Description of mixed-mode dynamics within the IVBM:

II. Orthosymplectic extension – the odd-even nuclei

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Outline

- Introduction
- Transition probabilities. Application
- The Inclusion of Spin
- Orthosymplectic extension. Algebraic structure
- Representations
- Application of the new dynamical symmetry
- Conclusions

Tensor properties

$$Sp(12,R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes U(1)$$

$$F^{[2]_6}_{[2]_3[2]_2} {}^{LM}_{11} = \sum_{m,k} C^{LM}_{1m1k} p^\dagger_m p^\dagger_k,$$

$$F^{[2]_6}_{[2]_3[2]_2} {}^{LM}_{1-1} = \sum_{m,k} C^{LM}_{1m1k} n^\dagger_m n^\dagger_k$$

$$F^{[2]_6}_{[2]_3[2]_2} {}^{LM}_{10} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{LM}_{1m1k} (p^\dagger_m n^\dagger_k - n^\dagger_m p^\dagger_k)$$

$$F^{[2]_6}_{[1,1]_3[0]_2} {}^{LM}_{00} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{LM}_{1m1k} (p^\dagger_m n^\dagger_k + n^\dagger_m p^\dagger_k)$$

$$A^{[1-1]_6}_{[210]_3[0]_2} {}^{1M}_{00} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{1M}_{1m1k} (p^\dagger_m p_k + n^\dagger_m n_k)$$

$$A^{[1-1]_6}_{[210]_3[0]_2} {}^{2M}_{00} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{2M}_{1m1k} (p^\dagger_m p_k + n^\dagger_m n_k)$$

H. G. Ganev and A. I. Georgieva,
Phys. Rev. C (accepted) [aeXiv:nucl-th/0708.0327].

Tensor product of two operators

$$T^{([\chi_1]_6[\chi_2]_6)}{}^{\omega[\chi]_6}_{[\lambda]_3[2T]_2}{}^{LM}_{TT_0} =$$

$$\sum T^{[\chi_1]_6}_{[\lambda_1]_3[2T_1]_2}{}^{L_1M_1}_{T_1(T_0)_1} T^{[\chi_2]_6}_{[\lambda_2]_3[2T_2]_2}{}^{L_2M_2}_{T_2(T_0)_2} \times$$

$$C^{[\chi_1]_6}_{[\lambda_1]_3[T_1]_2}{}^{[\chi_2]_6}_{[\lambda_2]_3[T_2]_2}{}^{\omega[\chi]_6}_{[\lambda]_3[2T]_2} C^{[\lambda_1]_3}_{(L_1)_3}{}^{[\lambda_2]_3}_{(L_2)_3}{}^{[\lambda]_3}_{(L)_3} \times$$

$$C^{L_1}_{M_1}{}^{L_2}_{M_2}{}^{L}_{M} C^{T_1}_{(T_0)_1}{}^{T_2}_{(T_0)_2}{}^{T}_{T_0}$$

TABLE I. Tensor products of two raising operators.

$[2]_6$ $[\lambda_1]_3[2T_1]_2$	$[2]_6$ $[\lambda_2]_3[2T_1]_2$	$[4]_6$ $[\lambda]_3[2T]_2$	$O(3)$ K; L	$U(2)$ T	$U(1)$ T_0
$(2,0)[2]_2$	$(2,0)[2]_2$	$(4,0)[4]_2$	0;0, 2, 4	2	$0,\pm1,\pm2$
$(2,0)[2]_2$	$(2,0)[2]_2$	$(2,1)[2]_2$	1;1, 2, 3	1	$0,\pm1$
$(2,0)[2]_2$	$(2,0)[2]_2$	$(0,2)[0]_2$	0;0, 2	0	0
$(2,0)[2]_2$	$(0,1)[0]_2$	$(2,1)[2]_2$	1;1, 2, 3	1	$0,\pm1$
$(0,1)[0]_2$	$(0,1)[0]_2$	$(0,2)[0]_2$	0;0, 2	0	0

Symplectic basis

$$Sp(12,R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes (U(1) \otimes U(1))$$

$$| [N]_6;(\lambda,\mu);KLM;TT_0 \rangle = [F\times\ldots\times F]^{[\chi]_6}{}_{[\lambda]_3[2T]_2}{}^{LM}{}_{TT_0} | \Omega \rangle$$

$$G^{[\chi]_6}{}_{[\lambda]_3[2T]_2}{}^{LM}{}_{TT_0} | \Omega \rangle = 0 , \qquad | \Omega \rangle - \text{LWS}$$

$$| \Omega \rangle = | 0 \rangle \quad \text{or} \quad | \Omega \rangle = u^{+}_{k}(\alpha) | 0 \rangle$$

$$[\chi]_6 = [N]_6$$

Symplectic basis

N \ T_3	5	4	3	2	1	0	−1	−2	−3	−4	−5
0						(0,0)					
2					(2,0)	(2,0) (0,1)	(2,0)				
4				(4,0)	(4,0) (2,1)	(4,0) (2,1) (0,2)	(4,0) (2,1)	(4,0)			
6			(6,0)	(6,0) (4,1)	(6,0) (4,1) (2,2)	(6,0) (4,1) (2,2) (0,3)	(6,0) (4,1) (2,2)	(6,0) (4,1)	(6,0)		
8		(8,0)	(8,0) (6,1)	(8,0) (6,1) (4,2)	(8,0) (6,1) (4,2) (2,3)	(8,0) (6,1) (4,2) (2,3) (0,4)	(8,0) (6,1) (4,2) (2,3)	(8,0) (6,1) (4,2)	(8,0) (6,1)	(8,0)	
10	(10,0)	(10,0) (8,1)	(10,0) (8,1) (6,2)	(10,0) (8,1) (6,2) (4,3)	(10,0) (8,1) (6,2) (4,3) (2,4)	(10,0) (8,1) (6,2) (4,3) (2,4) (0,5)	(10,0) (8,1) (6,2) (4,3) (2,4)	(10,0) (8,1) (6,2) (4,3)	(10,0) (8,1) (6,2)	(10,0) (8,1)	(10,0)
- - --	- - -	- - --	- - - -	- - -- -	- - - -	- - - -	- - - -	- - - -	- - - -	- -- -	- --- -

Matrix elements

Wigner–Eckart theorem

$$\langle [N']\,(\lambda',\mu');K'L'M';T'T_0'|T^{[\chi]_6\quad lm}_{[\sigma]_3[2t]_2\quad tt_0}|[N]\,(\lambda,\mu);KLM;TT_0\rangle$$

$$=\langle [N'](\lambda',\mu');K'L'||T^{[\chi]_6\quad lm}_{[\sigma]_3[2t]_2\quad tt_0}||[N](\lambda,\mu);KL\rangle C^{L'M'}_{LMlm}C^{T'T_0'}_{TT_0tt_0}$$

Reduced matrix elements

$$\langle [N']\,(\lambda',\mu');K'L'||T^{[\chi]_6\quad lm}_{[\sigma]_3[2t]_2\quad tt_0}||[N]\,(\lambda,\mu);KL\rangle$$

$$=\langle [N']|||T^{[\chi]_6}_{[\sigma]_3[2t]_2}|||[N]\rangle C^{[N]_6\quad [\chi]_6\quad [N']_6}_{(\lambda,\mu)[2T]_2\quad [\sigma]_3[2t]_2\quad (\lambda',\mu')[2T']_2} C^{(\lambda,\mu)\quad [\lambda]_3\quad (\lambda',\mu')}_{KL\quad k(l)_3\quad K'L'}$$

Transition probabilities

Transition operator

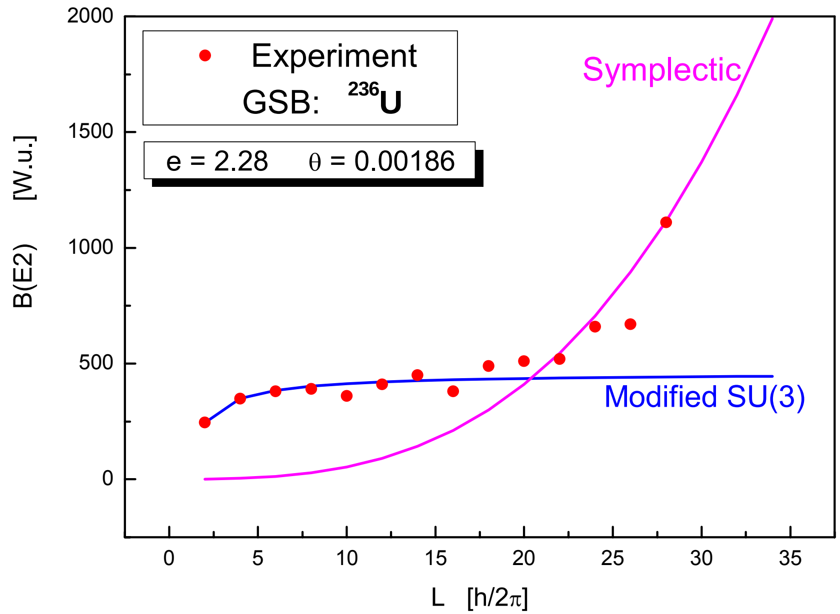
$$T^{E2} = e\left[A^{[1-1]_6}_{[210]_3[0]_2}\ {}^{20}_{00} + \theta([F\times F]^{[4]_6}_{(0,2)[0]_2}\ {}^{20}_{00} + [G\times G]^{[-4]_6}_{(2,0)[0]_2}\ {}^{20}_{00})\right]$$

SU(3) term

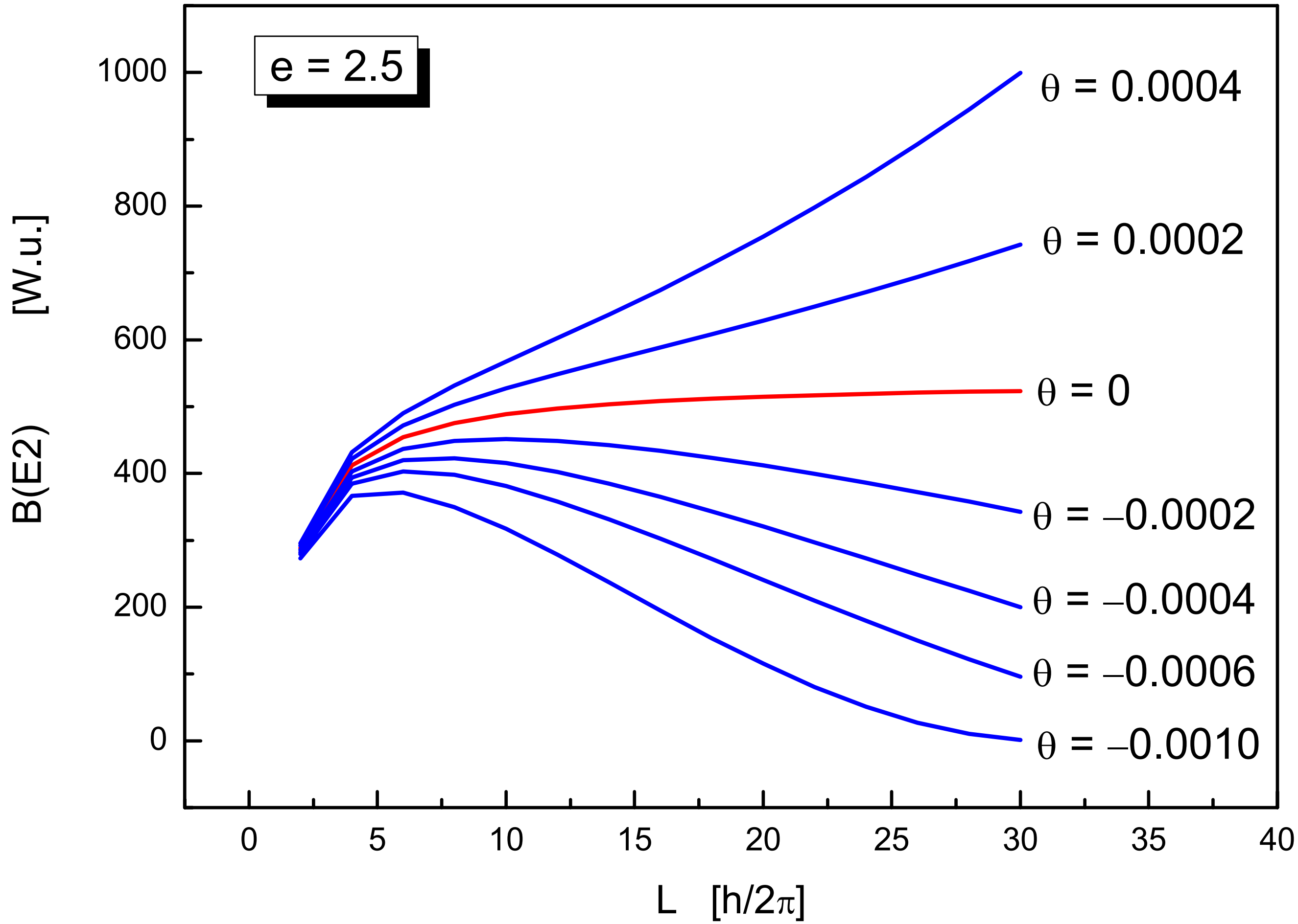
Symplectic term

Transition rates

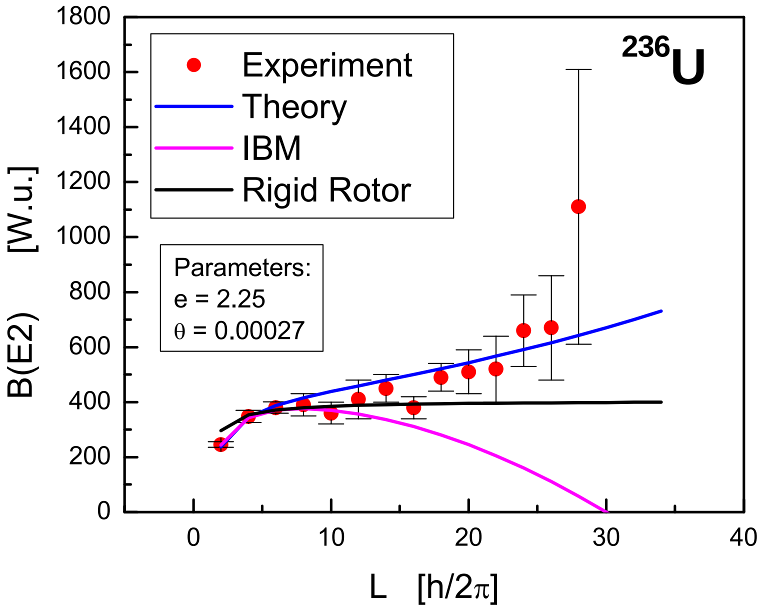
$$B(E2; L_i \to L_f) = \frac{1}{2L_i+1}\,|\,\langle\ f \,\|\, T^{E2} \,\|\, i\ \rangle\,|^2$$

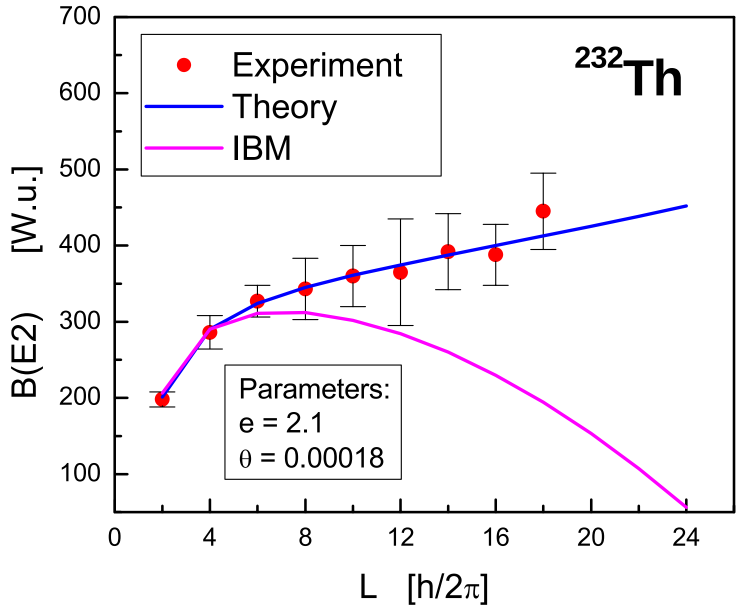


Transition probabilities

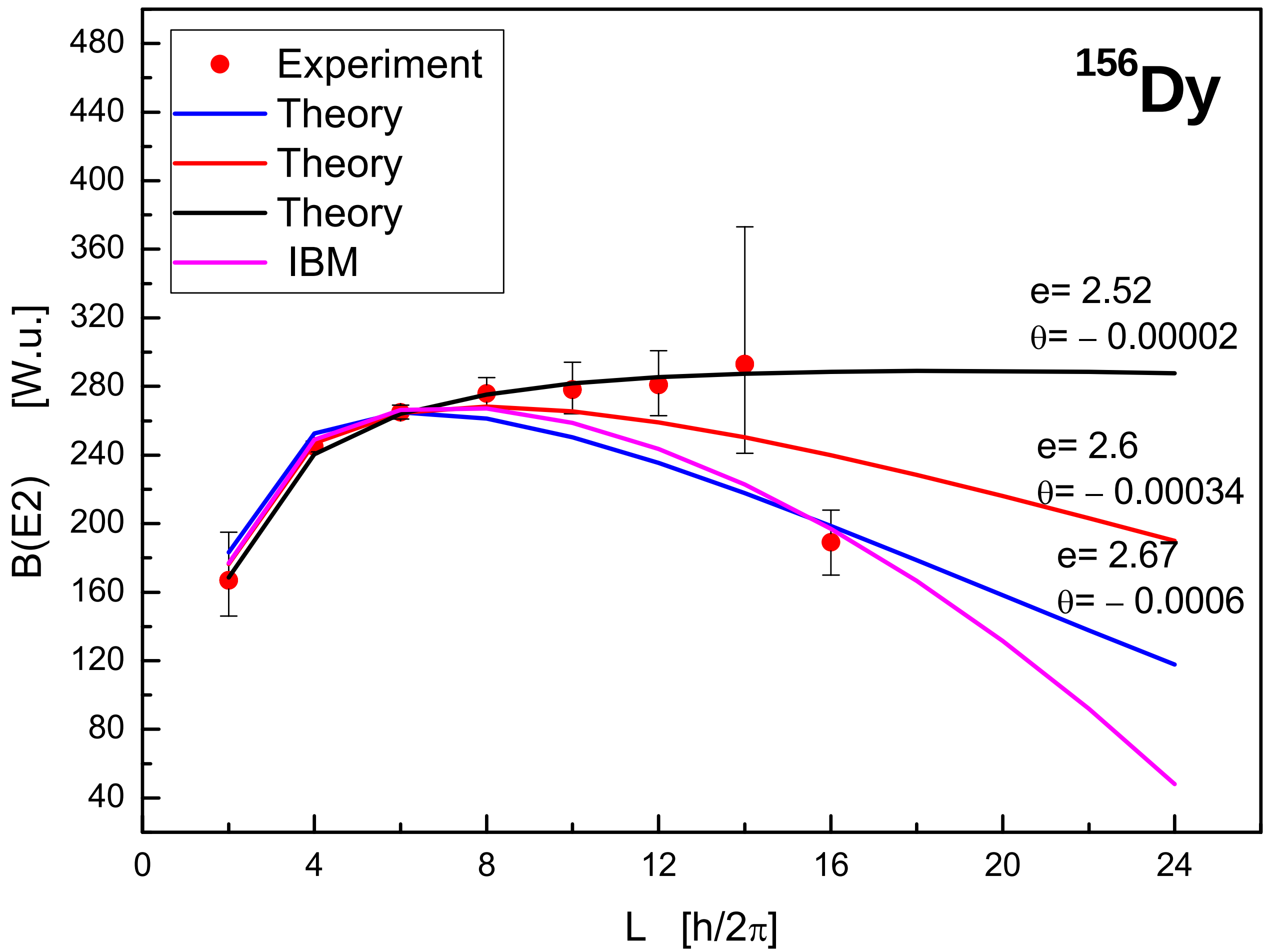


Transition probabilities





Transition probabilities



Odd mass nuclei

The Inclusion of Spin

- Consider a particle with spin S = 1/2 $\hbar$

Fermion operators:

$$\{a_i^\dagger, a_j^\dagger\} = \{a_i, a_j\} = 0,$$
$$\{a_i, a_j^\dagger\} = \delta_{ij}.$$

Generators

$$f_{ij} = a_i^\dagger a_j^\dagger,$$
$$g_{ij} = a_i a_j; \quad i \neq j,$$
$$C_{ij} = (a_i^\dagger a_j - a_j a_i^\dagger)/2$$

→ so(4) algebra

- The group of the spin - $SU^F(2)$
- Consider Core + particle picture
- Embedding $SU^F(2) \subset SO(4)$

Orthosymplectic extension

$$OSp(4/12,R) \supset SO^F(4) \otimes Sp^B(12,R)$$

$$\supset SU^F(2) \otimes U^B(6)$$

$$\quad S \qquad\qquad N$$

$$\supset SU^F(2) \otimes SU^B(3) \otimes U^B(2)$$

$$\qquad (\lambda,\mu) \qquad (N,T)$$

$$\supset SU^F(2) \otimes SO^B(3) \otimes U^B(1)$$

$$\qquad L \qquad\qquad T_0$$

$$\supset \mathrm{Spin}^{BF}(3) \supset \mathrm{Spin}^{BF}(2)$$

$$\quad J \qquad\qquad J_0$$

osp(4/12,R) Lie algebra

Superoscillators:

$$\xi_A^\dagger = \begin{pmatrix} u^\dagger{}_k(\alpha) \\ a_j^\dagger \end{pmatrix}, \qquad k = 0, \pm 1;\ \alpha = \pm 1/2; \quad j = \pm 1/2$$

➡ $u(6/2)$

$$\xi_A = (\xi_A^\dagger)^\dagger$$

Generators:

$$F_{AB} = \xi_A^\dagger \xi_B^\dagger$$
$$G_{AB} = \xi_A \xi_B$$
$$A_{AB} = \xi_A^\dagger \xi_B + (-1)^{\deg A \deg B} \xi_B \xi_A^\dagger$$

where *deg A* = 0 or 1 depending on whether *A* is a bosonic or a fermionic index

Limiting cases:

$$(A = k, \alpha) \longleftrightarrow Sp(12, R),$$
$$(A = j) \longleftrightarrow SO(4).$$

Representations of osp(4/12,R)

Jordan decomposition

$$n = n_- \oplus n_0 \oplus n_+$$ → u(6/2)

G_{AB} $\quad$ A_{AB} $\quad$ F_{AB}

Lowest weight state (LWS):

$$|\,\Omega\,\rangle$$

$$G_{AB}\,|\,\Omega\,\rangle = 0.$$

$$R = \{|\,\Omega\,\rangle \oplus F_{AB}\,|\,\Omega\,\rangle \oplus F_{AB}F_{CD}\,|\,\Omega\,\rangle \oplus \;\ldots\}$$ → Super Fock

U(6/2) content

$$F_{AB} \approx \boxslash\boxslash\,,$$

$$F_{AB}F_{CD} \approx (\boxslash\boxslash \otimes \boxslash\boxslash)_S = \boxslash\boxslash\boxslash\boxslash\,,$$

$$\vdots$$

$$\underbrace{F_{AB}\ldots F_{CD}}_{k\ \ times} \approx \underbrace{(\boxslash\boxslash \otimes \ldots \otimes \boxslash\boxslash)}_{k\ \ times}{}_S = \underbrace{\boxslash\boxslash\ldots\boxslash\boxslash}_{k\ \ times}$$

Representations of osp(4/12,R)

The irreducible LWS of osp(4/12):

1) $| \Omega \rangle = | 0 \rangle_{SF}$

2) $| \Omega \rangle = \xi^{\dagger}_{A} | 0 \rangle_{SF}$

Even subalgebra: sp(12,R) ⊕ so(4)

Lowest weight vectors:

$$| LWS \rangle = | \Omega_{LWS} \rangle_B \otimes | \Omega_{WLS} \rangle_F$$

$$[N]_6 \otimes (r_1, r_2)_{GZ} \qquad [N]_6 \qquad (r_1, r_2)_{GZ}$$

The IR of osp(4/12,R) with the lowest weight vector $| 0 \rangle_{SF}$ has lowest weight vectors of sp(12,R) ⊕ so(4):

(1) $| 0 \rangle$ → Even-even nuclei

(2) $u^{\dagger}_{k}(\alpha) a^{\dagger}_{j} | 0 \rangle$

The IR of osp(4/12,R) with the lowest weight vector $\xi^{+}_{A} | 0 \rangle_{SF}$ has lowest weight vectors of sp(12,R) ⊕ so(4):

(3) $u^{\dagger}_{k}(\alpha) | 0 \rangle \approx (\Box, 1)$

(4) $a^{\dagger}_{j} | 0 \rangle \approx (1, \Box)$ → Odd–A nuclei

The energy spectrum

The Hamiltonian

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + a_1 T_0^{\,2} + \gamma J^2 + \xi J_0^{\,2}$$

The Basis

$$|\, [N]_6; (N,T); KL; S; JJ_0; T_0 \,\rangle$$

The Energies

$$E([N]_6; (N,T); KL; S; JJ_0; T_0) =$$

$$= aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + \alpha_1 T_0^2 +$$

$$+ \gamma J(J+1) + \xi J_0^{\,2}$$

Basis states

N	T	SU(3)	K L	J	K_J
0	0	(0,0)	K=0 L=0	1/2	1/2
2	1	(2,0)	K=0 L=0,2	1/2; 3/2, 5/2	1/2
	0	(0,1)	K=0 L=1	1/2, 3/2	1/2
4	2	(4,0)	K=0 L=0,2,4	1/2; 3/2, 5/2; 7/2, 9/2	1/2
	1	(2,1)	K=1 L=1,2,3	1/2, 3/2; 3/2, 5/2; 5/2,7/2	1/2;3/2
	0	(0,2)	K=0 L=0,2	1/2; 3/2, 5/2	1/2
6	3	(6,0)	K=0 L=0,2,4,6	1/2; 3/2, 5/2; 7/2, 9/2; 11/2, 13/2	1/2
	2	(4,1)	K=1 L=1,2,3,4,5	1/2, 3/2; 3/2, 5/2; 5/2, 7/2; 7/2, 9/2; 9/2, 11/2	1/2;3/2
	1	(2,2)	K=2 L=2,3,4	3/2, 5/2; 5/2, 7/2; 7/2, 9/2	3/2;5/2
			K=0 L=0,2	1/2; 3/2, 5/2	1/2
	0	(0,3)	K=0 L=1,3	1/2, 3/2; 5/2, 7/2	1/2
8	4	(8,0)	K=0 L=0,2,4,6,8	1/2; 3/2, 5/2; 7/2, 9/2; 11/2, 13/2; 15/2, 17/2	1/2
	3	(6,1)	K=1 L=1,2,3,4,5,6,7	1/2,3/2; 3/2,5/2; 5/2,7/2; 7/2, 9/2; 9/2,11/2; 11/2, 13/2; 13/2, 15/2	1/2;3/2
	2	(4,2)	K=2 L=2,3,4,5,6	3/2, 5/2; 5/2, 7/2; 7/2, 9/2; 9/2, 11/2; 11/2, 13/2	3/2;5/2
			K=0 L=0,2,4	1/2; 3/2, 5/2; 7/2, 9/2	1/2
	1	(2,3)	K=2 L=2,3,4,5	3/2, 5/2; 5/2, 7/2; 7/2, 9/2; 9/2, 11/2	3/2;5/2
			K=0 L=1,3	1/2, 3/2; 5/2, 7/2	1/2
	0	(0,4)	K=0 L=0,2,4	1/2; 3/2, 5/2; 7/2, 9/2	1/2
⋮	⋮	⋮	⋮ ⋮	⋮	⋮

Application

- Parity: $\pi=(-1)^{T}$

  This allow us to describe both positive and negative parity states.

- Algebraic definition for yrast states:

$$E = \min \quad \text{– with respect to } N$$

$$\Rightarrow \quad N \leftrightarrow J$$

- GSB: $K=1/2^{+} \quad \rightarrow \quad N = 2J - 1$

  $K=3/2^{-} \quad \rightarrow \quad N = 2J + 3$

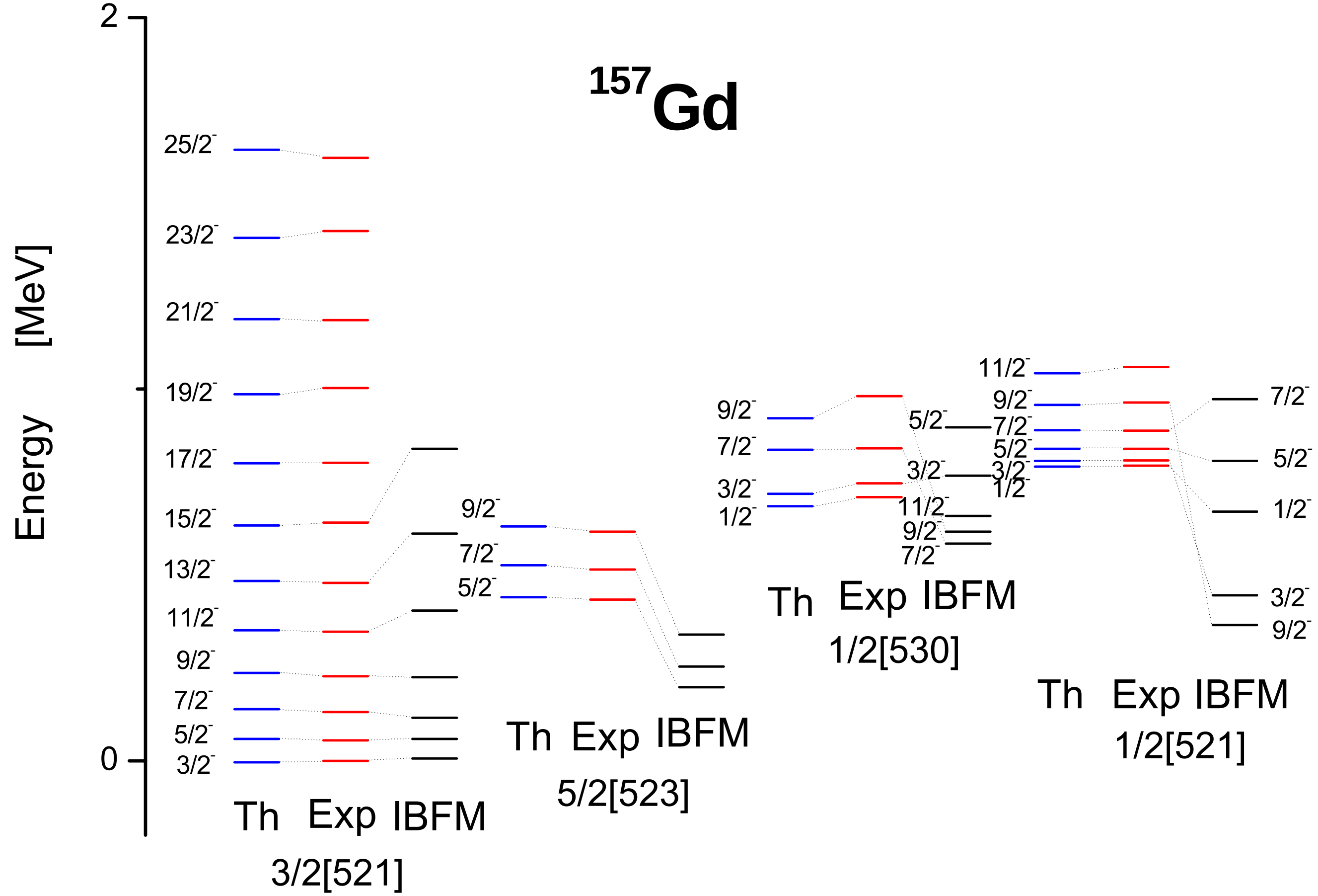
- Excited bands $\Rightarrow$ Band head structure: N_{ini}

The energy spectrum

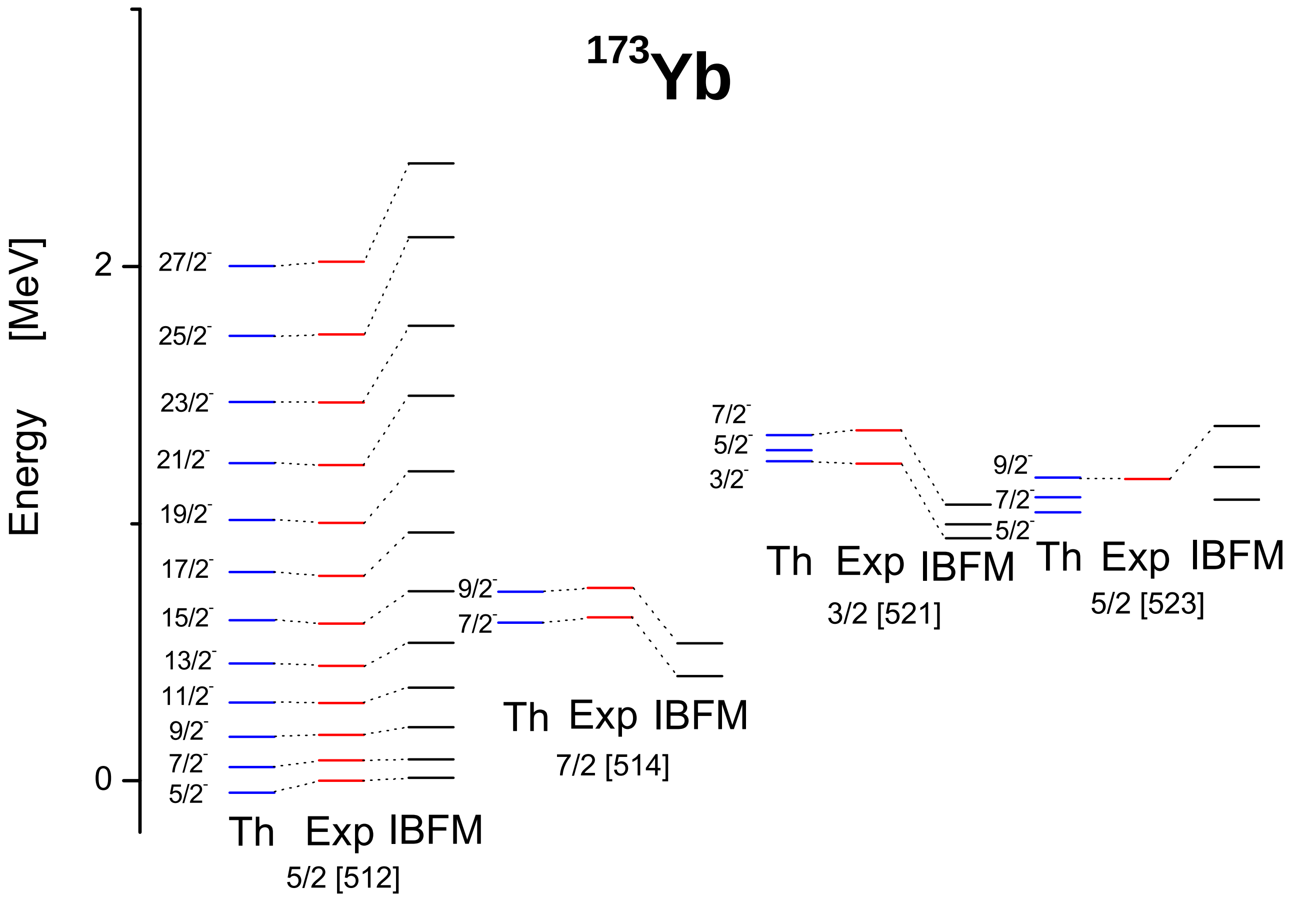
N	T	SU(3)	K	L	J	K_J
0	0	(0,0)	K=0	L=0	1/2	1/2
2	1	(2,0)	K=0	L=0,2	1/2; 3/2, 5/2	1/2
	0	(0,1)	K=0	L=1	1/2, 3/2	1/2
4	2	(4,0)	K=0	L=0,2,4	1/2; 3/2, 5/2; 7/2, 9/2	1/2
	1	(2,1)	K=1	L=1,2,3	1/2, 3/2; 3/2, 5/2; 5/2, 7/2	1/2;3/2
	0	(0,2)	K=0	L=0,2	1/2; 3/2, 5/2	1/2
6	3	(6,0)	K=0	L=0,2,4,6	1/2; 3/2, 5/2; 7/2, 9/2; 11/2, 13/2	1/2
	2	(4,1)	K=1	L=1,2,3,4,5	1/2, 3/2; 3/2, 5/2; 5/2, 7/2; 7/2, 9/2; 9/2, 11/2	1/2;3/2
	1	(2,2)	K=2	L=2,3,4	3/2, 5/2; 5/2, 7/2; 7/2, 9/2	3/2;5/2
			K=0	L=0,2	1/2; 3/2, 5/2	1/2
	0	(0,3)	K=0	L=1,3	1/2, 3/2; 5/2, 7/2	1/2
8	4	(8,0)	K=0	L=0,2,4,6,8	1/2; 3/2, 5/2; 7/2, 9/2; 11/2, 13/2; 15/2, 17/2	1/2
	3	(6,1)	K=1	L=1,2,3,4,5,6,7	1/2,3/2; 3/2,5/2; 5/2,7/2; 7/2, 9/2; 9/2,11/2; 11/2, 13/2; 13/2, 15/2	1/2;3/2
	2	(4,2)	K=2	L=2,3,4,5,6	3/2, 5/2; 5/2, 7/2; 7/2, 9/2; 9/2, 11/2; 11/2, 13/2	3/2;5/2
			K=0	L=0,2,4	1/2; 3/2, 5/2; 7/2, 9/2	1/2
	1	(2,3)	K=2	L=2,3,4,5	3/2, 5/2; 5/2, 7/2; 7/2, 9/2; 9/2, 11/2	3/2;5/2
			K=0	L=1,3	1/2, 3/2; 5/2, 7/2	1/2
	0	(0,4)	K=0	L=0,2,4	1/2; 3/2, 5/2; 7/2, 9/2	1/2
⋮	⋮	⋮	⋮	⋮	⋮	⋮

GSB: $K^{\pi}=1/2^{+}$

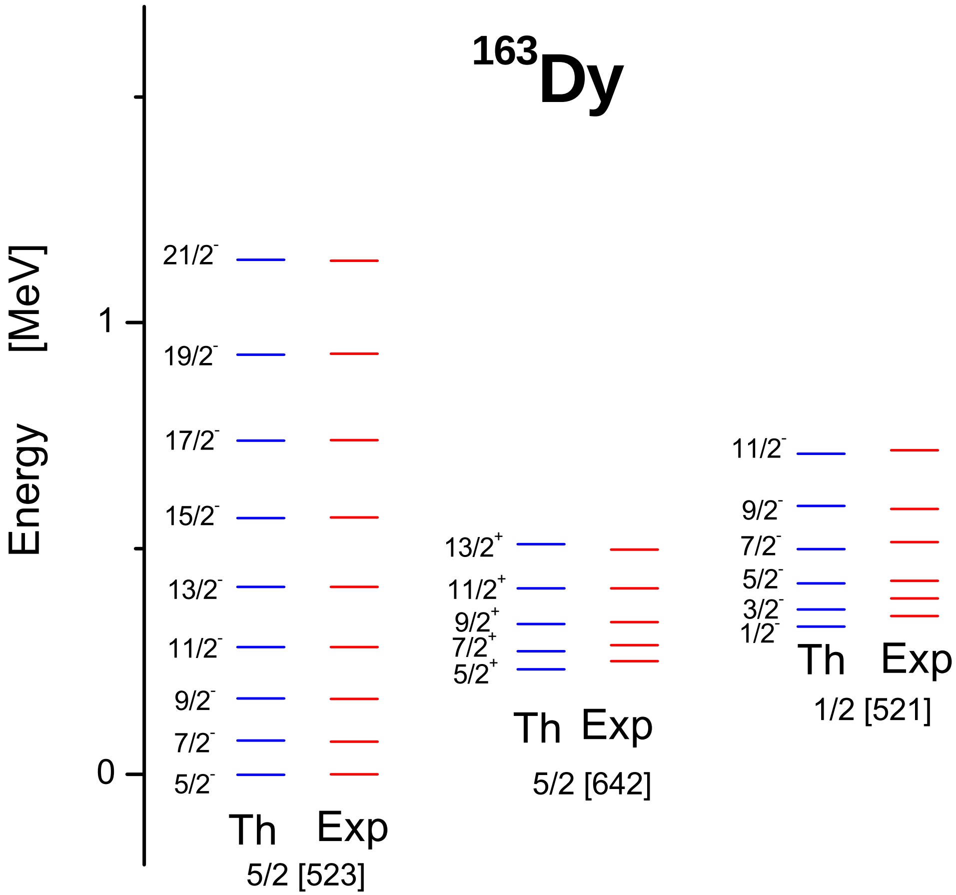
The energy spectrum



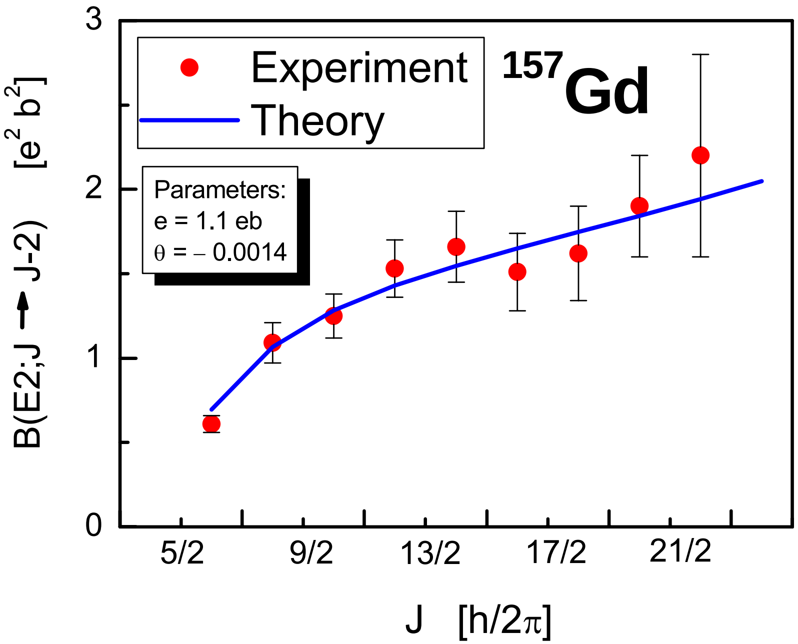
The energy spectrum

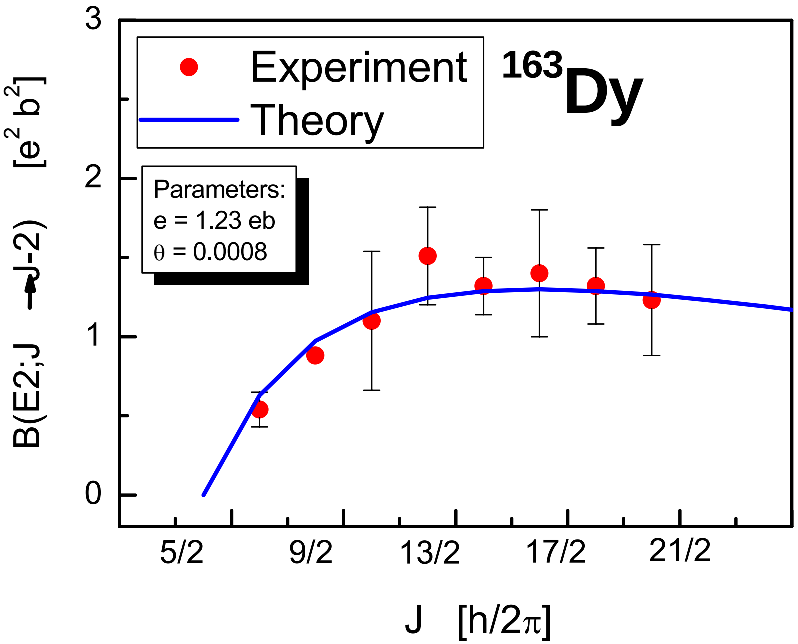


The energy spectrum



Transition probabilities





Conclusions

- The applicability of the model is further confirmed by the reproduction of the B(E2) behavior of the transition probabilities in the GSB for some even-even and odd - even nuclei. Analyzing the terms in the transition operator, the important role of the symplectic extension is revealed.
- The mixing of the different collective modes within the symplectic and orthosymplectic structures remains the main reason for the good reproduction of the experimental data.
- The model can be further used for the description and systematics of other collective bands.
- Critical phase/shape phenomena can be analyzed within the IVBM.
- Orthosymplectic extension of the IVBM can be used to examine the manifestation and the gross features of nuclear supersymmetry.

Thank you