Microscopic nuclear structure calculations with unitarily transformed effective interactions

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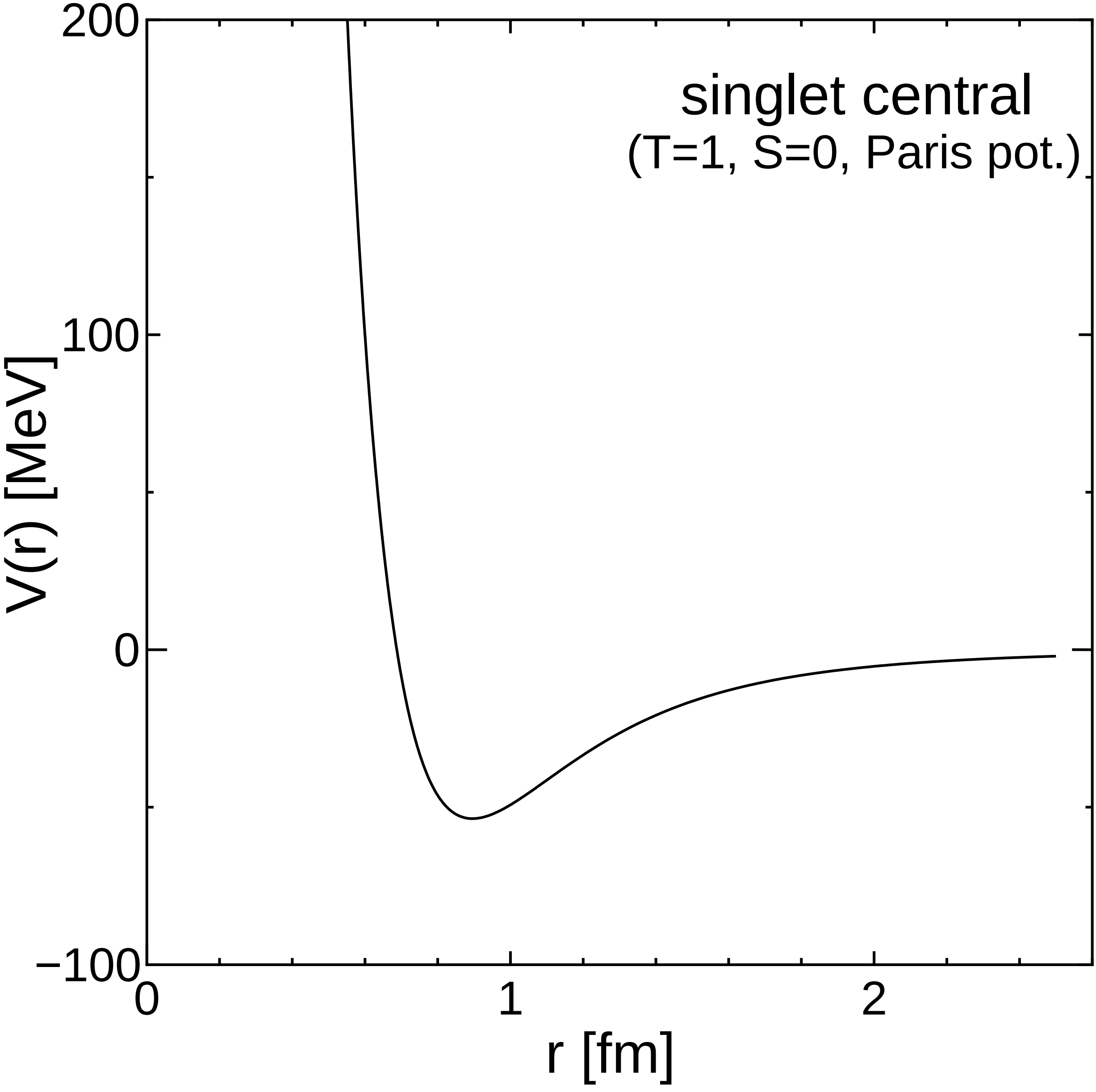
1. Unitary transformation
2. Unitary-model-operator approach (UMOA)
3. “No-core” shell model
4. Results
   - g.s. energies, s.p. levels in *p*- and *sd*-shell nuclei
   - neutron-rich C isotopes , ^{18}O
5. Summary

INT Seminar, Seattle, Sep. 27, 2007



singlet central
(T=1, S=0, Paris pot.)
200
100
0
−100
V(r) [MeV]
0
1
2
r [fm]

Derivation of effective interaction (Hamiltonian) by means of unitary transformation

Hamiltonian

$$H = H_0 + V$$

Unitary transformation of H

$$\tilde{H} = U^{-1}HU$$

$$U = e^S, \quad (S : \text{anti-Hermitian}, S^\dagger = -S)$$

Decoupling equation

$$Q(e^{-S}He^S)P = 0$$

Solution

$$S = \operatorname{arctanh}(\omega - \omega^\dagger), \quad \omega = Q\omega P$$

(with the restrictive condition $PSP = QSQ = 0$)

K. Suzuki, Prog. Theor. Phys. **68** (1982), 246

Effective Hamiltonian

$$H_{\text{eff}} = P\tilde{H}P$$

Effective interaction

$$V_{\text{eff}} = P\tilde{H}P - PH_0P$$

Unitary transformation operator U in terms of ω

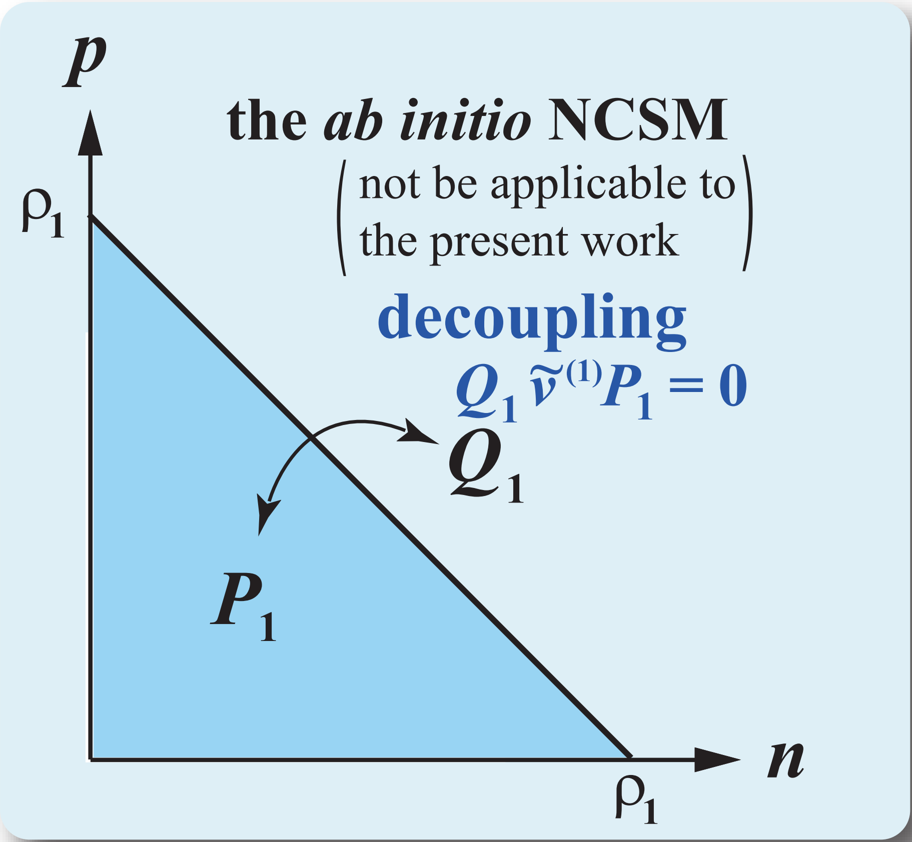
$$U = (1 + \omega - \omega^\dagger)(1 + \omega^\dagger\omega + \omega\omega^\dagger)^{-1/2}$$

$$= \begin{pmatrix} P(1 + \omega^\dagger\omega)^{-1/2}P & -P\omega^\dagger(1 + \omega\omega^\dagger)^{-1/2}Q \\ Q\omega(1 + \omega^\dagger\omega)^{-1/2}P & Q(1 + \omega\omega^\dagger)^{-1/2}Q \end{pmatrix}$$

S. Ōkubo, Prog. Theor. Phys. **12** (1954), 603

Derivation of effective interaction

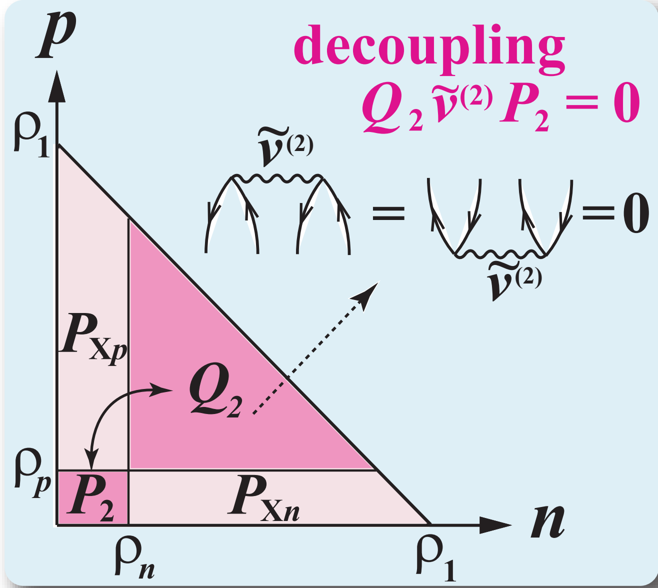
Eff. int. in a huge model space



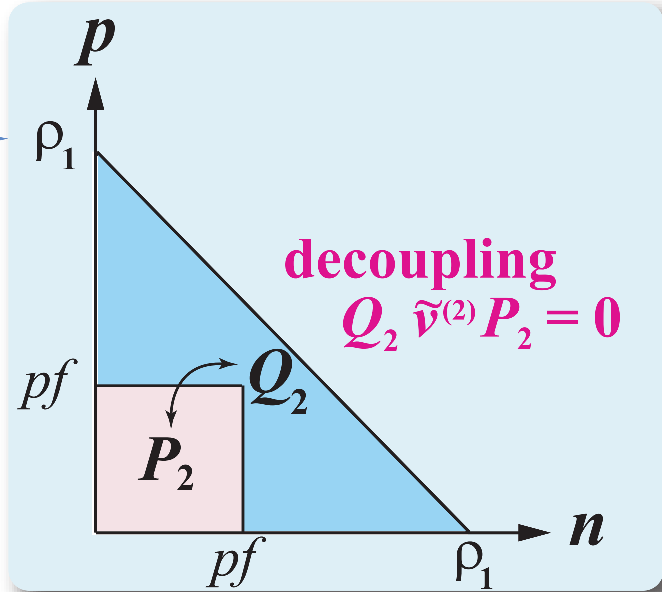
$\rho_1 = 2n_a + l_a + 2n_b + l_b$ ($\{n_a, l_a\}$ and $\{n_b, l_b\}$: sets of h.o. quantum numbers of the two-body states)

(For details,
- S. F., T. Mizusaki, T. Otsuka, T. Sebe, and A. Arima, Phys. Lett. **B650**, 9 (2007).
- S. F., R. Okamoto, and K. Suzuki, Phys. Rev. C **69**, 034328 (2004).)

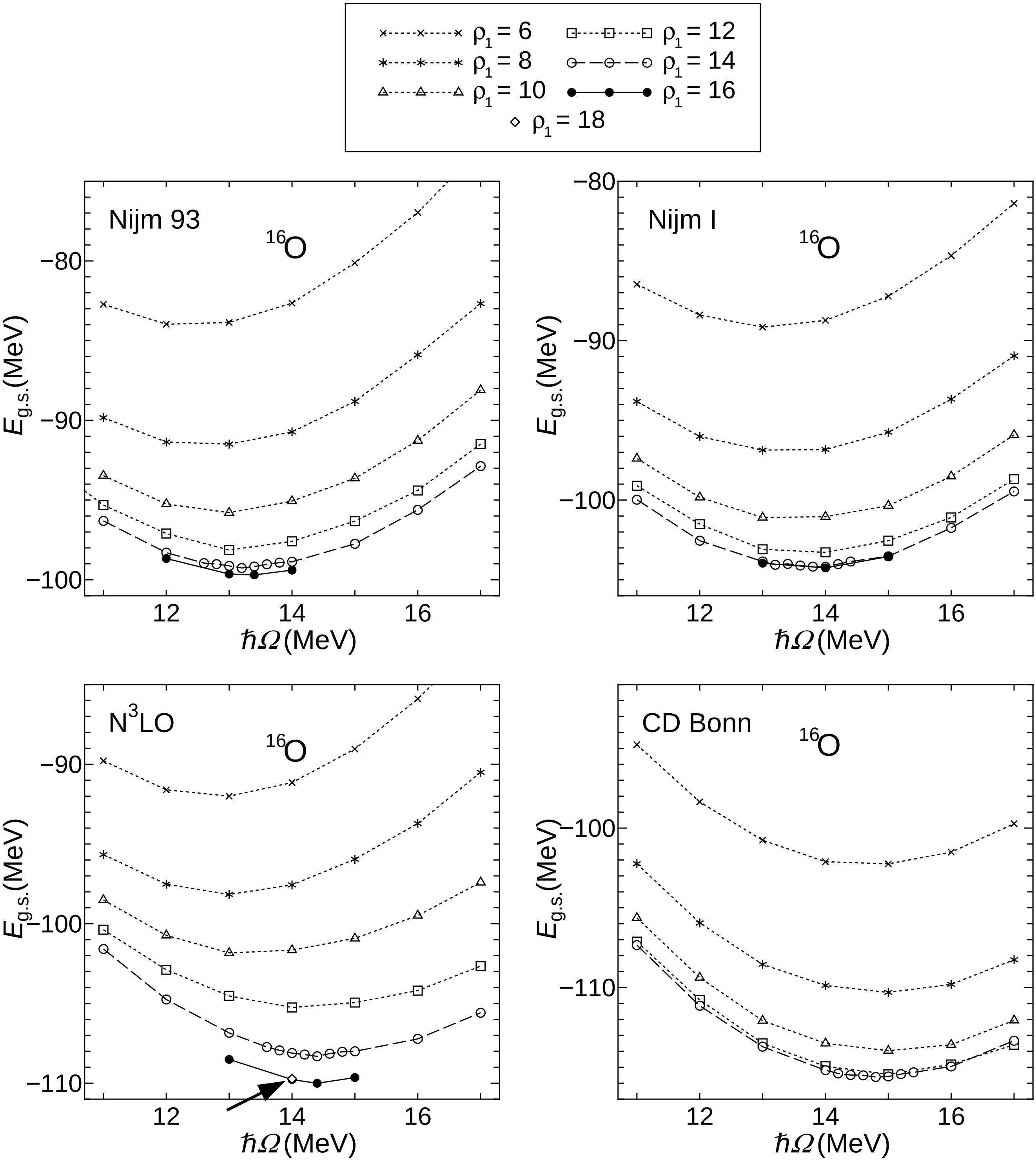
Eff. int. for the UMOA



Eff. int. for the 0*s*−1*p*0*f* shell model

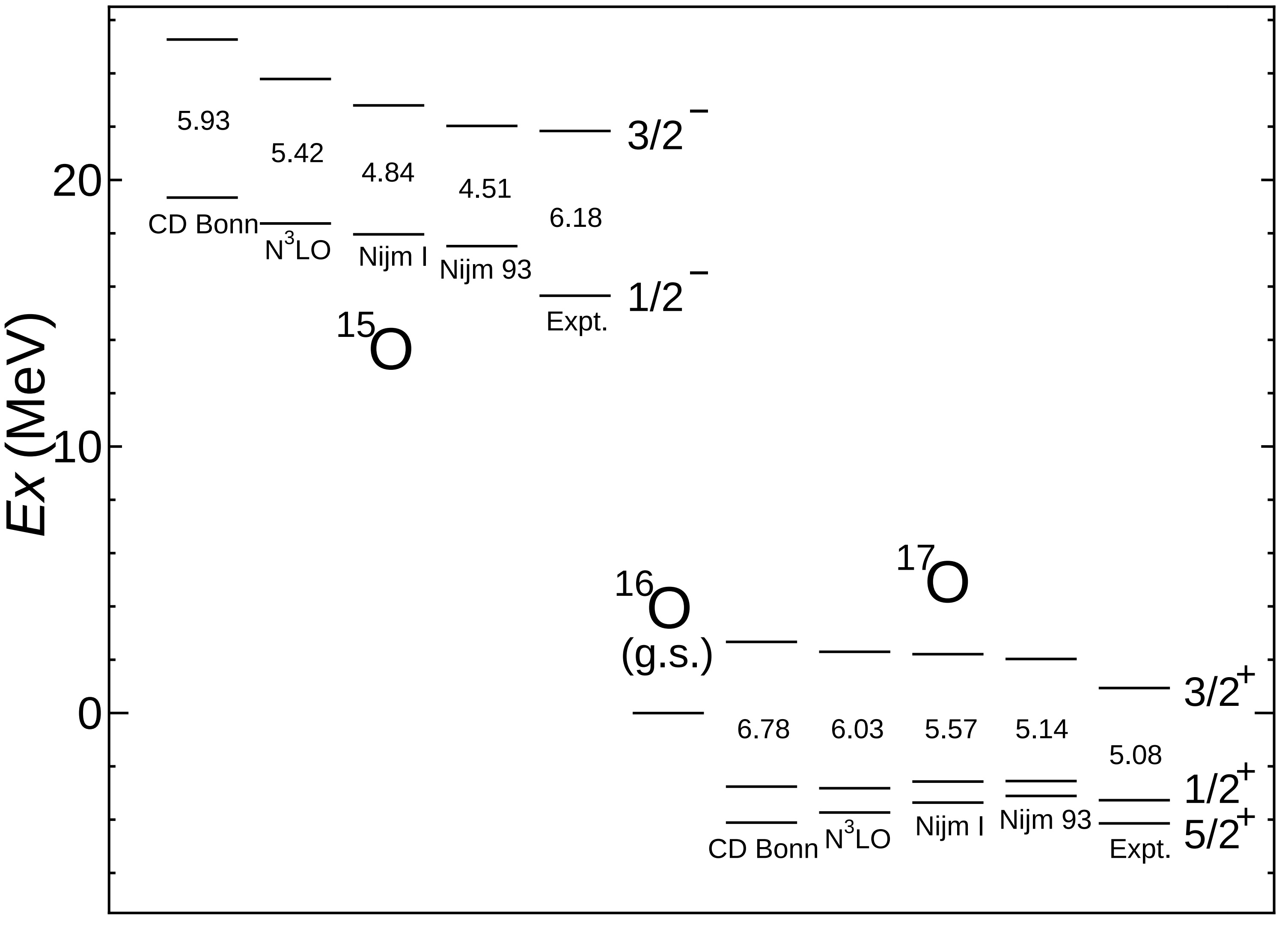


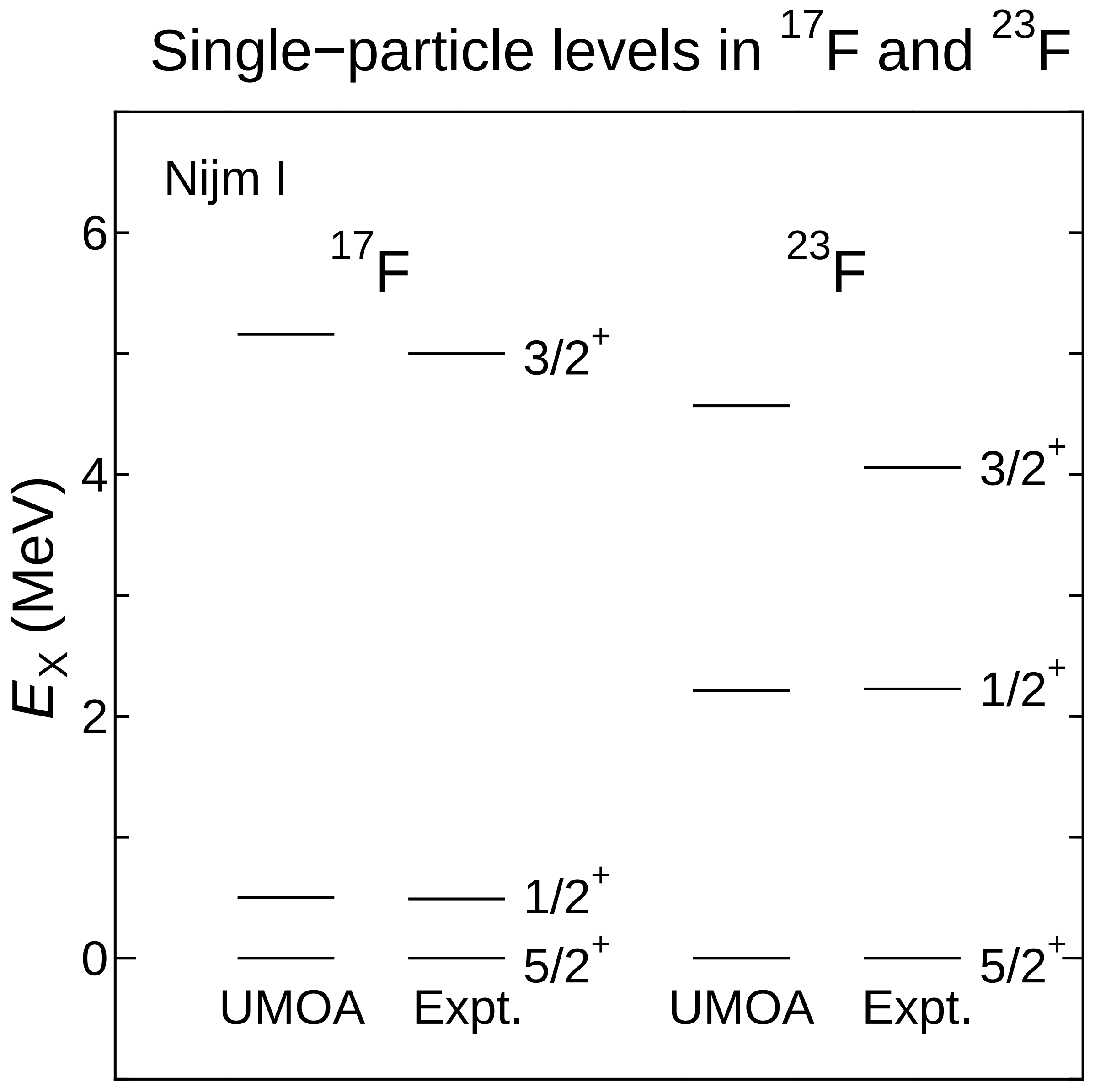
Ground−state energies of ^{16}O



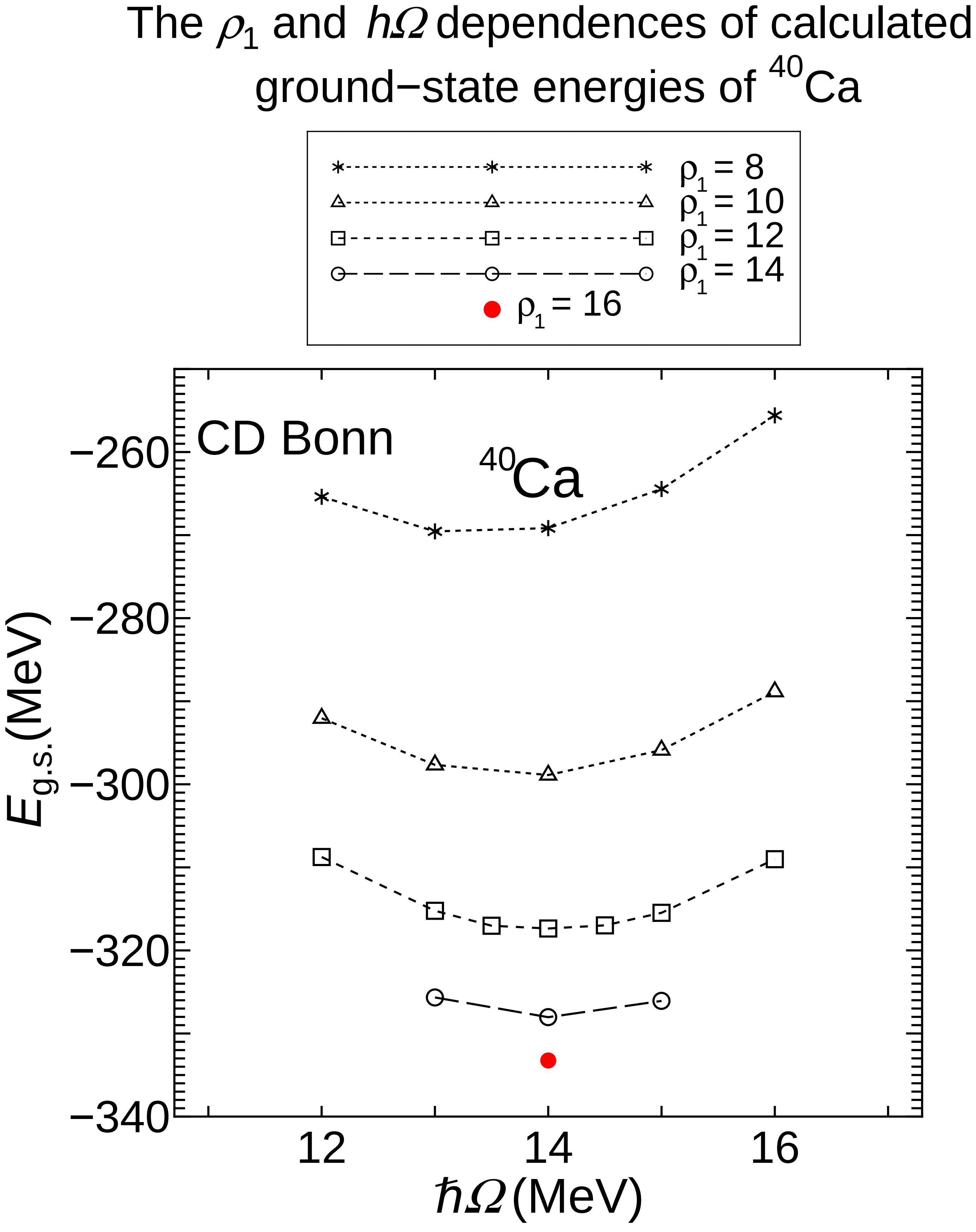
	Nijm 93	Nijm I	N^3LO	CD Bonn	Expt.	
$E_{g.s.}$	−99.69	−104.25	−110.00	−115.62	−127.62	
BE/A	6.23	6.52	6.88	7.23	7.98	(in MeV)

Comparison of Expt. and UMOA results from modern NN interactions

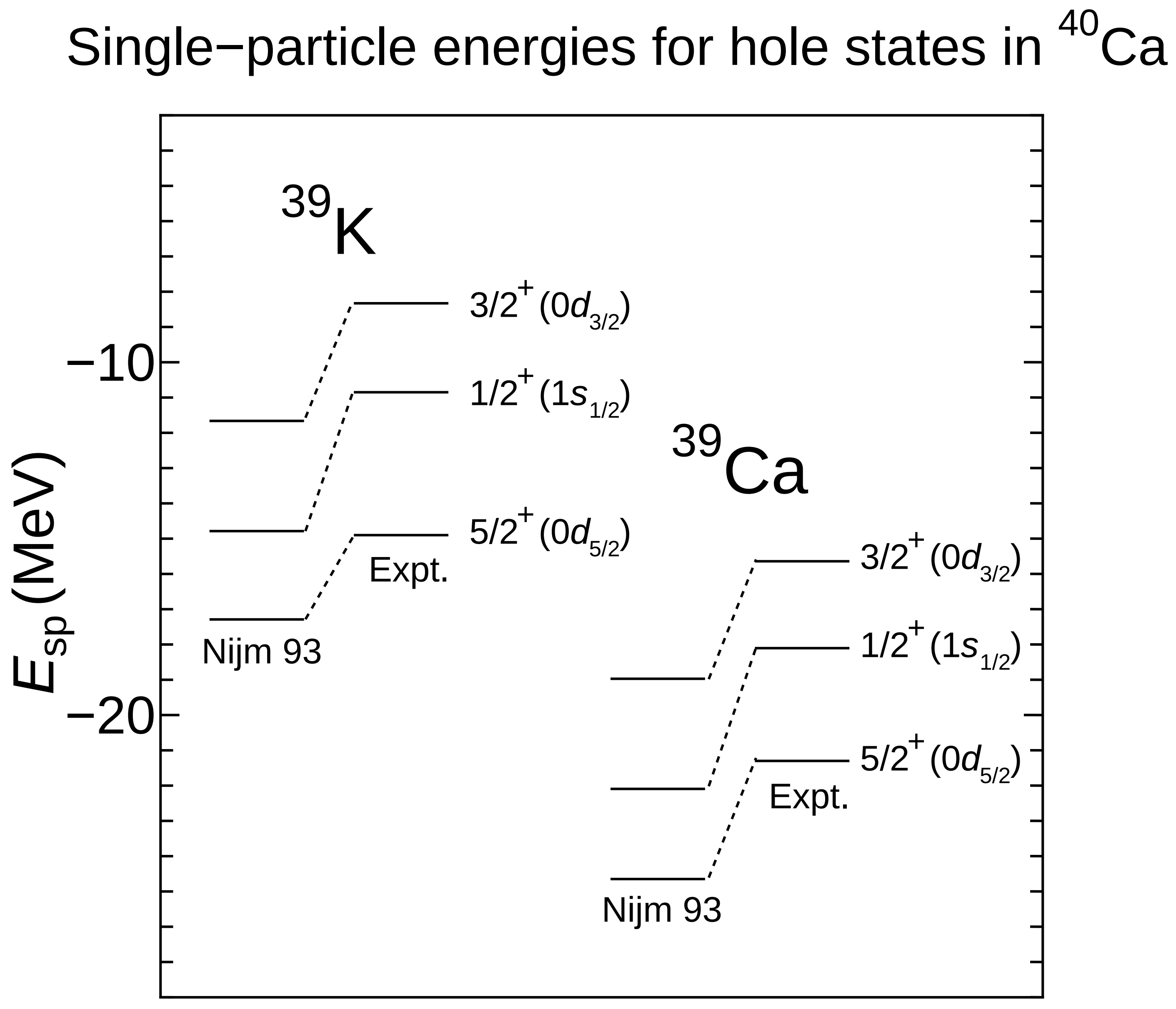




Single−particle levels in 17F and 23F
Nijm I
17F
23F
E_X (MeV)
6
4
2
0
3/2+
1/2+
5/2+
3/2+
1/2+
5/2+
UMOA
Expt.
UMOA
Expt.



The ρ_1 and $\hbar\Omega$ dependences of calculated ground−state energies of ^{40}Ca
$\rho_1 = 8$
$\rho_1 = 10$
$\rho_1 = 12$
$\rho_1 = 14$
$\rho_1 = 16$
CD Bonn
^{40}Ca
−260
−280
−300
−320
−340
$E_{g.s.}$(MeV)
12
14
16
$\hbar\Omega$ (MeV)


Single−particle energies for hole states in 40Ca
39K
3/2+ (0d3/2)
1/2+ (1s1/2)
5/2+ (0d5/2)
Expt.
Nijm 93
39Ca
3/2+ (0d3/2)
1/2+ (1s1/2)
5/2+ (0d5/2)
Expt.
Nijm 93
−10
−20
Esp (MeV)

New approach to neutron-rich C isotopes

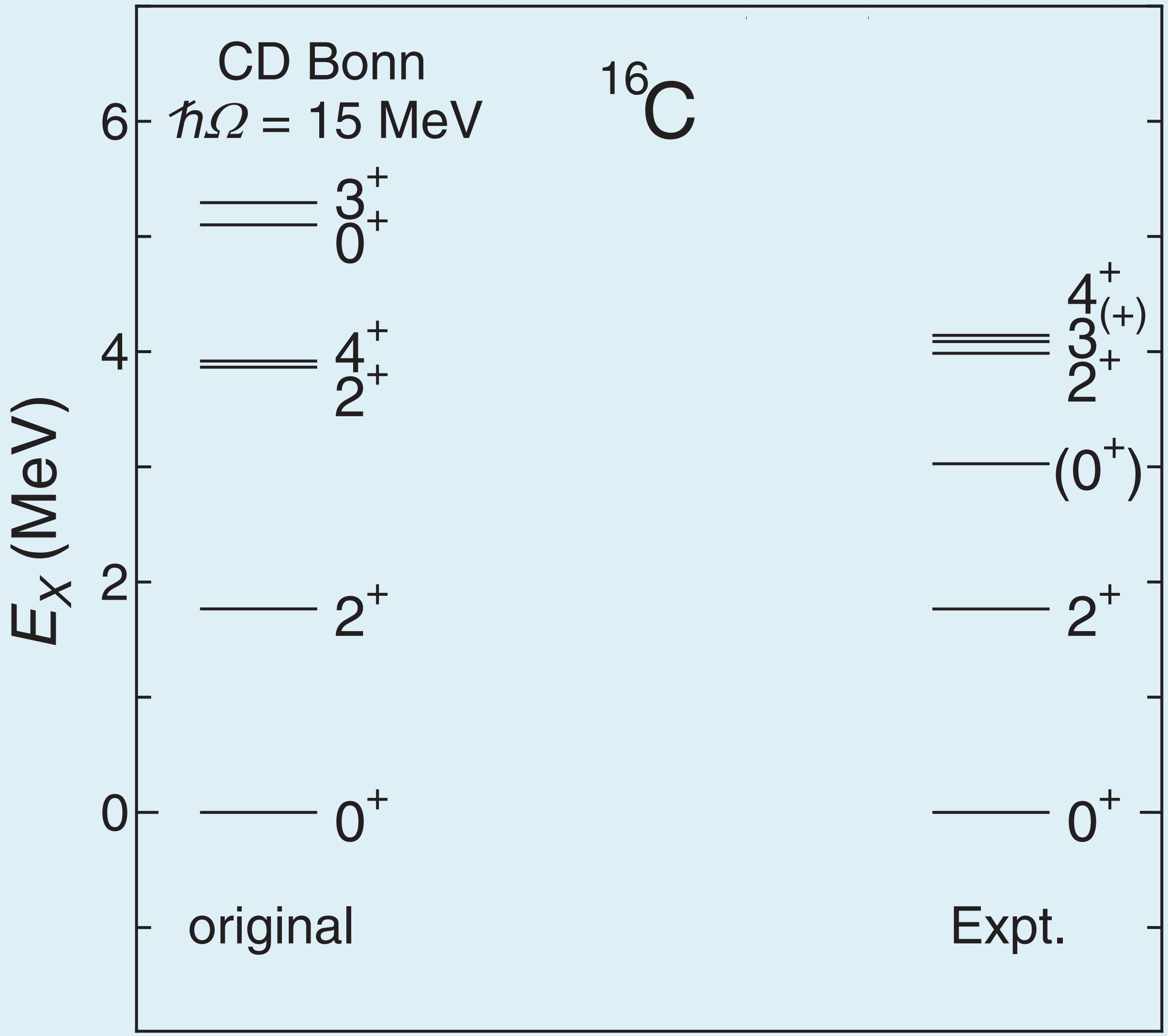
- **Large-scale shell model**

  - Code: newly developed version of MSHELL
  - Model space: the 0*s* − 1*p*0*f* shells
  - Nucleon excitation: up to 2 nucleons from the occupied shells for ^{14}C
    up to 2 nucleons to the 1*p*0*f* shells
  - Bare transition operator

- **Microscopic effective interaction**

  Derived from a high-precision NN interaction (CD Bonn, ⋯) and the Coulomb force in the neutron-proton formalism for the given model space through a unitary-transformation theory

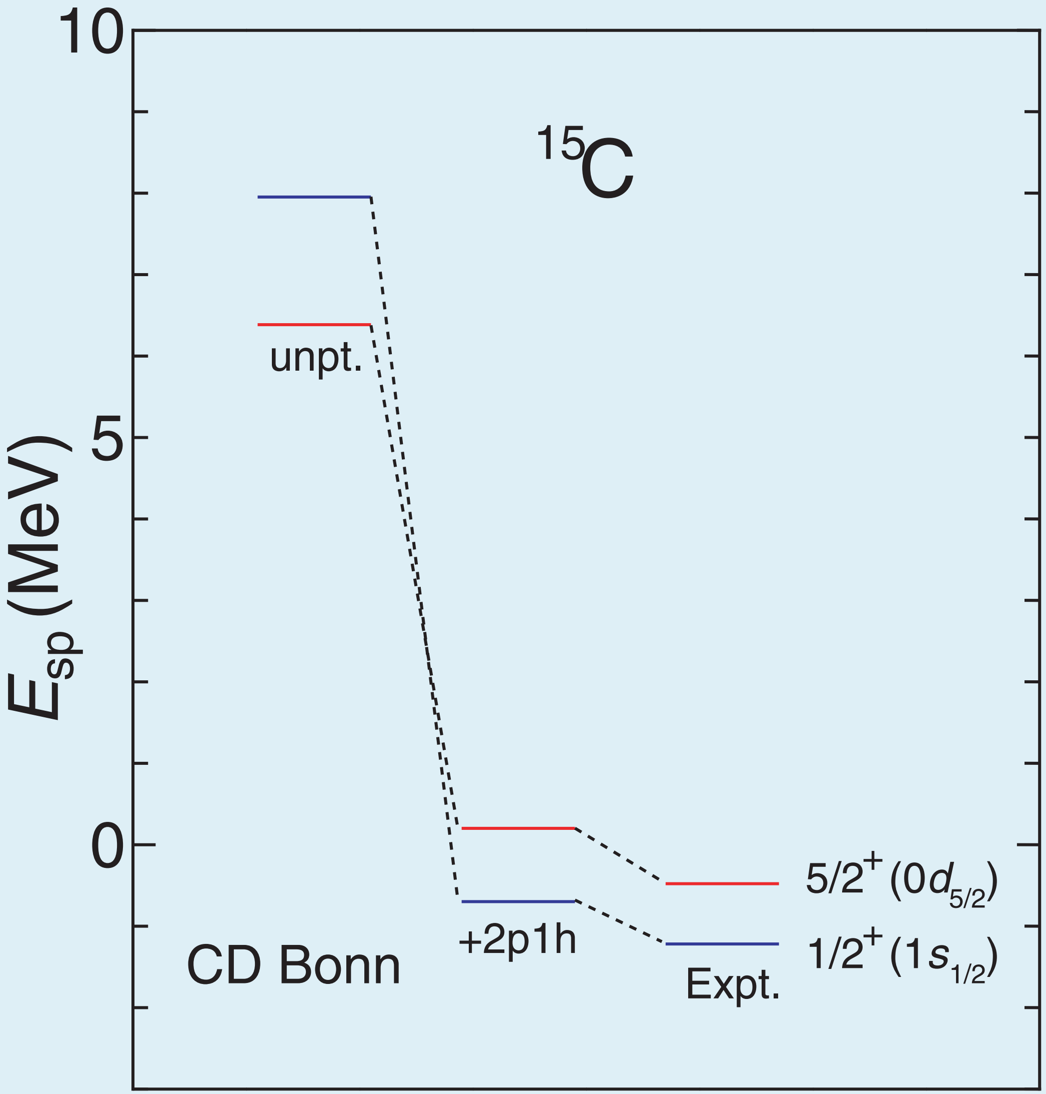
Low-lying energy levels in ^{16}C



$B(E2;\, 2_1^+ \rightarrow 0_1^+)$ in $e^2\mathrm{fm}^4$

original	Expt.
1.30	$0.63^{\pm 0.11(\mathrm{stat})}_{\pm 0.16(\mathrm{syst})}$

Single-particle energies in ^{15}C



Calculated results by the unitary-model-oparator approach (UMOA)

In the present shell model without any adjustable parameters

→ wrong ordering for the $1/2^+$ and $5/2^+$ states in ^{15}C due to the *small* model-space size

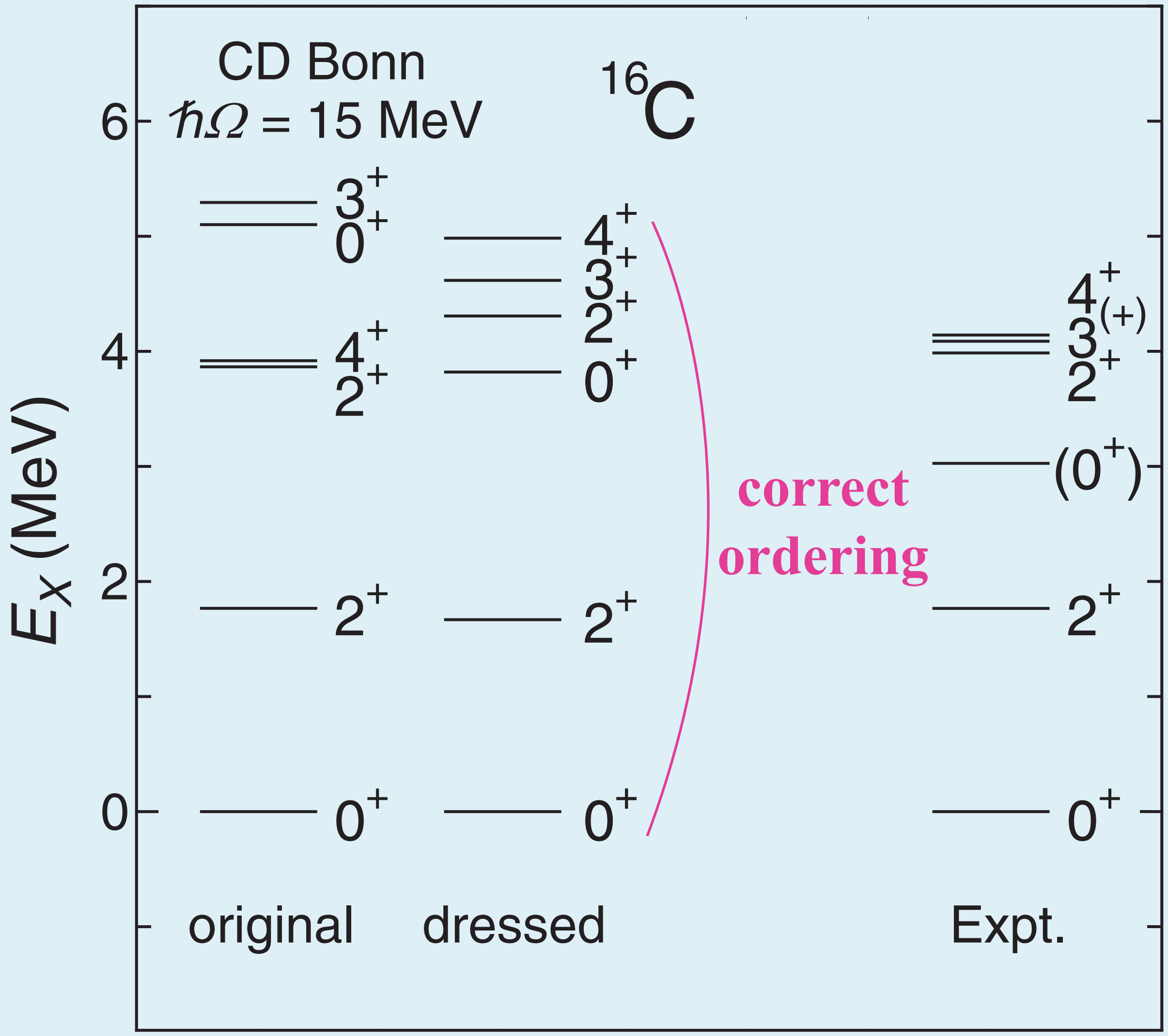
To remedy the wrong ordering and reproduce the binding energies for the $1/2^+$ and $5/2^+$ states of the UMOA results

→ introduce a minimal refinement of the one-body energies for the $0d_{5/2}$ and $1s_{1/2}$ orbits of the neutron

The calculated results are denoted by

"dressed"

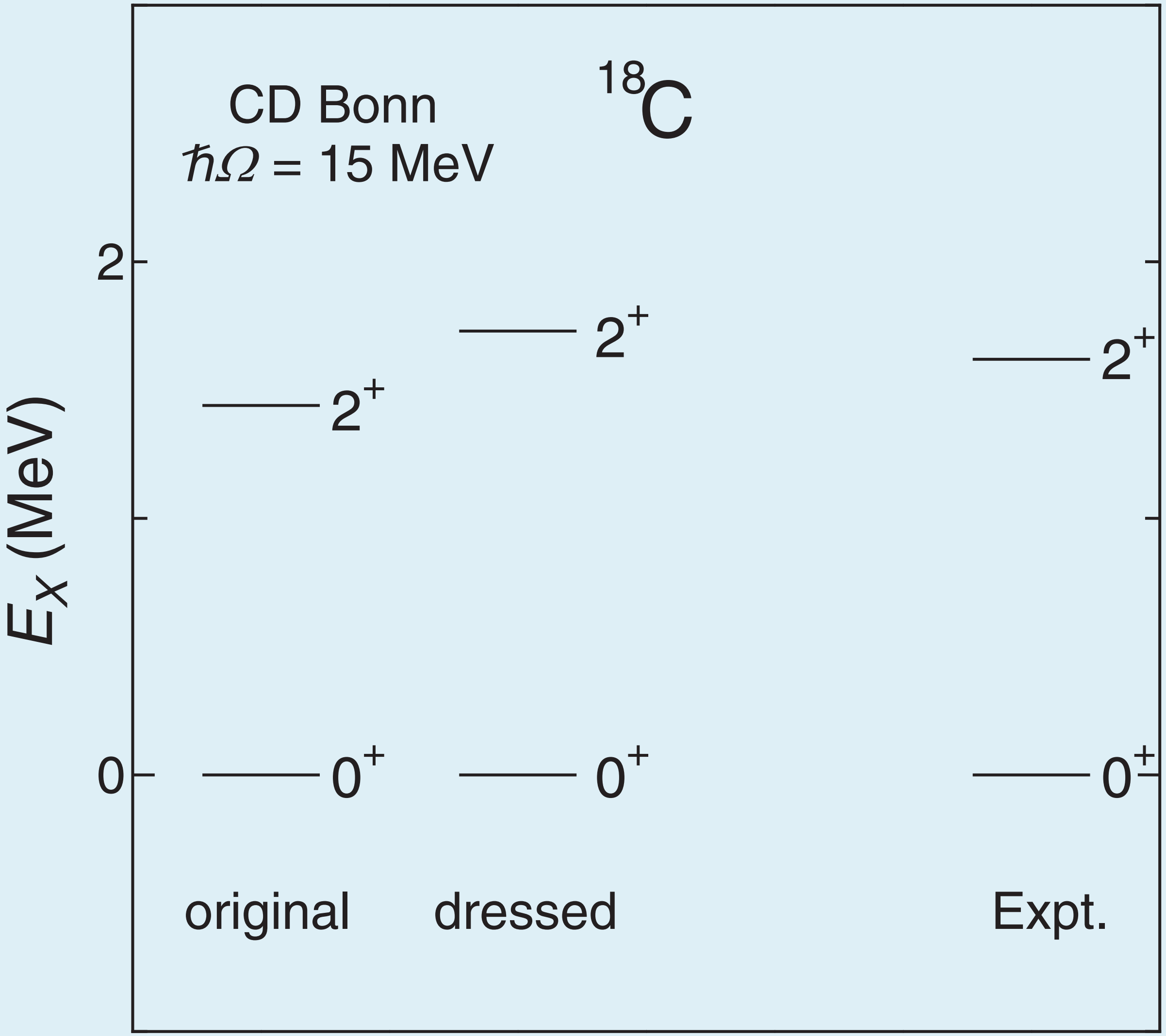
Low-lying energy levels in ^{16}C



$B(E2;\, 2_1^+ \rightarrow 0_1^+)$ in $e^2 fm^4$

original	dressed	Expt.
1.30	0.84	$0.63^{\pm 0.11 (stat)}_{\pm 0.16 (syst)}$

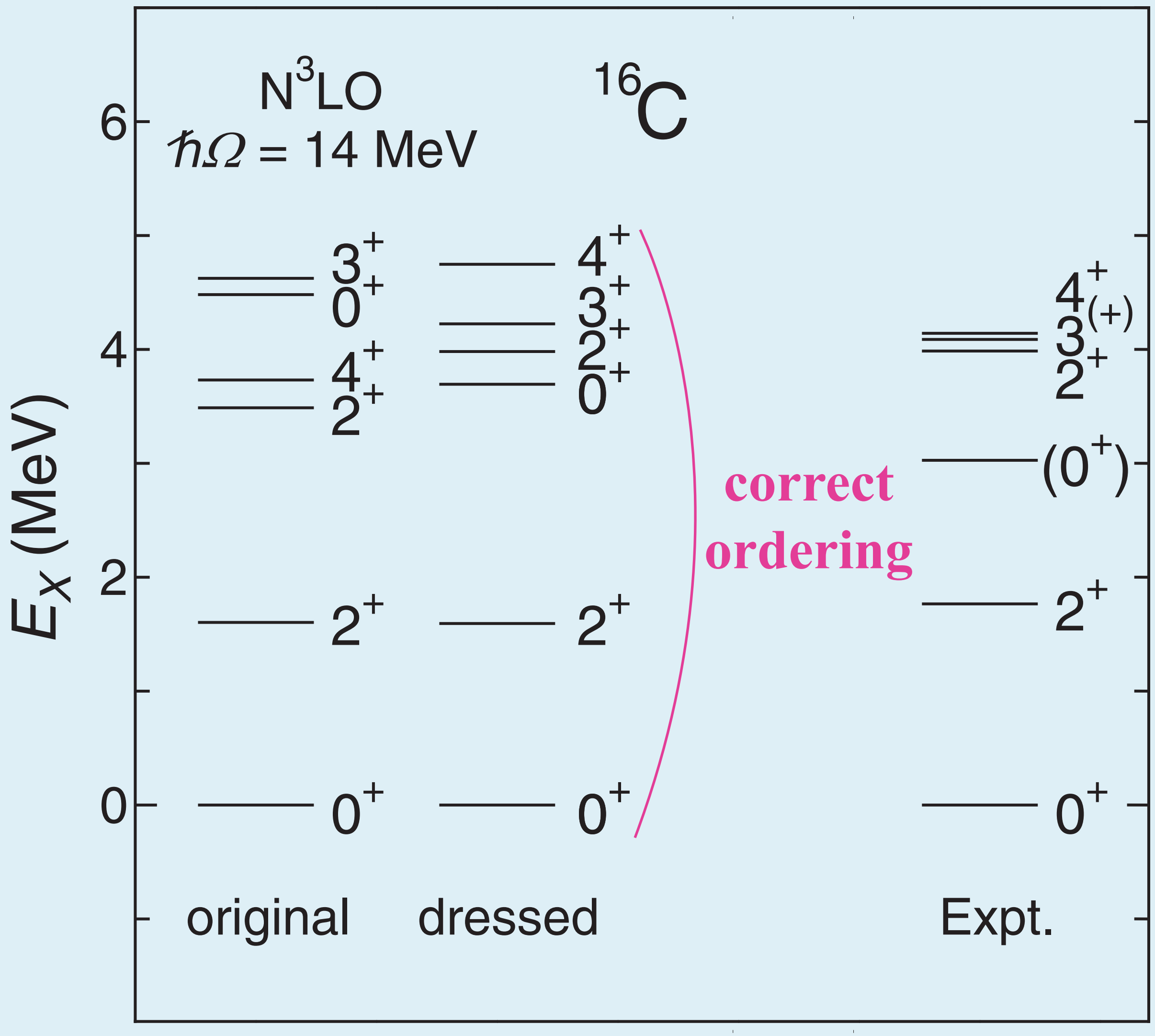
Energy differences between the 2_1^+ and 0_1^+ states in ^{18}C



$B(E2; 2_1^+ \rightarrow 0_1^+)$ in e^2fm^4

1.19 2.10 ?

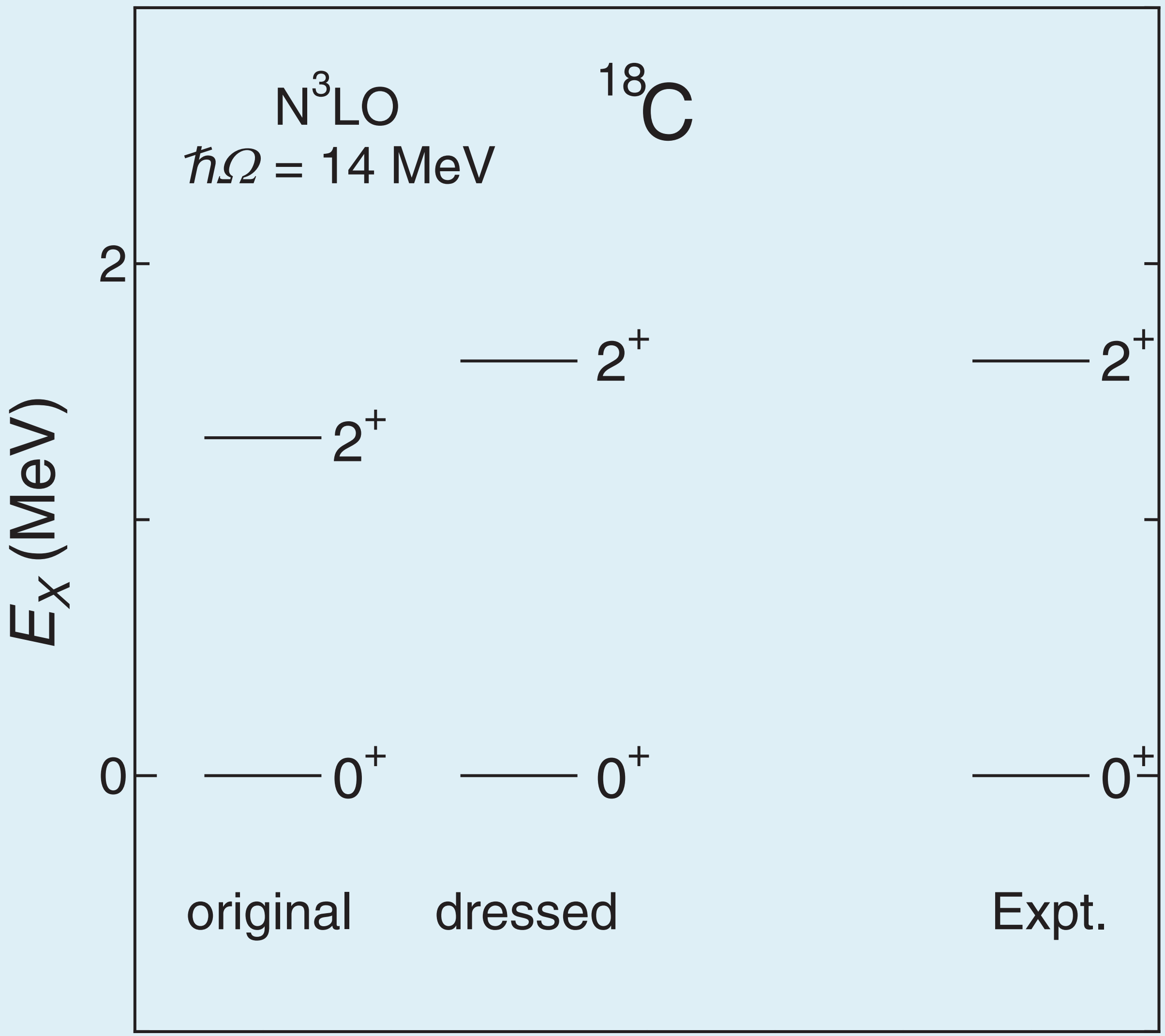
Low-lying energy levels in ^{16}C



$B(E2; 2_1^+ \rightarrow 0_1^+)$ in e^2fm^4

1.38 0.91 $0.63^{\pm 0.11(stat)}_{\pm 0.16(syst)}$

Energy differences between the 2_1^+ and 0_1^+ states in ^{18}C



$B(E2;\, 2_1^+ \rightarrow 0_1^+)$ in $e^2\text{fm}^4$

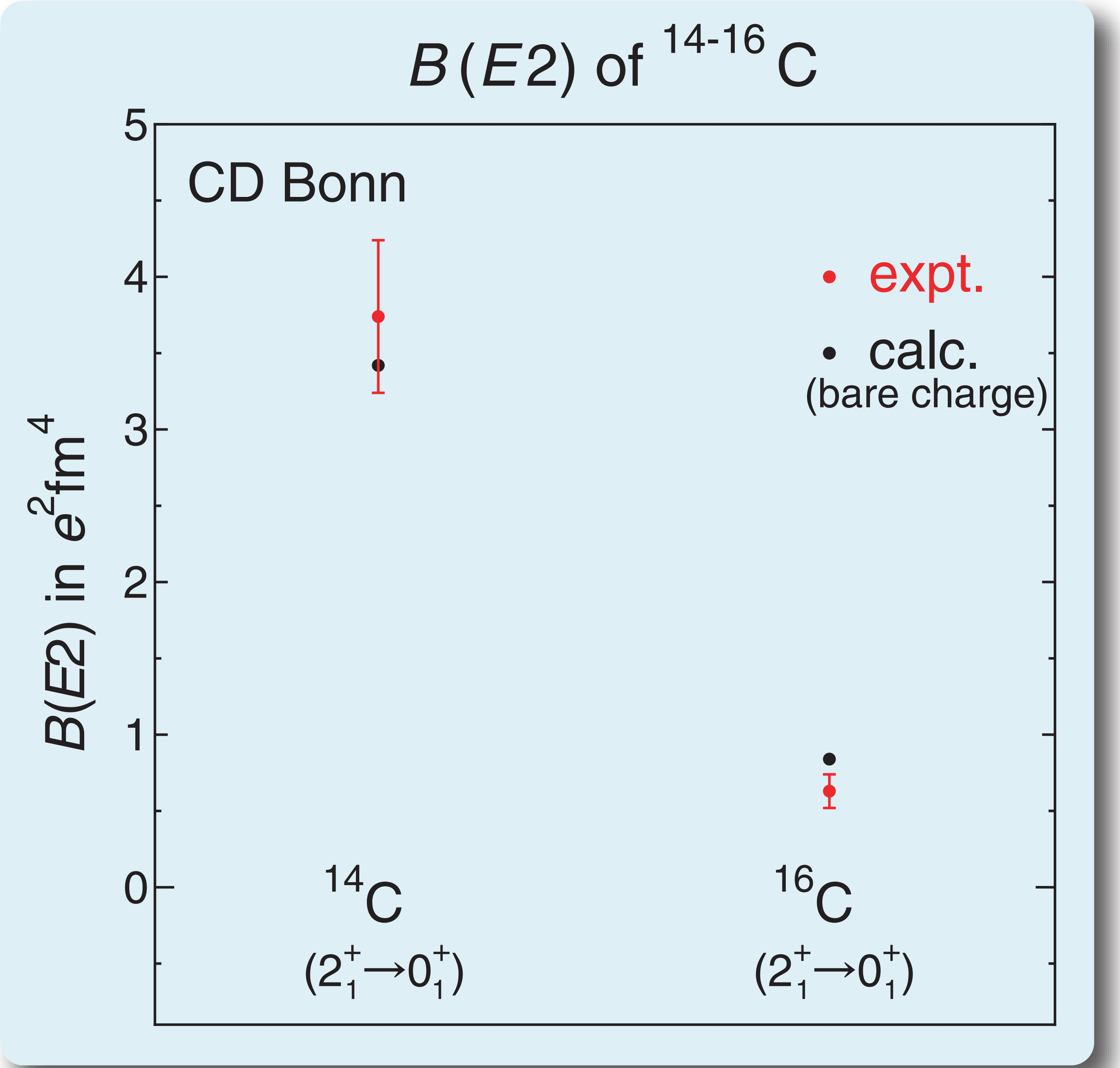
original	dressed	Expt.
1.38	2.47	?

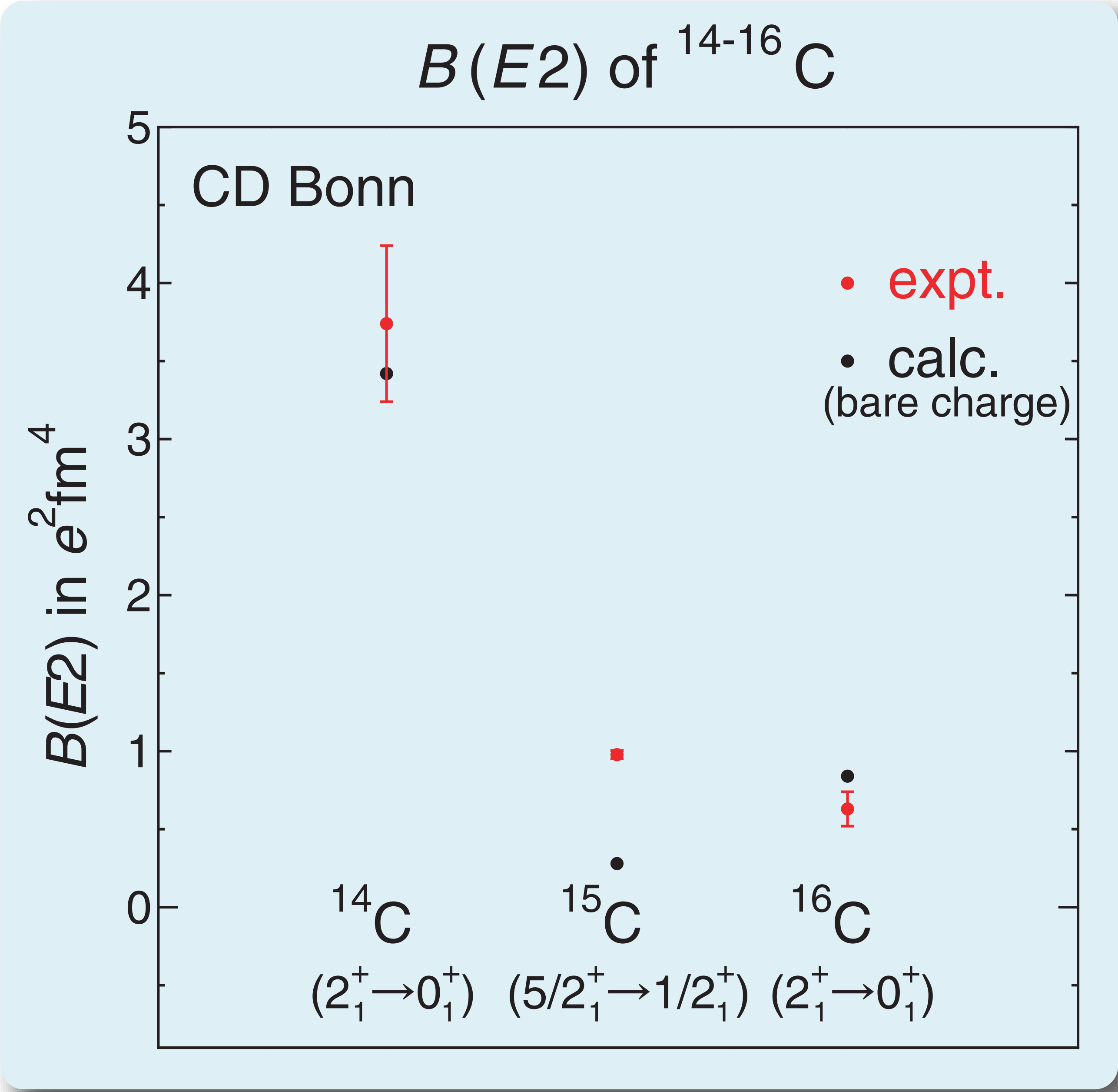
$B(E2;\, 2_1^+ \rightarrow 0_1^+)$ in $^{14,16,18}C$

for "dressed"

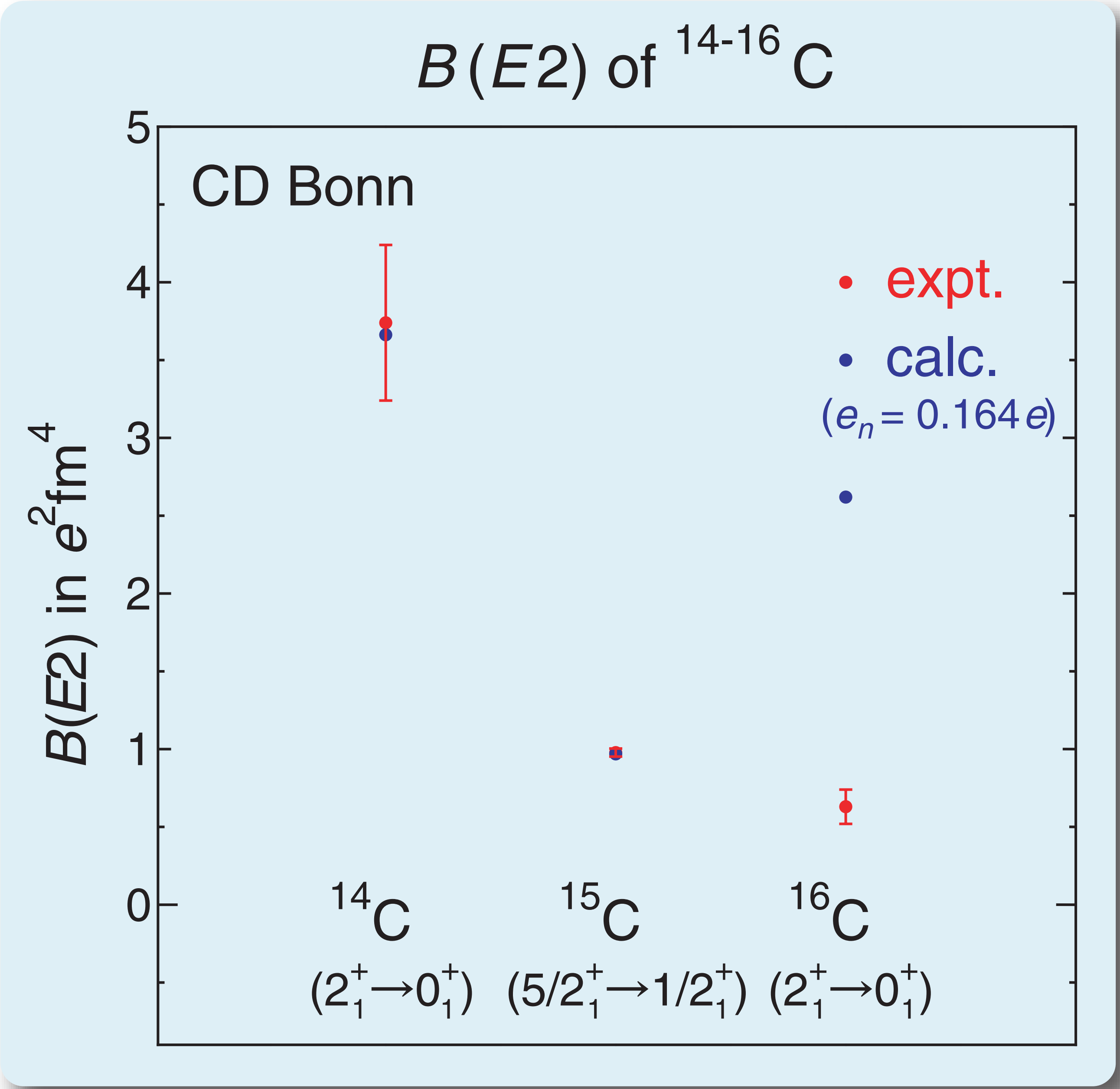
	CD Bonn	N^3LO	Expt.
^{14}C	3.42	4.11	3.74±0.50
^{16}C	0.84	0.91	$0.63 \pm 0.11(\text{stat}) \pm 0.16(\text{syst})$
^{18}C	2.10	2.47	?

in e^2fm^4

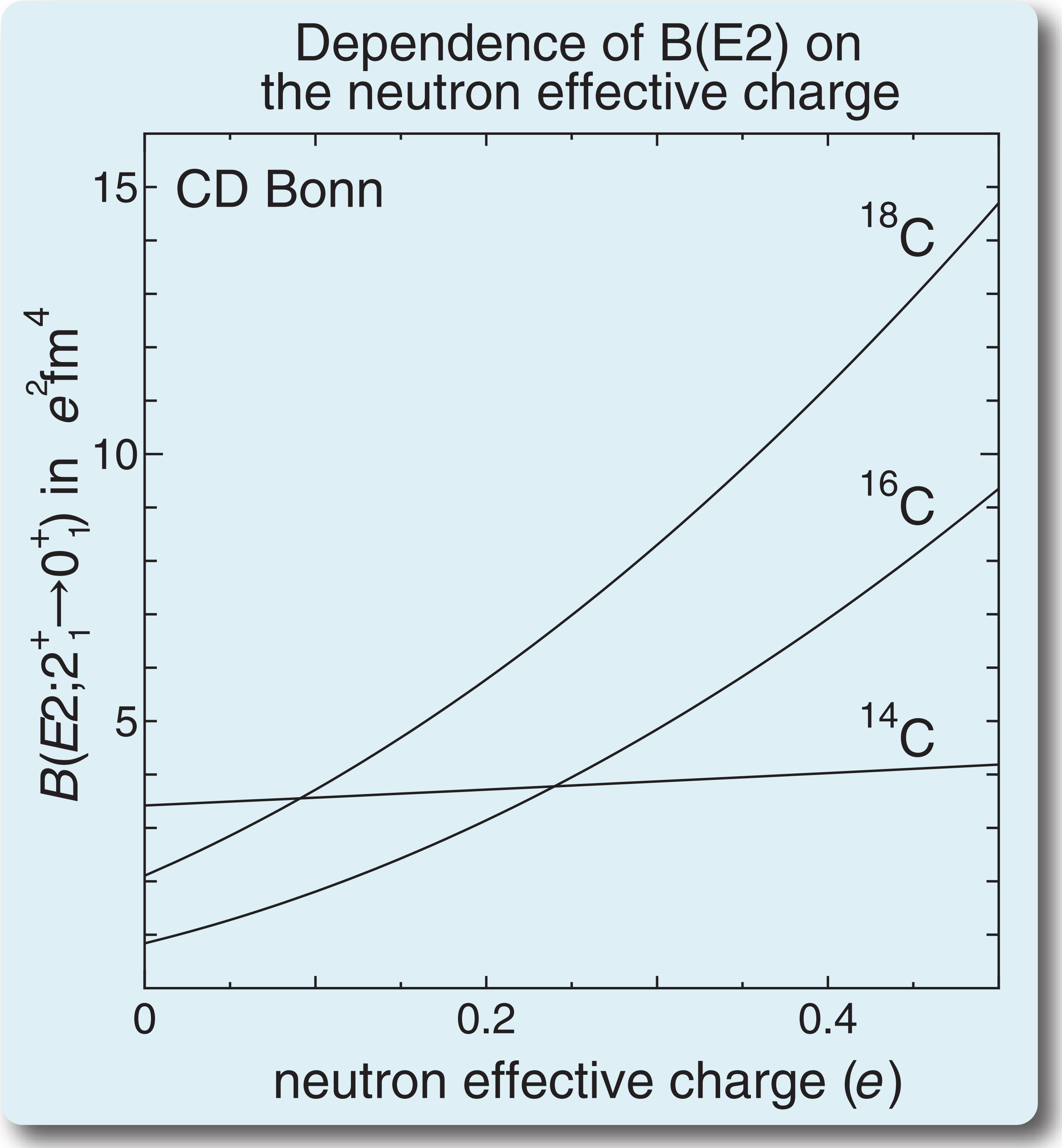

B(E2) of 14-16 C
CD Bonn
expt.
calc.
(bare charge)
B(E2) in e2fm4
0
1
2
3
4
5
14C
(2+1→0+1)
16C
(2+1→0+1)



B(E2) of 14-16C
CD Bonn
expt.
calc.
(bare charge)
B(E2) in e2fm4
0
1
2
3
4
5
14C
(2+1→0+1)
15C
(5/2+1→1/2+1)
16C
(2+1→0+1)



B(E2) of 14-16C
CD Bonn
B(E2) in e2fm4
5
4
3
2
1
0
expt.
calc.
(en = 0.164 e)
14C
(2+1→0+1)
15C
(5/2+1→1/2+1)
16C
(2+1→0+1)



Dependence of B(E2) on
the neutron effective charge
CD Bonn
18C
16C
14C
15
10
5
0
0.2
0.4
B(E2;2+1→0+1) in e2fm4
neutron effective charge (e)

$B(E2; 2_1^+ \rightarrow 0_1^+)$ in $^{14,16,18}C$

CD Bonn

for "dressed"

	$e_n = 0$	$e_n = 0.164e$	Expt.
^{14}C	3.42	3.66	3.74±0.50
^{16}C	0.84	2.62	0.63 ± 0.11(stat) ± 0.16(syst)
^{18}C	2.10	4.98	?

in e^2fm^4

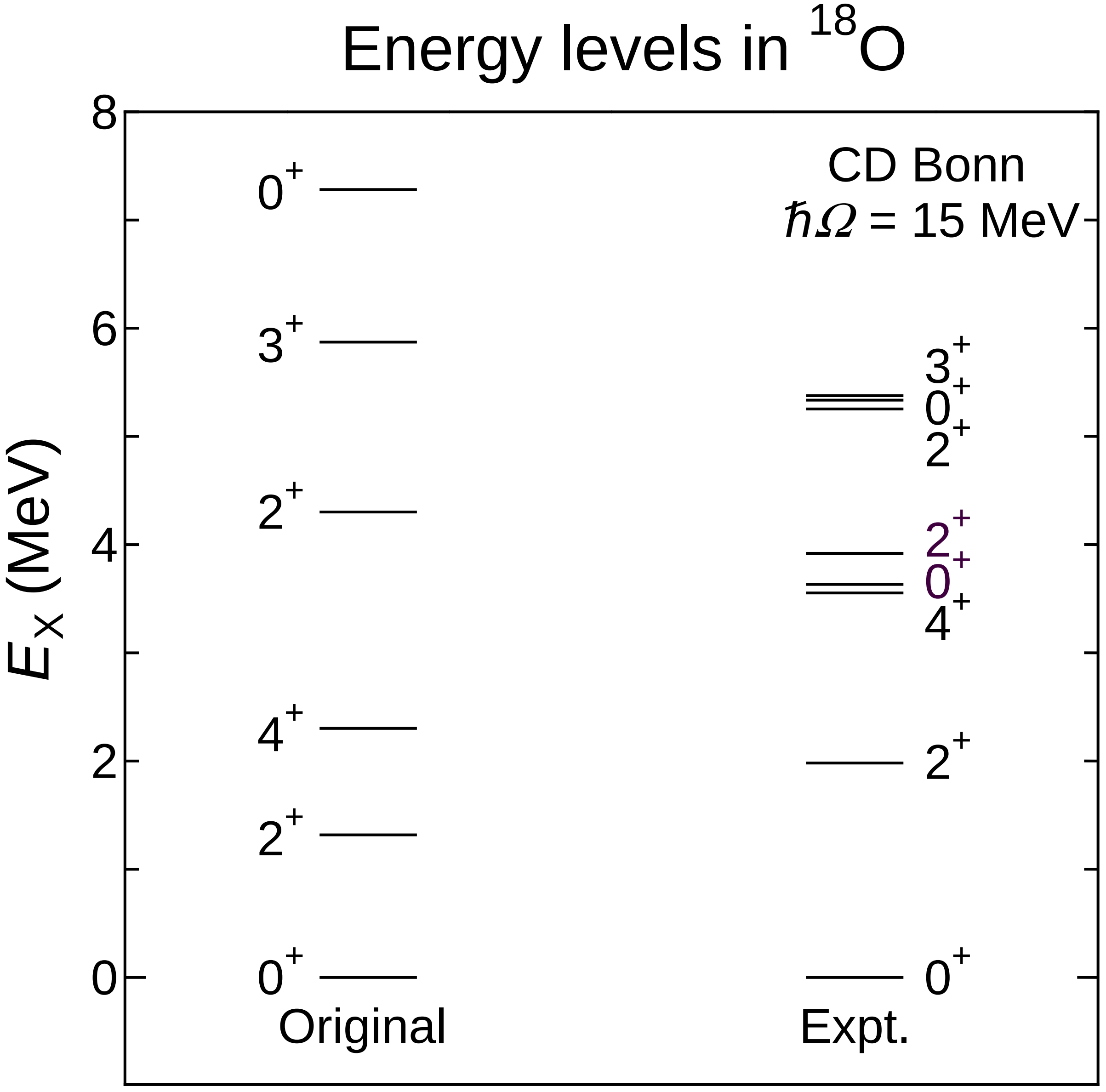
$B(E2; 2_1^+ \rightarrow 0_1^+)$ in $^{14,16,18}C$

CD Bonn

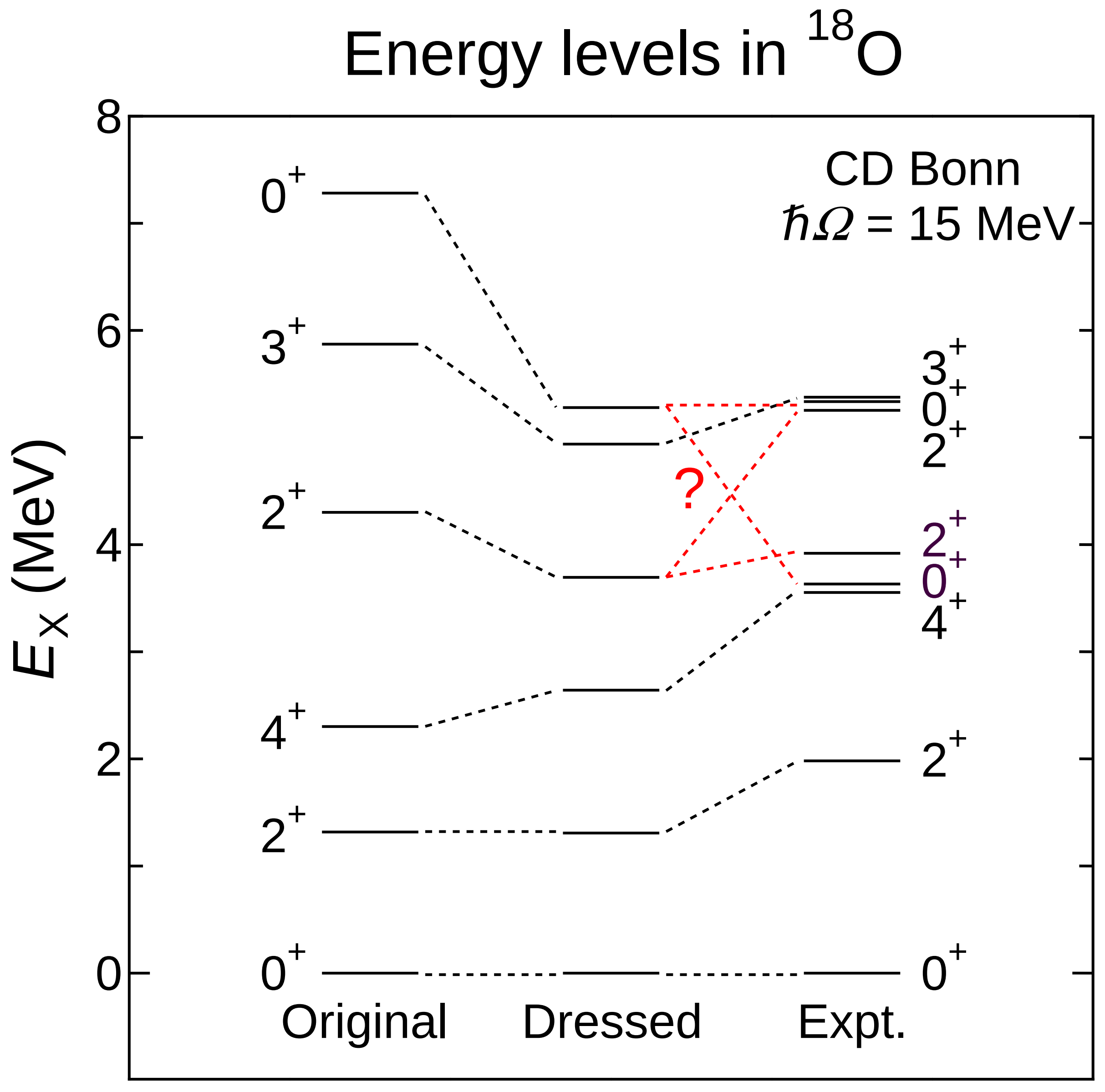
for "dressed"

	$e_n = 0$	$e_n = 0.164e$	Expt.
^{14}C	3.42	3.66	3.74±0.50
^{16}C	0.84	2.62	$0.63 \pm 0.11(\text{stat}) \pm 0.16(\text{syst})$
^{18}C	2.10	4.98	$4.7^{+0.8}_{-0.6}(\text{stat})^{+1.4}_{-0.9}(\text{syst})$

in $e^2 fm^4$



Energy levels in ¹⁸O
CD Bonn
ħΩ = 15 MeV
E_X (MeV)
8
6
4
2
0
0+
3+
2+
4+
2+
0+
3+
0+
2+
2+
0+
4+
2+
0+
Original
Expt.



Energy levels in ^{18}O
CD Bonn
$\hbar\Omega$ = 15 MeV
E_X (MeV)
8
6
4
2
0
0^+
3^+
2^+
4^+
2^+
0^+
?
3^+
0^+
2^+
2^+
0^+
4^+
2^+
0^+
Original
Dressed
Expt.

Summary

- **Derived the effective interaction for a given model space from the high-precision modern NN force through a unitary transformation theory toward the microscopic description of structure of exotic nuclei**

  - **Unitary-model-operator approach (UMOA)**

    g.s. energy for closed-shell nuclei, s.p. energy, ⋯
  - **Large-scale shell model (with information about s.p. states obtained by the UMOA)**

    complicated structure, transition, ⋯

- **Including the genuine three-body force and diminishing the approximations in the calculation**

Collaborators

UMOA

Ryoji Okamoto (Kyushu Inst. of Tech.)

Kenji Suzuki (Kyushu Inst. of Tech.)

"No-core" shell model

Takahiro Mizusaki (Senshu Univ.)

Takaharu Otsuka (Univ. of Tokyo)

Takashi Sebe (Hosei Univ.)

Akito Arima (Japan Science Foundation)

Application of the "No-core" shell model to ^{18}O

Bruce R. Barrett (Univ. of Arizona)