

# **Higgs Physics from the Lattice**

## **Lecture 3: Vacuum Instability and Higgs Mass Lower Bound**

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# Outline of Lecture Series: Higgs Physics from the Lattice

## 1. Standard Model Higgs Physics

- Outlook for the Higgs Particle
- Standard Model Review
- Expectations from the Renormalization Group
- Nonperturbative Lattice Issues?

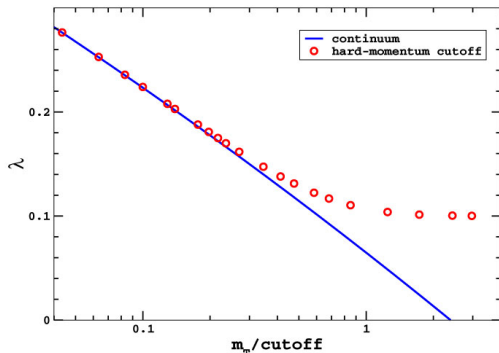
## 2. Triviality and Higgs Mass Upper Bound

- Renormalization Group and Triviality in Lattice Higgs and Yukawa couplings
- Higgs Upper Bound in 1-component  $\phi^4$  Lattice Model
- Higgs Upper Bound in O(4) Lattice Model
- Strongly Interacting Higgs Sector?
- Higgs Resonance on the Lattice

## 3. Vacuum Instability and Higgs Mass Lower Bound

- Vacuum Instability and Triviality in Top-Higgs Yukawa Models
- Chiral Lattice Fermions
- Top-Higgs and Top-Higgs-QCD sectors with Chiral Lattice Fermions
- Higgs mass lower bound
- Running couplings in 2-loop continuum Renormalization Group

## Large $N_F$ Effective Higgs Potential



running Higgs coupling (Holland,JK)

The continuum RG predicts that  $\lambda < 0$  as the blue continues and this has been used as an indication that the ground state of the theory is unstable.

Where  $\lambda$  vanishes is then used as a prediction of the energy scale of new physics.

The true RG flow with the full cutoff dependence saturates at  $\lambda_0$  and never turns negative.

Using the continuum RG flow when  $m_T/\Lambda$  is not small simply gives the wrong prediction. The apparent instability is an artifact of neglecting the necessary finite cutoff. Not only is that shown in the RG flow of  $\lambda$ , it can also be demonstrated directly in the Higgs effective potential.

## Effective Higgs potential

Let us review the argument that a light Higgs leads to an unstable vacuum:

For Higgs-Yukawa model with  $N_F$  fermions the 1-loop renormalized effective potential, including now the Higgs-loop contribution is

$$\begin{aligned}U_{\text{eff}} &= \frac{1}{2}m^2\phi^2 + \frac{1}{24}\lambda\phi^4 + \frac{1}{2}\delta m^2\phi^2 + \frac{1}{24}\delta\lambda\phi^4 - 2N_F \int_k \ln[1 + y^2\phi^2/k^2] \\ &+ \frac{1}{2} \int_k (\ln[k^2 + V''(\phi)] - \ln[k^2 + V''(0)]), \\ V &= \frac{1}{2}m^2\phi^2 + \frac{1}{24}\lambda\phi^4.\end{aligned}$$

Because  $\delta y$  and  $\delta z_{\psi}$  are non-zero (we no longer impose the large  $N_F$  limit), we specify two additional renormalization conditions. The fermion inverse propagator is

$$\begin{aligned}G_{\psi\psi}^{-1}(p) &= p_{\mu}\gamma_{\mu} + yv + \delta z_{\psi}p_{\mu}\gamma_{\mu} + \delta yv - \Sigma_F(p), \\ \Sigma_F(p) &= y^2 \int_k \frac{-k_{\mu}\gamma_{\mu} + yv}{(k^2 + y^2v^2)((k-p)^2 + m_H^2)},\end{aligned}$$

the radiative correction coming from a single Higgs-loop diagram, and we require that

$$G_{\psi\psi}^{-1}(p \rightarrow 0) = p_{\mu}\gamma_{\mu} + yv.$$

## Effective Higgs potential

The requirement  $G_{\psi\psi}^{-1}(p \rightarrow 0) = p_\mu \gamma_\mu + yv$  gives two renormalization conditions

$$\delta yv - \Sigma_F(p \rightarrow 0) = 0, \quad \delta z_\psi - \left. \frac{d\Sigma_F}{d(p_\mu \gamma_\mu)} \right|_{p \rightarrow 0} = 0.$$

We regulate the momentum integrals using e.g. a hard-momentum cutoff. The counterterms and the renormalized effective potential are calculated exactly using a finite cutoff.

We then take the naive limit  $\phi/\Lambda \rightarrow 0$  to remove all cutoff dependence. This ignores the fact that a finite and possibly low cutoff is required to maintain  $\lambda, y \neq 0$ . This will be the crucial point why the instability does not occur.

The continuum 1-loop renormalized effective potential is then

$$\begin{aligned} U_{\text{eff}} &= \frac{1}{2}m\phi^2 + \frac{1}{24}\lambda\phi^4 - \frac{N_F y^4}{16\pi^2} \left[ -\frac{3}{2}\phi^4 + 2v^2\phi^2 + \phi^4 \ln \frac{\phi^2}{v^2} \right] \\ &+ \frac{1}{16\pi^2} \left[ \frac{1}{16}(\lambda^2\phi^4 - 2\lambda\phi^2 m_H^2) \ln \frac{m^2 + \lambda\phi^2/2}{m_H^2} \right. \\ &\left. + \frac{1}{16}m_H^4 \ln \frac{m^2 + \lambda\phi^2/2}{m^2} - \frac{3}{32}\lambda^2\phi^4 + \frac{7}{16}\lambda\phi^2 m_H^2 \right], \end{aligned}$$

where  $m_H^2 = \lambda v^2/3$  and the tree-level vev  $v = \sqrt{3m_H^2/\lambda}$  is not shifted. The large  $N_F$  limit can be recovered by omitting the Higgs-loop terms.

## Effective Higgs potential Vacuum instability

The stability of the ground state is determined by  $U_{\text{eff}}$  for large  $\phi$  with the dominant terms  $\lambda^2 \phi^4 \ln(\phi^2/v^2)$  and  $-N_F y^4 \phi^4 \ln(\phi^2/v^2)$  from the relative values of  $\lambda^2$  and  $y^4$  (related to  $m_H$  and  $m_T$ ).

If the fermionic term dominates at large  $\phi$ , the minimum at  $v$  is only a local one and will decay. If we believe that the vacuum is absolutely stable, then new degrees of freedom must enter at the scale where  $U_{\text{eff}}(\phi)$  first becomes unstable. For given values of  $m_H$  and  $m_T$ , this predicts the emergence of new physics.

Turning this around, let us fix  $m_T$  and ask that no new stabilizing degrees of freedom are needed for  $\phi \leq E$ . Then we obtain a lower bound  $m_H(E)$ : if the Higgs is lighter than this,  $U_{\text{eff}}$  is already unstable for  $\phi$  below  $E$  because the fermion term dominates even earlier.

Vacuum instability is improved using the 1-loop RG equations

$$\begin{aligned}\mu \frac{dy}{d\mu} &= \frac{1}{8\pi^2} (3 + 2N_F)y^4, \\ \mu \frac{d\lambda}{d\mu} &= \frac{1}{16\pi^2} (3\lambda^2 + 8N_F\lambda y^2 - 48N_F y^4).\end{aligned}$$

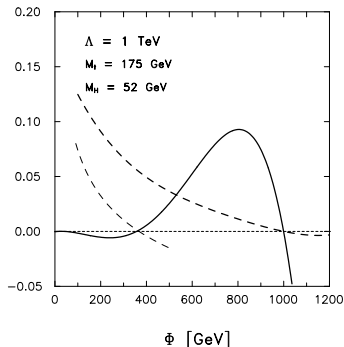
We can set the initial conditions  $\lambda(\mu = v) = 3m_H^2/v^2$  and  $y(\mu = v) = m_T/v$ . If  $m_T$  is sufficiently heavy relative to  $m_H$ , the Yukawa coupling dominates the RG flow and  $d\lambda/d\mu < 0$ . The renormalized Higgs coupling eventually becomes negative at some  $\mu = E$ .

## Effective Higgs potential    RG improved Vacuum instability

If the instability occurs at very large  $\phi/v$ , large logarithmic terms  $\ln(\phi/v)$  in  $U_{\text{eff}}$  might spoil the perturbative expansion. This can be reduced using renormalization group improvement to resum the leading large logarithms.

1-loop RG improved effective potential is  $U_{\text{eff}} \approx -\frac{1}{4}m_H^2(t)\Phi^2 + \frac{\lambda(t)}{4!}\Phi^4$ , with  $t = \ln \frac{\Phi}{\mu}$ .

### RG improved effective Higgs potential



The dominant terms of  $U_{\text{eff}}$  at large  $\phi$  then become  $\lambda(\mu)\phi^4(\mu)$ . Hence  $\lambda(E) = 0$  indicates that the ground state is just about to become unstable.

**We can calculate the effective potential non-perturbatively, using lattice simulations and compare with the RG predictions of vacuum instability**

## Effective Higgs potential    Lattice simulations

After the fermion field is integrated out in Top-Higgs Yukawa lattice field theory, the constraint effective potential in a finite euclidean volume  $\Omega$  is defined by

$$\exp(-\Omega U_{\Omega}(\Phi)) = \prod_x \int d\phi(x) \delta\left(\Phi - \frac{1}{\Omega} \sum_x \phi(x)\right) \exp(-S_{eff}[\phi]).$$

The delta function enforces the constraint that the scalar field  $\phi$  fluctuates around a fixed average  $\Phi$ . The constraint effective potential  $U_{\Omega}(\Phi)$  has a very physical interpretation. If the constraint is not imposed, the probability that the system generates a configuration where the average field takes the value  $\Phi$  is

$$P(\Phi) = \frac{1}{Z} \exp(-\Omega U_{\Omega}(\Phi)), \quad Z = \int d\Phi' \exp(-\Omega U_{\Omega}(\Phi')).$$

This is in close analogy to the probability distribution for the magnetization in a spin system. The scalar expectation value  $vev = \langle \phi \rangle$  is the value of  $\Phi$  for which  $U_{\Omega}$  has an absolute minimum. In a finite volume, the constraint effective potential is non-convex and can have multiple local minima. In a finite volume, it is convenient to work with the constraint effective potential, where multiple minima can be observed and the transition between the Higgs and symmetric phases is clear. It is also more natural, as the probability distribution  $P(\Phi)$  can be directly observed in lattice simulations. In what follows, we drop the subscript  $\Omega$ .



## Effective Higgs potential Lattice simulations

An accurate simulation method is available to calculate the derivative of the effective potential. For the Top-Higgs Yukawa model with  $N_F$  degenerate fermions, the derivative is

$$\frac{dU_{\text{eff}}}{d\Phi} = m^2\Phi + \frac{1}{6}\lambda\langle\phi^3\rangle_{\Phi} - N_F y\langle\bar{\psi}\psi\rangle_{\Phi}, \quad \langle\bar{\psi}\psi\rangle_{\Phi} = \langle\text{Tr}(D[\phi]^{-1})\rangle_{\Phi}.$$

The expectation values  $\langle\dots\rangle_{\Phi}$  mean that, in the lattice simulations, the scalar field fluctuates around some fixed average value  $\Phi$ . A separate lattice simulation has to be run for every value of  $\Phi$ . This is the method we propose to use in our investigation of the vacuum instability. Configurations are generated using the Hybrid Monte Carlo algorithm, where a fictitious time  $t$  and momenta  $\pi(x, t)$  are introduced. New configurations are generated from the equations of motion which we will write (for the scalar field only) as

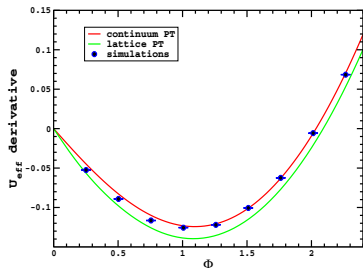
$$\begin{aligned}\dot{\phi}(x, t) &= \pi(x, t) \\ \dot{\pi}(x, t) &= -\left[ \frac{\partial S_{\text{eff}}}{\partial\phi(x, t)} - \frac{1}{\Omega} \sum_y \frac{\partial S_{\text{eff}}}{\partial\phi(y, t)} \right],\end{aligned}$$

where the effective action  $S_{\text{eff}}$  is obtained after fermion integration. The only difference from the usual HMC algorithm is the second term in  $\dot{\pi}(x, t)$ . This is a Lagrange multiplier which is present to enforce the constraints

$$\frac{1}{\Omega} \sum_y \phi(y, t) = \Phi, \quad \sum_y \pi(y, t) = 0.$$

This method requires extreme lattice computing if Ginsburg-Wilson chiral fermions are used!

# Effective Higgs potential Fate of the vacuum instability



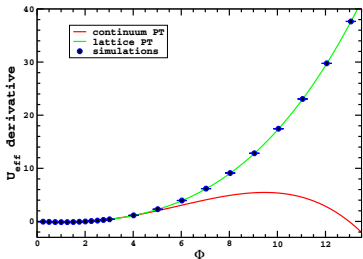
The derivative of the effective potential  $dU_{\text{eff}}/d\Phi$  for the bare couplings  $y_0 = 0.5$ ,  $\lambda_0 = 0.1$ ,  $m_0^2 = 0.1$ , for which the vev is  $av = 2.035(1)$ . The upper plot is a close-up of the behavior near the origin. The circles are the results of the simulations and the curves are given by continuum and lattice renormalized perturbation theory.

1-loop RG improved effective potential:

$$U_{\text{eff}} \approx -\frac{1}{4}m_H^2(t)\Phi^2 + \frac{\lambda(t)}{4!}\Phi^4$$

with  $t = \ln \frac{\Phi}{\mu}$ .

**2-loop RG improved effective potential is the most accurate state of the art today**



effective Higgs potential (Holland,JK)

## Ginsparg-Wilson overlap fermions

The Ginsparg-Wilson overlap fermion operator was adopted to represent the chiral Yukawa coupling between the Top quark fermion field and the Higgs scalar field. Staggered and Wilson fermions would not work.

Lattice Dirac operator  $D$  is required to satisfy the Ginsparg-Wilson (GW) algebraic relation

$$\gamma_5(\gamma_5 D) + (\gamma_5 D)\gamma_5 = 2R a(\gamma_5 D)^2$$

and  $R = 1$  will be chosen. The corresponding Wilson-Dirac operator is given by

$$D_w = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) - \frac{1}{2}a\nabla_\mu^*\nabla_\mu,$$

where  $\nabla_\mu$  ( $\nabla_\mu^*$ ) is the forward (backward) lattice derivative with or without the SU(3) gauge link matrices from QCD (note that  $\gamma_5 D_w \gamma_5 = D_w^\dagger$ ).

Define now

$$A = \frac{1}{2} - aD_w,$$

where  $D_w$  is the standard massless Wilson-Dirac operator.

Then define

$$D = \frac{1}{2a} \left( 1 - A \frac{1}{\sqrt{A^\dagger A}} \right).$$

With  $R = 1$ ,  $D$  satisfies the GW relation

$$\gamma_5 D + D \gamma_5 = 2a D \gamma_5 D.$$

## Ginsparg-Wilson overlap fermions

With  $\hat{\gamma}_5 = \gamma_5(1 - 2D)$  we define two projection operators

$$P_{\pm} = \frac{1}{2}(1 \pm \gamma_5), \quad \hat{P}_{\pm} = \frac{1}{2}(1 \pm \hat{\gamma}_5),$$

and chiral components

$$\bar{\psi}_{L,R} = \bar{\psi}P_{\pm}, \quad \psi_{R,L} = \hat{P}_{\pm}\psi.$$

With  $\Gamma_5 = \gamma_5(1 - D)$  the scalar and pseudoscalar densities are defined as

$$\begin{aligned} S(x) &= \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L = \bar{\psi}\gamma_5\Gamma_5\psi, \\ P(x) &= \bar{\psi}_L\psi_R - \bar{\psi}_R\psi_L = \bar{\psi}\Gamma_5\psi. \end{aligned}$$

The most natural Lagrangian consistent with lattice U(1) chiral symmetry  $\delta\psi = i\epsilon\hat{\gamma}_5\psi$ ,  $\delta\bar{\psi} = \bar{\psi}i\epsilon\gamma_5$  and  $\delta\phi = -2i\epsilon\phi$ , is defined by

$$\begin{aligned} \mathcal{L}_{\text{fermion}} &= \bar{\psi}D\psi + +2g_y\bar{\psi}(P_+\phi\hat{P}_+ + P_-\phi^\dagger\hat{P}_-)\psi \\ &= \bar{\psi}_RD\psi_R + \bar{\psi}_LD\psi_L + 2g_y[\bar{\psi}_L\phi\psi_R + \bar{\psi}_R\phi^\dagger\psi_L]. \end{aligned}$$

The full Lagrangian for a real scalar field has an exact discrete chiral symmetry when the fermion field is rotated by  $\pi$  under overlap chiral transformation and simultaneously  $\phi$  is flipped into  $-\phi$ . The fermion scalar density will change sign under this rotation which is compensated by the  $\phi \rightarrow -\phi$  reflection in the real scalar field. This symmetry is spontaneously broken in the Higgs vacuum. It is straightforward to incorporate the four-component Higgs field with O(4) chiral symmetric Yukawa coupling.

## Ginsparg-Wilson overlap fermions    Top-Bottom Doublet Coupling

Although difficult in simulations, it is possible to include the third, heaviest generation of quarks consisting of the left-handed  $SU(2)$  top-bottom doublet

$$Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix},$$

and the corresponding right-handed  $SU(2)$  singlets  $t_R, b_R$ . The complex  $SU(2)$  doublet Higgs field  $\Phi(x)$  with  $U(1)$  hypercharge  $Y = 1$  is

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},$$

where the suffixes  $+,0$  characterize the electric charge  $+1, 0$  of the components. Since  $\phi^+$  and  $\phi^0$  are complex, we can introduce four real components,

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ i\phi_3 + \phi_4 \end{pmatrix},$$

and the Higgs potential will have  $O(4)$  symmetry, with broken custodial  $O(3)$  symmetry, if  $y_t$  and  $y_b$  are chosen to be different.

## Ginsparg-Wilson overlap fermions    Top-Bottom Doublet Coupling

The Higgs potential in the complex doublet notation has the form,

$$V(\Phi) = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2.$$

It is parametrized such that the Higgs field acquires a vacuum expectation value responsible for the spontaneous electroweak symmetry breakdown

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \frac{m}{\sqrt{\lambda}}.$$

The numerical value of  $v$  is given in terms of the Fermi constant  $v = (\sqrt{2}G_F)^{-1/2} = 246.2 \text{ GeV}$ .

Of the four Higgs degrees of freedom three are Goldstone degrees of freedom, furnishing the longitudinal degrees of freedom for the massive weak gauge bosons, thus providing the W boson mass  $m_W = vg_2/2$ . The remaining one corresponds to the physical Higgs boson field

$$h = \sqrt{2}(\text{Re}\phi^0 - v/\sqrt{2}).$$

We do not use the Higgs mechanism in the limit of zero weak gauge couplings and keep all four Higgs field components where the  $\phi_1, \phi_2, \phi_3$  fluctuations represent Goldstone particles with the symmetry breaking in the  $\phi_4$  direction.

## Ginsparg-Wilson overlap fermions Top-Bottom Doublet Coupling

In the Lagrangian all 4 components are treated on equal footing where  $\mathcal{L}_{\text{Yukawa}}$  describes the interactions of the  $SU(2)_L$  doublet Higgs field with the quark fields

$$\mathcal{L}_{\text{Yukawa}} = y_t \cdot \bar{Q}_L \Phi^c t_R + y_b \cdot \bar{Q}_L \Phi b_R + \text{h.c.}$$

$\Phi^c = i\tau_2 \Phi^*$  is the charge conjugate of  $\Phi$ ,  $\tau_2$  the second Pauli matrix,  $y_t, y_b$  are the top and bottom Yukawa couplings, respectively. When they are equal, the  $O(3)$  custodial symmetry of the Higgs potential is preserved after symmetry breaking. For unequal couplings, only the  $SU(2)_L$  symmetry of the Lagrangian is maintained.

It is easy to write out the Yukawa couplings in components:

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & y_t \{ \bar{t}_L (\phi_4 - i\phi_3) t_R + \bar{b}_L (i\phi_2 - \phi_1) t_R \} + \\ & y_b \{ \bar{t}_L (\phi_1 + i\phi_2) b_R + \bar{b}_L (i\phi_3 + \phi_4) b_R \} + \text{h.c.} \end{aligned}$$

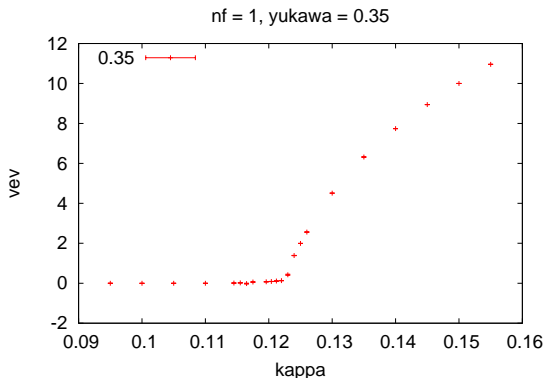
All masses are proportional to  $v$  as they are induced by spontaneous symmetry breaking. The weak gauge boson masses allow to determine  $v$  which was already introduced. The tree level top, bottom, and Higgs masses are given in terms of the vacuum expectation value  $v$  and their respective couplings  $m_t = g_t \frac{v}{\sqrt{2}}$ ,  $m_b = g_b \frac{v}{\sqrt{2}}$ ,  $m_H = \sqrt{2\lambda} v$ .

It is easy to readjust coupling constant and field normalizations with right and left handed quark fields as chiral overlap components.

# Chiral Top-Higgs Model: Phase diagram

Fodor,Holland,JK,Nogradi,Schroder

Top-Higgs phase diagram:



**1-component model**

**Large N hints full phase diagram**

hopping parameter notation for lattice Lagrangian:

$$\mathcal{L} = -2\kappa \sum_{\mu=1}^4 \phi(x)\phi(x + a\hat{\mu}) + \phi(x)^2 + \lambda[\phi(x)^2 - 1]^2,$$

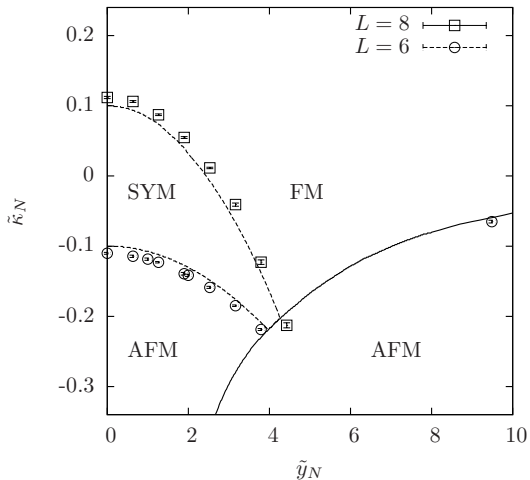
$$a\phi_0(x) = (2\kappa)^{1/2}\phi(x), \quad a^2m_0^2 = \frac{1-2\lambda}{\kappa} - 8, \quad g_0 = \frac{6\lambda}{\kappa^2}.$$



# Chiral Top-Higgs Model: Phase diagram

Gerhold and Jansen

Top-Higgs doublet phase diagram:



**Fermion doublet model**

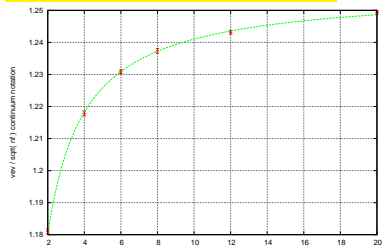
**Compared with large N**

**Very similar to 1-component model**

# Chiral Top-Higgs Model: Large N test

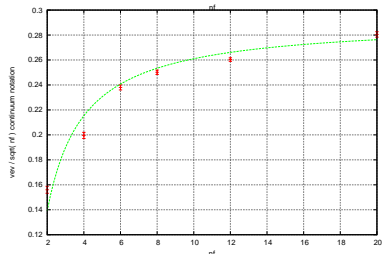
Fodor,Holland,JK,Nogradi,Schroder

vev with subleading  $\frac{1}{N}$  corrections:



Higgs vacuum expectation value

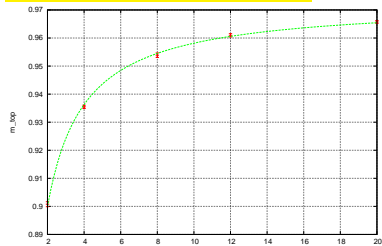
Subleading  $1/N$  correction  
extrapolates correctly



# Chiral Top-Higgs Model: Large N test

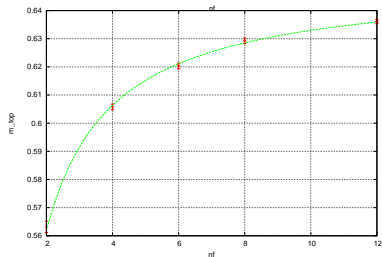
Fodor,Holland,JK,Nogradi,Schroder

subleading  $m_{\text{Top}}$   $\frac{1}{N}$  corrections:



Top quark mass

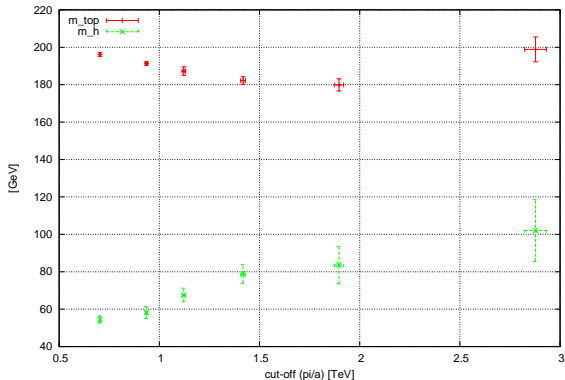
Subleading  $1/N$  correction  
extrapolates correctly



# Chiral Top-Higgs Model: Higgs mass lower bound

Fodor, Holland, JK, Nogradi, Schroder

Cutoff dependence of Higgs lower bound:

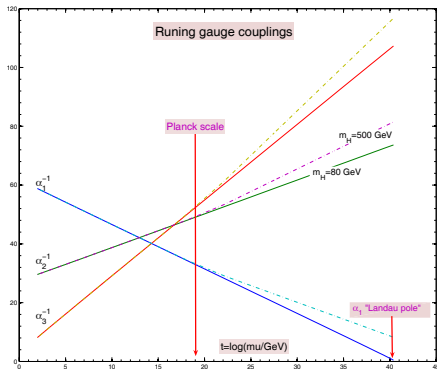


**Preliminary!**

**Extreme lattice computing: USQCD, Wuppertal(GPU)**

Higgs physics as a video game

## 2-loop continuum SM RG: Running gauge couplings



Gauge sector (2-loop):

$$\begin{aligned}
 (4\pi)^4 g_1^{-3} \beta_{g_1}^{(2)} &= \frac{199}{50} g_1^2 + \frac{27}{10} g_2^2 + \frac{44}{5} g_3^2 - \sum_g \left( \frac{17}{5} y_{u_g}^2 + y_{d_g}^2 + 3y_{e_g}^2 \right), \\
 (4\pi)^4 g_2^{-3} \beta_{g_2}^{(2)} &= \frac{9}{10} g_1^2 + \frac{35}{6} g_2^2 + 12g_3^2 - \sum_g (3y_{u_g}^2 + 3y_{d_g}^2 + y_{e_g}^2), \\
 (4\pi)^4 g_3^{-3} \beta_{g_3}^{(2)} &= \frac{11}{10} g_1^2 + \frac{9}{2} g_2^2 - 26g_3^2 - 4 \sum_g (y_{u_g}^2 + y_{d_g}^2).
 \end{aligned}$$

2-loop RG with 1-loop matching

$\mu = M_Z$  initial conditions:

$$\alpha_1(M_Z) = 0.0102$$

$$\alpha_2(M_Z) = 0.0338$$

$$\alpha_3(M_Z) = 0.123$$

Higgs and Yukawa couplings:

$$m_f = y_f v$$

$$m_H = \lambda^{1/2} v$$

$$v = 246.22 \text{ GeV}$$

$\alpha_1$  is  $U(1)_Y$  coupling

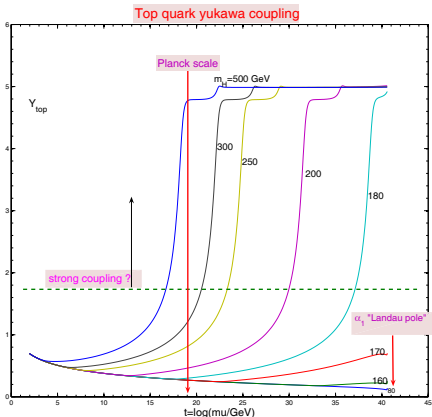
“Landau pole” in  $\alpha_1$

$$\approx 10^{41} \text{ GeV for } m_H = 80 \text{ GeV}$$

inherited from QED

shifted to  $\approx 10^{48} \text{ GeV}$  for  
 $m_H = 500 \text{ GeV}$

## 2-loop continuum SM RG: Running Top coupling



### Top yukawa coupling $y_t$

For  $m_H < 160\text{GeV}$  monotone decrease in  $y_t$

Lower Higgs mass bound (and vacuum instability?) might remain perturbative with running  $\lambda$  for  $m_H < 160\text{GeV}$

$y_t \approx 5$  pseudo-fixed point

At  $m_H = 500\text{GeV}$  strong yukawa coupling below the Planck scale

$m_H = 500\text{GeV}$  Higgs mass range is becoming strongly interacting

### Upper bound with heavy Higgs is strong coupling problem!

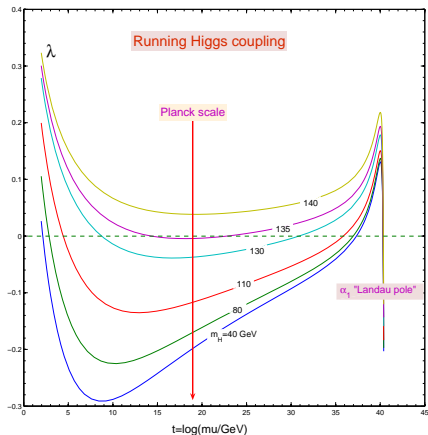
Yukawa sector (1-loop):

$$(4\pi)^2 y_\tau^{-1} \beta_{y_\tau}^{(1)} = 3y_\tau^2 + 2 \sum_g (3y_{u_g}^2 + 3y_{d_g}^2 + y_{e_g}^2) - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2,$$

$$(4\pi)^2 y_t^{-1} \beta_{y_t}^{(1)} = 3y_t^2 - 3y_b^2 + 2 \sum_g (3y_{u_g}^2 + 3y_{d_g}^2 + y_{e_g}^2) - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2,$$

$$(4\pi)^2 y_b^{-1} \beta_{y_b}^{(1)} = 3y_b^2 - 3y_t^2 + 2 \sum_g (3y_{u_g}^2 + 3y_{d_g}^2 + y_{e_g}^2) - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2.$$

## 2-loop continuum SM RG: Running Higgs coupling for light Higgs



Higgs sector (1-loop):

$$(4\pi)^2 \beta_\lambda^{(1)} = 12\lambda^2 + 8\lambda \sum_g (3y_{u_g}^2 + 3y_{d_g}^2 + y_{e_g}^2) - 9\lambda \left( \frac{1}{5}g_1^2 + g_2^2 \right) - 16 \sum_g (3y_{u_g}^4 + 3y_{d_g}^4 + y_{e_g}^4) + \frac{9}{4} \left( \frac{3}{25}g_1^4 + g_2^4 + \frac{2}{5}g_1^2 g_2^2 \right),$$

Lower bound and running  $\lambda$  coupling

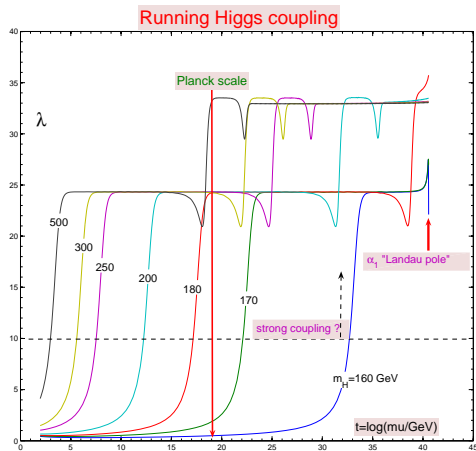
At low Higgs masses negative  $\lambda$  would suggest vacuum instability

Large changes in  $U_{eff}(\phi)$  from 1-loop to 2-loop would call for nonperturbative calculations

Running gauge couplings are important!

Problem is on borderline of perturbation theory, at best

## 2-loop continuum SM RG: Running Higgs coupling for heavy Higgs



Upper bound and running coupling  $\lambda$

At large Higgs masses growing  $\lambda$  requires strong coupling

Call for nonperturbative calculations

Running gauge couplings are important!

Is large  $m_H$  consistent with EW data?

What is the Standard Model beyond PT?



## Conclusions

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- ▶ Urgency of LHC: We do not have the luxury of many years to explore