pm \mu)} \; \Phi_{k,\mu\pm1}(\vec{r})$$

$$\mu = \; \pm 1/2$$

Realistic Relativistic Mean Fields



QCD SUM RULES

$$\frac{V_S}{V_V} \approx -\frac{\sigma_N}{8m_q}$$

σ_N is the chiral symmetry breaking nucleon sigma term

m_q is the average quark mass

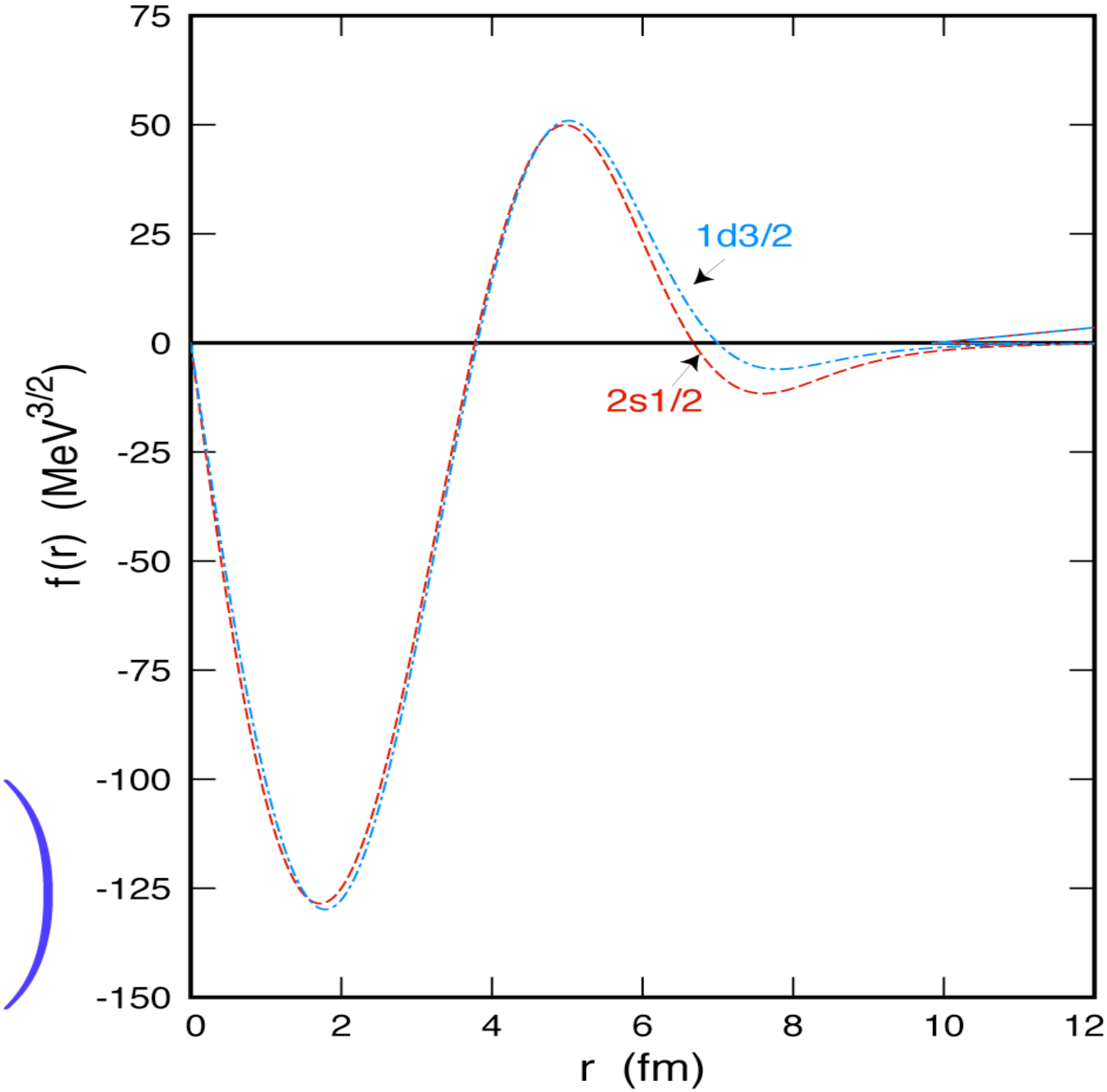
$$\sigma_N \approx 45\ MeV, m_q \approx 5\ MeV$$

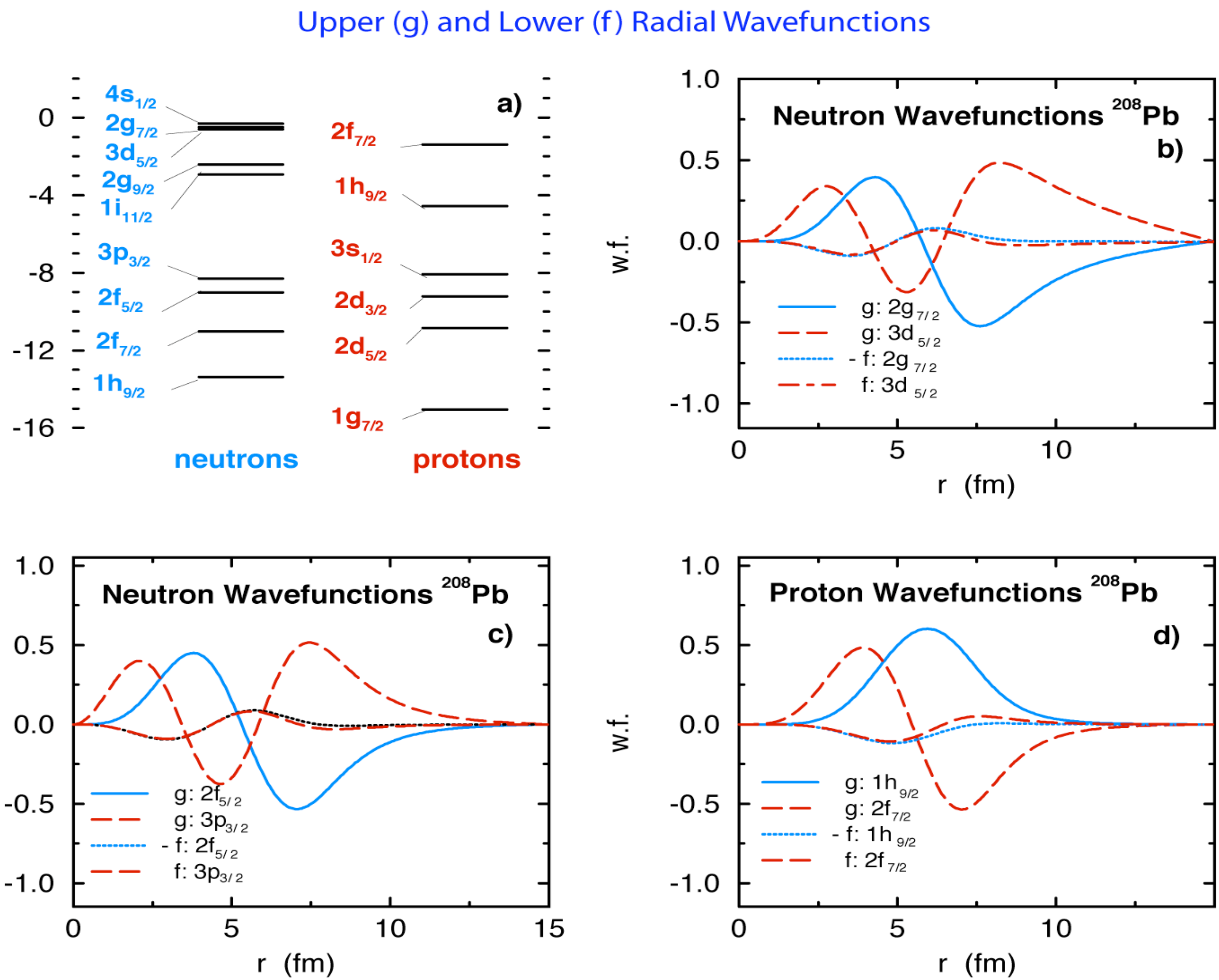
$$\frac{V_S}{V_V} \approx -1.1$$

Uncannily close to the ratio of central values of mean field potentials

Pseudospin Symmetry predicts that the lower spatial amplitudes of the two eigenstates in the doublets are equal

$$\hat{S}_i = \begin{pmatrix} \hat{\tilde{s}}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}$$





Upper (g) and Lower (f) Radial Wavefunctions
a)
4s1/2
2g7/2
3d5/2
2g9/2
1i11/2
3p3/2
2f5/2
2f7/2
1h9/2
2f7/2
1h9/2
3s1/2
2d3/2
2d5/2
1g7/2
neutrons
protons
Neutron Wavefunctions 208Pb
b)
w.f.
r (fm)
g: 2g7/2
g: 3d5/2
- f: 2g7/2
f: 3d5/2
Neutron Wavefunctions 208Pb
c)
g: 2f5/2
g: 3p3/2
- f: 2f5/2
f: 3p3/2
Proton Wavefunctions 208Pb
d)
g: 1h9/2
g: 2f7/2
- f: 1h9/2
f: 2f7/2

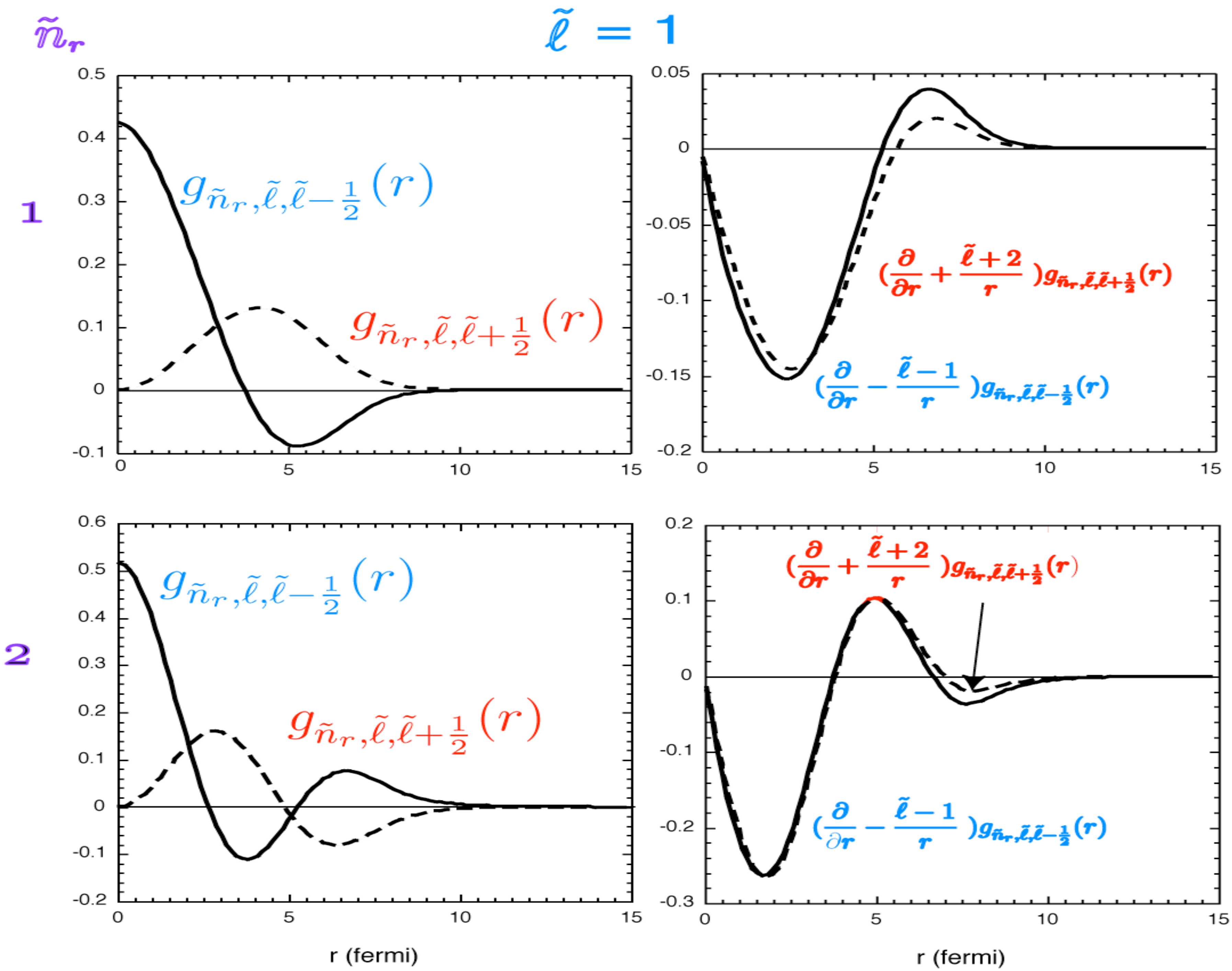
$$\tilde{S}_z \; \Phi^{ps}_{k,\tilde{\mu}}(\vec{r}) = \tilde{\mu} \; \Phi^{ps}_{k,\tilde{\mu}}(\vec{r})$$

$$\tilde{S}_\pm \; \Phi^{ps}_{k,\tilde{\mu}}(\vec{r}) = \sqrt{\left(\frac{1}{2} \mp \tilde{\mu}\right) (\frac{3}{2} \pm \tilde{\mu})} \; \Phi^{ps}_{k,\tilde{\mu}\pm 1}(\vec{r})$$

Pseudospin symmetry implies that the upper components are related by a differential relation

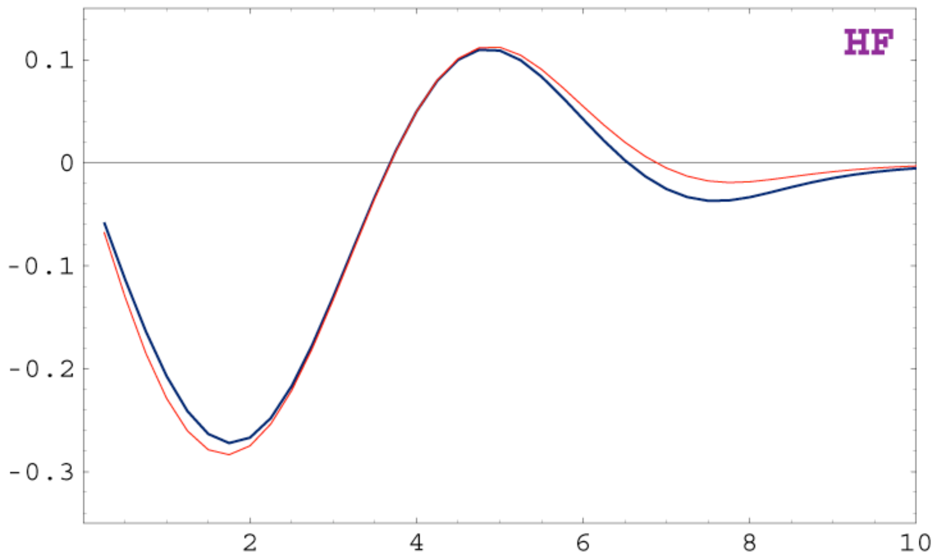
$$\left(\frac{\partial}{\partial r} + \frac{\tilde{\ell}+2}{r}\right) g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}+\frac{1}{2}}(r) =$$

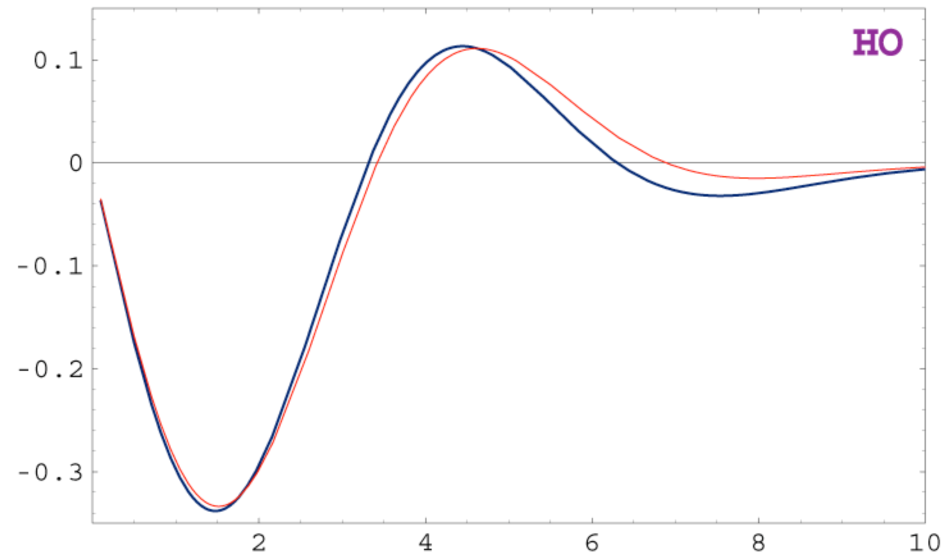
$$\left(\frac{\partial}{\partial r} - \frac{\tilde{\ell}-1}{r}\right) g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}-\frac{1}{2}}(r)$$

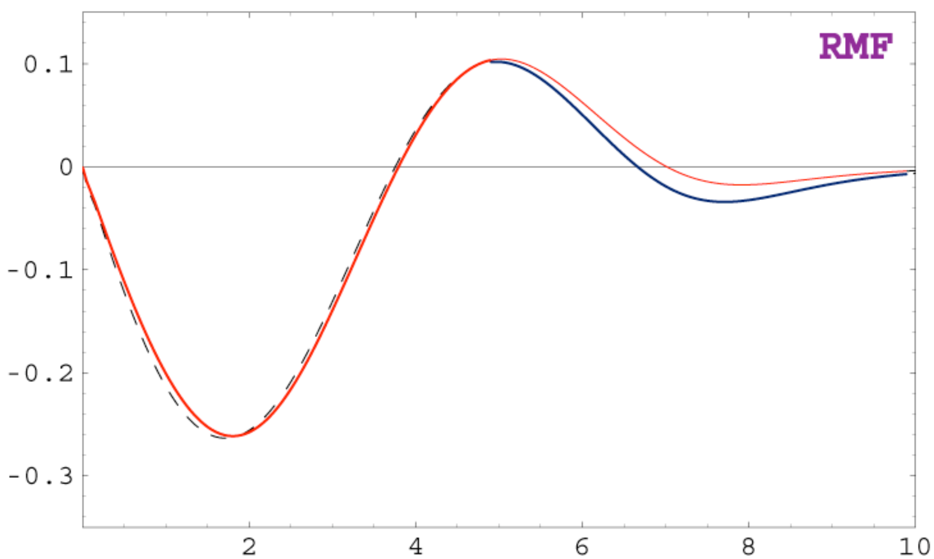


$\tilde{n}_r$
$\tilde{\ell} = 1$
1
2
$g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}-\frac{1}{2}}(r)$
$g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}+\frac{1}{2}}(r)$
$(\frac{\partial}{\partial r} + \frac{\tilde{\ell}+2}{r})g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}+\frac{1}{2}}(r)$
$(\frac{\partial}{\partial r} - \frac{\tilde{\ell}-1}{r})g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}-\frac{1}{2}}(r)$
r (fermi)

$2s_{\frac{1}{2}}$ $1d_{\frac{3}{2}}$







r (fermi)

$$\tilde{\ell} = 1$$

$$\left(\frac{\partial}{\partial r} + \frac{3}{r}\right) g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}+\frac{1}{2}}(r) =$$

$$\frac{\partial}{\partial r}\, g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}-\frac{1}{2}}(r)$$

Non-Spherical Nuclei

$$H_{ps}\ \Phi^{ps}_{k,\tilde{\mu}}(\vec{r}) = E_k \Phi^{ps}_{k,\tilde{\mu}}(\vec{r})$$

$$\Phi_{k,\tilde{\mu}}(\vec{r}) = \begin{pmatrix} g^{+}_{k,\tilde{\mu}}(\vec{r}) \\ g^{-}_{k,\tilde{\mu}}(\vec{r}) \\ i f^{+}_{k,\tilde{\mu}}(\vec{r}) \\ i f^{-}_{k,\tilde{\mu}}(\vec{r}) \end{pmatrix}$$

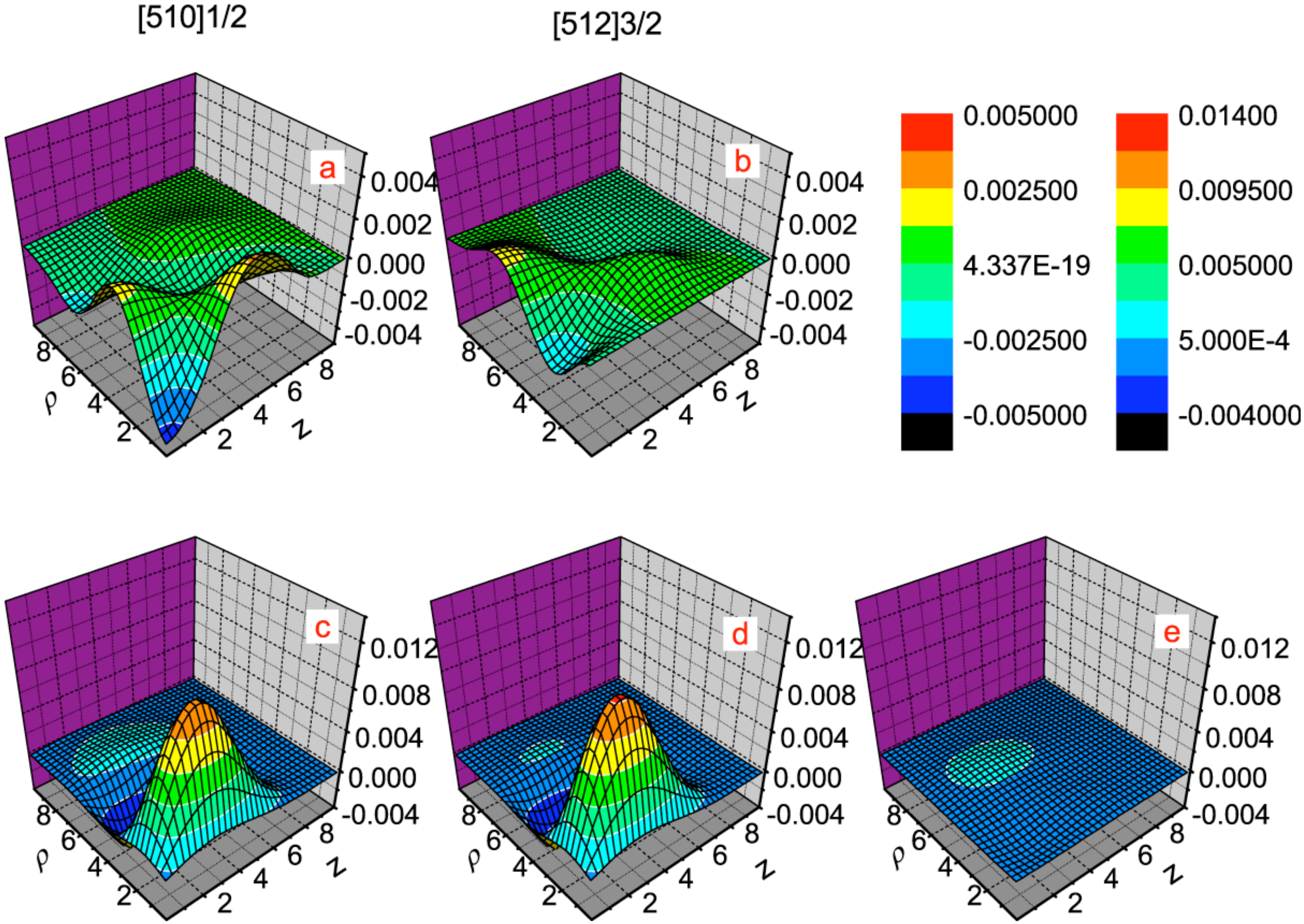
Pseudospin Conditions on Lower Deformed Components

$$k = \tilde{N}, \tilde{n}_3, \tilde{\Lambda}$$

$$f^{+}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho, z) = f^{-}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\rho, z) = f_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho, z),$$

$$f^{+}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\rho, z) = f^{-}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho, z) = 0$$

Lower Components



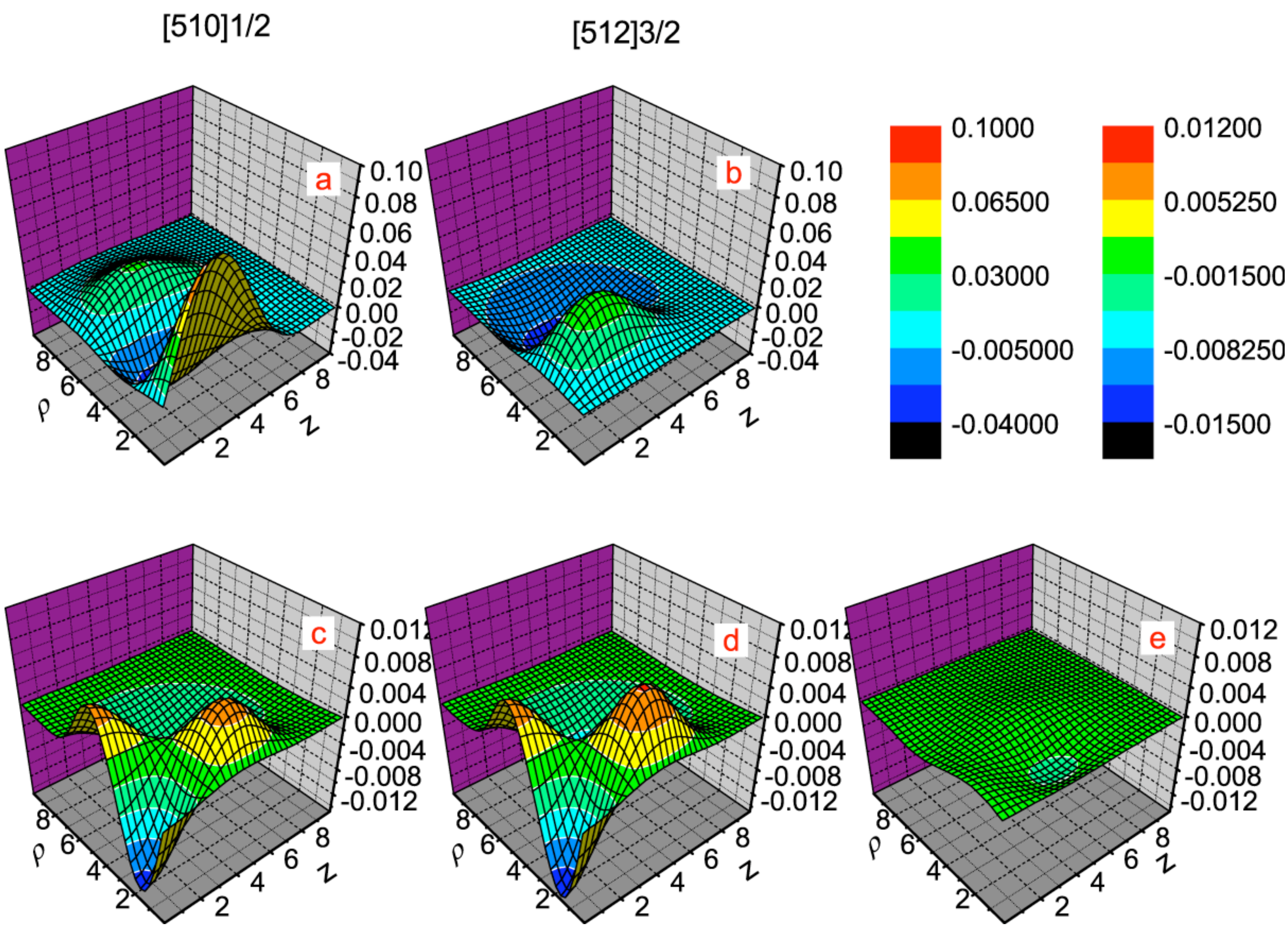
Pseudospin Conditions on Upper Deformed Components

$$g^{+}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho, z) = -g^{-}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho, z) = g_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho, z),$$

$$\Phi^{ps}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\vec{r}) = \begin{pmatrix} g_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho, z)\ e^{i\tilde{\Lambda}\phi} \\ g^{-}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho, z)\ e^{i(\tilde{\Lambda}+1)\phi} \\ i f_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho, z)\ e^{i\tilde{\Lambda}\phi} \\ 0 \end{pmatrix}, \quad \Phi^{ps}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\vec{r}) = \begin{pmatrix} g^{+}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\rho, z)\ e^{i(\tilde{\Lambda}-1)\phi} \\ -g_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho, z)\ e^{i\tilde{\Lambda}\phi} \\ 0 \\ i f_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho, z)\ e^{i\tilde{\Lambda}\phi} \end{pmatrix},$$

$$\Omega' = \tilde{\Lambda} + \frac{1}{2} \qquad\qquad \Omega = \tilde{\Lambda} - \frac{1}{2}$$

Upper Components



$$\tilde{S}_z \; \Phi^{ps}_{k,\tilde{\mu}}(\vec{r}) = \tilde{\mu} \; \Phi^{ps}_{k,\tilde{\mu}}(\vec{r})$$

$$\tilde{S}_\pm \; \Phi^{ps}_{k,\tilde{\mu}}(\vec{r}) = \sqrt{\left(\frac{1}{2} \mp \tilde{\mu}\right) (\frac{3}{2} \pm \tilde{\mu})} \; \Phi^{ps}_{k,\tilde{\mu}\pm 1}(\vec{r})$$

$$(\frac{\partial}{\partial\rho}+\frac{\tilde{\Lambda}+1}{\rho})g^{-}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho,z)=(\frac{\partial}{\partial\rho}-\frac{\tilde{\Lambda}-1}{\rho})g^{+}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\rho,z),$$

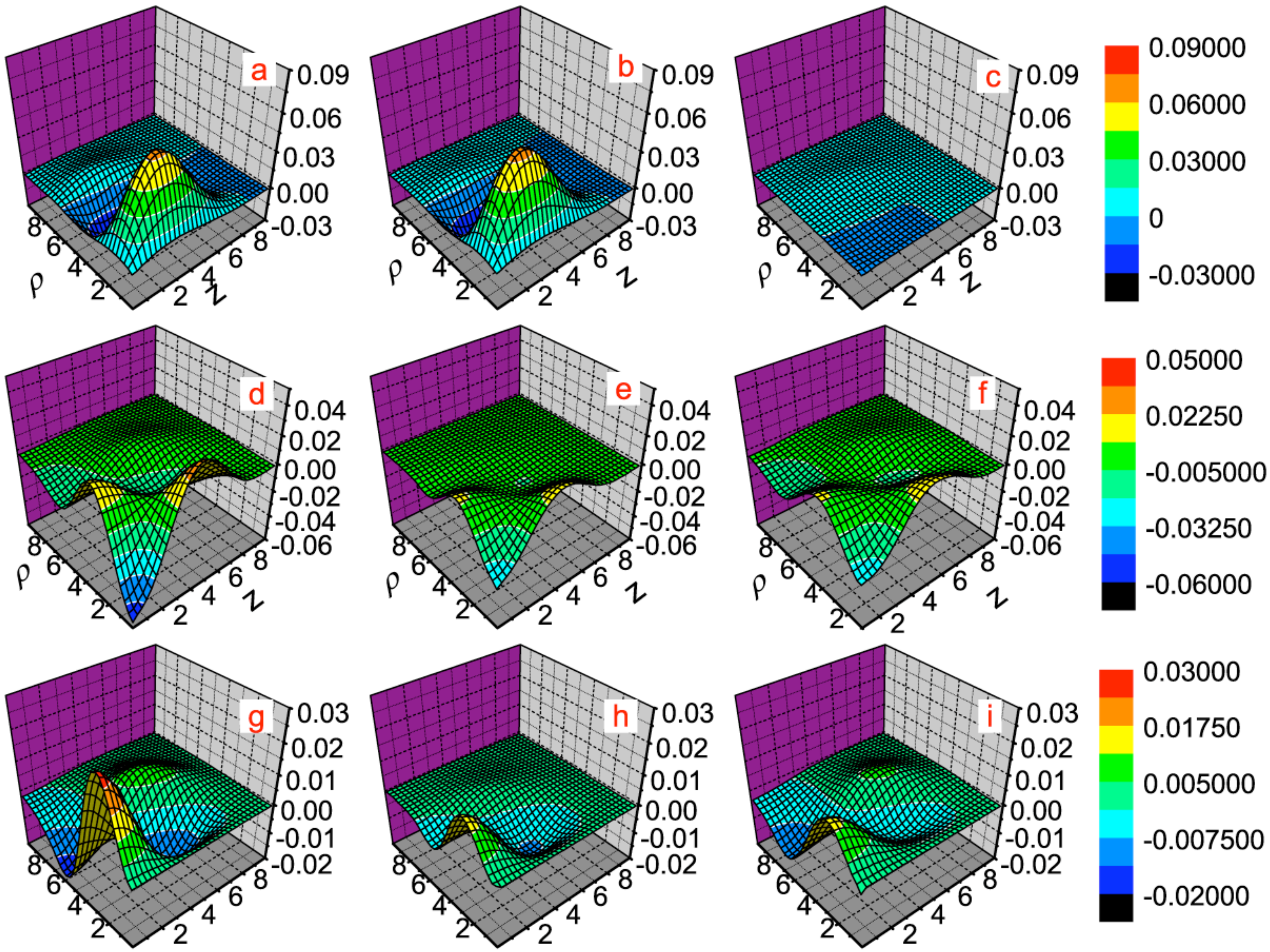
$$\frac{\partial}{\partial z}g^{\pm}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\mp\frac{1}{2}}(\rho,z)=(\frac{\partial}{\partial\rho}\pm\frac{\tilde{\Lambda}}{\rho})g_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho,z).$$

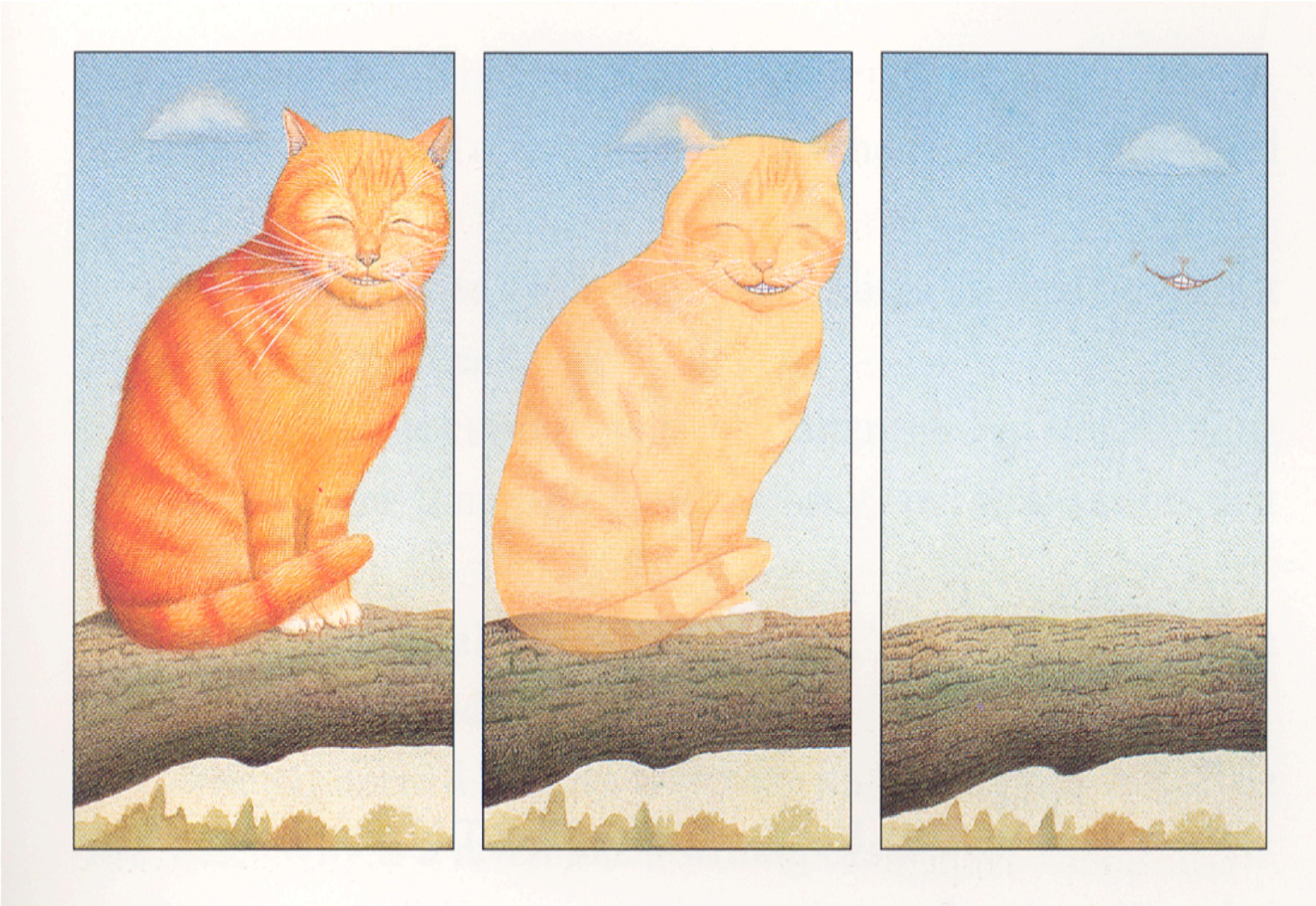
$$\Phi^{ps}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\vec{r})=\begin{pmatrix} g_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho,z)\ e^{i\tilde{\Lambda}\phi} \\ g^{-}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},\frac{1}{2}}(\rho,z)\ e^{i(\tilde{\Lambda}+1)\phi} \\ if_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho,z)\ e^{i\tilde{\Lambda}\phi} \\ 0 \end{pmatrix},\quad \Phi^{ps}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\vec{r})=\begin{pmatrix} g^{+}_{\tilde{N},\tilde{n}_3,\tilde{\Lambda},-\frac{1}{2}}(\rho,z)\ e^{i(\tilde{\Lambda}-1)\phi} \\ -g_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho,z)\ e^{i\tilde{\Lambda}\phi} \\ 0 \\ if_{\tilde{N},\tilde{n}_3,\tilde{\Lambda}}(\rho,z)\ e^{i\tilde{\Lambda}\phi} \end{pmatrix},$$

$$\Omega'=\tilde{\Lambda}+\frac{1}{2} \qquad\qquad \Omega=\tilde{\Lambda}-\frac{1}{2}$$

Differential Relations

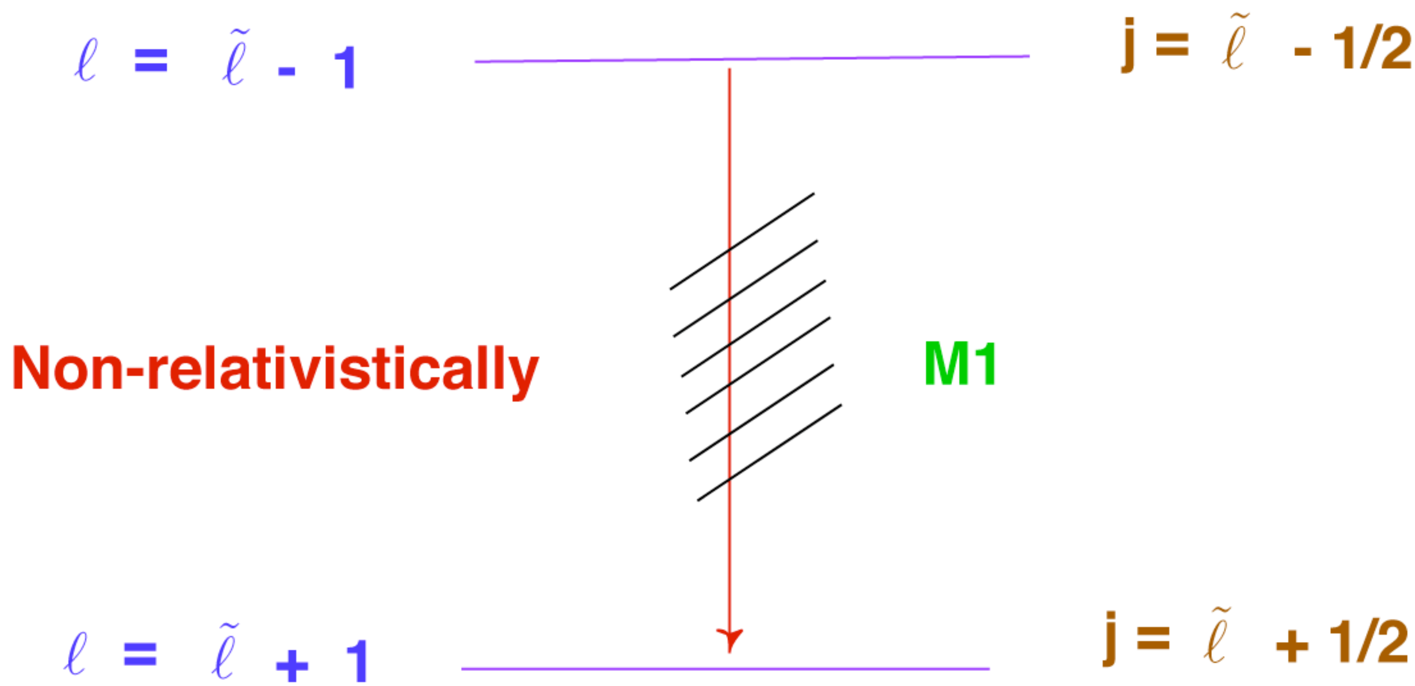
[510]1/2 & [512]3/2





“Well! I’ve often seen a cat without a grin,” thought Alice, “but a grin without a cat! It’s the most curious thing I ever saw in all my life!”
Lewis Carroll, *Alice’s Adventures in Wonderland*

Magnetic Transitions between Pseudospin doublets



Magnetic Transitions for Pseudospin Symmetry

$$B(M1:j' \rightarrow j)_\nu = \frac{j+1}{2j+1}\left[\mu_{j,\nu} - \mu_{A,\nu}\right]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1:j' \rightarrow j)_\nu = \frac{2j+1}{j+1}\left[\frac{j+2}{2j+3}\left(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu}\right)\right]^2 \qquad j = \tilde{\ell} - 1/2$$

Magnetic Transitions for Pseudospin Symmetry

$$B(M1 : j' \rightarrow j\ )_\nu = \frac{j+1}{2j+1}\,[\mu_{j,\nu} - \mu_{A,\nu}]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1 : j' \rightarrow j\ )_\nu = \frac{2j+1}{j+1}\left[\frac{j+2}{2j+3}\,(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu})\right]^2 \qquad j = \tilde{\ell} - 1/2$$

$$\frac{B(M1; j' \rightarrow j)_\nu}{S_j S_{j'}} = \frac{j+1}{2j+1}\left(\frac{\mu_{j,\nu}}{S} - \mu_{A,\nu}\right)^2 \qquad j' = \tilde{\ell} + 1/2$$

$$\frac{B(M1; j' \rightarrow j)_\nu}{S_j S_{j'}} = \frac{2j+1}{j+1}\left(\frac{j+2}{2j+3}\right)^2\left(\frac{\mu_{j'}}{S_{j'}} + \frac{j+1}{j+2}\,\mu_{A,\nu}\right)^2 \qquad j = \tilde{\ell} - 1/2$$

S_j $S_{j'}$ spectroscopic factors

Magnetic Transitions for Pseudospin Symmetry

$$B(M1: j' \rightarrow j\ )_\nu = \frac{j+1}{2j+1}\left[\mu_{j,\nu} - \mu_{A,\nu}\right]^2 \qquad j' = \tilde{\ell} + 1/2$$

$$B(M1: j' \rightarrow j\ )_\nu = \frac{2j+1}{j+1}\left[\frac{j+2}{2j+3}\,(\mu_{j',\nu} + \frac{j+1}{j+2}\mu_{A,\nu})\right]^2 \qquad j = \tilde{\ell} - 1/2$$

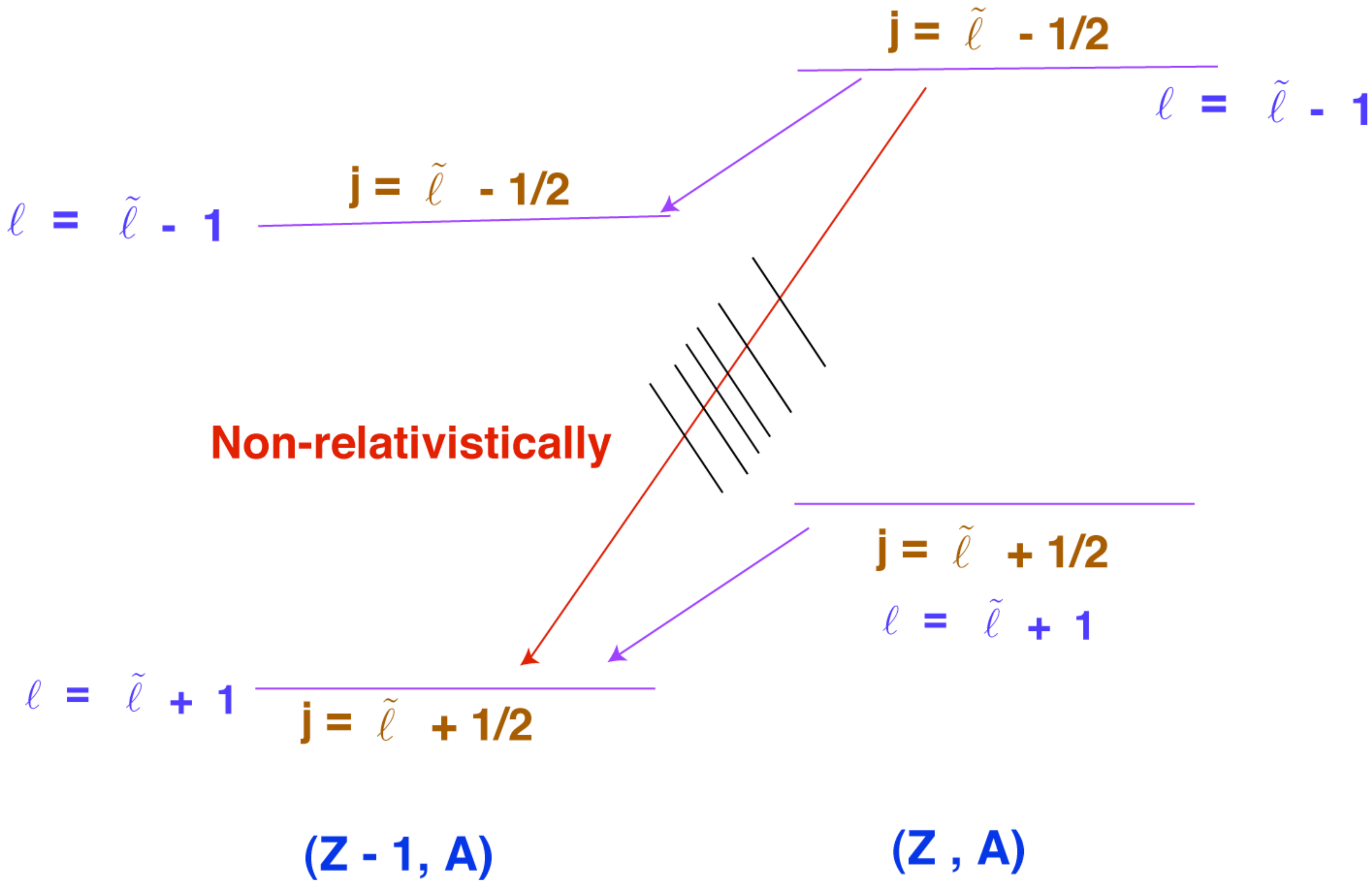
$$\frac{B(M1; j' \rightarrow j)_\nu}{S_j S_{j'}} = \frac{j+1}{2j+1}\left(\frac{\mu_{j,\nu}}{S} - \mu_{A,\nu}\right)^2 \qquad j' = \tilde{\ell} + 1/2$$

$$\frac{B(M1; j' \rightarrow j)_\nu}{S_j\, S_{j'}} = \frac{2j+1}{j+1}\left(\frac{j+2}{2j+3}\right)^2\left(\frac{\mu_{j'}}{S_{j'}} + \frac{j+1}{j+2}\,\mu_{A,\nu}\right)^2 \qquad j = \tilde{\ell} - 1/2$$

Predicted "ℓ forbidden" magnetic dipole transition in ^{39}Ca.

	$B(M1: j, \nu \rightarrow j', \nu)$
Predicted Equation	0.0166
Predicted Equation	0.0121
EXP	0.0121 (14)

Gamow-Teller Transitions between Pseudospin doublets



Quadrupole Operator in Pseudospin Basis

$$\tilde{Q}_\mu(r) = \Sigma_{i=1}^{A}\tilde{Q}_\mu(r_i) = \Sigma_{i=1}^{A}\sigma_i \cdot \hat{p} Q_\mu(r_i)\sigma_i \cdot \hat{p} =$$

$$\Sigma_{i=1}^{A} Q_\mu(r_i) + \Sigma_{i=1}^{A}[\sigma_i \cdot \hat{p}, Q_\mu(r_i)]\sigma_i \cdot \hat{p}$$

$$\Sigma_{i=1}^{A}[\sigma_i \cdot \hat{p}, Q_\mu(r_i)]\sigma_i \cdot \hat{p} = q_\mu^0 + \Sigma_{i=1}^{A} q_{\mu,i}^1 \sigma_i$$

The second term will be of order 1/A because

$$\vec{\sigma}_i = \frac{1}{A}\Sigma_{i=1}^{A}\vec{\sigma}_i = \frac{2}{A}\vec{S}_i$$

$$\tilde{Q}_\mu(r) \approx Q_\mu(r) + q_\mu^0$$

Since the quadrupole operator is approximately independent of the pseudospin, the B(E2)'s between doublets are related and the quadrupole moments are related to the B(E2) with a doublet. For example:

$$\frac{B(E2: \tilde{L},\ J=\tilde{L}-1/2 \rightarrow \tilde{L}-1,\ J'=\tilde{L}-1/2)}{B(E2: \tilde{L},\ J=\tilde{L}-1/2 \rightarrow \tilde{L}-1,\ J'=\tilde{L}-3/2)} = \frac{3}{(\tilde{L}+1)(2\tilde{L}-3)}$$

$$\frac{B(E2: \tilde{L},\ J=\tilde{L}+1/2 \rightarrow \tilde{L}-1,\ J'=\tilde{L}-3/2)}{B(E2: \tilde{L},\ J=\tilde{L}+1/2 \rightarrow \tilde{L}-1,\ J'=\tilde{L}-1/2)} = \frac{4}{(2\tilde{L}-3)(2\tilde{L}+3)}$$

$$\frac{B(E2: \tilde{L},\ J=\tilde{L}-1/2 \rightarrow \tilde{L}-2,\ J'=\tilde{L}-3/2)}{B(E2: \tilde{L},\ J=\tilde{L}+1/2 \rightarrow \tilde{L}-2,\ J'=\tilde{L}-3/2)} = \frac{1}{\tilde{L}(\tilde{L}-2)}$$

$$\frac{B(E2: \tilde{L},\ J=\tilde{L}-1/2 \rightarrow \tilde{L}-2,\ J'=\tilde{L}-3/2)}{B(E2: \tilde{L},\ J=\tilde{L}-1/2 \rightarrow \tilde{L}-2,\ J'=\tilde{L}-5/2)} = \frac{4}{(2\tilde{L}+1)(2\tilde{L}-3)}$$

$$\frac{B(E2: \tilde{L},\ J=\tilde{L}-1/2 \rightarrow \tilde{L},\ J'=\tilde{L}+1/2)}{[Q(\tilde{L},\ J=\tilde{L}+1/2)]^2} = \frac{15(\tilde{L}+1)}{\tilde{L}^2(2\tilde{L}-1)(2\tilde{L}+1)^2}$$

$$\frac{Q(\tilde{L},\ J=\tilde{L}-1/2)}{Q(\tilde{L},\ J=\tilde{L}+1/2)} = \frac{(\tilde{L}-1)(2\tilde{L}+3)}{\tilde{L}(2\tilde{L}+1)}$$

^{189}Os

	ps	exp
$\dfrac{B(E2: \tilde{L}=3,\ J=7/2 \to \tilde{L}=2,\ J'=3/2)}{B(E2: \tilde{L}=3,\ J=7/2 \to \tilde{L}=2,\ J'=5/2)}$	0.148	0.183 (0.073)
$\dfrac{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=1,\ J'=3/2)}{B(E2: \tilde{L}=3,\ J=7/2 \to \tilde{L}=1,\ J'=3/2)}$	0.333	0.281 (0.118)
$\dfrac{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=2,\ J'=5/2)}{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=2,\ J'=3/2)}$	0.250	0.438 (0.280)
$\dfrac{B(E2: \tilde{L}=2,\ J=3/2 \to \tilde{L}=1,\ J'=3/2)}{B(E2: \tilde{L}=2,\ J=3/2 \to \tilde{L}=1,\ J'=1/2)}$	1.000	0.355 (0.110)

^{187}Os

	ps	exp
$\dfrac{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=2,\ J'=5/2)}{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=2,\ J'=3/2)}$	0.250	0.293 (0.077)
$\dfrac{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=1,\ J'=3/2)}{B(E2: \tilde{L}=3,\ J=5/2 \to \tilde{L}=1,\ J'=1/2)}$	0.286	0.410 (0.094)

Isovector Effects

σ, ω isoscalar mesons

ρ isovector meson

$$V_V^{\pi} = V_{\omega} - (N - Z) V_{\rho} ,$$

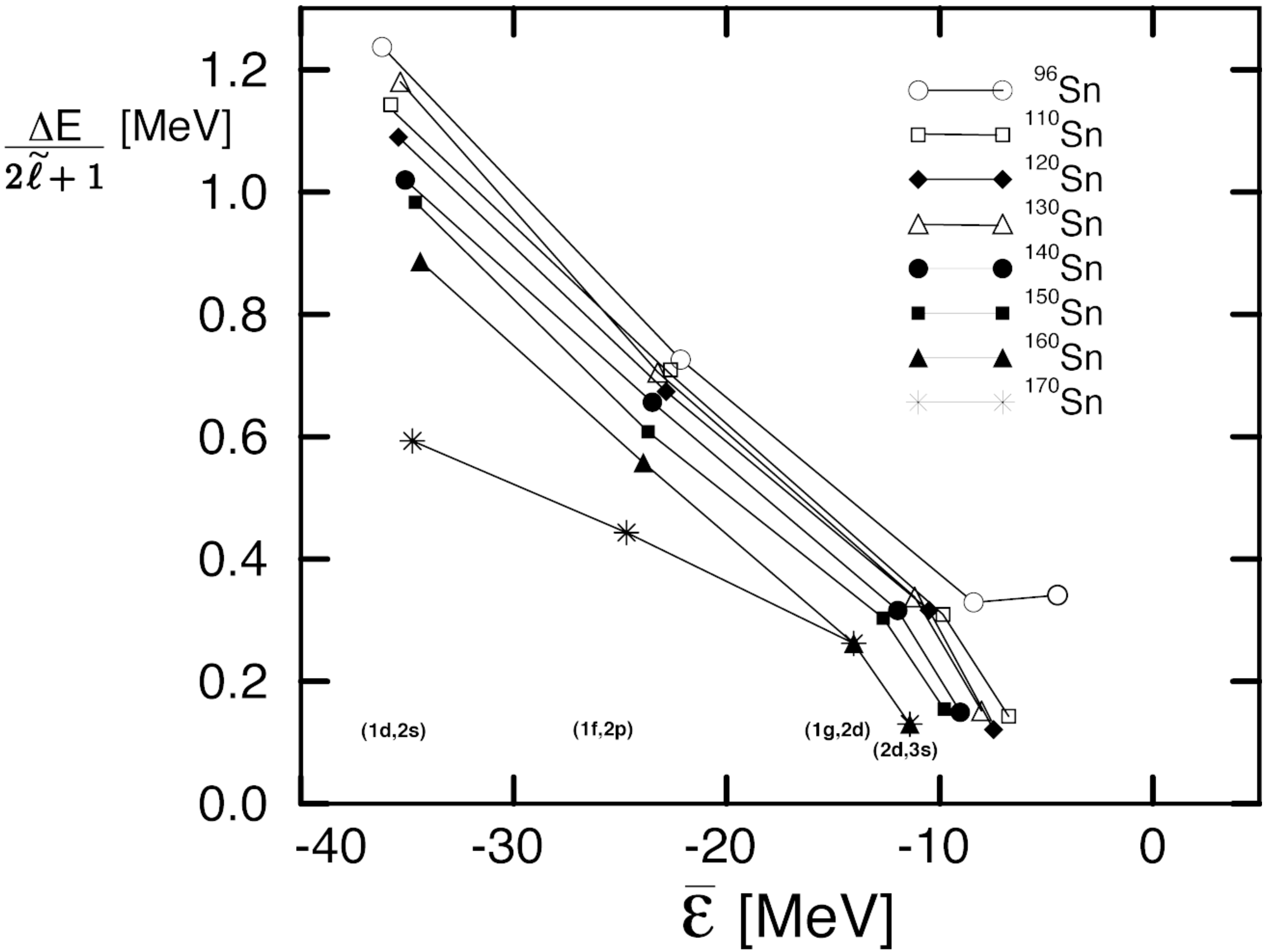
$$V_V^{\nu} = V_{\omega} + (N - Z) V_{\rho}$$

As neutron excess increases

$V_V^{\pi} + V_S^{\pi}$ **increases**

$V_V^{\nu} + V_S^{\nu}$ **decreases**

Pseudospin symmetry may improve for neutrons for neutron rich nuclei measured in RIA experiments!



$\frac{\Delta E}{2\tilde{\ell}+1}$ [MeV]
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-40
-30
-20
-10
0
$\bar{\varepsilon}$ [MeV]
^{96}Sn
^{110}Sn
^{120}Sn
^{130}Sn
^{140}Sn
^{150}Sn
^{160}Sn
^{170}Sn
(1d,2s)
(1f,2p)
(1g,2d)
(2d,3s)

Nucleon-Nucleon Scattering Matrix

$$S^{(J)} = \begin{pmatrix} e^{2i\delta_{0,J,J}} & 0 & 0 & 0 \\ 0 & e^{2i\delta_{1,J,J}} & 0 & 0 \\ 0 & 0 & e^{2i\delta_{1,J-1,J}} \cos(2\epsilon_J) & i\ e^{i\delta_J^{(+)}} \sin(2\epsilon_J) \\ 0 & 0 & i\ e^{i\delta_J^{(+)}} \sin(2\epsilon_J) & e^{2i\delta_{1,J+1,J}} \cos(2\epsilon_J) \end{pmatrix}$$

$\delta_{\mathrm{S,L},J}$ **phase shifts**

$$\delta_J^{(\pm)} = (\delta_{1,J-1,J} \pm \delta_{1,J+1,J})$$

ϵ_J **mixing angles**

between $L_> = J + 1$ and $L_< = J - 1$

Two Nucleons

$$\tilde{S}_\mu = \sum_{i=1,2} \tilde{S}_\mu(i) = \begin{pmatrix} \tilde{s}_\mu & 0 \\ 0 & s_\mu \end{pmatrix}$$

$$\tilde{s}_\mu = \sum_{i=1,2} \tilde{s}_\mu(i), \qquad s_\mu = \sum_{i=1,2} s_\mu(i)$$

$$\tilde{s}_\mu = U_{p,1} U_{p,2} s_\mu U_{p,1} U_{p,2}$$

Scattering Matrix in Pseudospin Basis

$$\mathcal{U} = \sigma(1)\cdot\hat{p}\ \sigma(2)\cdot\hat{p}$$

$$\tilde{S}^{(J)} = \mathcal{U}\, S^{(J)}\, \mathcal{U}$$

$$\tilde{S}^{(J)} = \begin{pmatrix} e^{2i\tilde{\delta}_{0,J,J}} & 0 & 0 & 0 \\ 0 & e^{2i\tilde{\delta}_{1,J,J}} & 0 & 0 \\ 0 & 0 & e^{2i\tilde{\delta}_{1,J-1,J}}\cos(2\tilde{\epsilon}_J) & i\, e^{i\tilde{\delta}_J^{(+)}}\sin(2\tilde{\epsilon}_J) \\ 0 & 0 & i\, e^{i\tilde{\delta}_J^{(+)}}\sin(2\tilde{\epsilon}_J) & e^{2i\tilde{\delta}_{1,J+1,J}}\cos(2\tilde{\epsilon}_J) \end{pmatrix}$$

Transformation from Spin to Pseudospin

$$\tilde{\delta}_{S,J,J} = \delta_{S,J,J}$$

$$\tilde{\delta}_J^{(+)} = \delta_J^{(+)}$$

$$\cos(\tilde{\delta}_J^{(-)}) \cos(2\tilde{\epsilon}_J) = \cos(\delta_J^{(-)}) \cos(2\epsilon_J)$$

$$\tan(\tilde{\delta}^{(-)}) = \frac{2\sqrt{N^2-1}\ \sin(2\epsilon_J) - (N^2-2)\ \sin(\delta^{(-)})\ \cos(2\epsilon_J)}{N^2\ \cos(\delta_J^{(-)}) \cos(2\epsilon_J)}$$

$$\sin(2\tilde{\epsilon}_J) = \frac{2\sqrt{N^2-1}\ \sin(\delta^{(-)})\ \cos(2\epsilon_J) + (N^2-2)\ \sin(2\epsilon_J)}{N^2}$$

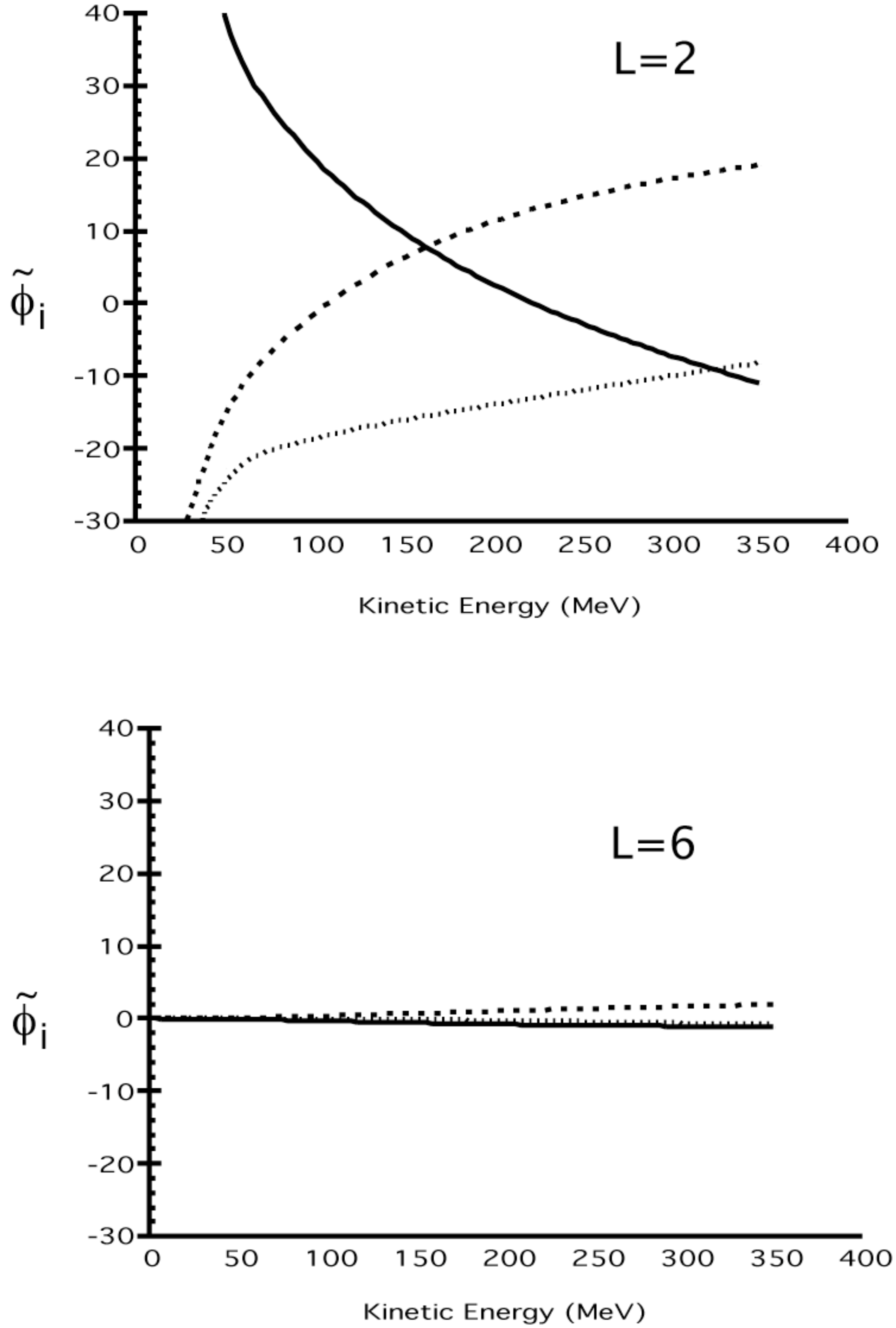
$$N = 2J + 1$$

Conditions for Pseudospin Symmetry

$$\tilde{\epsilon}_J = 0$$

$$\tilde{\delta}_{1,\tilde{L},J} = \tilde{\delta}_{1,\tilde{L},J+1} = \tilde{\delta}_{1,\tilde{L},J-1}$$

$$\tilde{\phi}_1 = \tilde{\delta}_{1,\tilde{L},J-1} - (\tilde{\delta}_{1,\tilde{L},J-1} + \tilde{\delta}_{1,\tilde{L},J} + \tilde{\delta}_{1,\tilde{L},J+1})/3 = 0$$
$$\tilde{\phi}_2 = \tilde{\delta}_{1,\tilde{L},J} - (\tilde{\delta}_{1,\tilde{L},J-1} + \tilde{\delta}_{1,\tilde{L},J} + \tilde{\delta}_{1,\tilde{L},J+1})/3 = 0$$
$$\tilde{\phi}_3 = \tilde{\delta}_{1,\tilde{L},J+1} - (\tilde{\delta}_{1,\tilde{L},J-1} + \tilde{\delta}_{1,\tilde{L},J} + \tilde{\delta}_{1,\tilde{L},J+1})/3 = 0$$

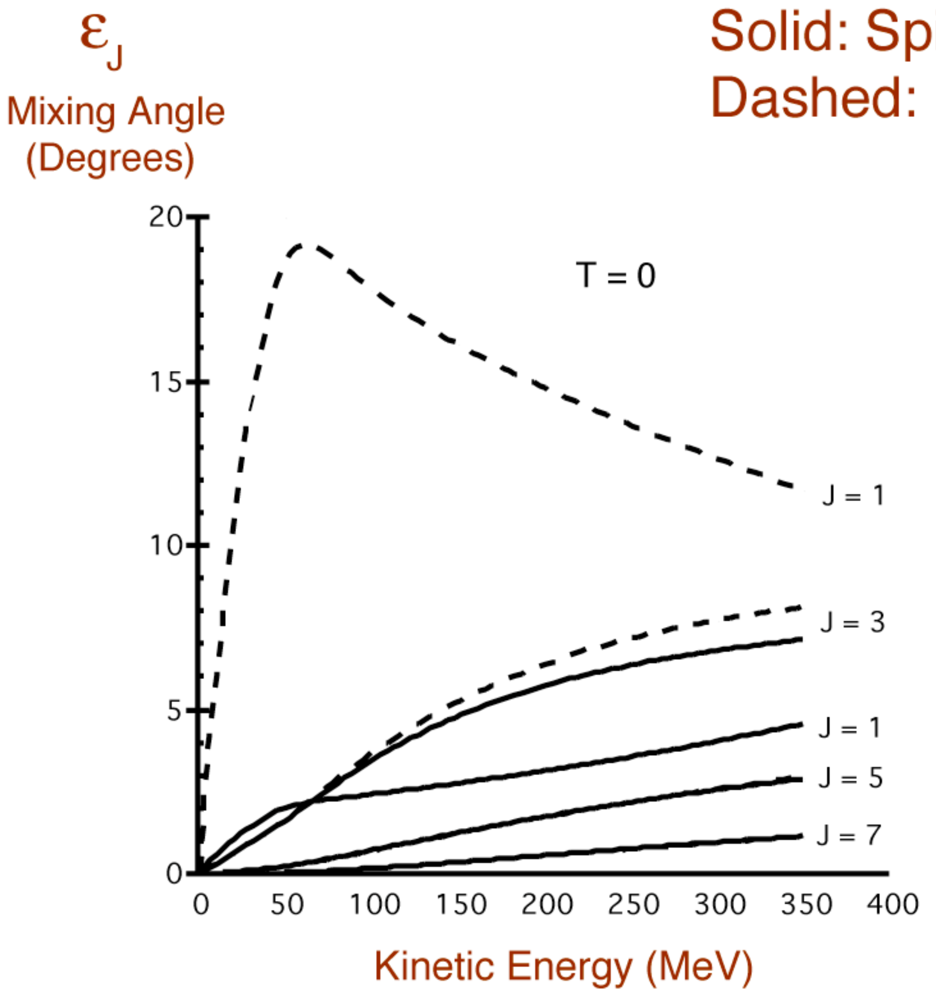


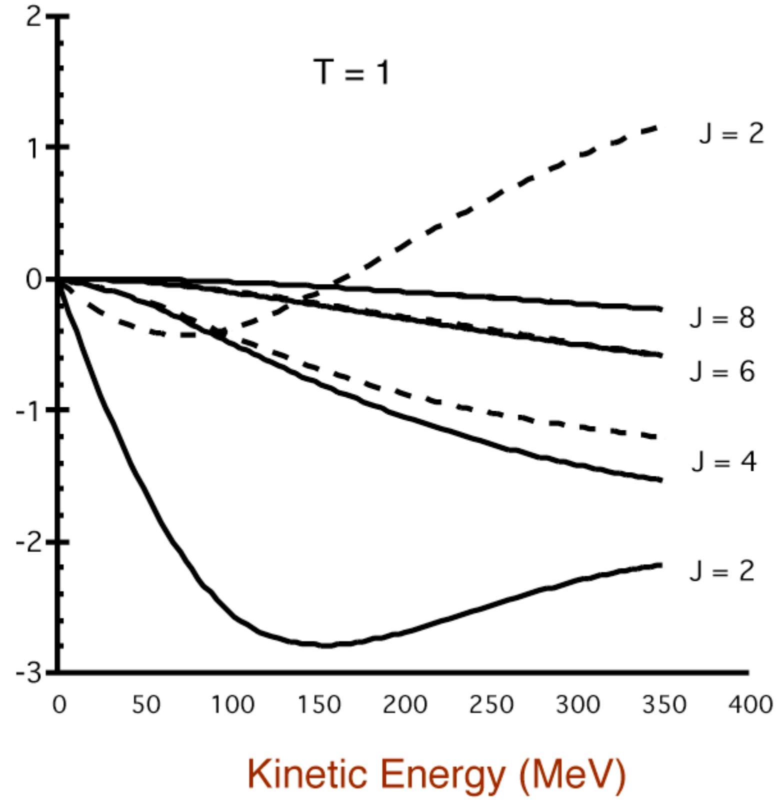
$\tilde{\phi}_i$

1 Solid
2 Dashed
3 Dotted

Mixing Angles

Solid: Spin
Dashed: Pseudospin





Effective Field Theory

In EFT expansion why aren't pseudospin operators on equal standing as spin operators?

For example

$$a_s \; s_1 \cdot s_2 + a_{ps} \; \tilde{s}_1 \cdot \tilde{s}_2$$

Effective Field Theory

$$\tilde{s}_i = U_p \; s_i \; U_p = \sigma \cdot \hat{p} \; \hat{p}_i - s_i , \;\; i = 1, 2, 3$$

where p is the relative momentum

For two nucleons:

$$U_p = \sigma_1 \cdot \hat{p} \; \sigma_2 \cdot \hat{p}$$

$$\tilde{s}_i = U_p \; s_i \; U_p = \sigma \cdot \hat{p} \; \hat{p}_i - s_i , \;\; i = 1, 2, 3$$

Spin-Spin Interaction

$$\tilde{s}_1 \cdot \tilde{s}_2 = s_1 \cdot s_2$$

This is consistent with the study of the nucleon-nucleon interaction in which it is shown that the pseudospin transformation on two nucleons does not change the spin. However, this does not mean they are equivalent because the mixing angle between states of different pseudo-orbtial angular momentum is different than the mixing angle between states of different orbital angular momentum, which come about through other terms involving the pseudo-orbital angular momentum operator and the orbital angular momentum operator.

Tensor Interaction

$$\sigma_1 \cdot p \; \sigma_2 \cdot p \;\; =$$

$$p^2 \; [\tilde{s}_1 \cdot s_2 + s_1 \cdot \tilde{s}_2 + \tilde{s}_1 \cdot \tilde{s}_2 + s_1 \cdot s_2]$$

That is, the tensor interaction is symmetrical in pseudospin and spin and it is an interaction between the spin and pseudospin, which is an interesting insight.

Spin - Orbit Interaction

$$b_s[s_1 + s_2] \cdot L + b_{ps}[\tilde{s}_1 + \tilde{s}_2] \cdot \tilde{L}$$

$$(\tilde{s}_1 + \tilde{s}_2) \cdot \tilde{L} = -(s_1 + s_2) \cdot L - 4$$

$$+2\ \sigma_1 \cdot \sigma_2 + 2\ \sigma_1 \cdot \hat{p}\ \sigma_2 \cdot \hat{p}$$

Antinucleon Spectrum

Charge Conjugation

$$\overline{V}_S(\vec{r}) = C'V_S(\vec{r})C = V_S(\vec{r})$$

$$\overline{V}_V(\vec{r}) = C'\overline{V}_V(\vec{r})C = -\overline{V}_V(\vec{r})$$

$$\therefore \overline{V}_S(\vec{r}) \approx \overline{V}_V(\vec{r})$$

$$\Rightarrow \text{Spin Symmetry for Antinucleons}$$

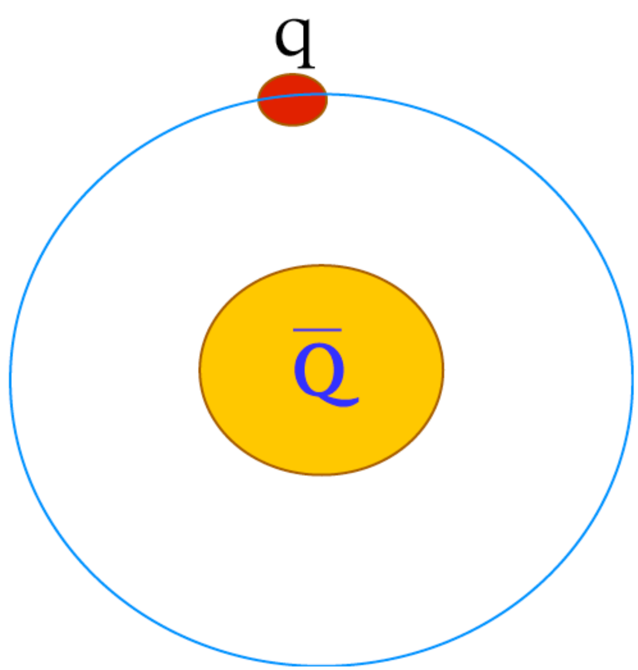
Spin polarization in antiproton scattering from Carbon is almost zero supporting this prediction, but data set is limited (PLB 151, 473 (1985)).

Perhaps additional antiproton scattering will be forthcoming at GSI

Dirac Hamiltonian has a spin symmetry if

$$V_S(x) = V_V(x) + C_S$$

Explains spin degeneracies of Light Quark - Heavy Quark Mesons



Phys. Rev. Lett. 86, 204 (2001); hep-ph/0002094

SUMMARY

1) Pseudospin symmetry is a relativistic symmetry of the Dirac Hamiltonian for

$$[\, p_i \,, V_S(\vec{r}) + V_V(\vec{r}) \,] = 0$$

2) Pseudo-spin symmetry is approximately conserved for realistic mean fields,

$$V_S(\vec{r}) \approx - V_V(\vec{r})$$

3) QCD sum rules suggest a connection with spontaneously broken chiral symmetry

4) Relativistic spherical and deformed mean field eigenfunctions satisfy approximately the conditions for pseudospin symmetry as do realistic non-relativistic eigenfunctions.

5) Pseudospin symmetry M1 predictions approximately valid.

6) Pseudospin symmetry approximately conserved in medium energy nucleon-nucleus scattering.

7) Pseudospin symmetry may improve for neutron rich nuclei.

8) Charge conjugation predicts that an antinucleon imbedded in a nucleus will have spin symmety.

Summary

9) Spin and Tensor Interaction are equivalent to P-Spin and P-Tensor interactions.

10) Spin-Orbital and P-Spin-P-Orbital interactions are not equivalent. In fact P-Spin-P-Orbital interaction produces a P- spin dynamical symmetry but violates spin symmetry.

Future

More fundamental rationale for pseudospin symmetry

1) Test the nucleon-nucleon interaction for generalized pseudospin symmetry conservation.

2) What is the connection between QCD and pseudospin symmetry suggested by QCD sum rules?

3) Why do mesons with one large quark have spin symmetry whereas nuclei have pseudospin symmetry?

Future

4) Effective Field Theory expansions with spin and P-spin on same footing.

5) Rewriting the tensor interaction in terms of the two body spin-orbit interaction and the two body pseudo spin-orbit interaction and determining the coefficients of each.