Description of mixed-mode dynamics within the Interacting Vector Boson Model:
1. Symplectic extension - the even-even nuclei

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Boson creation and annihilation operators

$$u_m^+(\alpha) = \frac{1}{\sqrt{2}}(x_m(\alpha) - i q_m(\alpha)),$$
$$u_m(\alpha) = (u_m^+(\alpha))^\dagger),$$

$$x_{\pm 1}(\alpha) = \mp \frac{1}{\sqrt{2}}(x_1(\alpha) \pm i x_2(\alpha)),\ x_0(\alpha) = x_3(\alpha)$$

$$q_m(\alpha) = -i\partial/\partial x^m(\alpha)$$

$u^+_k(\alpha)$, $u_m(\alpha)$

$\alpha = \pm 1/2$, k,m=0, ± 1

l=1, k,m= -1,0,1 vectors

$u^+_k(\alpha=1/2)=p^+_m$

$u^+_k(\alpha=-1/2)=n^+_m$

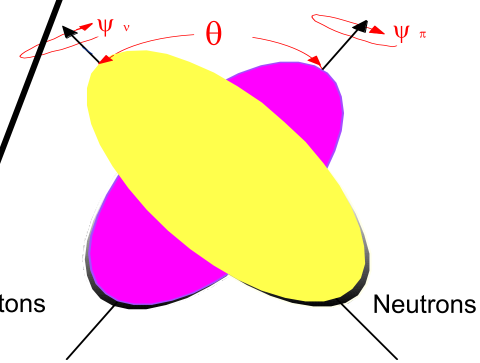
n boson

$(T_0 = -1/2)$

T=1/2

p boson

$(T_0 = 1/2)$



$[u_k(\alpha), u^+_m(\beta)] = \delta^k_m\, \delta_{\alpha\beta}$

$$L_M = -\mathrm{i} \sum_{\alpha,k,m} C^{1M}_{1k\,1m} x_m(\alpha) q_k(\alpha)$$

$su(2) \otimes su(3)$

$$Q_M = \sum_{\alpha,k,m} C^{2M}_{1k\,1m} x_k(\alpha) x_m(\alpha)$$

Angular momentum

U(6)

Quadrupole momentum

New degree of freedom – the T- spin

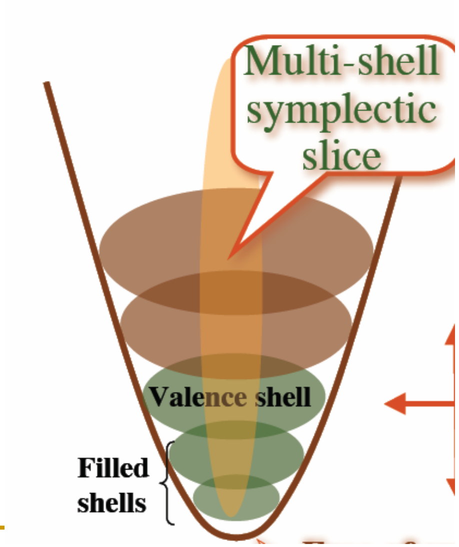
Applications of symplectic algebras in nuclear structure physics

$$Sp(12,R)$$

The **symplectic extenstion takes into account the change of the number of phonon excitations** (bosons) in the nuclear system.

$$\begin{array}{ccccccc} u(6) & \supset su(3) & \otimes\, u(2) & \supset & so(3) & \otimes & u(1) \\ [N] & (\lambda,\mu) & (N,T) & K & L & & T_0 \end{array}$$

- **Larger spaces** (infinite dimensional)
- **The complete spectrum** of the system can be calculated only trough **the diagonalization of the Hamiltonian** in **the subspaces of *all* the UIR of *U(6)*,** belonging to a given UIR of ***Sp(12,R)*.**



The symplectic algebras in nuclear structure physics

$(u \otimes u)$ l=0,1,2 ; t=0,1

- S (l=0;t=1)
- P (l=1;t=0)
- D (l=2;t=1)

IBM 3.5

Generators of *sp*(12,R)

sp(12,R):

$$F^L{}_M(\alpha,\beta) = \sum_{k,m=0,\pm1} (1k1m|LM)\, u^+{}_k(\alpha) u^+{}_m(\beta)$$

Create a pair

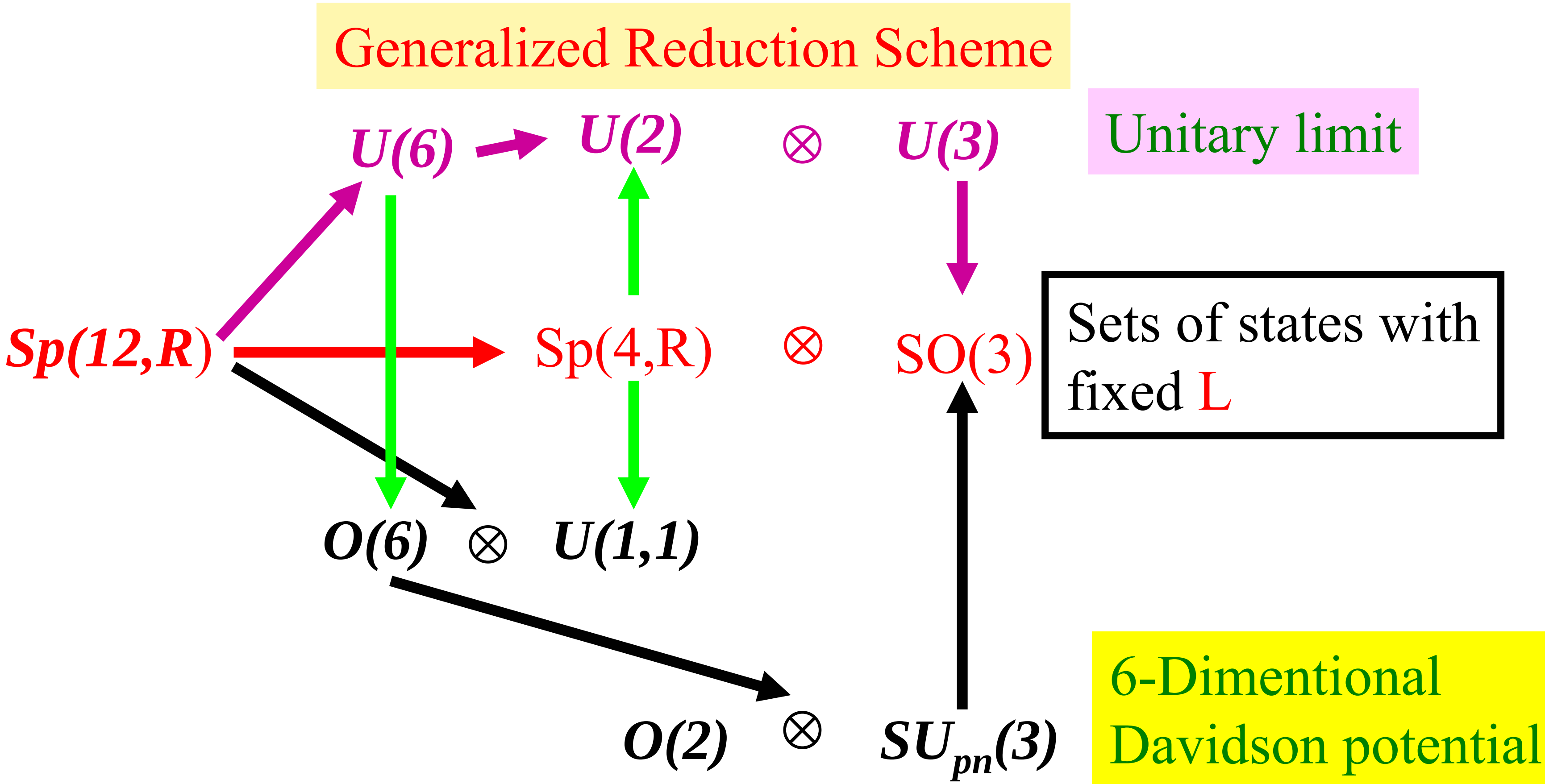
$$G^L{}_M(\alpha,\beta) = \sum_{k,m=0,\pm1} (1k1m|LM)\, u_k(\alpha) u_m(\beta)$$

Destroy a pair

$$A^L{}_M(\alpha,\beta) = \sum_{k,m=0,\pm1} (1k1m|LM) u^+{}_k(\alpha) u{}_m(\beta)$$

***u*(6)**

The new reduction chains



K=min(λ,μ), min(λ,μ) –2, …, 0(1)

L=max(λ,μ), max(λ,μ) –2, …, 0(1) K=0

L=K, K+1, …, K+ max(λ,μ) K≠0

su(3) ⊃ so(3)

The unitary limit

$sp(12,R) \supset u(6) \longleftrightarrow A^{L}{}_{M}(\alpha,\beta)$; L=0,1,2

$$N = -\sqrt{3}\sum_{\alpha} A^{0}(\alpha,\alpha)$$

$$Q_M = -\sqrt{6}\sum A^{2}{}_{M}(\alpha,\alpha)$$

$$L_M = -\sqrt{2}\sum_{\alpha} A^{1}{}_{M}(\alpha,\alpha)$$

$$u(6) \supset su(3) \otimes u(2) \supset so(3) \otimes u(1)$$

$[N]$ (λ,μ) (N,T) K L T_0

Generators: $T_{\pm 1}, T_0, N$

T= λ/2; N= λ + 2 μ

Labeling the collective bands K^{π}

The *u(6)* basis vectors:

$| [N]_6 ; (\lambda,\mu); KLM; T=T_0 > \equiv | N ,L,T,T0 >$

Parity of the states $\pi=(-1)^T$

Classification scheme

H_+

N\T$_3$	5	4	3	2	1	0	-1	-2	-3	-4	-5
0 0						(0,0)					
2 2 0					(2,0)	(2,0) (0,1)	(2,0)				
4 4 2 0				(4,0)	(4,0) (2,1)	(4,0) (2,1) (0,2)	(4,0) (2,1)	(4,0)			
6 6 4 2 0			(6,0)	(6,0) (4,1)	(6,0) (4,1) (2,2)	(6,0) (4,1) (2,2) (0,3)	(6,0) (4,1) (2,2)	(6,0) (4,1)	(6,0)		
8 8 6 4 2 0		(8,0)	(8,0) (6,1)	(8,0) (6,1) (4,2)	(8,0) (6,1) (4,2) (2,3)	(8,0) (6,1) (4,2) (2.3) (0,4)	(8,0) (6.1) (4,2) (2,3)	(8,0) (6,1) (4,2)	(8,0) (6,1)	(8,0)	
10 10 8 6 4 2 0	(10,0)	(10,0) (8,1)	(10,0) (8,1) (6,2)	(10,0) (8,1) (6,2) (4,3)	(10,0) (8,1) (6,2) (4,3) (2,4)	(10,0) (8,1) (6,2) (4,3) (2,4) (0,5)	(10,0) (8,1) (6,2) (4,3) (2,4)	(10,0) (8,1) (6,2) (4,3)	(10,0) (8,1) (6,2)	(10,0) (8,1)	(10,0)
- - - - -	- - - - -	- - - - -	- - - - -	- - - - -	- - - - -	- - - - -	- - - - - -	- - - - - -	- - - - - -	- - - - - -	- - - - - -

Ground Band

Octupole Band

The **Hamiltonian** of the system can be expressed in terms of the first and second order Casimir operators of the subalgebra from a chain.

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2$$

The energy spectrum: The eigenvalues of H :

$$E(N,L,T,T_0) = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2$$

increasing function of N

$$T = \frac{N}{2}, \frac{N}{2}-1, \frac{N}{2}-2, \ldots, 0 \vee 1$$
$$T_0 = -T, -T+1, \ldots, T-1, T$$

The successful reproduction of the experimental energies for each L is achieved as a result of their consideration as ***functions*** of the number of phonon excitation $N = -\sqrt{3}\,[\,A^0(p,p) + A^0(n,n)]$ that build the collective states $|\,N, L, T, T_0 >$.

Application: Energy levels of the ground $K^{\pi}=0^{+}$ and octupole bands $K^{\pi}=0^{-}$

$\pi=(-1)^{T}$

Algebraic yrast bands:

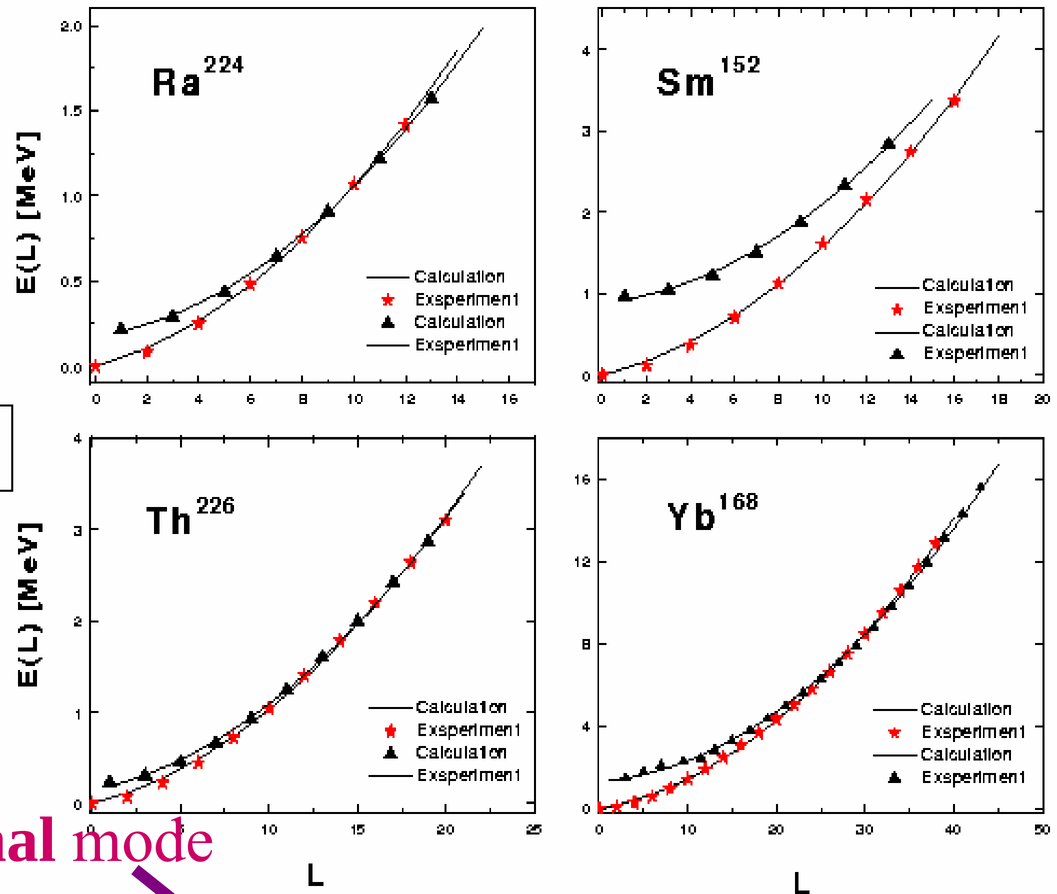
E = min

–with respect to N

ground band **L = N/2**

octupole band **L = N/2 –1**

From the reduction rules and the yrast condition



vibrational mode

$$E(L)=\beta L(L+1)+(\gamma+\eta)L+\xi$$

rotational mode

Interaction ηL between the bands

K from the su(3)⊃so(3) reduction

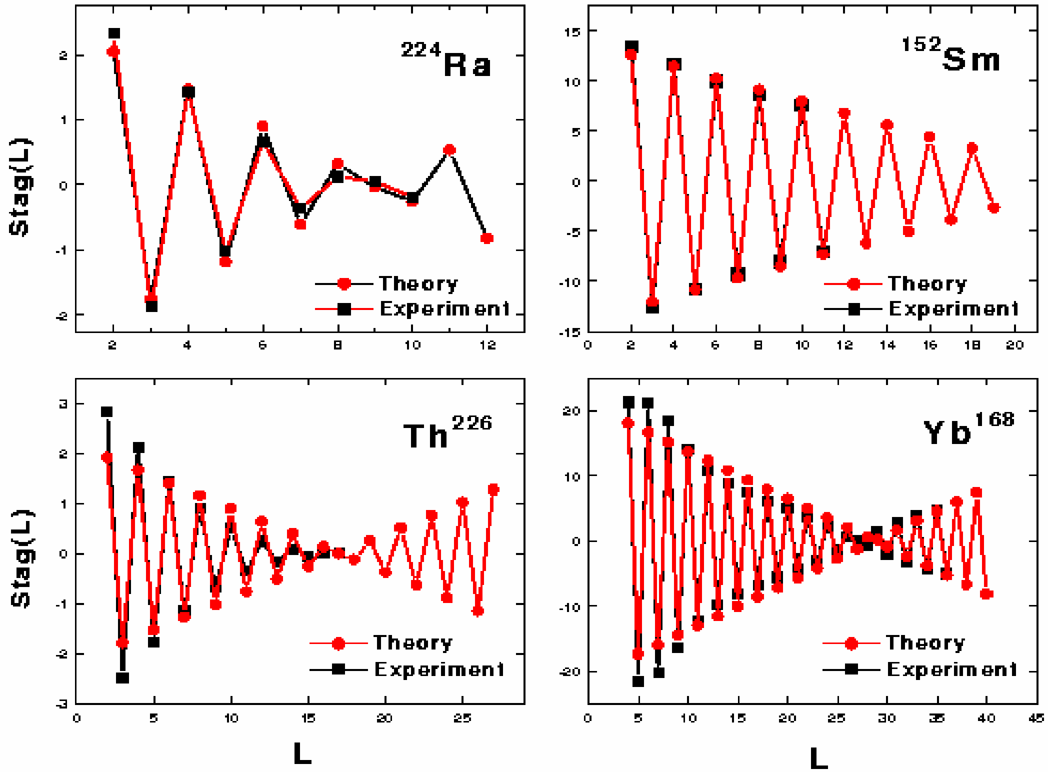
Odd-even staggering

The Staggering: **Odd-even staggering between ground and octupole bands is defined through following function:**

$$Stg(L)=6\Delta E(L) - 4\Delta E(L-1) - 4\Delta E(L+1) + \Delta E(L+2) + \Delta E(L-2)$$

Beat patterns

Crossing
of the two bands



6-dimensional Davidson Potential

- Takes into account rotation – vibration interaction in a many body system from the vibrational point of view
- Algebraically solvable – provides meaningful basis for more realistic approximations

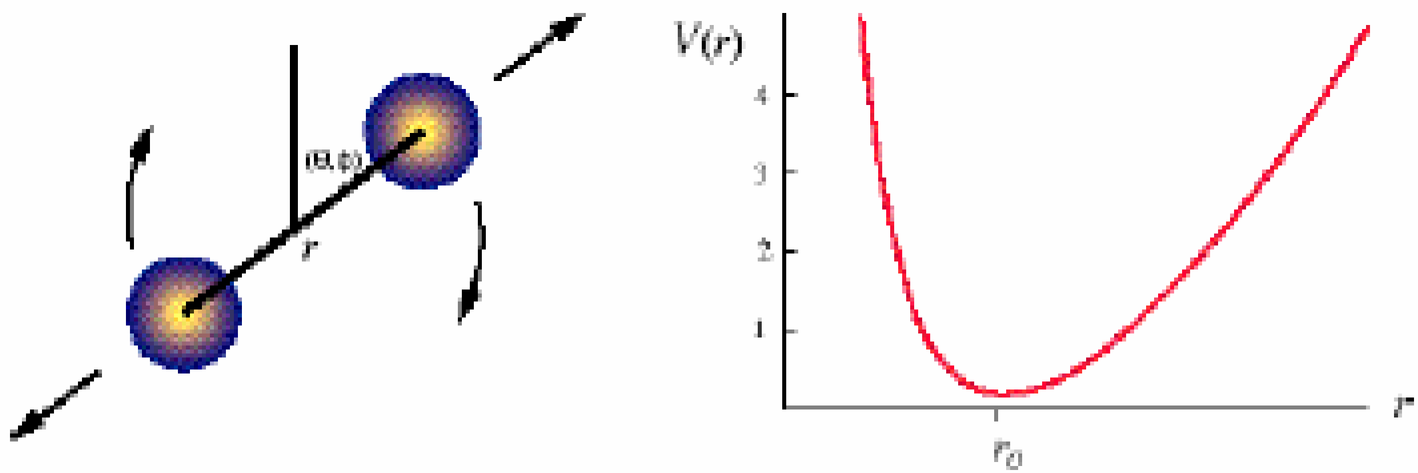


Figure 3: The Davidson potential for a diatomic molecule.

Algebraic constructions that survive the addition of Davidson potentials

The Collective Model

Sp (2,R) ≈SU(1,1)

A 5- dimensional Davidson Potential

The eigenstates can be classified according to the chain

$$\mathbf{SU(1,1) \times SO(5) \supset U(1) \times SO(5) \supset SO(3)}$$

υ $\quad$ $\mathbf{n}$ $\quad$ α $\quad$ $\mathbf{L}$

$| \, \mathbf{n} \, \upsilon \, \alpha \, \mathbf{L} \, >$

$$E_{n\upsilon} = [2n+1+ \ (\upsilon + 3/2)^2 \ + \varepsilon] \ \eta\omega$$

The spherical harmonic oscillator

$$\mathbf{SU(1,1) \times SO(3)}$$

$$E_{nl} = [2n+1+ \ (l+½)^2 \ + \varepsilon]$$

The 6-dimensional Davidson Potential is naturally contained in the IVBM trough the reduction chain

$$sp(12,R) \supset sp(2,R) \otimes \overset{\omega}{so(6)}$$

$$\cap$$

$$\underset{N}{su(1,1)} \otimes \underset{(\lambda,\mu)}{su(3)} \otimes \underset{\nu}{o(2)} \supset \underset{L}{so(3)}$$

Generators:

sp(2,R)

$$F = \sum_{\alpha=\pm 1/2} F^0(\alpha,\alpha) \qquad G = \sum G^0(\alpha,\alpha) \qquad A = \sum A^0(\alpha,\alpha)$$

$$\Lambda^L{}_M(\alpha,\beta) = A^L{}_M(\alpha,\beta) - (-1)^L A^L{}_M(\alpha,\beta);$$

L=0,1,2

so(6)

Completting the state labeling

The reduction along the chain

$$so(6) \supset su(3) \otimes o(2) \supset so(3)$$
$$\omega \quad (\lambda,\mu) \quad \nu \quad L$$

Generators:

$$X_M = i(A^2_M(p,n) - A^2_M(n,p);$$
$$\sqrt{2}\, Y_M = A^1_M(p,p) + A^1_M(n,n) = -L_M$$

→ su(3) ⊃ so(3)

$$M^0 = A^0(p,n) - A^0(n,p);$$

→ o(2)

Casimir invariants

$$\Lambda^2 = \sum_{L,\alpha,\beta} (-1)^M \Lambda^L_M(\alpha,\beta)\, \Lambda^L_{-M}(\alpha,\beta)$$

so(6)

$$C_2(o(2)) = M^2$$

$$C_2(su(3)) = \sum (-1)^M [X_M X_{-M} + Y_M Y_{-M}]$$

$$\Lambda^2 = 2C_2(su(3)) - 1/3 M^2$$

$$(\omega)_6 = \sum (\lambda,\mu)_3 \otimes (\nu)_2$$

Hamiltonian from the first and second order Casimir operators of the subalgebras from the chain.

$$H = aN + bN^2 + \alpha_6 \Lambda^2 + \alpha_1 M_2^{\,2} + \beta_3 L^2$$

The energy spectrum: The eigenvalues of H :

$$E(N, \omega, \nu, L) = aN + bN^2 + \alpha_6 \omega(\omega+4) + \alpha_2 \nu^2 + \beta_3 L(L+1)$$

The basis vectors:

$$| N\omega; (\lambda`,\mu`)\nu; KL >$$

Relations to the u(6) limit $T = \omega/2$ $T_0 = \nu/2$

Parity of the states $\pi = (-1)^T$

Excited β ($K^\pi = 0^+$) and γ ($K^\pi = 2^+$) bands

H_+

O(6)- limit

$\omega = N, N-2, \ldots, 0$; $\nu = \omega, \omega-2, \ldots, -\omega$;
$(\lambda = \omega+\nu/2,\ \mu = \omega-\nu/2)$

N \ ν	10	8	6	4	2	0	-2	-4	-6	-8	-10
0						(0,0)					
2					(2,0)	(1,1) (0,0)	(0,2)				
4				(4,0)	(3,1) (2,0)	(2,2) (1,1) (0,0)	(1,3) (0,2)	(0,4)			
6			(6,0)	(5,1) (4,0)	(4,2) (3,1) (2,0)	(3,3) (2,2) (1,1) (0,0)	(2,4) (1,3) (0,2)	(1,5) (0,4)	(0,6)		
8		(8,0)	(7,1) (6,0)	(6,2) (5,1) (4,0)	(5,3) (4,2) (3,1) (2,0)	(4,4) (3,3) (2,2) (1,1) (0,0)	(3,5) (2,4) (1,3) (0,2)	(2,6) (1,5) (0,4)	(1,7) (0,6)	(0,8)	
10	(10,0)	(9,1) (8,0)	(8,2) (7,1) (6,0)	(7,3) (6,2) (5,1) (4,0)	(6,4) (5,3) (4,2) (3,1) (2,0)	(5,5) (4,4) (3,3) (2,2) (1,1) (0,0)	(4,6) (3,5) (2,4) (1,3) (0,2)	(3,7) (2,6) (1,5) (0,4)	(2,8) (1,7) (0,6)	(1,9) (0,8)	(0,10)
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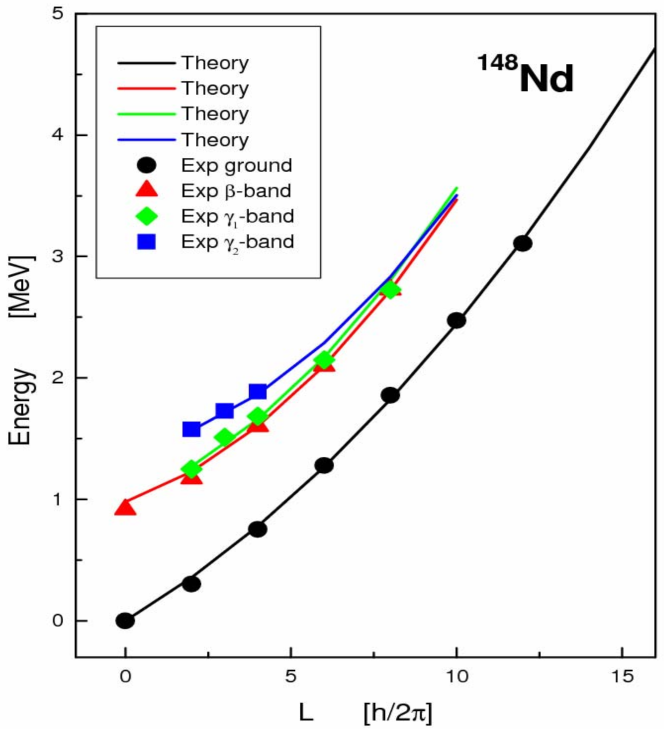
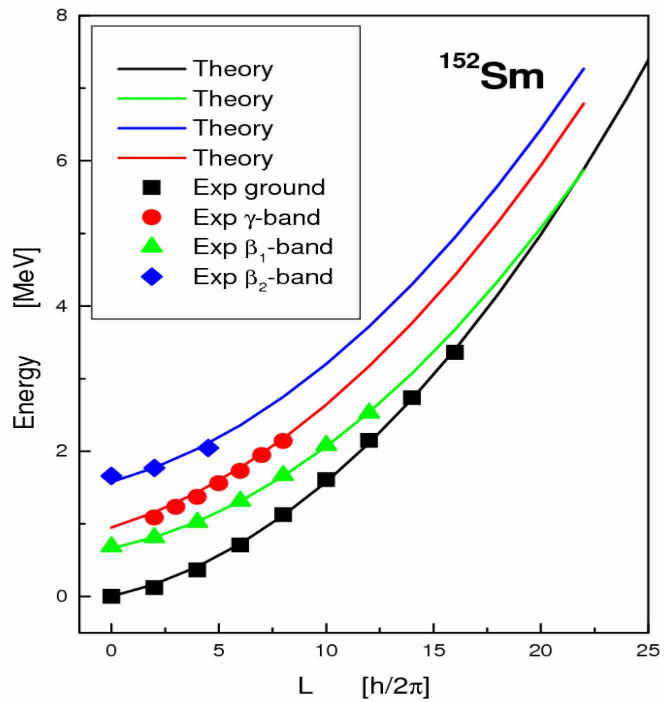
Ground band

Application to transitional nuclei

O(6)-limit of IBM



X(5)-critical point



rotational mode **vibrational** mode

$$E(L)=\beta^{`}L(L+1)+\eta^{`}L+\xi^{`}$$

Alternative form

 From the ***N*** $\Longleftrightarrow$ ***L*** Reduction rules and yrast condition

The reduction through the noncompact *sp(4,R)*

$$sp(12,R) \supset sp(4,R) \otimes so(3)$$

Generators:

$$F^0(\alpha,\beta),\ G^0(\alpha,\beta),\ A^0(\alpha,\beta)$$
$$\alpha,\beta = \pm 1/2$$

$$L_M = -\sqrt{2} \sum_{\alpha} A^1{}_M(\alpha,\alpha)$$

The quantum numbers $L = N_{min}/2$ of the algebra ***so(3)*** characterize the representations of ***sp(4,R)***.

Describes sequences of states with the same angular momentum

Realizations of the unitary subalgebras of $sp(4,R)$

$sp(4,R)$

$u(1,1)$ — noncompact

$u(2) \supset u(1) \otimes su(2)$ — compact

$$T_{+1} = \sqrt{3/2}\, A^0(p,n)$$

$$T_{-1} = -\sqrt{3/2}\, A^0(n,p)$$

$$T_0 = -\sqrt{3/2}\,[\, A^0(p,p) - A^0(n,n)]$$

$$N = -\sqrt{3}\,[\, A^0(p,p) + A^0(n,n)]$$

$$N = N_p + N_n$$

$$[T_{+1}, T_{-1}] = 2T_0$$

$$[N, T_{\pm 1}] = 0, \quad [N, T_0] = 0$$

The correspondence to the U(6) limit

$$\begin{array}{ccccc} sp(12,R) & \supset & sp(4,R) & \otimes & so(3) \\ \cup & & \cup & & \cap \\ u(6) & \supset & u(2) & \otimes & su(3) \end{array}$$

allows the use of the same Hamiltonian

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2$$

$$L = N_{min}/2$$

The parabolic dependence on N

Basis states of sp(4,R):

$$| T, T_0 > = N \left(F^0_M(\alpha,\beta)\right)^{r/2} | \mathrm{lwL} >$$

r=0,2,4,6,…..

Reduction of the *sp*(4,R) irreps [L] to an infinite number of *su*(2) irreps

$$[L]_2 \otimes \left(\bigoplus_{i=0}^{\langle \frac{r}{2} \rangle} [r_1 - 2i, i] \right)$$

TABLE I: $L = 0$

… $T=5$	$T=4$	$T=3$	$T=2$	$T=1$	$T=0$	T/N
					$[0]_2(0,0)$	$N=0$
				$[2]_2(2,0)$		$N=2$
			$[4]_2(4,0)$		$[0]_2(0,2)$	$N=4$
		$[6]_2(6,0)$		$[2]_2(2,2)$		$N=6$
…	$[8]_2(8,0)$		$[4]_2(4,2)$		$[0]_2(0,4)$	$N=8$
$[10]_2(10,0)$		$[6]_2(6,2)$		$[2]_2(2,4)$		$N=10$
…	…	…	…	…	…	…

Classification scheme

$[k]_2$ labels the ***su(2)*** representations.

The **columns** are defined by the quantum number $T = k/2$ and the **rows** by the eigenvalues of $N = k_{max} + r$, $r = 0,2,4,6\ldots$

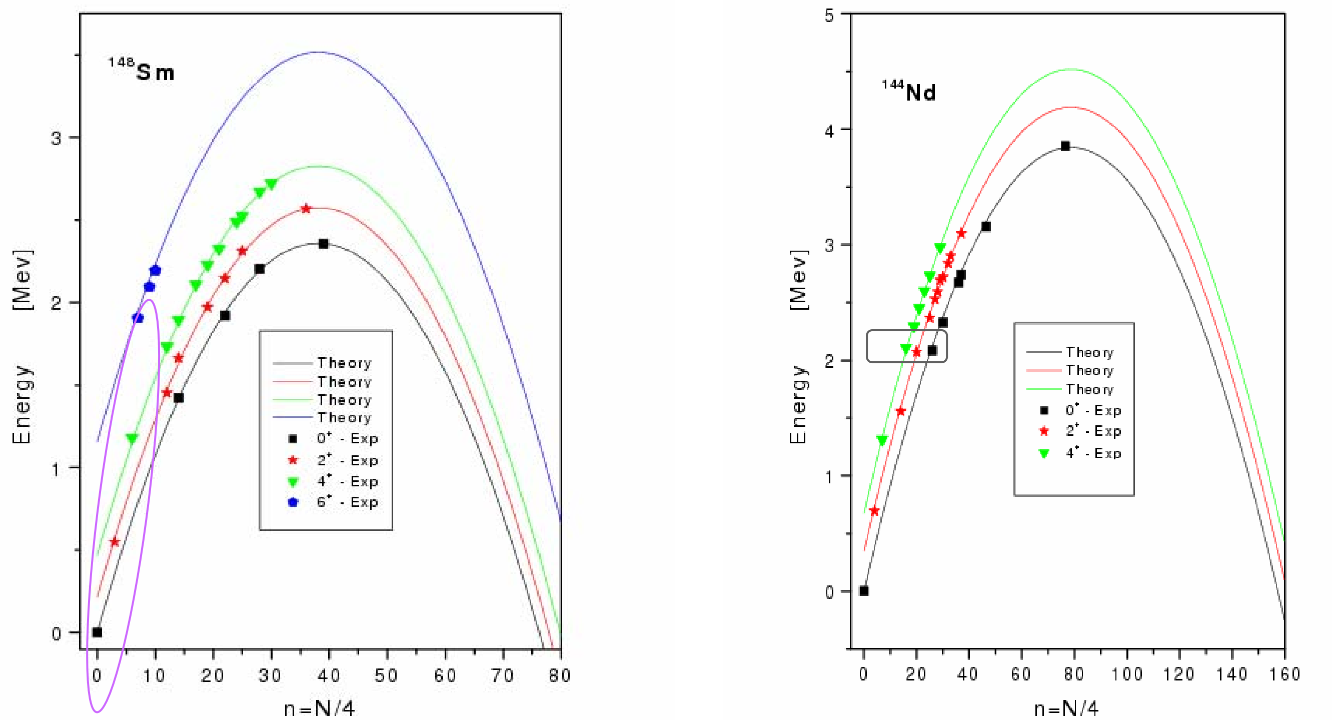
TABLE II: $L = 2$

$\ldots$	$T=5$	$T=4$	$T=3$	$T=2$	$T=1$	$T=0$	T/N
					$[2]_2(2,0)$		$N=2$
				$[4]_2(4,0)$	$[2]_2(2,1)$	$[0]_2(0,2)$	$N=4$
			$[6]_2(6,0)$	$[4]_2(4,1)$	$2\times[2]_2(2,2)$	$[0]_2(0,3)$	$N=6$
		$[8](8,0)$	$[6]_2(6,1)$	$2\times[4]_2(4,2)$	$2\times[2]_2(2,3)$	$[0]_2(0,4)$	$N=8$
	$[10]_2(10,0)$	$[8]_2(8,1)$	$2\times[6]_2(6,2)$	$2\times[4]_2(4,3)$	$2\times[2]_2(2,4)$	$[0]_2(0,5)$	$N=10$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots.$	$\ldots$	$\ldots$	$\ldots$

$\lambda = k;\ \mu = N - k/2$ Relation to the su(3) irreps

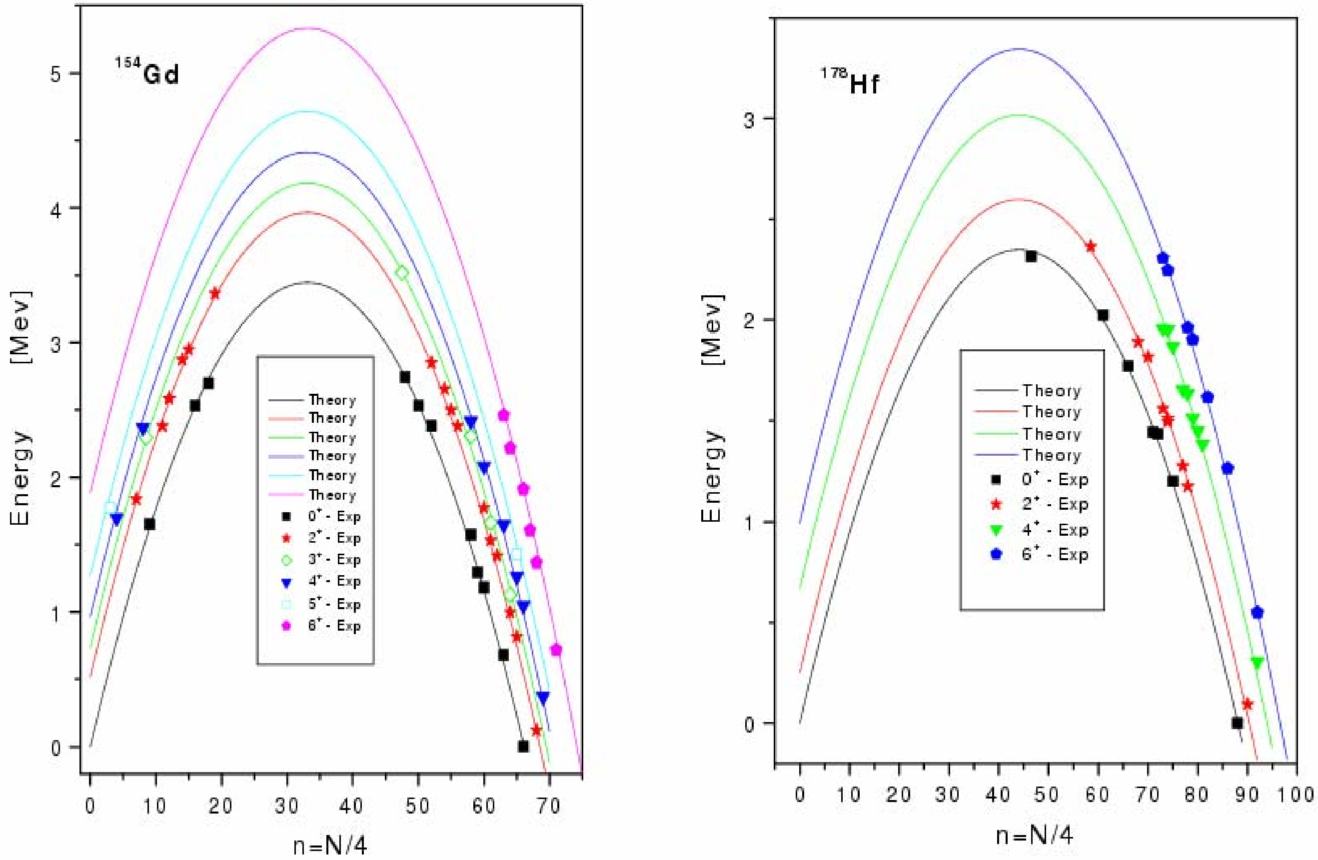
Vibrational spectra of nearly spherical nuclei

ΔT →big ; for a given ***L***, ***n*** increases with the increase of ***E***, left side of the parabolas, bands and the triplet of ***0^+ ,2^+ ,4^+***



Rotational spectra of well deformed nuclei

n decreases with the increase of ***E***, right side of the parabolas



Conclusion

U(6) limit

H. Ganev, V. P. Garistov, and A. I. Georgieva, Phys. Rev. **C 69**, 014305 (2004)

- Negative parity states
- Yrast bands
- Mixing of rotational and vibrational modes
- Staggering behavior

6-Dimentional Davidson potential

H. G. Ganev, A. I. Georgieva, and J. P. Draayer, Phys. Rev. **C 71**, 054317 (2005)

- Mixing of rotational and vibrational modes – transitional and critical point symmetries
- The role of the proton-neutron interactions
- Consideration of yrast and non-yrast collective bands – the importance of N for the band head states

Conclusion

Sets of states with fixed L

H. G. Ganev, V. P. Garistov, A. I. Georgieva, and J. P. Draayer, Phys. Rev. **C 70**, 054317 (2004)

- Distinguishes rotational and vibrational modes
- Parabolic behavior of the states
- Band head configurations

Relationships between the subgroups from different reduction chains

- The pseudospin and its third projection
- The exactly solvable Hamiltonians
- Tensor structure of the physical operators

Thank you