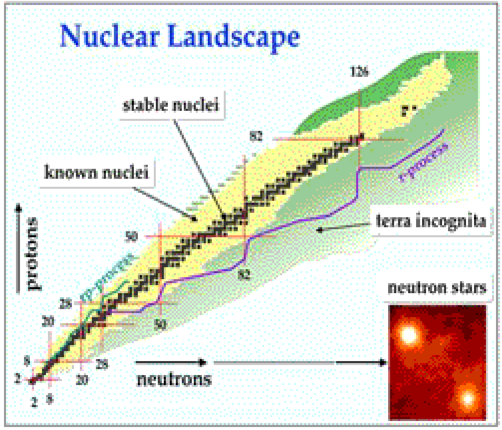
Global View on Pairing

Sven Åberg, Lund University, Sweden



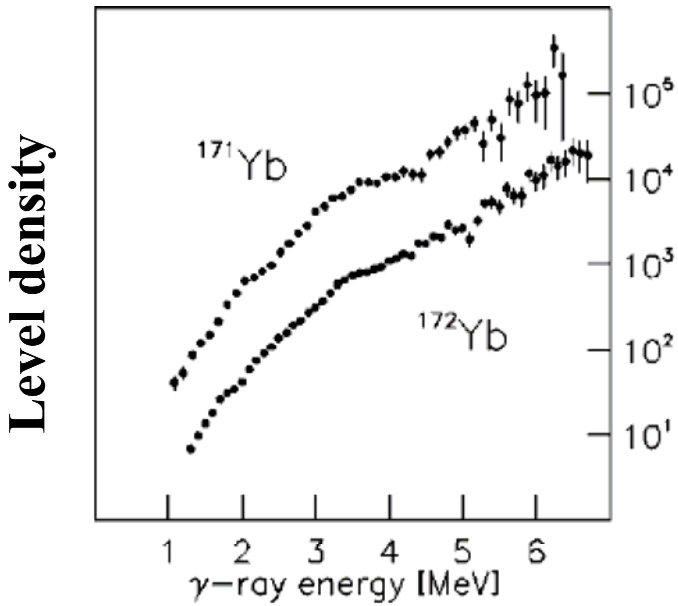
Nuclear Many-Body Approaches for the 21st Century

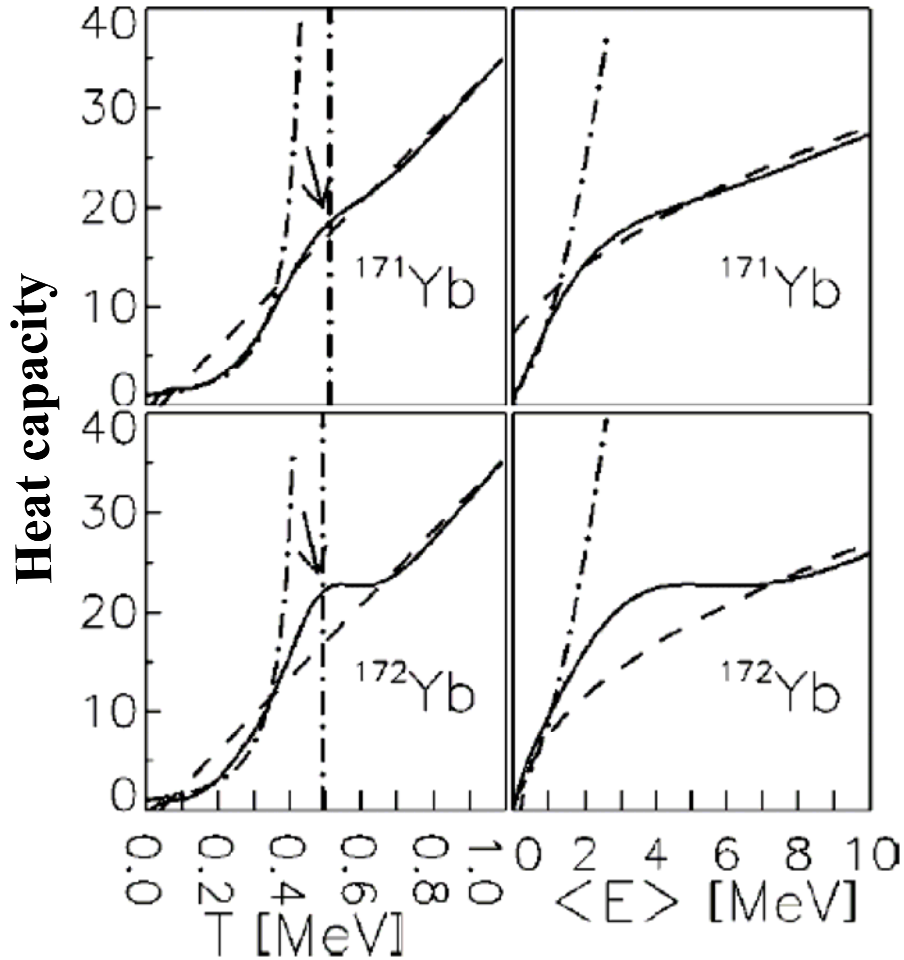
Institute for Nuclear Theory,

Sept 24 – Nov 30, 2007

Exp. sign for a pairing phase transition vs temperature [1]

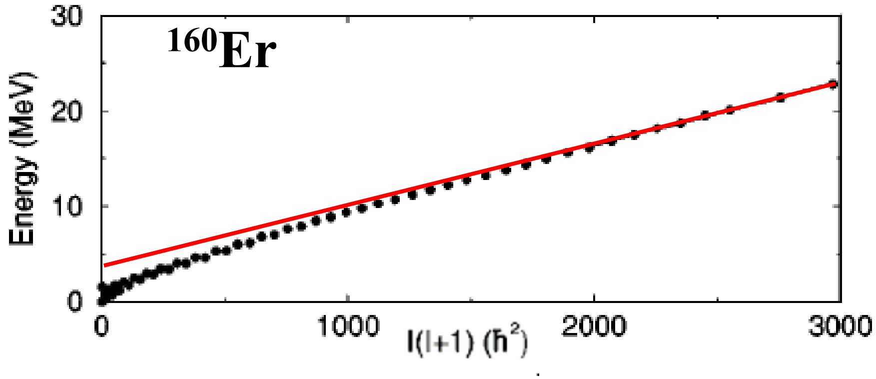
$$\text{BCS}: T_C = 0.57\Delta \approx 0.5 MeV$$



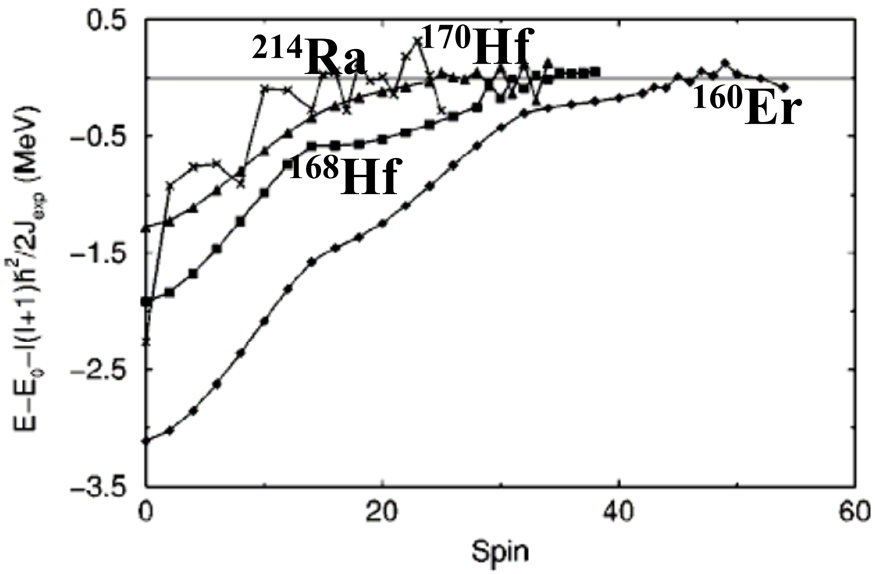


[1] A. Schiller et al, Phys Rev **C63** (2001) 021306(R)

Exp. sign for a pairing phase transition vs spin [1]



Moment of inertia supressed due to pairing



Pairing decreases gradually with temperature and spin

[1] M.A. Deleplanque et al, Phys Rev **C69** (2004) 044309

Global View on Pairing

I. Role of pairing in backbending and signature splitting

II. Nuclear masses and odd-even mass difference

III. Mesoscopic fluctuations of the pairing gap

- **a. Periodic orbit description of pairing**
- **b. Fluctuation of pairing gap in nuclei**
- **c. Shell structure in pairing gap from periodic orbit theory**

V. Pairing fluctuations in other finite fermi systems

- **a. Nanosized metallic grains**
- **b Ultracold fermionic gases**

A. Juodagalvis, I. Ragnarsson and S. Åberg, Phys Rev C73 (2006) 044327
H. Olofsson, S. Åberg and P. Leboeuf, submitted

I. Role of pairing in backbending and signature splitting [1]

Study A=50 region!

Collective phenomena observed:
rotation, backbending, triaxiality, band-termination, ...

Large-scale shell model calculations
full pf-shell, KB3-interaction, ANTOINE code

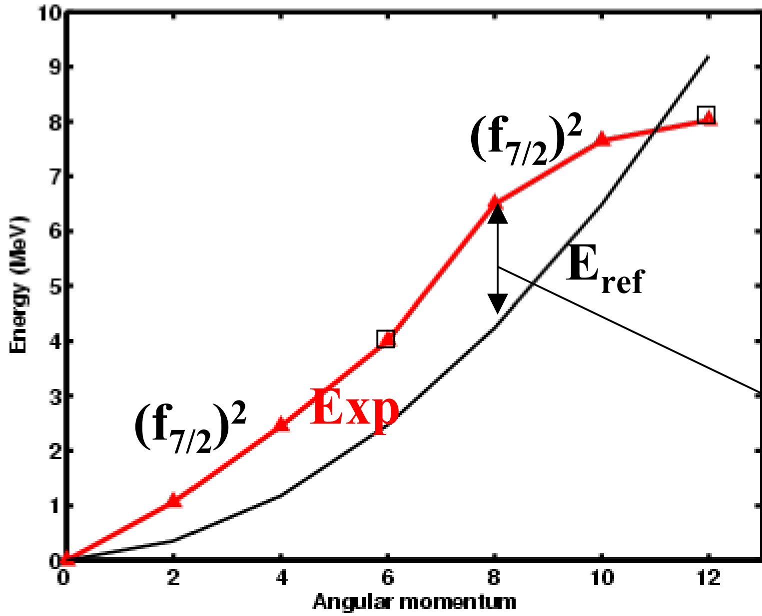
Pairing in SM [2]:
Difference between full H and $H\text{-}GP^{+}P$ energy eigenvalues

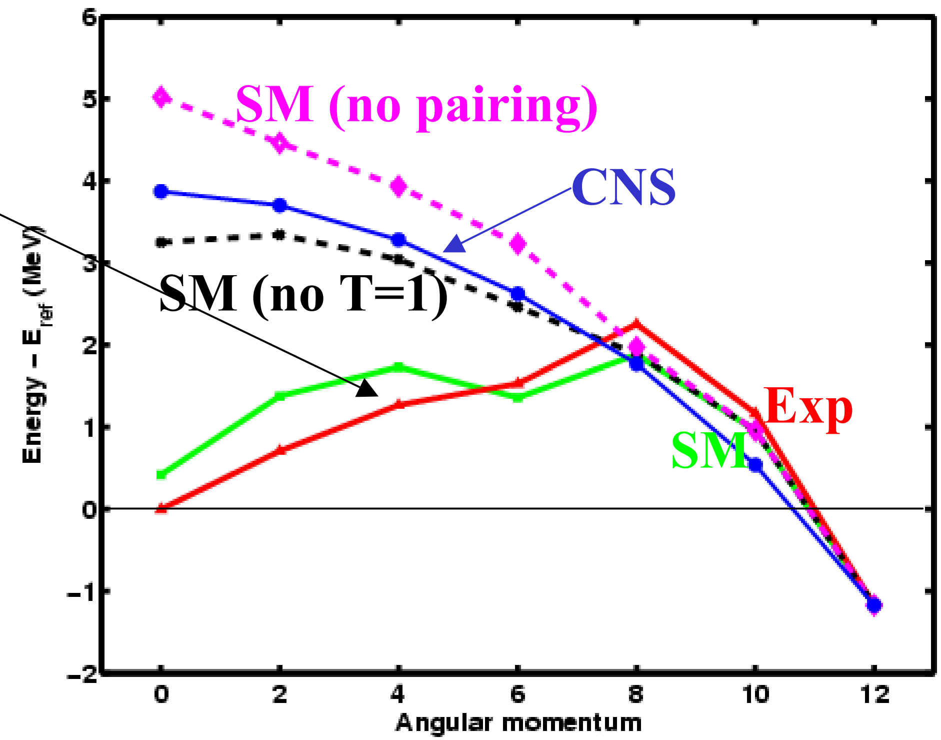
Compare SM to simple mean field calculations,
cranked Nilsson-Strutinsky (CNS) without pairing,
only including deformation and rotation

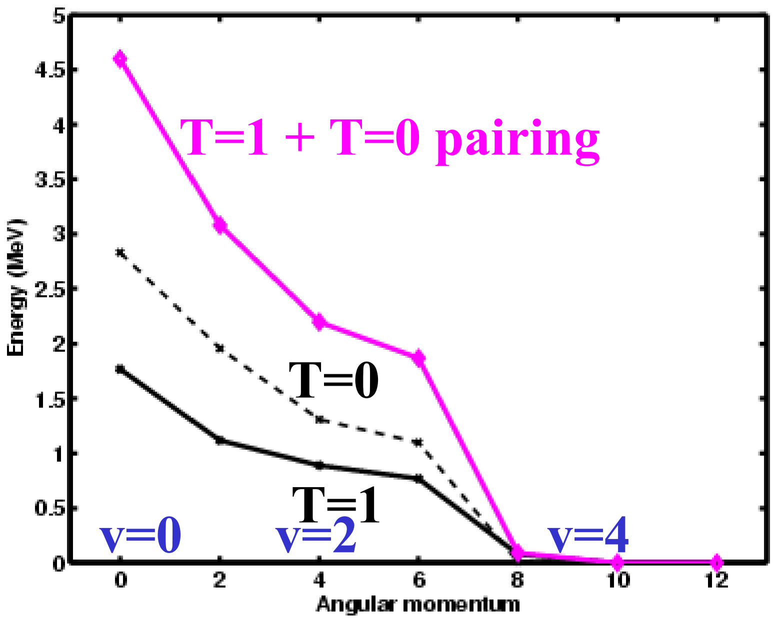
[1] A. Juodagalvis, I. Ragnarsson and S. Åberg, Phys Rev C73 (2006) 044327
[2] M. Dufour and A.P. Zhuker, PRC54 (1996) 1641

Ground-state band in $^{44}Ti_{22}$

^{44}Ti: 2p + 2n in $f_{7/2}$ shell

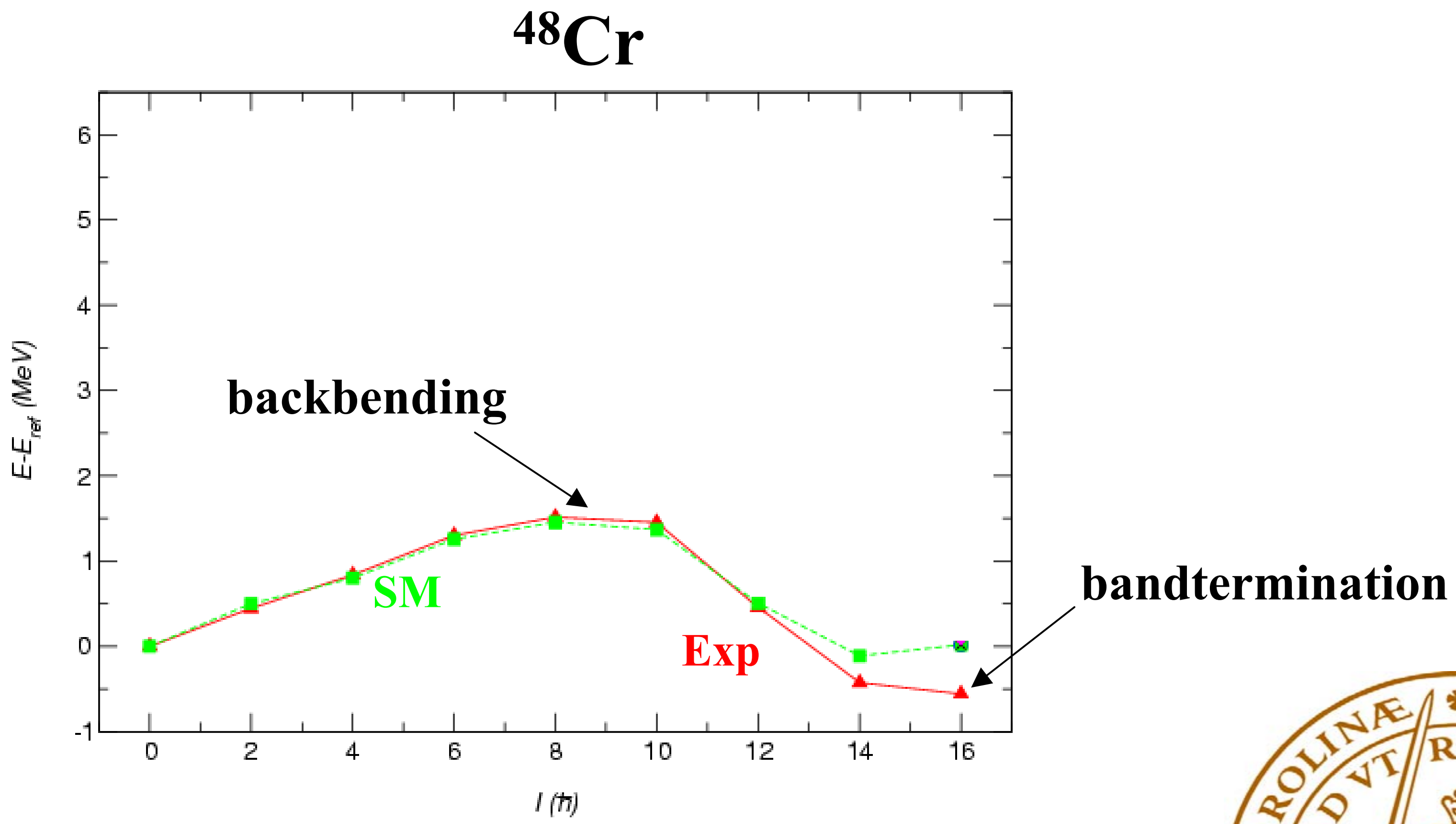






Gradual drop of pairing as seniority increases

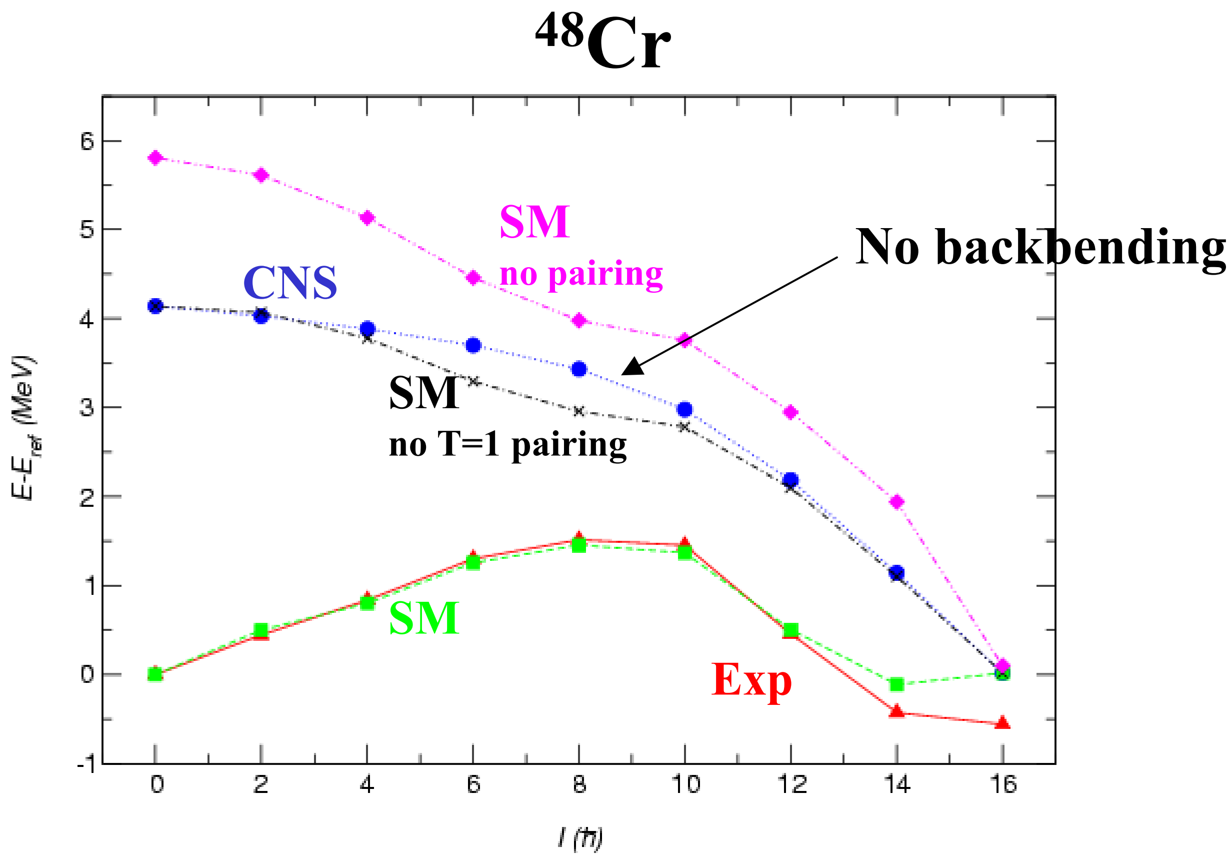
Rotation, backbending, bandtermination in ^{48}Cr



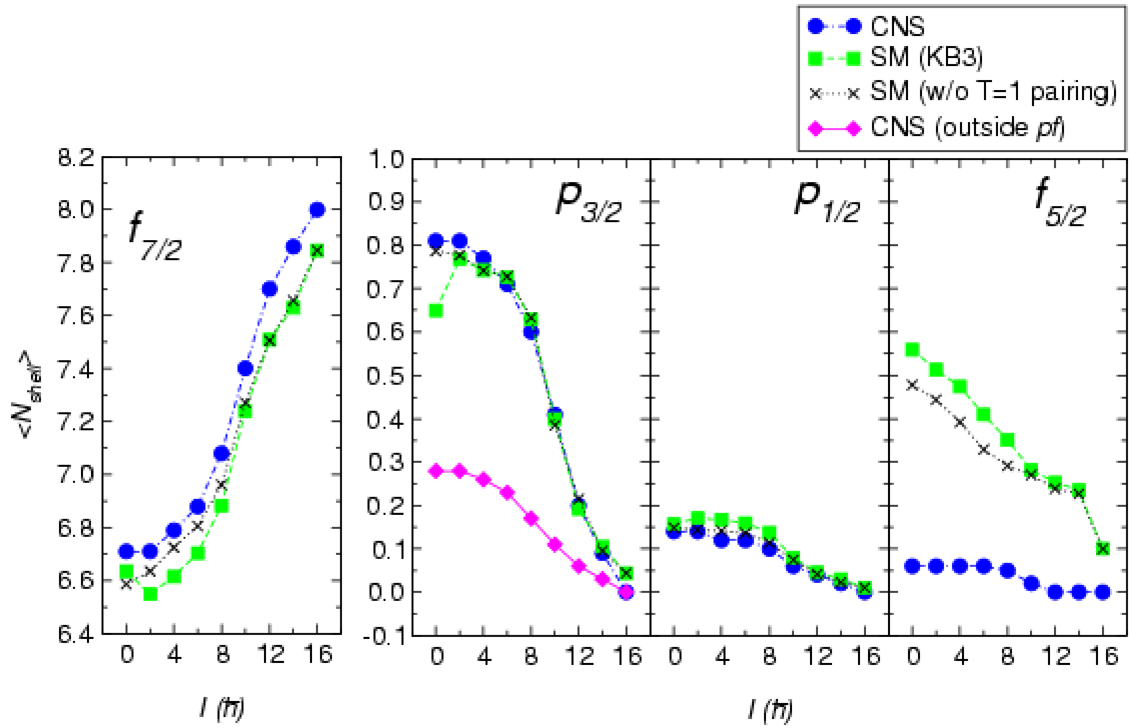
Exp: F. Brandolini et al, Nucl Phys **A693** (2001) 517.

SM: E. Caurier et al, PRL **75** (1995) 2466.

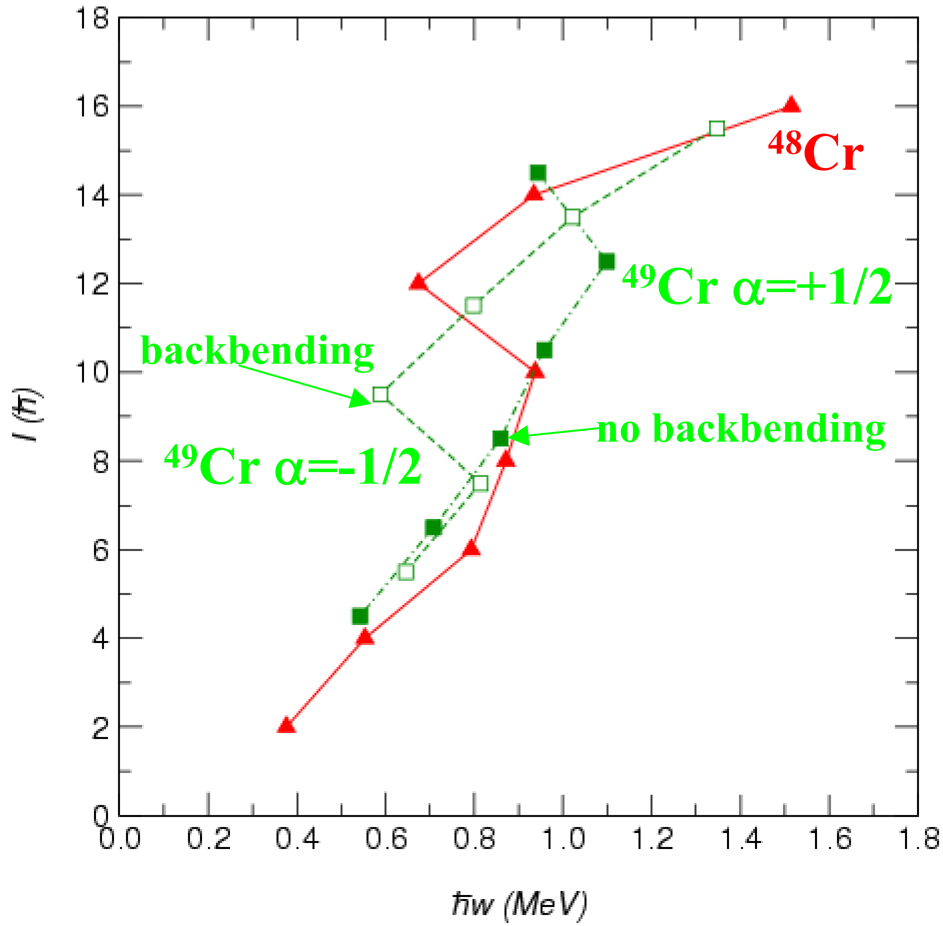
Role of pairing for backbending



Role of pairing for occupation numbers

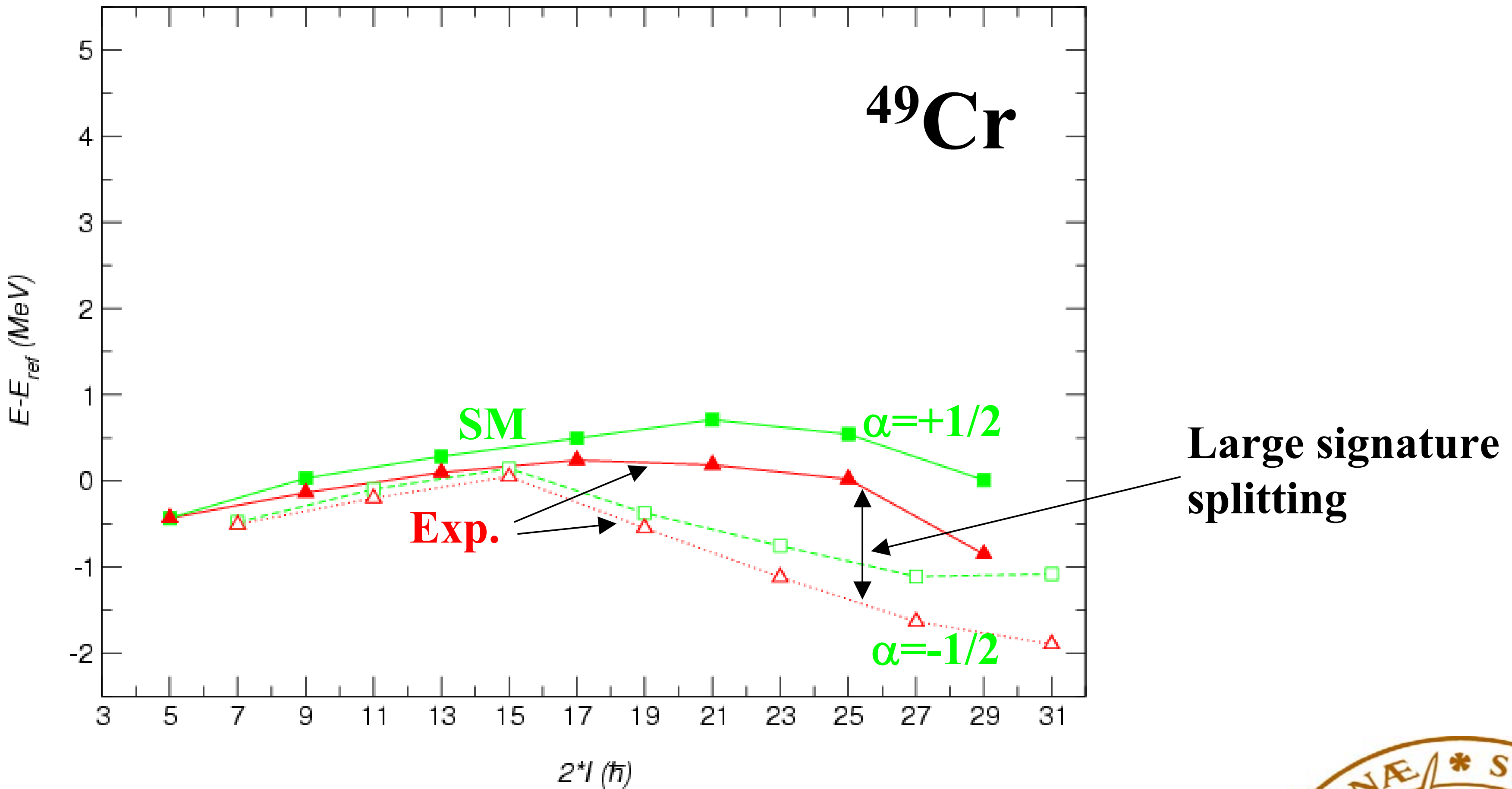


Backbending in ^{49}Cr – signature effects

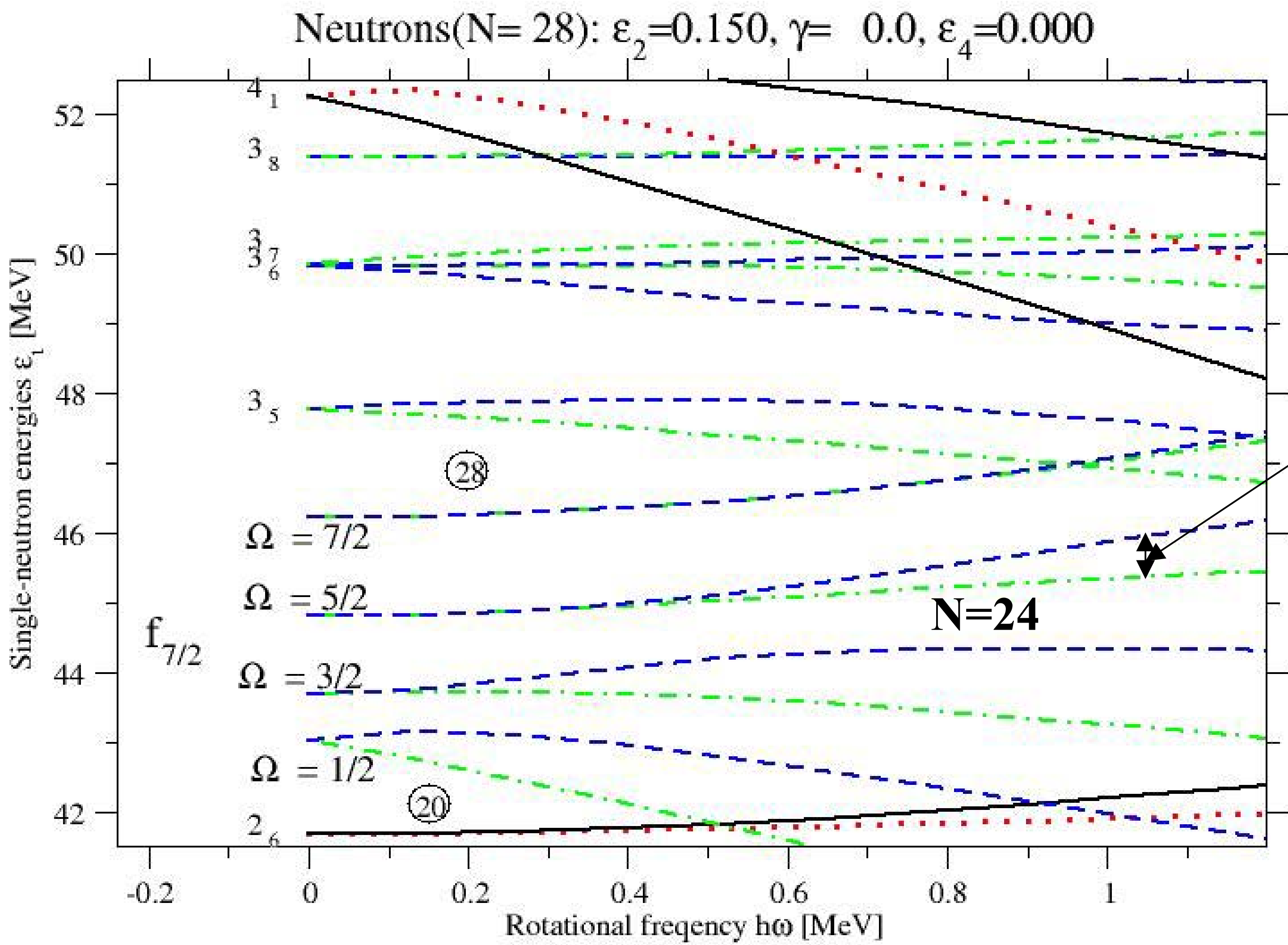


Exp. Data from: F. Brandolini et al, Nucl Phys **A693** (2001) 517**.**

Backbending in ^{49}Cr – signature effects



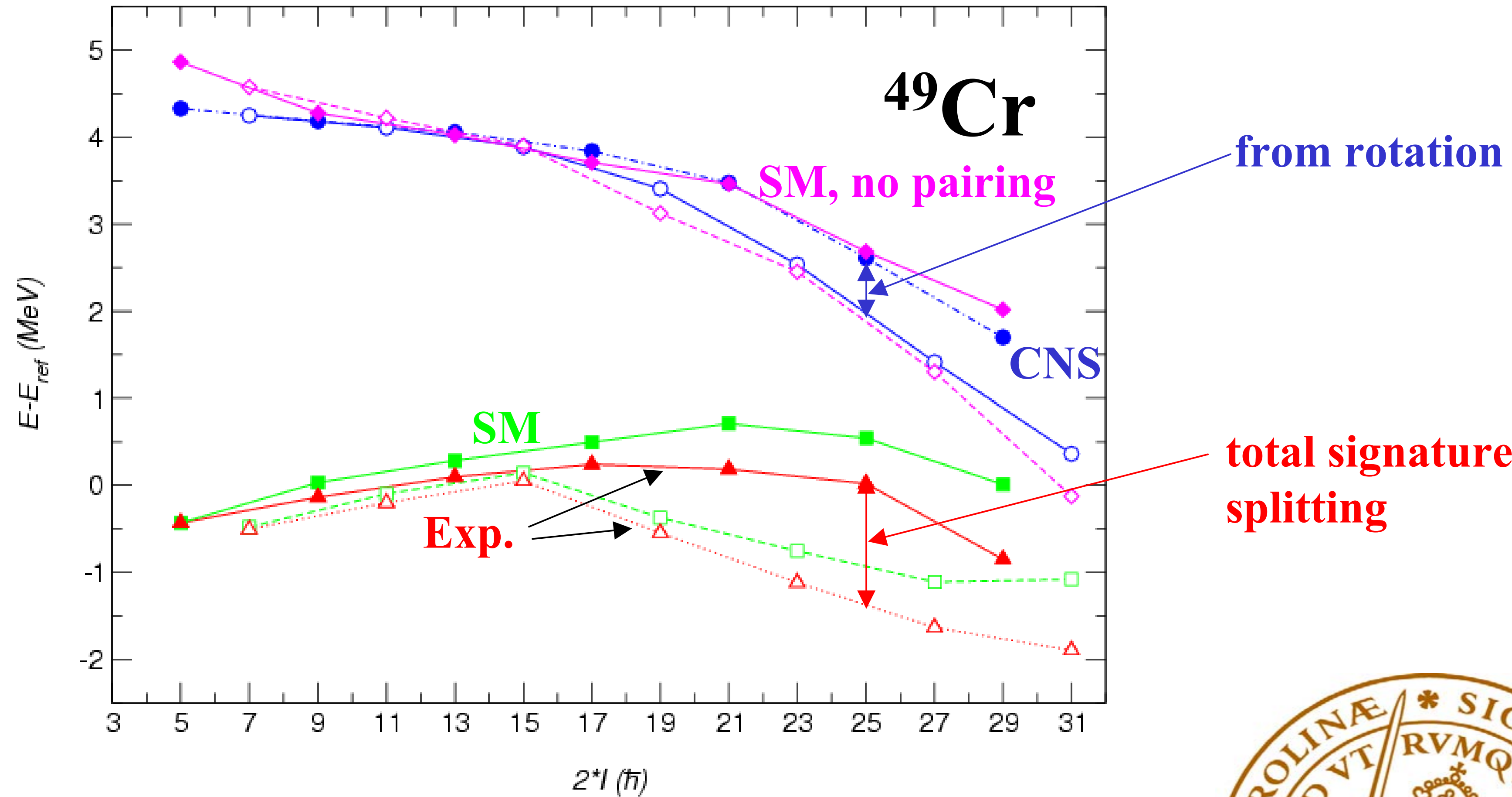
Signature splitting from rotation



Signature splitting from rotational coupling for N=25

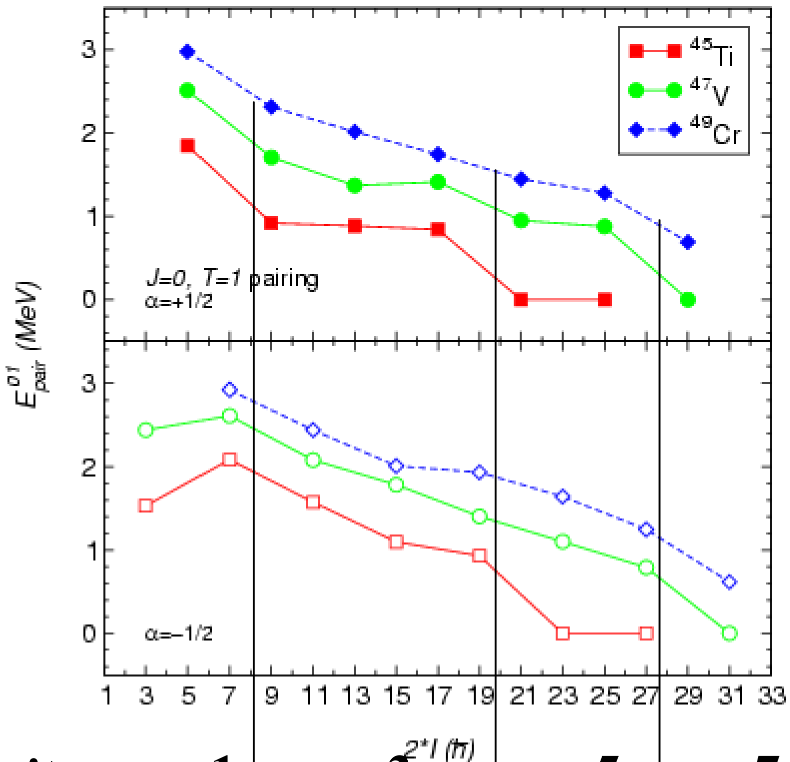
I. Ragnarsson

Signature splitting from rotation and pairing

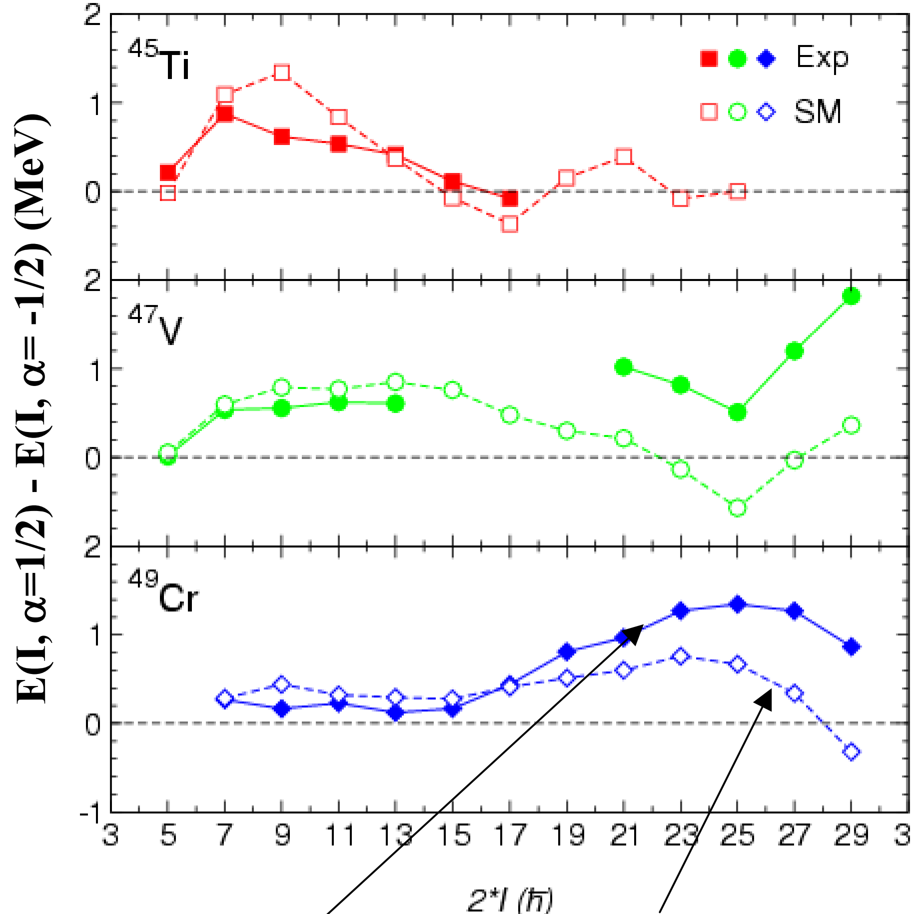


Role of pairing for signature splitting

Pairing energy (T=1)



Seniority: v=1 v=3 v=5 v=7

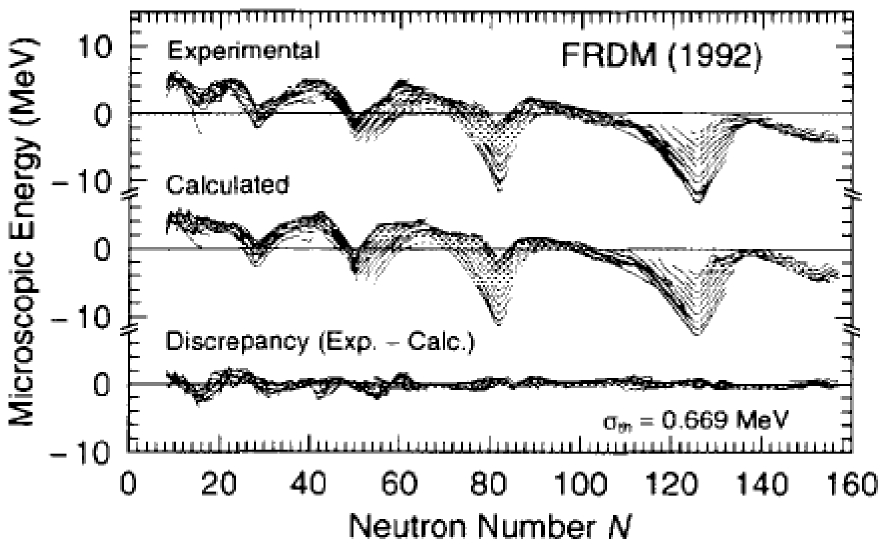


Pairing signature splitting

Total signature splitting (experimental)

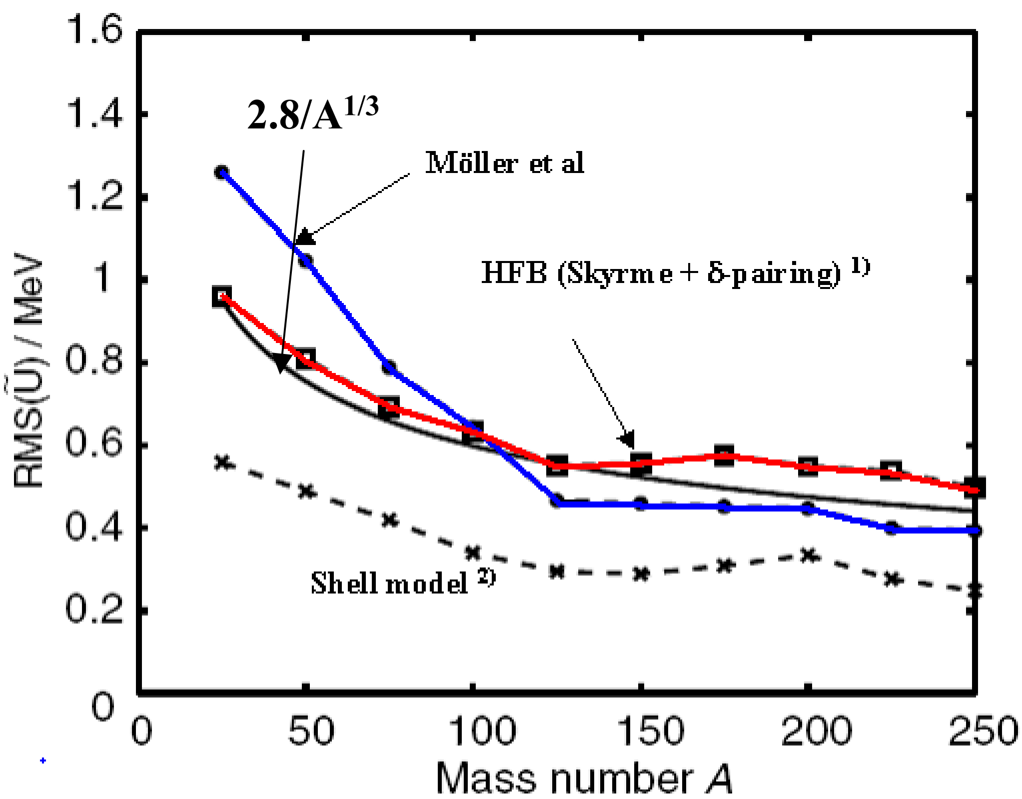
II. Nuclear masses and o-e mass difference

Shell energy $\widetilde{E}$ **versus neutron number**



P. Möller et al, Atomic data and nucl data tables **59** (1995) 185

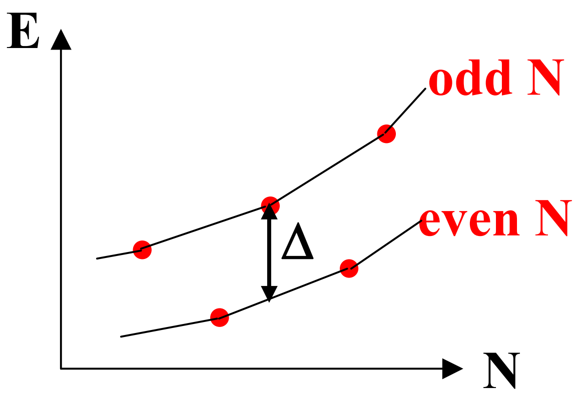
Error in mass formulae



1) Samyn, Goriely, Bender, Pearson, PRC 70 (2004) 044309
2) Duflo, Zuker, PRC 52 (1995) R23

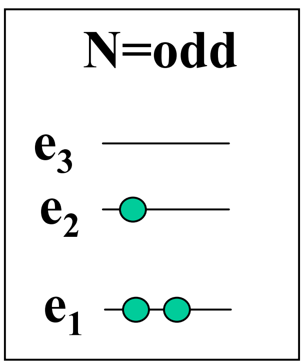
Odd-even mass difference

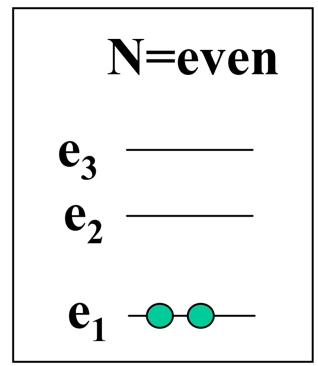
Extraction of pairing contribution from masses (binding energies)



$$\Delta^{(3)}(N) = \pm\left[B(N) - 0.5\left(B(N+1) + B(N-1)\right)\right]$$
$$\approx \frac{1}{2}\frac{\partial^2 B}{\partial N^2}$$

Contribution to Δ from mean field (assuming no pairing):



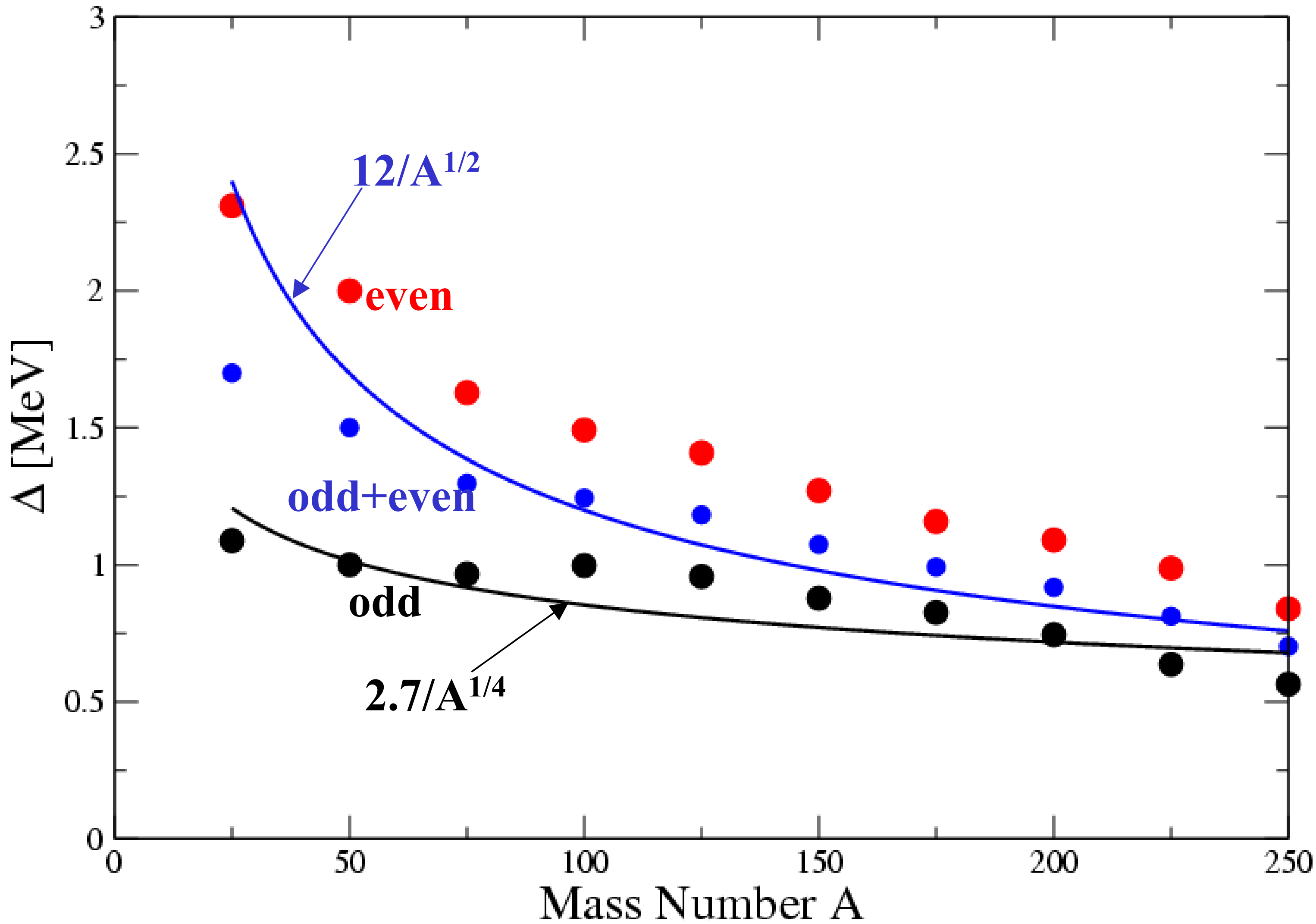


$\Delta^{(3)}(N=3)=(2e_1+e_2)-0.5(2e_1+2e_2+2e_1) = \boxed{0}$ $\Delta^{(3)}(N=2)=-(2e_1-0.5(2e_1+e_2+e_1)) = \boxed{0.5(e_2-e_1)}$

$\Delta^{(3)}$(odd N) is a good approximation to nuclear pairing gaps

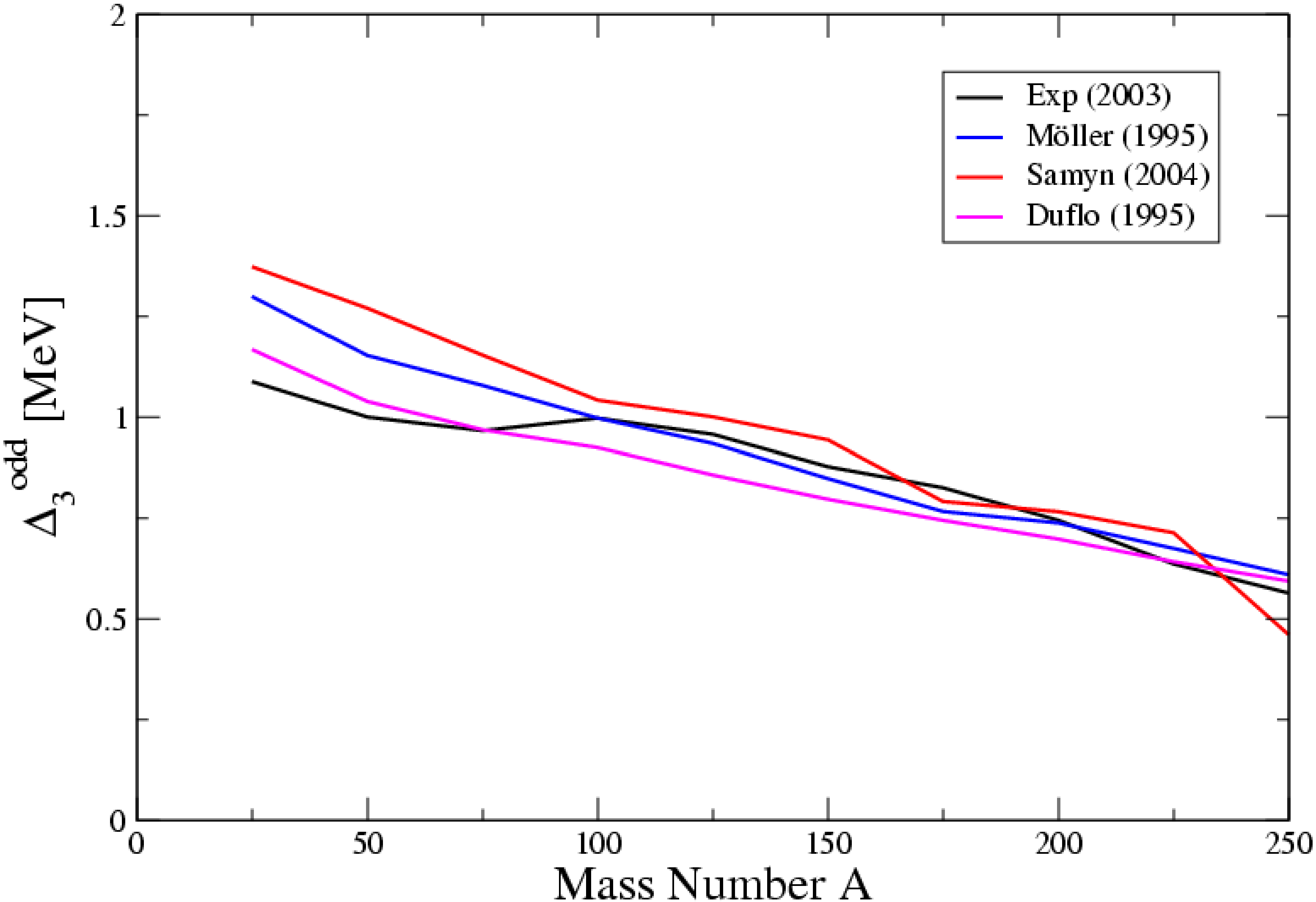
W. Satula, J. Dobaczewski and W. Nazarewicz, PRL 81 (1998) 3599

Odd-even mass difference from data [1]



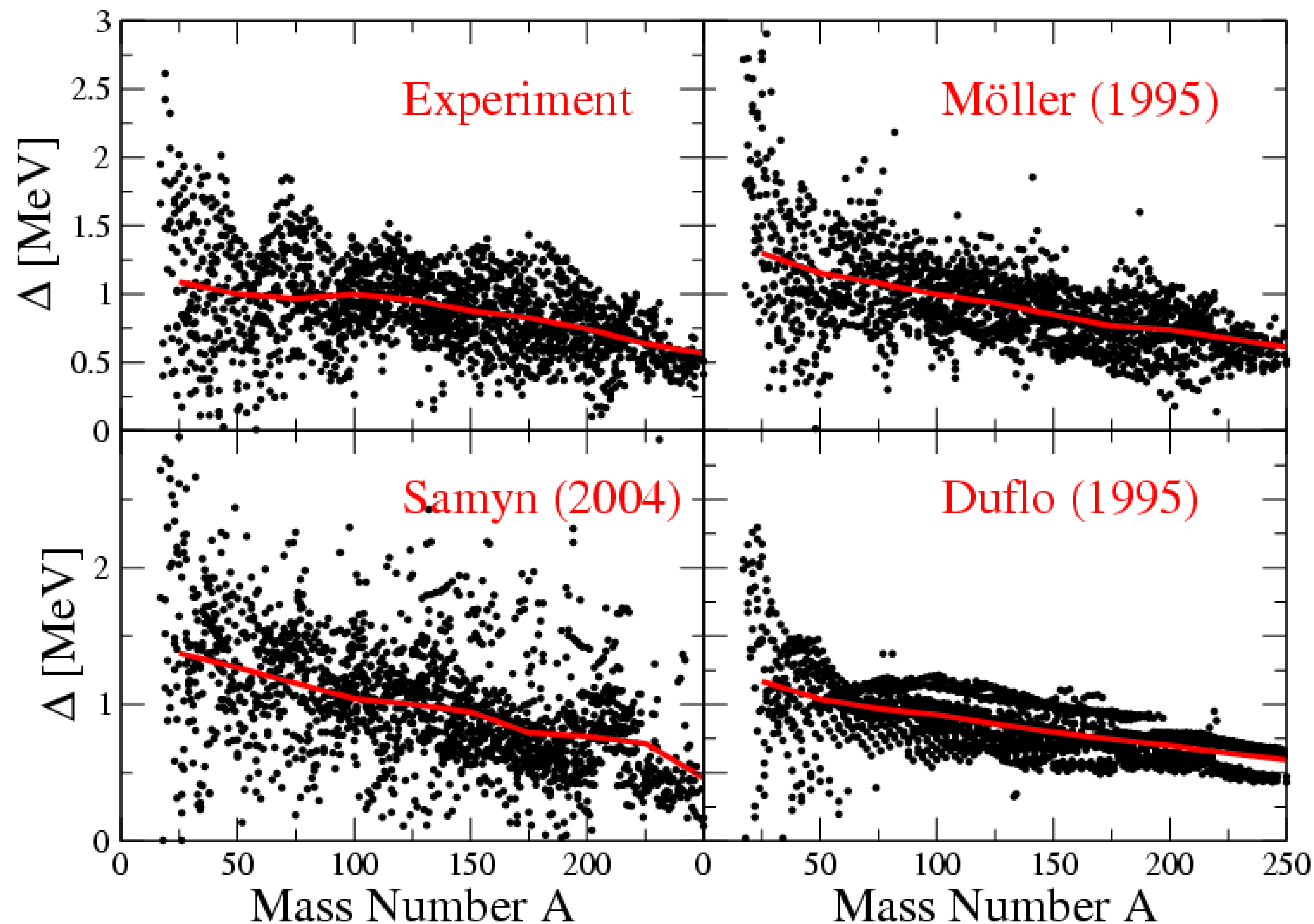
[1] H. Olofsson

Pairing gap (Δ_3(odd N)) from different mass models



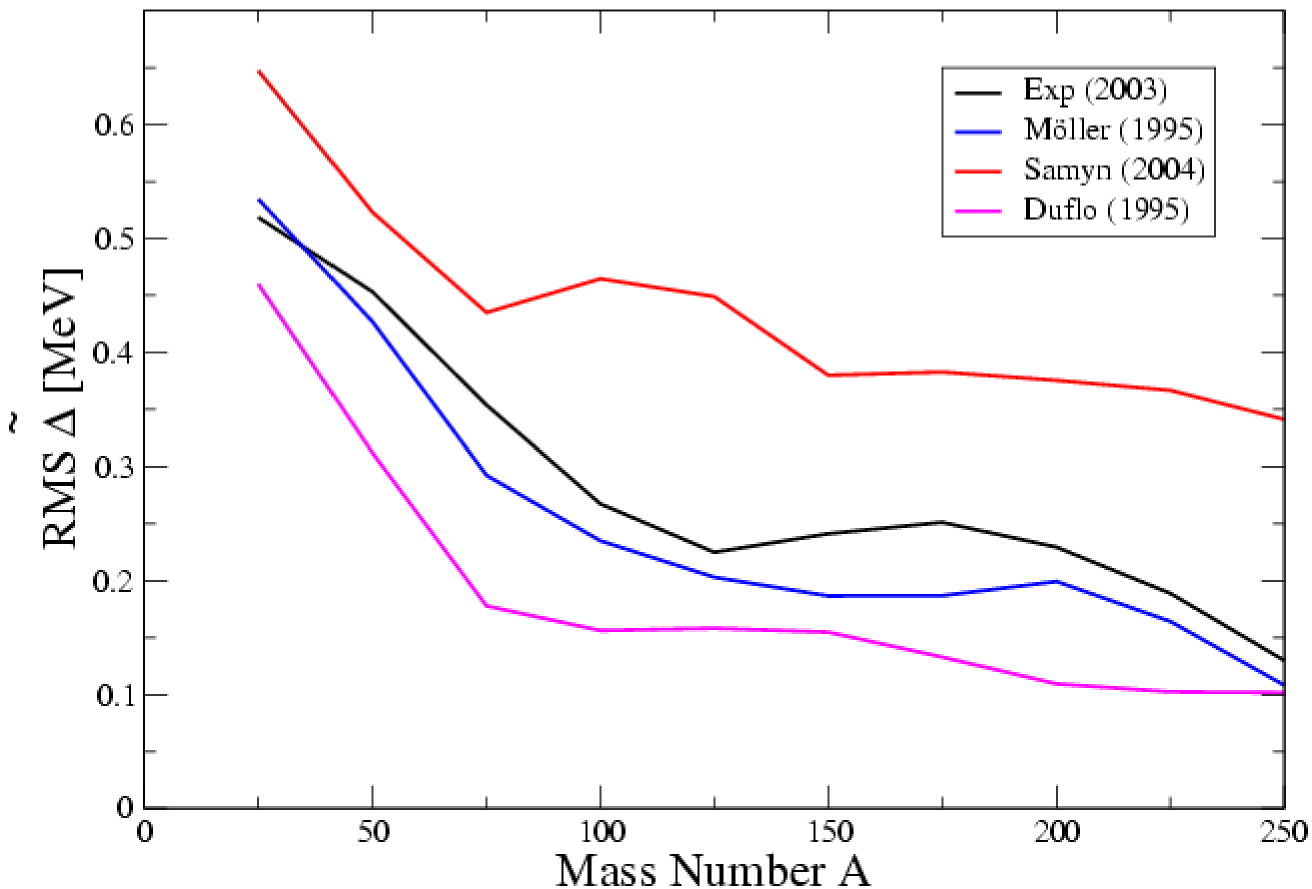
Mass models all seem to provide pairing gaps in good agreement with exp.

Pairing gap from different mass models



Quite different!

Fluctuations of the pairing gap



III. Mesoscopic Fluctuations of the Pairing Gap [1]

III.a Periodic orbit description of pairing

III.b. Fluctuations of pairing gap

- simple closed expressions

III.c. Description of pairing variation with particle number

[1] H. Olofsson, S. Åberg and P. Leboeuf, submitted

BCS theory

Hamiltonian:

$$H = \sum_k e_k a_k^+ a_k - G \sum_{kl} a_k^+ a_{\bar{k}}^+ a_{\bar{l}} a_l$$

Mean field approximation (in pairing space):

Pairing gap ("pairing deformation"):

$$\Delta = \left\langle G \sum_k a_k^+ a_{\bar{k}}^+ \right\rangle$$

is determined by gap equation:

$$\frac{2}{G} = \sum_\mu \frac{1}{\sqrt{(e_\mu - \lambda)^2 + \Delta^2}} \rightarrow \int_{-L}^{L} \frac{\rho(e)de}{\sqrt{e^2 + \Delta^2}}$$

Periodic orbit theory

The fluctuating part of the **level density,** $\rho(e) = \bar{\rho} + \tilde{\rho}$, is given by:

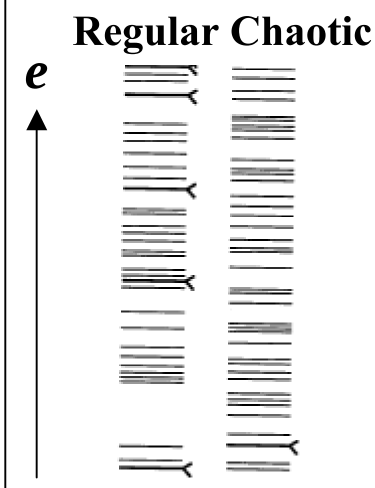
$$\tilde{\rho}(e) = \sum_{\substack{\text{periodic} \\ \text{orbits}, p}} \sum_{r=1}^{\infty} A_{p,r} \cdot \cos(rS_p / \hbar + \nu_{p,r})$$

$A_{p,r}$: stability amplitude

$\nu_{\mathrm{p,r}}$: Maslov index

$\mathrm{S_p} = \oint pdq$: action of periodic orbit p

$\tau_{\mathrm{p}} = \partial S_p / \partial E$: period of p.o



III.a Periodic orbit description of pairing

Divide pairing gap in smooth and fluctuating parts:

$$\Delta = \overline{\Delta} + \tilde{\Delta} \qquad \overline{\Delta} \approx 2L \exp\left(-\frac{1}{\overline{\rho} G}\right)$$

Insert into gap equation and assuming $\overline{\Delta} << L$
Expand to lowest order in fluctuations gives

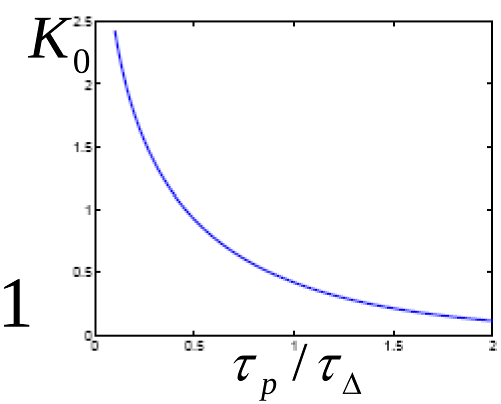
$$\tilde{\Delta} = 2\frac{\overline{\Delta}}{\overline{\rho}} \sum_{p,r} A_{p,r} K_0(r\tau_p/\tau_\Delta)\cos\left(rS_p(e)/\hbar + \nu_{p,r}\right)$$

where

$$\tau_\Delta = \frac{h}{2\pi\overline{\Delta}}$$ **is "pairing time"**

$$K_0(x) = \int_0^\infty \frac{\cos(xt)}{\sqrt{1+t^2}}dt$$

$$\rightarrow \exp(-x)/\sqrt{x}, \;\; x >> 1$$



i.e. no contribution from orbits with

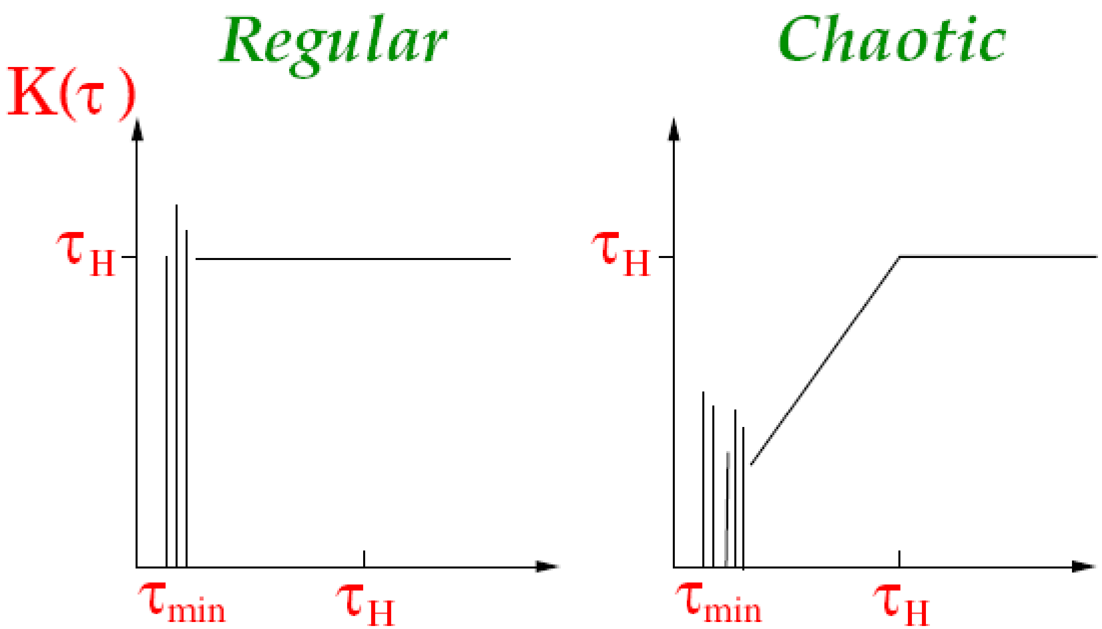
$$\tau_p >> \tau_\Delta$$

III.b Fluctuations of pairing

Fluctuations **of pairing gap become**

$$\left\langle \tilde{\Delta}^2 \right\rangle = 2\frac{\overline{\Delta}^2}{\tau_H^2}\int_0^\infty d\tau\, K_o^2(\tau/\tau_\Delta)K(\tau)$$

where K is the spectral form factor (Fourier transform of 2-point corr. function):

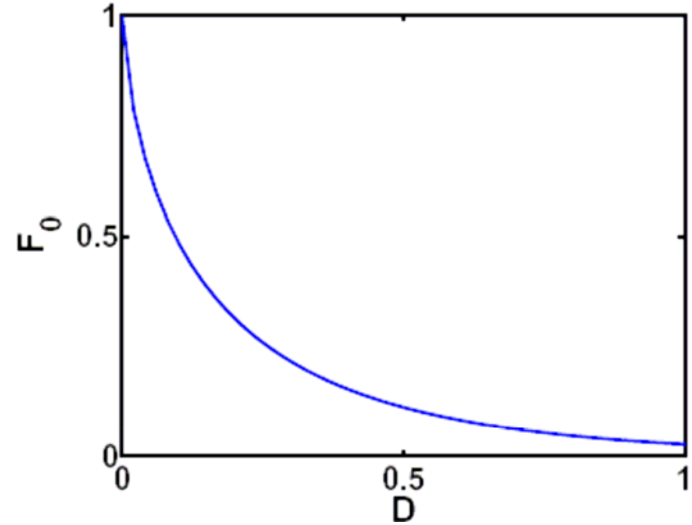


τ_{min} **is shortest periodic orbit,** $\tau_H = h\overline{\rho} = h/\delta$ **is Heisenberg time**

Fluctuations of pairing – simple expressions

Fluctuations of pairing, expressed in single-particle mean level spacing, δ:

$$\sigma = \sqrt{\left\langle \tilde{\Delta}^2 \right\rangle} / \delta$$



If regular: $$\sigma_{reg}^2 = \frac{\pi}{4} \frac{\overline{\Delta}}{\delta} F_0(D)$$

If chaotic: $$\sigma_{ch}^2 = \frac{1}{2\pi^2} F_1(D)$$

$$F_n(D) = 1 - \frac{\int_0^D x^n K_0^2(x) dx}{\int_0^\infty x^n K_0^2(x) dx}$$

$$D = \frac{\tau_{\min}}{\tau_\Delta}$$

Size of system: **2R**

Correlation length of Cooper pair: $\xi_0 = \hbar v_F / 2\Delta$

Dimensionless ratio: $D = 2R/\xi_0$

Fluctuations of pairing in nuclei

Nuclei:

- Mainly regular dynamics in gound state
- Size of D:

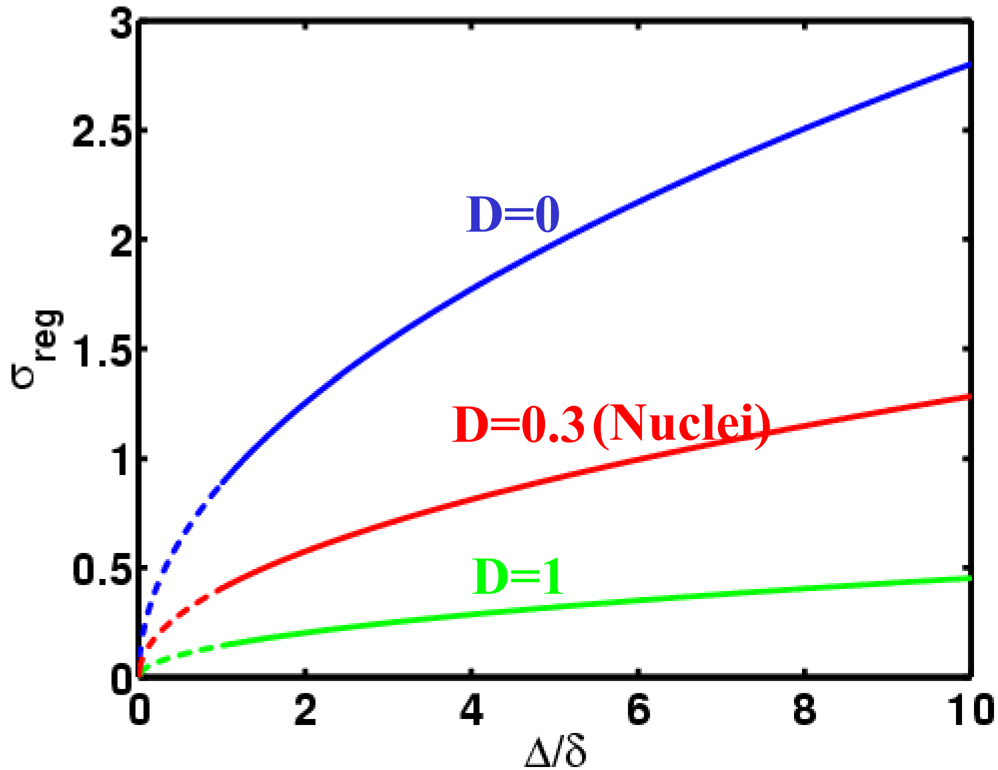
Size of system: $2R = 2*1.2A^{1/3}$ fm

Pairing length: $\xi_0 = \hbar v_F / 2\Delta = 11.3\ A^{-1/4}$ fm

$\Rightarrow$ $D = 2R/\xi_0 = 0.22A^{1/12} = 0.27 - 0.33$ (A=25-250)

Cooper pairs non-localized in nuclei

Fluctuations of pairing



If pairing correlation length < system size (D>1): small fluctuations

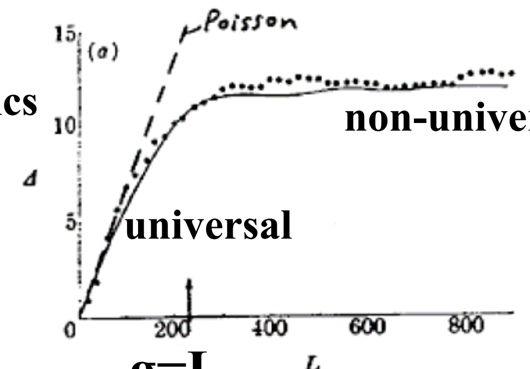
Universal/non-universal fluctuations

$$D = \frac{\tau_{\min}}{\tau_{\Delta}} = \frac{2\pi}{g}\frac{\overline{\Delta}}{\delta}$$

$$g = \frac{\tau_H}{\tau_{\min}}$$ **"dimensionless conductance"**

Non-universal spectrum fluctuations for energy distances larger than g:

Δ_3 statistics



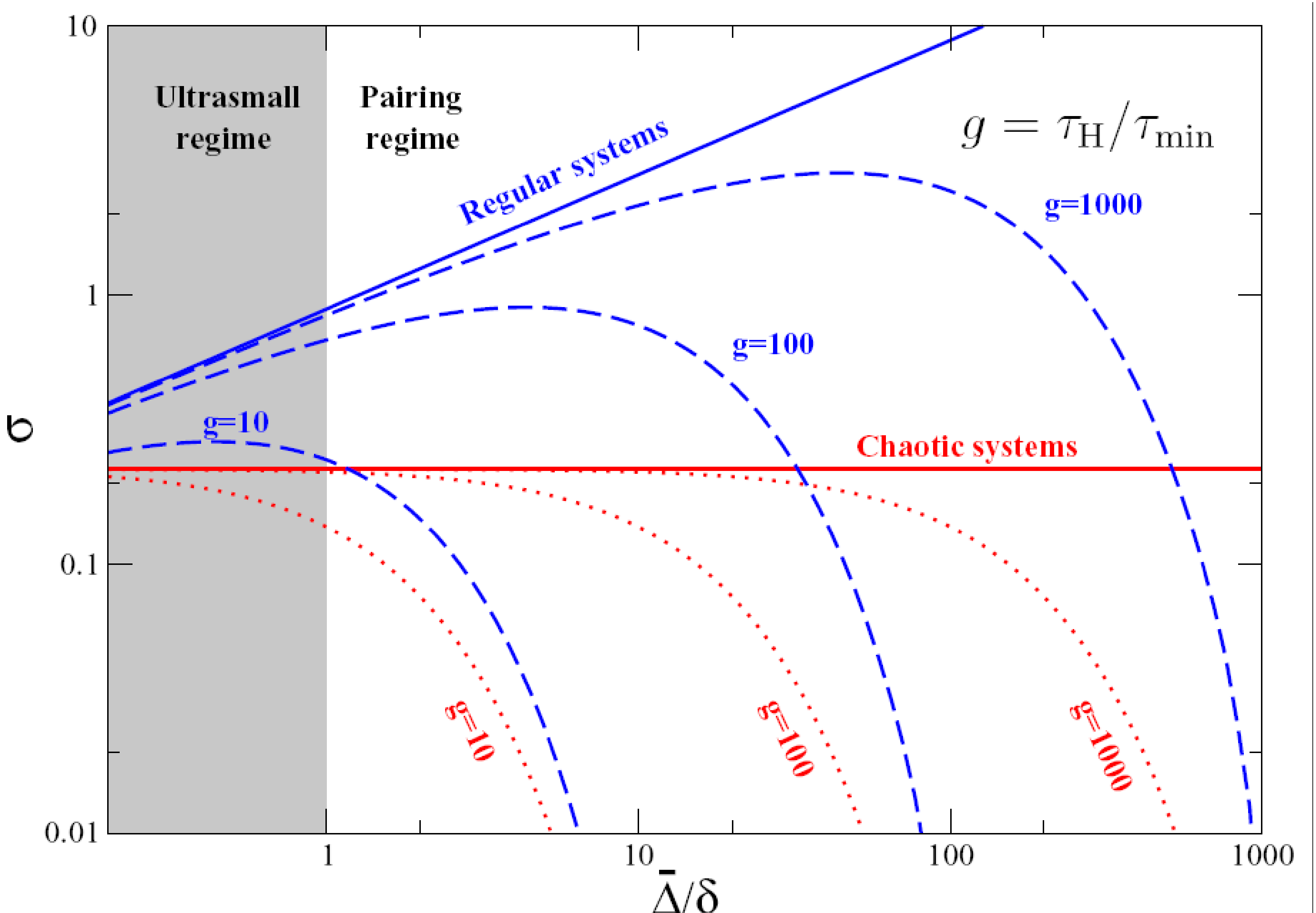
non-universal

universal

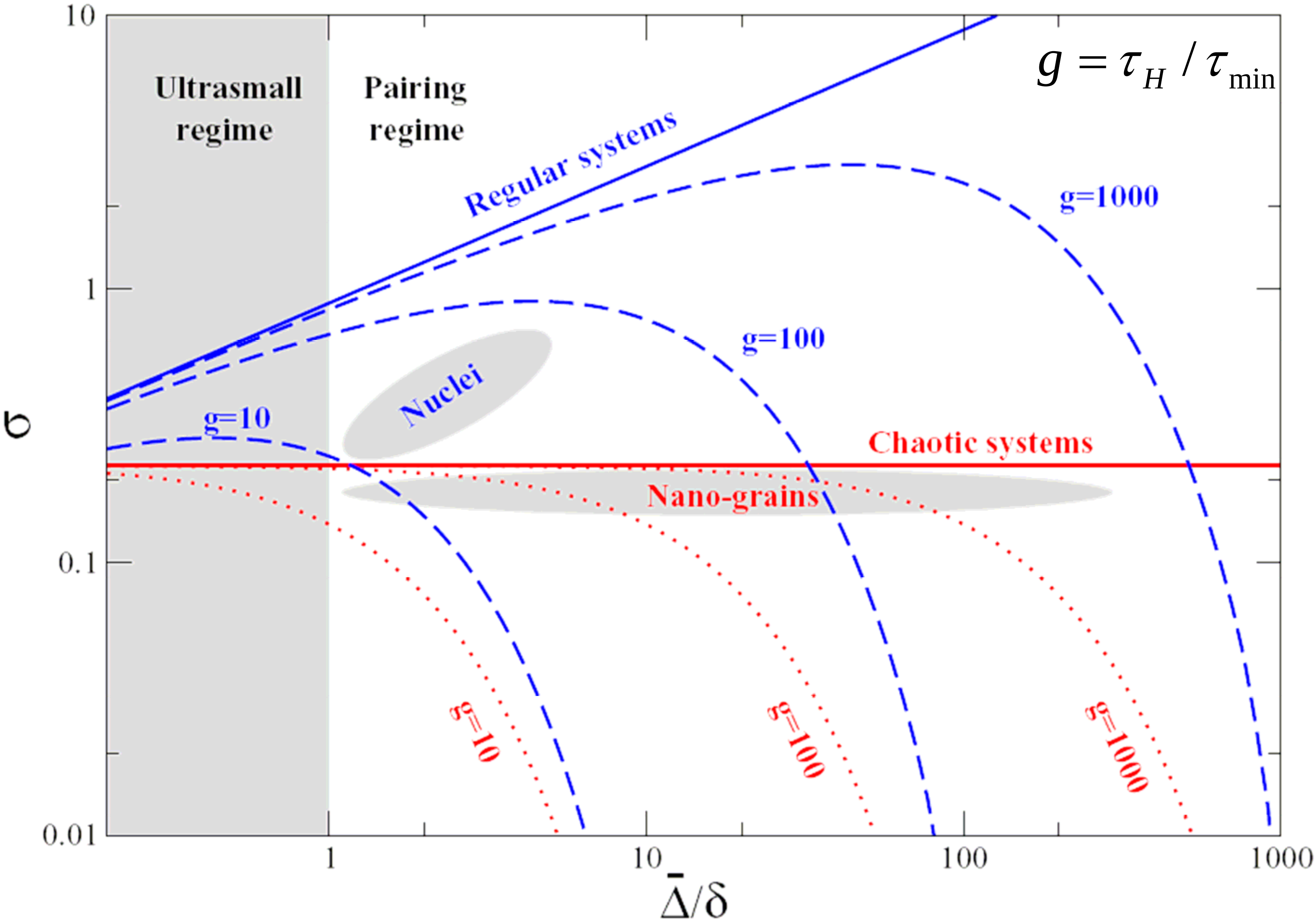
$g = L_{max}$

Random matrix limit: g → ∞ (i.e. D = 0) corresponding to pure GOE spectrum (chaotic) or pure Poisson spectrum (regular)

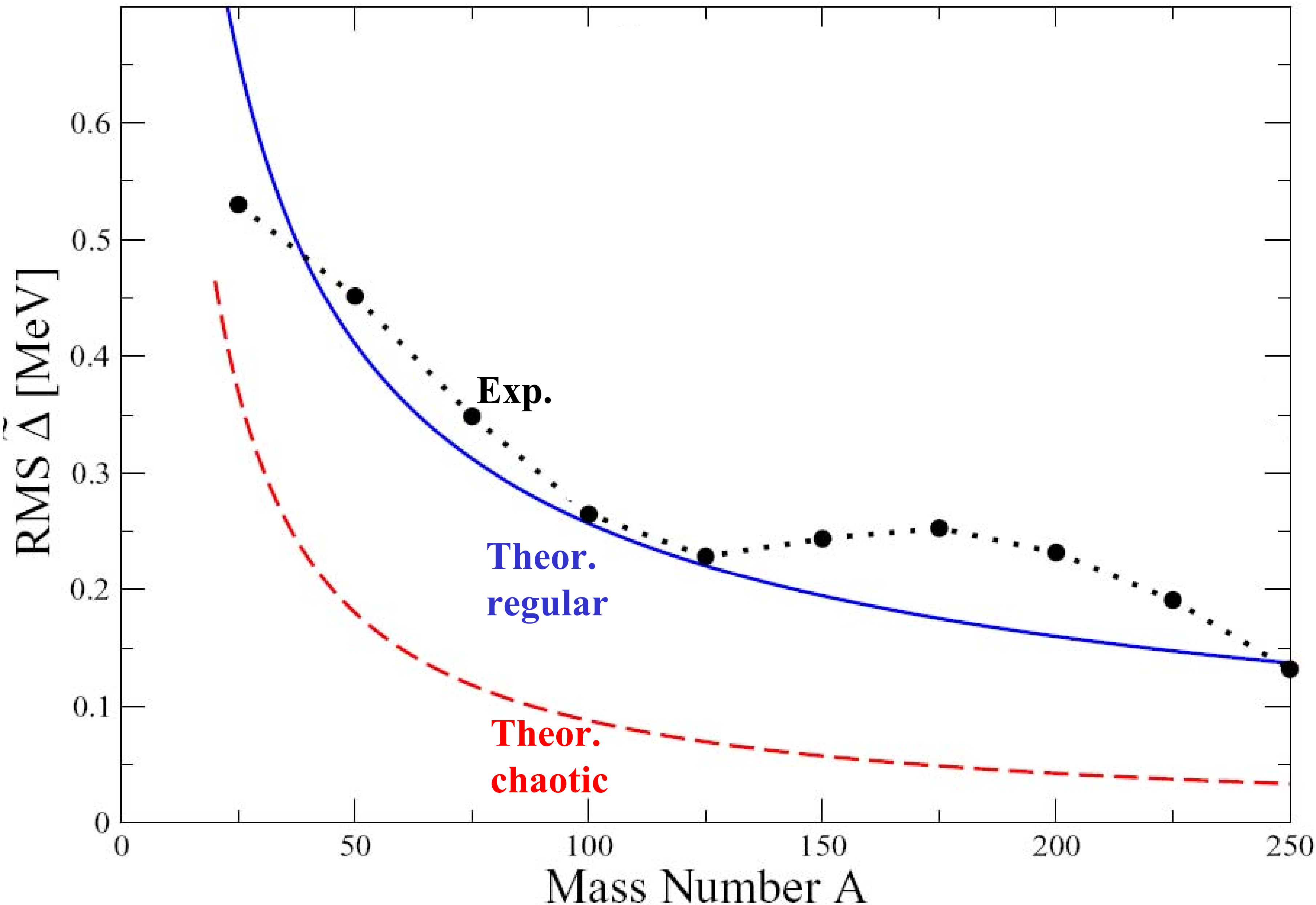
Generic behavior of pairing fluctuations



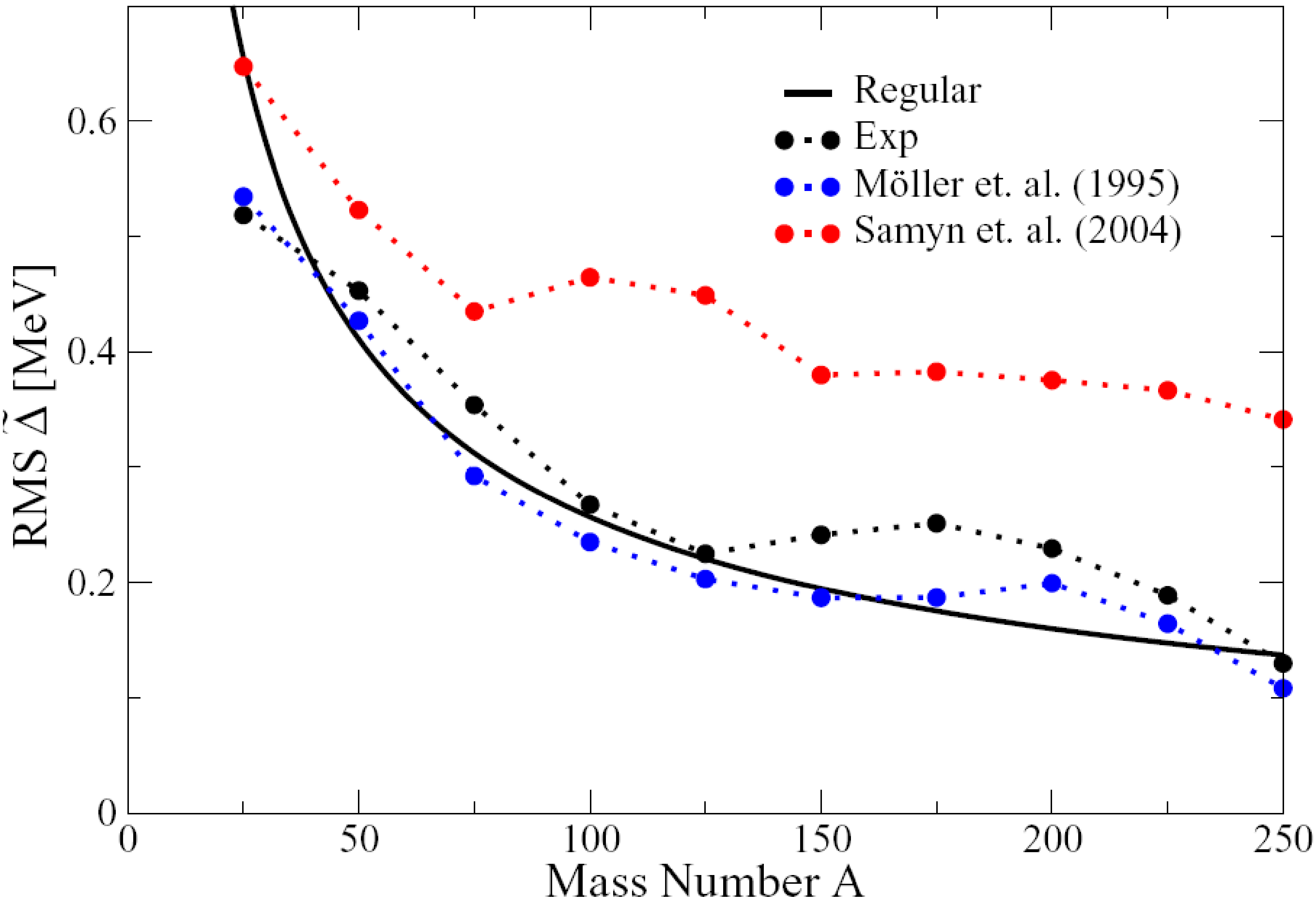
Generic behavior of pairing fluctuations



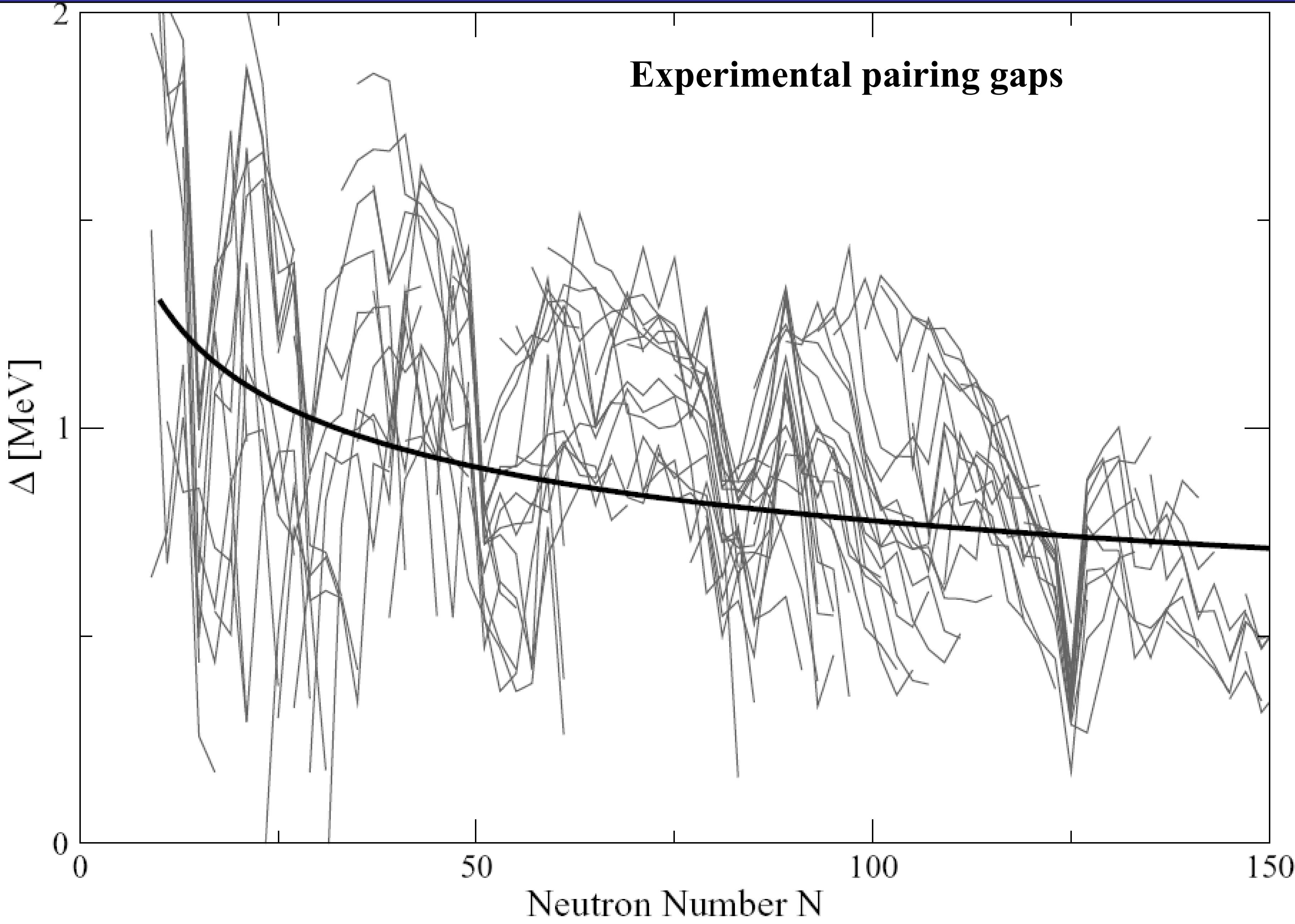
Fluctuations of nuclear pairing gap

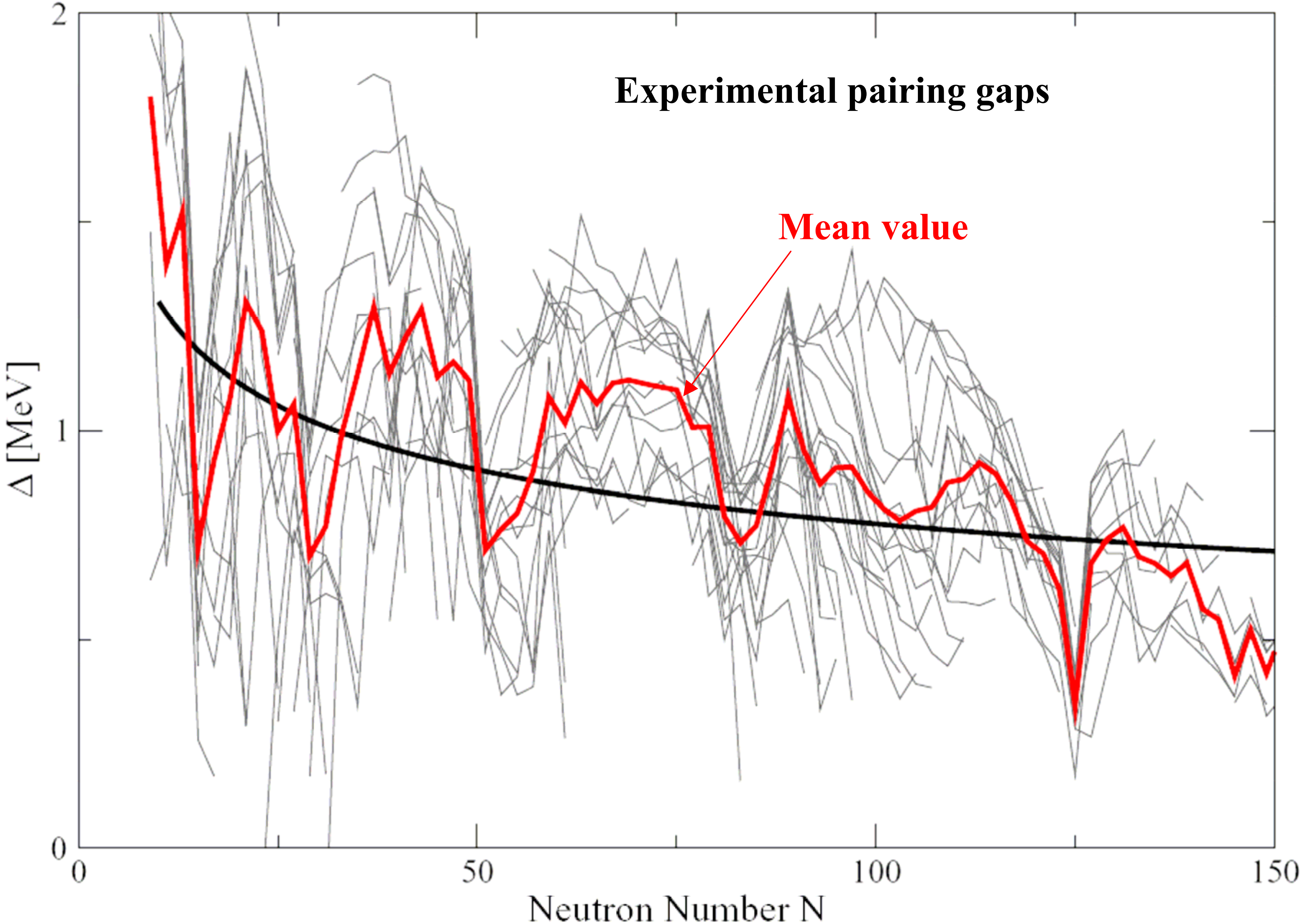


Fluctuations of nuclear pairing gap from mass models



III.c Shell structure in pairing gap from periodic orbit theory





Experimental pairing gaps
Mean value
Δ [MeV]
Neutron Number N
0
1
2
0
50
100
150

Periodic orbit description of pairing gap

$$\widetilde{\Delta} = \frac{\bar{\Delta}}{\bar{\rho} E_0} \sum_{\upsilon,\omega} A_{\upsilon\omega} M_{\upsilon\omega}(x) \kappa_\xi(\ell_{\upsilon\omega}) K_0 \left(\frac{\ell_{\upsilon\omega} \bar{\Delta}}{2 \bar{k}_\mathrm{F} R E_0} \right) \sin(\bar{k}_\mathrm{F} R \ell_{\upsilon\omega} + \nu_{\upsilon\omega} \pi / 2)$$

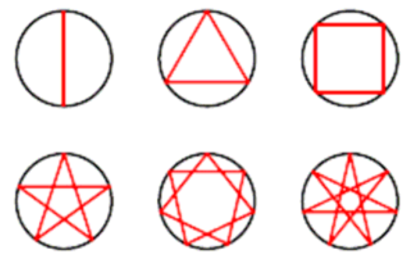
$M_{\upsilon\omega}(x)$ - Modulation factor for perturbative deformation

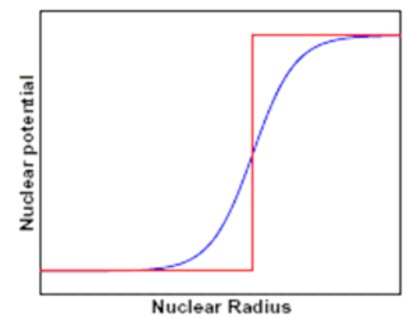
$\kappa_\xi(\ell_{\upsilon\omega})$ - Modulation factor for inelastic scattering

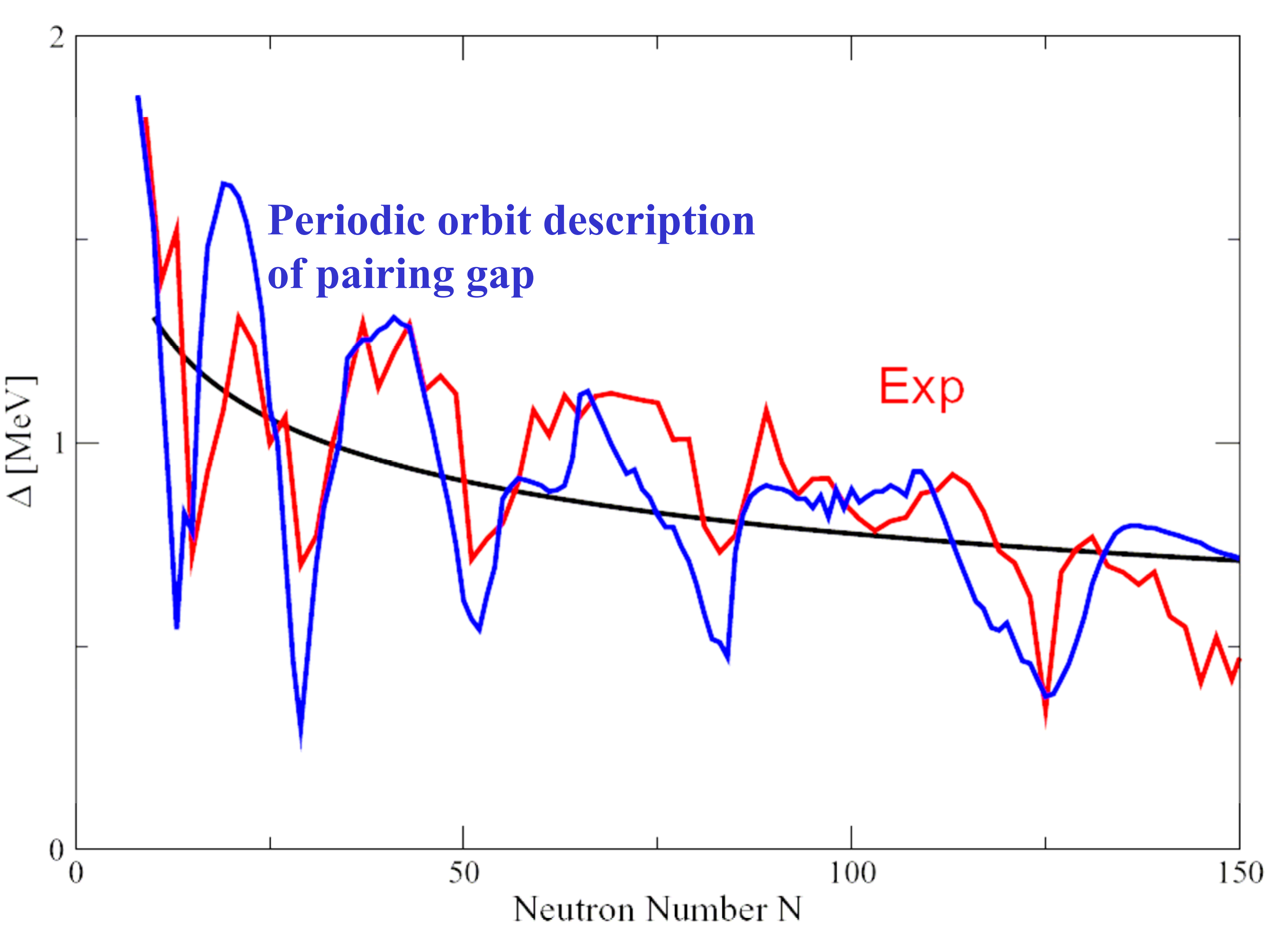
Deformations $x = (\varepsilon_2, \varepsilon_3, \varepsilon_4)$

- Quadrupole ε_2
- Octupole ε_3
- Hexadecapole ε_4

Periodic orbits in cavity:





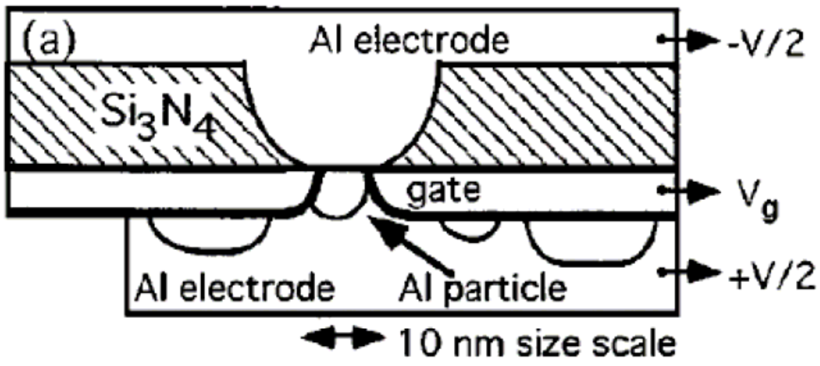


Periodic orbit description of pairing gap
Exp
Δ [MeV]
Neutron Number N
0
1
2
0
50
100
150

IV. Applications to other finite fermi systems

IV.a Nanosized metallic grains

- **Discrete exc. spectrum**
- **Irregular shape of grain ⇒ chaotic dynamics**
  - No symmetries – only time-rev. symm.
  - Energy level statistics described by GOE
- **Excitation gap – pairing gap (>>δ) observed for even *N***
- **Applied B-field ⇒ gap disappears**



$$N \sim 10^3 - 10^5$$

$$\bar{\Delta} \approx 0.38 \times 10^{-3} eV \qquad \delta = 2.1/N \text{ eV} \qquad g = 2.6 N^{2/3}$$

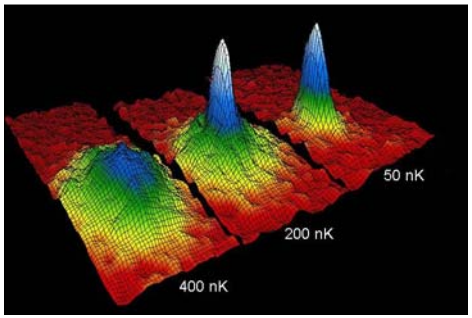
$$D = \frac{2\pi}{g}\frac{\bar{\Delta}}{\delta} \approx 0.0004 \quad \Rightarrow \quad F_1 \approx 1$$

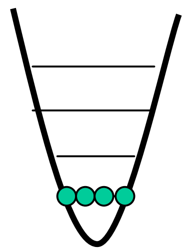
⇒ **Universal pairing fluctuations:** $\sigma_{ch}^2 = \frac{1}{2\pi^2}$

IV.b Ultracold fermionic gases

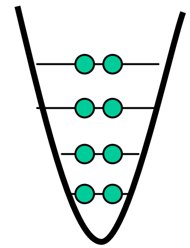
Trapped atomic quantum gases of bosons or fermions

$T \approx 0$





Bose condensate



Degenerate fermi gas

gives possibilities to study new phenomena in physics of finite many-body systems

Neutral atoms: # electrons = # protons

$\Rightarrow$ # neutrons determines quantum statistics

e.g. : $^{6}Li_3$ fermionic

$^{7}Li_4$ bosonic

Ultracold fermionic gases

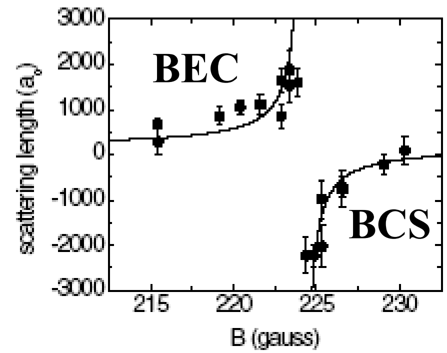
Atom-atom interaction is short-ranged (1-10 Å) and much smaller than interparticle range (~ 10^{-6} m) (dilute gas)

⇒ Approximate int. with:

$$V(r_1 - r_2) = 4\pi \frac{\hbar^2 a}{m} \delta^{(3)}(r_1 - r_2)$$

***a*=scattering length (s-wave)**

Via Feshbach resonance one can experimentally control size and sign of interaction (via external magnetic field):



C.A. Regal, D.S. Jin, PRL 90 (2003) 230404

Two free experimental parameters:
Particle number and interaction strength

Ultracold fermionic gases

In dilute BCS region: $\overline{\Delta}/\delta = (2/e)^{7/3}\frac{3N}{2}\exp\left(-\frac{\pi}{2k_F|a|}\right)$

$$D = \frac{2R}{\xi_0} = 2\pi(2/e)^{7/3}(3N)^{1/3}\exp\left(-\frac{\pi}{2k_F|a|}\right)$$

Recent experiments [1] using ^{6}Li reach $k_F|a| = 0.8$ and about 100 000 atoms gives Δ/δ=100 000, D=60 and:
negligible fluctuations of the pairing gap

However, for example, for $k_F|a| = 0.2$ and 50 000 atoms gives Δ/δ=12, D=0.06 and:

$\frac{\Delta}{\overline{\Delta}} = 1 \pm 0.24$ **for regular system** $\frac{\Delta}{\overline{\Delta}} = 1 \pm 0.02$ **for chaotic system**

Making the system chaotic strongly supresses pairing fluctuations!

$$\tilde{\Delta}_{RMS,regular} / \tilde{\Delta}_{RMS,chaotic} \approx 4\sqrt{\overline{\Delta}/\delta}$$

[1] C.H. Schunck et al, PRL **98** (2007) 050404

SUMMARY

- **no pairing SM: similar to mean field (energies, E2's and occ. numbers)**
- **Pairing contr. to signature splitting**
- **$\Delta = 2.7/A^{1/4}$ MeV**
- **Periodic orbit description of pairing gap in finite fermi systems**
- **Acurate, parameter free description of fluctuations of *nuclear* pairing gaps**
- **Prediction of pairing fluctuations (BCS-gaps) in nanosized *metallic grains and* ultracold *Fermi gases***