Bulk Thermodynamics in SU(3) gauge theory

In Monte-Carlo simulations ln $Z(T)$ cannot be determined but only its derivatives

$$\frac{\partial \ln Z(\beta)}{\partial \beta} = \frac{1}{Z(\beta)}\frac{\partial}{\partial\beta}\int \mathcal{D}U e^{-\beta S_g(U)} = - < S_g >$$

$$\frac{p(T)}{T^4} = \int_{\beta_0}^{\beta(T)} d\beta' \left(\frac{\partial \ln Z(T)}{\partial \beta'} - \frac{\partial \ln Z(T=0)}{\partial \beta'}\right) = \int_{\beta_0}^{\beta(T)} d\beta' \left(\langle S_g\rangle_0 - \langle S_g\rangle_T\right)$$

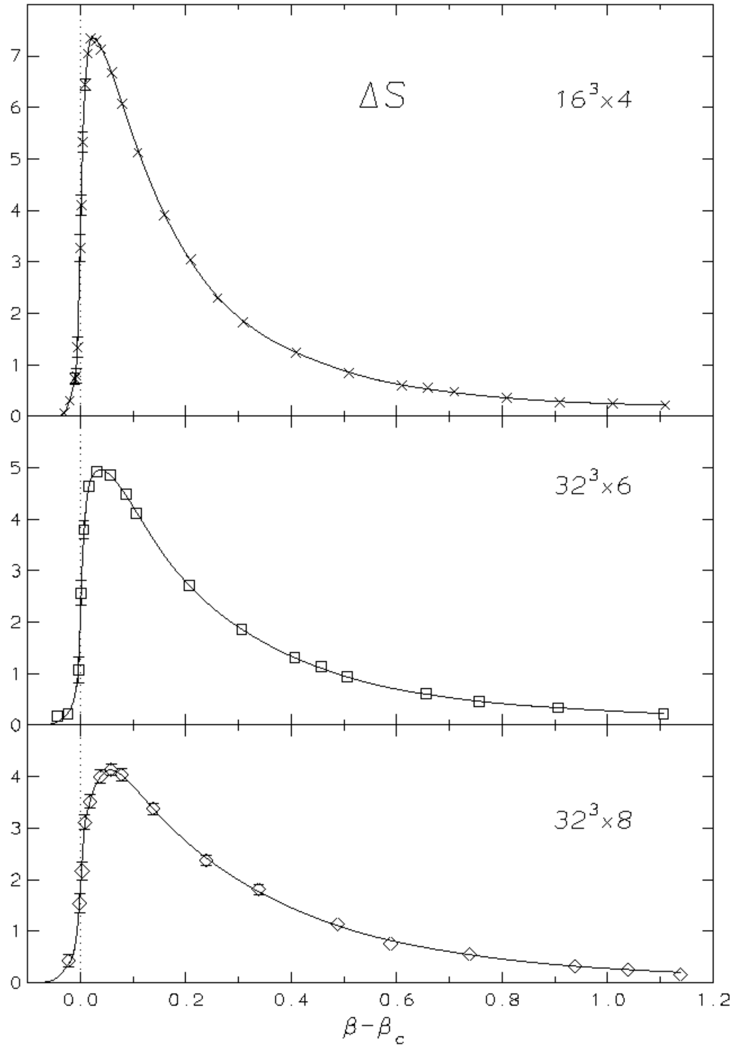
$$s = (\epsilon + p)/T = \frac{\partial p}{\partial T}$$

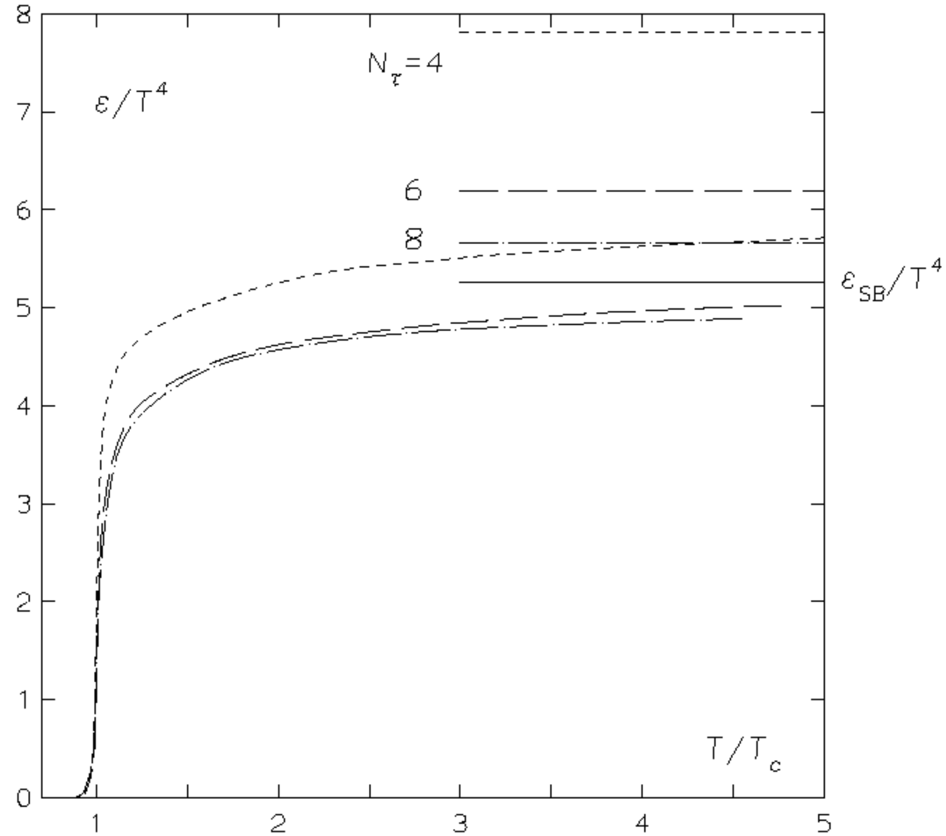
$$\frac{\epsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) = T\frac{d\beta}{dT}\frac{\partial p/T^4}{\partial \beta} = -\left(a\frac{d\beta}{da}\right)\frac{\partial p/T^4}{\partial\beta}$$

computational cost go as N_τ^4

large cutoff effects !

Boyd et al., Nucl. Phys. B496 (1996) 167





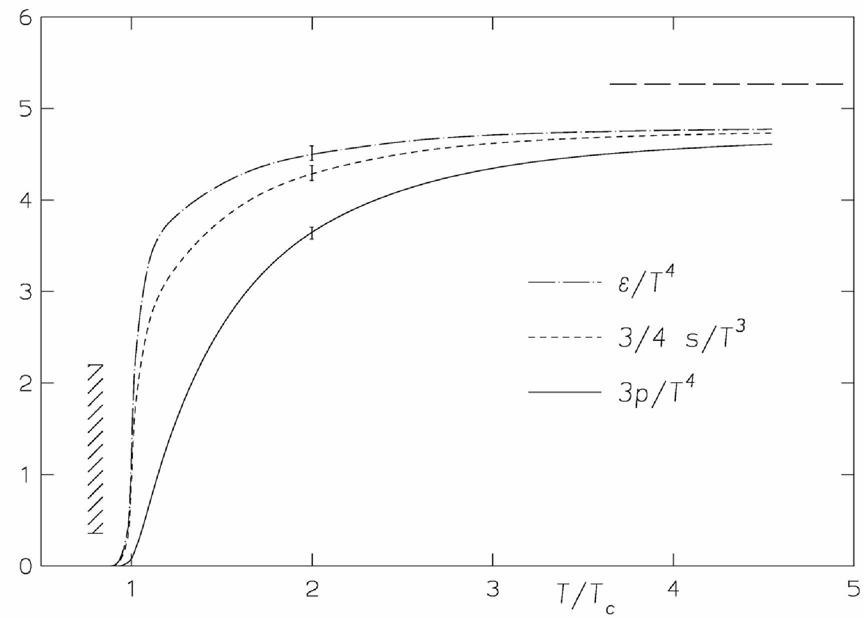
the free gas limit overestimates cutoff effects

Wilson gauge action a^2 discretization errors => $1/N_\tau^2$ corrections to the pressure

Boyd et al., Nucl. Phys. B496 (1996) 167

Wilson gauge action

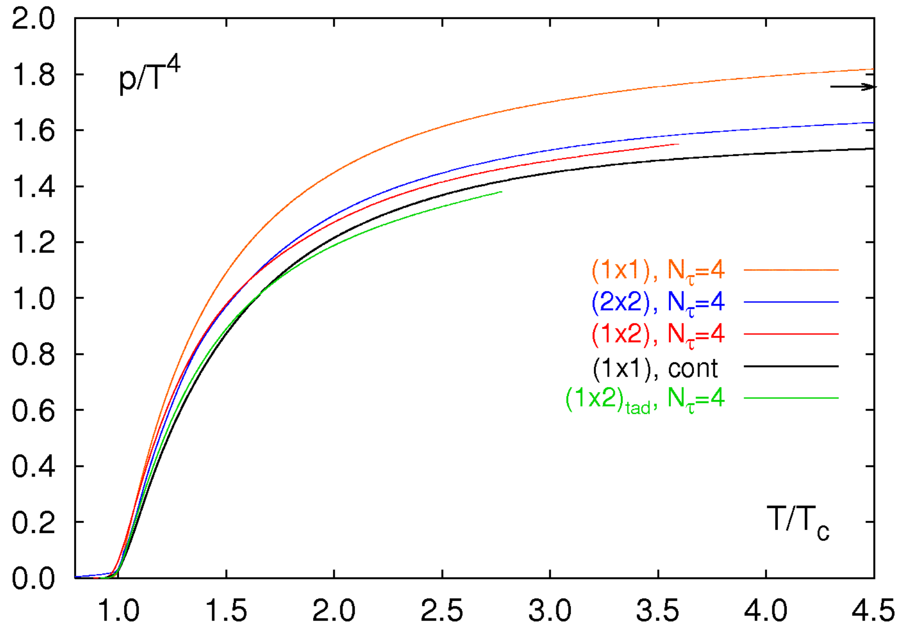
continuum extrapolation



Karsch et al, EPJ C 6 (99) 133

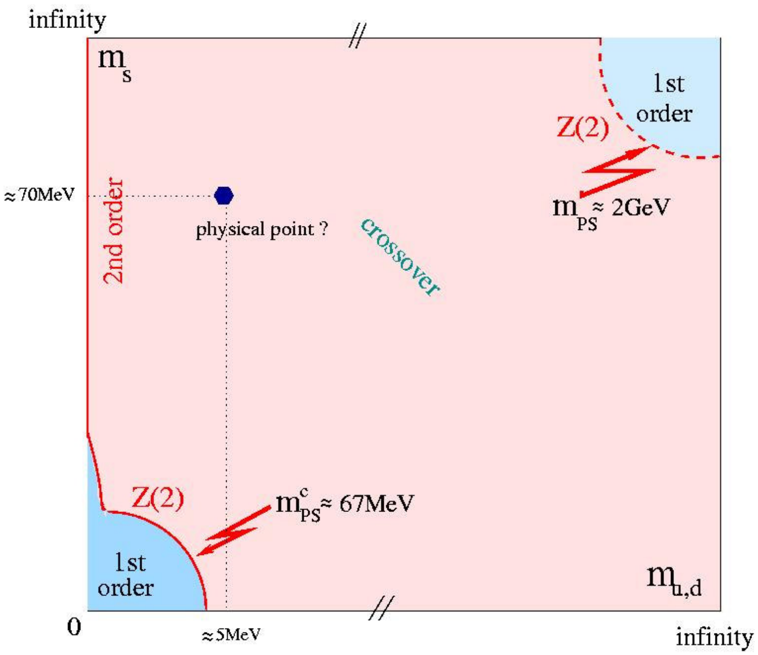
Luescher-Weisz gauge action:

large reduction of cutoff effects



QCD Phase diagram and EoS

At which temperature T_c does the transition occur ? What is the nature of transition deconfinement or chiral symmetrt restoring ?



$\sim \frac{1}{m_q} ReL \rightarrow$ crossover

Pisarski, Wilczek, PRD 29 (84) 338

chiral symmetry

$$\frac{\chi_{\psi\bar{\psi}}}{T^2} = N_\sigma^3 \left(\langle(\psi\bar{\psi})^2\rangle - \langle\psi\bar{\psi}\rangle^2 \right) = \langle(\delta\bar{\psi}\psi)^2\rangle$$

Rooted staggered quarks : U(1) chiral symmetry : no mass renormalization easy to fix LCP, can study chiral aspects of the transition, the most inexpensive

Sharpe, Rooted staggered fermions: Good, bad or ugly? PoS LAT2006:022,2006 (32cites)

Thermodynamics : >10,000 trajectories at >10 values of the gauge coupling !

Improved lattice action for QCD thermodynamics

Naïve (standard) discretisation : $S = \sum_{x,\mu} \eta_\mu(x) \, (\overline{\psi}_x U_\mu(x) \psi_{x+\mu} - \overline{\psi}_x U_\mu^+(x) \psi_{x-\mu})$

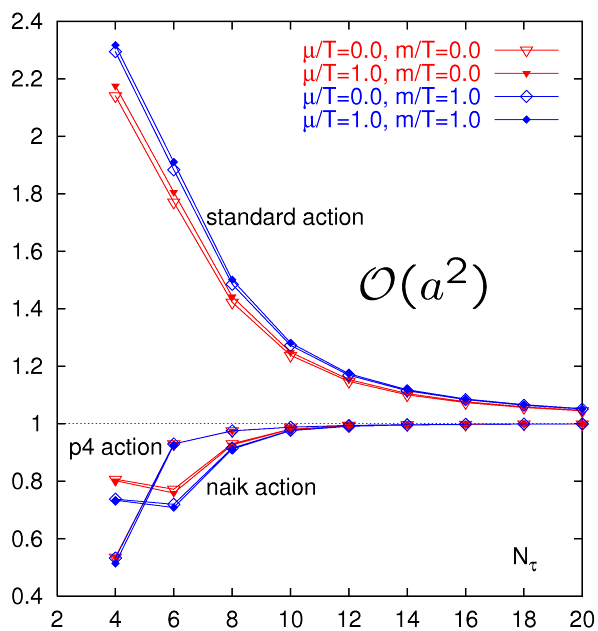
$\mathcal{O}(a^2)$ errors

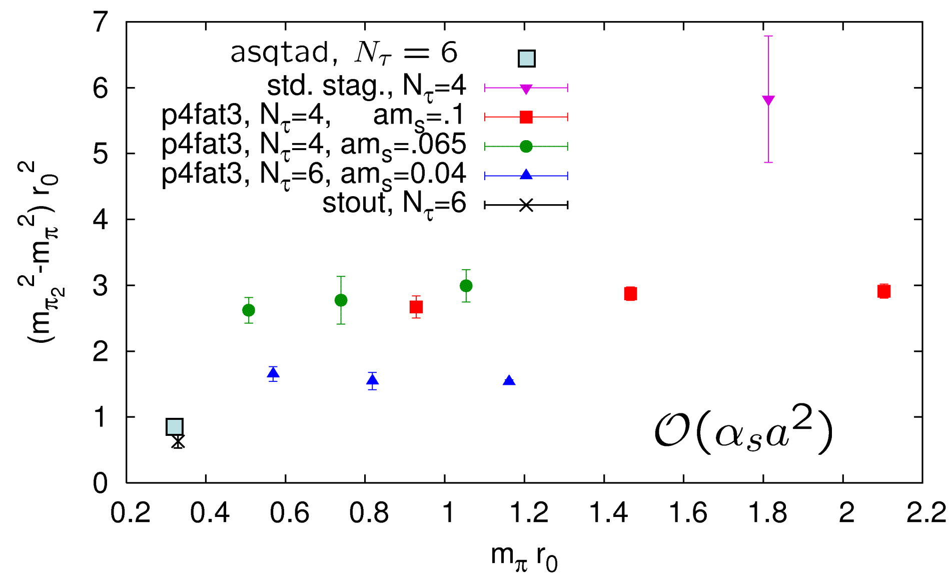
reduced discretization errors by
finite difference scheme with next-to-nearest
neighbor interaction

$SU(4)_V \times SU(4)_A \rightarrow U(1)$

$a = 1/(N_\tau T)$

$U_\mu(x) \Rightarrow U_\mu^{fat}(x)$



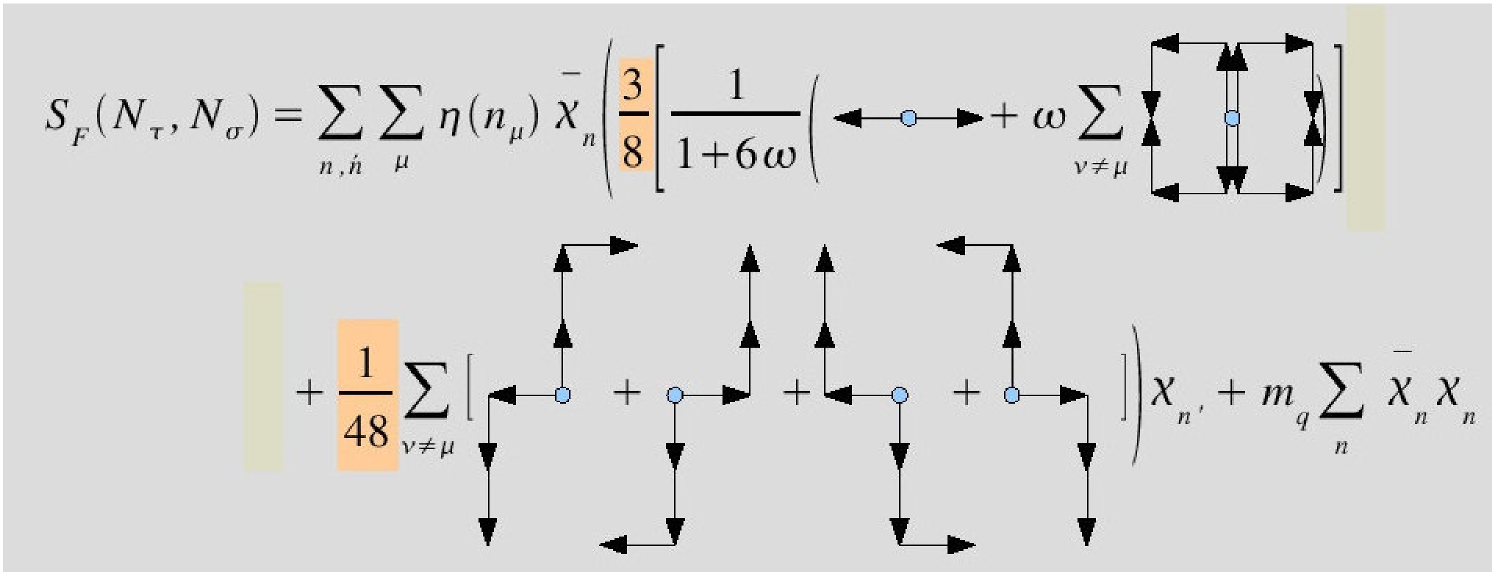


Improved staggered action: p4fat3, asqtad, stout

Algorithmic improvement : R-algorithm ⇨ RHMC algorithm : x 20 speedup at small *m*

Improved lattice actions

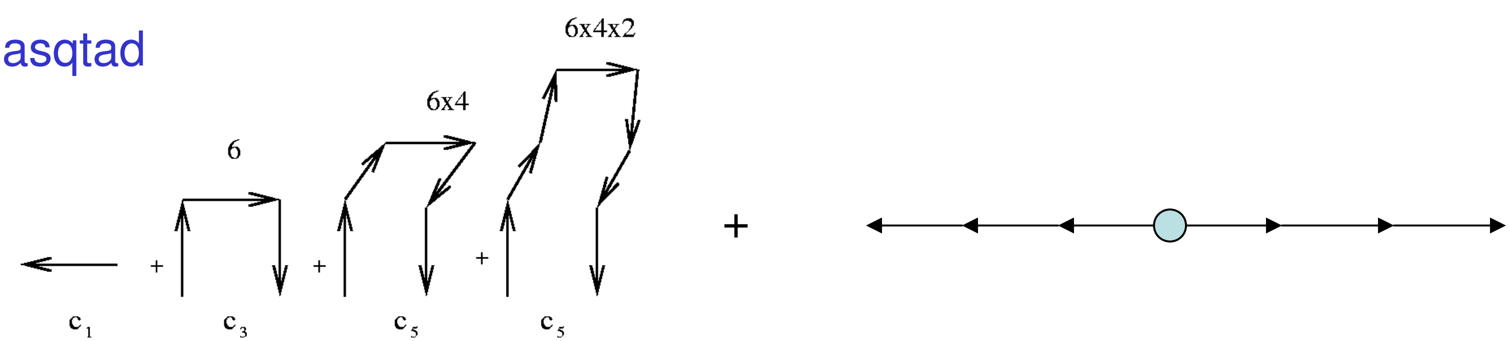
p4fat3



next-to-nearest neighbors
(p4) rotational symmetry
to $\mathcal{O}(p^4 a^4)$

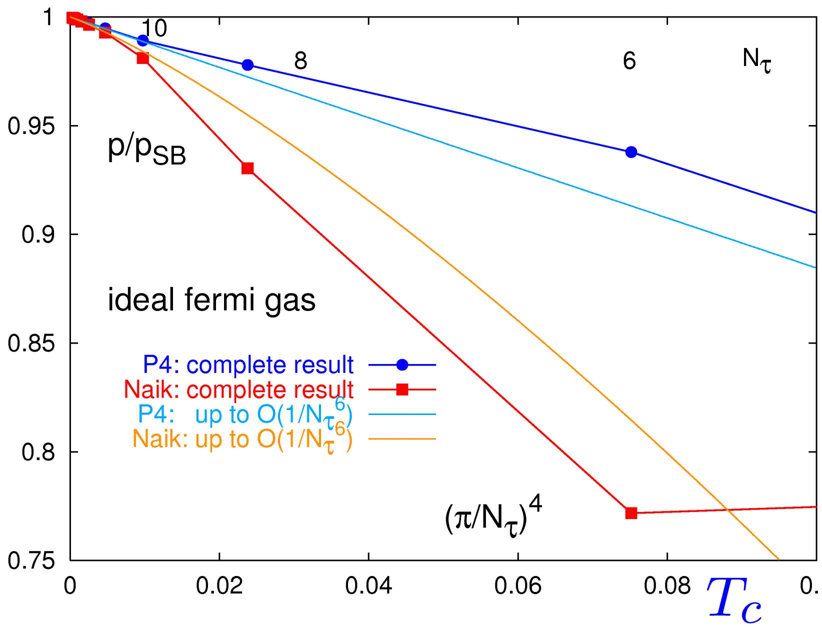
fat3 link improvement
of flavor symmetry

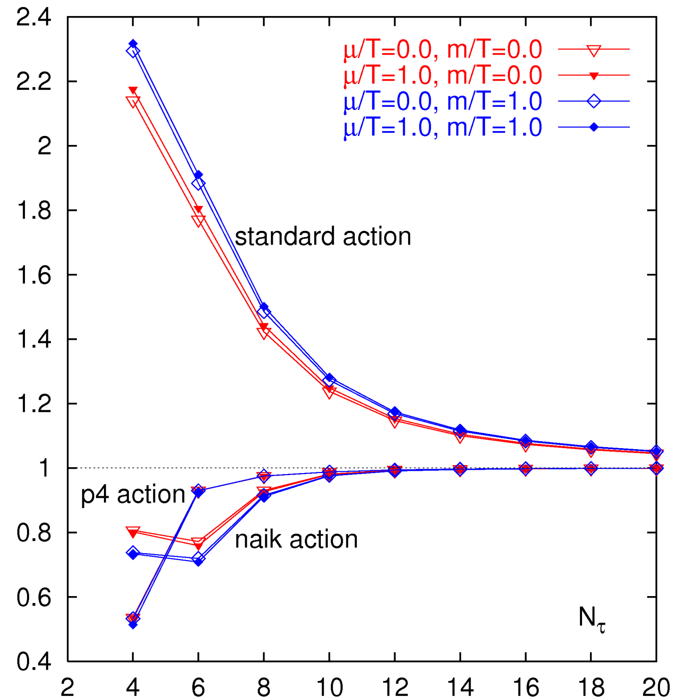
asqtad



no taste breaking at $\mathcal{O}(g^2 a^2)$

Why improved actions ?

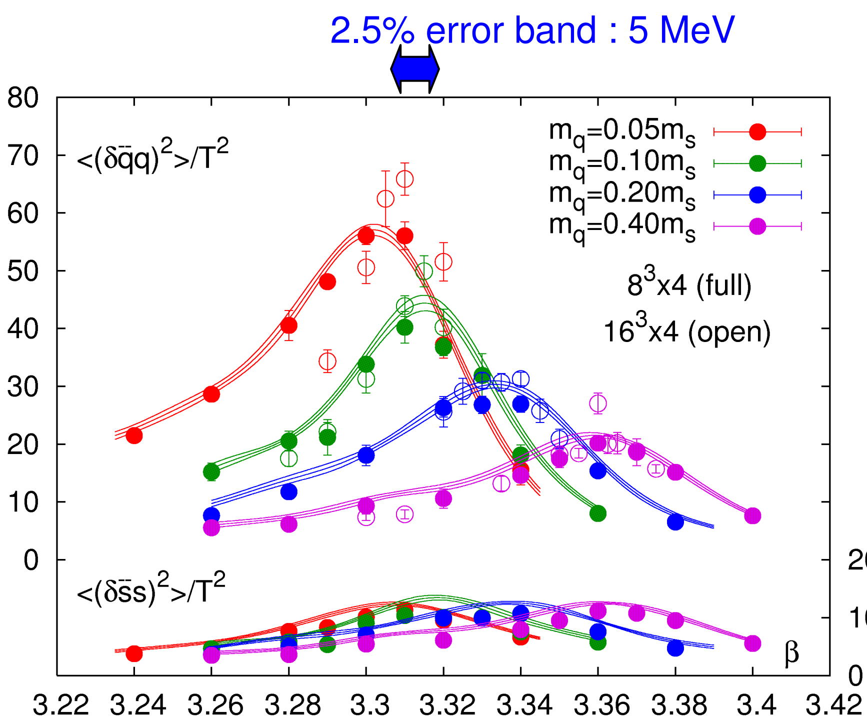


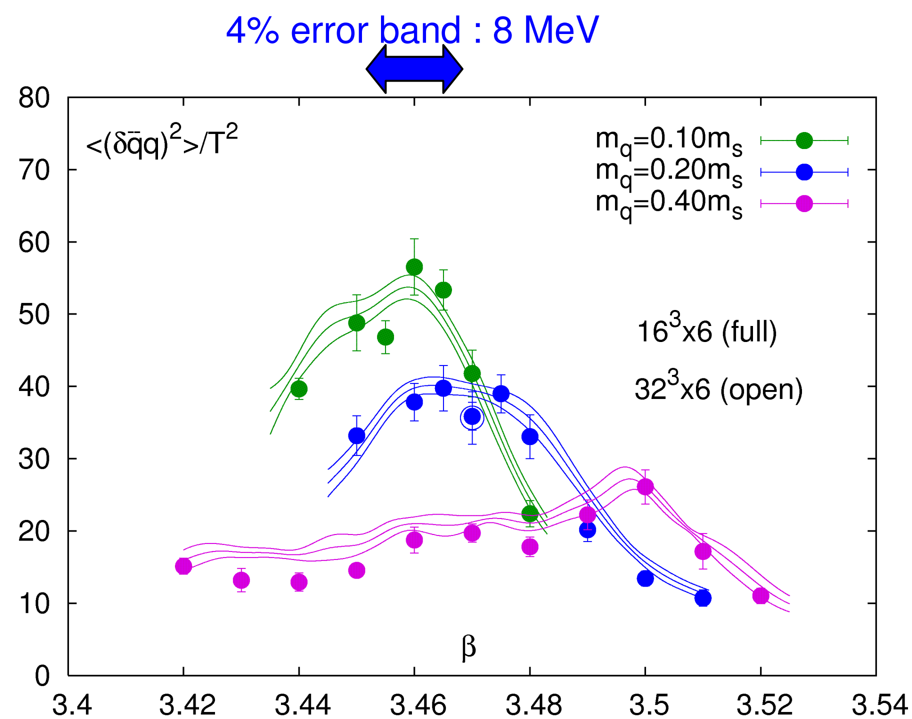


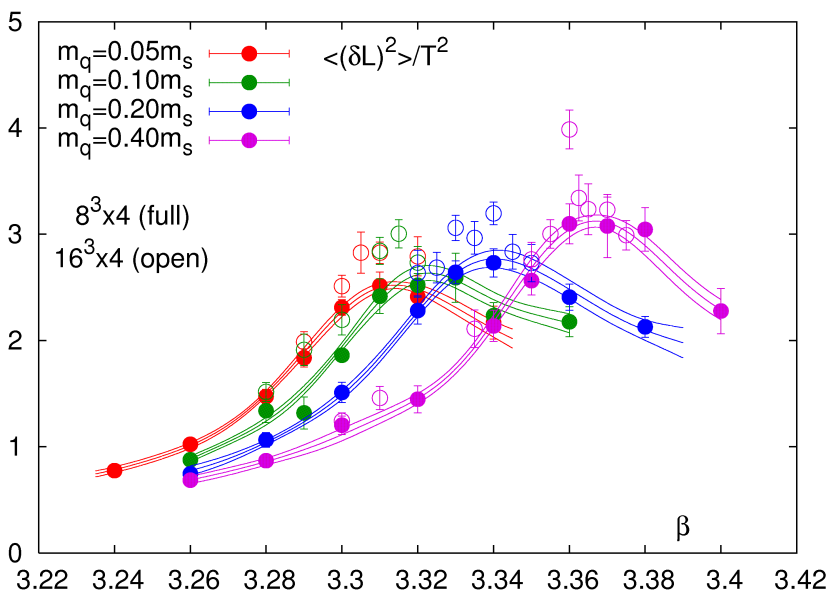
$$\frac{p(N_t)}{p_{cont}} = 1 - \frac{1143}{980}\left(\frac{\pi}{N_t}\right)^4 + \frac{73}{2079}(1+6528c_{30})\left(\frac{\pi}{N_t}\right)^6 + \mathcal{O}(N_t^{-8})$$

$c_{30} = 0$ for p4, $c_{30} = -1/48$ for Naik

$\alpha_s a^2$ corrections are smaller for p4 than for Naik



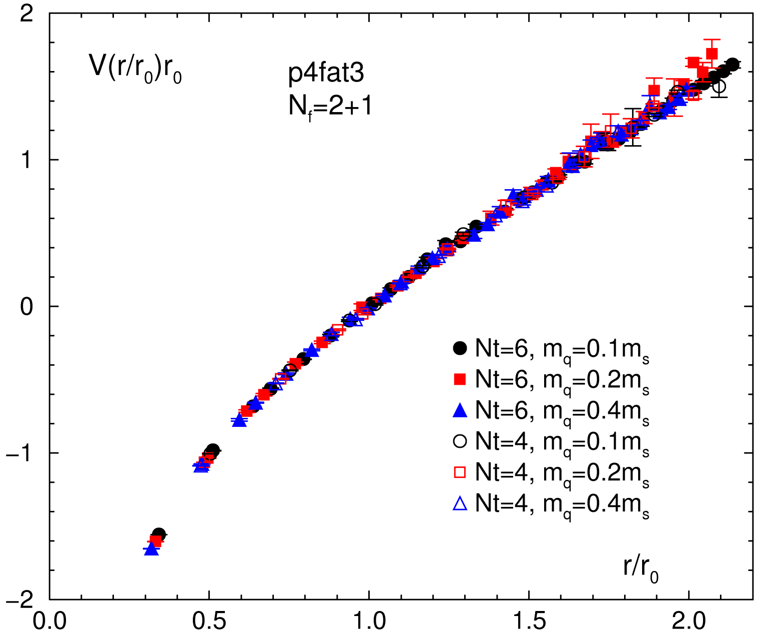




- the peak position in $\chi_{\psi\bar{\psi}}$ and χ_L is the same withing errors: deconfinement and chiral transition happen at the same temperature

- no significant volume dependence of the peak position

- the finite volume behavior is inconsistent with 1st ordre phase transition



Sommer scale:

$$r^2 \frac{dV}{dr}|_{r=r_0} = 1.65$$

quarkonia spectroscopy:

$r_0 = 0.469(11)(4)$ fm $\rightarrow \sqrt{\sigma} \simeq 460$ MeV

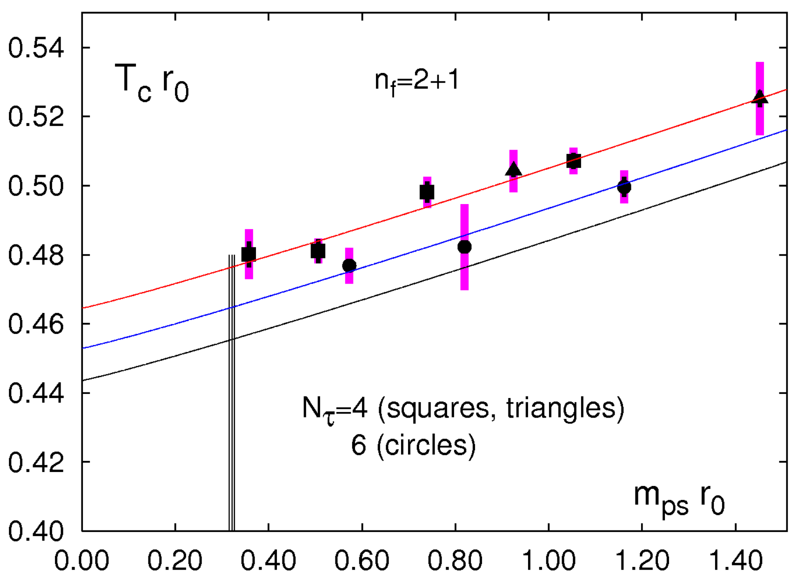
Gray et al, PRD 72 (2006) 094507

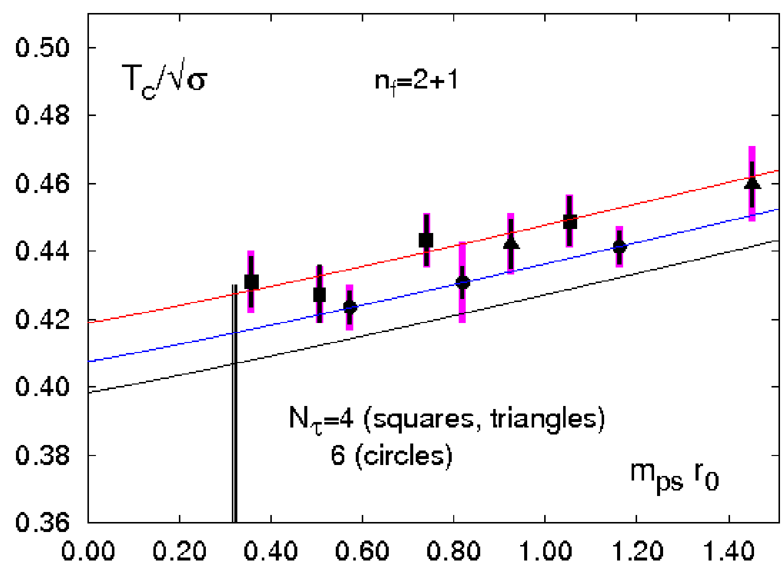
Combined $m_q - N_\tau$ extrapolation

$$r_0 T_c(m_\pi, N_\tau) = (r_0 T_c)^{chiral}_{cont} + b(m_\pi r_0)^d + c/N_\tau^2$$

$$T_c = 192(7)(4) \text{ MeV}$$

Cheng et al, PRD 74 (2006) 054507





Equation of State

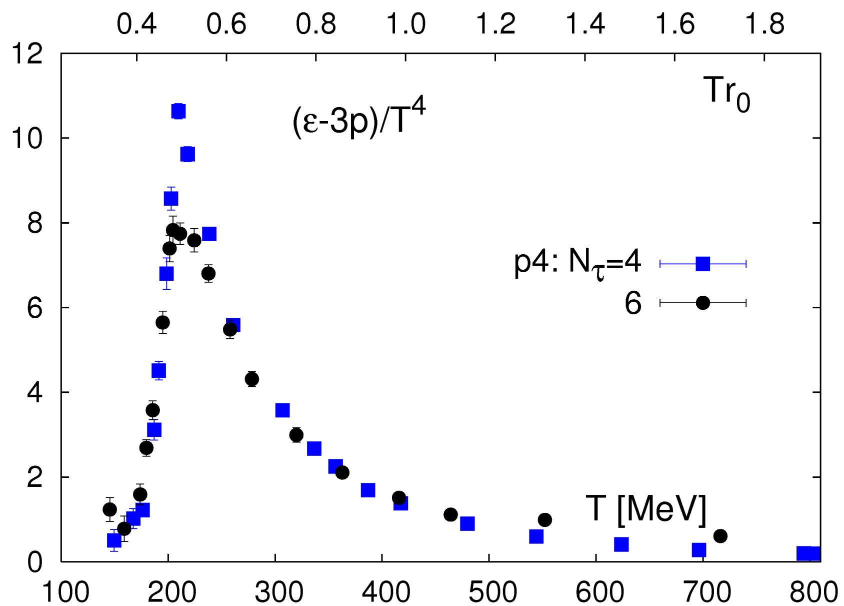
- Calculations : $16^3 \times 4$, $24^3 \times 6$ and $32^3 \times 6$ lattices, $m_q = 0.1 m_s \leftrightarrow m_\pi \simeq 200\text{MeV}$ and along the line of constant physics (LCP) :

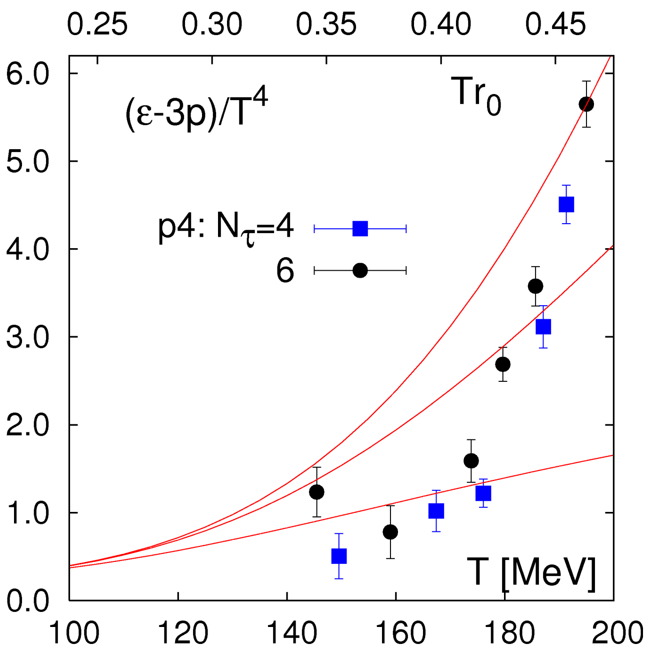
$\beta,\ m_q, m_s$ are varied in the way physical quantities, e.g. $m_\pi \cdot r_0$ and $m_\eta \cdot r_0$

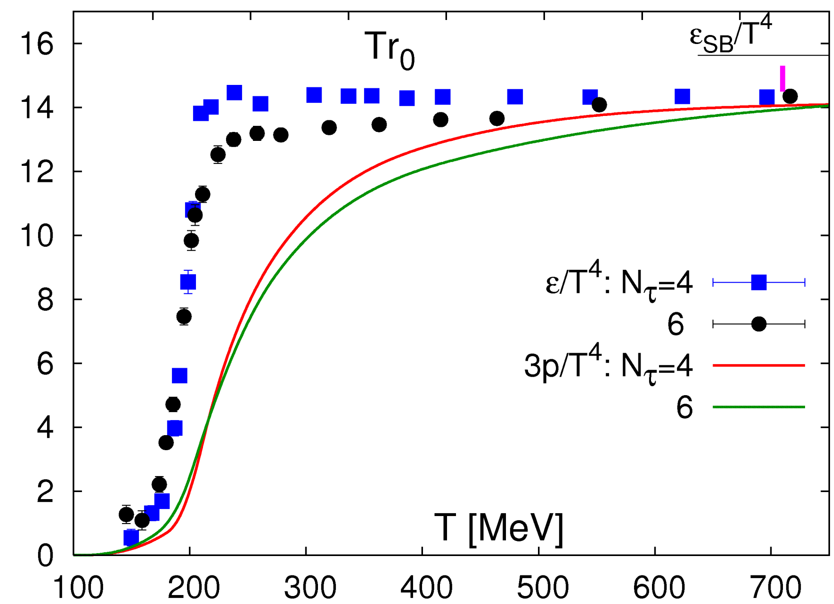
⇨ $m_q = m_q(\beta),\ m_s = m_s(\beta)$

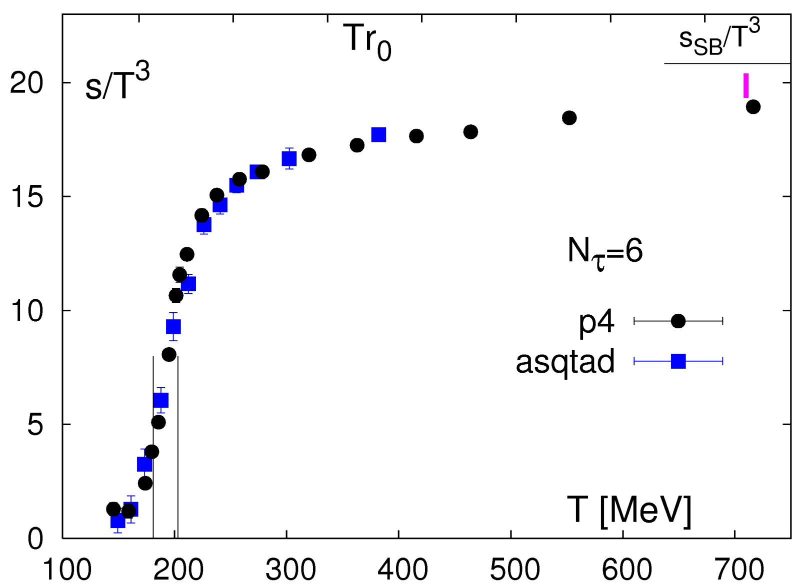
$$\frac{p(T)}{T^4} = \int_{\beta_0}^{\beta'(T)} d\beta [(\langle S_g \rangle_0 - \langle S_g \rangle_T) - m_q \left(\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T \right) \left(\frac{\partial m_s/m_q}{\partial \beta} \right)_{m_q} - \left(2(\langle \bar{q}q \rangle_0 - \langle \bar{q}q \rangle) + \frac{m_s}{m_q} (\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T) \right) \left(\frac{\partial m_q}{\partial \beta} \right)_{m_q/m_s}]$$

$$\frac{\epsilon - 3p}{T^4} = T \frac{d}{dT} \left(\frac{p}{T^4} \right) = - \left(a \frac{d\beta}{da} \right)_{LCP} \frac{\partial p/T^4}{\partial \beta'}$$



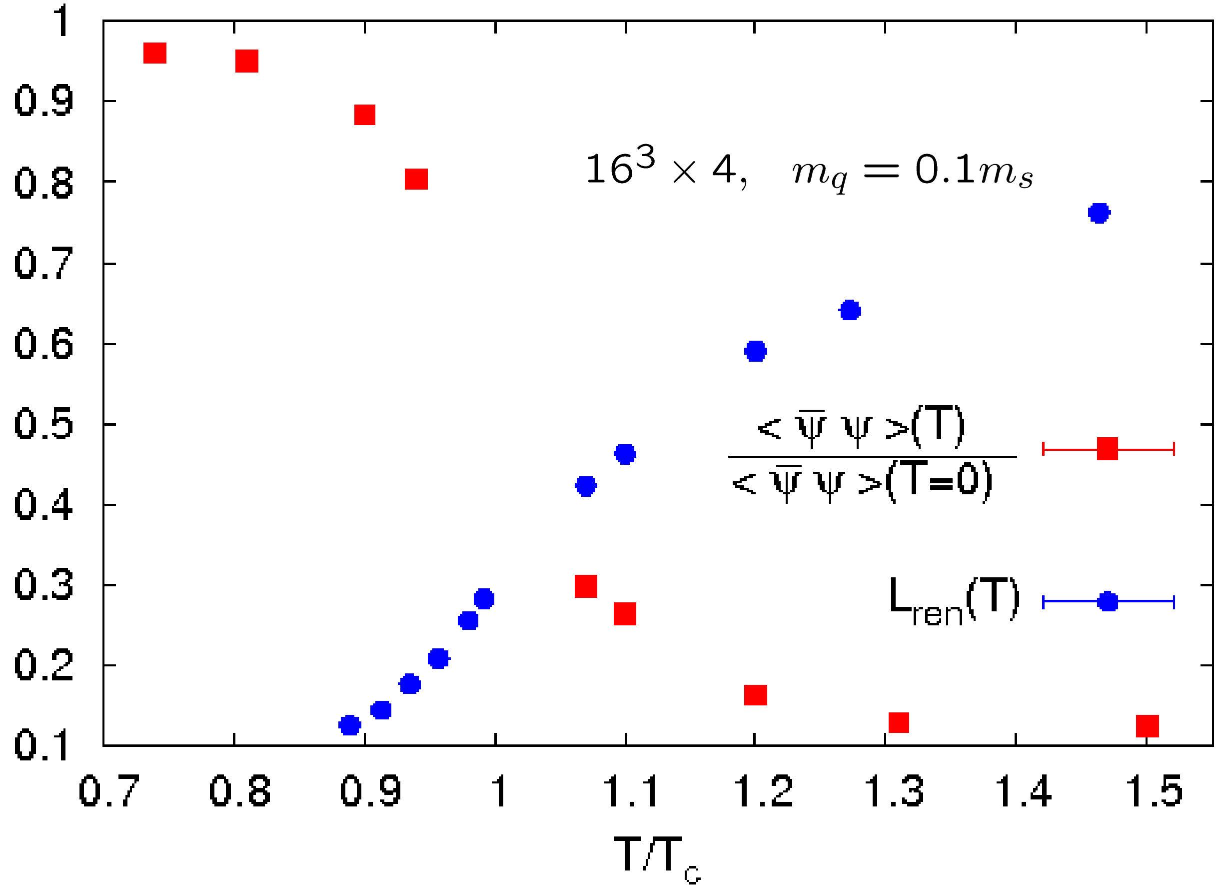






- rise in the entropy and energy density happens at the transition temperature determined from chiral condensate and Polyakov loops
- no large cutoff dependence in the pressure
- deviation from ideal gas limit is about 10% at high *T*, qualitative agreement with resonance gas at low *T*

Deconfinement and chiral transition ?



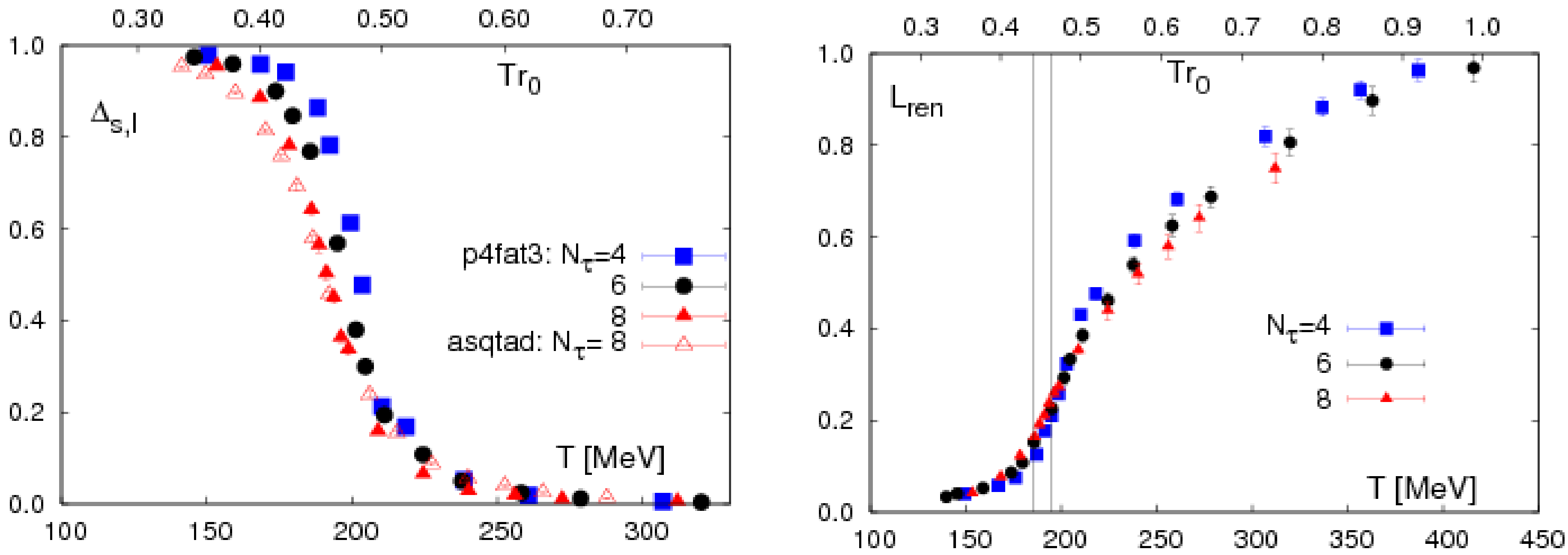
Chiral and deconfinement transitions happen at one temperature

Problem : chiral condensate has power divergence at non-zero quark mass : m/a^2

see Gasser, Leutwyler, Phys. Rept. 87 (82) 77

Power divergenices are present in the light and strange chiral condensates and proportional to the quark mass:

$$\Delta_l = \frac{\langle \bar{q} q \rangle_T - \frac{m_q}{m_s} \langle \bar{s} s \rangle_T}{\langle \bar{q} q \rangle_{T=0} - \frac{m_q}{m_s} \langle \bar{s} s \rangle_{T=0}}$$

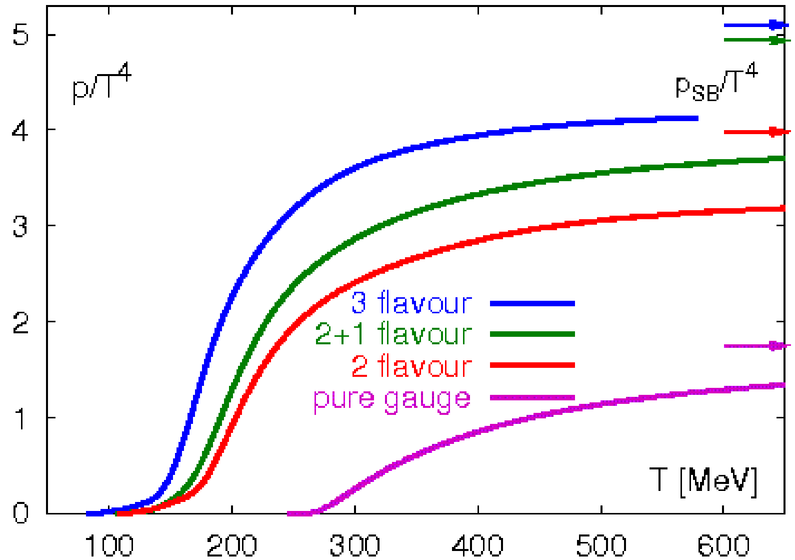


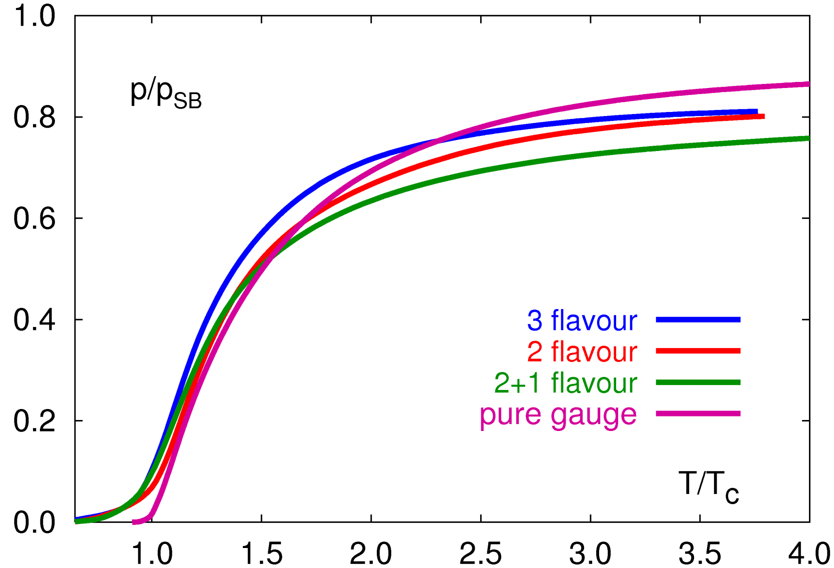
Hot QCD Collaboration, talk by R. Gupta, Lattice 2007

Flavor dependence of EoS

Karsch, Laermann, Peikert, PLB 478 (00) 447

$16^3 \times 4$ lattice, $a \simeq 0.25$ fm,
$m_q/T = 0.4 \leftrightarrow\ m_\pi \simeq 800$ MeV at T_c





Note on screened perturbation theory

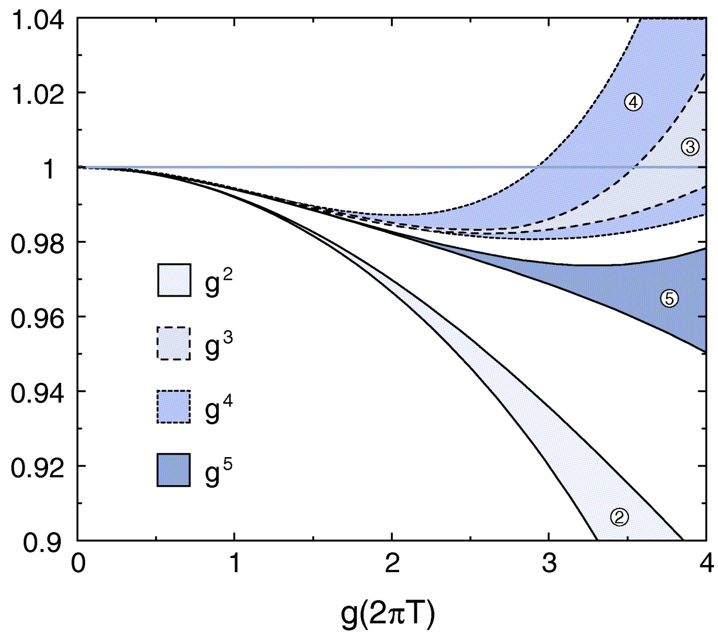
Consider the scalar field theory : resummation of ring diagrams is equivalent to claculation with massive propagators

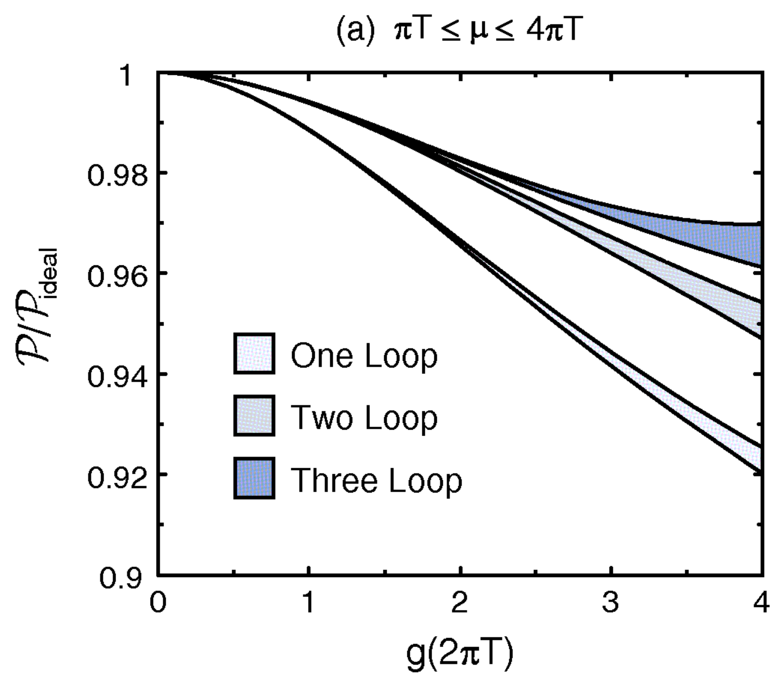
Karsch et al, PLB 401 (97) 69, Braaten et al, PRD 63 (01) 105008

1-loop results for the pressure

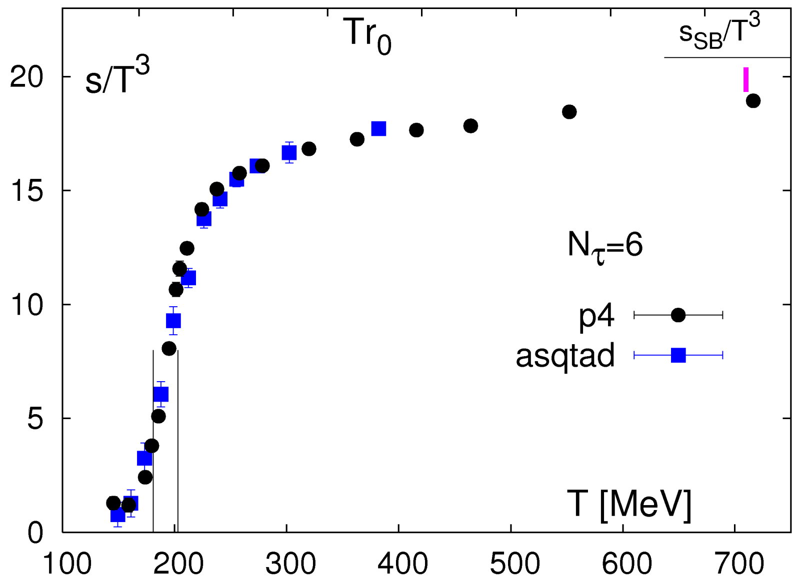
$$p(T) = \frac{1}{2} T \sum_n \int \frac{d^3p}{(2\pi)^3} \ln(\omega_n^2 + p^2 + m^2) = \frac{\pi^2 T^4}{90} - \frac{1}{24} m^2 T^2 + \frac{m^3}{12\pi} + \frac{m^4}{64\pi^2} \left(\ln\left(\frac{\mu}{4\pi T}\right) + 2\gamma_E \right)$$

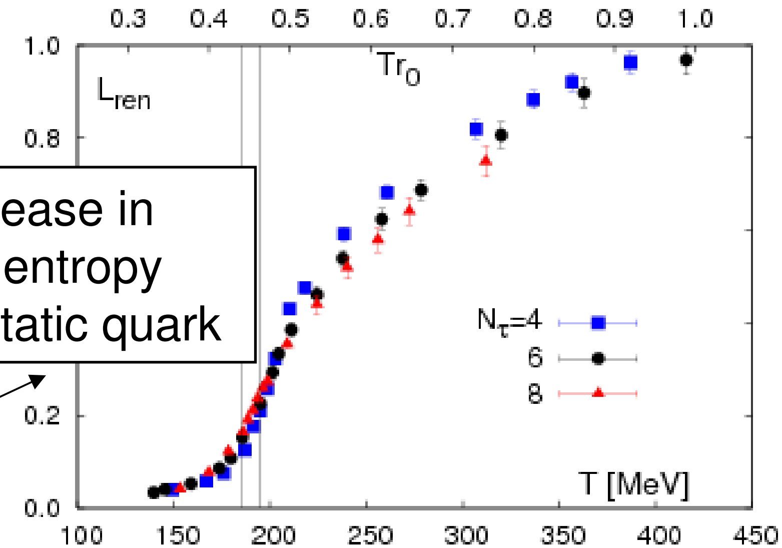
$m^2 \simeq \frac{g^2 T^2}{24}$ also contains contributions which are higher odrer in $\lambda = g^2$



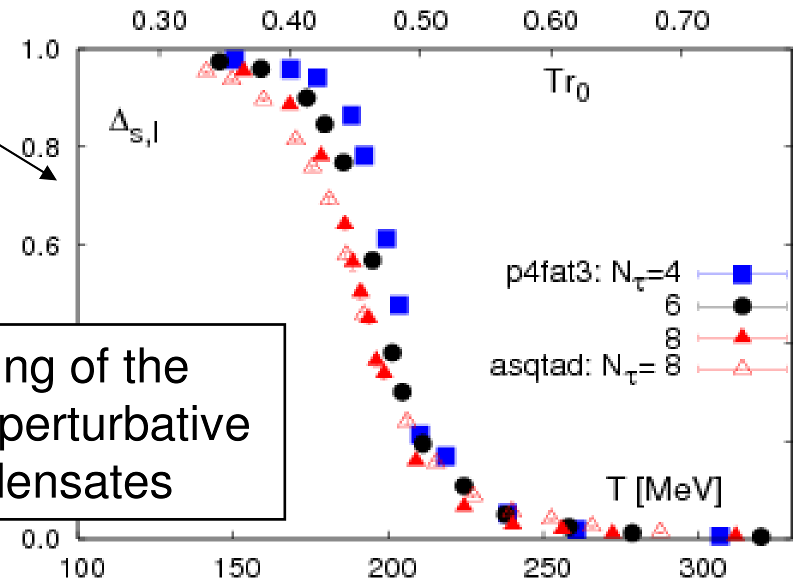


Is QCD transition deconfinement or chiral ?



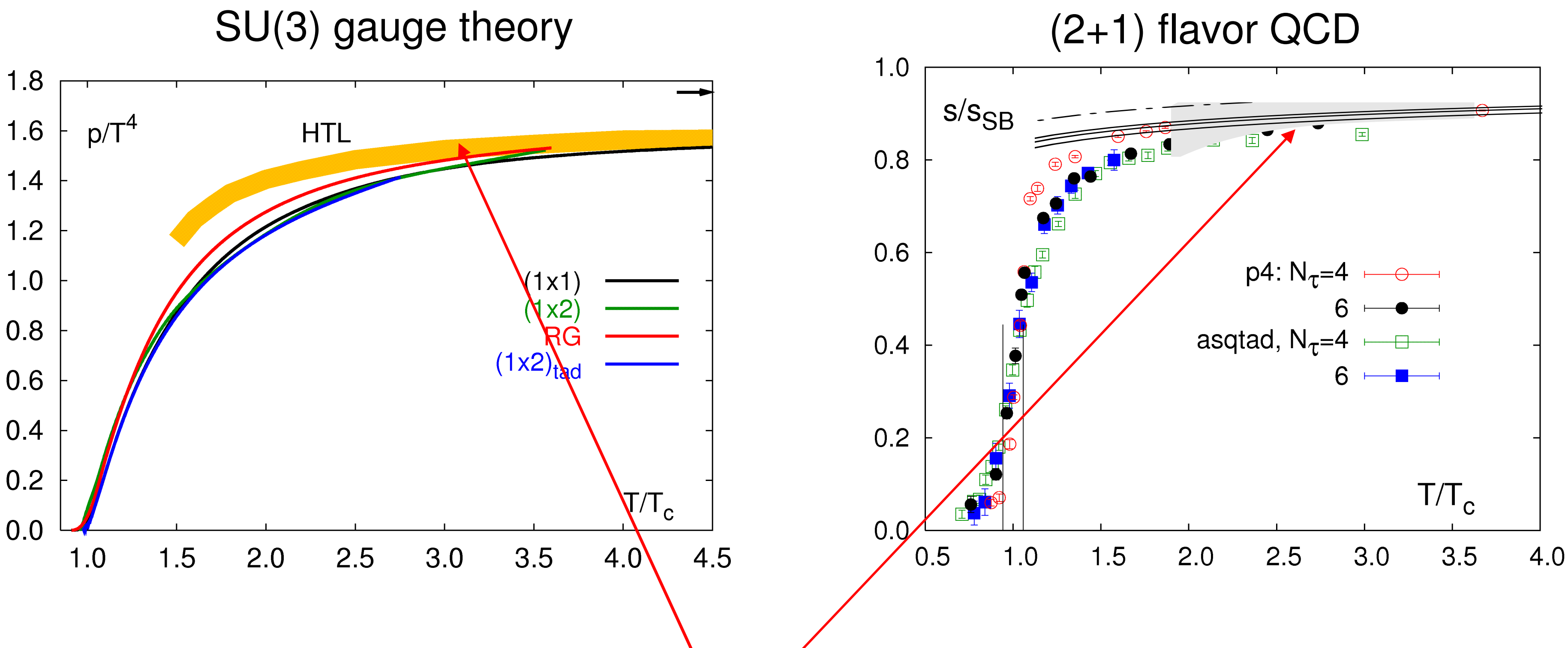


increase in the entropy of static quark



melting of the non-perturbative condensates

Comparison with resummed perturbation theory



Resummed perturbative calculations from :
Blaizot, Iancu, Rebhan, hep-ph/0303185

Lattice data on pressure and entropy density at high temperatures can be described by re-summed perturbation theory

Dimensional reduction at high temperatures

Decomposition in Matsubara modes

$$\phi(\tau, x) = \sum_n e^{i\omega_n \tau} \phi_n(x)$$

$$S_E = \int_0^\beta d\tau \int d^3x [(\partial_\mu \phi)^2 + V(\phi)] \rightarrow \int d^3x (\sum_n (\partial_i \phi_n(x))^2 + (2\pi T n)^2 \phi_n(x)) + V(\phi_n))$$

integrate out all $n \neq 0$ modes

Effective hight T theory for QCD $2\pi T \gg gT \gg g^2 T$:

$$S_{eff} = \int d^3x \left(\frac{1}{2} Tr F_{ij}^2 + Tr(D_i A_0)^2 + m_D^2 Tr A_0^2 + \lambda_3 (Tr A_0^2)^2 \right)$$

$$F_{ij} = \partial_i A_j - \partial_j A_i + i g_3 [A_i, A_j], \;\; D_i A_0 = \partial_i A_0 + i g_3 [A_i, A_0]$$

the parameters $g_3^2 \sim g^2 T$, $m_D \sim gT$ and $\lambda_3 \sim g^4 T$ can be computed perturbatively to any order.

The effective theory is confining and non-perturbative at momentum scales $< g_3^2$ but can be solved on the lattice to calculate the weak coupling expansion of the pressure and other quantities

Braaten, Nieto, PRD 51 (95) 6990, PRD 53 (96) 3421
Kajantie et al, NPB 503 (97) 357, PRD 67 (03) 105008

The spatial string tension

- Non-perturbative, vanishes in high-T perturbation theory:

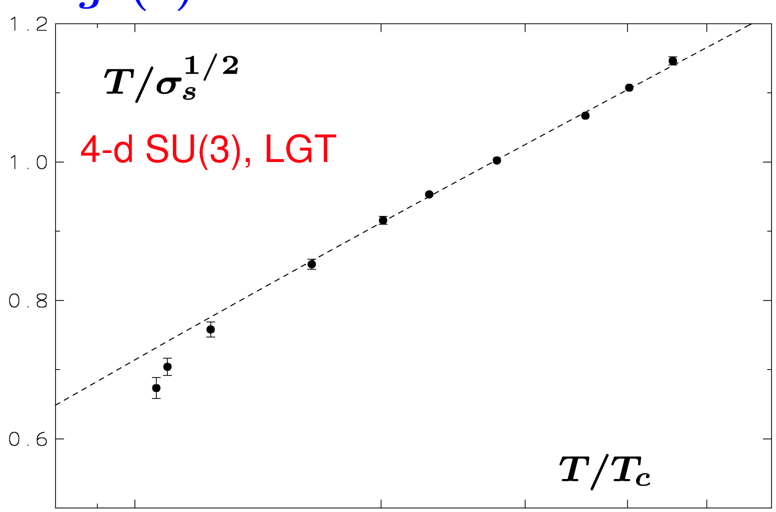
$$\sqrt{\sigma_s} = - \lim_{R_x, R_y \to \infty} \ln \frac{W(R_x, R_y)}{R_x R_y}$$

M. Teper, PRD 59 (99) 014512

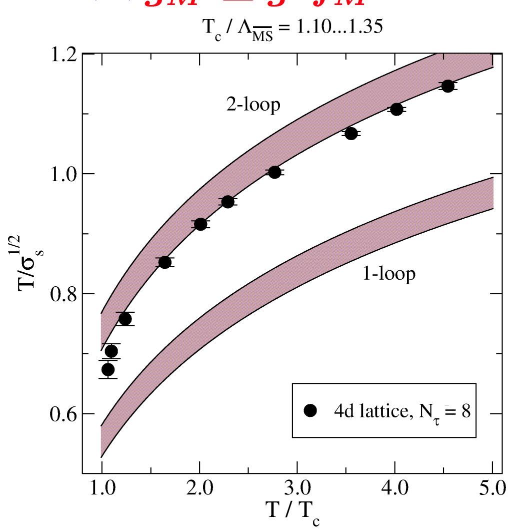
c_M : 3-d SU(3), LGT

- $$\frac{\sqrt{\sigma_s}}{g^2(2)T} = c_M f_M(g(T)) \; , \; c_M = 0.553(1)$$

$g_M \equiv g^2 f_M$: dim. red. pert. th.



G. Boyd et al. Nucl. Phys. B469 (1996) 419



M. Laine, Y. Schröder, JHEP 0503 (2005) 067

At which temperature lattice data meet the perturbative prediction ?

A new method to calculate the pressure

Fodor, Szabó, Lattice 2007, Regensburg, June 30-August 4, 2007