The Symmetry Energy in Nuclei and Nuclear Matter

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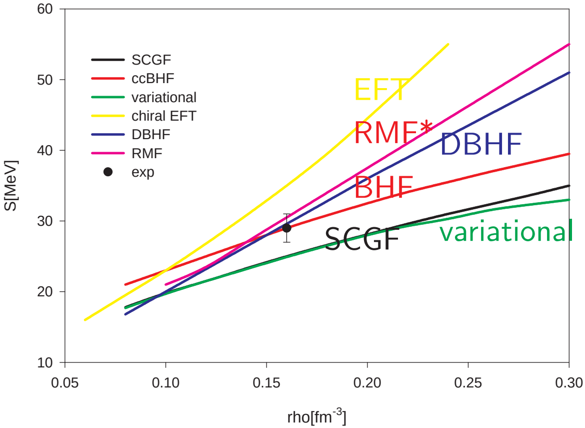
Density dependence of SE in nm

$$S(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}|_{\alpha=0} = S_v + \frac{p_0}{\rho_s^2}(\rho-\rho_s) + \frac{\Delta K}{18\rho_s^2}(\rho-\rho_s)^2 + \ldots$$



$\alpha = (N-Z)/A$

ρ_s : saturation dens

two-body NN only

Main uncertainty:

three-body force

at higher densities

What about constraints from nuclei?

Relevance: Neutron stars: $P(\rho \sim \rho_s) \approx \rho_s^2 S'(\rho_s)$ and $x_s(\rho) \sim (S/\hbar c)^3$

Nuclear symmetry energy from nuclei

Issue: Density dependence of symmetry energy at subsaturation densities

Two methods: Microscopic ("mean field" using effective Lagrangian)
Phenomenological approach (this talk)

Content:

- Extended Liquid Drop Model formula
- Volume and Surface Symmetry Energy
- Shell effects, Wigner energy, Coulomb energy
- Information from Neutron skin
- SE in Nuclear matter

Recent work by
Steiner et al, Danielewicz, Jänecke, ...

Symmetry energy

Energy density $E(\rho, \alpha = (N-Z)/A)$

Expand in asymmetry $E(\rho,\alpha) = E(\rho,0) + S(\rho)\alpha^2 + O(\alpha^4) + ...,$

Expand around saturation density ρ_s

$E(\rho,0) = E_0 + \frac{K}{18\rho_s^2}(\rho-\rho_s)^2 + ...,$

$$S(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}|_{\alpha=0} = S_v + \frac{p_0}{\rho_s^2}(\rho-\rho_s) + \frac{\Delta K}{18\rho_s^2}(\rho-\rho_s)^2 + ...$$

$S_v \approx 30 \pm 2\text{MeV}$; p_0: little info

ΔK : recently from ISGMR in Sn isotopes: $E_{GMR} = \sqrt{\frac{\hbar^2 K_A}{mR^2}}$

compressibility $K_A(N,Z) = K_\infty + \frac{(N-Z)^2}{A^2}K_\tau + K_{Coul}\frac{Z^2}{A^{4/3}} + ..,$

exp: $K_\tau = -440 \pm 40$ MeV (Garg et al)

hence $\Delta K = K_\tau + 9p_0/\rho_s \approx -340$ MeV

Bethe-Weizsäcker formula

- Conventional Bethe-von Weizsäcker (liquid drop) formula

$$E_A = -a_B A + a_{surf} A^{2/3} + a_{sym}(N - Z)^2/A + a_C \frac{Z^2}{A^{1/3}} + E_{pair}$$

- Incomplete form of symmetry energy
  (noted by Bohr-Mottelson, Myers, Danielewicz,..)
  Need to introduce volume and surface symmetry energy
- Coulomb needs to be refined
- shell corrections need to be added

Symmetry energy in Bethe-Weizsäcker

- Improved BW: $E_{sym} = E(vol, surf)$
  Decompose $N - Z = N_s - Z_s + N_v - Z_v$
  $E^A_{surf} = E^0_{surf} + S_{surf}(N_s - Z_s)^2/A^{2/3}$
  $E^A_{vol} = a_B A + S_{vol}\frac{(N_v - Z_v)^2}{A}$
  minimize $(E_v + E_s)$ under fixed $N - Z$:
  $\frac{N_s - Z_s}{N_v - Z_v} = A^{-1/3} S_v/S_s = A^{-1/3} y$
  $$E_A = -a_B A + a_{surf} A^{2/3} + \frac{S_v}{1 + yA^{-1/3}}(N - Z)^2/A + \ldots$$
- LDM yields also relation between skin ΔR, and S_s, S_v
  $$\frac{R_n - R_p}{R} = \frac{A(N_S - Z_S)}{6NZ} = \frac{A}{6NZ}\frac{N - Z - a_c Z A^{2/3}}{1 + A^{1/3}/y}$$
  note: depends on y only
  Coulomb term!

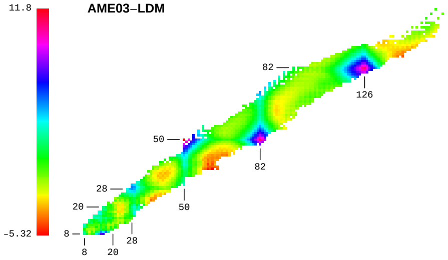
Danielewicz, NPA 727(2003)233; Steiner et al, Phys Rep 411,325

LDM: shell corrections

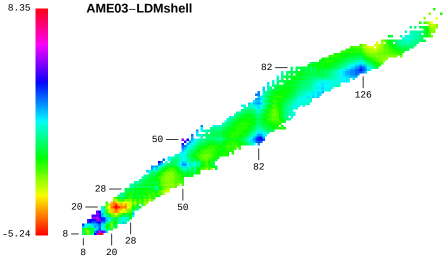
Further corrections to LDM:

1. shell effects: simple parametrization (x_v : # valence nucleons) works

$E_{shell} = a(n_v + z_v) + b(n_v + z_v)^2 + c(z_v . n_v)$ $c \approx 0 \rightarrow 2$ par's a, b



BE(exp)-BE(LDM-fit)



BE(exp)-BE(LDM+ Shell corr)

data base: Audi-Wapstra(2003) (2200 nuclei)

Similar (but simpler) to Duflo-Zuker; rms dev. reduced 50% to 1.3MeV

magic numbers: $N_0, Z_0 = 8, 20, 28, 50, 82, 126$ (note: $Z = 40$)

beyond midshell particles are counted as holes

LDM: shell corrections(2)

- Similar to Interacting Boson Model (IBA) (F-spin)
  $\Delta E_{sh} = aF_{max} + bF_{max}^2$ with $F_{max} = n_v + z_v$
- Can be improved by introducing two types of c.s. (Duflo - Zuker)
  (i)harmonic oscillator $(N_0, Z_0 = 2, 8, 20, 40, ..)$
  (ii) L.S $(N_0', Z_0' = 2, 6, 14, 28, 50, ..)$
  (the "high-spin", $j = l_m + 1/2$, intruder orbital included in lower shell)

Example: Replacement of "20" by "14" improves fit by 10%

LDM: Wigner energy

2. Treatment of neutron-proton correlations require more care.
Replace $(N-Z)^2$ by $4\hat{T}^2 = 4T(T+1)$??

Additional linear term, known as Wigner energy, can be present

$$B_{\rm w}(N,Z) = -W(A)|N-Z| - d(A)\delta_{N,Z}\pi_{\rm np}$$

Origin: overlap neutron and proton wave functions maximal if $N = Z$

small A: In supermultiplet theory (SU(4), no $L.S$) : $E \sim T(T+4)$
compared to "charge independence" (SU(2)): $E \sim T(T+1)$
large A: $(N-Z)^2 \rightarrow T(T+r)$ with r parameter (Jänecke)
(found to vary with region, in cases $r \sim 4$)
affects value of $a_{sym} = E/T(T+r)$

Inclusion of shell effects leads to $r \approx 1$

Wigner energy vs shell effects

Distinguish 2 situations

1) neutrons are particles and protons holes (or vv)

e.g. $66 < Z < 82, \quad 82 < N < 104$

shell corrections expressed in terms of isospin:

$n_v = n_n + n_p = \Omega - (N - Z) = \Omega - 2T$

Hence $\Delta E_{\rm shell}(N, Z) = an_v + bn_v^2 + c$
can be absorbed into $a_{\rm sym} T(T + r)$.

2) For pp (hh) situation: No net correction

Coulomb energy

3. Coulomb energy $E_C = \frac{Z^2}{R_c}$ needs to be refined
corrections for exchange, diffuseness etc
Various approaches

- charge radius $R_c = r_0 A^{1/3} \rightarrow R_c = r_0 A^{1/3}(1 + a\frac{N-Z}{A} + ..)$
(Duflo-Zuker; a fitted to charge radii)
- $Z \rightarrow Z_v + Z_s$ then $E_C = \frac{e^2}{R}(\frac{3}{5}Z_v^2 + Z_v Z_s + \frac{1}{2}Z_s^2) + E_{diff}$ (Danielewicz)
uniform sphere ↑ ↑ uniform shell
leads to $E_c \approx \frac{Z(Z-1)}{A^{1/3}(1+\Delta)}$ $\quad \Delta \approx \frac{N-Z}{A}\frac{1}{1+yA^{1/3}}$

* qualitatively similar
* improves fit to masses (rms dev by 10% ,no additional par's)
* if pure isospin: $\Delta R = R_n - R_p \approx 2a\frac{N-Z}{A}$
* important constraint: Coulomb displacement energies (CDE)

IAS energies

Fit to complete LDM formula not practical : too many par's...

eliminate isoscalar, pairing, Coulomb, .. by using differences of masses
Two ways to proceed

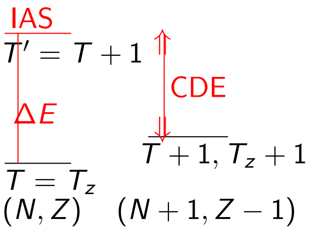
1. Isobaric analogue states (IAS) (Danielewicz, Jänecke)

$$E_A \sim \frac{S_v}{1+yA^{-1/3}} \cdot 4T(T+1)$$

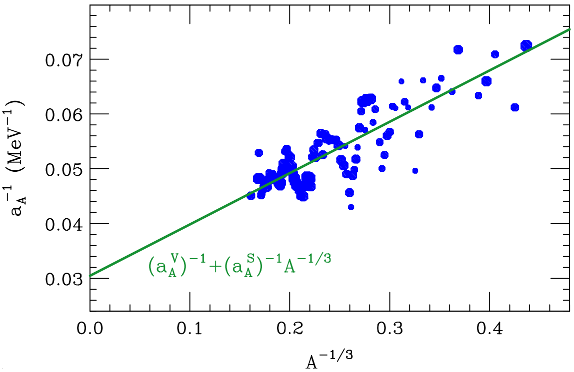
Take difference, invert

$${S_v}^{-1} + {S_s}^{-1} A^{-1/3} = \frac{4[T'(T'+1) - T(T+1)]}{A \Delta E}$$

(assuming charge independence, r=1)



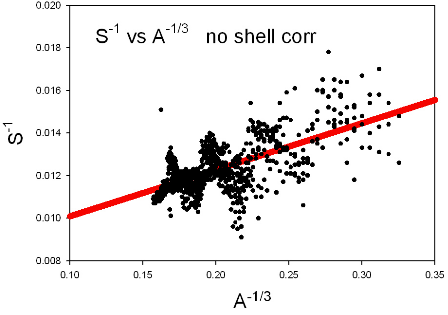
S_{vol}, S_s from fit to IAS

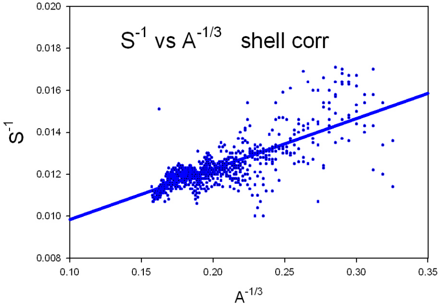


$S_A^{-1} = 1/S_v + A^{-1/3}/S_s$ vs $A^{-1/3}$ from Danielewicz Nucl-th/0411115

$S_{vol} \sim 31 \pm 2$ MeV (cf fit to gs energies 29 ± 2MeV
Values of S_v and S_s correlated!

Computed IAS energies from estimated Coulomb displacement energies





$S_v = 31.4 \pm 1.9$MeV

$S_v = 33.6 \pm 1.4$ MeV

Use of isovector chem potential

2. Use n-p separation energies (Steiner)

Isovector chemical potential $\frac{dE}{d\rho_n} - \frac{dE}{d\rho_p}$

$$\mu_a = \mu_n - \mu_p = \frac{1}{2}[B(N+1,Z) - B(N,Z+1) + B(N-1,Z) - B(N,Z-1)]$$

From LDM formula: $E \approx \frac{(N-Z)^2}{A} S_A + a_C(\frac{Z^2}{A^{1/3}} + ...)$

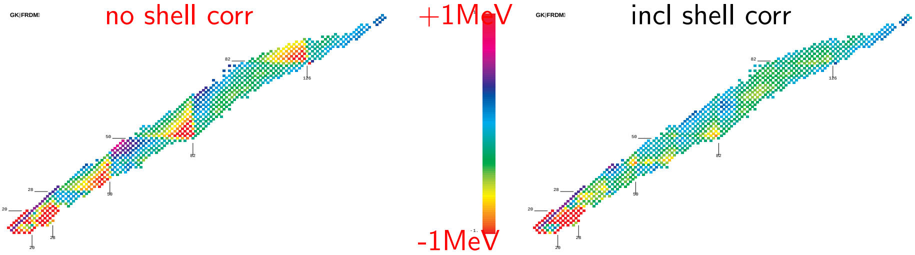
$$\mu_a = 2\frac{N-Z}{A} S_A + 2a_c[\frac{Z-1}{(A-1)^{1/3}} + \frac{Z+1}{(A+1)^{1/3}}]$$

Leads to 2-par fit: $S_A = \frac{S_v}{1+yA^{-1/3}}$

Add shell corrections, and Wigner term $(N - Z \rightarrow N - Z + 1)$

result for separation energies

$$\mu_a(exp) - \mu_a(fit) \text{ (MeV)}$$

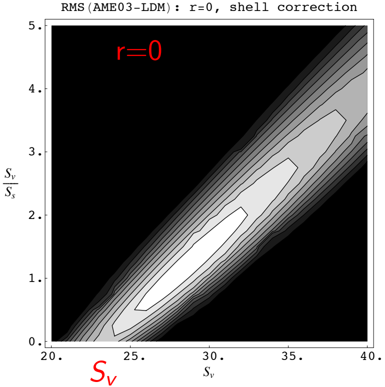


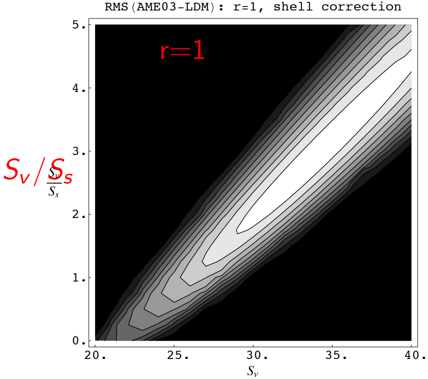
$S_v = 30.1 \pm 0.7$ MeV, $y = 2.8$ $\quad$ $S_v = 31.6 \pm 0.8$ MeV $y = 2.4$

$$S_A = \frac{S_v}{1+yA^{-1/3}} \approx \frac{A}{N-Z}\mu_a + E_C$$

Correlations

Correlations between S_s and S_v from LDM fit to masses





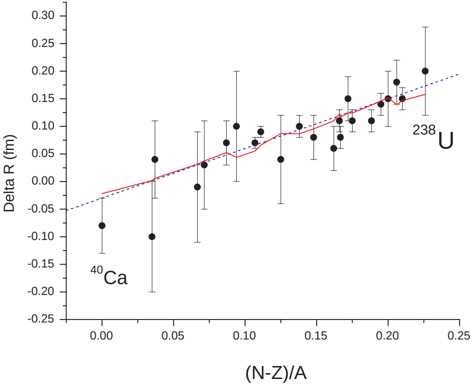
each contour: 100keV increase

Explanation $\frac{S_v}{1+S_v A^{-1/3}/S_s} \rightarrow < A^{1/3} \approx 6 > \rightarrow S_v/S_s = a + bS_v$

Way out: use neutron skin data: $y = S_v/S_s \sim \Delta R$

In LDM direct relation between neutron skin and surface asymmetry

$$\frac{R_\mathrm{n} - R_\mathrm{p}}{R} = \frac{A(N_\mathrm{s} - Z_\mathrm{s})}{6NZ} = \frac{A}{6NZ}\,\frac{N - Z - a_c Z A^{2/3}(12S_\mathrm{v})^{-1}}{1 + y^{-1}A^{1/3}}$$



data from anti-protonic atoms (Trzcinska et al, PRL 87) ..... $\Delta R = -.03 + .90\frac{N-Z}{A}$

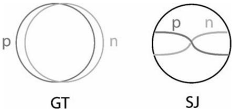
— $y = 1.9 \pm 0.2$ (our fit)

shell effects ignored

isovector GDR

Isovector dipole resonances contain info on SE

Two extreme models Steinwebel-Jensen (volume oscillation, $S_s = 0$) and Goldhaber-Teller (surface osc, $S_v = 0$) modes.

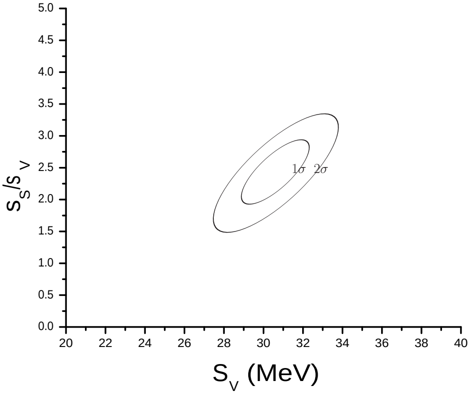


$$E_{\mathrm{GDR}} = \sqrt{\frac{6(1+K)S_v}{m\langle r^2\rangle(1+\frac{5}{3}y_s A^{-1/3})}}$$

Main limitation: Exp uncertainty in E_{GDR}
New Pygmy GDR: oscillation of neutron excess wrt core?

Result for S_v, S_s

Combined results from sep. energies and skin



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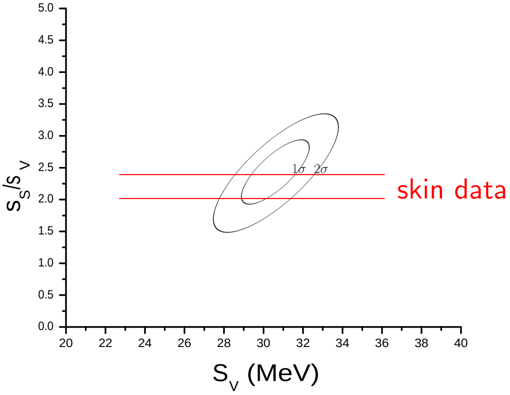

present: $S_v = 31.5 \pm 1.5$ MeV, $y = 2.0 \pm 0.2$

cf Danielewicz $S_v = 31 \pm 1.3$MeV, $y = 2.8 \pm 0.2$ (different data for ΔR)

Clearly S_s gives one more constraint on $S_{nm}(\rho)$

Use LDA: ratio S_s/S_v is integral over $\rho(r)$

$$\frac{S_{\rm v}}{S_{\rm s}} \approx \frac{3}{R\rho_0} \int \rho(r) \left(\frac{S(\rho_s)}{S(\rho(r))} - 1 \right) dr$$

Follows from the minimization of the second-order energy functional (in $\frac{N-Z}{A}$)
$E = E_0 + \int d^3 r S(\rho(r)) \rho(r) (\rho_{\rm a}/\rho)^2$ with $\rho_a \equiv \rho_{\rm n} - \rho_{\rm p}$
under fixed particle numbers $A = \int \rho dr, \;\; N - Z = \int \rho_{\rm a} dr$

Note: if S independent of ρ then $S_v/S_s \to 0$

Results for nuclear matter

In practice take $S_{nm}(\rho) = S_v(\rho/\rho_s)^{\gamma}$

Danielewicz: $0.55 < \gamma < 0.79$
Theory (RMF) : Piekarewicz: $\gamma = 0.98$(NL3), 0.64 (FSUGold)
B.-A. Li (exp) (isospin diffusion) $0.7 < \gamma < 1.1$
Presnt: $\gamma = 0.55 \pm 0.1$ Soft EOS

Summary

- For analysis of Symmetry energy
  LDM must be extended with surface term
- Shell effects affect values of SE
- Data on neutron skin are needed
- Analysis suggests "soft EOS "

Remarks

* Recent Asymmetry compressiblity ΔK :

from ISGMR in Sn isotopes: $E_{GMR} = \sqrt{\frac{\hbar^2 K_A}{mR^2}}$

Decompose $K_A(N,Z) = K_\infty + \frac{(N-Z)^2}{A^2} K_\tau + K_{Coul} \frac{Z^2}{A^{4/3}} + ..,$

exp: $K_\tau = -440 \pm 40$ MeV (Garg et al)

$\Delta K = K_\tau + 9 p_0 / \rho_s \approx -340$ MeV

* Pygmy GDR : low-energy oscillation of neutron excess wrt core?
correlation between relative dipole strength and neutron skin (Piekarewicz)

*Heavy-ion reactions (n-p flow) address $\rho > \rho_s$