Toward a Unified Description of 4n N=Z Light Nuclei in the “Ab initio” Symplectic No Core Shell Model

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Outline

1. Motivation

2. Overview of symplectic symmetry

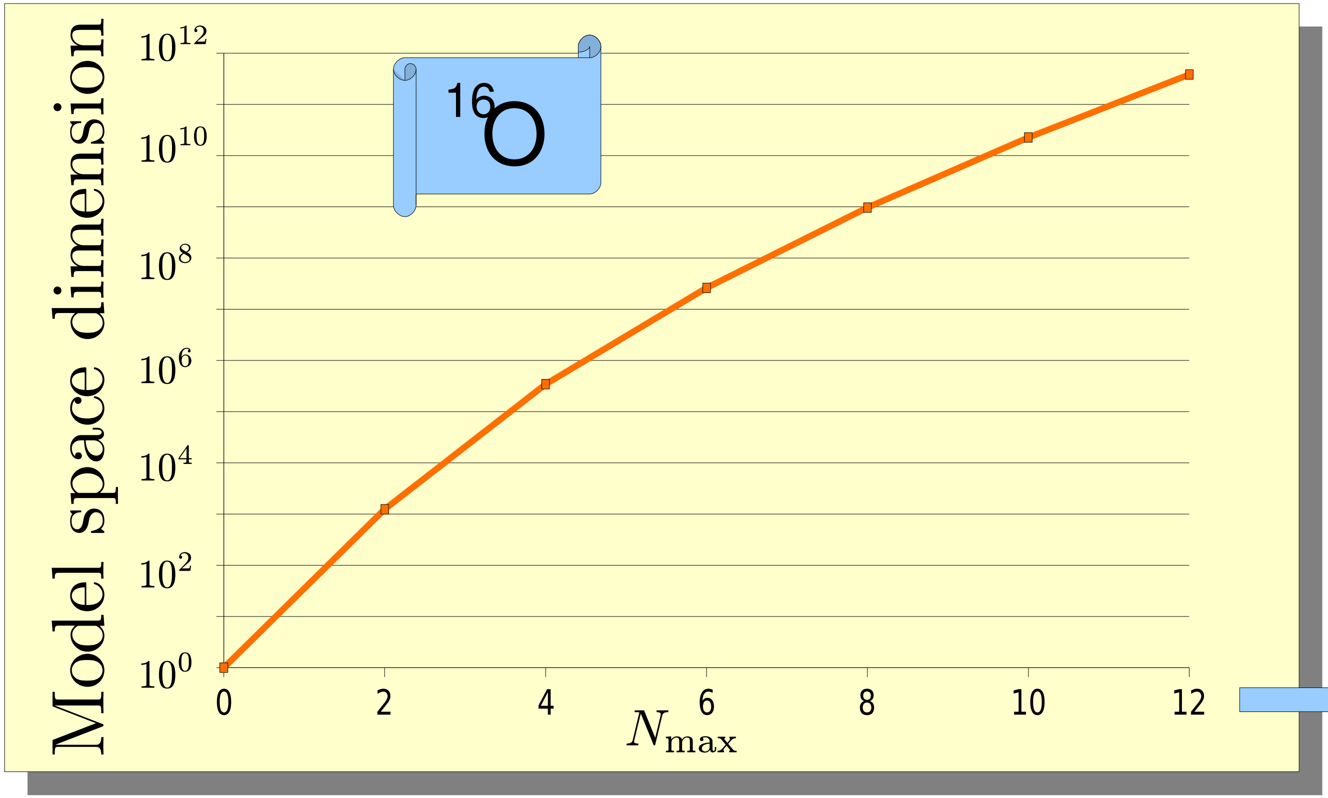
- Relation to α-cluster model wave functions
- Inclusion of highly deformed particle-hole configurations within symplectic space
- Discussion on center of mass spuriousity

3. Expansion of symplectic states in shell model basis

4. “proof-of-principle” results

Motivation

Scale Explosion: combinatorial growth in dimensionality of basis for heavier nuclei and increasing $N\hbar\Omega$ model spaces



Yet even larger model spaces are needed!

High $N\hbar\Omega$ configurations essential for:

- improving overall convergence of the spectrum
- reproducing B(E2) without effective charges
- modelling deformed and cluster modes

Solution to Scale Explosion

Symplectic $\mathrm{Sp}(3,\mathbb{R})$ symmetry-adapted basis

G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. 38, 10 (1977)

Properties of symplectic basis:

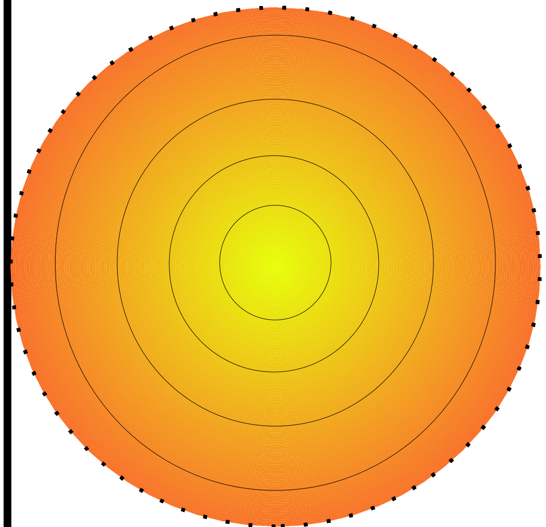
- ✔Complete
- ✔Translationally invariant
- ✔Natural for description of many-body collective dynamics
  - quadrupole and monopole vibrations
  - microscopic realization of Bohr-Mottelson model is embedded within Sp(3,R)
  - rotational dynamics in continuous range from rigid rotor to irrotational flow
- ✔Appropriate for description of α-clusters

In the classical limit symplectic symmetry underpins rotation of deformed stars and galaxies!

G. Rosensteel, Astrophys. J. 416, 291 (1993)

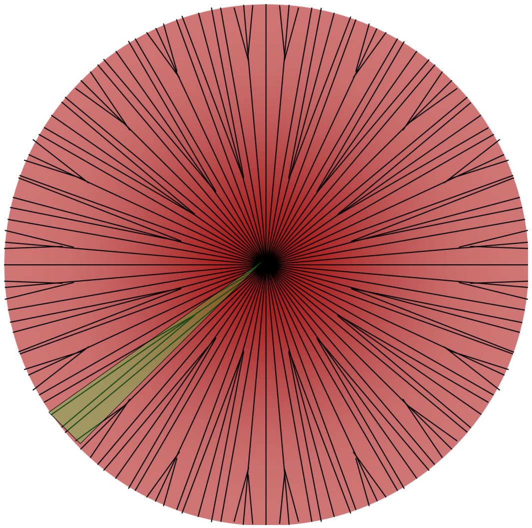
Reduction of Model Space

spherical harmonic oscillator basis

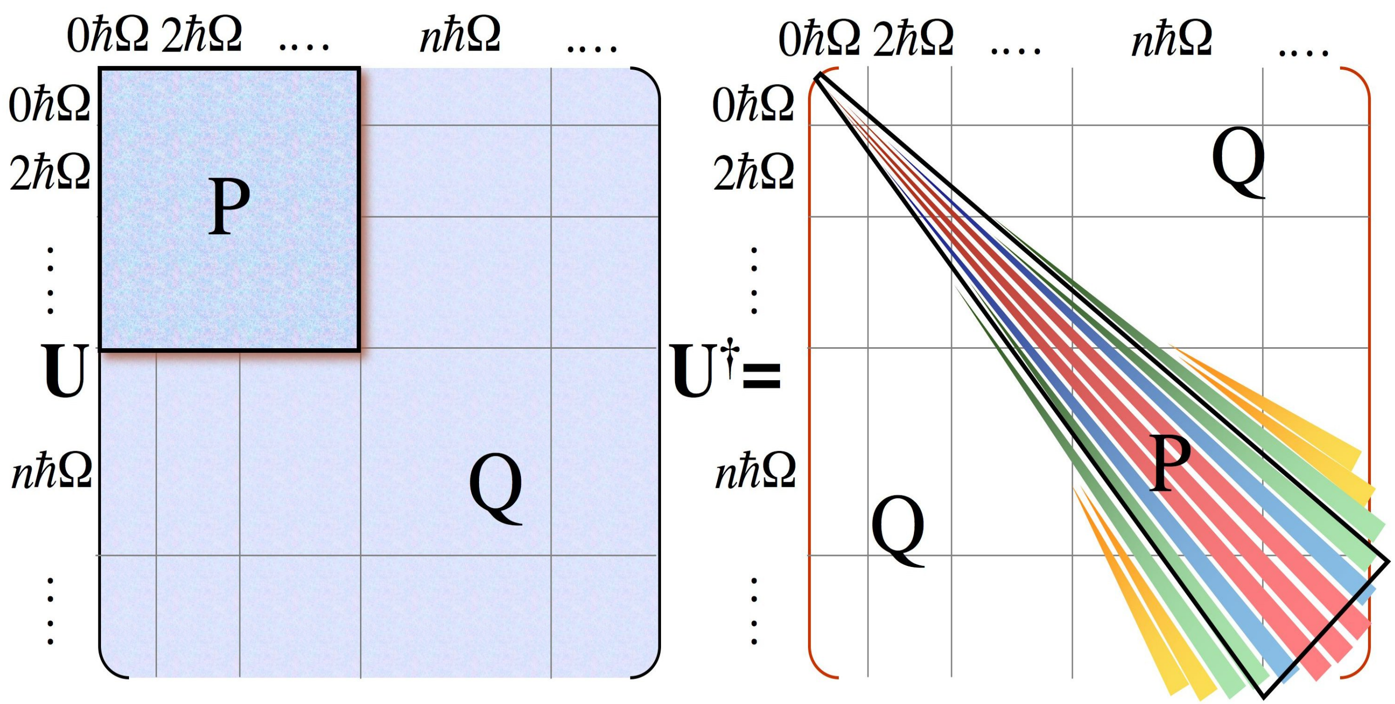


symplectic symmetry-adapted basis

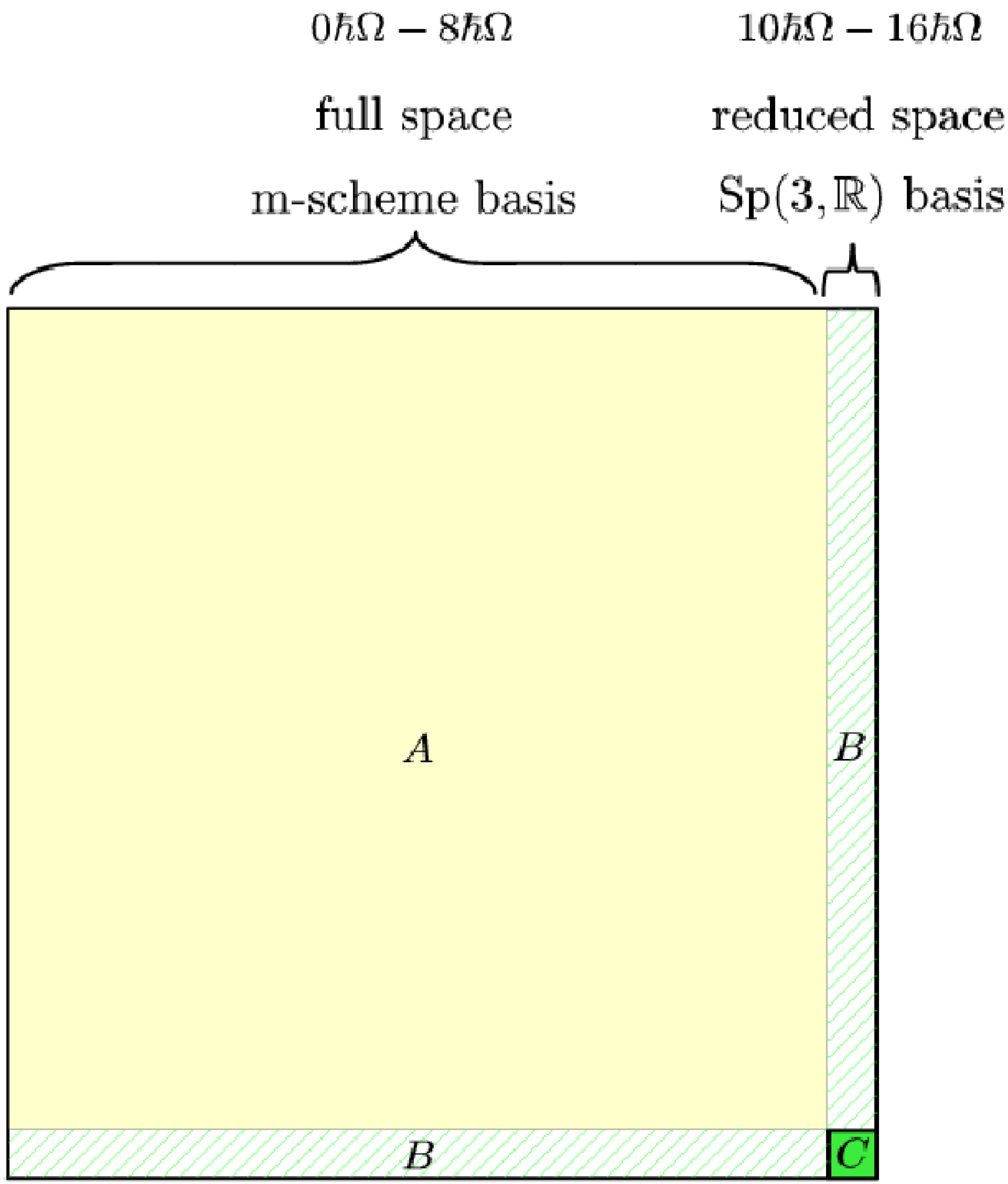
space splits into infinite number of vertical slices



Only fraction of vertical slices are expected to be important

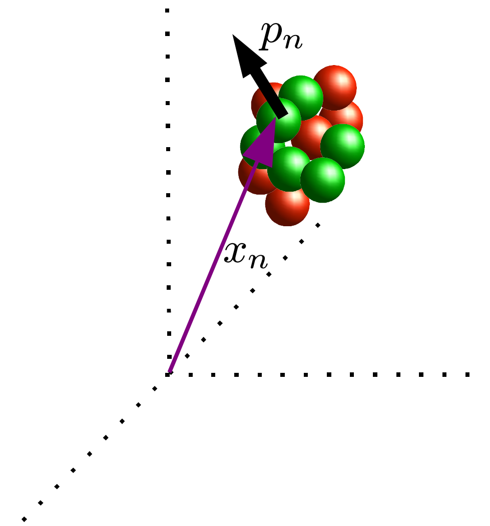


Sp-NCSM approach



Overview of Symplectic Sp(3,R) Symmetry

$\sum_n x_{ni} x_{nj}$	mass quadrupole moments
$\sum_n x_{ni} p_{nj} \pm x_{nj} p_{ni}$	(-) angular momentum
$\sum_n p_{ni} p_{nj}$	many particle kinetic energy

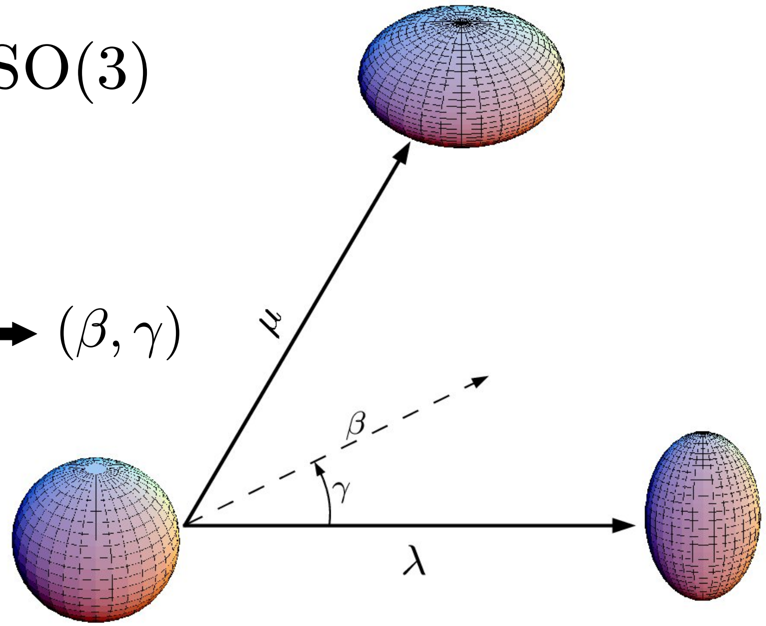


Collective model chain $\mathrm{Sp}(3,\mathbb{R}) \supset \mathrm{GCM}(3) \supset \mathrm{ROT}(3)$

- impractical for expansion in terms of shell model basis

Shell model chain $\mathrm{Sp}(3,\mathbb{R}) \supset \mathrm{SU}(3) \supset \mathrm{SO}(3)$

- expandable in harmonic oscillator basis
- labeled by Elliot's SU(3) quantum numbers $(\lambda\,\mu) \longleftrightarrow (\beta,\gamma)$



Shell Model Chain Basis

Translationally invariant generators of Sp(3,R) can be expressed in terms of harmonic oscillator raising and lowering operators: $b^{\dagger}_{ni} = \frac{1}{\sqrt{2}}(x_{ni} - ip_{ni}) \qquad b_{ni} = \frac{1}{\sqrt{2}}(x_{ni} + ip_{ni})$

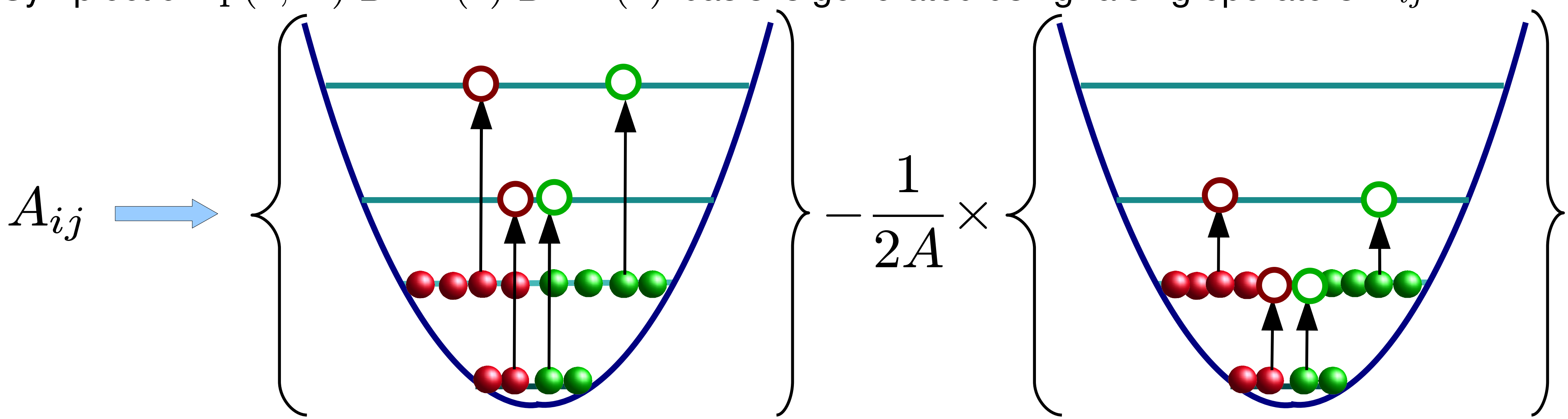
$2\hbar\Omega$ - raising operators $$A_{ij} = \frac{1}{2}\sum_{n=1}^{A} b^{\dagger}_{ni} b^{\dagger}_{nj} - \frac{1}{2A}\sum_{s,t=1} b^{\dagger}_{si} b^{\dagger}_{tj}$$

$2\hbar\Omega$ - lowering operators $$B_{ij} = \frac{1}{2}\sum_{n=1}^{A} b_{ni} b_{nj} - \frac{1}{2A}\sum_{s,t=1} b_{si} b_{tj}$$

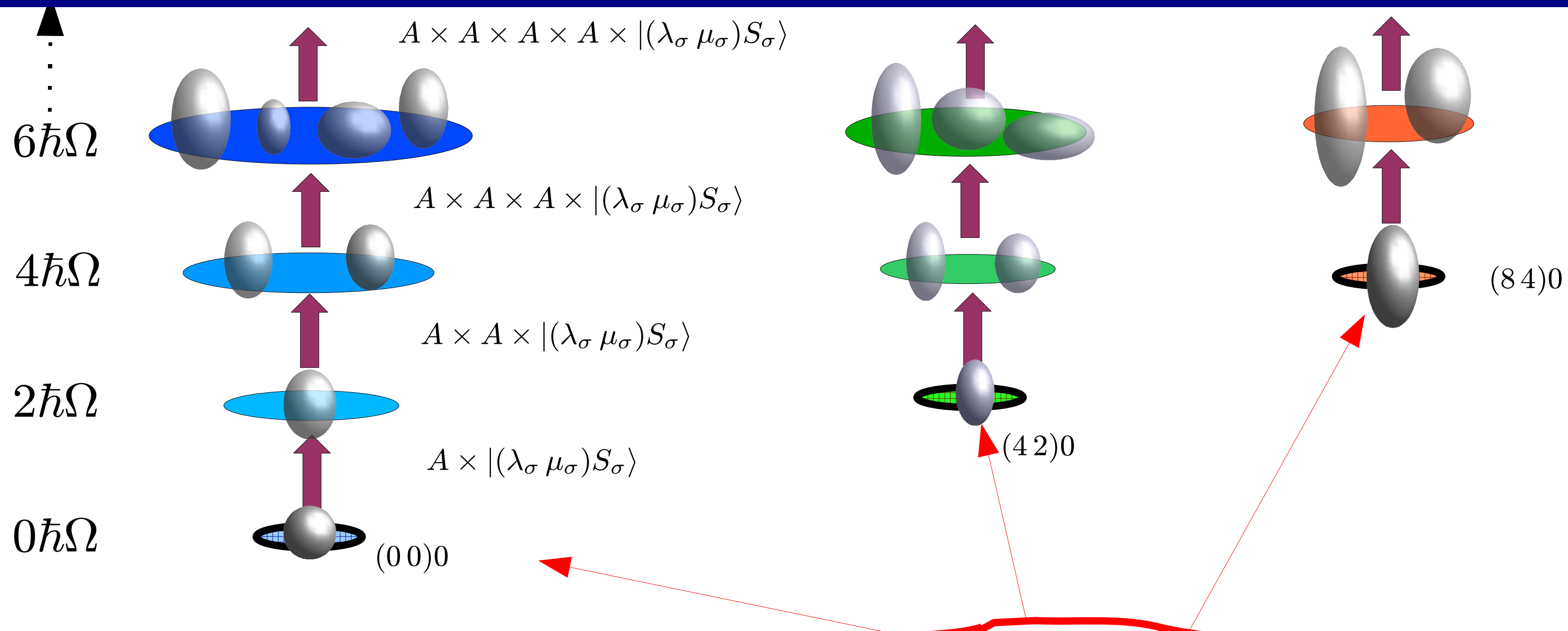
U(3) operators $$C_{ij} = \frac{1}{2}\sum_{n=1}^{A} (b^{\dagger}_{ni} b_{nj} + b_{nj} b^{\dagger}_{ni}) - \frac{1}{2A}\sum_{s,t=1}^{A} (b^{\dagger}_{si} b_{tj} + b_{tj} b^{\dagger}_{si})$$

center of mass removed

Symplectic $\mathrm{Sp}(3,\mathrm{R}) \supset \mathrm{SU}(3) \supset \mathrm{SO}(3)$ basis is generated using raising operators A_{ij}



Construction of Shell Model Chain Basis



Basis states in symplectic "slice" are built over symplectic bandhead by action of raising operators

$$\underbrace{|(\lambda_\sigma\,\mu_\sigma)n(\lambda\,\mu)\kappa\;\; LS_\sigma JM_J\rangle}_{\text{symplectic basis state}} = \big[\underbrace{\mathcal{P}^n(A_{ij})}_{\text{polynomial in raising operatos}} \times \underbrace{|(\lambda_\sigma\,\mu_\sigma)S_\sigma\rangle}_{\text{symplectic bandhead}}\big]^{(\lambda\,\mu)}_{\kappa\;LS_\sigma JM_J}$$

symplectic basis state

polynomial in raising operatos

symplectic bandhead

Symplectic bandhead:

- Expandable in m-scheme basis; labeled by $(\lambda_\sigma\,\mu_\sigma)S_\sigma$
- spurious center of mass excitation free
- annihilated by the symplectic lowering operators $B_{ij}|(\lambda_\sigma\,\mu_\sigma)S_\sigma\rangle = 0$

Expanding Symplectic Bandheads in m-scheme Basis

Single fermion creation operator is $SU(3)\times SU(2)$ irreducible tensor: $a^{\dagger}_{\eta l j m_j} = a^{\dagger(\eta\, 0)}_{l j m_j}$

$$\left[a^{\dagger(\eta_1\, 0)}_{\pi} \times \cdots \times a^{\dagger(\eta_Z\, 0)}_{\pi}\right]^{(\lambda_\pi\, \mu_\pi)}_{S_\pi} \times \left[a^{\dagger(\eta'_1\, 0)}_{\nu} \times \cdots \times a^{\dagger(\eta'_N\, 0)}_{\nu}\right]^{(\lambda_\nu\, \mu_\nu)}_{S_\nu} \Rightarrow \mathcal{P}^{(\lambda_\sigma\, \mu_\sigma)}_{\kappa(L_\sigma S_\sigma) J_\sigma M_\sigma}$$

... Act with $\mathcal{P}$ on vacuum state: $|(\lambda_\sigma\, \mu_\sigma)\kappa(L_\sigma S_\sigma)J_\sigma M_\sigma\rangle = \mathcal{P}^{(\lambda_\sigma\, \mu_\sigma)}_{\kappa(L_\sigma S_\sigma) J_\sigma M_\sigma}|0\rangle$

to obtain a “candidate” on symplectic bandhead ... test whether: $B_{ij}|(\lambda_\sigma\, \mu_\sigma)S_\sigma\rangle = 0$

Expanding Symplectic Bandheads in m-scheme Basis

This procedure does not generate translationally invariant $SU(3)\times SU(2)$ bandheads!

$$\sum_{n=0}^{N} \psi_{cm}(n) \otimes \psi_{int}(N-n)$$

Quick Fix: project out center of mass spuriosity excitations by symmetry preserving operator.

$$\hat{P}(N) = \prod_{k=1}^{N} \left(1 - \frac{\mathcal{B}^{\dagger}_{cm} \cdot \mathcal{B}_{cm}}{k}\right)$$

center-of-mass HO raising and lowering operators

Result: center-of-mass spuriosity free bandhead ... $\psi_{cm}(0) \otimes \psi_{int}(N)$

with the same symmetry

Proof Of Principle

- Calculations performed in symplectic basis achieved good description of low-lying spectra and B(E2) values ... **BUT** ... with simplistic or symmetry preserving phenomenological interactions.

How badly will symplectic symmetry be broken when realistic interactions are employed?

- Project NCSM eigenstates onto symplectic $\mathrm{Sp}(3,\mathrm{R}) \supset \mathrm{SU}(3) \supset \mathrm{SO}(3)$ basis.

✔Trivial task if we find expansion of symplectic states in terms of m-scheme basis

Example: ^{4}He

proton single particle states

neutron single particle states

$$|2\hbar\Omega\ (2\,0) L{=}2 J{=}2 M_J{=}2\rangle \Rightarrow \frac{1}{2}\,|0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}; 0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 2\,2\,\tfrac{5}{2}\,\tfrac{5}{2}\rangle$$
$$-\sqrt{\frac{1}{5}}\,|0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}; 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}, 2\,2\,\tfrac{3}{2}\,\tfrac{3}{2}\rangle$$
$$-\sqrt{\frac{1}{20}}\,|0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}; 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}, 2\,2\,\tfrac{5}{2}\,\tfrac{3}{2}\rangle$$
$$+\frac{1}{2}\,|0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 2\,2\,\tfrac{5}{2}\,\tfrac{5}{2}; 0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$
$$-\sqrt{\frac{1}{5}}\,|0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}, 2\,2\,\tfrac{3}{2}\,\tfrac{3}{2}; 0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$
$$-\sqrt{\frac{1}{20}}\,|0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}, 2\,2\,\tfrac{5}{2}\,\tfrac{3}{2}; 0\,0\,\tfrac{1}{2}\,-\tfrac{1}{2}, 0\,0\,\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

Example: ^{4}He

$0\hbar\Omega$

Start with symplectic bandhead (0 0)S=0

$$\left|(0\,0)L=0\,J=0\,M_J=0\right\rangle = \left|0\,0\tfrac{1}{2}\,\tfrac{1}{2},0\,0\tfrac{1}{2}\,-\tfrac{1}{2};0\,0\tfrac{1}{2}\,\tfrac{1}{2},0\,0\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$

$2\hbar\Omega$

Construction formula is trivial:

$$\left|(2\,0)L=2\,J=2\,M_J=2\right\rangle = A^{(2\,0)}_{2\,2}\left|(0\,0)L=0\,J=0\,M_J=0\right\rangle$$

$$\vdots$$

$$\left|(2\,0)L=0\,J=0\,M_J=0\right\rangle = A^{(2\,0)}_{0\,0}\left|(0\,0)L=0\,J=0\,M_J=0\right\rangle$$

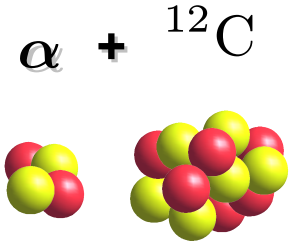
$4\hbar\Omega$

Apply raising operator on symplectic states generated at $2\hbar\Omega$ subspace

$$\begin{aligned}\left|(4\,0)L=2\,J=2\,M_J=2\right\rangle = & -\sqrt{\frac{4}{63}}\,A^{(2\,0)}_{2\,0}\left|(2\,0)L=2\,J=2\,M_J=2\right\rangle \\ & +\sqrt{\frac{2}{21}}\,A^{(2\,0)}_{2\,1}\left|(2\,0)L=2\,J=2\,M_J=1\right\rangle \\ & +\sqrt{\frac{7}{18}}\,A^{(2\,0)}_{0\,0}\left|(2\,0)L=2\,J=2\,M_J=2\right\rangle \\ & + A^{(2\,0)}_{2\,2}\left(-\sqrt{\frac{4}{63}}\left|(2\,0)L=2\,J=2\,M_J=0\right\rangle+\sqrt{\frac{7}{18}}\left|(2\,0)L=0\,J=0\,M_J=0\right\rangle\right)\end{aligned}$$

Symplectic states within $N\hbar\Omega$ subspace are generated using $(N-2)\hbar\Omega$ symplectic states

$\alpha-$ Cluster Model of ^{16}O



- Constituent clusters "frozen" to ground states.
- Relative motion of clusters carries Q oscillator quanta.

Few facts about symplectic states and α -cluster model wave functions:

- Deformed symplectic states possess appreciable overlaps with cluster wave functions.
- Overlap 100% for the most deformed symplectic bandheads.
- "0p-0h" Sp(3,$\mathbb{R}$) "slices" are not sufficient to reproduce $\alpha-$cluster modes.

We need to incorporate Sp(3,$\mathbb{R}$) "slices" build over highly deformed symplectic bandheads.

Y. Suzuki, Nucl. Phys. **A448**, 395 (1986).

Results

NCSM eigenstates: obtained with JISP16 interaction, $N_{\max}=6$ model space; ^{16}O and ^{12}C

(1) The ground-state band in ^{12}C

(2) The ground state of ^{16}O

- reasonably well converged
- $0\hbar\Omega$ configurations dominate

(3) The first 0^{+} excited state of ^{16}O

- not converged yet
- $2\hbar\Omega$ configurations dominate
- only a test of symplectic symmetry

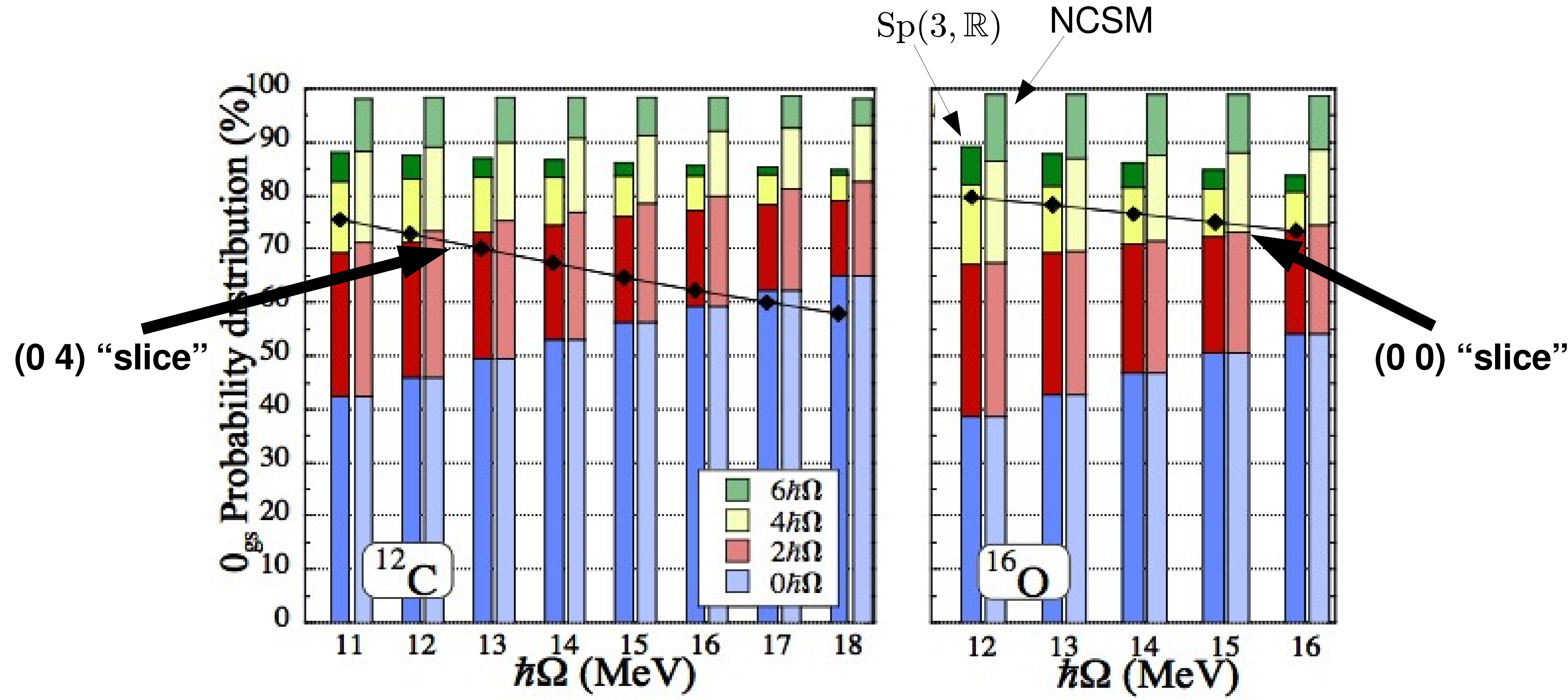
Sp(3,$\mathbb{R}$) model space includes:

All symplectic "slices" built over $0\hbar\Omega$ (0p-0h) and $2\hbar\Omega$ (2p-2h) bandheads

"Slice" built over the most deformed $4\hbar\Omega$ (4p-4h) bandhead

Generated up to $N_{\max}=6$ model space

Probability Distribution: Ground State 85%-90%



^{12}C

^{16}O

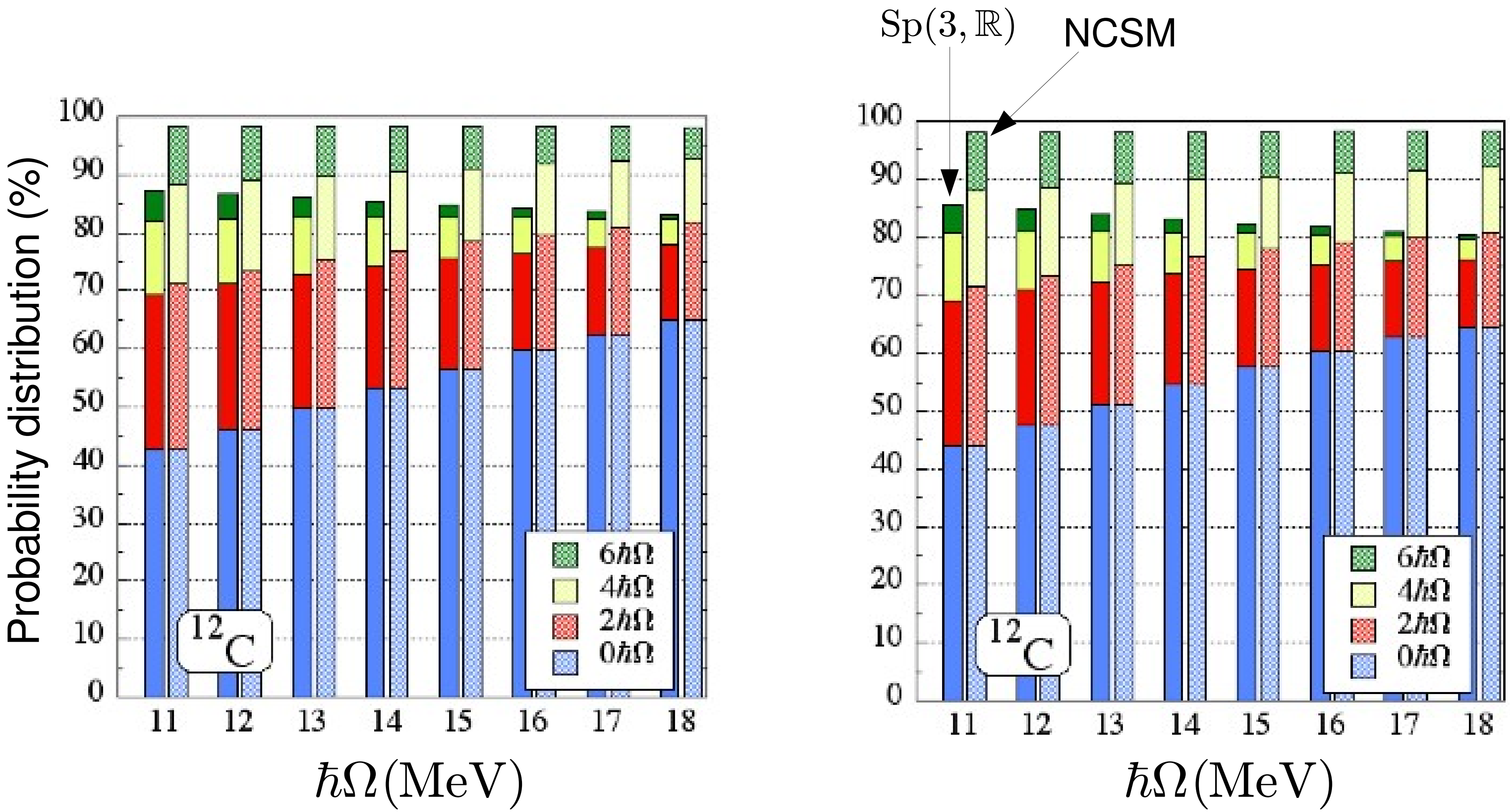
Only 3 "slices" built over 0p-0h bandheads: 80%

"slices" built over 2p-2h bandheads: 5%

Single (0 0) "slice": 75%

"slices" built over 2p-2h bandheads : 10%

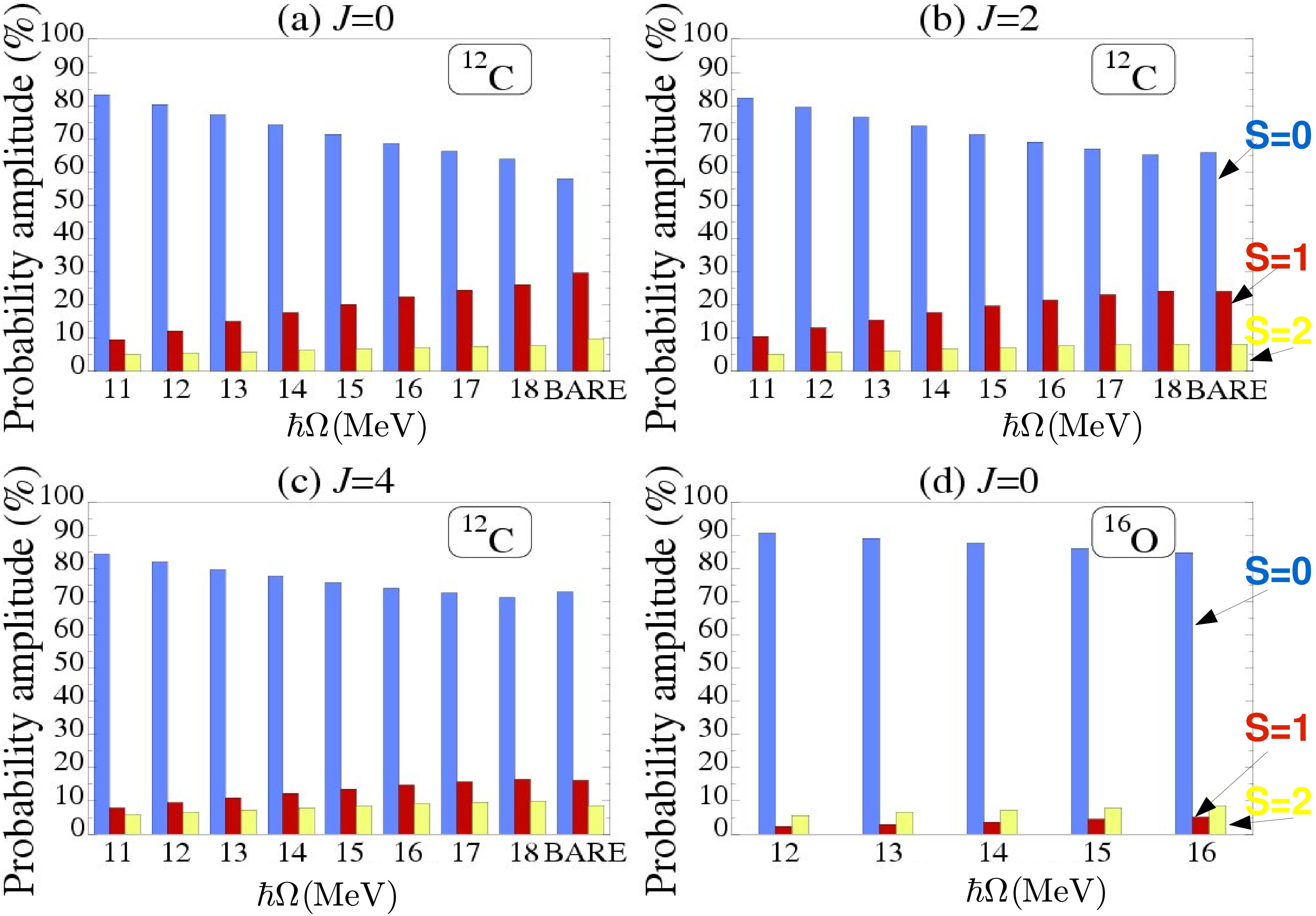
Probability Distribution: 2^+ and 4^+



Only 3 “slices” built over 0p-0h bandheads: 80%

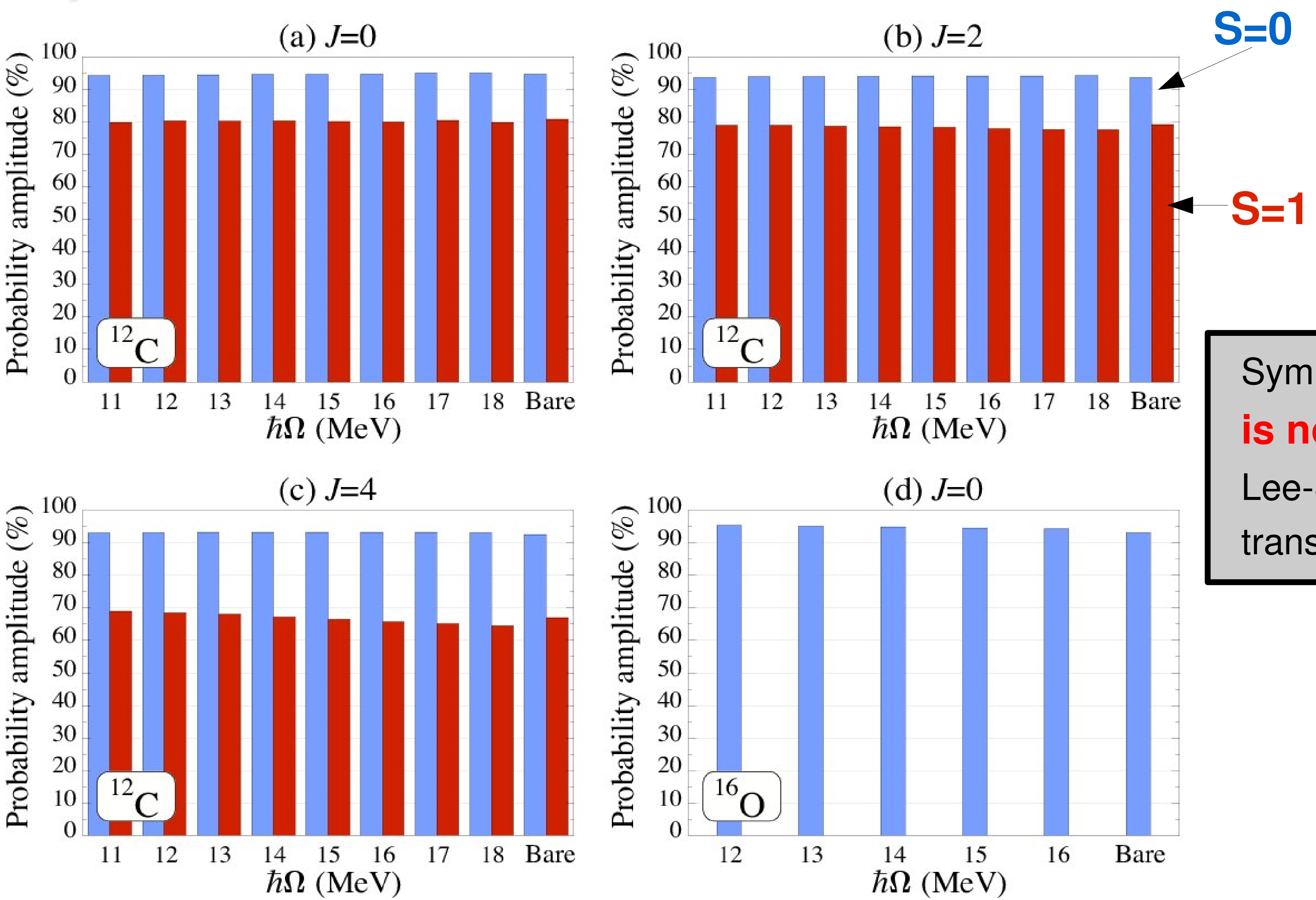
“slices” built over 2p-2h bandheads: 4%

Spin Distribution in NCSM eigenstates



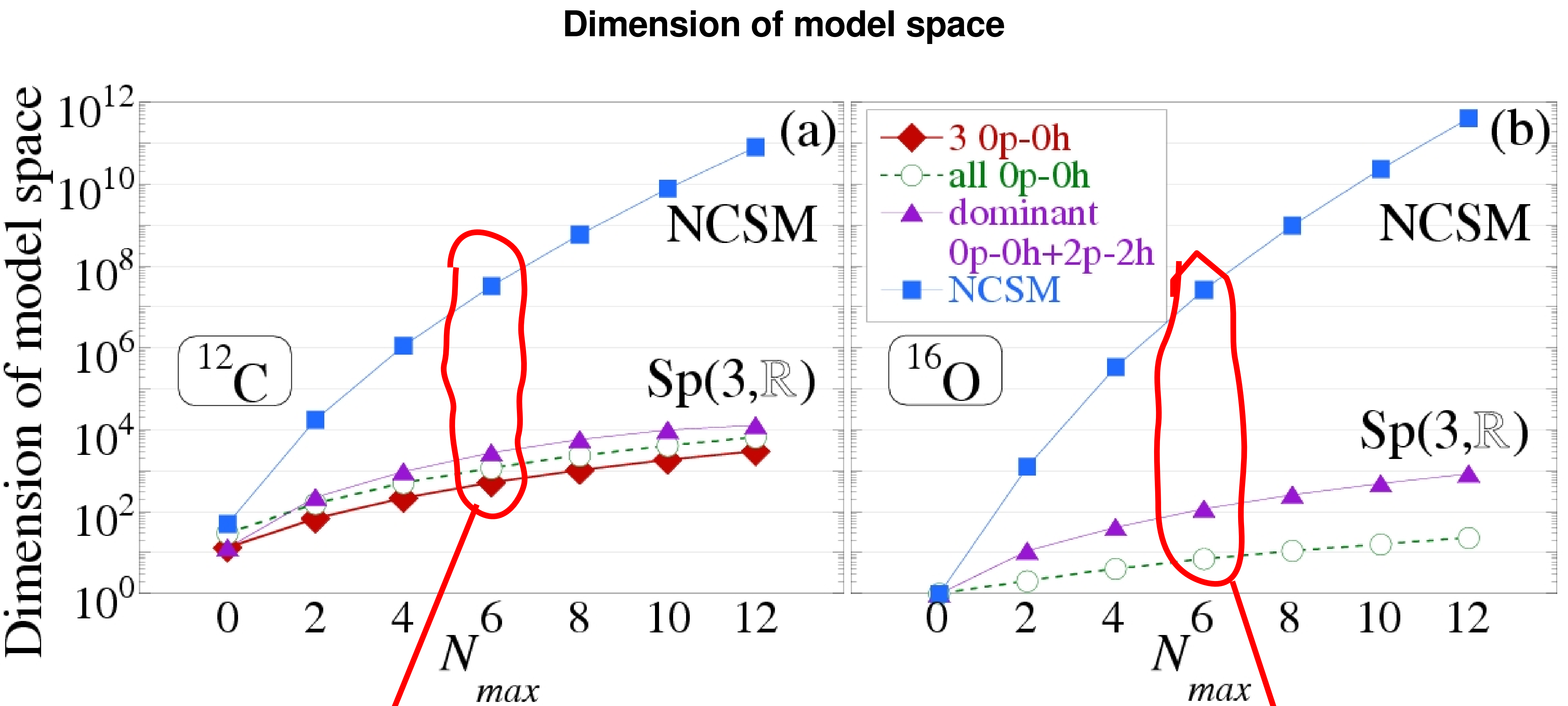
Independence of Oscillator Strength

- 6 spin S=0 symplectic "slices" compose 95% of S=0 component of NCSM eigenstates
- Independent of oscillator strength
- Same results for the bare interaction



Symplectic symmetry **is not altered** by Lee-Suzuki transformations

Major Reduction in Model Space



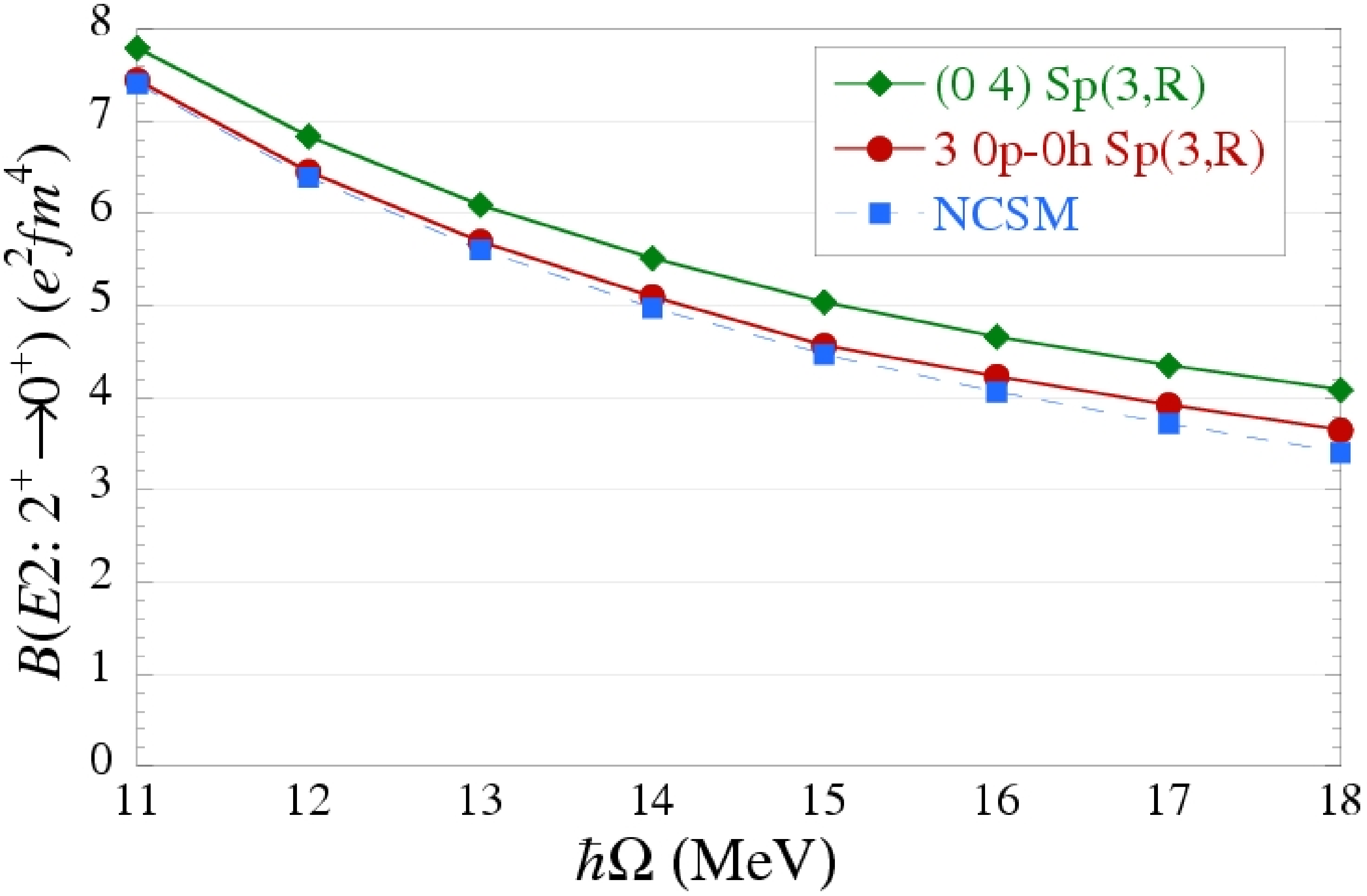
Reduction of model space size:

10^{-5} **for** $^{12}\mathrm{C}$

10^{-6} **for** $^{16}\mathrm{O}$

... and gets even better for higher model spaces ....

B(E2:$2^+ \to 0^+$)

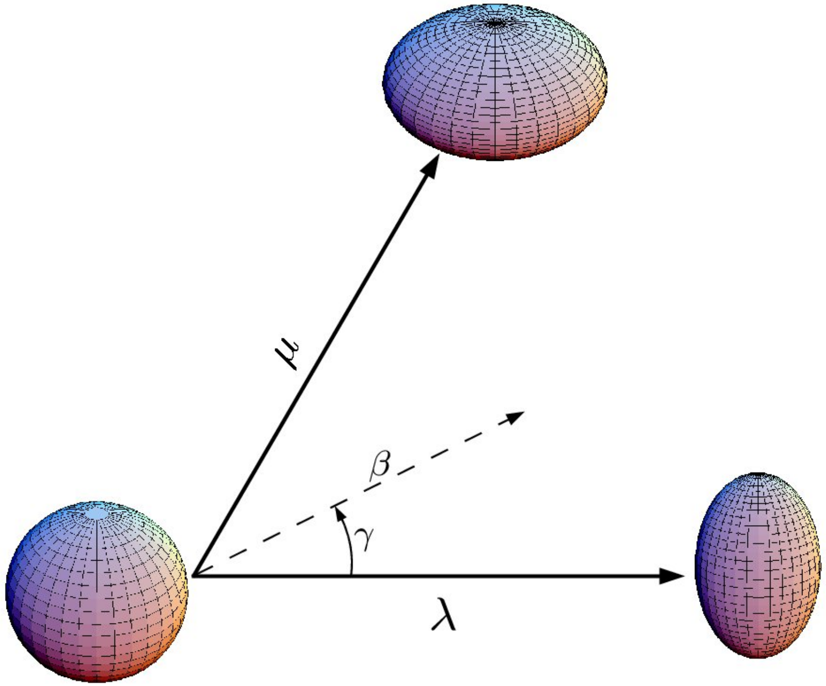


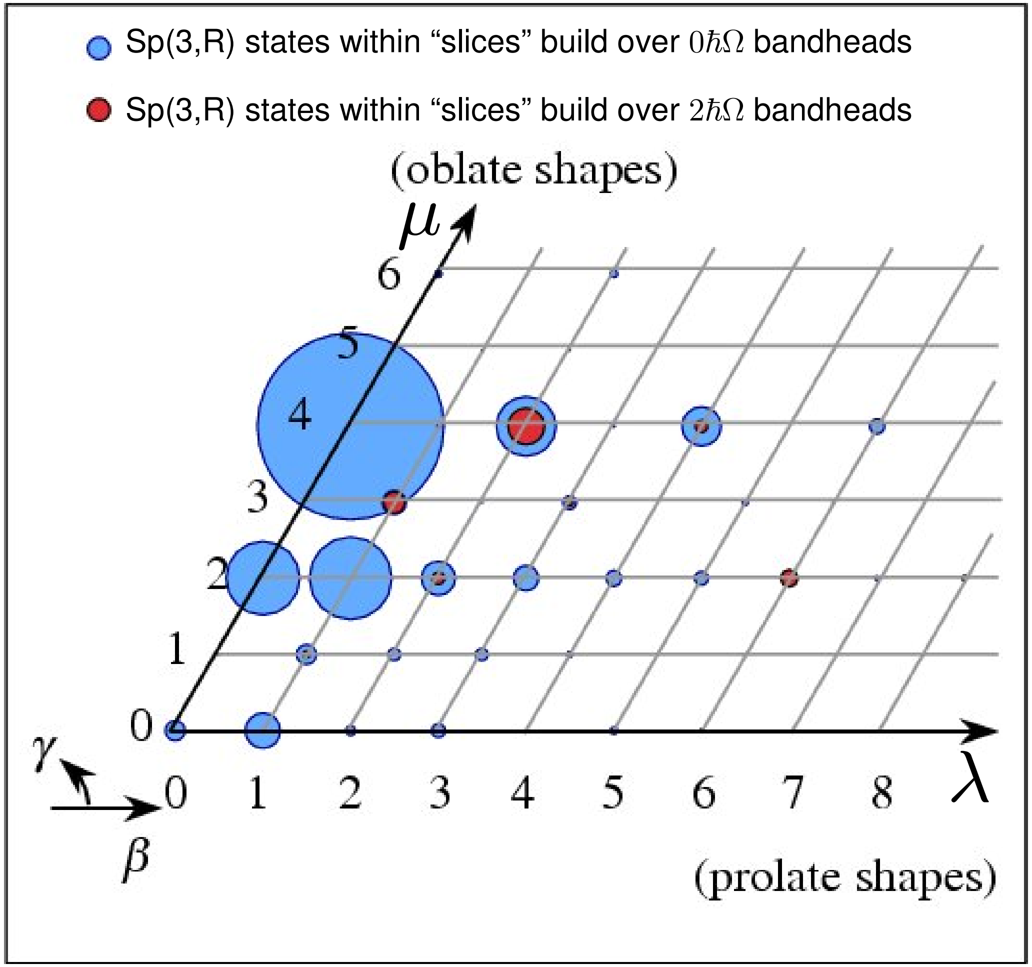
Model space: $N_{\mathrm{max}} = 6$

the most dominant Sp(3,R) slices reproduce NCSM results

Deformations present within NCSM eigenstates

ground state of ^{12}C



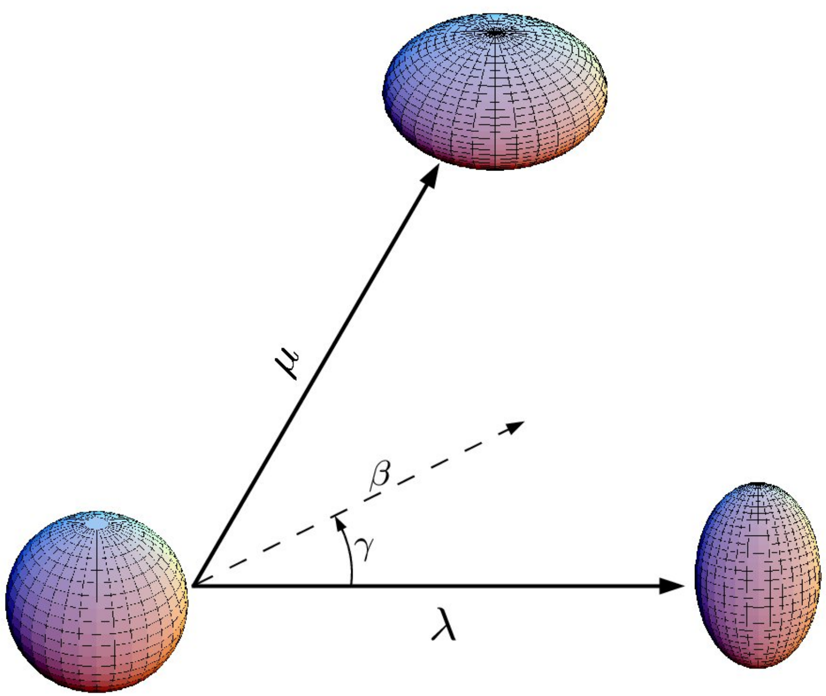


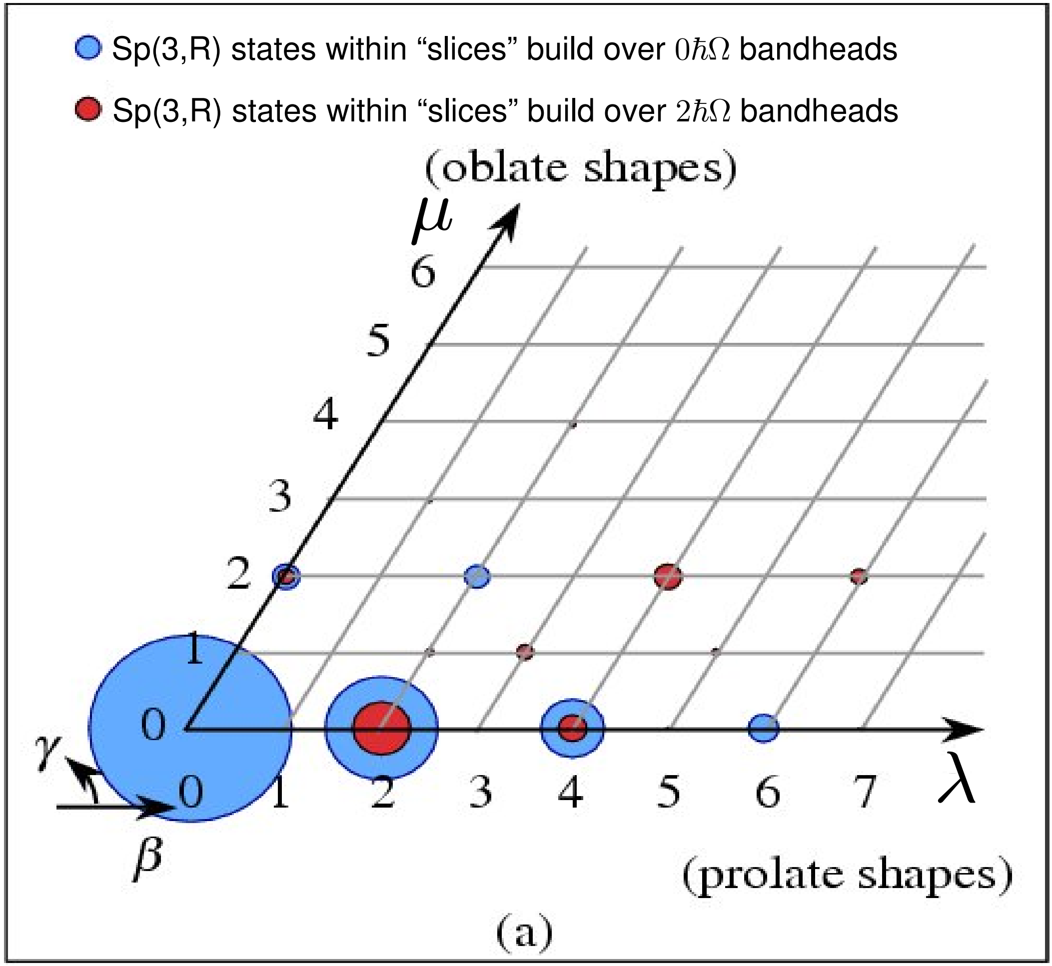
Near oblate deformed shapes dominate:
(0 4), (1 2), (0 2), (2 4)

Area $\propto$ probability of given symplectic states

Deformations present within NCSM eigenstates

ground state of ^{16}O





Spherical shape dominates: (0 0)

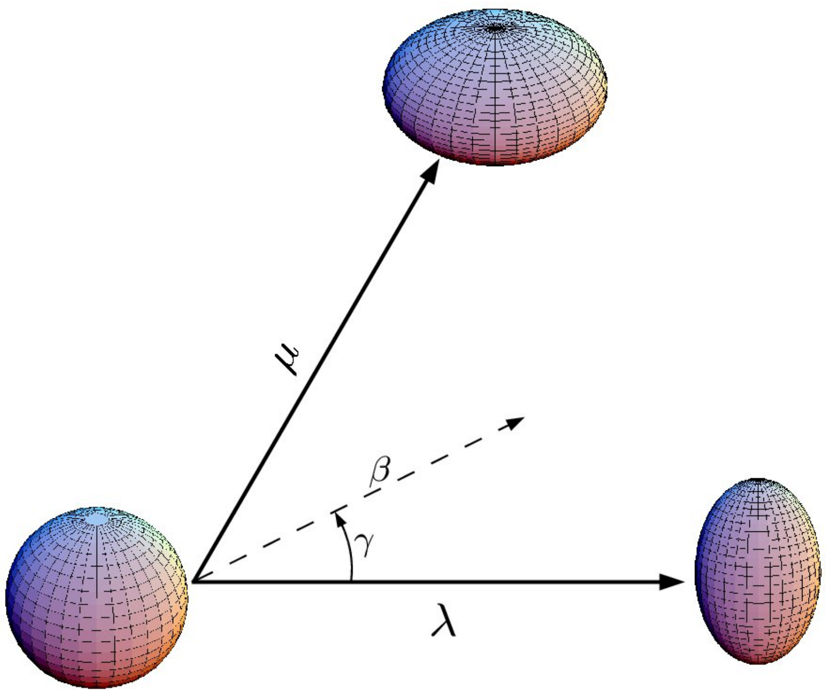
Prolate deformation present:

(2 0), (4 0) and (6 0)

Area $\propto$ probability of given symplectic states

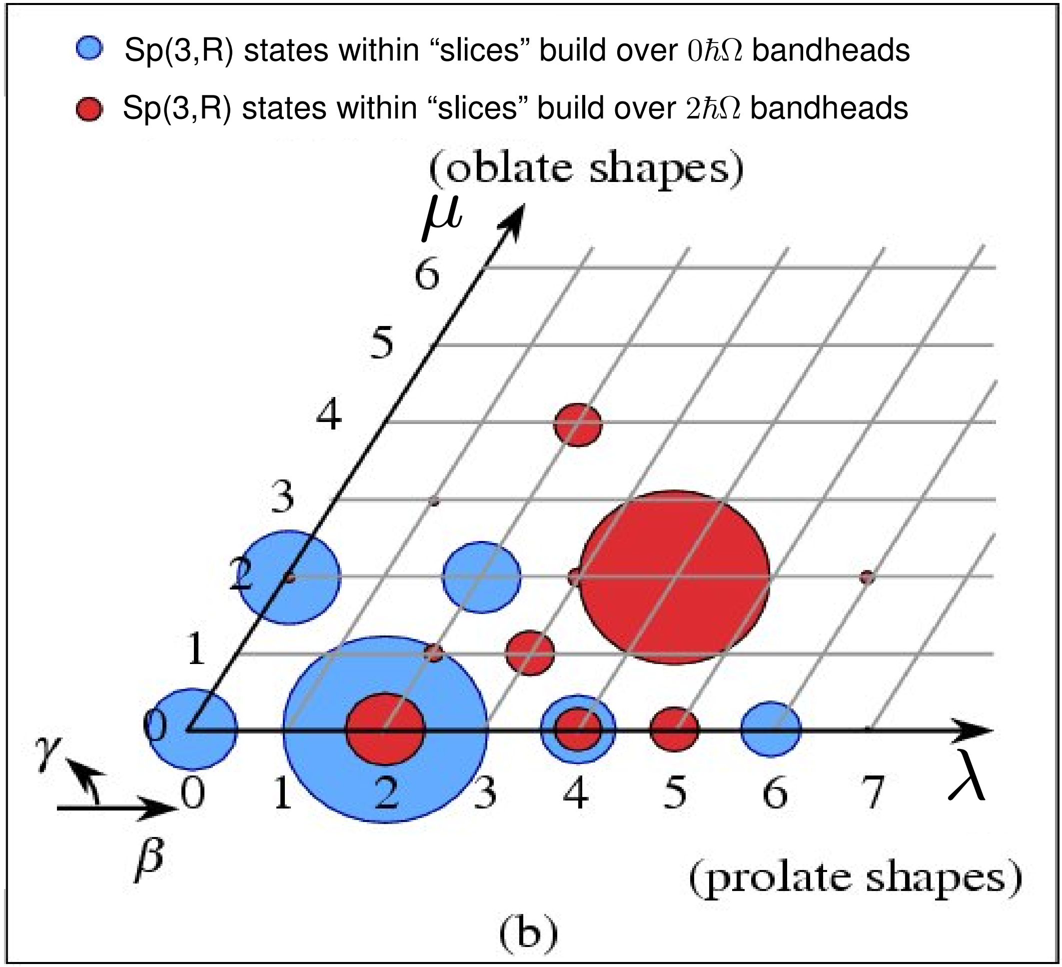
Deformations present within NCSM eigenstates

first 0^+ excited state of ^{16}O



Interplay between $0\hbar\Omega$ (blue) and $2\hbar\Omega$ symplectic "slices" (red)

4p-4h symplectic slices negligible



Area $\propto$ probability of given symplectic states

Summary

Ab-initio No Core Shell Model: successfully reproduces (low-lying) features of the deuteron, alpha particle, $^{12}\mathrm{C}$ and even $^{12}\mathrm{O}$

Comparison of converged NCSM eigenstates with Sp(3,R)-symmetric states shows:

- Reproduction of NCSM results by a few Sp(3,R) states
  - ✔85%-90% overlaps
  - ✔100% $B(E2; 2^+_1 \rightarrow 0^+_1)$
- Dramatic reduction in model space (several orders of magnitude)

Symplectic-NCSM: effective model space reduction scheme.